Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.2-d-x-^m-a+b-x^2+c-x^4-^p

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May 13, 2020 Compiled on May 13, 2020 at 10:11pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1126]. This is test number [39].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1126)	% 0. (0)
Mathematica	% 100. (1126)	% 0. (0)
Maple	% 94.32 (1062)	% 5.68 (64)
Maxima	% 35.08 (395)	% 64.92 (731)
Fricas	% 74.96 (844)	% 25.04 (282)
Sympy	% 43.16 (486)	% 56.84 (640)
Giac	% 65.63 (739)	% 34.37 (387)

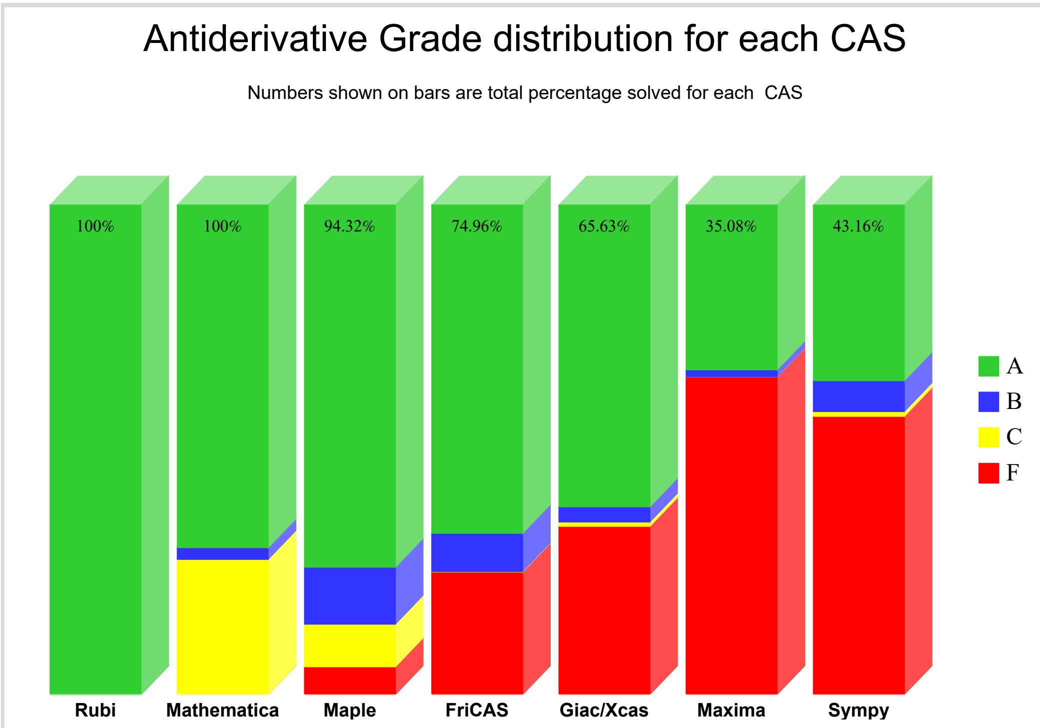
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

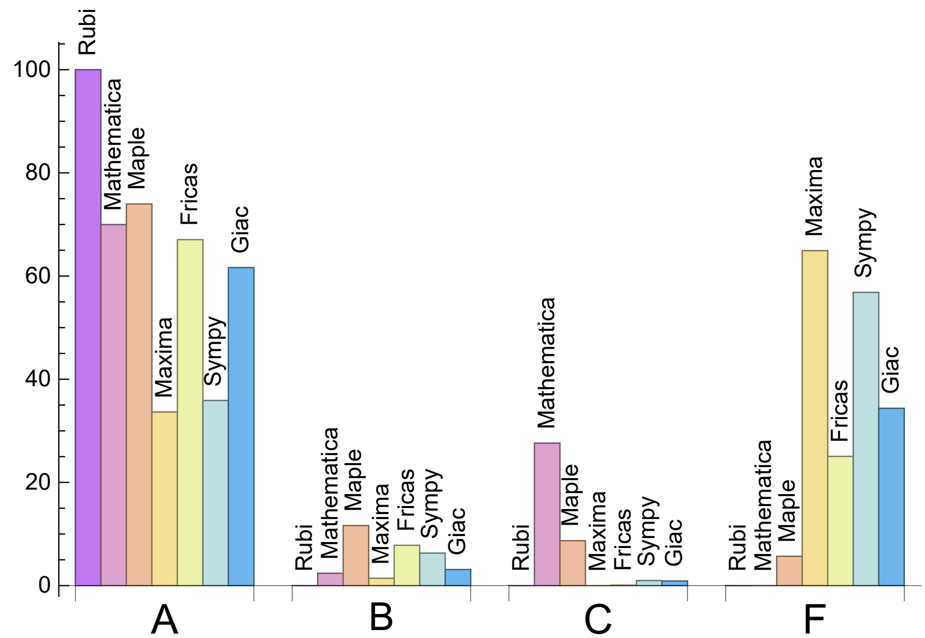
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	69.98	2.4	27.62	0.
Maple	73.98	11.63	8.7	5.68
Maxima	33.66	1.42	0.	64.92
Fricas	67.05	7.82	0.09	25.04
Sympy	35.88	6.31	0.98	56.84
Giac	61.63	3.11	0.89	34.37

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.15	144.69	1.	90.	1.
Mathematica	0.1	94.6	0.88	61.	0.89
Maple	0.13	149.27	1.03	78.	0.87
Maxima	1.01	65.27	1.15	58.	1.12
Fricas	2.01	717.75	4.1	186.	2.31
Sympy	4.69	122.84	1.46	53.	0.96
Giac	1.71	201.65	1.58	92.	1.17

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {14, 17, 18, 19, 23, 24, 25, 27, 28, 30, 31, 32, 36, 37, 38, 40, 41, 43, 44, 45, 46, 49, 50, 51, 53, 54, 56, 57, 58, 62, 63, 64, 78, 79, 81, 124, 125, 126, 128, 129, 130, 132, 133, 918, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

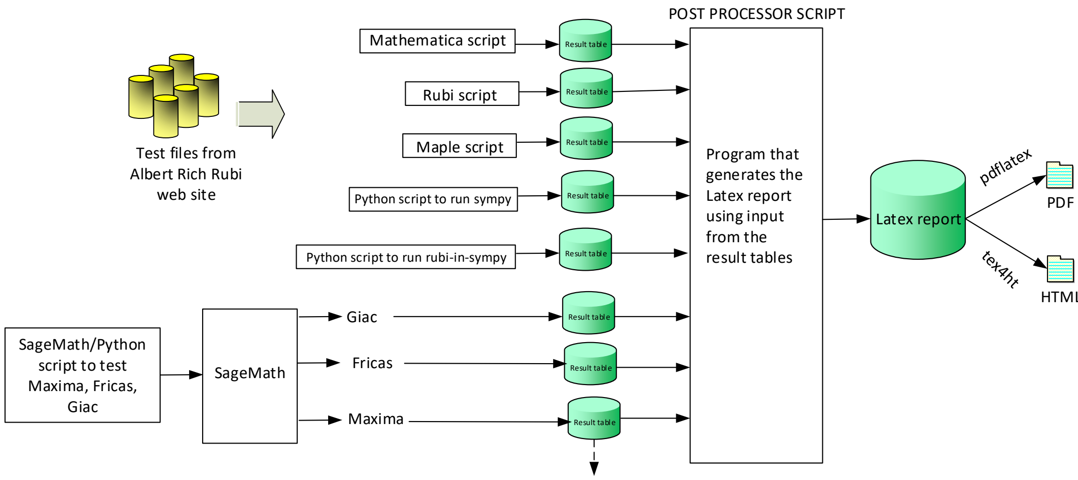
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 12, 21, 22, 33, 34, 42, 46, 47, 55, 60, 61, 76, 77, 92, 93, 94, 96, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 238, 239, 240, 241, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 327, 329, 331, 333, 339, 341, 343, 345, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 688, 690, 692, 696, 698, 700, 702, 704, 706, 710, 712, 714, 716, 718, 720, 722, 724, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 757, 759, 761, 763, 765, 769, 771, 773, 775, 777, 779, 781, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 915, 916, 917, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 955, 956, 957, 958, 959, 960, 961, 962, 968, 969, 970, 971, 972, 973, 974, 975, 981, 982, 983, 984, 985, 986, 987, 988, 994, 995, 996, 997, 998, 999, 1000, 1001, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1097, 1098, 1099, 1105, 1106, 1107, 1111, 1112, 1113, 1114, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

B grade: { 11, 26, 39, 95, 97, 108, 109, 171, 438, 449, 467, 515, 1040, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1100, 1101, 1102, 1103, 1104, 1110 }

C grade: { 8, 10, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 40, 41, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 236, 237, 242, 243, 255, 257, 258, 259, 272, 283, 284, 285, 322, 323, 324, 325, 326, 328, 330, 332, 334, 335, 336, 337, 338, 340, 342, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 659, 660, 661, 662, 663, 664, 687, 689, 691, 693, 694, 695, 697, 699, 701, 703, 705, 707, 708, 709, 711, 713, 715, 717, 719, 721, 723, 725, 726, 727, 754, 755, 756, 758, 760, 762, 764, 766, 767, 768, 770, 772, 774, 776, 778, 780, 782, 783, 784, 909, 910, 911, 912, 913, 914, 918, 919, 930, 931, 932, 933, 934, 935, 947, 948, 949, 950, 951, 952, 953, 954, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 989, 990, 991, 992, 993, 1002, 1003, 1005, 1007, 1009, 1011, 1039, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1108, 1109, 1115, 1116, 1117 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 4, 5, 6, 7, 9, 12, 13, 14, 66, 76, 77, 78, 79, 81, 92, 93, 94, 95, 96, 97, 108, 109, 119, 122, 124, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 766, 768, 787, 791, 792, 793, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 857, 858, 859, 861, 862, 863, 864, 870, 877, 878, 888, 889, 890, 891, 892, 894, 895, 896, 897, 898, 899, 900, 901, 903, 904, 906, 907, 908, 909, 910, 915, 916, 917, 918, 920, 921, 922, 923, 924, 925, 930, 931, 932, 933, 934, 935, 940, 941, 942, 943, 947, 948, 949, 950, 951, 952, 953, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 999, 1000, 1001, 1002, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1107 }

B grade: { 8, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 125, 126, 127, 128, 129, 130, 131, 132, 133, 161, 171, 243, 270, 287, 403, 426, 438, 449, 451, 467, 496, 515, 591, 598, 757, 758, 759, 760, 761, 762, 763, 764, 765, 767, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 855, 856, 860, 865, 866, 867, 868, 869, 871, 872, 873, 874, 875, 876, 879, 880, 881, 882, 883, 884, 885, 886, 887, 893, 902, 905, 911, 912, 913, 914, 919, 926, 927, 928, 929, 936, 937, 938, 939, 944, 945, 946, 954, 981, 982, 998, 1039, 1105, 1106 }

C grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 123, 1003, 1005, 1007, 1009, 1011, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088 }

F grade: { 3, 406, 407, 408, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.4 Maxima

A grade: { 12, 13, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 175, 177, 179, 181, 183, 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 206, 208, 212, 214, 216, 218, 220, 231, 232, 233, 249, 250, 251, 252, 253, 268, 269, 276, 277, 281, 282, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 403, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 493, 494, 495, 497, 499, 500, 501, 512, 513, 514, 517, 520, 521, 522, 535, 536, 537, 538, 539, 540, 550, 551, 552, 553, 554, 555, 556, 557, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 634, 635, 636, 637, 645, 646, 648, 649, 650, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 791, 792, 793, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 915, 916, 917, 994, 996, 1004, 1010, 1015, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061

B grade: { 11, 161, 171, 210, 426, 438, 449, 451, 467, 496, 498, 515, 516, 518, 519, 647 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 174, 176, 178, 180, 182, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 205, 207, 209, 211, 213, 215, 217, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 278, 279, 280, 283, 284, 285, 290, 291, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 484, 485, 486, 487, 488, 489, 490, 491, 492, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 541, 542, 543, 544, 545, 546, 547, 548, 549, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 627, 628, 629, 630, 631, 632, 633, 638, 639, 640, 641, 642, 643, 644, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 995, 997, 998, 999, 1000, 1001, 1002, 1003, 1005, 1006, 1007, 1008, 1009, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1020, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 12, 13, 15, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 517, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 787, 791, 792, 793, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 888, 889, 890, 891, 892, 897, 898, 899, 900, 901, 906, 907, 908, 909, 910, 911, 912, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 955, 956, 957, 958, 959, 960, 961, 962, 968, 969, 970, 971, 972, 973, 974, 975, 981, 982, 983, 984, 985, 987, 988, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1107 }

B grade: { 8, 10, 11, 14, 161, 171, 210, 403, 426, 438, 449, 451, 467, 496, 498, 499, 500, 515, 516, 518, 519, 520, 521, 522, 598, 647, 785, 786, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 893, 894, 895, 896, 902, 903, 904, 905, 913, 914, 918, 986, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1073, 1074, 1075, 1076, 1105, 1106 }

C grade: { 287 }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 930, 931, 932, 933, 934, 935, 947, 948, 949, 950, 951, 952, 953, 954, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 989, 990, 991, 992, 993, 1003, 1005, 1007, 1009, 1011, 1039, 1040, 1071, 1072, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.6 Sympy

A grade: { 8, 10, 12, 13, 14, 15, 21, 34, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 179, 181, 183, 185, 187, 189, 190, 191, 192, 193, 194, 195, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 212, 213, 214, 215, 216, 217, 218, 219, 220, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 319, 320, 321, 322, 323, 324, 325, 403, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 490, 491, 492, 493, 494, 495, 497, 499, 500, 501, 502, 503, 504, 505, 508, 509, 510, 511, 512, 513, 514, 517, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 691, 705, 752, 785, 786, 787, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 855, 856, 857, 858, 859, 860, 867, 868, 869, 870, 871, 872, 893, 894, 895, 896, 899, 902, 903, 904, 905, 911, 912, 913, 914, 915, 916, 917, 918, 919, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1105, 1106, 1107 }

B grade: { 9, 11, 148, 161, 171, 178, 180, 182, 184, 186, 188, 196, 198, 210, 211, 426, 438, 449, 451, 467, 469, 488, 489, 496, 498, 506, 507, 515, 516, 518, 519, 848, 849, 850, 851, 852, 853, 854, 861, 862, 863, 864, 865, 866, 873, 874, 875, 876, 877, 878, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 897, 898, 900, 901, 906, 907, 908, 909, 910 }

C grade: { 47, 60, 71, 87, 102, 114, 1003, 1005, 1007, 1009, 1011 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 316, 317, 318, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 686, 687, 688, 689, 690, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 879, 880, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1024, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.7 Giac

A grade: { 1, 3, 4, 9, 12, 13, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 254, 255, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 269, 270, 275, 276, 277, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 793, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 861, 862, 863, 864, 865, 866, 873, 874, 875, 876, 877, 878, 879, 880, 888, 889, 890, 891, 892, 897, 898, 899, 900, 901, 906, 907, 908, 909, 910, 915, 916, 917, 920, 921, 922, 923, 936, 937, 938, 939, 955, 956, 957, 958, 968, 969, 970, 971, 972, 973, 974, 975, 983, 984, 985, 995, 996, 997, 998, 999, 1001, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061 }

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C grade: { 855, 856, 857, 858, 859, 860, 893, 894, 895, 896 }

F grade: { 2, 5, 6, 7, 8, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 260, 261, 268, 271, 272, 273, 274, 278, 279, 280, 283, 284, 285, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 867, 868, 869, 870, 871, 872, 881, 882, 883, 884, 885, 886, 887, 902, 903, 904, 905, 911, 912, 913, 914, 918, 919, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 959, 960, 961, 962, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1000, 1002, 1003, 1005, 1007, 1009, 1011, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	97	77	0	408	0	117
normalized size	1	1.	0.76	0.6	0.	3.19	0.	0.91
time (sec)	N/A	0.031	0.064	0.199	0.	1.42	0.	1.216

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	59	58	0	347	0	0
normalized size	1	1.	0.65	0.64	0.	3.81	0.	0.
time (sec)	N/A	0.02	0.038	0.175	0.	1.201	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	49	0	0	217	0	36
normalized size	1	1.	0.82	0.	0.	3.62	0.	0.6
time (sec)	N/A	0.012	0.015	0.2	0.	1.29	0.	1.158

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	25	33	0	72	0	32
normalized size	1	1.	0.74	0.97	0.	2.12	0.	0.94
time (sec)	N/A	0.008	0.011	0.054	0.	1.282	0.	1.163

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	70	40	44	0	123	0	0
normalized size	1	1.03	0.59	0.65	0.	1.81	0.	0.
time (sec)	N/A	0.016	0.012	0.044	0.	1.284	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	107	51	55	0	170	0	0
normalized size	1	1.02	0.49	0.52	0.	1.62	0.	0.
time (sec)	N/A	0.024	0.015	0.046	0.	1.426	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	148	62	66	0	216	0	0
normalized size	1	1.1	0.46	0.49	0.	1.6	0.	0.
time (sec)	N/A	0.043	0.024	0.045	0.	1.378	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	81	1099	0	1206	63	0
normalized size	1	1.	0.27	3.68	0.	4.03	0.21	0.
time (sec)	N/A	0.312	0.044	0.199	0.	1.411	0.737	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	32	0	744	257	42
normalized size	1	1.	0.91	0.68	0.	15.83	5.47	0.89
time (sec)	N/A	0.026	0.023	0.09	0.	1.374	0.599	1.134

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	52	1073	0	1629	48	0
normalized size	1	1.	0.17	3.59	0.	5.45	0.16	0.
time (sec)	N/A	0.308	0.031	0.155	0.	1.379	0.496	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	26	34	95	26	39
normalized size	1	1.	2.18	1.53	2.	5.59	1.53	2.29
time (sec)	N/A	0.008	0.007	0.049	0.978	1.252	0.169	1.143

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	18	23	70	20	23
normalized size	1	1.	1.	0.75	0.96	2.92	0.83	0.96
time (sec)	N/A	0.015	0.011	0.049	1.448	1.349	0.145	1.164

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	91	54	72	193	70	72
normalized size	1	1.	1.36	0.81	1.07	2.88	1.04	1.07
time (sec)	N/A	0.05	0.071	0.046	1.469	1.253	0.233	1.131

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	77	57	0	529	63	0
normalized size	1	1.	1.04	0.77	0.	7.15	0.85	0.
time (sec)	N/A	0.049	0.071	0.059	0.	1.455	0.203	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	41	386	0	787	20	0
normalized size	1	1.	0.23	2.19	0.	4.47	0.11	0.
time (sec)	N/A	0.162	0.035	0.076	0.	1.379	0.552	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	0	0	0
normalized size	1	1.	6.5	5.1	0.	0.	0.	0.
time (sec)	N/A	0.016	0.026	0.059	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	48	48	49	84	0	0	0	0
normalized size	1	1.	1.02	1.75	0.	0.	0.	0.
time (sec)	N/A	0.134	0.056	0.24	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	48	48	53	80	0	0	0	0
normalized size	1	1.	1.1	1.67	0.	0.	0.	0.
time (sec)	N/A	0.104	0.059	0.25	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	49	84	0	0	0	0
normalized size	1	1.	1.11	1.91	0.	0.	0.	0.
time (sec)	N/A	0.067	0.043	0.237	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	63	41	0	0	0	0
normalized size	1	1.	5.25	3.42	0.	0.	0.	0.
time (sec)	N/A	0.012	0.023	0.049	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	54	0	0	37	0
normalized size	1	1.	1.	3.	0.	0.	2.06	0.
time (sec)	N/A	0.01	0.022	0.191	0.	0.	0.625	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	49	0	0	0	0
normalized size	1	1.	1.	2.45	0.	0.	0.	0.
time (sec)	N/A	0.011	0.026	0.059	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	42	42	51	84	0	0	0	0
normalized size	1	1.	1.21	2.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.044	0.227	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	55	80	0	0	0	0
normalized size	1	1.	1.2	1.74	0.	0.	0.	0.
time (sec)	N/A	0.089	0.064	0.256	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	48	48	49	84	0	0	0	0
normalized size	1	1.	1.02	1.75	0.	0.	0.	0.
time (sec)	N/A	0.094	0.058	0.24	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	54	50	0	0	0	0
normalized size	1	1.	3.	2.78	0.	0.	0.	0.
time (sec)	N/A	0.015	0.023	0.055	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	52	84	0	0	0	0
normalized size	1	1.	1.16	1.87	0.	0.	0.	0.
time (sec)	N/A	0.065	0.044	0.255	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	43	84	0	0	0	0
normalized size	1	1.	0.98	1.91	0.	0.	0.	0.
time (sec)	N/A	0.133	0.052	0.24	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	0	0	0
normalized size	1	1.	6.5	5.1	0.	0.	0.	0.
time (sec)	N/A	0.011	0.025	0.051	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	51	84	0	0	0	0
normalized size	1	1.	1.16	1.91	0.	0.	0.	0.
time (sec)	N/A	0.077	0.054	0.211	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	50	84	0	0	0	0
normalized size	1	1.	1.11	1.87	0.	0.	0.	0.
time (sec)	N/A	0.067	0.049	0.214	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	49	84	0	0	0	0
normalized size	1	1.	1.11	1.91	0.	0.	0.	0.
time (sec)	N/A	0.062	0.042	0.213	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	47	0	0	0	0
normalized size	1	1.	1.	2.35	0.	0.	0.	0.
time (sec)	N/A	0.011	0.025	0.064	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	54	0	0	37	0
normalized size	1	1.	1.	3.	0.	0.	2.06	0.
time (sec)	N/A	0.006	0.021	0.148	0.	0.	0.64	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	65	43	0	0	0	0
normalized size	1	1.	5.42	3.58	0.	0.	0.	0.
time (sec)	N/A	0.011	0.022	0.049	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	42	42	51	84	0	0	0	0
normalized size	1	1.	1.21	2.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.044	0.208	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	43	43	52	84	0	0	0	0
normalized size	1	1.	1.21	1.95	0.	0.	0.	0.
time (sec)	N/A	0.031	0.044	0.211	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	51	84	0	0	0	0
normalized size	1	1.	1.16	1.91	0.	0.	0.	0.
time (sec)	N/A	0.068	0.054	0.209	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	54	50	0	0	0	0
normalized size	1	1.	3.	2.78	0.	0.	0.	0.
time (sec)	N/A	0.012	0.024	0.056	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	42	42	45	84	0	0	0	0
normalized size	1	1.	1.07	2.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.055	0.229	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	52	84	0	0	0	0
normalized size	1	1.	1.16	1.87	0.	0.	0.	0.
time (sec)	N/A	0.048	0.044	0.23	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	53	0	0	0	0
normalized size	1	1.	0.81	0.79	0.	0.	0.	0.
time (sec)	N/A	0.008	0.022	0.052	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	81	84	0	0	0	0
normalized size	1	1.	0.57	0.6	0.	0.	0.	0.
time (sec)	N/A	0.035	0.052	0.181	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	83	84	0	0	0	0
normalized size	1	1.	0.57	0.58	0.	0.	0.	0.
time (sec)	N/A	0.039	0.065	0.176	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	83	84	0	0	0	0
normalized size	1	1.	0.59	0.6	0.	0.	0.	0.
time (sec)	N/A	0.03	0.052	0.178	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	48	43	0	0	0	0
normalized size	1	1.	0.76	0.68	0.	0.	0.	0.
time (sec)	N/A	0.008	0.03	0.046	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	40	56	0	0	34	0
normalized size	1	1.	0.35	0.49	0.	0.	0.3	0.
time (sec)	N/A	0.016	0.024	0.166	0.	0.	0.667	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	53	0	0	0	0
normalized size	1	1.	0.92	0.82	0.	0.	0.	0.
time (sec)	N/A	0.007	0.023	0.05	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	81	84	0	0	0	0
normalized size	1	1.	0.55	0.57	0.	0.	0.	0.
time (sec)	N/A	0.026	0.045	0.179	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	81	84	0	0	0	0
normalized size	1	1.	0.53	0.55	0.	0.	0.	0.
time (sec)	N/A	0.03	0.067	0.176	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	81	84	0	0	0	0
normalized size	1	1.	0.55	0.57	0.	0.	0.	0.
time (sec)	N/A	0.029	0.064	0.182	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	53	0	0	0	0
normalized size	1	1.	1.03	0.84	0.	0.	0.	0.
time (sec)	N/A	0.007	0.024	0.057	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	80	84	0	0	0	0
normalized size	1	1.	0.54	0.57	0.	0.	0.	0.
time (sec)	N/A	0.029	0.053	0.18	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	77	84	0	0	0	0
normalized size	1	1.	0.52	0.57	0.	0.	0.	0.
time (sec)	N/A	0.04	0.065	0.184	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	53	0	0	0	0
normalized size	1	1.	0.81	0.79	0.	0.	0.	0.
time (sec)	N/A	0.008	0.023	0.049	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	83	84	0	0	0	0
normalized size	1	1.	0.56	0.57	0.	0.	0.	0.
time (sec)	N/A	0.027	0.069	0.181	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	80	84	0	0	0	0
normalized size	1	1.	0.55	0.58	0.	0.	0.	0.
time (sec)	N/A	0.027	0.058	0.188	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	83	84	0	0	0	0
normalized size	1	1.	0.58	0.59	0.	0.	0.	0.
time (sec)	N/A	0.023	0.052	0.18	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	51	0	0	0	0
normalized size	1	1.	1.	0.81	0.	0.	0.	0.
time (sec)	N/A	0.007	0.022	0.067	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	40	56	0	0	34	0
normalized size	1	1.	0.36	0.5	0.	0.	0.3	0.
time (sec)	N/A	0.015	0.024	0.173	0.	0.	0.625	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	45	0	0	0	0
normalized size	1	1.	0.78	0.69	0.	0.	0.	0.
time (sec)	N/A	0.007	0.023	0.052	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	81	84	0	0	0	0
normalized size	1	1.	0.54	0.56	0.	0.	0.	0.
time (sec)	N/A	0.021	0.046	0.185	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	78	84	0	0	0	0
normalized size	1	1.	0.51	0.55	0.	0.	0.	0.
time (sec)	N/A	0.024	0.057	0.185	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	83	84	0	0	0	0
normalized size	1	1.	0.54	0.54	0.	0.	0.	0.
time (sec)	N/A	0.024	0.065	0.183	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	53	0	0	0	0
normalized size	1	1.	1.03	0.84	0.	0.	0.	0.
time (sec)	N/A	0.006	0.025	0.052	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	58	44	0	0	0	0
normalized size	1	1.	1.12	0.85	0.	0.	0.	0.
time (sec)	N/A	0.007	0.025	0.053	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.021	0.086	0.764	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.
time (sec)	N/A	0.016	0.116	0.825	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.
time (sec)	N/A	0.016	0.081	0.76	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	142	85	0	0	0	0
normalized size	1	1.	1.61	0.97	0.	0.	0.	0.
time (sec)	N/A	0.014	0.084	0.758	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	25	66	0	0	36	0
normalized size	1	1.	0.35	0.92	0.	0.	0.5	0.
time (sec)	N/A	0.007	0.025	0.196	0.	0.	0.684	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.015	0.078	0.786	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.015	0.076	0.79	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.016	0.107	0.801	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0
normalized size	1	1.	1.64	0.99	0.	0.	0.	0.
time (sec)	N/A	0.012	0.081	0.755	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	53	42	0	0	0	0
normalized size	1	1.	0.58	0.46	0.	0.	0.	0.
time (sec)	N/A	0.015	0.023	0.047	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	85	82	0	0	0	0
normalized size	1	1.	0.94	0.91	0.	0.	0.	0.
time (sec)	N/A	0.016	0.075	0.218	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	97	82	0	0	0	0
normalized size	1	1.	0.88	0.75	0.	0.	0.	0.
time (sec)	N/A	0.091	0.072	0.236	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	98	82	0	0	0	0
normalized size	1	1.	0.89	0.75	0.	0.	0.	0.
time (sec)	N/A	0.076	0.079	0.231	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	61	50	0	0	0	0
normalized size	1	1.	1.02	0.83	0.	0.	0.	0.
time (sec)	N/A	0.007	0.028	0.054	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	90	82	0	0	0	0
normalized size	1	1.	0.87	0.79	0.	0.	0.	0.
time (sec)	N/A	0.064	0.075	0.229	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	58	50	0	0	0	0
normalized size	1	1.	1.12	0.96	0.	0.	0.	0.
time (sec)	N/A	0.006	0.027	0.055	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.016	0.095	0.737	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	142	87	0	0	0	0
normalized size	1	1.	1.54	0.95	0.	0.	0.	0.
time (sec)	N/A	0.01	0.102	0.745	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.
time (sec)	N/A	0.01	0.085	0.778	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	140	85	0	0	0	0
normalized size	1	1.	1.59	0.97	0.	0.	0.	0.
time (sec)	N/A	0.012	0.072	0.752	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	25	66	0	0	36	0
normalized size	1	1.	0.35	0.92	0.	0.	0.5	0.
time (sec)	N/A	0.007	0.024	0.193	0.	0.	0.691	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0
normalized size	1	1.	1.58	0.97	0.	0.	0.	0.
time (sec)	N/A	0.01	0.068	0.737	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.01	0.082	0.748	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0
normalized size	1	1.	1.58	0.97	0.	0.	0.	0.
time (sec)	N/A	0.009	0.104	0.74	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0
normalized size	1	1.	1.64	0.99	0.	0.	0.	0.
time (sec)	N/A	0.01	0.086	0.74	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	53	42	0	0	0	0
normalized size	1	1.	0.58	0.46	0.	0.	0.	0.
time (sec)	N/A	0.009	0.026	0.048	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	81	82	0	0	0	0
normalized size	1	1.	0.9	0.91	0.	0.	0.	0.
time (sec)	N/A	0.01	0.075	0.202	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	58	49	0	0	0	0
normalized size	1	1.	0.63	0.53	0.	0.	0.	0.
time (sec)	N/A	0.014	0.023	0.051	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	58	48	0	0	0	0
normalized size	1	1.	3.05	2.53	0.	0.	0.	0.
time (sec)	N/A	0.012	0.023	0.049	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	81	82	0	0	0	0
normalized size	1	1.	1.84	1.86	0.	0.	0.	0.
time (sec)	N/A	0.061	0.027	0.173	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	53	50	0	0	0	0
normalized size	1	1.	3.79	3.57	0.	0.	0.	0.
time (sec)	N/A	0.012	0.025	0.051	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0
normalized size	1	1.	1.64	0.99	0.	0.	0.	0.
time (sec)	N/A	0.01	0.052	0.669	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0
normalized size	1	1.	1.58	0.97	0.	0.	0.	0.
time (sec)	N/A	0.01	0.099	0.667	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.01	0.072	0.667	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	140	85	0	0	0	0
normalized size	1	1.	1.59	0.97	0.	0.	0.	0.
time (sec)	N/A	0.009	0.067	0.671	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	47	66	0	0	39	0
normalized size	1	1.	0.65	0.92	0.	0.	0.54	0.
time (sec)	N/A	0.007	0.028	0.168	0.	0.	0.603	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0
normalized size	1	1.	1.58	0.97	0.	0.	0.	0.
time (sec)	N/A	0.01	0.063	0.711	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.
time (sec)	N/A	0.01	0.075	0.662	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	142	87	0	0	0	0
normalized size	1	1.	1.54	0.95	0.	0.	0.	0.
time (sec)	N/A	0.01	0.1	0.663	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.009	0.085	0.67	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	63	44	0	0	0	0
normalized size	1	1.	1.19	0.83	0.	0.	0.	0.
time (sec)	N/A	0.016	0.024	0.053	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	85	82	0	0	0	0
normalized size	1	1.	2.02	1.95	0.	0.	0.	0.
time (sec)	N/A	0.052	0.082	0.175	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	53	42	0	0	0	0
normalized size	1	1.	8.83	7.	0.	0.	0.	0.
time (sec)	N/A	0.01	0.026	0.049	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0
normalized size	1	1.	1.64	0.99	0.	0.	0.	0.
time (sec)	N/A	0.01	0.084	0.701	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.009	0.111	0.693	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.014	0.081	0.697	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	142	85	0	0	0	0
normalized size	1	1.	1.61	0.97	0.	0.	0.	0.
time (sec)	N/A	0.009	0.081	0.693	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	47	66	0	0	39	0
normalized size	1	1.	0.65	0.92	0.	0.	0.54	0.
time (sec)	N/A	0.008	0.028	0.174	0.	0.	0.613	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.01	0.08	0.696	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.
time (sec)	N/A	0.01	0.082	0.442	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.
time (sec)	N/A	0.01	0.115	0.699	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.
time (sec)	N/A	0.009	0.087	0.445	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	63	50	0	0	0	0
normalized size	1	1.	1.21	0.96	0.	0.	0.	0.
time (sec)	N/A	0.017	0.024	0.054	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.
time (sec)	N/A	0.026	0.114	0.749	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	147	87	0	0	0	0
normalized size	1	1.	1.63	0.97	0.	0.	0.	0.
time (sec)	N/A	0.019	0.088	0.743	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	58	44	0	0	0	0
normalized size	1	1.	1.12	0.85	0.	0.	0.	0.
time (sec)	N/A	0.007	0.02	0.046	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	48	0	0	0	0
normalized size	1	1.	1.	0.83	0.	0.	0.	0.
time (sec)	N/A	0.007	0.022	0.053	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	103	76	0	0	0	0
normalized size	1	1.	0.95	0.7	0.	0.	0.	0.
time (sec)	N/A	0.042	0.082	0.226	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	48	48	55	80	0	0	0	0
normalized size	1	1.	1.15	1.67	0.	0.	0.	0.
time (sec)	N/A	0.098	0.062	0.241	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	52	76	0	0	0	0
normalized size	1	1.	1.16	1.69	0.	0.	0.	0.
time (sec)	N/A	0.062	0.046	0.228	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	0	0	0
normalized size	1	1.	6.5	5.1	0.	0.	0.	0.
time (sec)	N/A	0.012	0.023	0.046	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	0	0	0
normalized size	1	1.	1.14	1.63	0.	0.	0.	0.
time (sec)	N/A	0.104	0.062	0.246	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	48	48	52	80	0	0	0	0
normalized size	1	1.	1.08	1.67	0.	0.	0.	0.
time (sec)	N/A	0.104	0.058	0.24	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	0	0	0
normalized size	1	1.	1.14	1.63	0.	0.	0.	0.
time (sec)	N/A	0.078	0.05	0.243	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	65	43	0	0	0	0
normalized size	1	1.	5.42	3.58	0.	0.	0.	0.
time (sec)	N/A	0.012	0.025	0.053	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	52	76	0	0	0	0
normalized size	1	1.	1.16	1.69	0.	0.	0.	0.
time (sec)	N/A	0.07	0.043	0.229	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	0	0	0
normalized size	1	1.	1.14	1.63	0.	0.	0.	0.
time (sec)	N/A	0.079	0.047	0.237	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.006	0.002	0.043	0.986	1.086	0.066	1.244

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.006	0.001	0.04	0.944	1.01	0.064	1.293

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.003	0.	0.042	0.956	1.095	0.074	1.216

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	31	12	18
normalized size	1	1.	1.	0.82	1.06	1.82	0.71	1.06
time (sec)	N/A	0.005	0.001	0.042	0.963	1.208	0.059	1.265

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.004	0.	0.041	0.969	1.111	0.063	1.252

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	19	30	10	19
normalized size	1	1.	1.	0.92	1.46	2.31	0.77	1.46
time (sec)	N/A	0.005	0.001	0.043	0.973	1.274	0.089	1.158

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	20	5	14
normalized size	1	1.	1.	1.1	1.4	2.	0.5	1.4
time (sec)	N/A	0.005	0.001	0.045	1.006	1.231	0.253	1.123

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	19	41	10	27
normalized size	1	1.	1.	0.92	1.46	3.15	0.77	2.08
time (sec)	N/A	0.006	0.002	0.046	1.03	1.251	0.29	1.259

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	32	14	18
normalized size	1	1.	1.	0.93	1.2	2.13	0.93	1.2
time (sec)	N/A	0.006	0.002	0.046	0.975	1.215	0.299	1.308

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	32	14	18
normalized size	1	1.	1.	0.82	1.06	1.88	0.82	1.06
time (sec)	N/A	0.006	0.002	0.046	0.981	1.262	0.332	1.303

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	20	36	15	20
normalized size	1	1.	1.	0.82	1.18	2.12	0.88	1.18
time (sec)	N/A	0.006	0.002	0.046	0.993	1.225	0.35	1.263

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.013	0.001	0.043	0.97	1.066	0.066	1.279

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	24	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.8	1.07
time (sec)	N/A	0.025	0.001	0.043	0.984	1.259	0.078	1.27

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.017	0.001	0.04	0.986	1.201	0.07	1.257

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	32	55	24	32
normalized size	1	1.	1.	1.56	2.	3.44	1.5	2.
time (sec)	N/A	0.009	0.002	0.041	0.983	1.206	0.067	1.313

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	47	22	28
normalized size	1	1.	1.	0.88	1.12	1.88	0.88	1.12
time (sec)	N/A	0.013	0.001	0.043	0.99	1.158	0.066	1.196

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	32	49	20	32
normalized size	1	1.	1.	0.96	1.39	2.13	0.87	1.39
time (sec)	N/A	0.019	0.001	0.043	0.961	1.272	0.267	1.262

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	50	19	30
normalized size	1	1.	1.	0.96	1.25	2.08	0.79	1.25
time (sec)	N/A	0.017	0.001	0.046	0.963	1.215	0.265	1.252

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	32	59	24	43
normalized size	1	1.	1.	0.89	1.19	2.19	0.89	1.59
time (sec)	N/A	0.021	0.001	0.048	0.992	1.241	0.322	1.233

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	30	53	20	30
normalized size	1	1.	1.	0.96	1.3	2.3	0.87	1.3
time (sec)	N/A	0.018	0.001	0.046	1.009	1.227	0.311	1.274

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	35	62	22	46
normalized size	1	1.	1.	0.96	1.46	2.58	0.92	1.92
time (sec)	N/A	0.019	0.001	0.047	0.982	1.206	0.356	1.323

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	35	61	27	35
normalized size	1	1.	1.	0.89	1.25	2.18	0.96	1.25
time (sec)	N/A	0.016	0.001	0.049	0.977	1.198	0.359	1.324

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	32	54	26	32
normalized size	1	1.	1.58	1.32	1.68	2.84	1.37	1.68
time (sec)	N/A	0.01	0.001	0.049	0.996	1.131	0.385	1.295

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	63	27	35
normalized size	1	1.	1.	0.83	1.17	2.1	0.9	1.17
time (sec)	N/A	0.016	0.001	0.047	1.084	1.266	0.429	1.276

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	82	37	47
normalized size	1	1.	1.	0.84	1.09	1.91	0.86	1.09
time (sec)	N/A	0.024	0.002	0.043	1.016	1.185	0.072	1.237

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	43	36	47	82	37	47
normalized size	1	1.	1.26	1.06	1.38	2.41	1.09	1.38
time (sec)	N/A	0.039	0.002	0.044	0.994	1.249	0.073	1.285

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	80	39	47
normalized size	1	1.	1.	0.84	1.09	1.86	0.91	1.09
time (sec)	N/A	0.021	0.002	0.043	1.059	1.199	0.075	1.294

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	36	47	80	37	47
normalized size	1	1.	1.	2.25	2.94	5.	2.31	2.94
time (sec)	N/A	0.009	0.002	0.042	0.972	1.155	0.076	1.208

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	66	32	42
normalized size	1	1.	1.	0.91	1.2	1.89	0.91	1.2
time (sec)	N/A	0.017	0.001	0.041	1.06	1.419	0.08	1.297

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	49	78	37	49
normalized size	1	1.	1.	0.87	1.26	2.	0.95	1.26
time (sec)	N/A	0.026	0.004	0.043	0.987	1.432	0.299	1.285

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	43	73	29	43
normalized size	1	1.	1.	0.97	1.26	2.15	0.85	1.26
time (sec)	N/A	0.019	0.004	0.048	1.008	1.419	0.293	1.275

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	49	85	37	62
normalized size	1	1.	1.	0.88	1.22	2.12	0.92	1.55
time (sec)	N/A	0.029	0.006	0.046	1.01	1.503	0.326	1.28

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	46	72	34	46
normalized size	1	1.	1.	0.92	1.24	1.95	0.92	1.24
time (sec)	N/A	0.019	0.004	0.048	0.954	1.433	0.323	1.301

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	50	85	36	62
normalized size	1	1.	1.	0.88	1.25	2.12	0.9	1.55
time (sec)	N/A	0.026	0.004	0.05	0.935	1.423	0.369	1.231

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	45	76	32	45
normalized size	1	1.	1.	0.97	1.32	2.24	0.94	1.32
time (sec)	N/A	0.02	0.005	0.048	0.963	1.468	0.382	1.268

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	53	90	36	63
normalized size	1	1.	1.	0.87	1.36	2.31	0.92	1.62
time (sec)	N/A	0.026	0.004	0.048	0.966	1.485	0.425	1.145

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	50	84	39	50
normalized size	1	1.	1.	0.92	1.28	2.15	1.	1.28
time (sec)	N/A	0.021	0.004	0.049	0.973	1.462	0.426	1.289

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	47	76	37	47
normalized size	1	1.	2.26	1.89	2.47	4.	1.95	2.47
time (sec)	N/A	0.01	0.007	0.048	0.973	1.41	0.495	1.278

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	50	90	39	50
normalized size	1	1.	1.	0.84	1.16	2.09	0.91	1.16
time (sec)	N/A	0.021	0.004	0.049	0.972	1.457	0.488	1.255

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	43	36	50	85	39	50
normalized size	1	1.	1.08	0.9	1.25	2.12	0.98	1.25
time (sec)	N/A	0.023	0.005	0.051	0.96	1.401	0.482	1.211

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	0	336	107	88
normalized size	1	1.	1.	0.88	0.	4.94	1.57	1.29
time (sec)	N/A	0.038	0.026	0.046	0.	1.457	0.386	1.228

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	62	99	44	63
normalized size	1	1.	1.	0.87	1.17	1.87	0.83	1.19
time (sec)	N/A	0.045	0.006	0.045	1.03	1.49	0.332	1.283

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	0	278	95	74
normalized size	1	1.	1.	0.89	0.	5.05	1.73	1.35
time (sec)	N/A	0.033	0.027	0.048	0.	1.464	0.364	1.275

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	46	73	32	47
normalized size	1	1.	1.	0.88	1.15	1.82	0.8	1.18
time (sec)	N/A	0.034	0.005	0.045	1.006	1.422	0.357	1.289

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	217	80	54
normalized size	1	1.	1.	0.9	0.	5.17	1.9	1.29
time (sec)	N/A	0.027	0.019	0.045	0.	1.545	0.385	1.216

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	49	20	32
normalized size	1	1.	1.	0.89	1.15	1.81	0.74	1.19
time (sec)	N/A	0.027	0.005	0.045	0.99	1.443	0.311	1.295

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	0	165	56	35
normalized size	1	1.	1.	0.87	0.	5.32	1.81	1.13
time (sec)	N/A	0.018	0.008	0.043	0.	1.52	0.313	1.246

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	30	10	19
normalized size	1	1.	1.	0.93	1.2	2.	0.67	1.27
time (sec)	N/A	0.009	0.002	0.042	1.003	1.43	0.113	1.204

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	151	53	20
normalized size	1	1.	1.	0.67	0.	6.29	2.21	0.83
time (sec)	N/A	0.012	0.004	0.044	0.	1.434	0.137	1.286

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	31	49	15	30
normalized size	1	1.	1.	0.95	1.41	2.23	0.68	1.36
time (sec)	N/A	0.016	0.005	0.046	1.023	1.43	0.203	1.173

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	0	173	65	39
normalized size	1	1.	1.	0.88	0.	5.09	1.91	1.15
time (sec)	N/A	0.014	0.012	0.046	0.	1.509	0.377	1.216

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	45	80	31	58
normalized size	1	1.	1.	0.91	1.29	2.29	0.89	1.66
time (sec)	N/A	0.028	0.007	0.047	0.956	1.5	0.469	1.223

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	0	234	87	54
normalized size	1	1.	1.	0.91	0.	5.44	2.02	1.26
time (sec)	N/A	0.023	0.02	0.05	0.	1.397	0.403	1.274

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	63	108	42	77
normalized size	1	1.	1.	0.9	1.29	2.2	0.86	1.57
time (sec)	N/A	0.035	0.007	0.049	0.976	1.494	0.53	1.257

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	0	296	100	70
normalized size	1	1.	1.	0.9	0.	5.1	1.72	1.21
time (sec)	N/A	0.036	0.027	0.05	0.	1.462	0.55	1.238

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	78	134	56	95
normalized size	1	1.	1.	0.89	1.24	2.13	0.89	1.51
time (sec)	N/A	0.041	0.007	0.05	1.028	1.439	0.63	1.286

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	68	0	409	124	99
normalized size	1	1.	0.9	0.86	0.	5.18	1.57	1.25
time (sec)	N/A	0.039	0.052	0.052	0.	1.477	0.483	1.212

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	52	73	143	53	90
normalized size	1	1.	0.86	0.91	1.28	2.51	0.93	1.58
time (sec)	N/A	0.051	0.017	0.053	0.975	1.443	0.436	1.256

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	60	57	0	348	107	82
normalized size	1	1.	0.91	0.86	0.	5.27	1.62	1.24
time (sec)	N/A	0.035	0.041	0.052	0.	1.543	0.49	1.225

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	58	113	39	66
normalized size	1	1.	0.86	0.93	1.32	2.57	0.89	1.5
time (sec)	N/A	0.037	0.015	0.05	0.983	1.412	0.457	1.233

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	43	0	285	83	57
normalized size	1	1.	0.93	0.78	0.	5.18	1.51	1.04
time (sec)	N/A	0.024	0.032	0.05	0.	1.539	0.465	1.239

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	43	76	29	43
normalized size	1	1.	0.82	0.91	1.3	2.3	0.88	1.3
time (sec)	N/A	0.032	0.008	0.051	0.965	1.498	0.361	1.203

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	263	78	47
normalized size	1	1.	1.	0.8	0.	5.84	1.73	1.04
time (sec)	N/A	0.019	0.021	0.051	0.	1.536	0.421	1.216

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	20	30	15	19
normalized size	1	1.	1.	0.94	1.25	1.88	0.94	1.19
time (sec)	N/A	0.009	0.002	0.042	0.971	1.405	0.337	1.262

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	261	78	47
normalized size	1	1.	1.	0.8	0.	5.8	1.73	1.04
time (sec)	N/A	0.017	0.025	0.048	0.	1.484	0.39	1.263

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	50	108	34	49
normalized size	1	1.	0.87	0.92	1.32	2.84	0.89	1.29
time (sec)	N/A	0.035	0.013	0.054	0.966	1.485	0.492	1.209

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	0	288	90	63
normalized size	1	1.	0.95	0.81	0.	5.05	1.58	1.11
time (sec)	N/A	0.025	0.037	0.053	0.	1.499	0.525	1.268

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	70	157	49	68
normalized size	1	1.	0.84	0.94	1.43	3.2	1.	1.39
time (sec)	N/A	0.041	0.036	0.057	0.962	1.528	0.577	1.317

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	0	359	114	80
normalized size	1	1.	0.99	0.87	0.	5.28	1.68	1.18
time (sec)	N/A	0.029	0.038	0.055	0.	1.584	0.576	1.24

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	57	61	95	184	68	116
normalized size	1	1.	0.86	0.92	1.44	2.79	1.03	1.76
time (sec)	N/A	0.055	0.05	0.056	0.973	1.509	0.813	1.225

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	70	0	423	126	95
normalized size	1	1.	0.99	0.86	0.	5.22	1.56	1.17
time (sec)	N/A	0.045	0.044	0.055	0.	1.487	0.848	1.252

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	77	0	493	131	99
normalized size	1	1.	0.91	0.91	0.	5.8	1.54	1.16
time (sec)	N/A	0.042	0.047	0.051	0.	1.537	0.636	1.228

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	48	58	89	186	66	84
normalized size	1	1.	0.74	0.89	1.37	2.86	1.02	1.29
time (sec)	N/A	0.054	0.061	0.052	0.984	1.502	0.619	1.198

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	66	63	0	425	107	73
normalized size	1	1.	0.89	0.85	0.	5.74	1.45	0.99
time (sec)	N/A	0.032	0.046	0.052	0.	1.447	0.602	1.35

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	39	46	74	143	53	57
normalized size	1	1.	0.8	0.94	1.51	2.92	1.08	1.16
time (sec)	N/A	0.045	0.016	0.051	1.021	1.46	0.508	1.206

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	55	47	0	404	109	61
normalized size	1	1.	0.86	0.73	0.	6.31	1.7	0.95
time (sec)	N/A	0.026	0.043	0.048	0.	1.596	0.532	1.289

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	49	73	36	30
normalized size	1	1.	1.26	1.63	2.58	3.84	1.89	1.58
time (sec)	N/A	0.011	0.007	0.049	1.001	1.473	0.535	1.281

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	49	0	394	110	68
normalized size	1	1.	0.89	0.75	0.	6.06	1.69	1.05
time (sec)	N/A	0.025	0.03	0.051	0.	1.499	0.531	1.199

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	35	51	27	19
normalized size	1	1.	1.	0.94	2.19	3.19	1.69	1.19
time (sec)	N/A	0.009	0.002	0.043	0.987	1.493	0.426	1.197

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	51	0	401	105	61
normalized size	1	1.	0.89	0.82	0.	6.47	1.69	0.98
time (sec)	N/A	0.023	0.034	0.046	0.	1.518	0.518	1.308

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	81	196	56	80
normalized size	1	1.	0.8	0.91	1.5	3.63	1.04	1.48
time (sec)	N/A	0.044	0.031	0.056	0.985	1.469	0.621	1.311

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	68	66	0	428	114	77
normalized size	1	1.	0.89	0.87	0.	5.63	1.5	1.01
time (sec)	N/A	0.036	0.041	0.055	0.	1.534	0.783	1.273

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	59	62	104	247	78	89
normalized size	1	1.	0.88	0.93	1.55	3.69	1.16	1.33
time (sec)	N/A	0.061	0.057	0.06	0.989	1.626	0.98	1.283

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	79	79	0	504	138	96
normalized size	1	1.	0.91	0.91	0.	5.79	1.59	1.1
time (sec)	N/A	0.042	0.044	0.056	0.	1.803	0.928	1.226

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	74	79	124	274	90	107
normalized size	1	1.	0.86	0.92	1.44	3.19	1.05	1.24
time (sec)	N/A	0.072	0.046	0.056	0.978	1.502	1.271	1.285

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	89	0	560	150	108
normalized size	1	1.	0.9	0.89	0.	5.6	1.5	1.08
time (sec)	N/A	0.049	0.054	0.056	0.	1.558	1.756	1.302

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	85	90	139	308	104	149
normalized size	1	1.	0.89	0.95	1.46	3.24	1.09	1.57
time (sec)	N/A	0.085	0.073	0.058	1.031	1.563	2.051	1.3

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	114	124	0	427	0	136
normalized size	1	1.	0.96	1.04	0.	3.59	0.	1.14
time (sec)	N/A	0.131	0.076	0.05	0.	1.696	0.	1.322

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	103	104	0	371	0	115
normalized size	1	1.	1.13	1.14	0.	4.08	0.	1.26
time (sec)	N/A	0.1	0.063	0.049	0.	1.59	0.	1.329

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	90	84	0	315	0	93
normalized size	1	1.	1.32	1.24	0.	4.63	0.	1.37
time (sec)	N/A	0.064	0.05	0.049	0.	1.619	0.	1.258

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	64	64	0	261	0	70
normalized size	1	1.	1.16	1.16	0.	4.75	0.	1.27
time (sec)	N/A	0.071	0.025	0.046	0.	1.61	0.	1.179

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	60	86	0	261	0	82
normalized size	1	1.	1.15	1.65	0.	5.02	0.	1.58
time (sec)	N/A	0.077	0.093	0.049	0.	1.58	0.	1.256

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	0	62	0	85
normalized size	1	1.	1.	1.16	0.	2.48	0.	3.4
time (sec)	N/A	0.039	0.01	0.045	0.	1.529	0.	1.34

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	0	89	0	162
normalized size	1	1.	0.67	0.75	0.	1.71	0.	3.12
time (sec)	N/A	0.083	0.011	0.046	0.	1.549	0.	1.32

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	0	117	0	200
normalized size	1	1.	0.57	0.62	0.	1.46	0.	2.5
time (sec)	N/A	0.12	0.012	0.046	0.	1.546	0.	1.281

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	57	61	0	140	0	240
normalized size	1	1.	0.53	0.56	0.	1.3	0.	2.22
time (sec)	N/A	0.164	0.014	0.046	0.	1.586	0.	1.371

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	68	72	0	174	0	278
normalized size	1	1.	0.5	0.53	0.	1.28	0.	2.04
time (sec)	N/A	0.214	0.015	0.046	0.	1.674	0.	1.352

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	46	50	62	113	0	76
normalized size	1	1.	0.59	0.64	0.79	1.45	0.	0.97
time (sec)	N/A	0.094	0.023	0.046	1.001	1.592	0.	1.243

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	46	86	0	57
normalized size	1	1.	0.67	0.75	0.88	1.65	0.	1.1
time (sec)	N/A	0.049	0.018	0.047	0.998	1.539	0.	1.26

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	19	58	0	36
normalized size	1	1.	1.	1.16	0.76	2.32	0.	1.44
time (sec)	N/A	0.006	0.005	0.044	0.985	1.512	0.	1.304

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	60	65	0	262	0	92
normalized size	1	1.	1.2	1.3	0.	5.24	0.	1.84
time (sec)	N/A	0.049	0.035	0.047	0.	1.537	0.	1.264

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	63	85	0	300	0	61
normalized size	1	1.	1.12	1.52	0.	5.36	0.	1.09
time (sec)	N/A	0.052	0.042	0.049	0.	1.613	0.	1.331

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	46	106	0	356	0	86
normalized size	1	1.	0.55	1.26	0.	4.24	0.	1.02
time (sec)	N/A	0.098	0.015	0.048	0.	1.623	0.	1.3

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	46	128	0	410	0	111
normalized size	1	1.	0.41	1.14	0.	3.66	0.	0.99
time (sec)	N/A	0.145	0.014	0.051	0.	1.676	0.	1.347

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	126	142	0	481	0	155
normalized size	1	1.	1.02	1.15	0.	3.88	0.	1.25
time (sec)	N/A	0.132	0.101	0.052	0.	1.65	0.	1.204

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	115	122	0	423	0	134
normalized size	1	1.	1.14	1.21	0.	4.19	0.	1.33
time (sec)	N/A	0.092	0.102	0.048	0.	1.341	0.	1.288

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	104	102	0	373	0	113
normalized size	1	1.	1.18	1.16	0.	4.24	0.	1.28
time (sec)	N/A	0.099	0.08	0.049	0.	1.369	0.	1.299

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	84	0	321	0	92
normalized size	1	1.	0.89	1.05	0.	4.01	0.	1.15
time (sec)	N/A	0.1	0.112	0.048	0.	1.373	0.	1.292

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	54	107	0	315	0	107
normalized size	1	1.	0.71	1.41	0.	4.14	0.	1.41
time (sec)	N/A	0.111	0.014	0.048	0.	1.301	0.	1.311

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	56	129	0	312	0	165
normalized size	1	1.	0.75	1.72	0.	4.16	0.	2.2
time (sec)	N/A	0.104	0.018	0.054	0.	1.297	0.	1.798

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	0	84	0	124
normalized size	1	1.	1.	1.16	0.	3.36	0.	4.96
time (sec)	N/A	0.046	0.014	0.047	0.	1.268	0.	1.267

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	0	111	0	240
normalized size	1	1.	0.67	0.75	0.	2.13	0.	4.62
time (sec)	N/A	0.093	0.015	0.046	0.	1.363	0.	1.312

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	0	142	0	278
normalized size	1	1.	0.57	0.62	0.	1.78	0.	3.48
time (sec)	N/A	0.141	0.015	0.046	0.	1.342	0.	1.35

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	57	61	0	169	0	319
normalized size	1	1.	0.53	0.56	0.	1.56	0.	2.95
time (sec)	N/A	0.184	0.018	0.045	0.	1.471	0.	1.27

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	68	72	0	204	0	356
normalized size	1	1.	0.5	0.53	0.	1.5	0.	2.62
time (sec)	N/A	0.262	0.019	0.045	0.	1.675	0.	1.27

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	75	72	107	198	0	240
normalized size	1	1.	0.56	0.54	0.8	1.48	0.	1.79
time (sec)	N/A	0.253	0.038	0.046	1.042	1.386	0.	1.223

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	64	61	92	165	0	201
normalized size	1	1.	0.6	0.58	0.87	1.56	0.	1.9
time (sec)	N/A	0.196	0.032	0.047	1.04	1.285	0.	1.179

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	53	50	77	136	0	163
normalized size	1	1.	0.66	0.62	0.96	1.7	0.	2.04
time (sec)	N/A	0.111	0.025	0.046	1.044	1.339	0.	1.158

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	39	61	108	0	126
normalized size	1	1.	0.81	0.75	1.17	2.08	0.	2.42
time (sec)	N/A	0.054	0.019	0.046	1.042	1.312	0.	1.151

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	43	80	0	78
normalized size	1	1.	1.	1.16	1.72	3.2	0.	3.12
time (sec)	N/A	0.048	0.009	0.044	1.03	1.378	0.	1.134

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	76	78	0	313	0	119
normalized size	1	1.	1.04	1.07	0.	4.29	0.	1.63
time (sec)	N/A	0.11	0.053	0.046	0.	1.408	0.	1.157

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	44	102	0	327	0	80
normalized size	1	1.	0.56	1.29	0.	4.14	0.	1.01
time (sec)	N/A	0.118	0.017	0.047	0.	1.436	0.	1.203

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	125	0	360	0	85
normalized size	1	1.	0.99	1.54	0.	4.44	0.	1.05
time (sec)	N/A	0.115	0.05	0.048	0.	1.269	0.	1.232

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	46	145	0	414	0	111
normalized size	1	1.	0.42	1.33	0.	3.8	0.	1.02
time (sec)	N/A	0.164	0.019	0.05	0.	1.291	0.	1.188

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	46	165	0	462	0	130
normalized size	1	1.	0.34	1.2	0.	3.37	0.	0.95
time (sec)	N/A	0.203	0.017	0.055	0.	1.447	0.	1.345

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	46	186	0	528	0	149
normalized size	1	1.	0.28	1.13	0.	3.2	0.	0.9
time (sec)	N/A	0.255	0.018	0.071	0.	1.481	0.	1.23

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	100	105	0	378	0	0
normalized size	1	1.	0.88	0.92	0.	3.32	0.	0.
time (sec)	N/A	0.129	0.053	0.051	0.	1.347	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	89	85	0	327	0	0
normalized size	1	1.	1.03	0.99	0.	3.8	0.	0.
time (sec)	N/A	0.103	0.037	0.046	0.	1.386	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	73	64	0	265	0	80
normalized size	1	1.	1.26	1.1	0.	4.57	0.	1.38
time (sec)	N/A	0.083	0.028	0.048	0.	1.375	0.	1.248

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	52	44	0	174	0	53
normalized size	1	1.	1.68	1.42	0.	5.61	0.	1.71
time (sec)	N/A	0.054	0.014	0.046	0.	1.295	0.	1.186

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	0	41	0	19
normalized size	1	1.	1.	1.13	0.	1.78	0.	0.83
time (sec)	N/A	0.041	0.009	0.046	0.	1.23	0.	1.154

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	37	0	66	0	36
normalized size	1	1.	0.67	0.71	0.	1.27	0.	0.69
time (sec)	N/A	0.083	0.015	0.048	0.	1.308	0.	1.144

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	0	93	0	58
normalized size	1	1.	0.57	0.62	0.	1.16	0.	0.72
time (sec)	N/A	0.125	0.014	0.044	0.	1.298	0.	1.149

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	57	61	0	115	0	77
normalized size	1	1.	0.53	0.56	0.	1.06	0.	0.71
time (sec)	N/A	0.169	0.015	0.045	0.	1.324	0.	1.173

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	34	37	46	63	0	0
normalized size	1	1.	0.68	0.74	0.92	1.26	0.	0.
time (sec)	N/A	0.079	0.018	0.043	1.03	1.256	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	18	36	0	42
normalized size	1	1.	1.	1.18	0.82	1.64	0.	1.91
time (sec)	N/A	0.017	0.005	0.043	1.018	1.096	0.	1.131

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	52	50	0	186	0	62
normalized size	1	1.	1.73	1.67	0.	6.2	0.	2.07
time (sec)	N/A	0.009	0.01	0.043	0.	1.425	0.	1.161

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	68	73	0	306	0	0
normalized size	1	1.	1.15	1.24	0.	5.19	0.	0.
time (sec)	N/A	0.055	0.062	0.044	0.	1.664	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	44	94	0	366	0	0
normalized size	1	1.	0.51	1.08	0.	4.21	0.	0.
time (sec)	N/A	0.097	0.012	0.046	0.	1.629	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	88	87	0	451	0	0
normalized size	1	1.	0.81	0.8	0.	4.14	0.	0.
time (sec)	N/A	0.129	0.051	0.051	0.	1.605	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	76	73	0	389	0	0
normalized size	1	1.	0.94	0.9	0.	4.8	0.	0.
time (sec)	N/A	0.109	0.042	0.048	0.	1.294	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	66	62	0	325	0	55
normalized size	1	1.	1.2	1.13	0.	5.91	0.	1.
time (sec)	N/A	0.092	0.078	0.045	0.	1.177	0.	1.155

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	28	27	50	0	47
normalized size	1	1.	1.	1.27	1.23	2.27	0.	2.14
time (sec)	N/A	0.055	0.008	0.043	0.986	1.254	0.	1.209

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	29	37	55	78	0	38
normalized size	1	1.	1.04	1.32	1.96	2.79	0.	1.36
time (sec)	N/A	0.046	0.009	0.044	1.013	1.21	0.	1.208

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	48	45	0	104	0	0
normalized size	1	1.	0.65	0.61	0.	1.41	0.	0.
time (sec)	N/A	0.129	0.012	0.046	0.	1.271	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	57	59	0	128	0	0
normalized size	1	1.	0.56	0.58	0.	1.25	0.	0.
time (sec)	N/A	0.185	0.012	0.046	0.	1.341	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	68	72	0	158	0	0
normalized size	1	1.	0.52	0.55	0.	1.22	0.	0.
time (sec)	N/A	0.233	0.014	0.047	0.	1.365	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	29	37	30	74	0	70
normalized size	1	1.	0.62	0.79	0.64	1.57	0.	1.49
time (sec)	N/A	0.068	0.015	0.046	0.99	1.243	0.	1.17

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	29	19	54	0	23
normalized size	1	1.	1.	1.38	0.9	2.57	0.	1.1
time (sec)	N/A	0.019	0.005	0.044	1.007	1.281	0.	1.165

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	38	67	0	348	0	0
normalized size	1	1.	0.75	1.31	0.	6.82	0.	0.
time (sec)	N/A	0.062	0.009	0.046	0.	1.32	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	40	77	0	425	0	0
normalized size	1	1.	0.49	0.95	0.	5.25	0.	0.
time (sec)	N/A	0.065	0.008	0.046	0.	1.417	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	41	94	0	485	0	0
normalized size	1	1.	0.38	0.86	0.	4.45	0.	0.
time (sec)	N/A	0.153	0.011	0.046	0.	1.348	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	57	48	35	96	0	35
normalized size	1	1.	1.68	1.41	1.03	2.82	0.	1.03
time (sec)	N/A	0.058	0.019	0.046	1.448	1.257	0.	1.152

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	52	54	35	158	0	36
normalized size	1	1.	1.53	1.59	1.03	4.65	0.	1.06
time (sec)	N/A	0.057	0.017	0.047	1.473	1.156	0.	1.169

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	51	48	55	95	0	55
normalized size	1	1.	1.13	1.07	1.22	2.11	0.	1.22
time (sec)	N/A	0.056	0.01	0.046	1.459	1.232	0.	1.169

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	60	55	95	0	57
normalized size	1	1.	1.27	1.33	1.22	2.11	0.	1.27
time (sec)	N/A	0.057	0.012	0.046	1.442	1.248	0.	1.17

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	73	64	0	265	0	80
normalized size	1	1.	1.26	1.1	0.	4.57	0.	1.38
time (sec)	N/A	0.083	0.033	0.047	0.	1.3	0.	1.222

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	77	67	0	271	0	92
normalized size	1	1.	1.28	1.12	0.	4.52	0.	1.53
time (sec)	N/A	0.082	0.041	0.049	0.	1.268	0.	1.193

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	50	19	18
normalized size	1	1.	1.	0.76	0.86	2.38	0.9	0.86
time (sec)	N/A	0.005	0.006	0.042	0.972	1.245	14.179	1.142

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	50	19	18
normalized size	1	1.	1.	0.76	0.86	2.38	0.9	0.86
time (sec)	N/A	0.005	0.005	0.042	0.999	1.232	7.917	1.132

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	49	19	18
normalized size	1	1.	1.	0.76	0.86	2.33	0.9	0.86
time (sec)	N/A	0.005	0.005	0.044	0.969	1.205	3.257	1.115

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	47	19	18
normalized size	1	1.	1.	0.76	0.86	2.24	0.9	0.86
time (sec)	N/A	0.005	0.008	0.044	0.965	1.25	1.691	1.118

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	46	19	18
normalized size	1	1.	1.	0.76	0.86	2.19	0.9	0.86
time (sec)	N/A	0.005	0.005	0.043	0.959	1.287	0.762	1.104

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	43	19	18
normalized size	1	1.	1.	0.76	0.86	2.05	0.9	0.86
time (sec)	N/A	0.005	0.004	0.043	0.953	1.297	0.916	1.124

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	18	36	17	18
normalized size	1	1.	1.	0.79	0.95	1.89	0.89	0.95
time (sec)	N/A	0.005	0.005	0.043	0.961	1.202	1.177	1.116

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	18	36	17	18
normalized size	1	1.	1.	0.84	0.95	1.89	0.89	0.95
time (sec)	N/A	0.005	0.006	0.045	0.98	1.372	2.098	1.18

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	81	34	32
normalized size	1	1.	0.83	0.75	0.89	2.25	0.94	0.89
time (sec)	N/A	0.016	0.008	0.045	0.963	1.239	41.117	1.155

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	80	34	32
normalized size	1	1.	0.83	0.75	0.89	2.22	0.94	0.89
time (sec)	N/A	0.018	0.008	0.045	0.969	1.266	24.129	1.129

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	80	34	32
normalized size	1	1.	0.83	0.75	0.89	2.22	0.94	0.89
time (sec)	N/A	0.015	0.008	0.047	0.98	1.247	13.136	1.117

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	78	34	32
normalized size	1	1.	0.83	0.75	0.89	2.17	0.94	0.89
time (sec)	N/A	0.014	0.008	0.046	0.99	1.248	3.472	1.111

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	78	34	32
normalized size	1	1.	0.83	0.75	0.89	2.17	0.94	0.89
time (sec)	N/A	0.014	0.008	0.046	1.001	1.215	5.497	1.171

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	77	34	32
normalized size	1	1.	0.83	0.75	0.89	2.14	0.94	0.89
time (sec)	N/A	0.016	0.008	0.046	0.985	1.226	6.097	1.189

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	76	34	32
normalized size	1	1.	0.83	0.75	0.89	2.11	0.94	0.89
time (sec)	N/A	0.015	0.008	0.046	0.989	1.265	7.107	1.161

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	70	34	32
normalized size	1	1.	0.83	0.75	0.89	1.94	0.94	0.89
time (sec)	N/A	0.016	0.008	0.045	0.991	1.245	10.081	1.142

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	47	119	49	47
normalized size	1	1.	1.	0.75	0.92	2.33	0.96	0.92
time (sec)	N/A	0.021	0.014	0.047	0.969	1.269	112.55	1.158

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	47	117	49	47
normalized size	1	1.	1.	0.75	0.92	2.29	0.96	0.92
time (sec)	N/A	0.02	0.013	0.047	1.061	1.213	70.773	1.122

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	47	116	49	47
normalized size	1	1.	1.	0.75	0.92	2.27	0.96	0.92
time (sec)	N/A	0.02	0.011	0.046	0.989	1.217	41.785	1.151

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	112	49	47
normalized size	1	1.	0.8	0.75	0.92	2.2	0.96	0.92
time (sec)	N/A	0.019	0.01	0.047	0.984	1.295	7.645	1.137

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	112	49	47
normalized size	1	1.	0.8	0.75	0.92	2.2	0.96	0.92
time (sec)	N/A	0.02	0.011	0.047	1.007	1.255	18.705	1.161

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	111	49	47
normalized size	1	1.	0.8	0.75	0.92	2.18	0.96	0.92
time (sec)	N/A	0.02	0.012	0.046	0.992	1.214	20.329	1.115

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	109	49	47
normalized size	1	1.	0.8	0.75	0.92	2.14	0.96	0.92
time (sec)	N/A	0.019	0.011	0.048	1.015	1.278	22.706	1.137

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	107	49	47
normalized size	1	1.	0.8	0.75	0.92	2.1	0.96	0.92
time (sec)	N/A	0.022	0.01	0.046	0.969	1.217	28.351	1.115

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	89	158	0	429	0	266
normalized size	1	1.	0.41	0.73	0.	1.98	0.	1.23
time (sec)	N/A	0.232	0.045	0.051	0.	1.35	0.	1.186

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	203	152	0	393	0	265
normalized size	1	1.	0.94	0.71	0.	1.83	0.	1.23
time (sec)	N/A	0.194	0.066	0.051	0.	1.355	0.	1.154

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	78	143	0	377	0	240
normalized size	1	1.	0.38	0.7	0.	1.85	0.	1.18
time (sec)	N/A	0.186	0.021	0.051	0.	1.287	0.	1.144

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	189	140	0	312	177	240
normalized size	1	1.	0.94	0.69	0.	1.54	0.88	1.19
time (sec)	N/A	0.185	0.035	0.048	0.	1.328	130.819	1.153

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	54	132	0	332	170	246
normalized size	1	1.	0.28	0.69	0.	1.73	0.89	1.28
time (sec)	N/A	0.142	0.025	0.049	0.	1.33	72.706	1.131

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	146	132	0	329	170	246
normalized size	1	1.	0.76	0.69	0.	1.71	0.89	1.28
time (sec)	N/A	0.139	0.037	0.05	0.	1.34	35.305	1.172

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	27	140	0	343	180	257
normalized size	1	1.	0.13	0.69	0.	1.7	0.89	1.27
time (sec)	N/A	0.164	0.006	0.051	0.	1.475	24.345	1.192

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	29	143	0	387	184	240
normalized size	1	1.	0.14	0.7	0.	1.9	0.9	1.18
time (sec)	N/A	0.167	0.007	0.053	0.	1.394	41.386	1.14

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	29	152	0	433	196	270
normalized size	1	1.	0.13	0.71	0.	2.01	0.91	1.26
time (sec)	N/A	0.188	0.007	0.055	0.	1.367	67.553	1.132

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	29	158	0	441	197	259
normalized size	1	1.	0.13	0.73	0.	2.03	0.91	1.19
time (sec)	N/A	0.182	0.006	0.055	0.	1.475	151.415	1.168

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	29	169	0	477	0	269
normalized size	1	1.	0.13	0.73	0.	2.07	0.	1.17
time (sec)	N/A	0.22	0.007	0.055	0.	1.328	0.	1.153

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	220	172	0	524	0	292
normalized size	1	1.	0.91	0.71	0.	2.16	0.	1.2
time (sec)	N/A	0.21	0.211	0.058	0.	1.4	0.	1.177

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	57	161	0	556	0	265
normalized size	1	1.	0.25	0.7	0.	2.42	0.	1.15
time (sec)	N/A	0.182	0.017	0.061	0.	1.476	0.	1.196

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	221	158	0	443	0	265
normalized size	1	1.	0.96	0.69	0.	1.93	0.	1.15
time (sec)	N/A	0.182	0.109	0.061	0.	1.364	0.	1.235

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	43	149	0	447	0	269
normalized size	1	1.	0.2	0.68	0.	2.05	0.	1.23
time (sec)	N/A	0.166	0.016	0.056	0.	1.411	0.	1.188

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	198	158	0	467	0	269
normalized size	1	1.	0.91	0.72	0.	2.14	0.	1.23
time (sec)	N/A	0.16	0.101	0.056	0.	1.581	0.	1.167

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	29	158	0	467	0	269
normalized size	1	1.	0.13	0.72	0.	2.14	0.	1.23
time (sec)	N/A	0.163	0.006	0.054	0.	1.623	0.	1.164

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	199	149	0	441	0	269
normalized size	1	1.	0.91	0.68	0.	2.02	0.	1.23
time (sec)	N/A	0.169	0.112	0.055	0.	1.586	0.	1.158

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	27	158	0	512	0	284
normalized size	1	1.	0.12	0.69	0.	2.23	0.	1.23
time (sec)	N/A	0.19	0.007	0.06	0.	1.568	0.	1.189

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	29	161	0	518	0	265
normalized size	1	1.	0.13	0.7	0.	2.25	0.	1.15
time (sec)	N/A	0.185	0.007	0.058	0.	1.5	0.	1.186

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	29	172	0	605	0	297
normalized size	1	1.	0.12	0.71	0.	2.49	0.	1.22
time (sec)	N/A	0.217	0.007	0.06	0.	1.432	0.	1.159

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	29	178	0	564	0	286
normalized size	1	1.	0.12	0.73	0.	2.32	0.	1.18
time (sec)	N/A	0.207	0.006	0.062	0.	1.435	0.	1.134

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	29	189	0	648	0	296
normalized size	1	1.	0.11	0.73	0.	2.51	0.	1.15
time (sec)	N/A	0.235	0.007	0.06	0.	1.423	0.	1.156

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	220	178	0	579	0	281
normalized size	1	1.	0.88	0.71	0.	2.31	0.	1.12
time (sec)	N/A	0.211	0.262	0.06	0.	1.388	0.	1.223

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	66	161	0	585	0	282
normalized size	1	1.	0.28	0.67	0.	2.45	0.	1.18
time (sec)	N/A	0.203	0.02	0.058	0.	1.336	0.	1.188

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	242	170	0	599	0	282
normalized size	1	1.	1.01	0.71	0.	2.51	0.	1.18
time (sec)	N/A	0.189	0.111	0.06	0.	1.401	0.	1.143

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	45	169	0	613	0	286
normalized size	1	1.	0.19	0.7	0.	2.53	0.	1.18
time (sec)	N/A	0.188	0.015	0.061	0.	1.372	0.	1.168

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	223	169	0	608	0	285
normalized size	1	1.	0.92	0.7	0.	2.51	0.	1.18
time (sec)	N/A	0.183	0.116	0.06	0.	1.381	0.	1.134

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	29	175	0	602	0	282
normalized size	1	1.	0.12	0.73	0.	2.52	0.	1.18
time (sec)	N/A	0.185	0.005	0.054	0.	1.388	0.	1.157

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	220	166	0	576	0	282
normalized size	1	1.	0.92	0.69	0.	2.41	0.	1.18
time (sec)	N/A	0.186	0.086	0.054	0.	1.409	0.	1.144

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	27	178	0	672	0	297
normalized size	1	1.	0.11	0.71	0.	2.68	0.	1.18
time (sec)	N/A	0.217	0.006	0.063	0.	1.344	0.	1.158

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	29	181	0	645	0	281
normalized size	1	1.	0.12	0.72	0.	2.57	0.	1.12
time (sec)	N/A	0.214	0.007	0.066	0.	1.374	0.	1.162

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	29	192	0	778	0	313
normalized size	1	1.	0.11	0.73	0.	2.95	0.	1.19
time (sec)	N/A	0.233	0.007	0.064	0.	1.406	0.	1.185

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	29	198	0	689	0	302
normalized size	1	1.	0.11	0.75	0.	2.61	0.	1.14
time (sec)	N/A	0.231	0.006	0.065	0.	1.406	0.	1.199

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	29	209	0	828	0	312
normalized size	1	1.	0.1	0.75	0.	2.97	0.	1.12
time (sec)	N/A	0.268	0.008	0.064	0.	1.424	0.	1.168

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	29	209	0	744	0	328
normalized size	1	1.	0.1	0.75	0.	2.67	0.	1.18
time (sec)	N/A	0.26	0.008	0.067	0.	1.449	0.	1.159

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	102	237	0	0	0	0
normalized size	1	1.	0.32	0.73	0.	0.	0.	0.
time (sec)	N/A	0.383	0.079	0.183	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	102	157	0	0	0	0
normalized size	1	1.	0.58	0.89	0.	0.	0.	0.
time (sec)	N/A	0.249	0.059	0.189	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	86	226	0	0	0	0
normalized size	1	1.	0.29	0.77	0.	0.	0.	0.
time (sec)	N/A	0.307	0.039	0.186	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	86	145	0	0	0	0
normalized size	1	1.	0.59	0.99	0.	0.	0.	0.
time (sec)	N/A	0.185	0.034	0.182	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	57	213	0	0	0	0
normalized size	1	1.	0.22	0.81	0.	0.	0.	0.
time (sec)	N/A	0.238	0.014	0.184	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	55	130	0	0	0	0
normalized size	1	1.	0.47	1.1	0.	0.	0.	0.
time (sec)	N/A	0.133	0.013	0.186	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	55	202	0	0	0	0
normalized size	1	1.	0.22	0.8	0.	0.	0.	0.
time (sec)	N/A	0.238	0.014	0.186	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	57	125	0	0	0	0
normalized size	1	1.	0.48	1.06	0.	0.	0.	0.
time (sec)	N/A	0.132	0.014	0.183	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	57	224	0	0	0	0
normalized size	1	1.	0.19	0.76	0.	0.	0.	0.
time (sec)	N/A	0.299	0.013	0.192	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	57	142	0	0	0	0
normalized size	1	1.	0.39	0.97	0.	0.	0.	0.
time (sec)	N/A	0.182	0.015	0.181	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	57	239	0	0	0	0
normalized size	1	1.	0.18	0.74	0.	0.	0.	0.
time (sec)	N/A	0.366	0.016	0.19	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	57	156	0	0	0	0
normalized size	1	1.	0.32	0.89	0.	0.	0.	0.
time (sec)	N/A	0.234	0.014	0.186	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	101	248	0	0	0	0
normalized size	1	1.	0.29	0.71	0.	0.	0.	0.
time (sec)	N/A	0.434	0.062	0.198	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	101	168	0	0	0	0
normalized size	1	1.	0.5	0.83	0.	0.	0.	0.
time (sec)	N/A	0.289	0.06	0.189	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	90	237	0	0	0	0
normalized size	1	1.	0.28	0.74	0.	0.	0.	0.
time (sec)	N/A	0.37	0.048	0.187	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	90	157	0	0	0	0
normalized size	1	1.	0.52	0.91	0.	0.	0.	0.
time (sec)	N/A	0.236	0.039	0.183	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	58	226	0	0	0	0
normalized size	1	1.	0.2	0.78	0.	0.	0.	0.
time (sec)	N/A	0.299	0.017	0.188	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	56	145	0	0	0	0
normalized size	1	1.	0.39	1.01	0.	0.	0.	0.
time (sec)	N/A	0.185	0.016	0.184	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	56	216	0	0	0	0
normalized size	1	1.	0.2	0.76	0.	0.	0.	0.
time (sec)	N/A	0.301	0.016	0.185	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	58	130	0	0	0	0
normalized size	1	1.	0.41	0.91	0.	0.	0.	0.
time (sec)	N/A	0.188	0.017	0.187	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	58	221	0	0	0	0
normalized size	1	1.	0.2	0.77	0.	0.	0.	0.
time (sec)	N/A	0.305	0.017	0.19	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	58	140	0	0	0	0
normalized size	1	1.	0.41	0.98	0.	0.	0.	0.
time (sec)	N/A	0.183	0.017	0.185	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	58	239	0	0	0	0
normalized size	1	1.	0.18	0.75	0.	0.	0.	0.
time (sec)	N/A	0.36	0.019	0.192	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	58	156	0	0	0	0
normalized size	1	1.	0.34	0.9	0.	0.	0.	0.
time (sec)	N/A	0.237	0.018	0.19	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	58	250	0	0	0	0
normalized size	1	1.	0.17	0.71	0.	0.	0.	0.
time (sec)	N/A	0.426	0.019	0.204	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	58	167	0	0	0	0
normalized size	1	1.	0.29	0.82	0.	0.	0.	0.
time (sec)	N/A	0.296	0.019	0.206	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	97	148	0	0	0	0
normalized size	1	1.	0.54	0.83	0.	0.	0.	0.
time (sec)	N/A	0.244	0.035	0.19	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	86	217	0	0	0	0
normalized size	1	1.	0.29	0.73	0.	0.	0.	0.
time (sec)	N/A	0.298	0.034	0.19	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	86	137	0	0	0	0
normalized size	1	1.	0.58	0.92	0.	0.	0.	0.
time (sec)	N/A	0.183	0.035	0.183	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	70	206	0	0	0	0
normalized size	1	1.	0.26	0.77	0.	0.	0.	0.
time (sec)	N/A	0.237	0.025	0.211	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	70	123	0	0	0	0
normalized size	1	1.	0.58	1.02	0.	0.	0.	0.
time (sec)	N/A	0.131	0.027	0.177	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	57	131	0	0	0	0
normalized size	1	1.	0.25	0.57	0.	0.	0.	0.
time (sec)	N/A	0.176	0.012	0.174	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	55	106	0	0	0	0
normalized size	1	1.	0.61	1.18	0.	0.	0.	0.
time (sec)	N/A	0.089	0.013	0.183	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	55	195	0	0	0	0
normalized size	1	1.	0.21	0.75	0.	0.	0.	0.
time (sec)	N/A	0.232	0.016	0.18	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	57	119	0	0	0	0
normalized size	1	1.	0.47	0.98	0.	0.	0.	0.
time (sec)	N/A	0.133	0.015	0.184	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	57	215	0	0	0	0
normalized size	1	1.	0.19	0.73	0.	0.	0.	0.
time (sec)	N/A	0.294	0.014	0.191	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	57	134	0	0	0	0
normalized size	1	1.	0.38	0.9	0.	0.	0.	0.
time (sec)	N/A	0.182	0.014	0.184	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	57	230	0	0	0	0
normalized size	1	1.	0.17	0.71	0.	0.	0.	0.
time (sec)	N/A	0.359	0.013	0.188	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	57	147	0	0	0	0
normalized size	1	1.	0.32	0.82	0.	0.	0.	0.
time (sec)	N/A	0.233	0.013	0.187	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	86	144	0	0	0	0
normalized size	1	1.	0.49	0.83	0.	0.	0.	0.
time (sec)	N/A	0.242	0.029	0.191	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	72	213	0	0	0	0
normalized size	1	1.	0.25	0.73	0.	0.	0.	0.
time (sec)	N/A	0.296	0.026	0.187	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	73	131	0	0	0	0
normalized size	1	1.	0.5	0.9	0.	0.	0.	0.
time (sec)	N/A	0.185	0.027	0.186	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	61	200	0	0	0	0
normalized size	1	1.	0.24	0.77	0.	0.	0.	0.
time (sec)	N/A	0.234	0.022	0.216	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	60	120	0	0	0	0
normalized size	1	1.	0.5	1.01	0.	0.	0.	0.
time (sec)	N/A	0.138	0.021	0.206	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	60	203	0	0	0	0
normalized size	1	1.	0.23	0.78	0.	0.	0.	0.
time (sec)	N/A	0.237	0.015	0.187	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	60	123	0	0	0	0
normalized size	1	1.	0.51	1.04	0.	0.	0.	0.
time (sec)	N/A	0.136	0.02	0.207	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	58	203	0	0	0	0
normalized size	1	1.	0.2	0.71	0.	0.	0.	0.
time (sec)	N/A	0.298	0.018	0.201	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	60	127	0	0	0	0
normalized size	1	1.	0.41	0.88	0.	0.	0.	0.
time (sec)	N/A	0.185	0.019	0.217	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	60	222	0	0	0	0
normalized size	1	1.	0.19	0.69	0.	0.	0.	0.
time (sec)	N/A	0.364	0.017	0.197	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	60	141	0	0	0	0
normalized size	1	1.	0.35	0.82	0.	0.	0.	0.
time (sec)	N/A	0.233	0.017	0.217	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	60	237	0	0	0	0
normalized size	1	1.	0.17	0.68	0.	0.	0.	0.
time (sec)	N/A	0.427	0.018	0.202	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	59	181	103	375	758	356
normalized size	1	1.	0.81	2.48	1.41	5.14	10.38	4.88
time (sec)	N/A	0.05	0.04	0.048	1.012	1.59	5.306	1.168

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	96	74	200	352	190
normalized size	1	1.	0.83	1.85	1.42	3.85	6.77	3.65
time (sec)	N/A	0.039	0.034	0.049	1.009	1.587	2.189	1.16

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	39	46	84	119	76
normalized size	1	1.	0.79	1.15	1.35	2.47	3.5	2.24
time (sec)	N/A	0.014	0.016	0.044	1.015	1.579	0.711	1.212

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.011	0.342	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.012	0.334	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.013	0.345	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	24	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.8	1.07
time (sec)	N/A	0.01	0.001	0.041	0.992	1.291	0.06	1.128

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	55	26	32
normalized size	1	1.	1.	0.83	1.07	1.83	0.87	1.07
time (sec)	N/A	0.009	0.001	0.04	0.986	1.273	0.06	1.113

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	16	25	32	55	24	32
normalized size	1	1.	0.53	0.83	1.07	1.83	0.8	1.07
time (sec)	N/A	0.009	0.002	0.041	0.993	1.216	0.061	1.18

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	47	22	28
normalized size	1	1.	1.	0.88	1.12	1.88	0.88	1.12
time (sec)	N/A	0.005	0.	0.039	0.991	1.283	0.062	1.136

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	32	49	20	32
normalized size	1	1.	1.	0.96	1.39	2.13	0.87	1.39
time (sec)	N/A	0.007	0.001	0.043	0.981	1.448	0.247	1.156

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	50	19	30
normalized size	1	1.	1.	0.96	1.25	2.08	0.79	1.25
time (sec)	N/A	0.008	0.001	0.044	0.986	1.415	0.249	1.159

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	32	59	24	43
normalized size	1	1.	1.	0.89	1.19	2.19	0.89	1.59
time (sec)	N/A	0.009	0.001	0.056	0.985	1.458	0.274	1.136

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	30	53	20	30
normalized size	1	1.	1.	0.96	1.3	2.3	0.87	1.3
time (sec)	N/A	0.009	0.001	0.048	0.986	1.372	0.286	1.153

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	35	62	22	46
normalized size	1	1.	1.	0.96	1.46	2.58	0.92	1.92
time (sec)	N/A	0.009	0.001	0.047	0.971	1.444	0.32	1.177

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	35	61	27	35
normalized size	1	1.	1.	0.89	1.25	2.18	0.96	1.25
time (sec)	N/A	0.01	0.001	0.047	1.355	1.382	0.324	1.127

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	54	26	32
normalized size	1	1.	1.	0.83	1.07	1.8	0.87	1.07
time (sec)	N/A	0.009	0.001	0.048	0.994	1.399	0.339	1.145

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	63	27	35
normalized size	1	1.	1.	0.83	1.17	2.1	0.9	1.17
time (sec)	N/A	0.009	0.001	0.047	1.001	1.415	0.348	1.134

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	62	112	53	62
normalized size	1	1.	1.	0.84	1.11	2.	0.95	1.11
time (sec)	N/A	0.028	0.003	0.04	0.99	1.308	0.074	1.12

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	56	47	62	109	49	62
normalized size	1	1.	1.06	0.89	1.17	2.06	0.92	1.17
time (sec)	N/A	0.07	0.002	0.042	0.986	1.249	0.074	1.158

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	62	109	53	62
normalized size	1	1.	1.	0.84	1.11	1.95	0.95	1.11
time (sec)	N/A	0.027	0.003	0.043	0.993	1.502	0.071	1.127

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	56	47	62	108	53	62
normalized size	1	1.	1.65	1.38	1.82	3.18	1.56	1.82
time (sec)	N/A	0.04	0.002	0.041	0.994	1.465	0.075	1.137

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	62	107	53	62
normalized size	1	1.	1.	0.84	1.11	1.91	0.95	1.11
time (sec)	N/A	0.029	0.002	0.04	0.989	1.516	0.071	1.128

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	45	59	96	44	59
normalized size	1	1.	1.	2.81	3.69	6.	2.75	3.69
time (sec)	N/A	0.005	0.002	0.039	0.983	1.466	0.072	1.143

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	74	96	49	58
normalized size	1	1.	1.	0.86	1.45	1.88	0.96	1.14
time (sec)	N/A	0.024	0.001	0.04	0.977	1.489	0.068	1.162

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	63	100	49	63
normalized size	1	1.	1.	0.9	1.26	2.	0.98	1.26
time (sec)	N/A	0.034	0.004	0.041	0.977	1.675	0.275	1.111

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	59	104	44	59
normalized size	1	1.	1.	0.94	1.23	2.17	0.92	1.23
time (sec)	N/A	0.026	0.008	0.046	0.958	1.648	0.274	1.113

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	62	108	46	76
normalized size	1	1.	1.	0.94	1.29	2.25	0.96	1.58
time (sec)	N/A	0.037	0.005	0.046	0.996	1.76	0.312	1.136

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	61	104	48	61
normalized size	1	1.	1.	0.9	1.22	2.08	0.96	1.22
time (sec)	N/A	0.026	0.006	0.046	1.013	1.702	0.316	1.156

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	65	104	48	80
normalized size	1	1.	1.	0.94	1.33	2.12	0.98	1.63
time (sec)	N/A	0.036	0.005	0.046	0.997	1.702	0.36	1.184

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	63	104	48	63
normalized size	1	1.	1.	0.9	1.26	2.08	0.96	1.26
time (sec)	N/A	0.026	0.008	0.046	0.994	1.673	0.368	1.119

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	65	108	48	77
normalized size	1	1.	1.	0.94	1.33	2.2	0.98	1.57
time (sec)	N/A	0.033	0.005	0.046	0.981	1.656	0.419	1.108

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	62	107	46	62
normalized size	1	1.	1.	0.94	1.32	2.28	0.98	1.32
time (sec)	N/A	0.025	0.005	0.049	0.994	1.598	0.423	1.1

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	68	115	48	78
normalized size	1	1.	1.	0.9	1.36	2.3	0.96	1.56
time (sec)	N/A	0.033	0.005	0.046	0.993	1.705	0.489	1.117

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	47	65	115	51	65
normalized size	1	1.	1.	0.87	1.2	2.13	0.94	1.2
time (sec)	N/A	0.026	0.008	0.047	0.985	1.687	0.489	1.126

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	52	47	62	103	49	62
normalized size	1	1.	2.74	2.47	3.26	5.42	2.58	3.26
time (sec)	N/A	0.006	0.004	0.048	0.994	1.661	0.507	1.209

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	65	124	51	65
normalized size	1	1.	1.	0.84	1.16	2.21	0.91	1.16
time (sec)	N/A	0.026	0.007	0.049	0.994	1.712	0.528	1.158

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	47	65	108	51	65
normalized size	1	1.	1.4	1.18	1.62	2.7	1.27	1.62
time (sec)	N/A	0.025	0.004	0.048	0.98	1.697	0.547	1.141

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	65	128	51	65
normalized size	1	1.	1.	0.84	1.16	2.29	0.91	1.16
time (sec)	N/A	0.027	0.008	0.046	0.978	1.618	0.559	1.201

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	65	113	51	65
normalized size	1	1.	1.	0.84	1.16	2.02	0.91	1.16
time (sec)	N/A	0.036	0.004	0.047	0.993	1.671	0.592	1.224

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	65	131	51	65
normalized size	1	1.	1.	0.84	1.16	2.34	0.91	1.16
time (sec)	N/A	0.025	0.007	0.047	0.985	1.876	0.576	1.29

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	92	170	80	92
normalized size	1	1.	1.	0.84	1.12	2.07	0.98	1.12
time (sec)	N/A	0.046	0.003	0.042	0.98	1.722	0.082	1.177

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	82	69	92	166	78	92
normalized size	1	1.	1.14	0.96	1.28	2.31	1.08	1.28
time (sec)	N/A	0.117	0.002	0.043	0.985	1.72	0.081	1.152

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	90	162	76	90
normalized size	1	1.	1.	0.86	1.14	2.05	0.96	1.14
time (sec)	N/A	0.038	0.002	0.043	1.006	1.456	0.079	1.145

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	82	69	92	163	80	92
normalized size	1	1.	1.55	1.3	1.74	3.08	1.51	1.74
time (sec)	N/A	0.085	0.003	0.042	0.981	1.513	0.082	1.186

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	92	165	80	92
normalized size	1	1.	1.	0.84	1.12	2.01	0.98	1.12
time (sec)	N/A	0.04	0.003	0.043	0.985	1.49	0.076	1.158

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	77	68	90	153	75	90
normalized size	1	1.	2.26	2.	2.65	4.5	2.21	2.65
time (sec)	N/A	0.046	0.003	0.042	0.984	1.508	0.077	1.16

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	92	165	80	92
normalized size	1	1.	1.	0.84	1.12	2.01	0.98	1.12
time (sec)	N/A	0.038	0.003	0.041	0.983	1.432	0.078	1.173

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	69	92	158	78	92
normalized size	1	1.	1.	4.31	5.75	9.88	4.88	5.75
time (sec)	N/A	0.005	0.002	0.04	0.994	1.465	0.077	1.177

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	66	135	146	73	88
normalized size	1	1.	1.	0.9	1.85	2.	1.	1.21
time (sec)	N/A	0.034	0.001	0.04	0.972	1.472	0.075	1.155

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	67	93	157	76	93
normalized size	1	1.	1.	0.88	1.22	2.07	1.	1.22
time (sec)	N/A	0.055	0.004	0.043	1.015	1.753	0.303	1.129

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	89	165	70	89
normalized size	1	1.	1.	0.93	1.24	2.29	0.97	1.24
time (sec)	N/A	0.039	0.008	0.045	0.992	1.717	0.305	1.141

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	93	167	76	107
normalized size	1	1.	1.	0.88	1.21	2.17	0.99	1.39
time (sec)	N/A	0.056	0.008	0.047	0.999	1.603	0.342	1.193

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	67	90	159	73	90
normalized size	1	1.	1.	0.91	1.22	2.15	0.99	1.22
time (sec)	N/A	0.036	0.009	0.046	1.017	1.626	0.342	1.126

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	93	158	71	108
normalized size	1	1.	1.	0.93	1.29	2.19	0.99	1.5
time (sec)	N/A	0.052	0.005	0.049	1.018	1.716	0.396	1.146

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	90	157	71	90
normalized size	1	1.	1.	0.93	1.25	2.18	0.99	1.25
time (sec)	N/A	0.041	0.006	0.046	0.988	1.592	0.409	1.179

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	95	154	75	109
normalized size	1	1.	1.	0.86	1.2	1.95	0.95	1.38
time (sec)	N/A	0.051	0.005	0.05	0.989	1.793	0.479	1.14

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	93	157	71	93
normalized size	1	1.	1.	0.93	1.29	2.18	0.99	1.29
time (sec)	N/A	0.037	0.009	0.049	0.972	1.702	0.47	1.156

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	68	95	158	71	109
normalized size	1	1.	1.	0.93	1.3	2.16	0.97	1.49
time (sec)	N/A	0.052	0.008	0.049	0.984	1.698	0.528	1.165

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	67	93	159	71	93
normalized size	1	1.	1.	0.91	1.26	2.15	0.96	1.26
time (sec)	N/A	0.039	0.009	0.048	0.973	1.648	0.551	1.151

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	97	169	73	109
normalized size	1	1.	1.	0.88	1.26	2.19	0.95	1.42
time (sec)	N/A	0.051	0.005	0.049	1.002	1.632	0.621	1.117

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	66	92	169	70	92
normalized size	1	1.	1.	0.93	1.3	2.38	0.99	1.3
time (sec)	N/A	0.037	0.006	0.049	0.983	1.731	0.637	1.138

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	67	97	174	71	108
normalized size	1	1.	1.	0.88	1.28	2.29	0.93	1.42
time (sec)	N/A	0.049	0.005	0.049	0.983	1.593	0.718	1.157

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	69	95	178	75	95
normalized size	1	1.	1.	0.91	1.25	2.34	0.99	1.25
time (sec)	N/A	0.04	0.009	0.048	1.011	1.615	0.736	1.136

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	82	69	92	151	73	92
normalized size	1	1.	4.32	3.63	4.84	7.95	3.84	4.84
time (sec)	N/A	0.007	0.007	0.048	0.997	1.686	0.758	1.175

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	95	190	75	95
normalized size	1	1.	1.	0.84	1.16	2.32	0.91	1.16
time (sec)	N/A	0.038	0.01	0.05	1.011	1.663	0.751	1.187

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	78	69	95	163	75	95
normalized size	1	1.	1.95	1.72	2.38	4.08	1.88	2.38
time (sec)	N/A	0.025	0.005	0.053	0.976	1.726	0.817	1.127

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	95	198	75	95
normalized size	1	1.	1.	0.84	1.16	2.41	0.91	1.16
time (sec)	N/A	0.038	0.007	0.049	1.	1.588	0.793	1.119

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	82	69	95	166	75	95
normalized size	1	1.	1.32	1.11	1.53	2.68	1.21	1.53
time (sec)	N/A	0.039	0.004	0.049	0.977	1.653	0.862	1.12

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	69	95	203	75	95
normalized size	1	1.	1.	0.86	1.19	2.54	0.94	1.19
time (sec)	N/A	0.038	0.01	0.047	1.	1.634	0.862	1.145

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	69	95	174	75	95
normalized size	1	1.	0.98	0.82	1.13	2.07	0.89	1.13
time (sec)	N/A	0.056	0.008	0.048	1.009	1.692	0.902	1.14

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	95	208	75	95
normalized size	1	1.	1.	0.84	1.16	2.54	0.91	1.16
time (sec)	N/A	0.04	0.01	0.047	1.236	1.662	0.896	1.121

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	74	104	198	80	124
normalized size	1	1.	0.87	0.89	1.25	2.39	0.96	1.49
time (sec)	N/A	0.082	0.02	0.048	1.187	1.65	0.441	1.206

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	60	63	88	166	66	108
normalized size	1	1.	0.86	0.9	1.26	2.37	0.94	1.54
time (sec)	N/A	0.063	0.022	0.048	0.975	1.695	0.431	1.139

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	52	73	143	53	90
normalized size	1	1.	0.86	0.91	1.28	2.51	0.93	1.58
time (sec)	N/A	0.052	0.016	0.048	0.961	1.723	0.404	1.129

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	58	113	39	66
normalized size	1	1.	0.86	0.93	1.32	2.57	0.89	1.5
time (sec)	N/A	0.037	0.016	0.047	1.01	1.659	0.379	1.137

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	43	76	29	41
normalized size	1	1.	0.82	0.91	1.3	2.3	0.88	1.24
time (sec)	N/A	0.028	0.009	0.046	1.011	1.66	0.34	1.127

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	20	30	15	19
normalized size	1	1.	1.	0.94	1.25	1.88	0.94	1.19
time (sec)	N/A	0.005	0.003	0.046	1.041	1.576	0.294	1.152

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	50	108	34	63
normalized size	1	1.	0.87	0.92	1.32	2.84	0.89	1.66
time (sec)	N/A	0.038	0.013	0.056	0.99	1.688	0.441	1.147

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	70	157	49	69
normalized size	1	1.	0.84	0.94	1.43	3.2	1.	1.41
time (sec)	N/A	0.049	0.036	0.055	0.991	1.723	0.534	1.167

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	57	61	95	184	68	116
normalized size	1	1.	0.86	0.92	1.44	2.79	1.03	1.76
time (sec)	N/A	0.054	0.05	0.053	0.984	1.774	0.673	1.12

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	82	78	0	459	134	113
normalized size	1	1.	0.89	0.85	0.	4.99	1.46	1.23
time (sec)	N/A	0.056	0.053	0.051	0.	1.659	0.465	1.141

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	68	0	409	124	99
normalized size	1	1.	0.9	0.86	0.	5.18	1.57	1.25
time (sec)	N/A	0.045	0.046	0.051	0.	1.762	0.461	1.185

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	60	57	0	348	107	82
normalized size	1	1.	0.91	0.86	0.	5.27	1.62	1.24
time (sec)	N/A	0.038	0.041	0.051	0.	1.712	0.44	1.132

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	43	0	285	83	57
normalized size	1	1.	0.93	0.78	0.	5.18	1.51	1.04
time (sec)	N/A	0.027	0.032	0.05	0.	1.696	0.407	1.177

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	263	78	47
normalized size	1	1.	1.	0.8	0.	5.84	1.73	1.04
time (sec)	N/A	0.017	0.02	0.048	0.	1.685	0.35	1.183

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	261	78	47
normalized size	1	1.	1.	0.8	0.	5.8	1.73	1.04
time (sec)	N/A	0.017	0.024	0.047	0.	1.755	0.359	1.146

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	0	288	90	63
normalized size	1	1.	0.95	0.81	0.	5.05	1.58	1.11
time (sec)	N/A	0.027	0.036	0.053	0.	1.76	0.453	1.17

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	0	359	114	80
normalized size	1	1.	0.99	0.87	0.	5.28	1.68	1.18
time (sec)	N/A	0.036	0.038	0.054	0.	1.817	0.551	1.139

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	70	0	423	126	95
normalized size	1	1.	0.99	0.86	0.	5.22	1.56	1.17
time (sec)	N/A	0.046	0.044	0.055	0.	1.718	0.699	1.128

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	78	86	134	285	100	123
normalized size	1	1.	0.86	0.95	1.47	3.13	1.1	1.35
time (sec)	N/A	0.093	0.031	0.052	1.006	1.637	0.784	1.135

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	59	74	119	255	88	99
normalized size	1	1.	0.77	0.96	1.55	3.31	1.14	1.29
time (sec)	N/A	0.073	0.048	0.051	0.998	1.725	0.742	1.113

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	50	64	104	213	76	72
normalized size	1	1.	0.7	0.9	1.46	3.	1.07	1.01
time (sec)	N/A	0.064	0.018	0.049	1.005	1.723	0.654	1.164

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	35	48	78	116	60	45
normalized size	1	1.	1.84	2.53	4.11	6.11	3.16	2.37
time (sec)	N/A	0.007	0.013	0.049	0.979	1.582	0.582	1.136

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	63	96	48	30
normalized size	1	1.	0.71	0.91	1.85	2.82	1.41	0.88
time (sec)	N/A	0.029	0.007	0.047	0.994	1.643	0.556	1.211

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	50	73	39	19
normalized size	1	1.	1.	0.94	3.12	4.56	2.44	1.19
time (sec)	N/A	0.005	0.002	0.046	0.977	1.763	0.541	1.115

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	54	63	111	288	80	95
normalized size	1	1.	0.77	0.9	1.59	4.11	1.14	1.36
time (sec)	N/A	0.076	0.036	0.054	0.997	1.92	0.811	1.143

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	70	77	134	339	100	126
normalized size	1	1.	0.83	0.92	1.6	4.04	1.19	1.5
time (sec)	N/A	0.086	0.064	0.056	1.012	1.782	1.273	1.133

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	85	96	154	375	116	146
normalized size	1	1.	0.84	0.95	1.52	3.71	1.15	1.45
time (sec)	N/A	0.098	0.054	0.055	1.025	1.813	2.252	1.152

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	99	108	0	706	172	130
normalized size	1	1.	0.85	0.92	0.	6.03	1.47	1.11
time (sec)	N/A	0.072	0.057	0.053	0.	1.738	0.854	1.12

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	89	97	0	633	155	113
normalized size	1	1.	0.86	0.93	0.	6.09	1.49	1.09
time (sec)	N/A	0.059	0.046	0.053	0.	1.754	0.812	1.138

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	77	83	0	571	131	88
normalized size	1	1.	0.83	0.89	0.	6.14	1.41	0.95
time (sec)	N/A	0.047	0.044	0.054	0.	1.735	0.744	1.111

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	66	58	0	544	133	76
normalized size	1	1.	0.8	0.7	0.	6.55	1.6	0.92
time (sec)	N/A	0.04	0.037	0.049	0.	1.664	0.658	1.134

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	69	58	0	537	143	84
normalized size	1	1.	0.82	0.69	0.	6.39	1.7	1.
time (sec)	N/A	0.041	0.045	0.049	0.	1.804	0.618	1.159

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	69	58	0	537	139	84
normalized size	1	1.	0.81	0.68	0.	6.32	1.64	0.99
time (sec)	N/A	0.043	0.037	0.049	0.	1.73	0.611	1.162

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	66	0	541	129	76
normalized size	1	1.	0.84	0.84	0.	6.85	1.63	0.96
time (sec)	N/A	0.037	0.034	0.045	0.	1.79	0.622	1.151

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	79	86	0	574	138	92
normalized size	1	1.	0.83	0.91	0.	6.04	1.45	0.97
time (sec)	N/A	0.056	0.043	0.055	0.	1.863	0.976	1.117

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	91	99	0	644	162	111
normalized size	1	1.	0.86	0.93	0.	6.08	1.53	1.05
time (sec)	N/A	0.066	0.047	0.056	0.	1.766	1.627	1.156

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	101	110	0	725	173	126
normalized size	1	1.	0.85	0.92	0.	6.09	1.45	1.06
time (sec)	N/A	0.079	0.051	0.056	0.	1.774	3.055	1.154

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	114	120	193	450	150	153
normalized size	1	1.	0.86	0.9	1.45	3.38	1.13	1.15
time (sec)	N/A	0.143	0.024	0.055	1.171	1.636	1.469	1.157

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	101	109	178	412	136	128
normalized size	1	1.	0.86	0.92	1.51	3.49	1.15	1.08
time (sec)	N/A	0.114	0.027	0.056	1.239	1.737	1.433	1.139

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	72	98	163	367	124	101
normalized size	1	1.	0.66	0.9	1.5	3.37	1.14	0.93
time (sec)	N/A	0.102	0.023	0.056	1.206	1.691	1.267	1.136

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	57	81	138	212	107	74
normalized size	1	1.	3.	4.26	7.26	11.16	5.63	3.89
time (sec)	N/A	0.007	0.016	0.049	1.253	1.679	1.187	1.162

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	46	65	123	189	95	59
normalized size	1	1.	1.18	1.67	3.15	4.85	2.44	1.51
time (sec)	N/A	0.026	0.014	0.048	1.346	1.696	1.103	1.13

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	48	108	166	83	45
normalized size	1	1.	0.66	0.91	2.04	3.13	1.57	0.85
time (sec)	N/A	0.045	0.013	0.049	1.396	1.576	1.053	1.165

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	93	143	71	30
normalized size	1	1.	0.71	0.91	2.74	4.21	2.09	0.88
time (sec)	N/A	0.031	0.008	0.049	1.422	1.926	1.012	1.147

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	80	122	63	19
normalized size	1	1.	1.	0.94	5.	7.62	3.94	1.19
time (sec)	N/A	0.005	0.003	0.047	1.081	1.681	0.962	1.151

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	76	91	170	489	128	124
normalized size	1	1.	0.75	0.89	1.67	4.79	1.25	1.22
time (sec)	N/A	0.105	0.049	0.056	1.351	1.505	2.732	1.148

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	92	107	193	545	148	155
normalized size	1	1.	0.79	0.92	1.66	4.7	1.28	1.34
time (sec)	N/A	0.126	0.081	0.057	1.027	1.471	6.155	1.163

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	107	129	213	586	165	176
normalized size	1	1.	0.76	0.92	1.52	4.19	1.18	1.26
time (sec)	N/A	0.145	0.059	0.058	1.05	1.545	13.451	1.16

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	122	148	0	1049	218	158
normalized size	1	1.	0.79	0.95	0.	6.77	1.41	1.02
time (sec)	N/A	0.106	0.065	0.056	0.	1.552	1.531	1.139

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	111	137	0	990	202	143
normalized size	1	1.	0.78	0.96	0.	6.97	1.42	1.01
time (sec)	N/A	0.093	0.059	0.055	0.	1.538	1.467	1.166

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	100	123	0	903	178	117
normalized size	1	1.	0.76	0.94	0.	6.89	1.36	0.89
time (sec)	N/A	0.078	0.051	0.056	0.	1.509	1.387	1.147

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	88	80	0	861	180	105
normalized size	1	1.	0.73	0.66	0.	7.12	1.49	0.87
time (sec)	N/A	0.069	0.051	0.056	0.	1.513	1.278	1.174

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	91	80	0	855	194	113
normalized size	1	1.	0.75	0.66	0.	7.01	1.59	0.93
time (sec)	N/A	0.072	0.058	0.051	0.	1.512	1.212	1.135

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	91	78	0	838	196	113
normalized size	1	1.	0.74	0.63	0.	6.81	1.59	0.92
time (sec)	N/A	0.072	0.059	0.052	0.	1.495	1.15	1.158

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	91	78	0	838	196	113
normalized size	1	1.	0.73	0.63	0.	6.76	1.58	0.91
time (sec)	N/A	0.073	0.047	0.052	0.	1.501	1.111	1.178

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	91	80	0	855	190	113
normalized size	1	1.	0.73	0.64	0.	6.84	1.52	0.9
time (sec)	N/A	0.075	0.051	0.051	0.	1.474	1.105	1.16

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	89	96	0	859	177	105
normalized size	1	1.	0.79	0.85	0.	7.6	1.57	0.93
time (sec)	N/A	0.066	0.044	0.046	0.	1.511	1.12	1.149

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	101	126	0	906	185	122
normalized size	1	1.	0.76	0.95	0.	6.81	1.39	0.92
time (sec)	N/A	0.089	0.055	0.057	0.	1.535	4.079	1.136

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	113	139	0	1006	209	140
normalized size	1	1.	0.78	0.97	0.	6.99	1.45	0.97
time (sec)	N/A	0.103	0.059	0.062	0.	1.513	9.096	1.141

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	123	150	0	1073	221	155
normalized size	1	1.	0.78	0.96	0.	6.83	1.41	0.99
time (sec)	N/A	0.122	0.063	0.061	0.	1.539	19.37	1.12

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	20	55	12	20
normalized size	1	1.	0.84	0.84	1.05	2.89	0.63	1.05
time (sec)	N/A	0.003	0.005	0.044	1.488	1.482	0.098	1.125

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	22	8	12
normalized size	1	1.	1.	0.91	1.09	2.	0.73	1.09
time (sec)	N/A	0.002	0.001	0.043	0.992	1.456	0.08	1.145

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	55	12	20
normalized size	1	1.	1.	0.84	1.05	2.89	0.63	1.05
time (sec)	N/A	0.005	0.007	0.047	1.5	1.432	0.096	1.135

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	24	59	15	24
normalized size	1	1.	0.82	0.86	1.09	2.68	0.68	1.09
time (sec)	N/A	0.011	0.005	0.047	0.966	1.434	0.084	1.14

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	12	22	8	12
normalized size	1	1.	0.85	0.77	0.92	1.69	0.62	0.92
time (sec)	N/A	0.002	0.002	0.045	0.985	1.687	0.081	1.129

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	19	24	59	14	26
normalized size	1	1.	0.83	0.79	1.	2.46	0.58	1.08
time (sec)	N/A	0.015	0.006	0.049	0.983	1.663	0.089	1.158

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	0	31	12	39
normalized size	1	1.	0.49	0.46	0.	0.39	0.15	0.49
time (sec)	N/A	0.064	0.014	0.172	0.	1.688	0.093	1.144

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	39	36	0	31	12	31
normalized size	1	1.	0.58	0.54	0.	0.46	0.18	0.46
time (sec)	N/A	0.052	0.008	0.043	0.	1.587	0.092	1.127

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	31	12	30
normalized size	1	1.	1.06	0.97	0.	0.86	0.33	0.83
time (sec)	N/A	0.026	0.007	0.043	0.	1.458	0.091	1.143

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	37	34	0	30	10	41
normalized size	1	1.	0.49	0.45	0.	0.4	0.13	0.55
time (sec)	N/A	0.022	0.012	0.212	0.	1.488	0.107	1.126

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	39	38	0	41	10	61
normalized size	1	1.	0.52	0.51	0.	0.55	0.13	0.81
time (sec)	N/A	0.022	0.011	0.173	0.	1.508	0.274	1.153

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	34	0	32	14	41
normalized size	1	1.	0.95	0.87	0.	0.82	0.36	1.05
time (sec)	N/A	0.038	0.008	0.042	0.	1.482	0.291	1.16

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	39	36	0	36	15	42
normalized size	1	1.	0.54	0.5	0.	0.5	0.21	0.58
time (sec)	N/A	0.016	0.009	0.043	0.	1.429	0.307	1.149

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	0	36	15	42
normalized size	1	1.	0.49	0.46	0.	0.46	0.19	0.53
time (sec)	N/A	0.059	0.008	0.042	0.	1.438	0.327	1.163

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	0	38	15	42
normalized size	1	1.	0.49	0.46	0.	0.48	0.19	0.53
time (sec)	N/A	0.058	0.008	0.044	0.	1.431	0.345	1.151

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	18	31	12	39
normalized size	1	1.	0.49	0.46	0.23	0.39	0.15	0.49
time (sec)	N/A	0.023	0.007	0.042	0.993	1.487	0.092	1.136

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	18	31	12	39
normalized size	1	1.	0.49	0.46	0.23	0.39	0.15	0.49
time (sec)	N/A	0.023	0.007	0.042	0.997	1.485	0.091	1.109

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	36	33	14	23	8	27
normalized size	1	1.	0.49	0.45	0.19	0.31	0.11	0.36
time (sec)	N/A	0.014	0.007	0.042	1.002	1.494	0.088	1.1

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	35	34	18	20	5	35
normalized size	1	1.	0.49	0.47	0.25	0.28	0.07	0.49
time (sec)	N/A	0.02	0.007	0.042	1.01	1.5	0.258	1.166

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	37	34	18	32	14	41
normalized size	1	1.	0.48	0.44	0.23	0.42	0.18	0.53
time (sec)	N/A	0.023	0.007	0.041	1.011	1.428	0.286	1.134

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	36	15	42
normalized size	1	1.	0.49	0.46	0.25	0.46	0.19	0.53
time (sec)	N/A	0.021	0.007	0.042	1.015	1.472	0.307	1.16

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	36	15	42
normalized size	1	1.	0.49	0.46	0.25	0.46	0.19	0.53
time (sec)	N/A	0.022	0.008	0.042	0.994	1.418	0.316	1.099

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	36	15	42
normalized size	1	1.	0.49	0.46	0.25	0.46	0.19	0.53
time (sec)	N/A	0.022	0.008	0.042	1.014	1.48	0.343	1.113

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	0	89	0	90
normalized size	1	1.	0.37	0.35	0.	0.53	0.	0.54
time (sec)	N/A	0.114	0.019	0.169	0.	1.518	0.	1.109

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	0	86	0	90
normalized size	1	1.	0.37	0.35	0.	0.51	0.	0.54
time (sec)	N/A	0.114	0.015	0.171	0.	1.482	0.	1.14

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	119	61	58	0	85	0	90
normalized size	1	1.12	0.58	0.55	0.	0.8	0.	0.85
time (sec)	N/A	0.083	0.015	0.175	0.	1.492	0.	1.124

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	61	58	0	82	0	61
normalized size	1	1.	0.91	0.87	0.	1.22	0.	0.91
time (sec)	N/A	0.051	0.015	0.173	0.	1.479	0.	1.121

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	57	0	80	0	59
normalized size	1	1.	0.75	1.58	0.	2.22	0.	1.64
time (sec)	N/A	0.026	0.012	0.171	0.	1.472	0.	1.1

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	60	57	0	78	0	92
normalized size	1	1.	0.37	0.35	0.	0.48	0.	0.56
time (sec)	N/A	0.049	0.02	0.22	0.	1.541	0.	1.119

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	62	59	0	85	0	117
normalized size	1	1.	0.38	0.36	0.	0.52	0.	0.71
time (sec)	N/A	0.047	0.021	0.223	0.	1.488	0.	1.11

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	61	60	0	85	0	117
normalized size	1	1.	0.37	0.37	0.	0.52	0.	0.71
time (sec)	N/A	0.049	0.015	0.23	0.	1.421	0.	1.112

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	63	60	0	90	0	117
normalized size	1	1.	0.39	0.37	0.	0.55	0.	0.72
time (sec)	N/A	0.046	0.021	0.224	0.	1.487	0.	1.114

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	59	56	0	76	0	92
normalized size	1	1.	1.44	1.37	0.	1.85	0.	2.24
time (sec)	N/A	0.039	0.015	0.176	0.	1.428	0.	1.138

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	61	58	0	85	0	93
normalized size	1	1.	0.85	0.81	0.	1.18	0.	1.29
time (sec)	N/A	0.017	0.013	0.164	0.	1.451	0.	1.136

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	0	88	0	93
normalized size	1	1.	0.37	0.35	0.	0.53	0.	0.56
time (sec)	N/A	0.105	0.014	0.166	0.	1.469	0.	1.117

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	0	88	0	93
normalized size	1	1.	0.37	0.35	0.	0.53	0.	0.56
time (sec)	N/A	0.106	0.017	0.176	0.	1.445	0.	1.114

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	0	90	0	93
normalized size	1	1.	0.37	0.35	0.	0.54	0.	0.56
time (sec)	N/A	0.105	0.014	0.166	0.	1.477	0.	1.114

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	88	0	90
normalized size	1	1.	0.37	0.35	0.28	0.53	0.	0.54
time (sec)	N/A	0.042	0.015	0.164	1.008	1.438	0.	1.126

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	85	0	90
normalized size	1	1.	0.37	0.35	0.28	0.51	0.	0.54
time (sec)	N/A	0.041	0.015	0.167	0.998	1.478	0.	1.094

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	82	0	90
normalized size	1	1.	0.37	0.35	0.28	0.49	0.	0.54
time (sec)	N/A	0.043	0.015	0.164	1.007	1.508	0.	1.126

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	80	0	90
normalized size	1	1.	0.37	0.35	0.28	0.48	0.	0.54
time (sec)	N/A	0.041	0.012	0.166	1.042	1.468	0.	1.138

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	59	56	42	66	0	85
normalized size	1	1.	0.37	0.35	0.26	0.42	0.	0.53
time (sec)	N/A	0.033	0.012	0.042	1.003	1.474	0.	1.096

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	60	58	49	73	0	86
normalized size	1	1.	0.38	0.37	0.31	0.46	0.	0.54
time (sec)	N/A	0.04	0.015	0.166	1.006	1.422	0.	1.116

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	59	56	49	72	0	90
normalized size	1	1.	0.37	0.35	0.3	0.45	0.	0.56
time (sec)	N/A	0.04	0.013	0.166	1.017	1.455	0.	1.136

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	59	56	50	76	0	89
normalized size	1	1.	0.37	0.35	0.32	0.48	0.	0.56
time (sec)	N/A	0.04	0.015	0.165	1.095	1.474	0.	1.114

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	50	84	0	93
normalized size	1	1.	0.37	0.36	0.31	0.52	0.	0.57
time (sec)	N/A	0.043	0.013	0.166	1.204	1.452	0.	1.112

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	90	0	93
normalized size	1	1.	0.37	0.35	0.3	0.54	0.	0.56
time (sec)	N/A	0.039	0.013	0.162	1.02	1.489	0.	1.119

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	95	0	93
normalized size	1	1.	0.37	0.35	0.3	0.57	0.	0.56
time (sec)	N/A	0.044	0.017	0.16	1.006	1.426	0.	1.133

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	96	0	93
normalized size	1	1.	0.37	0.35	0.3	0.57	0.	0.56
time (sec)	N/A	0.04	0.013	0.164	1.042	1.465	0.	1.099

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	97	0	93
normalized size	1	1.	0.37	0.35	0.3	0.58	0.	0.56
time (sec)	N/A	0.041	0.014	0.163	0.998	1.498	0.	1.149

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	142	0	142
normalized size	1	1.	0.33	0.31	0.	0.56	0.	0.56
time (sec)	N/A	0.162	0.025	0.164	0.	1.425	0.	1.117

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	140	0	142
normalized size	1	1.	0.33	0.31	0.	0.55	0.	0.56
time (sec)	N/A	0.159	0.021	0.163	0.	1.462	0.	1.127

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	83	80	0	142	0	142
normalized size	1	1.	0.41	0.4	0.	0.71	0.	0.71
time (sec)	N/A	0.132	0.02	0.162	0.	1.513	0.	1.117

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	83	80	0	138	0	142
normalized size	1	1.	0.52	0.5	0.	0.86	0.	0.89
time (sec)	N/A	0.118	0.022	0.162	0.	1.475	0.	1.134

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	83	80	0	131	0	140
normalized size	1	1.	0.7	0.67	0.	1.1	0.	1.18
time (sec)	N/A	0.097	0.022	0.164	0.	1.442	0.	1.131

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	83	80	0	130	0	90
normalized size	1	1.	1.24	1.19	0.	1.94	0.	1.34
time (sec)	N/A	0.052	0.022	0.165	0.	1.439	0.	1.173

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	79	0	132	0	89
normalized size	1	1.	0.75	2.19	0.	3.67	0.	2.47
time (sec)	N/A	0.026	0.014	0.166	0.	1.46	0.	1.174

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	82	79	0	130	0	143
normalized size	1	1.	0.33	0.31	0.	0.52	0.	0.57
time (sec)	N/A	0.071	0.025	0.211	0.	1.552	0.	1.125

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	0	142	0	169
normalized size	1	1.	0.34	0.33	0.	0.57	0.	0.68
time (sec)	N/A	0.071	0.027	0.214	0.	1.513	0.	1.129

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	0	139	0	171
normalized size	1	1.	0.34	0.33	0.	0.56	0.	0.68
time (sec)	N/A	0.07	0.026	0.213	0.	1.468	0.	1.135

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	0	139	0	173
normalized size	1	1.	0.34	0.33	0.	0.56	0.	0.69
time (sec)	N/A	0.072	0.024	0.212	0.	1.515	0.	1.123

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	0	142	0	170
normalized size	1	1.	0.34	0.33	0.	0.57	0.	0.68
time (sec)	N/A	0.071	0.02	0.208	0.	1.469	0.	1.141

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	85	82	0	149	0	169
normalized size	1	1.	0.34	0.33	0.	0.59	0.	0.67
time (sec)	N/A	0.068	0.028	0.212	0.	1.487	0.	1.153

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	81	78	0	127	0	143
normalized size	1	1.	1.98	1.9	0.	3.1	0.	3.49
time (sec)	N/A	0.039	0.018	0.159	0.	1.476	0.	1.12

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	83	80	0	134	0	144
normalized size	1	1.	1.15	1.11	0.	1.86	0.	2.
time (sec)	N/A	0.017	0.02	0.173	0.	1.479	0.	1.117

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	83	80	0	140	0	144
normalized size	1	1.	0.65	0.62	0.	1.09	0.	1.12
time (sec)	N/A	0.091	0.018	0.165	0.	1.488	0.	1.132

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	143	0	144
normalized size	1	1.	0.33	0.31	0.	0.56	0.	0.56
time (sec)	N/A	0.155	0.019	0.163	0.	1.646	0.	1.123

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	149	0	144
normalized size	1	1.	0.33	0.31	0.	0.58	0.	0.56
time (sec)	N/A	0.151	0.018	0.167	0.	1.818	0.	1.139

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	150	0	144
normalized size	1	1.	0.33	0.31	0.	0.59	0.	0.56
time (sec)	N/A	0.153	0.018	0.163	0.	1.252	0.	1.121

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	151	0	144
normalized size	1	1.	0.33	0.31	0.	0.59	0.	0.56
time (sec)	N/A	0.151	0.021	0.163	0.	1.305	0.	1.134

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	146	0	142
normalized size	1	1.	0.33	0.31	0.3	0.57	0.	0.56
time (sec)	N/A	0.062	0.023	0.163	1.008	1.27	0.	1.126

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	144	0	142
normalized size	1	1.	0.33	0.31	0.3	0.56	0.	0.56
time (sec)	N/A	0.058	0.021	0.162	0.997	1.266	0.	1.134

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	142	0	142
normalized size	1	1.	0.33	0.31	0.3	0.56	0.	0.56
time (sec)	N/A	0.061	0.022	0.161	1.004	1.25	0.	1.109

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	140	0	142
normalized size	1	1.	0.33	0.31	0.3	0.55	0.	0.56
time (sec)	N/A	0.058	0.02	0.164	1.017	1.349	0.	1.116

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	139	0	142
normalized size	1	1.	0.33	0.31	0.3	0.55	0.	0.56
time (sec)	N/A	0.059	0.021	0.164	1.014	1.328	0.	1.129

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	76	131	0	140
normalized size	1	1.	0.33	0.32	0.3	0.52	0.	0.56
time (sec)	N/A	0.06	0.022	0.163	0.994	1.238	0.	1.131

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	81	78	73	122	0	138
normalized size	1	1.	0.33	0.31	0.29	0.49	0.	0.56
time (sec)	N/A	0.051	0.016	0.044	0.998	1.369	0.	1.12

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	80	131	0	139
normalized size	1	1.	0.34	0.32	0.32	0.53	0.	0.56
time (sec)	N/A	0.058	0.02	0.171	1.006	1.189	0.	1.11

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	83	80	80	131	0	140
normalized size	1	1.	0.34	0.33	0.33	0.53	0.	0.57
time (sec)	N/A	0.06	0.023	0.163	1.007	1.332	0.	1.157

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	83	80	80	131	0	143
normalized size	1	1.	0.33	0.32	0.32	0.53	0.	0.57
time (sec)	N/A	0.058	0.019	0.163	1.016	1.345	0.	1.119

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	80	131	0	143
normalized size	1	1.	0.34	0.32	0.32	0.53	0.	0.58
time (sec)	N/A	0.058	0.017	0.168	0.998	1.393	0.	1.13

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	83	80	80	134	0	142
normalized size	1	1.	0.34	0.33	0.33	0.54	0.	0.58
time (sec)	N/A	0.058	0.018	0.171	1.008	1.265	0.	1.132

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	80	144	0	144
normalized size	1	1.	0.33	0.32	0.32	0.57	0.	0.57
time (sec)	N/A	0.058	0.017	0.17	1.113	1.2	0.	1.127

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	80	154	0	144
normalized size	1	1.	0.33	0.32	0.32	0.61	0.	0.57
time (sec)	N/A	0.06	0.018	0.166	1.026	1.21	0.	1.133

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	159	0	144
normalized size	1	1.	0.33	0.31	0.31	0.62	0.	0.56
time (sec)	N/A	0.058	0.018	0.17	1.003	1.363	0.	1.123

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	165	0	144
normalized size	1	1.	0.33	0.31	0.31	0.65	0.	0.56
time (sec)	N/A	0.059	0.018	0.163	1.012	1.235	0.	1.141

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	169	0	144
normalized size	1	1.	0.33	0.31	0.31	0.66	0.	0.56
time (sec)	N/A	0.058	0.02	0.171	0.996	1.258	0.	1.137

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	169	0	144
normalized size	1	1.	0.33	0.31	0.31	0.66	0.	0.56
time (sec)	N/A	0.058	0.018	0.168	0.986	1.293	0.	1.131

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	174	0	144
normalized size	1	1.	0.33	0.31	0.31	0.68	0.	0.56
time (sec)	N/A	0.058	0.018	0.168	1.01	1.34	0.	1.153

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	55	52	62	73	32	80
normalized size	1	1.	0.43	0.41	0.49	0.57	0.25	0.63
time (sec)	N/A	0.102	0.024	0.211	0.999	1.227	0.328	1.139

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	44	41	63	49	20	45
normalized size	1	1.	0.59	0.55	0.84	0.65	0.27	0.6
time (sec)	N/A	0.056	0.012	0.21	1.	1.291	0.318	1.126

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	35	32	23	30	10	30
normalized size	1	1.	0.8	0.73	0.52	0.68	0.23	0.68
time (sec)	N/A	0.032	0.007	0.207	0.997	1.187	0.136	1.148

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	42	39	0	49	15	45
normalized size	1	1.	0.52	0.49	0.	0.61	0.19	0.56
time (sec)	N/A	0.034	0.011	0.21	0.	1.246	0.222	1.12

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	54	51	0	80	31	70
normalized size	1	1.	0.44	0.42	0.	0.66	0.25	0.57
time (sec)	N/A	0.05	0.015	0.214	0.	1.295	0.465	1.107

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	66	63	0	217	80	86
normalized size	1	1.	0.51	0.49	0.	1.68	0.62	0.67
time (sec)	N/A	0.046	0.025	0.217	0.	1.34	0.351	1.168

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	54	48	0	165	56	57
normalized size	1	1.	0.61	0.54	0.	1.85	0.63	0.64
time (sec)	N/A	0.032	0.014	0.213	0.	1.284	0.333	1.137

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	34	0	151	53	31
normalized size	1	1.	0.83	0.64	0.	2.85	1.	0.58
time (sec)	N/A	0.015	0.012	0.167	0.	1.255	0.157	1.123

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	56	50	0	173	65	50
normalized size	1	1.	0.61	0.54	0.	1.88	0.71	0.54
time (sec)	N/A	0.033	0.014	0.171	0.	1.357	0.363	1.134

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	70	69	0	234	87	68
normalized size	1	1.	0.54	0.53	0.	1.8	0.67	0.52
time (sec)	N/A	0.043	0.023	0.225	0.	1.285	0.422	1.115

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	81	103	198	186	0	0
normalized size	1	1.	0.51	0.65	1.25	1.18	0.	0.
time (sec)	N/A	0.131	0.03	0.227	1.324	1.344	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	61	81	86	143	0	0
normalized size	1	1.	0.54	0.72	0.76	1.27	0.	0.
time (sec)	N/A	0.098	0.022	0.237	1.226	1.195	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	69	39	32	65	73	0	0
normalized size	1	1.68	0.95	0.78	1.59	1.78	0.	0.
time (sec)	N/A	0.05	0.012	0.175	1.017	1.232	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	24	51	0	0
normalized size	1	1.	0.71	0.63	0.63	1.34	0.	0.
time (sec)	N/A	0.026	0.008	0.164	1.018	1.188	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	74	107	0	196	0	0
normalized size	1	1.	0.5	0.73	0.	1.33	0.	0.
time (sec)	N/A	0.083	0.026	0.231	0.	1.268	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	97	133	0	247	0	0
normalized size	1	1.	0.51	0.7	0.	1.31	0.	0.
time (sec)	N/A	0.096	0.039	0.223	0.	1.3	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	84	97	0	404	0	0
normalized size	1	1.	0.66	0.76	0.	3.16	0.	0.
time (sec)	N/A	0.053	0.031	0.224	0.	1.312	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	81	97	0	394	0	0
normalized size	1	1.	0.63	0.75	0.	3.05	0.	0.
time (sec)	N/A	0.049	0.028	0.225	0.	1.264	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	83	97	0	401	0	0
normalized size	1	1.	0.61	0.72	0.	2.97	0.	0.
time (sec)	N/A	0.038	0.025	0.221	0.	1.311	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	93	119	0	428	0	0
normalized size	1	1.	0.55	0.7	0.	2.53	0.	0.
time (sec)	N/A	0.065	0.032	0.239	0.	1.327	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	105	139	0	504	0	0
normalized size	1	1.	0.5	0.67	0.	2.41	0.	0.
time (sec)	N/A	0.079	0.038	0.235	0.	1.234	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	103	163	161	332	0	0
normalized size	1	1.	0.43	0.68	0.68	1.39	0.	0.
time (sec)	N/A	0.189	0.038	0.238	1.024	1.317	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	83	141	134	282	0	0
normalized size	1	1.	0.42	0.72	0.68	1.44	0.	0.
time (sec)	N/A	0.162	0.029	0.23	1.03	1.267	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	61	54	197	159	0	0
normalized size	1	1.	1.49	1.32	4.8	3.88	0.	0.
time (sec)	N/A	0.04	0.017	0.173	0.995	1.306	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	50	43	85	139	0	0
normalized size	1	1.	0.68	0.58	1.15	1.88	0.	0.
time (sec)	N/A	0.016	0.017	0.17	0.975	1.251	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	39	32	65	117	0	0
normalized size	1	1.	0.57	0.46	0.94	1.7	0.	0.
time (sec)	N/A	0.054	0.014	0.167	0.964	1.229	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	24	95	0	0
normalized size	1	1.	0.71	0.63	0.63	2.5	0.	0.
time (sec)	N/A	0.025	0.011	0.173	0.967	1.239	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	96	193	0	378	0	0
normalized size	1	1.	0.43	0.87	0.	1.7	0.	0.
time (sec)	N/A	0.121	0.039	0.23	0.	1.339	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	119	219	0	440	0	0
normalized size	1	1.	0.45	0.82	0.	1.65	0.	0.
time (sec)	N/A	0.142	0.047	0.23	0.	1.389	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	105	172	0	684	0	0
normalized size	1	1.	0.5	0.82	0.	3.24	0.	0.
time (sec)	N/A	0.085	0.039	0.235	0.	1.577	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	105	172	0	674	0	0
normalized size	1	1.	0.5	0.81	0.	3.18	0.	0.
time (sec)	N/A	0.088	0.041	0.228	0.	1.522	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	105	172	0	684	0	0
normalized size	1	1.	0.49	0.81	0.	3.21	0.	0.
time (sec)	N/A	0.084	0.034	0.225	0.	1.573	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	105	169	0	691	0	0
normalized size	1	1.	0.49	0.79	0.	3.24	0.	0.
time (sec)	N/A	0.072	0.038	0.22	0.	1.555	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	115	191	0	721	0	0
normalized size	1	1.	0.46	0.76	0.	2.87	0.	0.
time (sec)	N/A	0.109	0.044	0.233	0.	1.404	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	127	211	0	815	0	0
normalized size	1	1.	0.44	0.73	0.	2.8	0.	0.
time (sec)	N/A	0.125	0.048	0.224	0.	1.382	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	64	0	0	0	0	0
normalized size	1	1.	0.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.229	0.033	0.177	0.	0.	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	48	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.011	0.169	0.	0.	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	51	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.01	0.178	0.	0.	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	618	618	64	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.412	0.033	0.204	0.	0.	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	609	609	64	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.366	0.022	0.176	0.	0.	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	649	649	61	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.444	0.014	0.188	0.	0.	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	55	96	49	65
normalized size	1	1.	0.65	0.59	1.08	1.88	0.96	1.27
time (sec)	N/A	0.014	0.014	0.046	0.955	1.243	2.587	1.112

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	55	86	49	57
normalized size	1	1.	0.65	0.59	1.08	1.69	0.96	1.12
time (sec)	N/A	0.014	0.011	0.047	0.954	1.212	1.157	1.13

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	55	73	49	61
normalized size	1	1.	0.65	0.59	1.08	1.43	0.96	1.2
time (sec)	N/A	0.013	0.009	0.047	0.975	1.295	0.438	1.112

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	55	70	48	55
normalized size	1	1.	0.67	0.61	1.12	1.43	0.98	1.12
time (sec)	N/A	0.013	0.01	0.047	0.96	1.229	0.596	1.115

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	59	78	48	69
normalized size	1	1.	0.67	0.61	1.2	1.59	0.98	1.41
time (sec)	N/A	0.015	0.011	0.049	0.973	1.17	0.623	1.107

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	58	80	48	72
normalized size	1	1.	0.67	0.61	1.18	1.63	0.98	1.47
time (sec)	N/A	0.014	0.012	0.046	0.976	1.301	0.88	1.125

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	38	30	63	80	48	65
normalized size	1	1.	0.78	0.61	1.29	1.63	0.98	1.33
time (sec)	N/A	0.013	0.013	0.046	0.969	1.244	1.809	1.134

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	99	173	90	116
normalized size	1	1.	0.6	0.57	1.09	1.9	0.99	1.27
time (sec)	N/A	0.045	0.021	0.048	1.083	1.228	5.395	1.118

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	99	158	90	103
normalized size	1	1.	0.6	0.57	1.09	1.74	0.99	1.13
time (sec)	N/A	0.044	0.018	0.049	1.007	1.321	2.778	1.115

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	99	138	90	107
normalized size	1	1.	0.6	0.57	1.09	1.52	0.99	1.18
time (sec)	N/A	0.041	0.014	0.049	0.984	1.303	1.247	1.117

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	122	132	88	99
normalized size	1	1.	0.62	0.58	1.37	1.48	0.99	1.11
time (sec)	N/A	0.041	0.015	0.049	0.979	1.24	1.702	1.126

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	103	136	88	120
normalized size	1	1.	0.62	0.58	1.16	1.53	0.99	1.35
time (sec)	N/A	0.042	0.016	0.049	0.984	1.2	1.345	1.127

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	103	136	88	124
normalized size	1	1.	0.62	0.58	1.16	1.53	0.99	1.39
time (sec)	N/A	0.042	0.016	0.049	0.991	1.229	1.697	1.141

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	60	52	111	135	87	128
normalized size	1	1.	0.69	0.6	1.28	1.55	1.	1.47
time (sec)	N/A	0.042	0.019	0.047	0.984	1.215	2.304	1.112

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	77	74	142	261	129	167
normalized size	1	1.	0.6	0.57	1.1	2.02	1.	1.29
time (sec)	N/A	0.066	0.029	0.049	0.959	1.227	12.574	1.113

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	77	74	142	239	131	149
normalized size	1	1.	0.59	0.56	1.08	1.82	1.	1.14
time (sec)	N/A	0.061	0.025	0.05	0.983	1.245	5.618	1.152

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	77	74	142	211	131	153
normalized size	1	1.	0.59	0.56	1.08	1.61	1.	1.17
time (sec)	N/A	0.061	0.019	0.05	0.967	1.258	2.854	1.125

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	77	74	209	205	129	142
normalized size	1	1.	0.6	0.57	1.62	1.59	1.	1.1
time (sec)	N/A	0.06	0.022	0.049	0.988	1.273	2.695	1.119

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	77	74	146	205	126	171
normalized size	1	1.	0.62	0.59	1.17	1.64	1.01	1.37
time (sec)	N/A	0.06	0.022	0.048	0.974	1.201	2.776	1.159

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	77	74	146	204	128	176
normalized size	1	1.	0.61	0.58	1.15	1.61	1.01	1.39
time (sec)	N/A	0.062	0.023	0.049	1.017	1.302	3.339	1.124

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	82	74	154	201	128	180
normalized size	1	1.	0.65	0.58	1.21	1.58	1.01	1.42
time (sec)	N/A	0.064	0.025	0.05	0.992	1.192	5.085	1.127

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	235	242	0	636	0	406
normalized size	1	1.	0.74	0.77	0.	2.01	0.	1.28
time (sec)	N/A	0.383	0.319	0.088	0.	1.4	0.	1.164

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	63	226	0	640	0	359
normalized size	1	1.	0.21	0.76	0.	2.15	0.	1.2
time (sec)	N/A	0.299	0.018	0.062	0.	1.406	0.	1.141

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	244	223	0	549	0	362
normalized size	1	1.	0.82	0.75	0.	1.84	0.	1.21
time (sec)	N/A	0.295	0.155	0.059	0.	1.386	0.	1.155

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	54	209	0	552	0	355
normalized size	1	1.	0.19	0.74	0.	1.96	0.	1.26
time (sec)	N/A	0.28	0.015	0.06	0.	1.376	0.	1.2

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	210	212	0	525	0	355
normalized size	1	1.	0.75	0.75	0.	1.87	0.	1.26
time (sec)	N/A	0.26	0.166	0.061	0.	1.371	0.	1.2

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	32	210	0	531	78	367
normalized size	1	1.	0.11	0.74	0.	1.88	0.28	1.3
time (sec)	N/A	0.271	0.006	0.056	0.	1.387	6.396	1.257

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	211	207	0	536	0	363
normalized size	1	1.	0.75	0.73	0.	1.89	0.	1.28
time (sec)	N/A	0.265	0.161	0.055	0.	1.389	0.	1.253

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	30	223	0	655	0	397
normalized size	1	1.	0.1	0.74	0.	2.18	0.	1.32
time (sec)	N/A	0.323	0.011	0.109	0.	1.428	0.	1.291

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	32	226	0	678	0	373
normalized size	1	1.	0.11	0.75	0.	2.26	0.	1.24
time (sec)	N/A	0.299	0.012	0.064	0.	1.44	0.	1.269

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	37	242	0	767	0	414
normalized size	1	1.	0.12	0.76	0.	2.41	0.	1.3
time (sec)	N/A	0.343	0.011	0.063	0.	1.386	0.	1.259

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	347	306	0	905	0	459
normalized size	1	1.	0.94	0.83	0.	2.46	0.	1.25
time (sec)	N/A	0.418	0.226	0.065	0.	1.409	0.	1.252

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	87	290	0	917	0	412
normalized size	1	1.	0.25	0.83	0.	2.62	0.	1.18
time (sec)	N/A	0.393	0.029	0.066	0.	1.451	0.	1.317

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	324	287	0	826	0	414
normalized size	1	1.	0.93	0.82	0.	2.36	0.	1.18
time (sec)	N/A	0.382	0.145	0.067	0.	1.409	0.	1.311

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	83	271	0	845	0	408
normalized size	1	1.	0.25	0.81	0.	2.54	0.	1.23
time (sec)	N/A	0.345	0.026	0.066	0.	1.416	0.	1.241

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	299	280	0	841	0	412
normalized size	1	1.	0.9	0.84	0.	2.53	0.	1.24
time (sec)	N/A	0.345	0.131	0.064	0.	1.42	0.	1.325

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	74	277	0	875	0	412
normalized size	1	1.	0.22	0.82	0.	2.6	0.	1.23
time (sec)	N/A	0.351	0.022	0.065	0.	1.423	0.	1.371

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	279	277	0	844	0	416
normalized size	1	1.	0.83	0.82	0.	2.51	0.	1.24
time (sec)	N/A	0.341	0.164	0.063	0.	1.464	0.	1.282

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	60	277	0	898	0	409
normalized size	1	1.	0.18	0.83	0.	2.68	0.	1.22
time (sec)	N/A	0.348	0.021	0.065	0.	1.418	0.	1.287

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	260	271	0	821	0	409
normalized size	1	1.	0.78	0.81	0.	2.45	0.	1.22
time (sec)	N/A	0.347	0.132	0.063	0.	1.365	0.	1.256

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	32	272	0	864	252	417
normalized size	1	1.	0.1	0.81	0.	2.58	0.75	1.24
time (sec)	N/A	0.352	0.007	0.061	0.	1.413	42.883	1.317

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	253	269	0	814	0	416
normalized size	1	1.	0.76	0.8	0.	2.43	0.	1.24
time (sec)	N/A	0.348	0.115	0.062	0.	1.381	0.	1.291

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	30	285	0	1008	0	441
normalized size	1	1.	0.09	0.81	0.	2.86	0.	1.25
time (sec)	N/A	0.402	0.012	0.07	0.	1.541	0.	1.222

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	32	288	0	972	0	416
normalized size	1	1.	0.09	0.82	0.	2.76	0.	1.18
time (sec)	N/A	0.388	0.013	0.069	0.	1.416	0.	1.333

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	37	304	0	1135	0	471
normalized size	1	1.	0.1	0.82	0.	3.07	0.	1.27
time (sec)	N/A	0.447	0.012	0.073	0.	1.541	0.	1.302

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	432	370	0	1229	0	510
normalized size	1	1.	1.03	0.88	0.	2.93	0.	1.21
time (sec)	N/A	0.529	0.186	0.122	0.	1.691	0.	1.275

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	109	354	0	1258	0	463
normalized size	1	1.	0.27	0.88	0.	3.13	0.	1.15
time (sec)	N/A	0.469	0.035	0.074	0.	1.737	0.	1.304

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	408	351	0	1141	0	466
normalized size	1	1.	1.01	0.87	0.	2.84	0.	1.16
time (sec)	N/A	0.486	0.297	0.072	0.	1.696	0.	1.332

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	104	335	0	1158	0	459
normalized size	1	1.	0.27	0.87	0.	3.01	0.	1.19
time (sec)	N/A	0.448	0.037	0.072	0.	1.746	0.	1.171

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	381	344	0	1127	0	463
normalized size	1	1.	0.99	0.89	0.	2.93	0.	1.2
time (sec)	N/A	0.446	0.204	0.071	0.	1.713	0.	1.175

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	96	341	0	1175	0	463
normalized size	1	1.	0.25	0.88	0.	3.03	0.	1.19
time (sec)	N/A	0.449	0.031	0.069	0.	1.452	0.	1.176

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	359	341	0	1134	0	467
normalized size	1	1.	0.93	0.88	0.	2.92	0.	1.2
time (sec)	N/A	0.446	0.247	0.067	0.	1.489	0.	1.18

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	85	339	0	1181	0	463
normalized size	1	1.	0.22	0.87	0.	3.02	0.	1.18
time (sec)	N/A	0.479	0.031	0.073	0.	1.491	0.	1.197

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	337	339	0	1150	0	467
normalized size	1	1.	0.86	0.87	0.	2.94	0.	1.19
time (sec)	N/A	0.473	0.184	0.069	0.	1.419	0.	1.174

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	61	339	0	1242	0	463
normalized size	1	1.	0.15	0.86	0.	3.15	0.	1.18
time (sec)	N/A	0.461	0.027	0.07	0.	1.488	0.	1.267

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	317	339	0	1145	0	467
normalized size	1	1.	0.8	0.86	0.	2.91	0.	1.19
time (sec)	N/A	0.449	0.176	0.066	0.	1.504	0.	1.267

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	48	341	0	1231	0	460
normalized size	1	1.	0.12	0.88	0.	3.16	0.	1.18
time (sec)	N/A	0.455	0.021	0.066	0.	1.469	0.	1.21

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	298	335	0	1100	0	460
normalized size	1	1.	0.77	0.86	0.	2.83	0.	1.18
time (sec)	N/A	0.499	0.179	0.064	0.	1.451	0.	1.174

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	32	336	0	1199	0	468
normalized size	1	1.	0.08	0.87	0.	3.1	0.	1.21
time (sec)	N/A	0.49	0.006	0.067	0.	1.472	0.	1.226

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	295	333	0	1104	0	467
normalized size	1	1.	0.76	0.86	0.	2.85	0.	1.21
time (sec)	N/A	0.496	0.163	0.069	0.	1.473	0.	1.274

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	30	349	0	1411	0	493
normalized size	1	1.	0.07	0.86	0.	3.49	0.	1.22
time (sec)	N/A	0.529	0.013	0.076	0.	1.754	0.	1.296

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	32	352	0	1320	0	481
normalized size	1	1.	0.08	0.87	0.	3.27	0.	1.19
time (sec)	N/A	0.509	0.015	0.074	0.	1.741	0.	1.325

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	37	368	0	1557	0	489
normalized size	1	1.	0.09	0.87	0.	3.69	0.	1.16
time (sec)	N/A	0.554	0.014	0.077	0.	1.719	0.	1.355

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	44	39	30	61	0	61
normalized size	1	1.	0.47	0.42	0.32	0.66	0.	0.66
time (sec)	N/A	0.03	0.021	0.044	1.016	1.262	0.	1.165

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	44	39	30	54	0	55
normalized size	1	1.	0.47	0.42	0.32	0.58	0.	0.59
time (sec)	N/A	0.029	0.013	0.046	1.006	1.325	0.	1.268

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	44	39	30	46	0	59
normalized size	1	1.	0.47	0.42	0.32	0.49	0.	0.63
time (sec)	N/A	0.029	0.013	0.043	1.033	1.257	0.	1.192

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	43	38	32	42	0	54
normalized size	1	1.	0.47	0.42	0.35	0.46	0.	0.59
time (sec)	N/A	0.028	0.013	0.042	1.042	1.31	0.	1.212

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	43	39	32	50	0	55
normalized size	1	1.	0.47	0.43	0.35	0.55	0.	0.6
time (sec)	N/A	0.028	0.014	0.044	1.015	1.212	0.	1.26

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	42	37	34	53	0	57
normalized size	1	1.	0.46	0.41	0.37	0.58	0.	0.63
time (sec)	N/A	0.028	0.016	0.045	1.004	1.27	0.	1.168

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	42	37	32	54	0	59
normalized size	1	1.	0.46	0.41	0.35	0.59	0.	0.65
time (sec)	N/A	0.029	0.016	0.044	1.01	1.307	0.	1.28

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	66	61	112	131	0	134
normalized size	1	1.	0.34	0.31	0.57	0.67	0.	0.69
time (sec)	N/A	0.059	0.031	0.167	1.014	1.246	0.	1.274

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	66	61	112	117	0	123
normalized size	1	1.	0.34	0.31	0.57	0.6	0.	0.63
time (sec)	N/A	0.058	0.024	0.165	0.998	1.202	0.	1.209

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	66	61	112	101	0	127
normalized size	1	1.	0.34	0.31	0.57	0.52	0.	0.65
time (sec)	N/A	0.055	0.021	0.166	0.998	1.261	0.	1.296

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	66	61	117	99	0	120
normalized size	1	1.	0.34	0.32	0.61	0.51	0.	0.62
time (sec)	N/A	0.054	0.022	0.164	1.004	1.301	0.	1.235

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	66	61	117	101	0	138
normalized size	1	1.	0.35	0.32	0.61	0.53	0.	0.72
time (sec)	N/A	0.058	0.025	0.176	1.011	1.283	0.	1.333

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	66	61	116	105	0	142
normalized size	1	1.	0.34	0.32	0.6	0.54	0.	0.74
time (sec)	N/A	0.055	0.027	0.17	1.026	1.494	0.	1.282

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	66	61	116	104	0	144
normalized size	1	1.	0.35	0.32	0.61	0.54	0.	0.75
time (sec)	N/A	0.055	0.026	0.17	1.002	1.469	0.	1.206

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	88	83	198	215	0	207
normalized size	1	1.	0.3	0.28	0.67	0.72	0.	0.7
time (sec)	N/A	0.082	0.043	0.174	1.013	1.477	0.	1.25

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	88	83	198	196	0	190
normalized size	1	1.	0.3	0.28	0.67	0.66	0.	0.64
time (sec)	N/A	0.077	0.034	0.174	1.018	1.491	0.	1.276

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	88	83	198	170	0	194
normalized size	1	1.	0.3	0.28	0.67	0.57	0.	0.65
time (sec)	N/A	0.081	0.031	0.167	0.985	1.532	0.	1.233

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	88	83	204	166	0	185
normalized size	1	1.	0.3	0.28	0.7	0.57	0.	0.63
time (sec)	N/A	0.079	0.031	0.166	1.001	1.492	0.	1.27

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	88	83	204	167	0	211
normalized size	1	1.	0.3	0.28	0.69	0.57	0.	0.72
time (sec)	N/A	0.079	0.033	0.167	1.05	1.529	0.	1.279

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	88	83	204	167	0	215
normalized size	1	1.	0.3	0.28	0.7	0.57	0.	0.73
time (sec)	N/A	0.082	0.038	0.178	1.022	1.474	0.	1.147

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	88	83	203	166	0	219
normalized size	1	1.	0.3	0.28	0.69	0.56	0.	0.74
time (sec)	N/A	0.079	0.036	0.171	1.016	1.492	0.	1.265

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	238	239	0	498	0	374
normalized size	1	1.	0.52	0.52	0.	1.09	0.	0.82
time (sec)	N/A	0.325	0.091	0.227	0.	1.596	0.	1.26

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	110	221	0	486	0	325
normalized size	1	1.	0.27	0.54	0.	1.18	0.	0.79
time (sec)	N/A	0.286	0.054	0.223	0.	1.638	0.	1.296

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	221	214	0	390	0	327
normalized size	1	1.	0.54	0.52	0.	0.95	0.	0.8
time (sec)	N/A	0.275	0.073	0.227	0.	1.587	0.	1.275

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	85	183	0	390	41	339
normalized size	1	1.	0.23	0.5	0.	1.06	0.11	0.92
time (sec)	N/A	0.246	0.045	0.225	0.	1.629	152.039	1.283

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	178	182	0	400	0	339
normalized size	1	1.	0.48	0.49	0.	1.09	0.	0.92
time (sec)	N/A	0.238	0.049	0.227	0.	1.631	0.	1.292

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	50	224	0	464	0	356
normalized size	1	1.	0.12	0.54	0.	1.13	0.	0.86
time (sec)	N/A	0.279	0.013	0.227	0.	1.615	0.	1.162

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	52	239	0	525	0	346
normalized size	1	1.	0.13	0.58	0.	1.27	0.	0.84
time (sec)	N/A	0.276	0.014	0.227	0.	1.657	0.	1.271

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	52	251	0	574	0	383
normalized size	1	1.	0.11	0.55	0.	1.25	0.	0.83
time (sec)	N/A	0.327	0.014	0.236	0.	1.691	0.	1.288

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	498	737	0	774	0	571
normalized size	1	1.	0.9	1.34	0.	1.4	0.	1.04
time (sec)	N/A	0.399	0.171	0.229	0.	1.721	0.	1.38

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	504	504	88	679	0	783	0	524
normalized size	1	1.	0.17	1.35	0.	1.55	0.	1.04
time (sec)	N/A	0.369	0.035	0.236	0.	1.665	0.	1.313

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	504	504	484	696	0	693	0	527
normalized size	1	1.	0.96	1.38	0.	1.38	0.	1.05
time (sec)	N/A	0.371	0.169	0.235	0.	1.667	0.	1.39

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	458	458	84	612	0	707	0	497
normalized size	1	1.	0.18	1.34	0.	1.54	0.	1.09
time (sec)	N/A	0.333	0.033	0.238	0.	1.643	0.	1.332

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	458	458	447	666	0	695	0	501
normalized size	1	1.	0.98	1.45	0.	1.52	0.	1.09
time (sec)	N/A	0.322	0.151	0.238	0.	1.671	0.	1.381

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	73	617	0	745	0	498
normalized size	1	1.	0.16	1.34	0.	1.62	0.	1.08
time (sec)	N/A	0.333	0.027	0.24	0.	1.83	0.	1.384

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	272	668	0	679	0	497
normalized size	1	1.	0.59	1.46	0.	1.48	0.	1.08
time (sec)	N/A	0.327	0.217	0.237	0.	1.924	0.	1.4

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	54	617	0	714	0	506
normalized size	1	1.	0.12	1.34	0.	1.55	0.	1.1
time (sec)	N/A	0.333	0.015	0.225	0.	1.934	0.	1.341

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	272	638	0	682	0	505
normalized size	1	1.	0.59	1.39	0.	1.48	0.	1.1
time (sec)	N/A	0.336	0.193	0.233	0.	1.992	0.	1.272

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	506	506	52	645	0	837	0	554
normalized size	1	1.	0.1	1.27	0.	1.65	0.	1.09
time (sec)	N/A	0.384	0.015	0.237	0.	1.771	0.	1.404

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	506	506	54	707	0	826	0	541
normalized size	1	1.	0.11	1.4	0.	1.63	0.	1.07
time (sec)	N/A	0.376	0.015	0.236	0.	1.705	0.	1.274

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	54	687	0	961	0	583
normalized size	1	1.	0.1	1.24	0.	1.74	0.	1.05
time (sec)	N/A	0.432	0.016	0.247	0.	1.699	0.	1.337

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	647	647	401	1287	0	1067	0	622
normalized size	1	1.	0.62	1.99	0.	1.65	0.	0.96
time (sec)	N/A	0.509	0.318	0.236	0.	1.723	0.	1.363

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	600	600	110	1171	0	1080	0	575
normalized size	1	1.	0.18	1.95	0.	1.8	0.	0.96
time (sec)	N/A	0.468	0.05	0.246	0.	1.69	0.	1.48

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	600	600	384	1202	0	964	0	578
normalized size	1	1.	0.64	2.	0.	1.61	0.	0.96
time (sec)	N/A	0.463	0.294	0.24	0.	1.691	0.	1.388

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	554	106	1046	0	992	0	548
normalized size	1	1.	0.19	1.89	0.	1.79	0.	0.99
time (sec)	N/A	0.419	0.046	0.234	0.	1.708	0.	1.437

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	554	366	1134	0	975	0	552
normalized size	1	1.	0.66	2.05	0.	1.76	0.	1.
time (sec)	N/A	0.422	0.282	0.245	0.	1.698	0.	1.443

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	97	1051	0	1021	0	552
normalized size	1	1.	0.17	1.89	0.	1.83	0.	0.99
time (sec)	N/A	0.425	0.04	0.231	0.	1.714	0.	1.323

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	352	1136	0	984	0	556
normalized size	1	1.	0.63	2.04	0.	1.77	0.	1.
time (sec)	N/A	0.43	0.306	0.232	0.	1.734	0.	1.459

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	86	1051	0	1080	0	552
normalized size	1	1.	0.15	1.88	0.	1.93	0.	0.99
time (sec)	N/A	0.424	0.037	0.234	0.	1.729	0.	1.451

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	341	1136	0	998	0	556
normalized size	1	1.	0.61	2.03	0.	1.78	0.	0.99
time (sec)	N/A	0.438	0.314	0.24	0.	1.738	0.	1.433

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	73	1051	0	1062	0	549
normalized size	1	1.	0.13	1.89	0.	1.91	0.	0.99
time (sec)	N/A	0.454	0.032	0.237	0.	1.708	0.	1.379

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	557	324	1136	0	953	0	549
normalized size	1	1.	0.58	2.04	0.	1.71	0.	0.99
time (sec)	N/A	0.426	0.293	0.237	0.	1.668	0.	1.424

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	556	556	54	1051	0	1027	0	558
normalized size	1	1.	0.1	1.89	0.	1.85	0.	1.
time (sec)	N/A	0.434	0.013	0.234	0.	1.695	0.	1.391

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	556	556	319	1133	0	946	0	556
normalized size	1	1.	0.57	2.04	0.	1.7	0.	1.
time (sec)	N/A	0.429	0.137	0.227	0.	1.705	0.	1.419

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	602	602	52	1081	0	1211	0	605
normalized size	1	1.	0.09	1.8	0.	2.01	0.	1.
time (sec)	N/A	0.485	0.015	0.24	0.	1.85	0.	1.371

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	602	602	54	1183	0	1133	0	593
normalized size	1	1.	0.09	1.97	0.	1.88	0.	0.99
time (sec)	N/A	0.484	0.025	0.243	0.	1.889	0.	1.483

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	649	649	54	1129	0	1350	0	635
normalized size	1	1.	0.08	1.74	0.	2.08	0.	0.98
time (sec)	N/A	0.534	0.017	0.251	0.	1.847	0.	1.444

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	105	602	0	1239	3188	1143
normalized size	1	1.	0.7	4.01	0.	8.26	21.25	7.62
time (sec)	N/A	0.121	0.076	0.051	0.	1.618	6.464	1.261

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	73	292	0	576	1321	560
normalized size	1	1.	0.7	2.81	0.	5.54	12.7	5.38
time (sec)	N/A	0.075	0.035	0.051	0.	1.628	3.09	1.285

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	41	94	0	186	345	182
normalized size	1	1.	0.71	1.62	0.	3.21	5.95	3.14
time (sec)	N/A	0.023	0.029	0.049	0.	1.572	0.894	1.285

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.008	0.078	0.	0.	0.	0.

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.008	0.066	0.	0.	0.	0.

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.009	0.076	0.	0.	0.	0.

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	111	453	328	873	0	1215
normalized size	1	1.	0.35	1.45	1.05	2.79	0.	3.88
time (sec)	N/A	0.12	0.09	0.166	0.994	1.63	0.	1.3

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	131	199	161	352	0	518
normalized size	1	1.	0.64	0.97	0.79	1.72	0.	2.53
time (sec)	N/A	0.076	0.069	0.169	0.984	1.555	0.	1.271

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	53	56	47	77	0	112
normalized size	1	1.	0.55	0.58	0.48	0.79	0.	1.15
time (sec)	N/A	0.034	0.025	0.166	0.971	1.541	0.	1.288

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.019	0.266	0.	0.	0.	0.

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.023	0.231	0.	0.	0.	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.019	0.232	0.	0.	0.	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	77	66	0	0	0	0	0
normalized size	1	1.04	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.019	0.331	0.	0.	0.	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	110	150	155	335	0	506
normalized size	1	1.	0.63	0.86	0.89	1.93	0.	2.91
time (sec)	N/A	0.109	0.058	0.05	0.978	1.58	0.	1.249

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	77	96	107	223	0	317
normalized size	1	1.	0.59	0.74	0.82	1.72	0.	2.44
time (sec)	N/A	0.082	0.033	0.046	0.983	1.573	0.	1.217

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	51	60	73	142	0	178
normalized size	1	1.	0.61	0.71	0.87	1.69	0.	2.12
time (sec)	N/A	0.057	0.021	0.048	0.975	1.612	0.	1.325

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	29	40	41	80	0	78
normalized size	1	1.	0.71	0.98	1.	1.95	0.	1.9
time (sec)	N/A	0.025	0.004	0.044	0.994	1.609	0.	1.184

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	54	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.01	0.202	0.	0.	0.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.012	0.197	0.	0.	0.	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.007	0.213	0.	0.	0.	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.006	0.2	0.	0.	0.	0.

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	46	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.006	0.178	0.	0.	0.	0.

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.006	0.194	0.	0.	0.	0.

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.006	0.203	0.	0.	0.	0.

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.009	0.171	0.	0.	0.	0.

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.009	0.171	0.	0.	0.	0.

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.009	0.176	0.	0.	0.	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.009	0.198	0.	0.	0.	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.009	0.173	0.	0.	0.	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.007	0.002	0.039	0.975	1.295	0.062	1.121

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.007	0.001	0.041	0.965	1.274	0.062	1.226

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	39	15	22
normalized size	1	1.	1.	0.85	1.1	1.95	0.75	1.1
time (sec)	N/A	0.003	0.	0.04	0.97	1.249	0.059	1.207

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	27	46	17	27
normalized size	1	1.	1.	0.86	1.29	2.19	0.81	1.29
time (sec)	N/A	0.005	0.002	0.043	0.996	1.481	0.097	1.322

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	42	12	22
normalized size	1	1.	1.	0.94	1.22	2.33	0.67	1.22
time (sec)	N/A	0.007	0.002	0.045	0.996	1.427	0.269	1.286

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	27	51	17	35
normalized size	1	1.	1.	0.86	1.29	2.43	0.81	1.67
time (sec)	N/A	0.007	0.002	0.046	0.989	1.46	0.292	1.271

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	45	15	23
normalized size	1	1.	1.	0.94	1.28	2.5	0.83	1.28
time (sec)	N/A	0.007	0.004	0.046	0.959	1.428	0.304	1.225

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	28	54	17	36
normalized size	1	1.	1.	0.86	1.33	2.57	0.81	1.71
time (sec)	N/A	0.007	0.003	0.047	0.957	1.513	0.403	1.31

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	28	51	22	28
normalized size	1	1.	1.	0.87	1.22	2.22	0.96	1.22
time (sec)	N/A	0.007	0.002	0.048	0.95	1.399	0.434	1.244

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	28	50	22	28
normalized size	1	1.	1.	0.8	1.12	2.	0.88	1.12
time (sec)	N/A	0.007	0.002	0.046	0.951	1.398	0.487	1.245

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	28	55	22	28
normalized size	1	1.	1.	0.8	1.12	2.2	0.88	1.12
time (sec)	N/A	0.007	0.002	0.049	0.936	1.435	0.487	1.208

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	115	51	62
normalized size	1	1.	1.	0.83	1.09	2.13	0.94	1.15
time (sec)	N/A	0.03	0.008	0.044	0.957	1.28	0.074	1.284

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	59	115	46	62
normalized size	1	1.	0.89	0.83	1.09	2.13	0.85	1.15
time (sec)	N/A	0.038	0.008	0.041	0.953	1.279	0.077	1.352

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	61	104	48	58
normalized size	1	1.	1.	0.86	1.24	2.12	0.98	1.18
time (sec)	N/A	0.021	0.005	0.041	0.956	1.267	0.081	1.199

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	59	100	42	62
normalized size	1	1.	1.	0.94	1.26	2.13	0.89	1.32
time (sec)	N/A	0.041	0.011	0.045	0.966	1.466	0.315	1.123

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	57	111	44	59
normalized size	1	1.	1.	0.94	1.19	2.31	0.92	1.23
time (sec)	N/A	0.021	0.017	0.046	0.951	1.422	0.323	1.138

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	45	59	109	44	72
normalized size	1	1.	0.9	0.88	1.16	2.14	0.86	1.41
time (sec)	N/A	0.041	0.014	0.048	0.956	1.432	0.345	1.149

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	42	57	107	44	57
normalized size	1	1.	1.	0.89	1.21	2.28	0.94	1.21
time (sec)	N/A	0.024	0.018	0.047	0.967	1.419	0.342	1.099

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	43	61	105	42	81
normalized size	1	1.	0.91	0.96	1.36	2.33	0.93	1.8
time (sec)	N/A	0.038	0.019	0.047	0.961	1.464	0.534	1.124

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	49	43	61	107	46	63
normalized size	1	1.	1.02	0.9	1.27	2.23	0.96	1.31
time (sec)	N/A	0.023	0.021	0.047	0.978	1.374	0.596	1.132

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	46	61	109	46	73
normalized size	1	1.	0.98	0.9	1.2	2.14	0.9	1.43
time (sec)	N/A	0.036	0.017	0.046	0.972	1.439	0.908	1.111

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	49	42	59	113	44	62
normalized size	1	1.	1.04	0.89	1.26	2.4	0.94	1.32
time (sec)	N/A	0.024	0.023	0.048	0.984	1.46	0.874	1.142

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	45	65	115	46	78
normalized size	1	1.	1.04	0.94	1.35	2.4	0.96	1.62
time (sec)	N/A	0.035	0.027	0.049	0.968	1.425	1.464	1.131

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	45	62	115	49	65
normalized size	1	1.	0.96	0.87	1.19	2.21	0.94	1.25
time (sec)	N/A	0.025	0.02	0.047	0.965	1.402	1.584	1.119

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	45	62	111	49	65
normalized size	1	1.	0.98	0.83	1.15	2.06	0.91	1.2
time (sec)	N/A	0.037	0.015	0.048	0.957	1.432	2.136	1.152

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	56	45	62	124	49	65
normalized size	1	1.	1.04	0.83	1.15	2.3	0.91	1.2
time (sec)	N/A	0.025	0.024	0.048	0.961	1.561	2.048	1.118

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	45	62	113	49	65
normalized size	1	1.	0.93	0.83	1.15	2.09	0.91	1.2
time (sec)	N/A	0.035	0.015	0.048	0.956	1.589	2.808	1.11

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	111	109	217	97	117
normalized size	1	1.	1.	1.25	1.22	2.44	1.09	1.31
time (sec)	N/A	0.059	0.013	0.041	0.953	1.528	0.088	1.117

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	79	111	109	216	92	117
normalized size	1	1.	0.89	1.25	1.22	2.43	1.03	1.31
time (sec)	N/A	0.083	0.016	0.043	0.981	1.496	0.081	1.128

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	107	115	198	87	112
normalized size	1	1.	1.	1.32	1.42	2.44	1.07	1.38
time (sec)	N/A	0.045	0.01	0.043	0.963	1.544	0.081	1.111

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	85	111	189	92	117
normalized size	1	1.	1.	1.	1.31	2.22	1.08	1.38
time (sec)	N/A	0.074	0.021	0.045	0.979	1.755	0.367	1.148

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	84	105	201	82	112
normalized size	1	1.	1.	1.05	1.31	2.51	1.02	1.4
time (sec)	N/A	0.039	0.023	0.046	0.956	1.466	0.36	1.161

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	87	111	197	92	132
normalized size	1	1.	0.91	1.01	1.29	2.29	1.07	1.53
time (sec)	N/A	0.078	0.033	0.047	0.965	1.406	0.394	1.127

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	84	108	197	88	113
normalized size	1	1.	1.	1.01	1.3	2.37	1.06	1.36
time (sec)	N/A	0.041	0.025	0.049	0.97	1.4	0.406	1.129

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	93	142	0	675	391	124
normalized size	1	1.	0.93	1.42	0.	6.75	3.91	1.24
time (sec)	N/A	0.117	0.09	0.174	0.	1.583	2.089	1.208

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	556	316	101
normalized size	1	1.	0.96	1.37	0.	6.86	3.9	1.25
time (sec)	N/A	0.08	0.045	0.171	0.	1.533	1.705	1.18

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	443	223	80
normalized size	1	1.	0.98	0.95	0.	7.03	3.54	1.27
time (sec)	N/A	0.055	0.023	0.162	0.	1.541	0.875	1.156

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	290	131	47
normalized size	1	1.	1.08	1.	0.	8.06	3.64	1.31
time (sec)	N/A	0.034	0.008	0.162	0.	1.449	0.477	1.158

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	113	66	0	510	253	92
normalized size	1	1.	1.64	0.96	0.	7.39	3.67	1.33
time (sec)	N/A	0.07	0.071	0.169	0.	1.591	3.197	1.169

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	135	119	0	664	345	127
normalized size	1	1.	1.52	1.34	0.	7.46	3.88	1.43
time (sec)	N/A	0.131	0.124	0.17	0.	1.583	7.8	1.175

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	188	159	0	819	423	170
normalized size	1	1.	1.65	1.39	0.	7.18	3.71	1.49
time (sec)	N/A	0.196	0.223	0.171	0.	1.762	9.748	1.174

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	250	467	0	3217	194	5466
normalized size	1	1.	1.23	2.3	0.	15.85	0.96	26.93
time (sec)	N/A	0.67	0.153	0.187	0.	1.726	2.459	2.618

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	202	343	0	2168	129	4593
normalized size	1	1.	1.13	1.92	0.	12.11	0.72	25.66
time (sec)	N/A	0.269	0.108	0.186	0.	1.612	1.774	2.389

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	165	208	0	1206	75	5350
normalized size	1	1.	1.1	1.39	0.	8.04	0.5	35.67
time (sec)	N/A	0.109	0.083	0.179	0.	1.508	0.85	2.304

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	129	116	0	1323	87	1365
normalized size	1	1.	0.86	0.77	0.	8.82	0.58	9.1
time (sec)	N/A	0.086	0.076	0.177	0.	1.534	0.972	1.384

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	191	232	0	2279	148	4581
normalized size	1	1.	1.1	1.33	0.	13.1	0.85	26.33
time (sec)	N/A	0.222	0.385	0.192	0.	1.663	1.964	2.427

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	216	368	0	3343	211	6892
normalized size	1	1.	1.1	1.88	0.	17.06	1.08	35.16
time (sec)	N/A	0.423	0.137	0.191	0.	1.705	2.726	2.423

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	121	222	0	1412	745	205
normalized size	1	1.	0.92	1.68	0.	10.7	5.64	1.55
time (sec)	N/A	0.168	0.182	0.18	0.	1.591	3.864	23.448

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	93	104	0	864	282	130
normalized size	1	1.	1.19	1.33	0.	11.08	3.62	1.67
time (sec)	N/A	0.066	0.089	0.174	0.	1.769	1.983	20.65

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	77	0	778	267	111
normalized size	1	1.	1.05	1.03	0.	10.37	3.56	1.48
time (sec)	N/A	0.061	0.064	0.171	0.	1.746	1.743	21.038

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	79	75	0	783	267	111
normalized size	1	1.	1.07	1.01	0.	10.58	3.61	1.5
time (sec)	N/A	0.058	0.078	0.172	0.	1.823	1.744	20.623

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	207	253	0	1728	772	224
normalized size	1	1.	1.7	2.07	0.	14.16	6.33	1.84
time (sec)	N/A	0.198	0.312	0.184	0.	2.47	45.252	21.464

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	248	352	0	2103	906	246
normalized size	1	1.	1.53	2.17	0.	12.98	5.59	1.52
time (sec)	N/A	0.251	0.266	0.184	0.	2.586	78.989	20.868

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	327	844	0	6311	450	0
normalized size	1	1.	0.99	2.55	0.	19.07	1.36	0.
time (sec)	N/A	0.844	0.686	0.205	0.	2.441	7.154	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	282	602	0	4806	379	0
normalized size	1	1.	1.04	2.22	0.	17.73	1.4	0.
time (sec)	N/A	0.572	0.54	0.201	0.	1.897	4.929	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	235	452	0	3584	294	0
normalized size	1	1.	0.99	1.91	0.	15.12	1.24	0.
time (sec)	N/A	0.411	0.423	0.194	0.	1.676	3.576	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	222	342	0	3623	298	0
normalized size	1	1.	1.	1.55	0.	16.39	1.35	0.
time (sec)	N/A	0.26	0.443	0.232	0.	1.74	3.746	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	243	733	0	4918	394	0
normalized size	1	1.	0.96	2.91	0.	19.52	1.56	0.
time (sec)	N/A	0.513	0.448	0.22	0.	2.021	4.91	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	302	712	0	6460	481	0
normalized size	1	1.	0.98	2.31	0.	20.97	1.56	0.
time (sec)	N/A	1.443	0.631	0.209	0.	2.429	7.703	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	244	547	0	3467	1520	413
normalized size	1	1.	1.17	2.62	0.	16.59	7.27	1.98
time (sec)	N/A	0.401	0.341	0.184	0.	1.887	17.06	27.379

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	194	267	0	2033	554	286
normalized size	1	1.	1.6	2.21	0.	16.8	4.58	2.36
time (sec)	N/A	0.112	0.176	0.179	0.	1.597	12.341	27.326

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	137	230	0	1875	520	231
normalized size	1	1.	1.15	1.93	0.	15.76	4.37	1.94
time (sec)	N/A	0.104	0.201	0.175	0.	1.562	10.625	28.794

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	145	270	0	1920	580	217
normalized size	1	1.	1.12	2.08	0.	14.77	4.46	1.67
time (sec)	N/A	0.129	0.137	0.18	0.	1.615	10.438	28.973

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	114	142	0	1719	490	193
normalized size	1	1.	1.01	1.26	0.	15.21	4.34	1.71
time (sec)	N/A	0.09	0.104	0.184	0.	1.518	9.814	28.591

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	106	141	0	1709	481	194
normalized size	1	1.	0.94	1.25	0.	15.12	4.26	1.72
time (sec)	N/A	0.088	0.1	0.176	0.	1.58	9.569	27.711

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	342	822	0	4271	0	436
normalized size	1	1.	1.71	4.11	0.	21.36	0.	2.18
time (sec)	N/A	0.298	0.496	0.198	0.	4.542	0.	32.726

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	402	1002	0	4906	0	516
normalized size	1	1.	1.58	3.93	0.	19.24	0.	2.02
time (sec)	N/A	0.391	0.614	0.192	0.	6.096	0.	31.952

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	455	1141	0	9721	804	0
normalized size	1	1.	1.14	2.85	0.	24.3	2.01	0.
time (sec)	N/A	1.727	1.193	0.214	0.	3.588	23.413	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	381	953	0	8352	716	0
normalized size	1	1.	1.09	2.74	0.	24.	2.06	0.
time (sec)	N/A	0.886	0.979	0.211	0.	2.567	16.696	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	343	753	0	6966	627	0
normalized size	1	1.	1.15	2.53	0.	23.38	2.1	0.
time (sec)	N/A	0.683	0.874	0.204	0.	2.038	14.04	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	285	617	0	6978	644	0
normalized size	1	1.	0.99	2.13	0.	24.15	2.23	0.
time (sec)	N/A	0.705	0.717	0.2	0.	2.149	14.128	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	334	2958	0	8466	733	0
normalized size	1	1.	1.07	9.51	0.	27.22	2.36	0.
time (sec)	N/A	0.701	0.851	0.28	0.	2.705	15.885	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	372	3360	0	9839	818	0
normalized size	1	1.	1.05	9.46	0.	27.72	2.3	0.
time (sec)	N/A	1.838	1.052	0.261	0.	3.748	21.011	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	454	1567	0	11826	925	0
normalized size	1	1.	1.07	3.69	0.	27.83	2.18	0.
time (sec)	N/A	0.963	1.804	0.222	0.	5.671	77.12	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	80	116	0	556	311	105
normalized size	1	1.	0.98	1.41	0.	6.78	3.79	1.28
time (sec)	N/A	0.092	0.053	0.173	0.	1.568	1.606	1.345

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	63	0	444	223	84
normalized size	1	1.	1.02	0.98	0.	6.94	3.48	1.31
time (sec)	N/A	0.062	0.023	0.158	0.	1.477	0.842	1.369

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	41	38	0	290	131	50
normalized size	1	1.	1.17	1.09	0.	8.29	3.74	1.43
time (sec)	N/A	0.043	0.009	0.159	0.	1.446	0.472	1.404

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	117	69	0	512	253	96
normalized size	1	1.	1.67	0.99	0.	7.31	3.61	1.37
time (sec)	N/A	0.084	0.071	0.165	0.	1.561	3.012	1.338

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	139	123	0	664	350	128
normalized size	1	1.	1.56	1.38	0.	7.46	3.93	1.44
time (sec)	N/A	0.14	0.142	0.166	0.	1.599	7.504	1.441

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	208	343	0	2157	129	4250
normalized size	1	1.	1.16	1.92	0.	12.05	0.72	23.74
time (sec)	N/A	0.365	0.131	0.183	0.	1.617	1.684	2.622

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	137	208	0	1196	75	5098
normalized size	1	1.	0.91	1.39	0.	7.97	0.5	33.99
time (sec)	N/A	0.109	0.107	0.17	0.	1.525	0.8	2.639

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	137	116	0	1312	87	1296
normalized size	1	1.	0.91	0.77	0.	8.75	0.58	8.64
time (sec)	N/A	0.072	0.084	0.172	0.	1.585	0.943	1.565

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	199	232	0	2267	148	4238
normalized size	1	1.	1.16	1.35	0.	13.18	0.86	24.64
time (sec)	N/A	0.203	0.397	0.178	0.	1.631	1.897	2.703

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	62	86	0	374	138	81
normalized size	1	1.	0.9	1.25	0.	5.42	2.	1.17
time (sec)	N/A	0.084	0.038	0.045	0.	1.492	1.084	3.485

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	51	49	0	312	110	62
normalized size	1	1.	0.91	0.88	0.	5.57	1.96	1.11
time (sec)	N/A	0.049	0.017	0.043	0.	1.496	0.488	3.557

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	0	215	53	31
normalized size	1	1.	1.	0.84	0.	6.94	1.71	1.
time (sec)	N/A	0.028	0.008	0.043	0.	1.587	0.287	4.483

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	90	71	0	350	184	96
normalized size	1	1.	1.17	0.92	0.	4.55	2.39	1.25
time (sec)	N/A	0.072	0.048	0.049	0.	1.53	2.292	4.187

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	146	122	0	487	372	170
normalized size	1	1.	1.51	1.26	0.	5.02	3.84	1.75
time (sec)	N/A	0.141	0.094	0.049	0.	1.579	5.241	2.321

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	144	210	0	1287	105	0
normalized size	1	1.	1.26	1.84	0.	11.29	0.92	0.
time (sec)	N/A	0.165	0.087	0.17	0.	1.601	1.095	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	128	134	0	617	44	0
normalized size	1	1.	1.17	1.23	0.	5.66	0.4	0.
time (sec)	N/A	0.054	0.105	0.133	0.	1.473	0.373	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	105	74	0	1115	63	0
normalized size	1	1.	0.96	0.68	0.	10.23	0.58	0.
time (sec)	N/A	0.047	0.063	0.139	0.	1.506	0.64	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	143	180	0	3290	134	0
normalized size	1	1.	1.18	1.49	0.	27.19	1.11	0.
time (sec)	N/A	0.113	0.151	0.144	0.	1.674	2.266	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	62	84	0	374	144	78
normalized size	1	1.	0.9	1.22	0.	5.42	2.09	1.13
time (sec)	N/A	0.076	0.035	0.046	0.	1.497	1.112	3.512

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	47	0	312	117	57
normalized size	1	1.	0.91	0.87	0.	5.78	2.17	1.06
time (sec)	N/A	0.045	0.017	0.045	0.	1.567	0.509	3.418

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	0	217	60	28
normalized size	1	1.	1.	0.84	0.	7.	1.94	0.9
time (sec)	N/A	0.026	0.007	0.042	0.	1.486	0.278	3.379

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	105	63	0	347	194	82
normalized size	1	1.	1.52	0.91	0.	5.03	2.81	1.19
time (sec)	N/A	0.067	0.051	0.049	0.	1.548	2.269	3.487

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	163	110	0	490	386	169
normalized size	1	1.	1.83	1.24	0.	5.51	4.34	1.9
time (sec)	N/A	0.132	0.092	0.051	0.	1.557	5.188	3.407

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	432	432	164	1658	0	1303	105	0
normalized size	1	1.	0.38	3.84	0.	3.02	0.24	0.
time (sec)	N/A	0.891	0.093	0.23	0.	1.609	1.064	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	143	724	0	633	44	0
normalized size	1	1.	0.43	2.19	0.	1.91	0.13	0.
time (sec)	N/A	0.256	0.117	0.159	0.	1.528	0.356	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	119	913	0	1195	63	0
normalized size	1	1.	0.33	2.54	0.	3.33	0.18	0.
time (sec)	N/A	0.261	0.071	0.167	0.	1.521	0.617	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	174	3318	0	3301	134	0
normalized size	1	1.	0.4	7.66	0.	7.62	0.31	0.
time (sec)	N/A	0.519	0.141	0.173	0.	1.618	2.231	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	61	26	24
normalized size	1	1.	1.	0.95	1.2	3.05	1.3	1.2
time (sec)	N/A	0.02	0.005	0.042	1.468	1.469	0.102	1.365

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	14
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.
time (sec)	N/A	0.016	0.005	0.043	1.431	1.425	0.102	1.264

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	66	20	24
normalized size	1	1.	1.	0.83	1.04	2.87	0.87	1.04
time (sec)	N/A	0.012	0.013	0.049	1.442	1.441	0.134	1.234

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	94	57	0	529	63	0
normalized size	1	1.	1.27	0.77	0.	7.15	0.85	0.
time (sec)	N/A	0.05	0.142	0.053	0.	1.572	0.169	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	39	308	0	771	24	0
normalized size	1	1.	0.21	1.64	0.	4.1	0.13	0.
time (sec)	N/A	0.177	0.032	0.076	0.	1.585	0.462	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	164	296	0	859	0	248
normalized size	1	1.	0.96	1.73	0.	5.02	0.	1.45
time (sec)	N/A	0.155	0.149	0.175	0.	1.641	0.	1.382

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	136	247	0	698	0	197
normalized size	1	1.	0.89	1.61	0.	4.56	0.	1.29
time (sec)	N/A	0.129	0.07	0.167	0.	1.669	0.	1.437

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	101	139	0	552	0	132
normalized size	1	1.	0.94	1.29	0.	5.11	0.	1.22
time (sec)	N/A	0.081	0.046	0.158	0.	1.728	0.	1.38

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	101	0	463	0	103
normalized size	1	1.	1.	1.22	0.	5.58	0.	1.24
time (sec)	N/A	0.055	0.023	0.155	0.	1.594	0.	1.334

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	106	91	0	1332	0	0
normalized size	1	1.	0.97	0.83	0.	12.22	0.	0.
time (sec)	N/A	0.109	0.041	0.163	0.	1.825	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	140	0	1408	0	0
normalized size	1	1.	1.	1.25	0.	12.57	0.	0.
time (sec)	N/A	0.104	0.047	0.162	0.	1.864	0.	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	193	0	502	0	0
normalized size	1	1.	1.	2.19	0.	5.7	0.	0.
time (sec)	N/A	0.071	0.039	0.16	0.	1.676	0.	0.

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	108	222	0	594	0	0
normalized size	1	1.	0.93	1.91	0.	5.12	0.	0.
time (sec)	N/A	0.096	0.076	0.164	0.	1.79	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	141	387	0	743	0	0
normalized size	1	1.	0.88	2.4	0.	4.61	0.	0.
time (sec)	N/A	0.15	0.095	0.161	0.	2.034	0.	0.

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	173	442	0	906	0	0
normalized size	1	1.	0.87	2.22	0.	4.55	0.	0.
time (sec)	N/A	0.229	0.122	0.165	0.	2.669	0.	0.

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	538	476	0	0	0	0
normalized size	1	1.	1.36	1.21	0.	0.	0.	0.
time (sec)	N/A	0.251	1.553	0.273	0.	0.	0.	0.

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	479	417	0	0	0	0
normalized size	1	1.	1.4	1.22	0.	0.	0.	0.
time (sec)	N/A	0.141	1.242	0.213	0.	0.	0.	0.

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	445	379	0	0	0	0
normalized size	1	1.	1.44	1.23	0.	0.	0.	0.
time (sec)	N/A	0.101	0.829	0.206	0.	0.	0.	0.

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	435	381	0	0	0	0
normalized size	1	1.	1.44	1.26	0.	0.	0.	0.
time (sec)	N/A	0.084	0.816	0.217	0.	0.	0.	0.

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	459	404	0	0	0	0
normalized size	1	1.	1.35	1.18	0.	0.	0.	0.
time (sec)	N/A	0.155	0.895	0.216	0.	0.	0.	0.

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	397	397	530	452	0	0	0	0
normalized size	1	1.	1.34	1.14	0.	0.	0.	0.
time (sec)	N/A	0.261	1.34	0.226	0.	0.	0.	0.

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	192	534	0	1265	0	374
normalized size	1	1.	0.86	2.39	0.	5.67	0.	1.68
time (sec)	N/A	0.212	0.252	0.174	0.	1.759	0.	1.37

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	175	432	0	1061	0	311
normalized size	1	1.	0.86	2.12	0.	5.2	0.	1.52
time (sec)	N/A	0.182	0.159	0.174	0.	1.738	0.	1.274

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	149	316	0	837	0	232
normalized size	1	1.	0.99	2.11	0.	5.58	0.	1.55
time (sec)	N/A	0.117	0.142	0.167	0.	1.682	0.	1.226

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	126	242	0	684	0	182
normalized size	1	1.	1.02	1.95	0.	5.52	0.	1.47
time (sec)	N/A	0.086	0.083	0.178	0.	1.667	0.	1.266

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	143	192	0	1715	0	0
normalized size	1	1.	0.92	1.24	0.	11.06	0.	0.
time (sec)	N/A	0.184	0.141	0.168	0.	2.497	0.	0.

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	134	170	0	1667	0	0
normalized size	1	1.	0.89	1.13	0.	11.11	0.	0.
time (sec)	N/A	0.169	0.117	0.176	0.	2.187	0.	0.

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	134	174	0	1670	0	0
normalized size	1	1.	0.89	1.15	0.	11.06	0.	0.
time (sec)	N/A	0.163	0.173	0.171	0.	2.436	0.	0.

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	149	202	0	1804	0	0
normalized size	1	1.	0.91	1.24	0.	11.07	0.	0.
time (sec)	N/A	0.184	0.203	0.177	0.	2.806	0.	0.

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	138	260	0	726	0	0
normalized size	1	1.	1.04	1.95	0.	5.46	0.	0.
time (sec)	N/A	0.117	0.162	0.18	0.	2.4	0.	0.

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	167	337	0	887	0	0
normalized size	1	1.	1.03	2.08	0.	5.48	0.	0.
time (sec)	N/A	0.14	0.143	0.175	0.	3.054	0.	0.

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	206	457	0	1108	0	0
normalized size	1	1.	0.95	2.12	0.	5.13	0.	0.
time (sec)	N/A	0.217	0.21	0.184	0.	3.901	0.	0.

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	657	674	0	0	0	0
normalized size	1	1.	1.33	1.36	0.	0.	0.	0.
time (sec)	N/A	0.44	2.214	0.219	0.	0.	0.	0.

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	602	545	0	0	0	0
normalized size	1	1.	1.36	1.23	0.	0.	0.	0.
time (sec)	N/A	0.285	1.927	0.214	0.	0.	0.	0.

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	533	471	0	0	0	0
normalized size	1	1.	1.4	1.24	0.	0.	0.	0.
time (sec)	N/A	0.251	1.51	0.214	0.	0.	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	505	430	0	0	0	0
normalized size	1	1.	1.4	1.19	0.	0.	0.	0.
time (sec)	N/A	0.206	1.263	0.223	0.	0.	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	473	428	0	0	0	0
normalized size	1	1.	1.34	1.21	0.	0.	0.	0.
time (sec)	N/A	0.153	0.903	0.226	0.	0.	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	527	450	0	0	0	0
normalized size	1	1.	1.32	1.12	0.	0.	0.	0.
time (sec)	N/A	0.251	1.348	0.224	0.	0.	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	447	572	495	0	0	0	0
normalized size	1	1.	1.28	1.11	0.	0.	0.	0.
time (sec)	N/A	0.395	1.592	0.231	0.	0.	0.	0.

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	59	114	0	0	0	0
normalized size	1	1.	1.23	2.38	0.	0.	0.	0.
time (sec)	N/A	0.043	0.058	0.048	0.	0.	0.	0.

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	104	162	0	567	0	147
normalized size	1	1.	0.86	1.34	0.	4.69	0.	1.21
time (sec)	N/A	0.114	0.061	0.179	0.	1.68	0.	1.246

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	116	0	475	0	111
normalized size	1	1.	0.85	1.12	0.	4.57	0.	1.07
time (sec)	N/A	0.099	0.033	0.172	0.	1.605	0.	1.274

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	56	0	383	0	82
normalized size	1	1.	1.	0.82	0.	5.63	0.	1.21
time (sec)	N/A	0.054	0.014	0.172	0.	1.622	0.	1.197

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	0	285	0	54
normalized size	1	1.	1.	0.81	0.	6.63	0.	1.26
time (sec)	N/A	0.031	0.005	0.165	0.	1.611	0.	1.197

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	294	0	0
normalized size	1	1.	1.	0.89	0.	6.68	0.	0.
time (sec)	N/A	0.041	0.01	0.169	0.	1.633	0.	0.

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	63	0	423	0	0
normalized size	1	1.	1.	0.88	0.	5.88	0.	0.
time (sec)	N/A	0.061	0.023	0.166	0.	1.707	0.	0.

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	91	127	0	512	0	0
normalized size	1	1.	0.84	1.18	0.	4.74	0.	0.
time (sec)	N/A	0.105	0.049	0.168	0.	1.739	0.	0.

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	112	176	0	612	0	0
normalized size	1	1.	0.77	1.21	0.	4.22	0.	0.
time (sec)	N/A	0.166	0.08	0.172	0.	1.832	0.	0.

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	444	388	0	0	0	0
normalized size	1	1.	1.42	1.24	0.	0.	0.	0.
time (sec)	N/A	0.113	0.861	0.212	0.	0.	0.	0.

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	278	216	0	0	0	0
normalized size	1	1.	1.04	0.81	0.	0.	0.	0.
time (sec)	N/A	0.069	0.131	0.217	0.	0.	0.	0.

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	186	144	0	0	0	0
normalized size	1	1.	1.63	1.26	0.	0.	0.	0.
time (sec)	N/A	0.014	0.1	0.217	0.	0.	0.	0.

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	298	239	0	0	0	0
normalized size	1	1.	1.01	0.81	0.	0.	0.	0.
time (sec)	N/A	0.116	0.49	0.226	0.	0.	0.	0.

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	459	413	0	0	0	0
normalized size	1	1.	1.33	1.2	0.	0.	0.	0.
time (sec)	N/A	0.156	0.892	0.22	0.	0.	0.	0.

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	107	168	0	572	0	159
normalized size	1	1.	0.86	1.35	0.	4.61	0.	1.28
time (sec)	N/A	0.113	0.076	0.17	0.	1.593	0.	1.31

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	89	120	0	479	0	123
normalized size	1	1.	0.83	1.12	0.	4.48	0.	1.15
time (sec)	N/A	0.093	0.043	0.168	0.	1.605	0.	1.287

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	58	0	390	0	95
normalized size	1	1.	1.	0.83	0.	5.57	0.	1.36
time (sec)	N/A	0.058	0.016	0.161	0.	1.568	0.	1.248

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	0	288	0	61
normalized size	1	1.	1.	0.82	0.	6.55	0.	1.39
time (sec)	N/A	0.038	0.006	0.158	0.	1.539	0.	1.224

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	46	45	0	294	0	59
normalized size	1	1.	0.98	0.96	0.	6.26	0.	1.26
time (sec)	N/A	0.042	0.014	0.165	0.	1.626	0.	1.336

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	76	74	0	421	0	93
normalized size	1	1.	0.99	0.96	0.	5.47	0.	1.21
time (sec)	N/A	0.062	0.023	0.169	0.	1.644	0.	1.334

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	95	149	0	510	0	122
normalized size	1	1.	0.83	1.3	0.	4.43	0.	1.06
time (sec)	N/A	0.123	0.046	0.17	0.	1.719	0.	1.844

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	116	202	0	609	0	158
normalized size	1	1.	0.75	1.31	0.	3.95	0.	1.03
time (sec)	N/A	0.172	0.077	0.17	0.	1.879	0.	1.799

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	459	391	0	0	0	0
normalized size	1	1.	1.12	0.96	0.	0.	0.	0.
time (sec)	N/A	0.588	0.774	0.281	0.	0.	0.	0.

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	271	217	0	0	0	0
normalized size	1	1.	0.72	0.58	0.	0.	0.	0.
time (sec)	N/A	0.245	0.118	0.224	0.	0.	0.	0.

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	177	145	0	0	0	0
normalized size	1	1.	1.05	0.86	0.	0.	0.	0.
time (sec)	N/A	0.066	0.081	0.21	0.	0.	0.	0.

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	283	241	0	0	0	0
normalized size	1	1.	0.69	0.59	0.	0.	0.	0.
time (sec)	N/A	0.332	0.432	0.225	0.	0.	0.	0.

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	445	472	417	0	0	0	0
normalized size	1	1.	1.06	0.94	0.	0.	0.	0.
time (sec)	N/A	0.469	0.743	0.233	0.	0.	0.	0.

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	181	354	0	1272	0	490
normalized size	1	1.	0.95	1.86	0.	6.69	0.	2.58
time (sec)	N/A	0.237	0.193	0.173	0.	2.268	0.	1.381

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	137	264	0	980	0	359
normalized size	1	1.	1.02	1.97	0.	7.31	0.	2.68
time (sec)	N/A	0.111	0.12	0.173	0.	2.017	0.	1.377

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	107	149	0	833	0	255
normalized size	1	1.	0.93	1.3	0.	7.24	0.	2.22
time (sec)	N/A	0.092	0.1	0.171	0.	1.834	0.	1.288

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	0	139	0	59
normalized size	1	1.	1.	1.06	0.	3.86	0.	1.64
time (sec)	N/A	0.029	0.096	0.046	0.	1.718	0.	1.28

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	37	36	0	140	0	61
normalized size	1	1.	1.03	1.	0.	3.89	0.	1.69
time (sec)	N/A	0.023	0.021	0.046	0.	1.702	0.	1.327

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	99	0	837	0	0
normalized size	1	1.	1.	1.11	0.	9.4	0.	0.
time (sec)	N/A	0.081	0.107	0.166	0.	1.942	0.	0.

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	137	195	0	1025	0	0
normalized size	1	1.	0.99	1.4	0.	7.37	0.	0.
time (sec)	N/A	0.126	0.085	0.17	0.	2.175	0.	0.

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	179	314	0	1314	0	0
normalized size	1	1.	0.92	1.61	0.	6.74	0.	0.
time (sec)	N/A	0.211	0.13	0.173	0.	2.611	0.	0.

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	489	482	0	0	0	0
normalized size	1	1.	1.2	1.18	0.	0.	0.	0.
time (sec)	N/A	0.21	1.32	0.248	0.	0.	0.	0.

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	452	450	0	0	0	0
normalized size	1	1.	1.32	1.32	0.	0.	0.	0.
time (sec)	N/A	0.13	0.833	0.225	0.	0.	0.	0.

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	437	446	0	0	0	0
normalized size	1	1.	1.28	1.31	0.	0.	0.	0.
time (sec)	N/A	0.125	0.775	0.217	0.	0.	0.	0.

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	456	481	0	0	0	0
normalized size	1	1.	1.29	1.36	0.	0.	0.	0.
time (sec)	N/A	0.135	0.858	0.226	0.	0.	0.	0.

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	428	428	515	536	0	0	0	0
normalized size	1	1.	1.2	1.25	0.	0.	0.	0.
time (sec)	N/A	0.217	1.33	0.229	0.	0.	0.	0.

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	34	37	46	63	0	0
normalized size	1	1.	0.68	0.74	0.92	1.26	0.	0.
time (sec)	N/A	0.068	0.018	0.046	0.992	1.514	0.	0.

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	73	64	0	265	0	80
normalized size	1	1.	1.26	1.1	0.	4.57	0.	1.38
time (sec)	N/A	0.084	0.031	0.046	0.	1.476	0.	1.195

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	18	36	0	42
normalized size	1	1.	1.	1.18	0.82	1.64	0.	1.91
time (sec)	N/A	0.017	0.005	0.044	0.991	1.457	0.	1.143

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	52	44	0	174	0	53
normalized size	1	1.	1.68	1.42	0.	5.61	0.	1.71
time (sec)	N/A	0.049	0.011	0.043	0.	1.554	0.	1.18

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	52	50	0	186	0	62
normalized size	1	1.	1.73	1.67	0.	6.2	0.	2.07
time (sec)	N/A	0.01	0.011	0.044	0.	1.524	0.	1.137

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	0	41	0	19
normalized size	1	1.	1.	1.13	0.	1.78	0.	0.83
time (sec)	N/A	0.04	0.007	0.043	0.	1.454	0.	1.133

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	68	73	0	306	0	0
normalized size	1	1.	1.15	1.24	0.	5.19	0.	0.
time (sec)	N/A	0.056	0.063	0.045	0.	1.523	0.	0.

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	37	0	66	0	36
normalized size	1	1.	0.67	0.71	0.	1.27	0.	0.69
time (sec)	N/A	0.084	0.015	0.043	0.	1.502	0.	1.199

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	44	94	0	366	0	0
normalized size	1	1.	0.51	1.08	0.	4.21	0.	0.
time (sec)	N/A	0.1	0.012	0.047	0.	1.57	0.	0.

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	62	91	0	0	37	0
normalized size	1	1.	0.57	0.84	0.	0.	0.34	0.
time (sec)	N/A	0.025	0.022	0.176	0.	0.	0.717	0.

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	31	22	19
normalized size	1	1.	1.	0.83	1.06	1.72	1.22	1.06
time (sec)	N/A	0.004	0.003	0.044	0.968	1.441	0.458	1.119

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	51	97	0	0	37	0
normalized size	1	1.	0.24	0.46	0.	0.	0.18	0.
time (sec)	N/A	0.053	0.008	0.171	0.	0.	0.67	0.

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	0	163	20	34
normalized size	1	1.	1.	0.8	0.	5.43	0.67	1.13
time (sec)	N/A	0.016	0.006	0.152	0.	1.528	1.034	1.161

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	0	36	0
normalized size	1	1.	0.84	0.8	0.	0.	0.41	0.
time (sec)	N/A	0.01	0.035	0.174	0.	0.	0.631	0.

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	0	161	22	31
normalized size	1	1.	1.	1.07	0.	5.96	0.81	1.15
time (sec)	N/A	0.02	0.005	0.158	0.	1.492	1.075	1.154

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	49	115	0	0	39	0
normalized size	1	1.	0.21	0.5	0.	0.	0.17	0.
time (sec)	N/A	0.072	0.009	0.18	0.	0.	0.717	0.

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	41	20	19
normalized size	1	1.	1.	0.86	1.1	1.95	0.95	0.9
time (sec)	N/A	0.005	0.004	0.045	0.944	1.453	0.617	1.188

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	51	93	0	0	41	0
normalized size	1	1.	0.46	0.85	0.	0.	0.37	0.
time (sec)	N/A	0.022	0.009	0.174	0.	0.	0.85	0.

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	62	59	0	300	95	73
normalized size	1	1.	0.85	0.81	0.	4.11	1.3	1.
time (sec)	N/A	0.022	0.024	0.045	0.	1.604	3.776	1.225

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	25	0	53	44	36
normalized size	1	1.	0.75	0.69	0.	1.47	1.22	1.
time (sec)	N/A	0.023	0.014	0.043	0.	1.456	0.458	1.116

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	39	0	238	42	54
normalized size	1	1.	1.	0.8	0.	4.86	0.86	1.1
time (sec)	N/A	0.013	0.018	0.045	0.	1.643	2.118	1.139

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	26	20	18
normalized size	1	1.	1.	0.93	1.2	1.73	1.33	1.2
time (sec)	N/A	0.003	0.002	0.041	0.938	1.568	0.368	1.129

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	153	17	31
normalized size	1	1.	1.	0.84	0.	6.12	0.68	1.24
time (sec)	N/A	0.006	0.005	0.042	0.	1.585	1.017	1.213

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	0	154	19	30
normalized size	1	1.	1.	1.16	0.	6.16	0.76	1.2
time (sec)	N/A	0.017	0.005	0.045	0.	1.542	1.047	1.2

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	0	32	19	41
normalized size	1	1.	1.	0.95	0.	1.68	1.	2.16
time (sec)	N/A	0.004	0.004	0.043	0.	1.528	0.574	1.188

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	61	48	0	263	42	65
normalized size	1	1.	1.22	0.96	0.	5.26	0.84	1.3
time (sec)	N/A	0.027	0.048	0.046	0.	1.55	2.222	1.17

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	26	0	61	46	74
normalized size	1	1.	0.66	0.59	0.	1.39	1.05	1.68
time (sec)	N/A	0.01	0.006	0.044	0.	1.499	0.754	1.199

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	28	15	11
normalized size	1	1.	1.	0.81	1.	1.75	0.94	0.69
time (sec)	N/A	0.002	0.002	0.044	0.955	1.26	0.533	1.211

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	26	15	11
normalized size	1	1.	1.	0.81	1.	1.62	0.94	0.69
time (sec)	N/A	0.002	0.002	0.042	0.946	1.222	0.465	1.252

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	26	14	7
normalized size	1	1.	1.	0.92	1.15	2.	1.08	0.54
time (sec)	N/A	0.001	0.001	0.042	0.961	1.214	0.42	1.131

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	38	0	9
normalized size	1	1.	1.	0.93	1.2	2.53	0.	0.6
time (sec)	N/A	0.001	0.002	0.046	0.952	1.294	0.	1.18

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	30	14	11
normalized size	1	1.	1.	0.92	1.17	2.5	1.17	0.92
time (sec)	N/A	0.001	0.001	0.041	0.948	1.324	0.384	1.179

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	35	15	11
normalized size	1	1.	1.	0.77	0.92	2.69	1.15	0.85
time (sec)	N/A	0.001	0.001	0.042	0.941	1.276	0.423	1.186

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	35	17	11
normalized size	1	1.	1.	0.81	1.	2.19	1.06	0.69
time (sec)	N/A	0.001	0.002	0.041	0.947	1.191	0.483	1.156

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	16	35	19	11
normalized size	1	1.	1.06	0.81	1.	2.19	1.19	0.69
time (sec)	N/A	0.002	0.003	0.042	0.955	1.255	0.546	1.199

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	15	13	16	35	19	11
normalized size	1	1.	0.94	0.81	1.	2.19	1.19	0.69
time (sec)	N/A	0.002	0.002	0.041	0.943	1.318	0.624	1.128

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	23	8	11
normalized size	1	1.	1.	0.75	0.92	1.92	0.67	0.92
time (sec)	N/A	0.001	0.001	0.047	0.949	1.28	0.055	1.133

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	23	8	11
normalized size	1	1.	1.	0.75	0.92	1.92	0.67	0.92
time (sec)	N/A	0.001	0.001	0.04	0.95	1.241	0.055	1.176

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	23	8	11
normalized size	1	1.	1.	0.75	0.92	1.92	0.67	0.92
time (sec)	N/A	0.001	0.001	0.041	0.954	1.253	0.056	1.166

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	23	8	11
normalized size	1	1.	1.	0.75	0.92	1.92	0.67	0.92
time (sec)	N/A	0.001	0.	0.04	0.951	1.206	0.054	1.167

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	15	5	7
normalized size	1	1.	1.	0.86	1.	2.14	0.71	1.
time (sec)	N/A	0.001	0.	0.039	0.95	1.181	0.05	1.151

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	22	7	9
normalized size	1	1.	1.	0.88	1.	2.75	0.88	1.12
time (sec)	N/A	0.001	0.	0.04	0.944	1.258	0.061	1.213

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	22	8	11
normalized size	1	1.	1.	0.9	1.1	2.2	0.8	1.1
time (sec)	N/A	0.001	0.	0.04	0.951	1.125	0.061	1.2

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	27	12	11
normalized size	1	1.	1.	0.75	0.92	2.25	1.	0.92
time (sec)	N/A	0.002	0.	0.04	0.947	1.282	0.064	1.2

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	27	12	11
normalized size	1	1.	1.	0.75	0.92	2.25	1.	0.92
time (sec)	N/A	0.001	0.	0.05	0.94	1.209	0.062	1.229

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	0	0	0
normalized size	1	1.	1.5	3.58	0.	0.	0.	0.
time (sec)	N/A	0.01	0.017	0.044	0.	0.	0.	0.

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	82	0	0	0	0
normalized size	1	1.	2.23	2.1	0.	0.	0.	0.
time (sec)	N/A	0.073	0.107	0.209	0.	0.	0.	0.

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	22	26	69	29	26
normalized size	1	1.	0.81	0.71	0.84	2.23	0.94	0.84
time (sec)	N/A	0.006	0.006	0.045	0.99	1.222	5.769	1.164

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	22	26	66	29	26
normalized size	1	1.	0.81	0.71	0.84	2.13	0.94	0.84
time (sec)	N/A	0.007	0.006	0.043	0.965	1.305	2.732	1.153

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	22	26	62	29	26
normalized size	1	1.	0.81	0.71	0.84	2.	0.94	0.84
time (sec)	N/A	0.007	0.006	0.042	0.946	1.224	1.698	1.172

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	26	55	27	26
normalized size	1	1.	0.86	0.76	0.9	1.9	0.93	0.9
time (sec)	N/A	0.006	0.006	0.043	0.947	1.205	0.716	1.166

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	26	55	27	26
normalized size	1	1.	0.86	0.76	0.9	1.9	0.93	0.9
time (sec)	N/A	0.006	0.008	0.045	0.948	1.214	0.969	1.149

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	26	55	27	26
normalized size	1	1.	0.86	0.76	0.9	1.9	0.93	0.9
time (sec)	N/A	0.006	0.008	0.044	0.949	1.25	1.213	1.179

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	27	55	27	27
normalized size	1	1.	0.86	0.76	0.93	1.9	0.93	0.93
time (sec)	N/A	0.006	0.008	0.045	0.953	1.252	1.889	1.134

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	49	59	147	70	62
normalized size	1	1.	1.	0.77	0.92	2.3	1.09	0.97
time (sec)	N/A	0.023	3.547	0.047	0.974	1.208	21.315	1.185

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	66	49	59	142	70	62
normalized size	1	1.	1.03	0.77	0.92	2.22	1.09	0.97
time (sec)	N/A	0.022	0.056	0.046	0.954	1.329	11.83	1.147

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	50	49	59	135	63	62
normalized size	1	1.	0.78	0.77	0.92	2.11	0.98	0.97
time (sec)	N/A	0.023	3.324	0.045	0.956	1.25	3.665	1.147

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	63	49	65	130	68	62
normalized size	1	1.	1.02	0.79	1.05	2.1	1.1	1.
time (sec)	N/A	0.022	0.042	0.046	0.965	1.312	4.932	1.165

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	49	59	124	68	62
normalized size	1	1.	0.87	0.79	0.95	2.	1.1	1.
time (sec)	N/A	0.022	0.044	0.048	0.964	1.282	5.538	1.166

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	53	49	59	123	68	62
normalized size	1	1.	0.85	0.79	0.95	1.98	1.1	1.
time (sec)	N/A	0.022	0.042	0.046	0.972	1.327	6.446	1.149

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	50	49	61	126	68	63
normalized size	1	1.	0.81	0.79	0.98	2.03	1.1	1.02
time (sec)	N/A	0.023	0.045	0.044	0.97	1.309	8.949	1.18

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	103	90	109	257	129	117
normalized size	1	1.	1.	0.87	1.06	2.5	1.25	1.14
time (sec)	N/A	0.048	3.514	0.046	0.968	1.37	63.145	1.154

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	105	90	109	246	129	117
normalized size	1	1.	1.02	0.87	1.06	2.39	1.25	1.14
time (sec)	N/A	0.043	0.092	0.046	0.976	1.193	36.958	1.149

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	103	90	109	244	112	117
normalized size	1	1.	1.	0.87	1.06	2.37	1.09	1.14
time (sec)	N/A	0.042	3.316	0.046	0.977	1.296	7.62	1.158

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	102	90	119	225	128	117
normalized size	1	1.	1.01	0.89	1.18	2.23	1.27	1.16
time (sec)	N/A	0.044	0.071	0.045	0.968	1.235	19.396	1.133

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	100	90	109	228	126	117
normalized size	1	1.	1.01	0.91	1.1	2.3	1.27	1.18
time (sec)	N/A	0.043	0.081	0.046	0.961	1.193	20.86	1.137

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	103	90	109	224	128	117
normalized size	1	1.	1.02	0.89	1.08	2.22	1.27	1.16
time (sec)	N/A	0.044	0.072	0.045	0.968	1.25	23.65	1.128

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	100	90	111	215	124	119
normalized size	1	1.	1.01	0.91	1.12	2.17	1.25	1.2
time (sec)	N/A	0.042	0.073	0.046	0.968	1.266	29.167	1.191

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	80	65	0	14407	0	0
normalized size	1	1.	0.21	0.17	0.	37.04	0.	0.
time (sec)	N/A	0.86	0.05	0.312	0.	20.862	0.	0.

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	80	64	0	11297	0	0
normalized size	1	1.	0.21	0.17	0.	29.34	0.	0.
time (sec)	N/A	0.797	0.044	0.249	0.	7.276	0.	0.

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	48	45	0	8469	0	0
normalized size	1	1.	0.15	0.14	0.	25.59	0.	0.
time (sec)	N/A	0.442	0.028	0.246	0.	3.21	0.	0.

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	46	45	0	5355	0	0
normalized size	1	1.	0.14	0.14	0.	16.18	0.	0.
time (sec)	N/A	0.403	0.025	0.259	0.	1.767	0.	0.

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	47	45	0	5956	0	0
normalized size	1	1.	0.14	0.14	0.	17.99	0.	0.
time (sec)	N/A	0.366	0.028	0.257	0.	2.088	0.	0.

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	49	42	0	8471	0	0
normalized size	1	1.	0.15	0.13	0.	25.59	0.	0.
time (sec)	N/A	0.415	0.031	0.254	0.	3.049	0.	0.

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	78	65	0	11429	0	0
normalized size	1	1.	0.21	0.18	0.	30.81	0.	0.
time (sec)	N/A	0.567	0.046	0.26	0.	8.614	0.	0.

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	82	64	0	14453	0	0
normalized size	1	1.	0.22	0.17	0.	38.96	0.	0.
time (sec)	N/A	0.512	0.053	0.261	0.	15.29	0.	0.

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	107	82	0	17871	0	0
normalized size	1	1.	0.26	0.2	0.	43.38	0.	0.
time (sec)	N/A	0.981	0.074	0.261	0.	53.644	0.	0.

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	544	544	144	149	0	0	0	0
normalized size	1	1.	0.26	0.27	0.	0.	0.	0.
time (sec)	N/A	2.577	0.261	0.266	0.	0.	0.	0.

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	520	520	144	146	0	0	0	0
normalized size	1	1.	0.28	0.28	0.	0.	0.	0.
time (sec)	N/A	1.371	0.248	0.267	0.	0.	0.	0.

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	471	124	120	0	28295	0	0
normalized size	1	1.	0.26	0.25	0.	60.07	0.	0.
time (sec)	N/A	0.917	0.196	0.267	0.	169.428	0.	0.

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	483	483	127	118	0	22881	0	0
normalized size	1	1.	0.26	0.24	0.	47.37	0.	0.
time (sec)	N/A	1.032	0.201	0.263	0.	25.769	0.	0.

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	450	109	121	0	24712	0	0
normalized size	1	1.	0.24	0.27	0.	54.92	0.	0.
time (sec)	N/A	0.71	0.207	0.27	0.	63.12	0.	0.

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	111	118	0	26324	0	0
normalized size	1	1.	0.25	0.27	0.	59.56	0.	0.
time (sec)	N/A	0.705	0.175	0.264	0.	58.717	0.	0.

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	489	489	149	146	0	0	0	0
normalized size	1	1.	0.3	0.3	0.	0.	0.	0.
time (sec)	N/A	0.998	0.229	0.27	0.	0.	0.	0.

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	503	153	144	0	0	0	0
normalized size	1	1.	0.3	0.29	0.	0.	0.	0.
time (sec)	N/A	1.276	0.235	0.264	0.	0.	0.	0.

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	573	573	190	245	0	0	0	0
normalized size	1	1.	0.33	0.43	0.	0.	0.	0.
time (sec)	N/A	2.446	0.312	0.305	0.	0.	0.	0.

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	621	254	275	0	0	0	0
normalized size	1	1.	0.41	0.44	0.	0.	0.	0.
time (sec)	N/A	1.772	0.448	0.297	0.	0.	0.	0.

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	569	569	216	242	0	0	0	0
normalized size	1	1.	0.38	0.43	0.	0.	0.	0.
time (sec)	N/A	1.908	0.421	0.285	0.	0.	0.	0.

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	569	569	219	241	0	0	0	0
normalized size	1	1.	0.38	0.42	0.	0.	0.	0.
time (sec)	N/A	1.965	0.404	0.277	0.	0.	0.	0.

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	533	533	176	244	0	0	0	0
normalized size	1	1.	0.33	0.46	0.	0.	0.	0.
time (sec)	N/A	1.452	0.391	0.302	0.	0.	0.	0.

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	533	533	177	237	0	0	0	0
normalized size	1	1.	0.33	0.44	0.	0.	0.	0.
time (sec)	N/A	1.363	0.443	0.27	0.	0.	0.	0.

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	222	277	0	0	0	0
normalized size	1	1.	0.37	0.47	0.	0.	0.	0.
time (sec)	N/A	2.31	0.412	0.272	0.	0.	0.	0.

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	224	270	0	0	0	0
normalized size	1	1.	0.38	0.45	0.	0.	0.	0.
time (sec)	N/A	2.366	0.41	0.268	0.	0.	0.	0.

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	658	658	254	321	0	0	0	0
normalized size	1	1.	0.39	0.49	0.	0.	0.	0.
time (sec)	N/A	5.485	0.508	0.273	0.	0.	0.	0.

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	658	658	258	316	0	0	0	0
normalized size	1	1.	0.39	0.48	0.	0.	0.	0.
time (sec)	N/A	5.792	0.472	0.266	0.	0.	0.	0.

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	365	0	0	0	0	0
normalized size	1	1.	2.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.584	0.322	0.	0.	0.	0.

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	342	0	0	0	0	0
normalized size	1	1.	2.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.431	0.271	0.	0.	0.	0.

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	342	0	0	0	0	0
normalized size	1	1.	2.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.38	0.274	0.	0.	0.	0.

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	345	0	0	0	0	0
normalized size	1	1.	2.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.365	0.27	0.	0.	0.	0.

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	459	0	0	0	0	0
normalized size	1	1.	3.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.856	0.315	0.	0.	0.	0.

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	417	0	0	0	0	0
normalized size	1	1.	2.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.728	0.28	0.	0.	0.	0.

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	415	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.732	0.289	0.	0.	0.	0.

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	384	0	0	0	0	0
normalized size	1	1.	2.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.748	0.287	0.	0.	0.	0.

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	173	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.093	0.06	0.	0.	0.	0.

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	173	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.103	0.059	0.	0.	0.	0.

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	171	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.081	0.063	0.	0.	0.	0.

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	348	0	0	0	0	0
normalized size	1	1.	2.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.501	0.285	0.	0.	0.	0.

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	348	0	0	0	0	0
normalized size	1	1.	2.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.37	0.064	0.	0.	0.	0.

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	367	0	0	0	0	0
normalized size	1	1.	2.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.523	0.058	0.	0.	0.	0.

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	366	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.496	0.074	0.	0.	0.	0.

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	409	0	0	0	0	0
normalized size	1	1.	2.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.729	0.343	0.	0.	0.	0.

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	111	782	0	1474	4451	1528
normalized size	1	1.	0.71	5.01	0.	9.45	28.53	9.79
time (sec)	N/A	0.1	0.13	0.048	0.	1.46	7.35	1.181

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	0	587	1486	606
normalized size	1	1.	0.69	2.98	0.	5.81	14.71	6.
time (sec)	N/A	0.053	0.053	0.048	0.	1.477	2.764	1.132

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	35	78	0	159	314	161
normalized size	1	1.	0.67	1.5	0.	3.06	6.04	3.1
time (sec)	N/A	0.02	0.03	0.043	0.	1.293	0.875	1.107

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	59	0	0	0	0	0
normalized size	1	1.	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.251	0.042	0.065	0.	0.	0.	0.

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	78	0	0	0	0	0
normalized size	1	1.	0.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.696	0.08	0.061	0.	0.	0.	0.

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	357	0	0	0	0	0
normalized size	1	1.	2.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.278	0.051	0.	0.	0.	0.

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.1	0.051	0.	0.	0.	0.

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.109	0.052	0.	0.	0.	0.

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	221	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.172	0.05	0.	0.	0.	0.

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	179	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.244	0.1	0.	0.	0.	0.

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	257	162	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.373	0.242	0.087	0.	0.	0.	0.

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	162	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	0.233	0.079	0.	0.	0.	0.

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	162	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.196	0.072	0.	0.	0.	0.

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	135	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.099	0.059	0.	0.	0.	0.

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	152	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	0.213	0.053	0.	0.	0.	0.

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	163	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.21	0.065	0.	0.	0.	0.

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	159	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.225	0.073	0.	0.	0.	0.

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.185	0.069	0.	0.	0.	0.

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.161	0.064	0.	0.	0.	0.

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.178	0.053	0.	0.	0.	0.

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	164	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.161	0.062	0.	0.	0.	0.

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.171	0.069	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [327] had the largest ratio of [0.5263]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2	A	3	3	1.	22	0.136
3	A	2	2	1.	22	0.091
4	A	2	2	1.	22	0.091
5	A	3	3	1.03	22	0.136
6	A	4	3	1.02	22	0.136
7	A	5	3	1.1	22	0.136
8	A	9	5	1.	16	0.312
9	A	3	3	1.	16	0.188
10	A	9	5	1.	16	0.312
11	A	3	2	1.	12	0.167
12	A	3	2	1.	12	0.167
13	A	9	5	1.	12	0.417
14	A	9	5	1.	12	0.417
15	A	9	5	1.	12	0.417
16	A	2	2	1.	16	0.125
17	A	2	2	1.	16	0.125
18	A	2	2	1.	16	0.125
19	A	2	2	1.	16	0.125
20	A	2	2	1.	14	0.143
21	A	1	1	1.	11	0.091
22	A	2	2	1.	16	0.125
23	A	2	2	1.	16	0.125
24	A	2	2	1.	16	0.125
25	A	2	2	1.	16	0.125
26	A	2	2	1.	16	0.125
27	A	2	2	1.	16	0.125
28	A	2	2	1.	16	0.125
29	A	2	2	1.	16	0.125
30	A	2	2	1.	16	0.125
31	A	2	2	1.	16	0.125
32	A	2	2	1.	16	0.125
33	A	2	2	1.	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
34	A	1	1	1.	11	0.091
35	A	2	2	1.	16	0.125
36	A	2	2	1.	16	0.125
37	A	2	2	1.	16	0.125
38	A	2	2	1.	16	0.125
39	A	2	2	1.	16	0.125
40	A	2	2	1.	16	0.125
41	A	2	2	1.	16	0.125
42	A	1	1	1.	16	0.062
43	A	1	1	1.	16	0.062
44	A	1	1	1.	16	0.062
45	A	1	1	1.	16	0.062
46	A	1	1	1.	14	0.071
47	A	1	1	1.	11	0.091
48	A	1	1	1.	16	0.062
49	A	1	1	1.	16	0.062
50	A	1	1	1.	16	0.062
51	A	1	1	1.	16	0.062
52	A	1	1	1.	16	0.062
53	A	1	1	1.	16	0.062
54	A	1	1	1.	16	0.062
55	A	1	1	1.	16	0.062
56	A	1	1	1.	16	0.062
57	A	1	1	1.	16	0.062
58	A	1	1	1.	16	0.062
59	A	1	1	1.	14	0.071
60	A	1	1	1.	11	0.091
61	A	1	1	1.	16	0.062
62	A	1	1	1.	16	0.062
63	A	1	1	1.	16	0.062
64	A	1	1	1.	16	0.062
65	A	1	1	1.	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
66	A	1	1	1.	16	0.062
67	A	1	1	1.	16	0.062
68	A	1	1	1.	16	0.062
69	A	1	1	1.	16	0.062
70	A	1	1	1.	14	0.071
71	A	1	1	1.	11	0.091
72	A	1	1	1.	16	0.062
73	A	1	1	1.	16	0.062
74	A	1	1	1.	16	0.062
75	A	1	1	1.	16	0.062
76	A	1	1	1.	16	0.062
77	A	1	1	1.	16	0.062
78	A	1	1	1.	16	0.062
79	A	1	1	1.	16	0.062
80	A	1	1	1.	16	0.062
81	A	1	1	1.	16	0.062
82	A	1	1	1.	16	0.062
83	A	1	1	1.	16	0.062
84	A	1	1	1.	16	0.062
85	A	1	1	1.	16	0.062
86	A	1	1	1.	14	0.071
87	A	1	1	1.	11	0.091
88	A	1	1	1.	16	0.062
89	A	1	1	1.	16	0.062
90	A	1	1	1.	16	0.062
91	A	1	1	1.	16	0.062
92	A	1	1	1.	16	0.062
93	A	1	1	1.	16	0.062
94	A	1	1	1.	16	0.062
95	A	2	2	1.	16	0.125
96	A	2	2	1.	16	0.125
97	A	2	2	1.	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
98	A	1	1	1.	16	0.062
99	A	1	1	1.	16	0.062
100	A	1	1	1.	16	0.062
101	A	1	1	1.	14	0.071
102	A	1	1	1.	11	0.091
103	A	1	1	1.	16	0.062
104	A	1	1	1.	16	0.062
105	A	1	1	1.	16	0.062
106	A	1	1	1.	16	0.062
107	A	2	2	1.	16	0.125
108	A	2	2	1.	16	0.125
109	A	2	2	1.	16	0.125
110	A	1	1	1.	16	0.062
111	A	1	1	1.	16	0.062
112	A	1	1	1.	16	0.062
113	A	1	1	1.	14	0.071
114	A	1	1	1.	11	0.091
115	A	1	1	1.	16	0.062
116	A	1	1	1.	16	0.062
117	A	1	1	1.	16	0.062
118	A	1	1	1.	16	0.062
119	A	2	2	1.	16	0.125
120	A	1	1	1.	16	0.062
121	A	1	1	1.	16	0.062
122	A	1	1	1.	16	0.062
123	A	1	1	1.	16	0.062
124	A	1	1	1.	14	0.071
125	A	2	2	1.	16	0.125
126	A	2	2	1.	16	0.125
127	A	2	2	1.	16	0.125
128	A	2	2	1.	16	0.125
129	A	2	2	1.	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	2	2	1.	16	0.125
131	A	2	2	1.	16	0.125
132	A	2	2	1.	16	0.125
133	A	2	2	1.	16	0.125
134	A	2	1	1.	15	0.067
135	A	2	1	1.	13	0.077
136	A	1	0	1.	11	0.
137	A	2	1	1.	15	0.067
138	A	2	1	1.	15	0.067
139	A	2	1	1.	15	0.067
140	A	2	1	1.	15	0.067
141	A	2	1	1.	15	0.067
142	A	2	1	1.	15	0.067
143	A	2	1	1.	15	0.067
144	A	2	1	1.	15	0.067
145	A	3	2	1.	13	0.154
146	A	4	3	1.	17	0.176
147	A	3	2	1.	17	0.118
148	A	2	2	1.	17	0.118
149	A	3	2	1.	17	0.118
150	A	4	3	1.	17	0.176
151	A	3	2	1.	17	0.118
152	A	4	3	1.	17	0.176
153	A	3	2	1.	17	0.118
154	A	4	3	1.	17	0.176
155	A	3	2	1.	17	0.118
156	A	2	2	1.	17	0.118
157	A	3	2	1.	17	0.118
158	A	3	2	1.	17	0.118
159	A	4	3	1.	17	0.176
160	A	3	2	1.	17	0.118
161	A	2	2	1.	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
162	A	3	2	1.	17	0.118
163	A	4	3	1.	17	0.176
164	A	3	2	1.	17	0.118
165	A	4	3	1.	17	0.176
166	A	3	2	1.	17	0.118
167	A	4	3	1.	17	0.176
168	A	3	2	1.	17	0.118
169	A	4	3	1.	17	0.176
170	A	3	2	1.	17	0.118
171	A	2	2	1.	17	0.118
172	A	3	2	1.	17	0.118
173	A	4	4	1.	17	0.235
174	A	4	3	1.	17	0.176
175	A	4	3	1.	17	0.176
176	A	4	3	1.	17	0.176
177	A	4	3	1.	17	0.176
178	A	4	3	1.	17	0.176
179	A	4	3	1.	17	0.176
180	A	3	3	1.	17	0.176
181	A	2	2	1.	17	0.118
182	A	2	2	1.	17	0.118
183	A	5	5	1.	15	0.333
184	A	3	3	1.	13	0.231
185	A	4	3	1.	17	0.176
186	A	4	3	1.	17	0.176
187	A	4	3	1.	17	0.176
188	A	5	3	1.	17	0.176
189	A	4	3	1.	17	0.176
190	A	5	4	1.	17	0.235
191	A	4	3	1.	17	0.176
192	A	5	4	1.	17	0.235
193	A	4	3	1.	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
194	A	4	4	1.	17	0.235
195	A	4	3	1.	17	0.176
196	A	3	3	1.	17	0.176
197	A	2	2	1.	17	0.118
198	A	3	3	1.	17	0.176
199	A	4	3	1.	17	0.176
200	A	4	4	1.	17	0.235
201	A	4	3	1.	15	0.2
202	A	5	4	1.	13	0.308
203	A	4	3	1.	17	0.176
204	A	6	4	1.	17	0.235
205	A	6	4	1.	17	0.235
206	A	4	3	1.	17	0.176
207	A	5	4	1.	17	0.235
208	A	4	3	1.	17	0.176
209	A	4	3	1.	17	0.176
210	A	2	2	1.	17	0.118
211	A	4	4	1.	17	0.235
212	A	2	2	1.	17	0.118
213	A	4	3	1.	17	0.176
214	A	4	3	1.	17	0.176
215	A	5	4	1.	17	0.235
216	A	4	3	1.	17	0.176
217	A	6	4	1.	17	0.235
218	A	4	3	1.	15	0.2
219	A	7	4	1.	13	0.308
220	A	4	3	1.	17	0.176
221	A	6	6	1.	19	0.316
222	A	5	5	1.	19	0.263
223	A	4	4	1.	17	0.235
224	A	4	4	1.	19	0.21
225	A	4	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
226	A	1	1	1.	19	0.053
227	A	2	2	1.	19	0.105
228	A	3	2	1.	19	0.105
229	A	4	2	1.	19	0.105
230	A	5	2	1.	19	0.105
231	A	3	2	1.	19	0.105
232	A	2	2	1.	19	0.105
233	A	1	1	1.	15	0.067
234	A	3	3	1.	19	0.158
235	A	3	3	1.	19	0.158
236	A	4	4	1.	19	0.21
237	A	5	4	1.	19	0.21
238	A	6	5	1.	19	0.263
239	A	5	4	1.	17	0.235
240	A	5	5	1.	19	0.263
241	A	5	4	1.	19	0.21
242	A	5	5	1.	19	0.263
243	A	5	4	1.	19	0.21
244	A	1	1	1.	19	0.053
245	A	2	2	1.	19	0.105
246	A	3	2	1.	19	0.105
247	A	4	2	1.	19	0.105
248	A	5	2	1.	19	0.105
249	A	5	3	1.	19	0.158
250	A	4	3	1.	19	0.158
251	A	3	3	1.	19	0.158
252	A	2	2	1.	15	0.133
253	A	1	1	1.	19	0.053
254	A	4	3	1.	19	0.158
255	A	4	4	1.	19	0.21
256	A	4	3	1.	19	0.158
257	A	5	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
258	A	6	4	1.	19	0.21
259	A	7	4	1.	19	0.21
260	A	6	5	1.	19	0.263
261	A	5	5	1.	19	0.263
262	A	4	4	1.	19	0.21
263	A	3	3	1.	17	0.176
264	A	1	1	1.	19	0.053
265	A	2	2	1.	19	0.105
266	A	3	2	1.	19	0.105
267	A	4	2	1.	19	0.105
268	A	2	2	1.	19	0.105
269	A	1	1	1.	19	0.053
270	A	2	2	1.	15	0.133
271	A	3	3	1.	19	0.158
272	A	4	3	1.	19	0.158
273	A	6	6	1.	19	0.316
274	A	5	5	1.	19	0.263
275	A	4	4	1.	19	0.21
276	A	1	1	1.	19	0.053
277	A	2	2	1.	17	0.118
278	A	3	3	1.	19	0.158
279	A	4	3	1.	19	0.158
280	A	5	3	1.	19	0.158
281	A	2	2	1.	19	0.105
282	A	1	1	1.	19	0.053
283	A	3	3	1.	19	0.158
284	A	4	4	1.	15	0.267
285	A	5	4	1.	19	0.21
286	A	4	4	1.	19	0.21
287	A	4	4	1.	19	0.21
288	A	4	4	1.	19	0.21
289	A	4	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
290	A	4	4	1.	19	0.21
291	A	4	4	1.	20	0.2
292	A	2	1	1.	17	0.059
293	A	2	1	1.	17	0.059
294	A	2	1	1.	17	0.059
295	A	2	1	1.	17	0.059
296	A	2	1	1.	17	0.059
297	A	2	1	1.	17	0.059
298	A	2	1	1.	17	0.059
299	A	2	1	1.	17	0.059
300	A	3	2	1.	19	0.105
301	A	3	2	1.	19	0.105
302	A	3	2	1.	19	0.105
303	A	3	2	1.	19	0.105
304	A	3	2	1.	19	0.105
305	A	3	2	1.	19	0.105
306	A	3	2	1.	19	0.105
307	A	3	2	1.	19	0.105
308	A	3	2	1.	19	0.105
309	A	3	2	1.	19	0.105
310	A	3	2	1.	19	0.105
311	A	3	2	1.	19	0.105
312	A	3	2	1.	19	0.105
313	A	3	2	1.	19	0.105
314	A	3	2	1.	19	0.105
315	A	3	2	1.	19	0.105
316	A	13	9	1.	19	0.474
317	A	13	9	1.	19	0.474
318	A	12	9	1.	19	0.474
319	A	12	9	1.	19	0.474
320	A	11	8	1.	19	0.421
321	A	11	8	1.	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
322	A	12	9	1.	19	0.474
323	A	12	9	1.	19	0.474
324	A	13	9	1.	19	0.474
325	A	13	9	1.	19	0.474
326	A	14	9	1.	19	0.474
327	A	14	10	1.	19	0.526
328	A	13	10	1.	19	0.526
329	A	13	10	1.	19	0.526
330	A	12	9	1.	19	0.474
331	A	12	9	1.	19	0.474
332	A	12	9	1.	19	0.474
333	A	12	9	1.	19	0.474
334	A	13	10	1.	19	0.526
335	A	13	10	1.	19	0.526
336	A	14	10	1.	19	0.526
337	A	14	10	1.	19	0.526
338	A	15	10	1.	19	0.526
339	A	14	10	1.	19	0.526
340	A	13	9	1.	19	0.474
341	A	13	9	1.	19	0.474
342	A	13	10	1.	19	0.526
343	A	13	10	1.	19	0.526
344	A	13	9	1.	19	0.474
345	A	13	9	1.	19	0.474
346	A	14	10	1.	19	0.526
347	A	14	10	1.	19	0.526
348	A	15	10	1.	19	0.526
349	A	15	10	1.	19	0.526
350	A	16	10	1.	19	0.526
351	A	16	10	1.	19	0.526
352	A	8	7	1.	21	0.333
353	A	6	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
354	A	7	7	1.	21	0.333
355	A	5	5	1.	21	0.238
356	A	6	6	1.	21	0.286
357	A	4	4	1.	21	0.19
358	A	6	6	1.	21	0.286
359	A	4	4	1.	21	0.19
360	A	7	7	1.	21	0.333
361	A	5	5	1.	21	0.238
362	A	8	7	1.	21	0.333
363	A	6	5	1.	21	0.238
364	A	9	7	1.	21	0.333
365	A	7	5	1.	21	0.238
366	A	8	7	1.	21	0.333
367	A	6	5	1.	21	0.238
368	A	7	6	1.	21	0.286
369	A	5	4	1.	21	0.19
370	A	7	7	1.	21	0.333
371	A	5	5	1.	21	0.238
372	A	7	6	1.	21	0.286
373	A	5	4	1.	21	0.19
374	A	8	7	1.	21	0.333
375	A	6	5	1.	21	0.238
376	A	9	7	1.	21	0.333
377	A	7	5	1.	21	0.238
378	A	6	4	1.	21	0.19
379	A	7	6	1.	21	0.286
380	A	5	4	1.	21	0.19
381	A	6	6	1.	21	0.286
382	A	4	4	1.	21	0.19
383	A	5	5	1.	21	0.238
384	A	3	3	1.	21	0.143
385	A	6	6	1.	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
386	A	4	4	1.	21	0.19
387	A	7	6	1.	21	0.286
388	A	5	4	1.	21	0.19
389	A	8	6	1.	21	0.286
390	A	6	4	1.	21	0.19
391	A	6	5	1.	21	0.238
392	A	7	7	1.	21	0.333
393	A	5	5	1.	21	0.238
394	A	6	6	1.	21	0.286
395	A	4	4	1.	21	0.19
396	A	6	6	1.	21	0.286
397	A	4	4	1.	21	0.19
398	A	7	7	1.	21	0.333
399	A	5	5	1.	21	0.238
400	A	8	7	1.	21	0.333
401	A	6	5	1.	21	0.238
402	A	9	7	1.	21	0.333
403	A	4	3	1.	19	0.158
404	A	4	3	1.	19	0.158
405	A	2	1	1.	17	0.059
406	A	3	3	1.	19	0.158
407	A	3	3	1.	19	0.158
408	A	3	3	1.	19	0.158
409	A	2	1	1.	22	0.045
410	A	2	1	1.	22	0.045
411	A	2	1	1.	20	0.05
412	A	1	0	1.	18	0.
413	A	2	1	1.	22	0.045
414	A	2	1	1.	22	0.045
415	A	2	1	1.	22	0.045
416	A	2	1	1.	22	0.045
417	A	2	1	1.	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
418	A	2	1	1.	22	0.045
419	A	2	1	1.	22	0.045
420	A	2	1	1.	22	0.045
421	A	3	2	1.	24	0.083
422	A	4	3	1.	24	0.125
423	A	3	2	1.	24	0.083
424	A	4	3	1.	24	0.125
425	A	3	2	1.	24	0.083
426	A	2	2	1.	22	0.091
427	A	3	2	1.	20	0.1
428	A	4	3	1.	24	0.125
429	A	3	2	1.	24	0.083
430	A	4	3	1.	24	0.125
431	A	3	2	1.	24	0.083
432	A	4	3	1.	24	0.125
433	A	3	2	1.	24	0.083
434	A	4	3	1.	24	0.125
435	A	3	2	1.	24	0.083
436	A	4	3	1.	24	0.125
437	A	3	2	1.	24	0.083
438	A	2	2	1.	24	0.083
439	A	3	2	1.	24	0.083
440	A	4	4	1.	24	0.167
441	A	3	2	1.	24	0.083
442	A	4	3	1.	24	0.125
443	A	3	2	1.	24	0.083
444	A	3	2	1.	24	0.083
445	A	4	3	1.	24	0.125
446	A	3	2	1.	24	0.083
447	A	4	3	1.	24	0.125
448	A	3	2	1.	24	0.083
449	A	4	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
450	A	3	2	1.	24	0.083
451	A	2	2	1.	22	0.091
452	A	3	2	1.	20	0.1
453	A	4	3	1.	24	0.125
454	A	3	2	1.	24	0.083
455	A	4	3	1.	24	0.125
456	A	3	2	1.	24	0.083
457	A	4	3	1.	24	0.125
458	A	3	2	1.	24	0.083
459	A	4	3	1.	24	0.125
460	A	3	2	1.	24	0.083
461	A	4	3	1.	24	0.125
462	A	3	2	1.	24	0.083
463	A	4	3	1.	24	0.125
464	A	3	2	1.	24	0.083
465	A	4	3	1.	24	0.125
466	A	3	2	1.	24	0.083
467	A	2	2	1.	24	0.083
468	A	3	2	1.	24	0.083
469	A	4	4	1.	24	0.167
470	A	3	2	1.	24	0.083
471	A	5	4	1.	24	0.167
472	A	3	2	1.	24	0.083
473	A	6	4	1.	24	0.167
474	A	3	2	1.	24	0.083
475	A	4	3	1.	24	0.125
476	A	4	3	1.	24	0.125
477	A	4	3	1.	24	0.125
478	A	4	3	1.	24	0.125
479	A	4	3	1.	24	0.125
480	A	2	2	1.	22	0.091
481	A	4	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
482	A	4	3	1.	24	0.125
483	A	4	3	1.	24	0.125
484	A	5	4	1.	24	0.167
485	A	5	4	1.	24	0.167
486	A	5	4	1.	24	0.167
487	A	4	4	1.	24	0.167
488	A	3	3	1.	24	0.125
489	A	3	3	1.	20	0.15
490	A	4	4	1.	24	0.167
491	A	5	4	1.	24	0.167
492	A	6	4	1.	24	0.167
493	A	4	3	1.	24	0.125
494	A	4	3	1.	24	0.125
495	A	4	3	1.	24	0.125
496	A	2	2	1.	24	0.083
497	A	4	3	1.	24	0.125
498	A	2	2	1.	22	0.091
499	A	4	3	1.	24	0.125
500	A	4	3	1.	24	0.125
501	A	4	3	1.	24	0.125
502	A	7	4	1.	24	0.167
503	A	7	4	1.	24	0.167
504	A	6	4	1.	24	0.167
505	A	5	3	1.	24	0.125
506	A	5	4	1.	24	0.167
507	A	5	4	1.	24	0.167
508	A	5	3	1.	20	0.15
509	A	6	4	1.	24	0.167
510	A	7	4	1.	24	0.167
511	A	8	4	1.	24	0.167
512	A	4	3	1.	24	0.125
513	A	4	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
514	A	4	3	1.	24	0.125
515	A	2	2	1.	24	0.083
516	A	4	4	1.	24	0.167
517	A	4	3	1.	24	0.125
518	A	4	3	1.	24	0.125
519	A	2	2	1.	22	0.091
520	A	4	3	1.	24	0.125
521	A	4	3	1.	24	0.125
522	A	4	3	1.	24	0.125
523	A	9	4	1.	24	0.167
524	A	9	4	1.	24	0.167
525	A	8	4	1.	24	0.167
526	A	7	3	1.	24	0.125
527	A	7	4	1.	24	0.167
528	A	7	4	1.	24	0.167
529	A	7	4	1.	24	0.167
530	A	7	4	1.	24	0.167
531	A	7	3	1.	20	0.15
532	A	8	4	1.	24	0.167
533	A	9	4	1.	24	0.167
534	A	10	4	1.	24	0.167
535	A	3	3	1.	12	0.25
536	A	2	2	1.	14	0.143
537	A	3	3	1.	16	0.188
538	A	4	3	1.	16	0.188
539	A	2	2	1.	14	0.143
540	A	4	3	1.	16	0.188
541	A	4	3	1.	26	0.115
542	A	3	3	1.	26	0.115
543	A	2	2	1.	24	0.083
544	A	3	2	1.	26	0.077
545	A	3	2	1.	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
546	A	3	3	1.	26	0.115
547	A	1	1	1.	26	0.038
548	A	4	3	1.	26	0.115
549	A	4	3	1.	26	0.115
550	A	3	2	1.	26	0.077
551	A	3	2	1.	26	0.077
552	A	2	1	1.	22	0.045
553	A	3	2	1.	26	0.077
554	A	3	2	1.	26	0.077
555	A	3	2	1.	26	0.077
556	A	3	2	1.	26	0.077
557	A	3	2	1.	26	0.077
558	A	4	3	1.	26	0.115
559	A	4	3	1.	26	0.115
560	A	3	2	1.12	26	0.077
561	A	3	3	1.	26	0.115
562	A	2	2	1.	24	0.083
563	A	4	3	1.	26	0.115
564	A	4	3	1.	26	0.115
565	A	4	3	1.	26	0.115
566	A	4	3	1.	26	0.115
567	A	3	3	1.	26	0.115
568	A	1	1	1.	26	0.038
569	A	4	3	1.	26	0.115
570	A	4	3	1.	26	0.115
571	A	4	3	1.	26	0.115
572	A	3	2	1.	26	0.077
573	A	3	2	1.	26	0.077
574	A	3	2	1.	26	0.077
575	A	3	2	1.	26	0.077
576	A	3	2	1.	22	0.091
577	A	3	2	1.	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
578	A	3	2	1.	26	0.077
579	A	3	2	1.	26	0.077
580	A	3	2	1.	26	0.077
581	A	3	2	1.	26	0.077
582	A	3	2	1.	26	0.077
583	A	3	2	1.	26	0.077
584	A	3	2	1.	26	0.077
585	A	4	3	1.	26	0.115
586	A	4	3	1.	26	0.115
587	A	3	2	1.	26	0.077
588	A	4	3	1.	26	0.115
589	A	4	3	1.	26	0.115
590	A	3	3	1.	26	0.115
591	A	2	2	1.	24	0.083
592	A	4	3	1.	26	0.115
593	A	4	3	1.	26	0.115
594	A	4	3	1.	26	0.115
595	A	4	3	1.	26	0.115
596	A	4	3	1.	26	0.115
597	A	4	3	1.	26	0.115
598	A	3	3	1.	26	0.115
599	A	1	1	1.	26	0.038
600	A	5	4	1.	26	0.154
601	A	4	3	1.	26	0.115
602	A	4	3	1.	26	0.115
603	A	4	3	1.	26	0.115
604	A	4	3	1.	26	0.115
605	A	3	2	1.	26	0.077
606	A	3	2	1.	26	0.077
607	A	3	2	1.	26	0.077
608	A	3	2	1.	26	0.077
609	A	3	2	1.	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
610	A	3	2	1.	26	0.077
611	A	3	2	1.	22	0.091
612	A	3	2	1.	26	0.077
613	A	3	2	1.	26	0.077
614	A	3	2	1.	26	0.077
615	A	3	2	1.	26	0.077
616	A	3	2	1.	26	0.077
617	A	3	2	1.	26	0.077
618	A	3	2	1.	26	0.077
619	A	3	2	1.	26	0.077
620	A	3	2	1.	26	0.077
621	A	3	2	1.	26	0.077
622	A	3	2	1.	26	0.077
623	A	3	2	1.	26	0.077
624	A	4	3	1.	26	0.115
625	A	4	4	1.	26	0.154
626	A	3	3	1.	24	0.125
627	A	5	5	1.	26	0.192
628	A	4	3	1.	26	0.115
629	A	4	3	1.	26	0.115
630	A	3	3	1.	26	0.115
631	A	2	2	1.	22	0.091
632	A	3	3	1.	26	0.115
633	A	4	3	1.	26	0.115
634	A	4	3	1.	26	0.115
635	A	4	3	1.	26	0.115
636	A	3	3	1.68	26	0.115
637	A	2	2	1.	24	0.083
638	A	4	3	1.	26	0.115
639	A	4	3	1.	26	0.115
640	A	4	3	1.	26	0.115
641	A	4	4	1.	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
642	A	4	3	1.	22	0.136
643	A	5	4	1.	26	0.154
644	A	6	4	1.	26	0.154
645	A	4	3	1.	26	0.115
646	A	4	3	1.	26	0.115
647	A	3	3	1.	26	0.115
648	A	1	1	1.	26	0.038
649	A	3	3	1.	26	0.115
650	A	2	2	1.	24	0.083
651	A	4	3	1.	26	0.115
652	A	4	3	1.	26	0.115
653	A	6	4	1.	26	0.154
654	A	6	4	1.	26	0.154
655	A	6	4	1.	26	0.154
656	A	6	3	1.	22	0.136
657	A	7	4	1.	26	0.154
658	A	8	4	1.	26	0.154
659	A	4	4	1.	26	0.154
660	A	3	3	1.	22	0.136
661	A	4	4	1.	26	0.154
662	A	6	6	1.	26	0.231
663	A	6	6	1.	22	0.273
664	A	7	7	1.	26	0.269
665	A	2	1	1.	26	0.038
666	A	2	1	1.	26	0.038
667	A	2	1	1.	26	0.038
668	A	2	1	1.	26	0.038
669	A	2	1	1.	26	0.038
670	A	2	1	1.	26	0.038
671	A	2	1	1.	26	0.038
672	A	3	2	1.	28	0.071
673	A	3	2	1.	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
674	A	3	2	1.	28	0.071
675	A	3	2	1.	28	0.071
676	A	3	2	1.	28	0.071
677	A	3	2	1.	28	0.071
678	A	3	2	1.	28	0.071
679	A	3	2	1.	28	0.071
680	A	3	2	1.	28	0.071
681	A	3	2	1.	28	0.071
682	A	3	2	1.	28	0.071
683	A	3	2	1.	28	0.071
684	A	3	2	1.	28	0.071
685	A	3	2	1.	28	0.071
686	A	14	10	1.	28	0.357
687	A	13	10	1.	28	0.357
688	A	13	10	1.	28	0.357
689	A	12	9	1.	28	0.321
690	A	12	9	1.	28	0.321
691	A	12	9	1.	28	0.321
692	A	12	9	1.	28	0.321
693	A	13	10	1.	28	0.357
694	A	13	10	1.	28	0.357
695	A	14	10	1.	28	0.357
696	A	16	10	1.	28	0.357
697	A	15	10	1.	28	0.357
698	A	15	10	1.	28	0.357
699	A	14	9	1.	28	0.321
700	A	14	9	1.	28	0.321
701	A	14	10	1.	28	0.357
702	A	14	10	1.	28	0.357
703	A	14	10	1.	28	0.357
704	A	14	10	1.	28	0.357
705	A	14	9	1.	28	0.321

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
706	A	14	9	1.	28	0.321
707	A	15	10	1.	28	0.357
708	A	15	10	1.	28	0.357
709	A	16	10	1.	28	0.357
710	A	18	10	1.	28	0.357
711	A	17	10	1.	28	0.357
712	A	17	10	1.	28	0.357
713	A	16	9	1.	28	0.321
714	A	16	9	1.	28	0.321
715	A	16	10	1.	28	0.357
716	A	16	10	1.	28	0.357
717	A	16	10	1.	28	0.357
718	A	16	10	1.	28	0.357
719	A	16	10	1.	28	0.357
720	A	16	10	1.	28	0.357
721	A	16	10	1.	28	0.357
722	A	16	10	1.	28	0.357
723	A	16	9	1.	28	0.321
724	A	16	9	1.	28	0.321
725	A	17	10	1.	28	0.357
726	A	17	10	1.	28	0.357
727	A	18	10	1.	28	0.357
728	A	3	2	1.	30	0.067
729	A	3	2	1.	30	0.067
730	A	3	2	1.	30	0.067
731	A	3	2	1.	30	0.067
732	A	3	2	1.	30	0.067
733	A	3	2	1.	30	0.067
734	A	3	2	1.	30	0.067
735	A	3	2	1.	30	0.067
736	A	3	2	1.	30	0.067
737	A	3	2	1.	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
738	A	3	2	1.	30	0.067
739	A	3	2	1.	30	0.067
740	A	3	2	1.	30	0.067
741	A	3	2	1.	30	0.067
742	A	3	2	1.	30	0.067
743	A	3	2	1.	30	0.067
744	A	3	2	1.	30	0.067
745	A	3	2	1.	30	0.067
746	A	3	2	1.	30	0.067
747	A	3	2	1.	30	0.067
748	A	3	2	1.	30	0.067
749	A	13	9	1.	30	0.3
750	A	12	9	1.	30	0.3
751	A	12	9	1.	30	0.3
752	A	11	8	1.	30	0.267
753	A	11	8	1.	30	0.267
754	A	12	9	1.	30	0.3
755	A	12	9	1.	30	0.3
756	A	13	9	1.	30	0.3
757	A	15	10	1.	30	0.333
758	A	14	10	1.	30	0.333
759	A	14	10	1.	30	0.333
760	A	13	9	1.	30	0.3
761	A	13	9	1.	30	0.3
762	A	13	10	1.	30	0.333
763	A	13	10	1.	30	0.333
764	A	13	9	1.	30	0.3
765	A	13	9	1.	30	0.3
766	A	14	10	1.	30	0.333
767	A	14	10	1.	30	0.333
768	A	15	10	1.	30	0.333
769	A	17	10	1.	30	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
770	A	16	10	1.	30	0.333
771	A	16	10	1.	30	0.333
772	A	15	9	1.	30	0.3
773	A	15	9	1.	30	0.3
774	A	15	10	1.	30	0.333
775	A	15	10	1.	30	0.333
776	A	15	10	1.	30	0.333
777	A	15	10	1.	30	0.333
778	A	15	10	1.	30	0.333
779	A	15	10	1.	30	0.333
780	A	15	9	1.	30	0.3
781	A	15	9	1.	30	0.3
782	A	16	10	1.	30	0.333
783	A	16	10	1.	30	0.333
784	A	17	10	1.	30	0.333
785	A	3	2	1.	26	0.077
786	A	3	2	1.	26	0.077
787	A	2	1	1.	24	0.042
788	A	2	2	1.	26	0.077
789	A	2	2	1.	26	0.077
790	A	2	2	1.	26	0.077
791	A	3	2	1.	28	0.071
792	A	3	2	1.	28	0.071
793	A	3	2	1.	28	0.071
794	A	2	2	1.	28	0.071
795	A	2	2	1.	28	0.071
796	A	2	2	1.	28	0.071
797	A	2	2	1.04	26	0.077
798	A	4	3	1.	24	0.125
799	A	4	3	1.	24	0.125
800	A	4	3	1.	24	0.125
801	A	2	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
802	A	3	3	1.	24	0.125
803	A	3	3	1.	24	0.125
804	A	2	2	1.	24	0.083
805	A	2	2	1.	24	0.083
806	A	2	2	1.	20	0.1
807	A	2	2	1.	24	0.083
808	A	2	2	1.	24	0.083
809	A	2	2	1.	28	0.071
810	A	2	2	1.	28	0.071
811	A	2	2	1.	28	0.071
812	A	2	2	1.	28	0.071
813	A	2	2	1.	28	0.071
814	A	2	1	1.	16	0.062
815	A	2	1	1.	14	0.071
816	A	1	0	1.	12	0.
817	A	2	1	1.	16	0.062
818	A	2	1	1.	16	0.062
819	A	2	1	1.	16	0.062
820	A	2	1	1.	16	0.062
821	A	2	1	1.	16	0.062
822	A	2	1	1.	16	0.062
823	A	2	1	1.	16	0.062
824	A	2	1	1.	16	0.062
825	A	2	1	1.	18	0.056
826	A	3	2	1.	16	0.125
827	A	2	1	1.	14	0.071
828	A	3	2	1.	18	0.111
829	A	2	1	1.	18	0.056
830	A	3	2	1.	18	0.111
831	A	2	1	1.	18	0.056
832	A	3	2	1.	18	0.111
833	A	2	1	1.	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
834	A	3	2	1.	18	0.111
835	A	2	1	1.	18	0.056
836	A	3	2	1.	18	0.111
837	A	2	1	1.	18	0.056
838	A	3	2	1.	18	0.111
839	A	2	1	1.	18	0.056
840	A	3	2	1.	18	0.111
841	A	2	1	1.	18	0.056
842	A	3	2	1.	16	0.125
843	A	2	1	1.	14	0.071
844	A	3	2	1.	18	0.111
845	A	2	1	1.	18	0.056
846	A	3	2	1.	18	0.111
847	A	2	1	1.	18	0.056
848	A	7	6	1.	18	0.333
849	A	6	6	1.	18	0.333
850	A	5	5	1.	18	0.278
851	A	3	3	1.	16	0.188
852	A	7	7	1.	18	0.389
853	A	8	7	1.	18	0.389
854	A	8	7	1.	18	0.389
855	A	5	4	1.	18	0.222
856	A	4	3	1.	18	0.167
857	A	3	2	1.	18	0.111
858	A	3	2	1.	14	0.143
859	A	4	3	1.	18	0.167
860	A	5	4	1.	18	0.222
861	A	7	7	1.	18	0.389
862	A	4	4	1.	18	0.222
863	A	4	4	1.	18	0.222
864	A	4	4	1.	16	0.25
865	A	8	7	1.	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	8	7	1.	18	0.389
867	A	6	4	1.	18	0.222
868	A	5	4	1.	18	0.222
869	A	4	3	1.	18	0.167
870	A	4	3	1.	18	0.167
871	A	4	3	1.	14	0.214
872	A	5	4	1.	18	0.222
873	A	8	8	1.	18	0.444
874	A	5	4	1.	18	0.222
875	A	5	5	1.	18	0.278
876	A	5	5	1.	18	0.278
877	A	5	5	1.	18	0.278
878	A	5	4	1.	16	0.25
879	A	9	8	1.	18	0.444
880	A	9	8	1.	18	0.444
881	A	7	5	1.	18	0.278
882	A	6	5	1.	18	0.278
883	A	5	4	1.	18	0.222
884	A	5	4	1.	18	0.222
885	A	5	4	1.	18	0.222
886	A	5	4	1.	14	0.286
887	A	6	5	1.	18	0.278
888	A	6	6	1.	19	0.316
889	A	5	5	1.	19	0.263
890	A	3	3	1.	17	0.176
891	A	7	7	1.	19	0.368
892	A	8	7	1.	19	0.368
893	A	4	3	1.	19	0.158
894	A	3	2	1.	19	0.105
895	A	3	2	1.	15	0.133
896	A	4	3	1.	19	0.158
897	A	6	6	1.	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
898	A	5	5	1.	22	0.227
899	A	3	3	1.	20	0.15
900	A	7	7	1.	22	0.318
901	A	8	7	1.	22	0.318
902	A	4	3	1.	22	0.136
903	A	3	2	1.	22	0.091
904	A	3	2	1.	18	0.111
905	A	4	3	1.	22	0.136
906	A	6	6	1.	20	0.3
907	A	5	5	1.	20	0.25
908	A	3	3	1.	18	0.167
909	A	7	7	1.	20	0.35
910	A	8	7	1.	20	0.35
911	A	10	6	1.	20	0.3
912	A	9	5	1.	20	0.25
913	A	9	5	1.	16	0.312
914	A	10	6	1.	20	0.3
915	A	3	3	1.	12	0.25
916	A	3	3	1.	14	0.214
917	A	3	2	1.	16	0.125
918	A	9	6	1.	16	0.375
919	A	9	6	1.	16	0.375
920	A	6	6	1.	20	0.3
921	A	6	6	1.	20	0.3
922	A	5	5	1.	20	0.25
923	A	4	4	1.	18	0.222
924	A	7	6	1.	20	0.3
925	A	7	6	1.	20	0.3
926	A	4	4	1.	20	0.2
927	A	5	5	1.	20	0.25
928	A	6	6	1.	20	0.3
929	A	7	7	1.	20	0.35

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
930	A	5	5	1.	20	0.25
931	A	4	4	1.	20	0.2
932	A	4	4	1.	16	0.25
933	A	4	4	1.	20	0.2
934	A	5	5	1.	20	0.25
935	A	6	5	1.	20	0.25
936	A	7	6	1.	20	0.3
937	A	7	6	1.	20	0.3
938	A	6	5	1.	20	0.25
939	A	5	4	1.	18	0.222
940	A	8	7	1.	20	0.35
941	A	8	7	1.	20	0.35
942	A	8	7	1.	20	0.35
943	A	8	7	1.	20	0.35
944	A	5	4	1.	20	0.2
945	A	6	5	1.	20	0.25
946	A	7	6	1.	20	0.3
947	A	6	6	1.	20	0.3
948	A	5	5	1.	20	0.25
949	A	5	5	1.	16	0.312
950	A	5	5	1.	20	0.25
951	A	5	5	1.	20	0.25
952	A	6	6	1.	20	0.3
953	A	7	6	1.	20	0.3
954	A	5	5	1.	16	0.312
955	A	5	5	1.	20	0.25
956	A	5	5	1.	20	0.25
957	A	4	4	1.	20	0.2
958	A	3	3	1.	18	0.167
959	A	3	3	1.	20	0.15
960	A	4	4	1.	20	0.2
961	A	5	5	1.	20	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
962	A	6	6	1.	20	0.3
963	A	4	4	1.	20	0.2
964	A	3	3	1.	20	0.15
965	A	1	1	1.	16	0.062
966	A	5	5	1.	20	0.25
967	A	5	5	1.	20	0.25
968	A	5	5	1.	21	0.238
969	A	5	5	1.	21	0.238
970	A	4	4	1.	21	0.19
971	A	3	3	1.	19	0.158
972	A	3	3	1.	22	0.136
973	A	4	4	1.	22	0.182
974	A	5	5	1.	22	0.227
975	A	6	6	1.	22	0.273
976	A	5	5	1.	21	0.238
977	A	4	4	1.	21	0.19
978	A	2	2	1.	17	0.118
979	A	6	6	1.	21	0.286
980	A	6	6	1.	21	0.286
981	A	6	6	1.	20	0.3
982	A	5	5	1.	20	0.25
983	A	5	5	1.	20	0.25
984	A	2	2	1.	20	0.1
985	A	2	2	1.	18	0.111
986	A	5	5	1.	20	0.25
987	A	5	5	1.	20	0.25
988	A	6	6	1.	20	0.3
989	A	5	5	1.	20	0.25
990	A	4	4	1.	20	0.2
991	A	4	4	1.	20	0.2
992	A	4	4	1.	16	0.25
993	A	5	5	1.	20	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
994	A	3	3	1.	28	0.107
995	A	5	5	1.	28	0.179
996	A	2	2	1.	28	0.071
997	A	4	4	1.	26	0.154
998	A	3	3	1.	24	0.125
999	A	2	2	1.	28	0.071
1000	A	4	4	1.	28	0.143
1001	A	3	3	1.	28	0.107
1002	A	5	4	1.	28	0.143
1003	A	3	3	1.	29	0.103
1004	A	2	2	1.	29	0.069
1005	A	4	4	1.	29	0.138
1006	A	4	4	1.	27	0.148
1007	A	2	2	1.	25	0.08
1008	A	4	4	1.	29	0.138
1009	A	5	5	1.	29	0.172
1010	A	2	2	1.	29	0.069
1011	A	3	3	1.	29	0.103
1012	A	5	4	1.	29	0.138
1013	A	4	3	1.	29	0.103
1014	A	4	4	1.	29	0.138
1015	A	2	2	1.	27	0.074
1016	A	3	3	1.	25	0.12
1017	A	4	4	1.	29	0.138
1018	A	2	2	1.	29	0.069
1019	A	5	5	1.	29	0.172
1020	A	3	3	1.	29	0.103
1021	A	3	3	1.	23	0.13
1022	A	3	3	1.	23	0.13
1023	A	3	3	1.	23	0.13
1024	A	3	3	1.	21	0.143
1025	A	3	3	1.	19	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1026	A	3	3	1.	23	0.13
1027	A	3	3	1.	23	0.13
1028	A	3	3	1.	23	0.13
1029	A	3	3	1.	23	0.13
1030	A	3	3	1.	24	0.125
1031	A	3	3	1.	24	0.125
1032	A	3	3	1.	24	0.125
1033	A	3	3	1.	22	0.136
1034	A	2	2	1.	20	0.1
1035	A	3	3	1.	24	0.125
1036	A	3	3	1.	24	0.125
1037	A	3	3	1.	24	0.125
1038	A	3	3	1.	24	0.125
1039	A	2	2	1.	16	0.125
1040	A	2	2	1.	16	0.125
1041	A	2	1	1.	18	0.056
1042	A	2	1	1.	18	0.056
1043	A	2	1	1.	18	0.056
1044	A	2	1	1.	18	0.056
1045	A	2	1	1.	18	0.056
1046	A	2	1	1.	18	0.056
1047	A	2	1	1.	18	0.056
1048	A	2	1	1.	20	0.05
1049	A	2	1	1.	20	0.05
1050	A	2	1	1.	20	0.05
1051	A	2	1	1.	20	0.05
1052	A	2	1	1.	20	0.05
1053	A	2	1	1.	20	0.05
1054	A	2	1	1.	20	0.05
1055	A	2	1	1.	20	0.05
1056	A	2	1	1.	20	0.05
1057	A	2	1	1.	20	0.05

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1058	A	2	1	1.	20	0.05
1059	A	2	1	1.	20	0.05
1060	A	2	1	1.	20	0.05
1061	A	2	1	1.	20	0.05
1062	A	9	6	1.	20	0.3
1063	A	9	6	1.	20	0.3
1064	A	8	5	1.	20	0.25
1065	A	8	5	1.	20	0.25
1066	A	8	5	1.	20	0.25
1067	A	8	5	1.	20	0.25
1068	A	9	6	1.	20	0.3
1069	A	9	6	1.	20	0.3
1070	A	10	7	1.	20	0.35
1071	A	10	7	1.	20	0.35
1072	A	10	7	1.	20	0.35
1073	A	9	6	1.	20	0.3
1074	A	9	6	1.	20	0.3
1075	A	9	6	1.	20	0.3
1076	A	9	6	1.	20	0.3
1077	A	9	6	1.	20	0.3
1078	A	9	6	1.	20	0.3
1079	A	10	7	1.	20	0.35
1080	A	11	8	1.	20	0.4
1081	A	10	7	1.	20	0.35
1082	A	10	7	1.	20	0.35
1083	A	10	7	1.	20	0.35
1084	A	10	7	1.	20	0.35
1085	A	10	7	1.	20	0.35
1086	A	10	7	1.	20	0.35
1087	A	10	7	1.	20	0.35
1088	A	10	7	1.	20	0.35
1089	A	2	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1090	A	2	2	1.	24	0.083
1091	A	2	2	1.	24	0.083
1092	A	2	2	1.	24	0.083
1093	A	2	2	1.	24	0.083
1094	A	2	2	1.	24	0.083
1095	A	2	2	1.	24	0.083
1096	A	2	2	1.	24	0.083
1097	A	2	2	1.	24	0.083
1098	A	2	2	1.	24	0.083
1099	A	2	2	1.	24	0.083
1100	A	2	2	1.	24	0.083
1101	A	2	2	1.	24	0.083
1102	A	2	2	1.	24	0.083
1103	A	2	2	1.	24	0.083
1104	A	2	2	1.	24	0.083
1105	A	2	1	1.	20	0.05
1106	A	2	1	1.	20	0.05
1107	A	2	1	1.	18	0.056
1108	A	3	2	1.	20	0.1
1109	A	4	3	1.	20	0.15
1110	A	2	2	1.	22	0.091
1111	A	2	2	1.	22	0.091
1112	A	2	2	1.	22	0.091
1113	A	2	2	1.	22	0.091
1114	A	2	2	1.	20	0.1
1115	A	4	4	1.	18	0.222
1116	A	4	4	1.	18	0.222
1117	A	3	3	1.	18	0.167
1118	A	2	2	1.	16	0.125
1119	A	3	3	1.	18	0.167
1120	A	3	3	1.	18	0.167
1121	A	3	3	1.	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1122	A	2	2	1.	18	0.111
1123	A	2	2	1.	18	0.111
1124	A	2	2	1.	14	0.143
1125	A	2	2	1.	18	0.111
1126	A	2	2	1.	18	0.111

Chapter 3

Listing of integrals

3.1 $\int \left(a^2+2abx^2+b^2x^4\right)^{3/4} dx$

Optimal. Leaf size=128

$$\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)}+\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3\sqrt{a}\left(a^2+2abx^2+b^2x^4\right)^{3/4}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))/4 + (3*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4
)^(3/4))/(8*(a + b*x^2)) + (3*Sqrt[a]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)*Arc
Sinh[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[b]*(1 + (b*x^2)/a)^(3/2))
```

Rubi [A] time = 0.0308191, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 195, 215}

$$\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)}+\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3\sqrt{a}\left(a^2+2abx^2+b^2x^4\right)^{3/4}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))/4 + (3*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4
)^(3/4))/(8*(a + b*x^2)) + (3*Sqrt[a]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)*Arc
```

```
Sinh[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[b]*(1 + (b*x^2)/a)^(3/2))
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/4}\int \left(1+\frac{bx^2}{a}\right)^{3/2}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4} + \frac{\left(3\left(a^2+2abx^2+b^2x^4\right)^{3/4}\right)\int \sqrt{1+\frac{bx^2}{a}}\,dx}{4\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4} + \frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)} + \frac{\left(3\left(a^2+2abx^2+b^2x^4\right)^{3/4}\right)\int -}{8\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4} + \frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)} + \frac{3\sqrt{a}\left(a^2+2abx^2+b^2x^4\right)^{3/4}\sin}{8\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

Mathematica [A] time = 0.0640802, size = 97, normalized size = 0.76

$$\frac{\left(\left(a+bx^2\right)^2\right)^{3/4}\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x\left(5a+2bx^2\right)\sqrt{\frac{bx^2}{a}+1}\right)}{8\sqrt{b}\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

```
[Out] (((a + b*x^2)^2)^(3/4)*(Sqrt[b]*x*(5*a + 2*b*x^2)*Sqrt[1 + (b*x^2)/a] + 3*a
^(3/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/(8*Sqrt[b]*(a + b*x^2)*Sqrt[1 + (b*x^
2)/a])
```

Maple [A] time = 0.199, size = 77, normalized size = 0.6

$$\frac{x\left(2\,bx^2+5\,a\right)\left(bx^2+a\right)}{8}\frac{1}{\sqrt[4]{\left(bx^2+a\right)^2}}+\frac{3\,a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\sqrt{bx^2+a}\frac{1}{\sqrt{b}}\frac{1}{\sqrt[4]{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x)
```

```
[Out] 1/8*x*(2*b*x^2+5*a)*(b*x^2+a)/((b*x^2+a)^2)^(1/4)+3/8*a^2*ln(x*b^(1/2)+(b*x
^2+a)^(1/2))/b^(1/2)/((b*x^2+a)^2)^(1/4)*(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/4), x)
```

Fricas [A] time = 1.41979, size = 408, normalized size = 3.19

$$\left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{b}x-a\right)+2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\left(2\,b^2x^3+5\,abx\right)}{16\,b}, -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{(b^2}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqr
t(b)*x - a) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b^2*x^3 + 5*a*b*x))/b,
 -1/8*(3*a^2*sqrt(-b)*arctan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(
b*x^2 + a)) - (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b^2*x^3 + 5*a*b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4), x)
```

Giac [A] time = 1.21555, size = 117, normalized size = 0.91

$$-\frac{1}{8}\left(\frac{x^4\left(\frac{5\sqrt{-bx^2-a}\left(b+\frac{a}{x^2}\right)|x|}{x^2}-\frac{3\sqrt{-bx^2-a}b|x|}{x^2}\right)}{a^2}+\frac{3\arctan\left(\frac{\sqrt{-bx^2-a}|x|}{\sqrt{b}x^2}\right)}{\sqrt{b}}\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
```

```
[Out] -1/8*(x^4*(5*sqrt(-b*x^2 - a)*(b + a/x^2)*abs(x)/x^2 - 3*sqrt(-b*x^2 - a)*b
*abs(x)/x^2)/a^2 + 3*arctan(sqrt(-b*x^2 - a)*abs(x)/(sqrt(b)*x^2))/sqrt(b))
*a^2
```

3.2 $\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\sqrt[4]{a^2+2abx^2+b^2x^4}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}$$

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))/2 + (Sqrt[a]*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(1/4)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b]*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] time = 0.0195423, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 195, 215}

$$\frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\sqrt[4]{a^2+2abx^2+b^2x^4}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))/2 + (Sqrt[a]*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(1/4)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b]*Sqrt[1 + (b*x^2)/a])
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\int \sqrt{1+\frac{bx^2}{a}}\,dx}{\sqrt{1+\frac{bx^2}{a}}}$$
$$= \frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4} + \frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{2\sqrt{1+\frac{bx^2}{a}}}$$
$$= \frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4} + \frac{\sqrt{a}\sqrt[4]{a^2+2abx^2+b^2x^4}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{1+\frac{bx^2}{a}}}$$

Mathematica [A] time = 0.0376042, size = 59, normalized size = 0.65

$$\frac{1}{2}\sqrt[4]{\left(a+bx^2\right)^2}\left(\frac{a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}\sqrt{a+bx^2}}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

```
[Out] (((a + b*x^2)^2)^(1/4)*(x + (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(Sqrt[b]
*Sqrt[a + b*x^2])))/2
```

Maple [A] time = 0.175, size = 58, normalized size = 0.6

$$\frac{x}{2}\sqrt[4]{\left(bx^2+a\right)^2}+\frac{a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\sqrt[4]{\left(bx^2+a\right)^2}\frac{1}{\sqrt{b}}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

```
[Out] 1/2*x*((b*x^2+a)^2)^(1/4)+1/2*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)*((b*x
^2+a)^2)^(1/4)/(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

Fricas [A] time = 1.20101, size = 347, normalized size = 3.81

$$\left[\frac{a\sqrt{b}\log\left(-2\,bx^2 - 2\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{4}}\sqrt{b}x - a\right) + 2\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{4}}bx}{4\,b}, -\frac{a\sqrt{-b}\arctan\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{-b}x}{bx^2+a}\right)}{2\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(b)*
x - a) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*b*x)/b, -1/2*(a*sqrt(-b)*arcta
n((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(b*x^2 + a)) - (b^2*x^4 + 2*
a*b*x^2 + a^2)^(1/4)*b*x)/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[4]{a^2 + 2abx^2 + b^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

3.3 $\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

```
[Out] (Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(a^2 +
2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rubi [A] time = 0.0122802, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1089, 215}

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(a^2 +
2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0149033, size = 49, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

```
[Out] (Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*((a + b*x^2
)^2)^(1/4))
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

```
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/4), x)
```

Fricas [A] time = 1.28972, size = 217, normalized size = 3.62

$$\left[\frac{\log\left(-2\,bx^2-2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{b}x-a\right)}{2\,\sqrt{b}},-\frac{\sqrt{-b}\arctan\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\sqrt{-b}x}{bx^2+a}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(b)*x - a)/sqrt
(b), -sqrt(-b)*arctan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(b*x^2 +
 a))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/4), x)
```

Giac [A] time = 1.15798, size = 36, normalized size = 0.6

$$-\frac{\arctan\left(\frac{\sqrt{-bx^2-a}|x|}{\sqrt{b}x^2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(-b*x^2 - a)*abs(x)/(sqrt(b)*x^2))/sqrt(b)
```

3.4 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\,dx$

Optimal. Leaf size=34

$$\frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

```
[Out] (x*(a + b*x^2))/(a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))
```

Rubi [A] time = 0.0084038, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1089, 191}

$$\frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
```

```
[Out] (x*(a + b*x^2))/(a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/2}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{3/4}} = \frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Mathematica [A] time = 0.0110457, size = 25, normalized size = 0.74

$$\frac{x\left(a+bx^2\right)}{a\left(\left(a+bx^2\right)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
```

```
[Out] (x*(a + b*x^2))/(a*((a + b*x^2)^2)^(3/4))
```

Maple [A] time = 0.054, size = 33, normalized size = 1.

$$\frac{x\left(bx^2+a\right)}{a}\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x)
```

```
[Out] x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/4), x)
```

Fricas [A] time = 1.28183, size = 72, normalized size = 2.12

$$\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{4}}x}{abx^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="fricas")
```

```
[Out] (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*x/(a*b*x^2 + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/4), x)
```

Giac [A] time = 1.16254, size = 32, normalized size = 0.94

$$-\frac{x^2}{\sqrt{-bx^2 - a}a|x|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
```

```
[Out] -x^2/(sqrt(-b*x^2 - a)*a*abs(x))
```

3.5 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/4}}\,dx$

Optimal. Leaf size=68

$$\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x\left(a+bx^2\right)}{3a\left(a^2+2abx^2+b^2x^4\right)^{5/4}}$$

```
[Out] (x*(a + b*x^2))/(3*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/4)) + (2*x)/(3*a^2*(a^2
 + 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rubi [A] time = 0.0161971, antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 192, 191}

$$\frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
```

```
[Out] (2*x)/(3*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + x/(3*a*(a + b*x^2)*(a^2 +
 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/4}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/2}}\,dx}{a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{\left(2\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.012402, size = 40, normalized size = 0.59

$$\frac{x\left(3a+2bx^2\right)}{3a^2\left(a+bx^2\right)\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
```

```
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)*((a + b*x^2)^2)^(1/4))
```

Maple [A] time = 0.044, size = 44, normalized size = 0.7

$$\frac{\left(bx^2+a\right)x\left(2\,bx^2+3\,a\right)}{3\,a^2}\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x)
```

```
[Out] 1/3*(b*x^2+a)*x*(2*b*x^2+3*a)/a^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

Fricas [A] time = 1.28432, size = 123, normalized size = 1.81

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\left(2\,bx^3+3\,ax\right)}{3\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b*x^3 + 3*a*x)/(a^2*b^2*x^4 + 2*a^
3*b*x^2 + a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

3.6 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx$

Optimal. Leaf size=105

$$\frac{8x\left(a+bx^2\right)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{x\left(a+bx^2\right)}{5a\left(a^2+2abx^2+b^2x^4\right)^{7/4}}$$

```
[Out] (x*(a + b*x^2))/(5*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(7/4)) + (4*x)/(15*a^2*(a^
2 + 2*a*b*x^2 + b^2*x^4)^(3/4)) + (8*x*(a + b*x^2))/(15*a^3*(a^2 + 2*a*b*x^
2 + b^2*x^4)^(3/4))
```

Rubi [A] time = 0.0236083, antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 192, 191}

$$\frac{8x\left(a+bx^2\right)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] (4*x)/(15*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)) + x/(5*a*(a + b*x^2)*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(3/4)) + (8*x*(a + b*x^2))/(15*a^3*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/4))
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/2}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{7/2}}\,dx}{a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

$$= \frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}} + \frac{\left(4\left(1+\frac{bx^2}{a}\right)^{3/2}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/2}}\,dx}{5a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

$$= \frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}} + \frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}} + \frac{\left(8\left(1+\frac{bx^2}{a}\right)^{3/2}\right)\int}{15a^2\left(a^2+2abx^2\right.}$$

$$= \frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}} + \frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}} + \frac{8x\left(a+bx\right.}{15a^3\left(a^2+2abx^2\right.}$$

Mathematica [A] time = 0.0150957, size = 51, normalized size = 0.49

$$\frac{x\left(15a^2+20abx^2+8b^2x^4\right)}{15a^3\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] (x*(15*a^2 + 20*a*b*x^2 + 8*b^2*x^4))/(15*a^3*(a + b*x^2)*((a + b*x^2)^2)^(
3/4))
```

Maple [A] time = 0.046, size = 55, normalized size = 0.5

$$\frac{\left(bx^{2}+a\right)x\left(8\,b^{2}x^{4}+20\,abx^{2}+15\,a^{2}\right)}{15\,a^{3}}\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{-\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x)
```

```
[Out] 1/15*(b*x^2+a)*x*(8*b^2*x^4+20*a*b*x^2+15*a^2)/a^3/(b^2*x^4+2*a*b*x^2+a^2)^
(7/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

Fricas [A] time = 1.42588, size = 170, normalized size = 1.62

$$\frac{\left(8\,b^{2}x^{5}+20\,abx^{3}+15\,a^{2}x\right)\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{1}{4}}}{15\left(a^{3}b^{3}x^{6}+3\,a^{4}b^{2}x^{4}+3\,a^{5}bx^{2}+a^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/15*(8*b^2*x^5 + 20*a*b*x^3 + 15*a^2*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)/
(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(7/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-7/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

3.7 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{9/4}}\,dx$

Optimal. Leaf size=135

$$\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{8x\left(a+bx^2\right)}{35a^3\left(a^2+2abx^2+b^2x^4\right)^{5/4}}+\frac{6x}{35a^2\left(a^2+2abx^2+b^2x^4\right)^{5/4}}+\frac{x\left(a+bx^2\right)}{7a\left(a^2+2abx^2+b^2x^4\right)^{9/4}}$$

```
[Out] (x*(a + b*x^2))/(7*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(9/4)) + (6*x)/(35*a^2*(a^
2 + 2*a*b*x^2 + b^2*x^4)^(5/4)) + (8*x*(a + b*x^2))/(35*a^3*(a^2 + 2*a*b*x^
2 + b^2*x^4)^(5/4)) + (16*x)/(35*a^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rubi [A] time = 0.0432201, antiderivative size = 148, normalized size of antiderivative = 1.1, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 192, 191}

$$\frac{8x}{35a^3\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{}{35a^4\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

```
[Out] (16*x)/(35*a^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + x/(7*a*(a + b*x^2)^3*(a
^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + (6*x)/(35*a^2*(a + b*x^2)^2*(a^2 + 2*a*b
*x^2 + b^2*x^4)^(1/4)) + (8*x)/(35*a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(1/4))
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
```

```
&& NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{9/4}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{9/2}}\,dx}{a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{\left(6\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{7/2}}\,dx}{7a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{\left(24\sqrt{1+\frac{bx^2}{a}}\right.}{35a^4\sqrt[4]{a^2}}$$

$$= \frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{}{35a^3\left(a+b\right.}$$

$$= \frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+}}$$

Mathematica [A] time = 0.0236541, size = 62, normalized size = 0.46

$$\frac{x\left(70a^2bx^2+35a^3+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^3\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(a + b*x^2)
^3*((a + b*x^2)^2)^(1/4))
```

Maple [A] time = 0.045, size = 66, normalized size = 0.5

$$\frac{(bx^2+a)\,x\,(16\,b^3x^6+56\,b^2x^4a+70\,a^2bx^2+35\,a^3)}{35\,a^4}\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x)
```

```
[Out] 1/35*(b*x^2+a)*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/(b^2*x^4
+2*a*b*x^2+a^2)^(9/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

Fricas [A] time = 1.37772, size = 216, normalized size = 1.6

$$\frac{\left(16\,b^3x^7+56\,ab^2x^5+70\,a^2bx^3+35\,a^3x\right)\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}{35\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*(b^2*x^4 + 2*a*b
*x^2 + a^2)^(1/4)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^
2 + a^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(9/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-9/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

3.8 $\int \frac{1}{a^2+b+2ax^2+x^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\log\left(-\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\log\left(\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}+a}}+$$

```
[Out] -ArcTan[(Sqrt[-a + Sqrt[a^2 + b]] - Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/(2*
Sqrt[2]*Sqrt[a^2 + b]*Sqrt[a + Sqrt[a^2 + b]]) + ArcTan[(Sqrt[-a + Sqrt[a^2
 + b]] + Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/(2*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[
a + Sqrt[a^2 + b]]) - Log[Sqrt[a^2 + b] - Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*
x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[-a + Sqrt[a^2 + b]]) + Log[Sqrt[a^2
+ b] + Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*S
qrt[-a + Sqrt[a^2 + b]])
```

Rubi [A] time = 0.312186, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(-\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\log\left(\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[-a + Sqrt[a^2 + b]] - Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/(2*
Sqrt[2]*Sqrt[a^2 + b]*Sqrt[a + Sqrt[a^2 + b]]) + ArcTan[(Sqrt[-a + Sqrt[a^2
 + b]] + Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/(2*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[
a + Sqrt[a^2 + b]]) - Log[Sqrt[a^2 + b] - Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*
x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[-a + Sqrt[a^2 + b]]) + Log[Sqrt[a^2
+ b] + Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*S
qrt[-a + Sqrt[a^2 + b]])
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
```

```
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}-x}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}x+x^2}\,dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} + \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}+x}{\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}x+x^2}\,dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}}$$

$$= \frac{\int \frac{1}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}x+x^2}\,dx}{4\sqrt{a^2+b}} + \frac{\int \frac{1}{\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}x+x^2}\,dx}{4\sqrt{a^2+b}} - \frac{\int \frac{-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}+2x}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}x+x^2}\,dx}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} + \cdots$$

$$= -\frac{\log\left(\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}x+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} + \frac{\log\left(\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}x+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} - \cdots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{a^2+b}}-\sqrt{2}x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{a^2+b}}+\sqrt{2}x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}} - \frac{\log\left(\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2 \cdots}}\right.}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2 \cdots}}}$$

Mathematica [C] time = 0.0442712, size = 81, normalized size = 0.27

$$-\frac{i\left(\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-i\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+i\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{b}}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] ((-I/2)*(ArcTan[x/Sqrt[a - I*Sqrt[b]]]/Sqrt[a - I*Sqrt[b]] - ArcTan[x/Sqrt[
a + I*Sqrt[b]]]/Sqrt[a + I*Sqrt[b]]))/Sqrt[b]
```

Maple [B] time = 0.199, size = 1099, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2+b),x)
```

```
[Out] 1/8/b/(a^2+b)*ln(x^2+x*(2*(a^2+b)^(1/2)-2*a)^(1/2)+(a^2+b)^(1/2))*(2*(a^2+b
)^(1/2)-2*a)^(1/2)*a^2+1/8/b/(a^2+b)^(3/2)*ln(x^2+x*(2*(a^2+b)^(1/2)-2*a)^(
1/2)+(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^3+1/8/(a^2+b)*ln(x^2+x*(2
*(a^2+b)^(1/2)-2*a)^(1/2)+(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)+1/8/(a
^2+b)^(3/2)*ln(x^2+x*(2*(a^2+b)^(1/2)-2*a)^(1/2)+(a^2+b)^(1/2))*(2*(a^2+b)^
(1/2)-2*a)^(1/2)*a-1/2/b/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((
2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2+1/2/b/(a^
2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1
/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^4-1/2/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*
a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/
2))+3/2/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1
/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2+b/(a^2+b)^(3/2)/(2*(a^2+b)
^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)
+2*a)^(1/2))-1/8/b/(a^2+b)*ln(x*(2*(a^2+b)^(1/2)-2*a)^(1/2)-x^2-(a^2+b)^(1/
2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^2-1/8/b/(a^2+b)^(3/2)*ln(x*(2*(a^2+b)^(1/
2)-2*a)^(1/2)-x^2-(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^3-1/8/(a^2+b
)*ln(x*(2*(a^2+b)^(1/2)-2*a)^(1/2)-x^2-(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)
^(1/2)-1/8/(a^2+b)^(3/2)*ln(x*(2*(a^2+b)^(1/2)-2*a)^(1/2)-x^2-(a^2+b)^(1/2)
)*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a+1/2/b/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*a)^(
1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*
a^2-1/2/b/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2
)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^4+1/2/(a^2+b)^(1/2)/(2*(a^
2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(
1/2)+2*a)^(1/2))-3/2/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(
a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2-b/(a^2+b)^(3/
2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*
(a^2+b)^(1/2)+2*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 + 2\,ax^2 + a^2 + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + b), x)
```

Fricas [B] time = 1.41114, size = 1206, normalized size = 4.03

$$\frac{1}{4}\sqrt{\frac{\left(a^2b+b^2\right)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}+a}{a^2b+b^2}}\log\left(\left(\left(a^3b+ab^2\right)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}+b\right)\sqrt{\frac{\left(a^2b+b^2\right)\sqrt{-\frac{1}{a^4b+2\,a^2b^2+b^3}}+a}{a^2b+b^2}}+x\right)-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^
2))*log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((a^2
*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) - 1/4
*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2))
*log(-((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((a^2*b
 + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) - 1/4*s
qrt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2))*
log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)*sqrt(-((a^2*b
+ b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) + x) + 1/4*sq
rt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2))*l
og(-((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)*sqrt(-((a^2*b
+ b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) + x)
```

Sympy [A] time = 0.736991, size = 63, normalized size = 0.21

$$\operatorname{RootSum}\left(t^4\left(256a^2b^2+256b^3\right)-32t^2ab+1,\left(t\mapsto t\log\left(64t^3a^3b+64t^3ab^2-4ta^2+4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2+b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 + 256*b**3) - 32*_t**2*a*b + 1, Lambda(_t, _t*
log(64*_t**3*a**3*b + 64*_t**3*a*b**2 - 4*_t*a**2 + 4*_t*b + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.9 $\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

```
[Out] -ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1 + a]) - ArcTanh[x/Sqrt[1 - a]]/(2*Sqrt[1 -
 a])
```

Rubi [A] time = 0.0263606, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1093, 207, 203}

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1 + a]) - ArcTanh[x/Sqrt[1 - a]]/(2*Sqrt[1 -
 a])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = \frac{1}{2}\int \frac{1}{-1+a+x^2}\,dx - \frac{1}{2}\int \frac{1}{1+a+x^2}\,dx$$
$$= -\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+a}}\right)}{2\sqrt{1+a}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

Mathematica [A] time = 0.0232025, size = 43, normalized size = 0.91

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + a]]/(2*Sqrt[-1 + a]) - ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1 +
 a])
```

Maple [A] time = 0.09, size = 32, normalized size = 0.7

$$-\frac{1}{2}\arctan\left(x\frac{1}{\sqrt{1+a}}\right)\frac{1}{\sqrt{1+a}} + \frac{1}{2}\arctan\left(x\frac{1}{\sqrt{a-1}}\right)\frac{1}{\sqrt{a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2-1),x)
```

```
[Out] -1/2*arctan(x/(1+a)^(1/2))/(1+a)^(1/2)+1/2/(a-1)^(1/2)*arctan(x/(a-1)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37397, size = 744, normalized size = 15.83

$$\left[-\frac{(a-1)\sqrt{-a-1}\log\left(\frac{x^2+2\sqrt{-a-1}x-a-1}{x^2+a+1}\right)+(a+1)\sqrt{-a+1}\log\left(\frac{x^2-2\sqrt{-a+1}x-a+1}{x^2+a-1}\right)}{4\left(a^2-1\right)}, \frac{2\,(a+1)\sqrt{a-1}\arctan\left(\frac{x}{\sqrt{a-1}}\right)-(a-1)\sqrt{}}{4\left(a^2-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - 1)*sqrt(-a - 1)*log((x^2 + 2*sqrt(-a - 1)*x - a - 1)/(x^2 + a +
 1)) + (a + 1)*sqrt(-a + 1)*log((x^2 - 2*sqrt(-a + 1)*x - a + 1)/(x^2 + a -
 1)))/(a^2 - 1), 1/4*(2*(a + 1)*sqrt(a - 1)*arctan(x/sqrt(a - 1)) - (a - 1)
*sqrt(-a - 1)*log((x^2 + 2*sqrt(-a - 1)*x - a - 1)/(x^2 + a + 1)))/(a^2 - 1
), -1/4*(2*sqrt(a + 1)*(a - 1)*arctan(x/sqrt(a + 1)) + (a + 1)*sqrt(-a + 1)
*log((x^2 - 2*sqrt(-a + 1)*x - a + 1)/(x^2 + a - 1)))/(a^2 - 1), -1/2*(sqrt
(a + 1)*(a - 1)*arctan(x/sqrt(a + 1)) - (a + 1)*sqrt(a - 1)*arctan(x/sqrt(a
 - 1)))/(a^2 - 1)]
```

Sympy [B] time = 0.599136, size = 257, normalized size = 5.47

$$\frac{\sqrt{-\frac{1}{a-1}}\log\left(-a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}-a^2\sqrt{-\frac{1}{a-1}}+a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+x-\sqrt{-\frac{1}{a-1}}\right)}{4}-\frac{\sqrt{-\frac{1}{a-1}}\log\left(a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+a^2\sqrt{-\frac{1}{a-1}}-a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+x\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2-1),x)
```

```
[Out] sqrt(-1/(a - 1))*log(-a**3*(-1/(a - 1))**(3/2) - a**2*sqrt(-1/(a - 1)) + a*
(-1/(a - 1))**(3/2) + x - sqrt(-1/(a - 1)))/4 - sqrt(-1/(a - 1))*log(a**3*(
-1/(a - 1))**(3/2) + a**2*sqrt(-1/(a - 1)) - a*(-1/(a - 1))**(3/2) + x + sq
```

```
rt(-1/(a - 1)))/4 + sqrt(-1/(a + 1))*log(-a**3*(-1/(a + 1))**(3/2) - a**2*s
qrt(-1/(a + 1)) + a*(-1/(a + 1))**(3/2) + x - sqrt(-1/(a + 1)))/4 - sqrt(-1
/(a + 1))*log(a**3*(-1/(a + 1))**(3/2) + a**2*sqrt(-1/(a + 1)) - a*(-1/(a +
 1))**(3/2) + x + sqrt(-1/(a + 1)))/4
```

Giac [A] time = 1.13409, size = 42, normalized size = 0.89

$$-\frac{\arctan\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}+\frac{\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x/sqrt(a + 1))/sqrt(a + 1) + 1/2*arctan(x/sqrt(a - 1))/sqrt(a -
 1)
```

3.10 $\int \frac{1}{1+a^2+2ax^2+x^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\log\left(\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+1}+a}}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}+a}}+ -$$

```
[Out] -ArcTan[(Sqrt[-a + Sqrt[1 + a^2]] - Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/(2*
Sqrt[2]*Sqrt[1 + a^2]*Sqrt[a + Sqrt[1 + a^2]]) + ArcTan[(Sqrt[-a + Sqrt[1 +
 a^2]] + Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/(2*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[
a + Sqrt[1 + a^2]]) - Log[Sqrt[1 + a^2] - Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*
x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[-a + Sqrt[1 + a^2]]) + Log[Sqrt[1 +
a^2] + Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*S
qrt[-a + Sqrt[1 + a^2]])
```

Rubi [A] time = 0.307804, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\log\left(\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+1}+a}}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}+a}}+ -$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[-a + Sqrt[1 + a^2]] - Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/(2*
Sqrt[2]*Sqrt[1 + a^2]*Sqrt[a + Sqrt[1 + a^2]]) + ArcTan[(Sqrt[-a + Sqrt[1 +
 a^2]] + Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/(2*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[
a + Sqrt[1 + a^2]]) - Log[Sqrt[1 + a^2] - Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*
x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[-a + Sqrt[1 + a^2]]) + Log[Sqrt[1 +
a^2] + Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*S
qrt[-a + Sqrt[1 + a^2]])
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
```

```
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}-x}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}x+x^2}\,dx}{2\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} + \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}+x}{\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}x+x^2}\,dx}{2\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}}$$

$$= \frac{\int \frac{1}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}x+x^2}\,dx}{4\sqrt{1+a^2}} + \frac{\int \frac{1}{\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}x+x^2}\,dx}{4\sqrt{1+a^2}} - \frac{\int \frac{-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}+2x}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}x+x^2}\,dx}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} + \cdots$$

$$= -\frac{\log\left(\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}x+x^2\right)}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} + \frac{\log\left(\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}x+x^2\right)}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} - \cdots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{1+a^2}}-\sqrt{2}x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{1+a^2}}+\sqrt{2}x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}} - \frac{\log\left(\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+\cdots}}\right.}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+\cdots}}}$$

Mathematica [C] time = 0.0310982, size = 52, normalized size = 0.17

$$-\frac{1}{2}i\left(\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-i}}\right)}{\sqrt{a-i}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+i}}\right)}{\sqrt{a+i}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] (-I/2)*(ArcTan[x/Sqrt[-I + a]]/Sqrt[-I + a] - ArcTan[x/Sqrt[I + a]]/Sqrt[I
+ a])
```

Maple [B] time = 0.155, size = 1073, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2+1),x)
```

```
[Out] -1/8/(a^2+1)*ln(x*(2*(a^2+1)^(1/2)-2*a)^(1/2)-x^2-(a^2+1)^(1/2))*(2*(a^2+1)
^(1/2)-2*a)^(1/2)*a^2-1/8/(a^2+1)^(3/2)*ln(x*(2*(a^2+1)^(1/2)-2*a)^(1/2)-x^
2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)*a^3-1/8/(a^2+1)*ln(x*(2*(a^2+1
)^(1/2)-2*a)^(1/2)-x^2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)-1/8/(a^2+
1)^(3/2)*ln(x*(2*(a^2+1)^(1/2)-2*a)^(1/2)-x^2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/
2)-2*a)^(1/2)*a+1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a
^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2-1/2/(a^2+1)^(3
/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2
*(a^2+1)^(1/2)+2*a)^(1/2))*a^4+1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2
)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))-3/2
/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1
/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2-1/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+
2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(
1/2))+1/8/(a^2+1)*ln(x^2+x*(2*(a^2+1)^(1/2)-2*a)^(1/2)+(a^2+1)^(1/2))*(2*(a
^2+1)^(1/2)-2*a)^(1/2)*a^2+1/8/(a^2+1)^(3/2)*ln(x^2+x*(2*(a^2+1)^(1/2)-2*a)
^(1/2)+(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)*a^3+1/8/(a^2+1)*ln(x^2+x*
(2*(a^2+1)^(1/2)-2*a)^(1/2)+(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)+1/8/
(a^2+1)^(3/2)*ln(x^2+x*(2*(a^2+1)^(1/2)-2*a)^(1/2)+(a^2+1)^(1/2))*(2*(a^2+1
)^(1/2)-2*a)^(1/2)*a-1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((
2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2+1/2/(a^2+
1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2
))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^4-1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)
^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2)
)+3/2/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2
)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2+1/(a^2+1)^(3/2)/(2*(a^2+1)^(
1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2
*a)^(1/2))
```

Maximā [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 + 2\,ax^2 + a^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + 1), x)
```

Fricas [B] time = 1.37862, size = 1629, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*(a/sqrt(a^2 + 1) + 1)*log(x
^2 + sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(1/4) + sqrt(a
^2 + 1))/(a^2 + 1)^(1/4) - 1/8*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*
(a/sqrt(a^2 + 1) + 1)*log(x^2 - sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)
*x/(a^2 + 1)^(1/4) + sqrt(a^2 + 1))/(a^2 + 1)^(1/4) - 1/2*sqrt(2*a^2 - 2*(a
^3 + a)/sqrt(a^2 + 1) + 2)*(a^2 + 1)^(1/4)*arctan(-sqrt(a^4 + 2*a^2 + 1)*sq
rt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(5/4) + (a^3 + a)/sqr
t(a^4 + 2*a^2 + 1) + sqrt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^
2 + 1) + 2)*sqrt(x^2 + sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 +
 1)^(1/4) + sqrt(a^2 + 1))/(a^2 + 1)^(5/4) - sqrt(a^4 + 2*a^2 + 1)/sqrt(a^2
 + 1))/sqrt(a^4 + 2*a^2 + 1) - 1/2*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) +
 2)*(a^2 + 1)^(1/4)*arctan(-sqrt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/
sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(5/4) - (a^3 + a)/sqrt(a^4 + 2*a^2 + 1) + sq
rt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*sqrt(x^2 -
sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(1/4) + sqrt(a^2 +
1))/(a^2 + 1)^(5/4) + sqrt(a^4 + 2*a^2 + 1)/sqrt(a^2 + 1))/sqrt(a^4 + 2*a^2
 + 1)
```

Sympy [A] time = 0.496145, size = 48, normalized size = 0.16

$$\operatorname{RootSum}\left(t^4\left(256a^2+256\right)-32t^2a+1,\left(t\mapsto t\log\left(64t^3a^3+64t^3a-4ta^2+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2+1),x)
```

```
[Out] RootSum(_t**4*(256*a**2 + 256) - 32*_t**2*a + 1, Lambda(_t, _t*log(64*_t**3
*a**3 + 64*_t**3*a - 4*_t*a**2 + 4*_t + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4+2\,ax^2+a^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + 1), x)
```

3.11 $\int \frac{1}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}\tanh^{-1}(x)-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -ArcTanh[x/2]/6 + ArcTanh[x]/3
```

Rubi [A] time = 0.0079561, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 207}

$$\frac{1}{3}\tanh^{-1}(x)-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 5*x^2 + x^4)^(-1),x]

[Out] -ArcTanh[x/2]/6 + ArcTanh[x]/3
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-5x^2+x^4} \, dx &= \frac{1}{3}\int \frac{1}{-4+x^2} \, dx-\frac{1}{3}\int \frac{1}{-1+x^2} \, dx\\ &= -\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.0072305, size = 37, normalized size = 2.18

$$-\frac{1}{6}\log(1-x)+\frac{1}{12}\log(2-x)+\frac{1}{6}\log(x+1)-\frac{1}{12}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 5*x^2 + x^4)^(-1),x]
```

```
[Out] -Log[1 - x]/6 + Log[2 - x]/12 + Log[1 + x]/6 - Log[2 + x]/12
```

Maple [B] time = 0.049, size = 26, normalized size = 1.5

$$-\frac{\ln(2+x)}{12}+\frac{\ln(1+x)}{6}-\frac{\ln(-1+x)}{6}+\frac{\ln(-2+x)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-5*x^2+4),x)
```

```
[Out] -1/12*ln(2+x)+1/6*ln(1+x)-1/6*ln(-1+x)+1/12*ln(-2+x)
```

Maxima [B] time = 0.97829, size = 34, normalized size = 2.

$$-\frac{1}{12}\log(x+2)+\frac{1}{6}\log(x+1)-\frac{1}{6}\log(x-1)+\frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x - 2)
```

Fricas [B] time = 1.25218, size = 95, normalized size = 5.59

$$-\frac{1}{12}\log(x+2)+\frac{1}{6}\log(x+1)-\frac{1}{6}\log(x-1)+\frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x - 2)
```

Sympy [B] time = 0.169447, size = 26, normalized size = 1.53

$$\frac{\log(x-2)}{12} - \frac{\log(x-1)}{6} + \frac{\log(x+1)}{6} - \frac{\log(x+2)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-5*x**2+4),x)
```

```
[Out] log(x - 2)/12 - log(x - 1)/6 + log(x + 1)/6 - log(x + 2)/12
```

Giac [B] time = 1.14287, size = 39, normalized size = 2.29

$$-\frac{1}{12}\log(|x+2|) + \frac{1}{6}\log(|x+1|) - \frac{1}{6}\log(|x-1|) + \frac{1}{12}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*log(abs(x + 2)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1)) + 1/12*lo
g(abs(x - 2))
```

3.12 $\int \frac{1}{3+4x^2+x^4} \, dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.0150571, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 203}

$$\frac{1}{2}\tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{3+4x^2+x^4}\,dx = \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \frac{1}{2}\int \frac{1}{3+x^2}\,dx$$
$$= \frac{1}{2}\tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.0110083, size = 24, normalized size = 1.

$$\frac{1}{2}\tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] time = 0.049, size = 18, normalized size = 0.8

$$\frac{\arctan(x)}{2} - \frac{\sqrt{3}}{6}\arctan\left(\frac{x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^2+3),x)
```

```
[Out] 1/2*arctan(x)-1/6*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.44783, size = 23, normalized size = 0.96

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

Fricas [A] time = 1.34928, size = 70, normalized size = 2.92

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

Sympy [A] time = 0.145124, size = 20, normalized size = 0.83

$$\frac{\operatorname{atan}\left(x\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**2+3),x)
```

```
[Out] atan(x)/2 - sqrt(3)*atan(sqrt(3)*x/3)/6
```

Giac [A] time = 1.16407, size = 23, normalized size = 0.96

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

3.13 $\int \frac{1}{9+5x^2+x^4} \, dx$

Optimal. Leaf size=67

$$-\frac{1}{12}\log\left(x^2-x+3\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)}{6\sqrt{11}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[11]]/(6*Sqrt[11]) + ArcTan[(1 + 2*x)/Sqrt[11]]/(6*Sq
rt[11]) - Log[3 - x + x^2]/12 + Log[3 + x + x^2]/12
```

Rubi [A] time = 0.049631, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{1}{12}\log\left(x^2-x+3\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)}{6\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 5*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[11]]/(6*Sqrt[11]) + ArcTan[(1 + 2*x)/Sqrt[11]]/(6*Sq
rt[11]) - Log[3 - x + x^2]/12 + Log[3 + x + x^2]/12
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{9+5x^2+x^4}\,dx &= \frac{1}{6}\int \frac{1-x}{3-x+x^2}\,dx+\frac{1}{6}\int \frac{1+x}{3+x+x^2}\,dx\\ &= \frac{1}{12}\int \frac{1}{3-x+x^2}\,dx-\frac{1}{12}\int \frac{-1+2x}{3-x+x^2}\,dx+\frac{1}{12}\int \frac{1}{3+x+x^2}\,dx+\frac{1}{12}\int \frac{1+2x}{3+x+x^2}\,dx\\ &= -\frac{1}{12}\log\left(3-x+x^2\right)+\frac{1}{12}\log\left(3+x+x^2\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{-11-x^2}\,dx,x,-1+2x\right)-\frac{1}{6}\operatorname{Subst}\left(\int\right.\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{11}}\right)}{6\sqrt{11}}-\frac{1}{12}\log\left(3-x+x^2\right)+\frac{1}{12}\log\left(3+x+x^2\right)\end{aligned}$$

Mathematica [C] time = 0.0714477, size = 91, normalized size = 1.36

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(5+i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5+i\sqrt{11}\right)}}-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(5-i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5-i\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 5*x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(5 - I*Sqrt[11])/2]])/Sqrt[(11*(5 - I*Sqrt[11]))/2] + (
I*ArcTan[x/Sqrt[(5 + I*Sqrt[11])/2]])/Sqrt[(11*(5 + I*Sqrt[11]))/2]
```

Maple [A] time = 0.046, size = 54, normalized size = 0.8

$$\frac{\ln\left(x^2+x+3\right)}{12}+\frac{\sqrt{11}}{66}\arctan\left(\frac{(1+2x)\sqrt{11}}{11}\right)-\frac{\ln\left(x^2-x+3\right)}{12}+\frac{\sqrt{11}}{66}\arctan\left(\frac{(2x-1)\sqrt{11}}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+5*x^2+9),x)
```

```
[Out] 1/12*ln(x^2+x+3)+1/66*arctan(1/11*(1+2*x)*11^(1/2))*11^(1/2)-1/12*ln(x^2-x+
3)+1/66*11^(1/2)*arctan(1/11*(2*x-1)*11^(1/2))
```

Maxima [A] time = 1.46939, size = 72, normalized size = 1.07

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="maxima")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

Fricas [A] time = 1.2528, size = 193, normalized size = 2.88

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="fricas")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

Sympy [A] time = 0.233039, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2-x+3\right)}{12}+\frac{\log\left(x^2+x+3\right)}{12}+\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}x}{11}-\frac{\sqrt{11}}{11}\right)}{66}+\frac{\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}x}{11}+\frac{\sqrt{11}}{11}\right)}{66}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+5*x**2+9),x)
```

```
[Out] -log(x**2 - x + 3)/12 + log(x**2 + x + 3)/12 + sqrt(11)*atan(2*sqrt(11)*x/1
1 - sqrt(11)/11)/66 + sqrt(11)*atan(2*sqrt(11)*x/11 + sqrt(11)/11)/66
```

Giac [A] time = 1.13146, size = 72, normalized size = 1.07

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="giac")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

3.14 $\int \frac{1}{1-x^2+x^4} dx$

Optimal. Leaf size=74

$$-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 - Log[1 - Sqrt[3]*x + x^
2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi [A] time = 0.0489245, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 - Log[1 - Sqrt[3]*x + x^
2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{1-x^2+x^4}\,dx = \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{2\sqrt{3}}$$

$$= \frac{1}{4}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx - \frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}$$

$$= -\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x\right) - \frac{1}{2}\text{Subst}\left(\int\right.$$

$$= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right) + \frac{1}{2}\tan^{-1}\left(\sqrt{3}+2x\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}$$

Mathematica [C] time = 0.0712979, size = 77, normalized size = 1.04

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-\sqrt{-1+i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - Sqrt[-1 + I*Sqrt[3
]]*ArcTan[((1 + I*Sqrt[3])*x)/2]))/Sqrt[6]
```

Maple [A] time = 0.059, size = 57, normalized size = 0.8

$$\frac{\arctan\left(2x-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x+\sqrt{3}\right)}{2}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^2+1),x)
```

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))-1/12*ln(1+x^2-x*3^(1/2))*3^
(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - x^2 + 1), x)
```

Fricas [B] time = 1.45485, size = 529, normalized size = 7.15

$$-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/6*sqrt(6)*
```

```
sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)
*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 1/24*sqrt(6)*sqrt(2)*log
(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/24*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2)
*x + 2*x^2 + 2)
```

Sympy [A] time = 0.203368, size = 63, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4 - x^2 + 1), x)
```

3.15 $\int \frac{1}{2+2x^2+x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{\log\left(x^2-\sqrt{2\left(\sqrt{2}-1\right)}x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\log\left(x^2+\sqrt{2\left(\sqrt{2}-1\right)}x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}-\frac{1}{4}\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{4}\sqrt{\sqrt{2}-1}$$

```
[Out] -(Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] - 2*x)/Sqrt[2*(1 + Sqrt
[2])]])/4 + (Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] + 2*x)/Sqrt[
2*(1 + Sqrt[2])]])/4 - Log[Sqrt[2] - Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqr
t[-1 + Sqrt[2]]) + Log[Sqrt[2] + Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqrt[-1
 + Sqrt[2]])
```

Rubi [A] time = 0.161705, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2\left(\sqrt{2}-1\right)}x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\log\left(x^2+\sqrt{2\left(\sqrt{2}-1\right)}x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}-\frac{1}{4}\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{4}\sqrt{\sqrt{2}-1}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] - 2*x)/Sqrt[2*(1 + Sqrt
[2])]])/4 + (Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] + 2*x)/Sqrt[
2*(1 + Sqrt[2])]])/4 - Log[Sqrt[2] - Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqr
t[-1 + Sqrt[2]]) + Log[Sqrt[2] + Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqrt[-1
 + Sqrt[2]])
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{2+2x^2+x^4}\,dx = \frac{\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{-1+\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}+x}{\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{-1+\sqrt{2}}}$$

$$=\frac{\int \frac{1}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{1}{\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{-\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{8\sqrt{-1+\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2} \cdots}{8\sqrt{-1+\sqrt{2}}}$$

$$=-\frac{\log\left(\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2\right)}{8\sqrt{-1+\sqrt{2}}}+\frac{\log\left(\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2\right)}{8\sqrt{-1+\sqrt{2}}}-\frac{\text{Subst}\left(\int \frac{1}{-2\left(1+\sqrt{2}\right)-x^2}\,dx \cdots\right.}{2\cdots}$$

$$=-\frac{\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{1+\sqrt{2}}}-\frac{\log\left(\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2\right)}{8\sqrt{-1+\sqrt{2}}}+\frac{\log\left(\sqrt{2}+\cdots\right.}{8\cdots}$$

Mathematica [C] time = 0.0351169, size = 41, normalized size = 0.23

$$\frac{1}{4}\left((1-i)^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{1+i}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] ((1 - I)^(3/2)*ArcTan[x/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTan[x/Sqrt[1 + I]])
/4
```

Maple [B] time = 0.076, size = 386, normalized size = 2.2

$$\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-2+2\sqrt{2}}\right)\sqrt{2}\sqrt{-2+2\sqrt{2}}}{16}+\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-2+2\sqrt{2}}\right)\sqrt{-2+2\sqrt{2}}}{8}-\frac{\sqrt{2}\left(-2+2\sqrt{2}\right)}{8\sqrt{2+2\sqrt{2}}}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*x^2+2),x)
```

```
[Out] 1/16*ln(x^2+2^(1/2)+x*(-2+2*2^(1/2))^(1/2))*2^(1/2)*(-2+2*2^(1/2))^(1/2)+1/
8*ln(x^2+2^(1/2)+x*(-2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)-1/8/(2+2*2^(1
/2))^(1/2)*arctan((2*x+(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*2^(1/2)*(
-2+2*2^(1/2))-1/4/(2+2*2^(1/2))^(1/2)*arctan((2*x+(-2+2*2^(1/2))^(1/2))/(2+
2*2^(1/2))^(1/2))*(-2+2*2^(1/2))+1/2/(2+2*2^(1/2))^(1/2)*arctan((2*x+(-2+2*
2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*2^(1/2)-1/16*ln(x^2+2^(1/2)-x*(-2+2*2^
(1/2))^(1/2))*2^(1/2)*(-2+2*2^(1/2))^(1/2)-1/8*ln(x^2+2^(1/2)-x*(-2+2*2^(1/
2))^(1/2))*(-2+2*2^(1/2))^(1/2)-1/8/(2+2*2^(1/2))^(1/2)*arctan((2*x-(-2+2*2
^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*2^(1/2)*(-2+2*2^(1/2))-1/4/(2+2*2^(1/2)
)^(1/2)*arctan((2*x-(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2
))+1/2/(2+2*2^(1/2))^(1/2)*arctan((2*x-(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^
(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^4+2\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*x^2 + 2), x)
```

Fricas [A] time = 1.37879, size = 787, normalized size = 4.47

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}\log\left(2^{\frac{3}{4}}x\sqrt{-2\sqrt{2}+4}+2x^2+2\sqrt{2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}\log\left(-2^{\frac{3}{4}}x\sqrt{-2\sqrt{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log(2^(3/4)*x*sqrt(-2*sqrt(
2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) +
 4)*log(-2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/4*2^(1/4)*
sqrt(-2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 1/2*2^(1/
4)*sqrt(2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(-2*sqrt(2)
 + 4) - sqrt(2) + 1) - 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)
*x*sqrt(-2*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(-2*sqrt(2) + 4)
+ 2*x^2 + 2*sqrt(2))*sqrt(-2*sqrt(2) + 4) + sqrt(2) - 1)
```

Sympy [A] time = 0.552499, size = 20, normalized size = 0.11

$$\mathrm{RootSum}\left(512t^4-32t^2+1,\left(t\mapsto t\log\left(128t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*x**2+2),x)
```

```
[Out] RootSum(512*_t**4 - 32*_t**2 + 1, Lambda(_t, _t*log(128*_t**3 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^4+2x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4 + 2*x^2 + 2), x)
```

3.16 $\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx$

Optimal. Leaf size=10

$$\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-6\right)$$

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Rubi [A] time = 0.0157512, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{12-6x^2}\sqrt{2+6x^2}}\,dx$$
$$= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

Mathematica [C] time = 0.0255884, size = 65, normalized size = 6.5

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{3x^2+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{3}\sqrt{-3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6]
)/(Sqrt[3]*Sqrt[2 + 5*x^2 - 3*x^4])
```

Maple [B] time = 0.059, size = 51, normalized size = 5.1

$$\frac{\sqrt{2}}{2}\sqrt{-2x^2+4}\sqrt{3x^2+1}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)\frac{1}{\sqrt{-3x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*Ellipti
cF(1/2*x*2^(1/2),I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+5\,x^2+2}}{3\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 5*x^2 + 2)/(3*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

3.17 $\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(\sqrt{10}-2\right)}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

```
[Out] Sqrt[(2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(-2 + Sqrt[10])/2]*x], (-7 - 2
*Sqrt[10])/3]
```

Rubi [A] time = 0.133588, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[(2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(-2 + Sqrt[10])/2]*x], (-7 - 2
*Sqrt[10])/3]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{4+2\sqrt{10}-6x^2}\sqrt{-4+2\sqrt{10}+6x^2}}\,dx$$
$$= \sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

Mathematica [C] time = 0.0562925, size = 49, normalized size = 1.02

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{2}}}x\right),\frac{1}{3}\left(2\sqrt{10}-7\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (-7 + 2*Sqrt[10])/3])/Sqr
t[2 + Sqrt[10]]
```

Maple [B] time = 0.24, size = 84, normalized size = 1.8

$$2\,\frac{\sqrt{1-\left(-1+1/2\,\sqrt{10}\right)x^2}\sqrt{1-\left(-1-1/2\,\sqrt{10}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-4+2\,\sqrt{10}},i/3\sqrt{6}+i/3\sqrt{15}\right)}{\sqrt{-4+2\,\sqrt{10}}\sqrt{-3\,x^4+4\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+4*x^2+2)^(1/2),x)
```

```
[Out] 2/(-4+2*10^(1/2))^(1/2)*(1-(-1+1/2*10^(1/2))*x^2)^(1/2)*(1-(-1-1/2*10^(1/2)
)*x^2)^(1/2)/(-3*x^4+4*x^2+2)^(1/2)*EllipticF(1/2*x*(-4+2*10^(1/2))^(1/2),1
/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-3\,x^4+4\,x^2+2}}{3\,x^4-4\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 4*x^2 + 2)/(3*x^4 - 4*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 4*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

3.18 $\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{\sqrt{33}-3}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x], (-7 - S
qrt[33])/4]
```

Rubi [A] time = 0.104378, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{33}-3}}F\left(\sin^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^2 - 3*x^4],x]
```



```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x], (-7 - S
qrt[33])/4]
```


Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{3+\sqrt{33}-6x^2}\sqrt{-3+\sqrt{33}+6x^2}}\,dx$$
$$= \sqrt{\frac{2}{-3+\sqrt{33}}}F\left(\sin^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Mathematica [C] time = 0.0590053, size = 53, normalized size = 1.1

$$-i\sqrt{\frac{2}{3+\sqrt{33}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{6}{\sqrt{33}-3}}x\right),\frac{1}{4}\left(\sqrt{33}-7\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 3*x^2 - 3*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(3 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x],
 (-7 + Sqrt[33])/4]
```

Maple [B] time = 0.25, size = 80, normalized size = 1.7

$$2\,\frac{\sqrt{1-\left(-3/4+1/4\sqrt{33}\right)x^2}\sqrt{1-\left(-3/4-1/4\sqrt{33}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-3+\sqrt{33}},i/4\sqrt{6}+i/4\sqrt{22}\right)}{\sqrt{-3+\sqrt{33}}\sqrt{-3\,x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 2/(-3+33^(1/2))^(1/2)*(1-(-3/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*33^(1/
2))*x^2)^(1/2)/(-3*x^4+3*x^2+2)^(1/2)*EllipticF(1/2*x*(-3+33^(1/2))^(1/2),1
/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+3\,x^2+2}}{3\,x^4-3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 3*x^2 + 2)/(3*x^4 - 3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

3.19 $\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

```
[Out] EllipticF[ArcSin[Sqrt[3/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt[-1 + Sqrt
[7]]
```

Rubi [A] time = 0.0674992, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right)\middle|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt[-1 + Sqrt
[7]]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{2+2\sqrt{7}-6x^2}\sqrt{-2+2\sqrt{7}+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{-1+\sqrt{7}}}$$

Mathematica [C] time = 0.0427984, size = 49, normalized size = 1.11

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{3}{\sqrt{7}-1}}x\right),\frac{1}{3}\left(\sqrt{7}-4\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3])/Sqr
t[1 + Sqrt[7]]
```

Maple [B] time = 0.237, size = 84, normalized size = 1.9

$$2\,\frac{\sqrt{1-\left(-1/2+1/2\,\sqrt{7}\right)x^2}\sqrt{1-\left(-1/2-1/2\,\sqrt{7}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-2+2\,\sqrt{7}},i/6\sqrt{6}+i/6\sqrt{42}\right)}{\sqrt{-2+2\,\sqrt{7}}\sqrt{-3\,x^4+2\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+2*x^2+2)^(1/2),x)
```

```
[Out] 2/(-2+2*7^(1/2))^(1/2)*(1-(-1/2+1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*7^(1/2
))*x^2)^(1/2)/(-3*x^4+2*x^2+2)^(1/2)*EllipticF(1/2*x*(-2+2*7^(1/2))^(1/2),1
/6*I*6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+2\,x^2+2}}{3\,x^4-2\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 2*x^2 + 2)/(3*x^4 - 2*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 2*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

3.20 $\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rubi [A] time = 0.0119981, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x^2 - 3*x^4],x]

[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{6-6x^2}\sqrt{4+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}(x)\middle| -\frac{3}{2}\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.0227184, size = 63, normalized size = 5.25

$$-\frac{i\sqrt{1-x^2}\sqrt{3x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-3x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], -2/3]
)/(Sqrt[3]*Sqrt[2 + x^2 - 3*x^4])
```

Maple [B] time = 0.049, size = 41, normalized size = 3.4

$$\frac{\text{EllipticF}\left(x,\frac{i}{2}\sqrt{6}\right)}{2}\sqrt{-x^2+1}\sqrt{6\,x^2+4}\frac{1}{\sqrt{-3\,x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(-3*x^4+x^2+2)^(1/2)*EllipticF(x,1/2*I*6
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+x^2+2}}{3\,x^4-x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + x^2 + 2)/(3*x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 + 2), x)
```

3.21 $\int \frac{1}{\sqrt{2-3x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),-1\right)}{\sqrt[4]{6}}$$

```
[Out] EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

Rubi [A] time = 0.0101381, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {221}

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^4}}\,dx=\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Mathematica [A] time = 0.0220728, size = 18, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

Maple [B] time = 0.191, size = 54, normalized size = 3.

$$\frac{\sqrt{2}6^{\frac{3}{4}}}{24}\sqrt{4-2x^{2}\sqrt{6}}\sqrt{4+2x^{2}\sqrt{6}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt[4]{6}}{2},i\right)\frac{1}{\sqrt{-3x^{4}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+2)^(1/2),x)
```

```
[Out] 1/24*2^(1/2)*6^(3/4)*(4-2*x^2*6^(1/2))^(1/2)*(4+2*x^2*6^(1/2))^(1/2)/(-3*x^
4+2)^(1/2)*EllipticF(1/2*x*2^(1/2)*6^(1/4),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^{4}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-3x^{4}+2}}{3x^{4}-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 2)/(3*x^4 - 2), x)
```

Sympy [A] time = 0.624791, size = 37, normalized size = 2.06

$$\frac{\sqrt{2}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{3x^4e^{2i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+2)**(1/2),x)
```

```
[Out] sqrt(2)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(2*I*pi)/2)/
(8*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2), x)
```

3.22 $\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx$

Optimal. Leaf size=20

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi [A] time = 0.0111094, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{4-6x^2}\sqrt{6+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.025868, size = 20, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [B] time = 0.059, size = 49, normalized size = 2.5

$$\frac{\sqrt{6}}{6}\sqrt{-6\,x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i}{3}\sqrt{6}\right)\frac{1}{\sqrt{-3\,x^4-x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-x^2+2)^(1/2),x)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-x^2+2)^(1/2)*EllipticF(1
/2*x*6^(1/2),1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-x^2+2}}{3\,x^4+x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - x^2 + 2)/(3*x^4 + x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 + 2), x)
```

3.23 $\int \frac{1}{\sqrt{2-2x^2-3x^4}} \, dx$

Optimal. Leaf size=42

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{\sqrt{7}-1}}x\right),\frac{1}{3}\left(\sqrt{7}-4\right)\right)}{\sqrt{1+\sqrt{7}}}$$

```
[Out] EllipticF[ArcSin[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqrt[1 + Sqrt
[7]]
```

Rubi [A] time = 0.064737, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 2*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqrt[1 + Sqrt
[7]]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-2+2\sqrt{7}-6x^2}\sqrt{2+2\sqrt{7}+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Mathematica [C] time = 0.0435436, size = 51, normalized size = 1.21

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 - 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3])/Sqrt
[-1 + Sqrt[7]]
```

Maple [B] time = 0.227, size = 84, normalized size = 2.

$$2\,\frac{\sqrt{1-\left(1/2\,\sqrt{7}+1/2\right)x^2}\sqrt{1-\left(-1/2\,\sqrt{7}+1/2\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{2+2\,\sqrt{7}},i/6\sqrt{42}-i/6\sqrt{6}\right)}{\sqrt{2+2\,\sqrt{7}}\sqrt{-3\,x^4-2\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-2*x^2+2)^(1/2),x)
```

```
[Out] 2/(2+2*7^(1/2))^(1/2)*(1-(1/2*7^(1/2)+1/2)*x^2)^(1/2)*(1-(-1/2*7^(1/2)+1/2)
*x^2)^(1/2)/(-3*x^4-2*x^2+2)^(1/2)*EllipticF(1/2*x*(2+2*7^(1/2))^(1/2),1/6*
I*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-2\,x^2+2}}{3\,x^4+2\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 2*x^2 + 2)/(3*x^4 + 2*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 2*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

3.24 $\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx$

Optimal. Leaf size=46

$$\sqrt{\frac{2}{3+\sqrt{33}}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{6}{\sqrt{33}-3}}\,x\right),\frac{1}{4}\left(\sqrt{33}-7\right)\right)$$

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + S
qrt[33])/4]
```

Rubi [A] time = 0.0889233, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)\middle|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 3*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + S
qrt[33])/4]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-3+\sqrt{33}-6x^2}\sqrt{3+\sqrt{33}+6x^2}}\,dx$$
$$= \sqrt{\frac{2}{3+\sqrt{33}}}F\left(\sin^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Mathematica [C] time = 0.0643541, size = 55, normalized size = 1.2

$$-i\sqrt{\frac{2}{\sqrt{33}-3}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 - 3*x^2 - 3*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-3 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqrt[33])]*x],
 -7/4 - Sqrt[33]/4]
```

Maple [B] time = 0.256, size = 80, normalized size = 1.7

$$2\,\frac{\sqrt{1-\left(1/4\sqrt{33}+3/4\right)x^2}\sqrt{1-\left(-1/4\sqrt{33}+3/4\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{3+\sqrt{33}},i/4\sqrt{22}-i/4\sqrt{6}\right)}{\sqrt{3+\sqrt{33}}\sqrt{-3\,x^4-3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-3*x^2+2)^(1/2),x)
```

```
[Out] 2/(3+33^(1/2))^(1/2)*(1-(1/4*33^(1/2)+3/4)*x^2)^(1/2)*(1-(-1/4*33^(1/2)+3/4
)*x^2)^(1/2)/(-3*x^4-3*x^2+2)^(1/2)*EllipticF(1/2*x*(3+33^(1/2))^(1/2),1/4*
I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-3\,x^4-3\,x^2+2}}{3\,x^4+3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 3*x^2 + 2)/(3*x^4 + 3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

3.25 $\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{1}{6}\left(\sqrt{10}-2\right)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right),\frac{1}{3}\left(2\sqrt{10}-7\right)\right)$$

```
[Out] Sqrt[(-2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(2 + Sqrt[10])/2]*x], (-7 + 2
*Sqrt[10])/3]
```

Rubi [A] time = 0.0938131, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{1}{6}\left(\sqrt{10}-2\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 4*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[(-2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(2 + Sqrt[10])/2]*x], (-7 + 2
*Sqrt[10])/3]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-4+2\sqrt{10}-6x^2}\sqrt{4+2\sqrt{10}+6x^2}}\,dx$$
$$= \sqrt{\frac{1}{6}\left(-2+\sqrt{10}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

Mathematica [C] time = 0.0582146, size = 49, normalized size = 1.02

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\sqrt{\frac{5}{2}}-1}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x], (-7 - 2*Sqrt[10])/3])/Sq
rt[-2 + Sqrt[10]]
```

Maple [B] time = 0.24, size = 84, normalized size = 1.8

$$2\,\frac{\sqrt{1-\left(1+1/2\,\sqrt{10}\right)x^2}\sqrt{1-\left(-1/2\,\sqrt{10}+1\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{4+2\,\sqrt{10}},i/3\sqrt{15}-i/3\sqrt{6}\right)}{\sqrt{4+2\,\sqrt{10}}\sqrt{-3\,x^4-4\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-4*x^2+2)^(1/2),x)
```

```
[Out] 2/(4+2*10^(1/2))^(1/2)*(1-(1+1/2*10^(1/2))*x^2)^(1/2)*(1-(-1/2*10^(1/2)+1)*
x^2)^(1/2)/(-3*x^4-4*x^2+2)^(1/2)*EllipticF(1/2*x*(4+2*10^(1/2))^(1/2),1/3*
I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 - 4\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-3\,x^4 - 4\,x^2 + 2}}{3\,x^4 + 4\,x^2 - 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 4*x^2 + 2)/(3*x^4 + 4*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4 - 4x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 4*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 - 4\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

3.26 $\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{6}}$$

```
[Out] EllipticF[ArcSin[Sqrt[3]*x], -1/6]/Sqrt[6]
```

Rubi [A] time = 0.0145935, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{3}x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 5*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3]*x], -1/6]/Sqrt[6]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{2-6x^2}\sqrt{12+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{3}x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

Mathematica [B] time = 0.0229124, size = 54, normalized size = 3.

$$\frac{\sqrt{1-3x^2}\sqrt{x^2+2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{6}\sqrt{-3x^4-5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 5*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[1 - 3*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x], -1/6])/(Sqrt[6]
*Sqrt[2 - 5*x^2 - 3*x^4])
```

Maple [B] time = 0.055, size = 50, normalized size = 2.8

$$\frac{\sqrt{3}\text{EllipticF}\left(x\sqrt{3},\frac{i}{6}\sqrt{6}\right)}{6}\sqrt{-3\,x^2+1}\sqrt{2\,x^2+4}\frac{1}{\sqrt{-3\,x^4-5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-5*x^2+2)^(1/2),x)
```

```
[Out] 1/6*3^(1/2)*(-3*x^2+1)^(1/2)*(2*x^2+4)^(1/2)/(-3*x^4-5*x^2+2)^(1/2)*Ellipti
cF(x*3^(1/2),1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-5\,x^2+2}}{3\,x^4+5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 5*x^2 + 2)/(3*x^4 + 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

3.27 $\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{73}-7}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

```
[Out] Sqrt[2/(-7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[7 + Sqrt[73]]], (-61 -
7*Sqrt[73])/12]
```

Rubi [A] time = 0.064989, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{73}-7}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 7*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[7 + Sqrt[73]]], (-61 -
7*Sqrt[73])/12]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{7+\sqrt{73}-4x^2}\sqrt{-7+\sqrt{73}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{-7+\sqrt{73}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Mathematica [C] time = 0.0442848, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{7+\sqrt{73}}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{\sqrt{73}-7}}\right),\frac{1}{12}\left(7\sqrt{73}-61\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 7*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(7 + Sqrt[73])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-7 + Sqrt[73]]],
 (-61 + 7*Sqrt[73])/12]
```

Maple [B] time = 0.255, size = 84, normalized size = 1.9

$$6\,\frac{\sqrt{1-\left(-7/6+1/6\sqrt{73}\right)x^2}\sqrt{1-\left(-1/6\sqrt{73}-7/6\right)x^2}\text{EllipticF}\left(1/6\,x\sqrt{-42+6\sqrt{73}},\frac{7i}{12}\sqrt{6}+i/12\sqrt{438}\right)}{\sqrt{-42+6\sqrt{73}}\sqrt{-2x^4+7x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+7*x^2+3)^(1/2),x)
```

```
[Out] 6/(-42+6*73^(1/2))^(1/2)*(1-(-7/6+1/6*73^(1/2))*x^2)^(1/2)*(1-(-1/6*73^(1/2
)-7/6)*x^2)^(1/2)/(-2*x^4+7*x^2+3)^(1/2)*EllipticF(1/6*x*(-42+6*73^(1/2))^(
1/2),7/12*I*6^(1/2)+1/12*I*438^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+7\,x^2+3}}{2\,x^4-7\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 7*x^2 + 3)/(2*x^4 - 7*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 7*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

3.28 $\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(\sqrt{15}-3\right)}x\right),-4-\sqrt{15}\right)$$

```
[Out] Sqrt[(3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(-3 + Sqrt[15])/3]*x], -4 - Sq
rt[15]]
```

Rubi [A] time = 0.132986, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right)\middle|-4-\sqrt{15}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 6*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[(3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(-3 + Sqrt[15])/3]*x], -4 - Sq
rt[15]]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6+2\sqrt{15}-4x^2}\sqrt{-6+2\sqrt{15}+4x^2}}\,dx$$
$$= \sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right)|-4-\sqrt{15}\right)$$

Mathematica [C] time = 0.0520907, size = 43, normalized size = 0.98

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{3}}}x\right),\sqrt{15}-4\right)}{\sqrt{3+\sqrt{15}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 6*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/3]]*x], -4 + Sqrt[15]])/Sqrt[3 +
Sqrt[15]]
```

Maple [B] time = 0.24, size = 84, normalized size = 1.9

$$3\,\frac{\sqrt{1-\left(-1+1/3\sqrt{15}\right)x^2}\sqrt{1-\left(-1-1/3\sqrt{15}\right)x^2}\text{EllipticF}\left(1/3\,x\sqrt{-9+3\sqrt{15}},i/2\sqrt{6}+i/2\sqrt{10}\right)}{\sqrt{-9+3\sqrt{15}}\sqrt{-2\,x^4+6\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+6*x^2+3)^(1/2),x)
```

```
[Out] 3/(-9+3*15^(1/2))^(1/2)*(1-(-1+1/3*15^(1/2))*x^2)^(1/2)*(1-(-1-1/3*15^(1/2)
)*x^2)^(1/2)/(-2*x^4+6*x^2+3)^(1/2)*EllipticF(1/3*x*(-9+3*15^(1/2))^(1/2),1
/2*I*6^(1/2)+1/2*I*10^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 + 6\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4 + 6\,x^2 + 3}}{2\,x^4 - 6\,x^2 - 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 6*x^2 + 3)/(2*x^4 - 6*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 + 6x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 6*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 + 6\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

3.29 $\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx$

Optimal. Leaf size=10

$$\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right),-6\right)$$

```
[Out] EllipticF[ArcSin[x/Sqrt[3]], -6]
```

Rubi [A] time = 0.0105851, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 5*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[3]], -6]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{12-4x^2}\sqrt{2+4x^2}}\,dx$$
$$= F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle| -6\right)$$

Mathematica [C] time = 0.0253669, size = 65, normalized size = 6.5

$$-\frac{i\sqrt{1-\frac{x^2}{3}}\sqrt{2x^2+1}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{2}\sqrt{-2x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 5*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/3]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/6]
)/(Sqrt[2]*Sqrt[3 + 5*x^2 - 2*x^4])
```

Maple [B] time = 0.051, size = 51, normalized size = 5.1

$$\frac{\sqrt{3}}{3}\sqrt{-3x^2+9}\sqrt{2x^2+1}\text{EllipticF}\left(\frac{x\sqrt{3}}{3},i\sqrt{6}\right)\frac{1}{\sqrt{-2x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(-2*x^4+5*x^2+3)^(1/2)*Ellipti
cF(1/3*x*3^(1/2),I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+5\,x^2+3}}{2\,x^4-5\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 5*x^2 + 3)/(2*x^4 - 5*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

3.30 $\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2/(2 + Sqrt[10])]*x], (-7 - 2*Sqrt[10])/3]/Sqrt[-2 +
Sqrt[10]]
```

Rubi [A] time = 0.0766136, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 4*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(2 + Sqrt[10])]*x], (-7 - 2*Sqrt[10])/3]/Sqrt[-2 +
Sqrt[10]]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{4+2\sqrt{10}-4x^2}\sqrt{-4+2\sqrt{10}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)\middle|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{-2+\sqrt{10}}}$$

Mathematica [C] time = 0.0544684, size = 51, normalized size = 1.16

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{\sqrt{10}-2}}x\right),\frac{2\sqrt{10}}{3}-\frac{7}{3}\right)}{\sqrt{2+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x], -7/3 + (2*Sqrt[10])/3
])/Sqrt[2 + Sqrt[10]]
```

Maple [B] time = 0.211, size = 84, normalized size = 1.9

$$3\,\frac{\sqrt{1-\left(-2/3+1/3\,\sqrt{10}\right)x^2}\sqrt{1-\left(-2/3-1/3\,\sqrt{10}\right)x^2}\,\text{EllipticF}\left(1/3\,x\sqrt{-6+3\,\sqrt{10}},i/3\sqrt{6}+i/3\sqrt{15}\right)}{\sqrt{-6+3\,\sqrt{10}}\sqrt{-2\,x^4+4\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+4*x^2+3)^(1/2),x)
```

```
[Out] 3/(-6+3*10^(1/2))^(1/2)*(1-(-2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3*10^(
1/2))*x^2)^(1/2)/(-2*x^4+4*x^2+3)^(1/2)*EllipticF(1/3*x*(-6+3*10^(1/2))^(1/
2),1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+4\,x^2+3}}{2\,x^4-4\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 4*x^2 + 3)/(2*x^4 - 4*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 4*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

3.31 $\int \frac{1}{\sqrt{3+3x^2-2x^4}} \, dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{33}-3}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right),\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[3 + Sqrt[33]]], (-7 - S
qrt[33])/4]
```

Rubi [A] time = 0.0672437, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{33}-3}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 3*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[3 + Sqrt[33]]], (-7 - S
qrt[33])/4]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{3+\sqrt{33}-4x^2}\sqrt{-3+\sqrt{33}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{-3+\sqrt{33}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Mathematica [C] time = 0.0488248, size = 50, normalized size = 1.11

$$-i\sqrt{\frac{2}{3+\sqrt{33}}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{\sqrt{33}-3}}\right),\frac{1}{4}\left(\sqrt{33}-7\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 3*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(3 + Sqrt[33])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + Sqrt[33]]],
 (-7 + Sqrt[33])/4]
```

Maple [B] time = 0.214, size = 84, normalized size = 1.9

$$6\,\frac{\sqrt{1-\left(-1/2+1/6\sqrt{33}\right)x^2}\sqrt{1-\left(-1/2-1/6\sqrt{33}\right)x^2}\text{EllipticF}\left(1/6\,x\sqrt{-18+6\sqrt{33}},i/4\sqrt{6}+i/4\sqrt{22}\right)}{\sqrt{-18+6\sqrt{33}}\sqrt{-2\,x^4+3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+3*x^2+3)^(1/2),x)
```

```
[Out] 6/(-18+6*33^(1/2))^(1/2)*(1-(-1/2+1/6*33^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*33^
(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*33^(1/2))^(
1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+3\,x^2+3}}{2\,x^4-3\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 3*x^2 + 3)/(2*x^4 - 3*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 3*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

3.32 $\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right),\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt[-1 + Sqrt
[7]]
```

Rubi [A] time = 0.0620457, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 2*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt[-1 + Sqrt
[7]]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{2+2\sqrt{7}-4x^2}\sqrt{-2+2\sqrt{7}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{-1+\sqrt{7}}}$$

Mathematica [C] time = 0.0416297, size = 49, normalized size = 1.11

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-1}}x\right),\frac{1}{3}\left(\sqrt{7}-4\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3])/Sqr
t[1 + Sqrt[7]]
```

Maple [B] time = 0.213, size = 84, normalized size = 1.9

$$3\,\frac{\sqrt{1-\left(-1/3+1/3\,\sqrt{7}\right)x^2}\sqrt{1-\left(-1/3-1/3\,\sqrt{7}\right)x^2}\,\text{EllipticF}\left(1/3\,x\sqrt{-3+3\,\sqrt{7}},i/6\sqrt{6}+i/6\sqrt{42}\right)}{\sqrt{-3+3\,\sqrt{7}}\sqrt{-2\,x^4+2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+2*x^2+3)^(1/2),x)
```

```
[Out] 3/(-3+3*7^(1/2))^(1/2)*(1-(-1/3+1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*7^(1/2
))*x^2)^(1/2)/(-2*x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*(-3+3*7^(1/2))^(1/2),1
/6*I*6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 + 2\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4 + 2\,x^2 + 3}}{2\,x^4 - 2\,x^2 - 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 2*x^2 + 3)/(2*x^4 - 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 + 2x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 2*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 + 2\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

3.33 $\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx$

Optimal. Leaf size=20

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Rubi [A] time = 0.0110002, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right)|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6-4x^2}\sqrt{4+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0253478, size = 20, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Maple [B] time = 0.064, size = 47, normalized size = 2.4

$$\frac{\sqrt{6}}{6}\sqrt{-6\,x^2+9}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i}{2}\sqrt{6}\right)\frac{1}{\sqrt{-2\,x^4+x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+x^2+3)^(1/2),x)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-2*x^4+x^2+3)^(1/2)*EllipticF(1
/3*x*6^(1/2),1/2*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+x^2+3}}{2\,x^4-x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + x^2 + 3)/(2*x^4 - x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 + 3), x)
```

3.34 $\int \frac{1}{\sqrt{3-2x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{\sqrt[4]{6}}$$

```
[Out] EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

Rubi [A] time = 0.0060963, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {221}

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x^4}}\,dx=\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Mathematica [A] time = 0.0210378, size = 18, normalized size = 1.

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

Maple [B] time = 0.148, size = 54, normalized size = 3.

$$\frac{\sqrt{3}6^{\frac{3}{4}}}{54}\sqrt{9-3x^2\sqrt{6}}\sqrt{9+3x^2\sqrt{6}}\mathrm{EllipticF}\left(\frac{x\sqrt{3}\sqrt[4]{6}}{3},i\right)\frac{1}{\sqrt{-2x^4+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+3)^(1/2),x)
```

```
[Out] 1/54*3^(1/2)*6^(3/4)*(9-3*x^2*6^(1/2))^(1/2)*(9+3*x^2*6^(1/2))^(1/2)/(-2*x^
4+3)^(1/2)*EllipticF(1/3*x*3^(1/2)*6^(1/4),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2x^4+3}}{2x^4-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 3)/(2*x^4 - 3), x)
```

Sympy [A] time = 0.639526, size = 37, normalized size = 2.06

$$\frac{\sqrt{3}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{2i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+3)**(1/2),x)
```

```
[Out] sqrt(3)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(2*I*pi)/3)/
(12*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3), x)
```

3.35 $\int \frac{1}{\sqrt{3-x^2-2x^4}} \, dx$

Optimal. Leaf size=12

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticF[ArcSin[x], -2/3]/Sqrt[3]
```

Rubi [A] time = 0.010715, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}(x)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -2/3]/Sqrt[3]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{4-4x^2}\sqrt{6+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}(x)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.0224095, size = 65, normalized size = 5.42

$$-\frac{i\sqrt{1-x^2}\sqrt{2x^2+3}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-2x^4-x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], -3/2]
)/(Sqrt[2]*Sqrt[3 - x^2 - 2*x^4])
```

Maple [B] time = 0.049, size = 43, normalized size = 3.6

$$\frac{\text{EllipticF}\left(x,\frac{i}{3}\sqrt{6}\right)}{3}\sqrt{-x^2+1}\sqrt{6\,x^2+9}\frac{1}{\sqrt{-2\,x^4-x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-x^2+3)^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(6*x^2+9)^(1/2)/(-2*x^4-x^2+3)^(1/2)*EllipticF(x,1/3*I*6
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-x^2+3}}{2\,x^4+x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - x^2 + 3)/(2*x^4 + x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 + 3), x)
```

3.36 $\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx$

Optimal. Leaf size=42

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-1}}\,x\right),\frac{1}{3}\left(\sqrt{7}-4\right)\right)}{\sqrt{1+\sqrt{7}}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqrt[1 + Sqrt
[7]]
```

Rubi [A] time = 0.0473294, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)\middle|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqrt[1 + Sqrt
[7]]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-2+2\sqrt{7}-4x^2}\sqrt{2+2\sqrt{7}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Mathematica [C] time = 0.0440687, size = 51, normalized size = 1.21

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right),-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3])/Sqrt
[-1 + Sqrt[7]]
```

Maple [B] time = 0.208, size = 84, normalized size = 2.

$$3\,\frac{\sqrt{1-\left(1/3\sqrt{7}+1/3\right)x^2}\sqrt{1-\left(-1/3\sqrt{7}+1/3\right)x^2}\text{EllipticF}\left(1/3\,x\sqrt{3+3\sqrt{7}},i/6\sqrt{42}-i/6\sqrt{6}\right)}{\sqrt{3+3\sqrt{7}}\sqrt{-2\,x^4-2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-2*x^2+3)^(1/2),x)
```

```
[Out] 3/(3+3*7^(1/2))^(1/2)*(1-(1/3*7^(1/2)+1/3)*x^2)^(1/2)*(1-(-1/3*7^(1/2)+1/3)
*x^2)^(1/2)/(-2*x^4-2*x^2+3)^(1/2)*EllipticF(1/3*x*(3+3*7^(1/2))^(1/2),1/6*
I*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4-2\,x^2+3}}{2\,x^4+2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 2*x^2 + 3)/(2*x^4 + 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 2*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

3.37 $\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx$

Optimal. Leaf size=43

$$\sqrt{\frac{2}{3+\sqrt{33}}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2x}{\sqrt{\sqrt{33}-3}}\right),\frac{1}{4}\left(\sqrt{33}-7\right)\right)$$

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[-3 + Sqrt[33]]], (-7 + S
qrt[33])/4]
```

Rubi [A] time = 0.0314177, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{3+\sqrt{33}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)\middle|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 3*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[-3 + Sqrt[33]]], (-7 + S
qrt[33])/4]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-3+\sqrt{33}-4x^2}\sqrt{3+\sqrt{33}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{3+\sqrt{33}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)\middle|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Mathematica [C] time = 0.0444487, size = 52, normalized size = 1.21

$$-i\sqrt{\frac{2}{\sqrt{33}-3}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right),-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 3*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-3 + Sqrt[33])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[33]]],
 -7/4 - Sqrt[33]/4]
```

Maple [B] time = 0.211, size = 84, normalized size = 2.

$$6\,\frac{\sqrt{1-\left(1/6\,\sqrt{33}+1/2\right)x^2}\sqrt{1-\left(-1/6\,\sqrt{33}+1/2\right)x^2}\text{EllipticF}\left(1/6\,x\sqrt{18+6\,\sqrt{33}},i/4\sqrt{22}-i/4\sqrt{6}\right)}{\sqrt{18+6\,\sqrt{33}}\sqrt{-2\,x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-3*x^2+3)^(1/2),x)
```

```
[Out] 6/(18+6*33^(1/2))^(1/2)*(1-(1/6*33^(1/2)+1/2)*x^2)^(1/2)*(1-(-1/6*33^(1/2)+
1/2)*x^2)^(1/2)/(-2*x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*(18+6*33^(1/2))^(1/2
),1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-3\,x^2+3}}{2\,x^4+3\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 3*x^2 + 3)/(2*x^4 + 3*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 3*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

3.38 $\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{\sqrt{10}-2}}x\right),\frac{1}{3}\left(2\sqrt{10}-7\right)\right)}{\sqrt{2+\sqrt{10}}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2/(-2 + Sqrt[10])]*x], (-7 + 2*Sqrt[10])/3]/Sqrt[2 +
Sqrt[10]]
```

Rubi [A] time = 0.0679591, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(-2 + Sqrt[10])]*x], (-7 + 2*Sqrt[10])/3]/Sqrt[2 +
Sqrt[10]]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-4+2\sqrt{10}-4x^2}\sqrt{4+2\sqrt{10}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Mathematica [C] time = 0.0536968, size = 51, normalized size = 1.16

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right),-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{\sqrt{10}-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x], -7/3 - (2*Sqrt[10])/3]
)/Sqrt[-2 + Sqrt[10]]
```

Maple [B] time = 0.209, size = 84, normalized size = 1.9

$$3\,\frac{\sqrt{1-\left(2/3+1/3\sqrt{10}\right)x^2}\sqrt{1-\left(2/3-1/3\sqrt{10}\right)x^2}\text{EllipticF}\left(1/3\,x\sqrt{6+3\sqrt{10}},i/3\sqrt{15}-i/3\sqrt{6}\right)}{\sqrt{6+3\sqrt{10}}\sqrt{-2\,x^4-4\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-4*x^2+3)^(1/2),x)
```

```
[Out] 3/(6+3*10^(1/2))^(1/2)*(1-(2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*10^(1/2
))*x^2)^(1/2)/(-2*x^4-4*x^2+3)^(1/2)*EllipticF(1/3*x*(6+3*10^(1/2))^(1/2),1
/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-4\,x^2+3}}{2\,x^4+4\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 4*x^2 + 3)/(2*x^4 + 4*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 4*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

3.39 $\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{6}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/6]/Sqrt[6]
```

Rubi [A] time = 0.0122699, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 5*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/6]/Sqrt[6]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{2-4x^2}\sqrt{12+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{6}}$$

Mathematica [B] time = 0.0243992, size = 54, normalized size = 3.

$$\frac{\sqrt{1-2x^2}\sqrt{x^2+3}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{6}\sqrt{-2x^4-5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/6])/(Sqrt[6]
*Sqrt[3 - 5*x^2 - 2*x^4])
```

Maple [B] time = 0.056, size = 50, normalized size = 2.8

$$\frac{\sqrt{2}\text{EllipticF}\left(x\sqrt{2},\frac{i}{6}\sqrt{6}\right)}{6}\sqrt{-2x^2+1}\sqrt{3x^2+9}\frac{1}{\sqrt{-2x^4-5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-5*x^2+3)^(1/2),x)
```

```
[Out] 1/6*2^(1/2)*(-2*x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-2*x^4-5*x^2+3)^(1/2)*Ellipti
cF(x*2^(1/2),1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-5\,x^2+3}}{2\,x^4+5\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 5*x^2 + 3)/(2*x^4 + 5*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 5*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

3.40 $\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx$

Optimal. Leaf size=42

$$\sqrt{\frac{1}{6}\left(\sqrt{15}-3\right)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right),\sqrt{15}-4\right)$$

```
[Out] Sqrt[(-3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(3 + Sqrt[15])/3]*x], -4 + Sq
rt[15]]
```

Rubi [A] time = 0.0958311, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{1}{6}\left(\sqrt{15}-3\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right)|-4+\sqrt{15}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 6*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[(-3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(3 + Sqrt[15])/3]*x], -4 + Sq
rt[15]]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-6+2\sqrt{15}-4x^2}\sqrt{6+2\sqrt{15}+4x^2}}\,dx$$
$$= \sqrt{\frac{1}{6}\left(-3+\sqrt{15}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right)\middle| -4+\sqrt{15}\right)$$

Mathematica [C] time = 0.0546513, size = 45, normalized size = 1.07

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\sqrt{\frac{5}{3}}-1}x\right),-4-\sqrt{15}\right)}{\sqrt{\sqrt{15}-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 6*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x], -4 - Sqrt[15]])/Sqrt[-3
+ Sqrt[15]]
```

Maple [B] time = 0.229, size = 84, normalized size = 2.

$$3\,\frac{\sqrt{1-\left(1+1/3\,\sqrt{15}\right)x^2}\sqrt{1-\left(1-1/3\,\sqrt{15}\right)x^2}\text{EllipticF}\left(1/3\,x\sqrt{9+3\,\sqrt{15}},i/2\sqrt{10}-i/2\sqrt{6}\right)}{\sqrt{9+3\,\sqrt{15}}\sqrt{-2\,x^4-6\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-6*x^2+3)^(1/2),x)
```

```
[Out] 3/(9+3*15^(1/2))^(1/2)*(1-(1+1/3*15^(1/2))*x^2)^(1/2)*(1-(1-1/3*15^(1/2))*x
^2)^(1/2)/(-2*x^4-6*x^2+3)^(1/2)*EllipticF(1/3*x*(9+3*15^(1/2))^(1/2),1/2*I
*10^(1/2)-1/2*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4-6\,x^2+3}}{2\,x^4+6\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 6*x^2 + 3)/(2*x^4 + 6*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 6*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

3.41 $\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{7+\sqrt{73}}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2x}{\sqrt{\sqrt{73}-7}}\right),\frac{1}{12}\left(7\sqrt{73}-61\right)\right)$$

```
[Out] Sqrt[2/(7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[-7 + Sqrt[73]]], (-61 +
7*Sqrt[73])/12]
```

Rubi [A] time = 0.0478908, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{7+\sqrt{73}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 7*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[-7 + Sqrt[73]]], (-61 +
7*Sqrt[73])/12]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-7+\sqrt{73}-4x^2}\sqrt{7+\sqrt{73}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{7+\sqrt{73}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

Mathematica [C] time = 0.0435045, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{\sqrt{73}-7}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 7*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-7 + Sqrt[73])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[7 + Sqrt[73]]],
 (-61 - 7*Sqrt[73])/12]
```

Maple [B] time = 0.23, size = 84, normalized size = 1.9

$$6\,\frac{\sqrt{1-\left(1/6\,\sqrt{73}+7/6\right)x^2}\sqrt{1-\left(7/6-1/6\,\sqrt{73}\right)x^2}\text{EllipticF}\left(1/6\,x\sqrt{42+6\,\sqrt{73}},i/12\sqrt{438}-\frac{7i}{12}\sqrt{6}\right)}{\sqrt{42+6\,\sqrt{73}}\sqrt{-2\,x^4-7\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-7*x^2+3)^(1/2),x)
```

```
[Out] 6/(42+6*73^(1/2))^(1/2)*(1-(1/6*73^(1/2)+7/6)*x^2)^(1/2)*(1-(7/6-1/6*73^(1/
2))*x^2)^(1/2)/(-2*x^4-7*x^2+3)^(1/2)*EllipticF(1/6*x*(42+6*73^(1/2))^(1/2)
,1/12*I*438^(1/2)-7/12*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-7\,x^2+3}}{2\,x^4+7\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 7*x^2 + 3)/(2*x^4 + 7*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 7*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

3.42 $\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{x^2+2}\sqrt{3x^2-1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{2}}x}{\sqrt{3x^2-1}}\right),\frac{6}{7}\right)}{\sqrt{7}\sqrt{3x^4+5x^2-2}}$$

```
[Out] (Sqrt[2 + x^2]*Sqrt[-1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7/2]*x)/Sqrt[-1 + 3*
x^2]], 6/7])/(Sqrt[7]*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Rubi [A] time = 0.0082686, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2+2}\sqrt{3x^2-1}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{2}}x}{\sqrt{3x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\sqrt{3x^4+5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x^2 + 3*x^4],x]
```


```
[Out] (Sqrt[2 + x^2]*Sqrt[-1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7/2]*x)/Sqrt[-1 + 3*
x^2]], 6/7])/(Sqrt[7]*Sqrt[-2 + 5*x^2 + 3*x^4])
```


Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx = \frac{\sqrt{2+x^2}\sqrt{-1+3x^2}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{2}}x}{\sqrt{-1+3x^2}}\right)|\frac{6}{7}\right)}{\sqrt{7}\sqrt{-2+5x^2+3x^4}}$$

Mathematica [A] time = 0.0219737, size = 54, normalized size = 0.81

$$\frac{\sqrt{1-3x^2}\sqrt{x^2+2}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{6}\sqrt{3x^4+5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[1 - 3*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x], -1/6])/(Sqrt[6]
*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Maple [C] time = 0.052, size = 53, normalized size = 0.8

$$-\frac{i}{2}\sqrt{2}\,\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},i\sqrt{6}\right)\sqrt{2\,x^2+4}\sqrt{-3\,x^2+1}\frac{1}{\sqrt{3\,x^4+5\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+5*x^2-2)^(1/2),x)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)/(3*x^4+5*x^2-2)^(1/2)*Ellip
ticF(1/2*I*x*2^(1/2),I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+5\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

3.43 $\int \frac{1}{\sqrt{-2+4x^2+3x^4}} \, dx$

Optimal. Leaf size=141

$$\frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-2}}\right),\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{3x^4+4x^2-2}}$$

```
[Out] (Sqrt[(2 - (2 - Sqrt[10])*x^2)/(2 - (2 + Sqrt[10])*x^2)]*Sqrt[-2 + (2 + Sqr
t[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 + (2 + Sqrt[10])*x
^2]], (5 + Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2 - (2 + Sqrt[10])*x^2)^(-1)]*S
qrt[-2 + 4*x^2 + 3*x^4])
```

Rubi [A] time = 0.0347325, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2 - (2 - Sqrt[10])*x^2)/(2 - (2 + Sqrt[10])*x^2)]*Sqrt[-2 + (2 + Sqr
t[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 + (2 + Sqrt[10])*x
^2]], (5 + Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2 - (2 + Sqrt[10])*x^2)^(-1)]*S
qrt[-2 + 4*x^2 + 3*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx = \frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{-2+\left(2+\sqrt{10}\right)x^2}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-2+\left(2+\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{-2+4x^2+3x^4}}$$

Mathematica [C] time = 0.0519145, size = 81, normalized size = 0.57

$$-\frac{i\sqrt{-3x^4-4x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\sqrt{\frac{5}{2}}-1}x\right),\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}\sqrt{3x^4+4x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 - 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x],
(-7 - 2*Sqrt[10])/3])/(Sqrt[-2 + Sqrt[10]]*Sqrt[-2 + 4*x^2 + 3*x^4])
```

Maple [C] time = 0.181, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-1/2\sqrt{10}+1\right)x^2}\sqrt{1-\left(1+1/2\sqrt{10}\right)x^2}\,\text{EllipticF}\left(1/2\sqrt{4-2\sqrt{10}}x,i/3\sqrt{6}+i/3\sqrt{15}\right)}{\sqrt{4-2\sqrt{10}}\sqrt{3x^4+4x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+4*x^2-2)^(1/2),x)
```

```
[Out] 2/(4-2*10^(1/2))^(1/2)*(1-(-1/2*10^(1/2)+1)*x^2)^(1/2)*(1-(1+1/2*10^(1/2))*
x^2)^(1/2)/(3*x^4+4*x^2-2)^(1/2)*EllipticF(1/2*(4-2*10^(1/2))^(1/2)*x,1/3*I
*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+4\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 4*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

3.44 $\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-4}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-4}}\right),\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{3x^4+3x^2-2}}$$

```
[Out] (Sqrt[(4 - (3 - Sqrt[33])*x^2)/(4 - (3 + Sqrt[33])*x^2)]*Sqrt[-4 + (3 + Sqr
t[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 + (3 + Sqrt[33])*
x^2]], (11 + Sqrt[33])/22])/(2*Sqrt[2]*33^(1/4)*Sqrt[(4 - (3 + Sqrt[33])*x^
2)^(-1)]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

Rubi [A] time = 0.0385048, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-4}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 3*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(4 - (3 - Sqrt[33])*x^2)/(4 - (3 + Sqrt[33])*x^2)]*Sqrt[-4 + (3 + Sqr
t[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 + (3 + Sqrt[33])*
x^2]], (11 + Sqrt[33])/22])/(2*Sqrt[2]*33^(1/4)*Sqrt[(4 - (3 + Sqrt[33])*x^
2)^(-1)]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx = \frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{-4+\left(3+\sqrt{33}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-4+\left(3+\sqrt{33}\right)x^2}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{-2+3x^2+3x^4}}$$

Mathematica [C] time = 0.0648364, size = 83, normalized size = 0.57

$$-\frac{i\sqrt{-6x^4-6x^2+4}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right),-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{\sqrt{33}-3}\sqrt{3x^4+3x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[4 - 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqrt[33])]*x]
, -7/4 - Sqrt[33]/4])/(Sqrt[-3 + Sqrt[33]]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

Maple [C] time = 0.176, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-1/4\sqrt{33}+3/4\right)x^2}\sqrt{1-\left(1/4\sqrt{33}+3/4\right)x^2}\,\text{EllipticF}\left(1/2\sqrt{3-\sqrt{33}}x,i/4\sqrt{6}+i/4\sqrt{22}\right)}{\sqrt{3-\sqrt{33}}\sqrt{3x^4+3x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+3*x^2-2)^(1/2),x)
```

```
[Out] 2/(3-33^(1/2))^(1/2)*(1-(-1/4*33^(1/2)+3/4)*x^2)^(1/2)*(1-(1/4*33^(1/2)+3/4
)*x^2)^(1/2)/(3*x^4+3*x^2-2)^(1/2)*EllipticF(1/2*(3-33^(1/2))^(1/2)*x,1/4*I
*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^{4}+3\,x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\sqrt{3\,x^{4}+3\,x^{2}-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^{4}+3x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 3*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^{4}+3\,x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

3.45 $\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{\frac{2-(1-\sqrt{7})x^2}{2-(1+\sqrt{7})x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{(1+\sqrt{7})x^2-2}}\right),\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-(1+\sqrt{7})x^2}}\sqrt{3x^4+2x^2-2}}$$

```
[Out] (Sqrt[(2 - (1 - Sqrt[7])*x^2)/(2 - (1 + Sqrt[7])*x^2)]*Sqrt[-2 + (1 + Sqrt[
7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[7])*x^2]]
, (7 + Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 - (1 + Sqrt[7])*x^2)^(-1)]*Sqrt[-2
+ 2*x^2 + 3*x^4])
```

Rubi [A] time = 0.0302293, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{2-(1-\sqrt{7})x^2}{2-(1+\sqrt{7})x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{(1+\sqrt{7})x^2-2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-(1+\sqrt{7})x^2}}\sqrt{3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 2*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2 - (1 - Sqrt[7])*x^2)/(2 - (1 + Sqrt[7])*x^2)]*Sqrt[-2 + (1 + Sqrt[
7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[7])*x^2]]
, (7 + Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 - (1 + Sqrt[7])*x^2)^(-1)]*Sqrt[-2
+ 2*x^2 + 3*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx = \frac{\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{-2+\left(1+\sqrt{7}\right)x^2}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-2+\left(1+\sqrt{7}\right)x^2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{-2+2x^2+3x^4}}$$

Mathematica [C] time = 0.0515857, size = 83, normalized size = 0.59

$$-\frac{i\sqrt{-3x^4-2x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right),-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}\sqrt{3x^4+2x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 - 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x],
 -4/3 - Sqrt[7]/3])/(Sqrt[-1 + Sqrt[7]]*Sqrt[-2 + 2*x^2 + 3*x^4])
```

Maple [C] time = 0.178, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-1/2\,\sqrt{7}+1/2\right)x^2}\sqrt{1-\left(1/2\,\sqrt{7}+1/2\right)x^2}\text{EllipticF}\left(1/2\,\sqrt{2-2\,\sqrt{7}}x,i/6\sqrt{6}+i/6\sqrt{42}\right)}{\sqrt{2-2\,\sqrt{7}}\sqrt{3\,x^4+2\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2*x^2-2)^(1/2),x)
```

```
[Out] 2/(2-2*7^(1/2))^(1/2)*(1-(-1/2*7^(1/2)+1/2)*x^2)^(1/2)*(1-(1/2*7^(1/2)+1/2)
*x^2)^(1/2)/(3*x^4+2*x^2-2)^(1/2)*EllipticF(1/2*(2-2*7^(1/2))^(1/2)*x,1/6*I
*6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+2\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 2*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

3.46 $\int \frac{1}{\sqrt{-2+x^2+3x^4}} \, dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2+1}\sqrt{3x^2-2}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{5}x}{\sqrt{3x^2-2}}\right),\frac{3}{5}\right)}{\sqrt{5}\sqrt{3x^4+x^2-2}}$$

```
[Out] (Sqrt[1 + x^2]*Sqrt[-2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-2 + 3*x^
2]], 3/5])/(Sqrt[5]*Sqrt[-2 + x^2 + 3*x^4])
```

Rubi [A] time = 0.0077939, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1097}

$$\frac{\sqrt{x^2+1}\sqrt{3x^2-2}F\left(\sin^{-1}\left(\frac{\sqrt{5}x}{\sqrt{3x^2-2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\sqrt{3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[-2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-2 + 3*x^
2]], 3/5])/(Sqrt[5]*Sqrt[-2 + x^2 + 3*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}} \, dx = \frac{\sqrt{1+x^2}\sqrt{-2+3x^2}F\left(\sin^{-1}\left(\frac{\sqrt{5}x}{\sqrt{-2+3x^2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\sqrt{-2+x^2+3x^4}}$$

Mathematica [A] time = 0.0298895, size = 48, normalized size = 0.76

$$\frac{\sqrt{\left(\frac{2}{3}-x^2\right)\left(x^2+1\right)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3x^4+x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2/3 - x^2)*(1 + x^2)]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/Sqrt[-2
+ x^2 + 3*x^4]
```

Maple [C] time = 0.046, size = 43, normalized size = 0.7

$$-\frac{i}{2}\text{EllipticF}\left(ix,\frac{i}{2}\sqrt{6}\right)\sqrt{x^2+1}\sqrt{-6\,x^2+4}\frac{1}{\sqrt{3\,x^4+x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+x^2-2)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(3*x^4+x^2-2)^(1/2)*EllipticF(I*x,1/2
*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 - 2), x)
```

3.47 $\int \frac{1}{\sqrt{-2+3x^4}}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\sqrt{6}x^2-2}\sqrt{\frac{\sqrt{6}x^2+2}{2-\sqrt{6}x^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-2}}\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}x^2}}\sqrt{3x^4-2}}$$

```
[Out] (Sqrt[-2 + Sqrt[6]*x^2]*Sqrt[(2 + Sqrt[6]*x^2)/(2 - Sqrt[6]*x^2)]*EllipticF
[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-2 + Sqrt[6]*x^2]], 1/2])/(2*6^(1/4)*Sqrt[
(2 - Sqrt[6]*x^2)^(-1)]*Sqrt[-2 + 3*x^4])
```

Rubi [A] time = 0.016283, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {223}

$$\frac{\sqrt{\sqrt{6}x^2-2}\sqrt{\frac{\sqrt{6}x^2+2}{2-\sqrt{6}x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-2}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}x^2}}\sqrt{3x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 3*x^4],x]
```


```
[Out] (Sqrt[-2 + Sqrt[6]*x^2]*Sqrt[(2 + Sqrt[6]*x^2)/(2 - Sqrt[6]*x^2)]*EllipticF
[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-2 + Sqrt[6]*x^2]], 1/2])/(2*6^(1/4)*Sqrt[
(2 - Sqrt[6]*x^2)^(-1)]*Sqrt[-2 + 3*x^4])
```


Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[(a - q*x^2)/(a + q*x^2)]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqr
t[(a + q*x^2)/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[a + b*x^4]*Sqrt[a/(a + q*x^2)]),
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx = \frac{\sqrt{-2+\sqrt{6}x^2}\sqrt{\frac{2+\sqrt{6}x^2}{2-\sqrt{6}x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{-2+\sqrt{6}x^2}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}x^2}}\sqrt{-2+3x^4}}$$

Mathematica [A] time = 0.0236983, size = 40, normalized size = 0.35

$$\frac{\sqrt{2-3x^4}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),-1\right)}{\sqrt[4]{6}\sqrt{3x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^4]*EllipticF[ArcSin[(3/2)^(1/4)*x], -1])/(6^(1/4)*Sqrt[-2 + 3
*x^4])
```

Maple [C] time = 0.166, size = 56, normalized size = 0.5

$$\frac{1}{2\sqrt{-2\sqrt{6}}}\sqrt{4+2x^2\sqrt{6}}\sqrt{4-2x^2\sqrt{6}}\,\text{EllipticF}\left(\frac{\sqrt{-2\sqrt{6}}x}{2},i\right)\frac{1}{\sqrt{3x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-2)^(1/2),x)
```

```
[Out] 1/2/(-2*6^(1/2))^(1/2)*(4+2*x^2*6^(1/2))^(1/2)*(4-2*x^2*6^(1/2))^(1/2)/(3*x
^4-2)^(1/2)*EllipticF(1/2*(-2*6^(1/2))^(1/2)*x,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 2), x)
```

Sympy [C] time = 0.667135, size = 34, normalized size = 0.3

$$-\frac{\sqrt{2}ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{3x^4}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-2)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4/2)/(8*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2), x)
```

3.48 $\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x^2-1}\sqrt{3x^2+2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{2}}x}{\sqrt{x^2-1}}\right),\frac{2}{5}\right)}{\sqrt{5}\sqrt{3x^4-x^2-2}}$$

```
[Out] (Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5/2]*x)/Sqrt[-1 + x^
2]], 2/5])/(Sqrt[5]*Sqrt[-2 - x^2 + 3*x^4])
```

Rubi [A] time = 0.006997, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2-1}\sqrt{3x^2+2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{2}}x}{\sqrt{x^2-1}}\right)|\frac{2}{5}\right)}{\sqrt{5}\sqrt{3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5/2]*x)/Sqrt[-1 + x^
2]], 2/5])/(Sqrt[5]*Sqrt[-2 - x^2 + 3*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx = \frac{\sqrt{-1+x^2}\sqrt{2+3x^2}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{2}}x}{\sqrt{-1+x^2}}\right)|\frac{2}{5}\right)}{\sqrt{5}\sqrt{-2-x^2+3x^4}}$$

Mathematica [C] time = 0.0232928, size = 60, normalized size = 0.92

$$-\frac{i\sqrt{1-x^2}\sqrt{3x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{9x^4-3x^2-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], -2/3]
)/Sqrt[-6 - 3*x^2 + 9*x^4]
```

Maple [C] time = 0.05, size = 53, normalized size = 0.8

$$-\frac{i}{6}\sqrt{6}\text{EllipticF}\left(\frac{i}{2}x\sqrt{6},\frac{i}{3}\sqrt{6}\right)\sqrt{6x^2+4}\sqrt{-x^2+1}\frac{1}{\sqrt{3x^4-x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-x^2-2)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+4)^(1/2)*(-x^2+1)^(1/2)/(3*x^4-x^2-2)^(1/2)*EllipticF
(1/2*I*x*6^(1/2),1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 - 2), x)
```

3.49 $\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{-\left(1-\sqrt{7}\right)x^2-2}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+2}{\left(1-\sqrt{7}\right)x^2+2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(1-\sqrt{7}\right)x^2-2}}\right),\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+2}}\sqrt{3x^4-2x^2-2}}$$

```
[Out] (Sqrt[-2 - (1 - Sqrt[7])*x^2]*Sqrt[(2 + (1 + Sqrt[7])*x^2)/(2 + (1 - Sqrt[7
])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[7])*x^2]]
, (7 - Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 + (1 - Sqrt[7])*x^2)^(-1)]*Sqrt[-2
- 2*x^2 + 3*x^4])
```

Rubi [A] time = 0.02631, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(1-\sqrt{7}\right)x^2-2}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+2}{\left(1-\sqrt{7}\right)x^2+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(1-\sqrt{7}\right)x^2-2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+2}}\sqrt{3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 2*x^2 + 3*x^4],x]
```


```
[Out] (Sqrt[-2 - (1 - Sqrt[7])*x^2]*Sqrt[(2 + (1 + Sqrt[7])*x^2)/(2 + (1 - Sqrt[7
])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[7])*x^2]]
, (7 - Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 + (1 - Sqrt[7])*x^2)^(-1)]*Sqrt[-2
- 2*x^2 + 3*x^4])
```


Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = \frac{\sqrt{-2-\left(1-\sqrt{7}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{7}\right)x^2}{2+\left(1-\sqrt{7}\right)x^2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-2-\left(1-\sqrt{7}\right)x^2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2+\left(1-\sqrt{7}\right)x^2}}\sqrt{-2-2x^2+3x^4}}$$

Mathematica [C] time = 0.0448102, size = 81, normalized size = 0.55

$$-\frac{i\sqrt{-3x^4+2x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{3}{\sqrt{7}-1}}x\right),\frac{1}{3}\left(\sqrt{7}-4\right)\right)}{\sqrt{1+\sqrt{7}}\sqrt{3x^4-2x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 + 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x]
, (-4 + Sqrt[7])/3])/(Sqrt[1 + Sqrt[7]]*Sqrt[-2 - 2*x^2 + 3*x^4])
```

Maple [C] time = 0.179, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-1/2-1/2\,\sqrt{7}\right)x^2}\sqrt{1-\left(-1/2+1/2\,\sqrt{7}\right)x^2}\,\text{EllipticF}\left(1/2\,\sqrt{-2-2\,\sqrt{7}}x,\,i/6\sqrt{42}-i/6\sqrt{6}\right)}{\sqrt{-2-2\,\sqrt{7}}\sqrt{3\,x^4-2\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-2*x^2-2)^(1/2),x)
```

```
[Out] 2/(-2-2*7^(1/2))^(1/2)*(1-(-1/2-1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2+1/2*7^(1/2
))*x^2)^(1/2)/(3*x^4-2*x^2-2)^(1/2)*EllipticF(1/2*(-2-2*7^(1/2))^(1/2)*x,1/
6*I*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 - 2\,x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{3\,x^4 - 2\,x^2 - 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 - 2x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 2*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 - 2\,x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

3.50 $\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{-\left(3-\sqrt{33}\right)x^2-4}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+4}{\left(3-\sqrt{33}\right)x^2+4}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(3-\sqrt{33}\right)x^2-4}}\right),\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+4}}\sqrt{3x^4-3x^2-2}}$$

```
[Out] (Sqrt[-4 - (3 - Sqrt[33])*x^2]*Sqrt[(4 + (3 + Sqrt[33])*x^2)/(4 + (3 - Sqrt
[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 - (3 - Sqrt[33])*
x^2]], (11 - Sqrt[33])/22])/(2*Sqrt[2]*33^(1/4)*Sqrt[(4 + (3 - Sqrt[33])*x^
2)^(-1)]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

Rubi [A] time = 0.030337, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(3-\sqrt{33}\right)x^2-4}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+4}{\left(3-\sqrt{33}\right)x^2+4}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(3-\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+4}}\sqrt{3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 3*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-4 - (3 - Sqrt[33])*x^2]*Sqrt[(4 + (3 + Sqrt[33])*x^2)/(4 + (3 - Sqrt
[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 - (3 - Sqrt[33])*
x^2]], (11 - Sqrt[33])/22])/(2*Sqrt[2]*33^(1/4)*Sqrt[(4 + (3 - Sqrt[33])*x^
2)^(-1)]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx = \frac{\sqrt{-4-\left(3-\sqrt{33}\right)x^2}\sqrt{\frac{4+\left(3+\sqrt{33}\right)x^2}{4+\left(3-\sqrt{33}\right)x^2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-4-\left(3-\sqrt{33}\right)x^2}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4+\left(3-\sqrt{33}\right)x^2}}\sqrt{-2-3x^2+3x^4}}$$

Mathematica [C] time = 0.0672875, size = 81, normalized size = 0.53

$$-\frac{i\sqrt{-6x^4+6x^2+4}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{6}{\sqrt{33}-3}}x\right),\frac{1}{4}\left(\sqrt{33}-7\right)\right)}{\sqrt{3+\sqrt{33}}\sqrt{3x^4-3x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[4 + 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x
], (-7 + Sqrt[33])/4])/(Sqrt[3 + Sqrt[33]]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

Maple [C] time = 0.176, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-3/4-1/4\sqrt{33}\right)x^2}\sqrt{1-\left(-3/4+1/4\sqrt{33}\right)x^2}\,\text{EllipticF}\left(1/2\sqrt{-\sqrt{33}-3}x,i/4\sqrt{22}-i/4\sqrt{6}\right)}{\sqrt{-\sqrt{33}-3}\sqrt{3x^4-3x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-3*x^2-2)^(1/2),x)
```

```
[Out] 2/(-33^(1/2)-3)^(1/2)*(1-(-3/4-1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4+1/4*33^(1/
2))*x^2)^(1/2)/(3*x^4-3*x^2-2)^(1/2)*EllipticF(1/2*(-33^(1/2)-3)^(1/2)*x,1/
4*I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-3\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 3*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

3.51 $\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{-\left(2-\sqrt{10}\right)x^2-2}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+2}{\left(2-\sqrt{10}\right)x^2+2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(2-\sqrt{10}\right)x^2-2}}\right),\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+2}}\sqrt{3x^4-4x^2-2}}$$

```
[Out] (Sqrt[-2 - (2 - Sqrt[10])*x^2]*Sqrt[(2 + (2 + Sqrt[10])*x^2)/(2 + (2 - Sqrt
[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 - (2 - Sqrt[10])*x
^2]], (5 - Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2 + (2 - Sqrt[10])*x^2)^(-1)]*S
qrt[-2 - 4*x^2 + 3*x^4])
```

Rubi [A] time = 0.0288026, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(2-\sqrt{10}\right)x^2-2}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+2}{\left(2-\sqrt{10}\right)x^2+2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(2-\sqrt{10}\right)x^2-2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+2}}\sqrt{3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 4*x^2 + 3*x^4],x]
```


```
[Out] (Sqrt[-2 - (2 - Sqrt[10])*x^2]*Sqrt[(2 + (2 + Sqrt[10])*x^2)/(2 + (2 - Sqrt
[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 - (2 - Sqrt[10])*x
^2]], (5 - Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2 + (2 - Sqrt[10])*x^2)^(-1)]*S
qrt[-2 - 4*x^2 + 3*x^4])
```


Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx = \frac{\sqrt{-2-\left(2-\sqrt{10}\right)x^2}\sqrt{\frac{2+\left(2+\sqrt{10}\right)x^2}{2+\left(2-\sqrt{10}\right)x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-2-\left(2-\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2+\left(2-\sqrt{10}\right)x^2}}\sqrt{-2-4x^2+3x^4}}$$

Mathematica [C] time = 0.0638357, size = 81, normalized size = 0.55

$$-\frac{i\sqrt{-3x^4+4x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{2}}}x\right),\frac{1}{3}\left(2\sqrt{10}-7\right)\right)}{\sqrt{2+\sqrt{10}}\sqrt{3x^4-4x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 + 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (
-7 + 2*Sqrt[10])/3])/(Sqrt[2 + Sqrt[10]]*Sqrt[-2 - 4*x^2 + 3*x^4])
```

Maple [C] time = 0.182, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-1-1/2\sqrt{10}\right)x^2}\sqrt{1-\left(-1+1/2\sqrt{10}\right)x^2}\,\text{EllipticF}\left(1/2\sqrt{-4-2\sqrt{10}}x,i/3\sqrt{15}-i/3\sqrt{6}\right)}{\sqrt{-4-2\sqrt{10}}\sqrt{3x^4-4x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-4*x^2-2)^(1/2),x)
```

```
[Out] 2/(-4-2*10^(1/2))^(1/2)*(1-(-1-1/2*10^(1/2))*x^2)^(1/2)*(1-(-1+1/2*10^(1/2)
)*x^2)^(1/2)/(3*x^4-4*x^2-2)^(1/2)*EllipticF(1/2*(-4-2*10^(1/2))^(1/2)*x,1/
3*I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{3\,x^4-4\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 4*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

3.52 $\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2-2}\sqrt{3x^2+1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{7}x}{\sqrt{x^2-2}}\right),\frac{1}{7}\right)}{\sqrt{7}\sqrt{3x^4-5x^2-2}}$$

```
[Out] (Sqrt[-2 + x^2]*Sqrt[1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-2 + x^2]
], 1/7])/(Sqrt[7]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Rubi [A] time = 0.0069716, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2-2}\sqrt{3x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{7}x}{\sqrt{x^2-2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\sqrt{3x^4-5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 + x^2]*Sqrt[1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-2 + x^2]
], 1/7])/(Sqrt[7]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx=\frac{\sqrt{-2+x^2}\sqrt{1+3x^2}F\left(\sin^{-1}\left(\frac{\sqrt{7}x}{\sqrt{-2+x^2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\sqrt{-2-5x^2+3x^4}}$$

Mathematica [C] time = 0.0240015, size = 65, normalized size = 1.03

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{3x^2+1}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{3}\sqrt{3x^4-5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6]
)/(Sqrt[3]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Maple [C] time = 0.057, size = 53, normalized size = 0.8

$$-\frac{i}{6}\sqrt{3}\text{EllipticF}\left(i\sqrt{3}x,\frac{i}{6}\sqrt{6}\right)\sqrt{3\,x^2+1}\sqrt{-2\,x^2+4}\frac{1}{\sqrt{3\,x^4-5\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-5*x^2-2)^(1/2),x)
```

```
[Out] -1/6*I*3^(1/2)*(3*x^2+1)^(1/2)*(-2*x^2+4)^(1/2)/(3*x^4-5*x^2-2)^(1/2)*Ellip
ticF(I*3^(1/2)*x,1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-5\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 5*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

3.53 $\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-6}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{\left(7+\sqrt{73}\right)x^2-6}}\right),\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{2x^4+7x^2-3}}$$

```
[Out] (Sqrt[(6 - (7 - Sqrt[73])*x^2)/(6 - (7 + Sqrt[73])*x^2)]*Sqrt[-6 + (7 + Sqr
t[73])*x^2]*EllipticF[ArcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[-6 + (7 + Sqrt[73])*
x^2]], (73 + 7*Sqrt[73])/146])/(2*Sqrt[3]*73^(1/4)*Sqrt[(6 - (7 + Sqrt[73])
*x^2)^(-1)]*Sqrt[-3 + 7*x^2 + 2*x^4])
```

Rubi [A] time = 0.02914, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-6}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{\left(7+\sqrt{73}\right)x^2-6}}\right)|\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{2x^4+7x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 - (7 - Sqrt[73])*x^2)/(6 - (7 + Sqrt[73])*x^2)]*Sqrt[-6 + (7 + Sqr
t[73])*x^2]*EllipticF[ArcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[-6 + (7 + Sqrt[73])*
x^2]], (73 + 7*Sqrt[73])/146])/(2*Sqrt[3]*73^(1/4)*Sqrt[(6 - (7 + Sqrt[73])
*x^2)^(-1)]*Sqrt[-3 + 7*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx = \frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{-6+\left(7+\sqrt{73}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{-6+\left(7+\sqrt{73}\right)x^2}}\right)|\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{-3+7x^2+2x^4}}$$

Mathematica [C] time = 0.052869, size = 80, normalized size = 0.54

$$-\frac{i\sqrt{-4x^4-14x^2+6}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right),\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)}{\sqrt{\sqrt{73}-7}\sqrt{2x^4+7x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 7*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[6 - 14*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[7 + Sqrt[73]]
], (-61 - 7*Sqrt[73])/12])/(Sqrt[-7 + Sqrt[73]]*Sqrt[-3 + 7*x^2 + 2*x^4])
```

Maple [C] time = 0.18, size = 84, normalized size = 0.6

$$6\,\frac{\sqrt{1-\left(7/6-1/6\sqrt{73}\right)x^2}\sqrt{1-\left(1/6\sqrt{73}+7/6\right)x^2}\,\text{EllipticF}\left(1/6\sqrt{42-6\sqrt{73}}x,\frac{7i}{12}\sqrt{6}+i/12\sqrt{438}\right)}{\sqrt{42-6\sqrt{73}}\sqrt{2x^4+7x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+7*x^2-3)^(1/2),x)
```

```
[Out] 6/(42-6*73^(1/2))^(1/2)*(1-(7/6-1/6*73^(1/2))*x^2)^(1/2)*(1-(1/6*73^(1/2)+7
/6)*x^2)^(1/2)/(2*x^4+7*x^2-3)^(1/2)*EllipticF(1/6*(42-6*73^(1/2))^(1/2)*x,
7/12*I*6^(1/2)+1/12*I*438^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+7\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4+7\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+7x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+7*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 7*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+7\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

3.54 $\int \frac{1}{\sqrt{-3+6x^2+2x^4}} \, dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-3}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{\left(3+\sqrt{15}\right)x^2-3}}\right),\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{2x^4+6x^2-3}}$$

```
[Out] (Sqrt[(3 - (3 - Sqrt[15])*x^2)/(3 - (3 + Sqrt[15])*x^2)]*Sqrt[-3 + (3 + Sqr
t[15])*x^2]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-3 + (3 + Sqrt[15])*
x^2]], (5 + Sqrt[15])/10])/(Sqrt[2]*3^(3/4)*5^(1/4)*Sqrt[(3 - (3 + Sqrt[15]
)*x^2)^(-1)]*Sqrt[-3 + 6*x^2 + 2*x^4])
```

Rubi [A] time = 0.0397224, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-3}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{\left(3+\sqrt{15}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{2x^4+6x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (3 - Sqrt[15])*x^2)/(3 - (3 + Sqrt[15])*x^2)]*Sqrt[-3 + (3 + Sqr
t[15])*x^2]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-3 + (3 + Sqrt[15])*
x^2]], (5 + Sqrt[15])/10])/(Sqrt[2]*3^(3/4)*5^(1/4)*Sqrt[(3 - (3 + Sqrt[15]
)*x^2)^(-1)]*Sqrt[-3 + 6*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{-3+\left(3+\sqrt{15}\right)x^2}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{-3+\left(3+\sqrt{15}\right)x^2}}\right)|\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{-3+6x^2+2x^4}}$$

Mathematica [C] time = 0.0645388, size = 77, normalized size = 0.52

$$-\frac{i\sqrt{-2x^4-6x^2+3}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\sqrt{\frac{5}{3}}-1}x\right),-4-\sqrt{15}\right)}{\sqrt{\sqrt{15}-3}\sqrt{2x^4+6x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 6*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 - 6*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x],
-4 - Sqrt[15]])/(Sqrt[-3 + Sqrt[15]]*Sqrt[-3 + 6*x^2 + 2*x^4])
```

Maple [C] time = 0.184, size = 84, normalized size = 0.6

$$3\,\frac{\sqrt{1-\left(1-1/3\,\sqrt{15}\right)x^2}\sqrt{1-\left(1+1/3\,\sqrt{15}\right)x^2}\text{EllipticF}\left(1/3\,\sqrt{9-3\,\sqrt{15}}x,i/2\sqrt{6}+i/2\sqrt{10}\right)}{\sqrt{9-3\,\sqrt{15}}\sqrt{2\,x^4+6\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+6*x^2-3)^(1/2),x)
```

```
[Out] 3/(9-3*15^(1/2))^(1/2)*(1-(1-1/3*15^(1/2))*x^2)^(1/2)*(1-(1+1/3*15^(1/2))*x
^2)^(1/2)/(2*x^4+6*x^2-3)^(1/2)*EllipticF(1/3*(9-3*15^(1/2))^(1/2)*x,1/2*I*
6^(1/2)+1/2*I*10^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4+6\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+6x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+6*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 6*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

3.55 $\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{x^2+3}\sqrt{2x^2-1}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{3}}x}{\sqrt{2x^2-1}}\right),\frac{6}{7}\right)}{\sqrt{7}\sqrt{2x^4+5x^2-3}}$$

```
[Out] (Sqrt[3 + x^2]*Sqrt[-1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7/3]*x)/Sqrt[-1 + 2*
x^2]], 6/7])/(Sqrt[7]*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Rubi [A] time = 0.0083515, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2+3}\sqrt{2x^2-1}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{3}}x}{\sqrt{2x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\sqrt{2x^4+5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 + x^2]*Sqrt[-1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7/3]*x)/Sqrt[-1 + 2*
x^2]], 6/7])/(Sqrt[7]*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx = \frac{\sqrt{3+x^2}\sqrt{-1+2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{3}}x}{\sqrt{-1+2x^2}}\right)|\frac{6}{7}\right)}{\sqrt{7}\sqrt{-3+5x^2+2x^4}}$$

Mathematica [A] time = 0.0226701, size = 54, normalized size = 0.81

$$\frac{\sqrt{1-2x^2}\sqrt{x^2+3}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{6}\sqrt{2x^4+5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/6])/(Sqrt[6]
*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Maple [C] time = 0.049, size = 53, normalized size = 0.8

$$-\frac{i}{3}\sqrt{3}\text{EllipticF}\left(\frac{i}{3}\sqrt{3}x,i\sqrt{6}\right)\sqrt{3x^2+9}\sqrt{-2x^2+1}\frac{1}{\sqrt{2x^4+5x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+5*x^2-3)^(1/2),x)
```

```
[Out] -1/3*I*3^(1/2)*(3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(2*x^4+5*x^2-3)^(1/2)*Ellip
ticF(1/3*I*3^(1/2)*x,I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+5x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+5\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+5x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

3.56 $\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{3-(2-\sqrt{10})x^2}{3-(2+\sqrt{10})x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-3}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{(2+\sqrt{10})x^2-3}}\right),\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-(2+\sqrt{10})x^2}}\sqrt{2x^4+4x^2-3}}$$

```
[Out] (Sqrt[(3 - (2 - Sqrt[10])*x^2)/(3 - (2 + Sqrt[10])*x^2)]*Sqrt[-3 + (2 + Sqr
t[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 + (2 + Sqrt[10])*x
^2]], (5 + Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3 - (2 + Sqrt[10])
*x^2)^(-1)]*Sqrt[-3 + 4*x^2 + 2*x^4])
```

Rubi [A] time = 0.0271006, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{3-(2-\sqrt{10})x^2}{3-(2+\sqrt{10})x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-3}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{(2+\sqrt{10})x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-(2+\sqrt{10})x^2}}\sqrt{2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 4*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (2 - Sqrt[10])*x^2)/(3 - (2 + Sqrt[10])*x^2)]*Sqrt[-3 + (2 + Sqr
t[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 + (2 + Sqrt[10])*x
^2]], (5 + Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3 - (2 + Sqrt[10])
*x^2)^(-1)]*Sqrt[-3 + 4*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{-3+\left(2+\sqrt{10}\right)x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-3+\left(2+\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{-3+4x^2+2x^4}}$$

Mathematica [C] time = 0.0687126, size = 83, normalized size = 0.56

$$-\frac{i\sqrt{-2x^4-4x^2+3}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right),-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{\sqrt{10}-2}\sqrt{2x^4+4x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 - 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x]
, -7/3 - (2*Sqrt[10])/3])/(Sqrt[-2 + Sqrt[10]]*Sqrt[-3 + 4*x^2 + 2*x^4])
```

Maple [C] time = 0.181, size = 84, normalized size = 0.6

$$3\,\frac{\sqrt{1-\left(2/3-1/3\,\sqrt{10}\right)x^2}\sqrt{1-\left(2/3+1/3\,\sqrt{10}\right)x^2}\,\text{EllipticF}\left(1/3\,\sqrt{6-3\,\sqrt{10}}x,\,i/3\sqrt{6}+i/3\sqrt{15}\right)}{\sqrt{6-3\,\sqrt{10}}\sqrt{2\,x^4+4\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+4*x^2-3)^(1/2),x)
```

```
[Out] 3/(6-3*10^(1/2))^(1/2)*(1-(2/3-1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3+1/3*10^(1/2
))*x^2)^(1/2)/(2*x^4+4*x^2-3)^(1/2)*EllipticF(1/3*(6-3*10^(1/2))^(1/2)*x,1/
3*I*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4+4\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 4*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

3.57 $\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{\frac{6-(3-\sqrt{33})x^2}{6-(3+\sqrt{33})x^2}}\sqrt{(3+\sqrt{33})\,x^2-6}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{(3+\sqrt{33})x^2-6}}\right),\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\;3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-(3+\sqrt{33})x^2}}\sqrt{2x^4+3x^2-3}}$$

```
[Out] (Sqrt[(6 - (3 - Sqrt[33])*x^2)/(6 - (3 + Sqrt[33])*x^2)]*Sqrt[-6 + (3 + Sqr
t[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 + (3 + Sqrt[33])*
x^2]], (11 + Sqrt[33])/22])/(2*3^(3/4)*11^(1/4)*Sqrt[(6 - (3 + Sqrt[33])*x^
2)^(-1)]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

Rubi [A] time = 0.0266953, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{6-(3-\sqrt{33})x^2}{6-(3+\sqrt{33})x^2}}\sqrt{(3+\sqrt{33})\,x^2-6}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{(3+\sqrt{33})x^2-6}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\;3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-(3+\sqrt{33})x^2}}\sqrt{2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 3*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 - (3 - Sqrt[33])*x^2)/(6 - (3 + Sqrt[33])*x^2)]*Sqrt[-6 + (3 + Sqr
t[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 + (3 + Sqrt[33])*
x^2]], (11 + Sqrt[33])/22])/(2*3^(3/4)*11^(1/4)*Sqrt[(6 - (3 + Sqrt[33])*x^
2)^(-1)]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx = \frac{\sqrt{\frac{6-(3-\sqrt{33})x^2}{6-(3+\sqrt{33})x^2}}\sqrt{-6+\left(3+\sqrt{33}\right)x^2}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-6+(3+\sqrt{33})x^2}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\ 3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-(3+\sqrt{33})x^2}}\sqrt{-3+3x^2+2x^4}}$$

Mathematica [C] time = 0.0584732, size = 80, normalized size = 0.55

$$-\frac{i\sqrt{-4x^4-6x^2+6}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right),-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{\sqrt{33}-3}\sqrt{2x^4+3x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[6 - 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[33]]]
, -7/4 - Sqrt[33]/4])/(Sqrt[-3 + Sqrt[33]]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

Maple [C] time = 0.188, size = 84, normalized size = 0.6

$$6\,\frac{\sqrt{1-\left(-1/6\sqrt{33}+1/2\right)x^2}\sqrt{1-\left(1/6\sqrt{33}+1/2\right)x^2}\,\text{EllipticF}\left(1/6\sqrt{18-6\sqrt{33}}x,i/4\sqrt{6}+i/4\sqrt{22}\right)}{\sqrt{18-6\sqrt{33}}\sqrt{2x^4+3x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+3*x^2-3)^(1/2),x)
```

```
[Out] 6/(18-6*33^(1/2))^(1/2)*(1-(-1/6*33^(1/2)+1/2)*x^2)^(1/2)*(1-(1/6*33^(1/2)+
1/2)*x^2)^(1/2)/(2*x^4+3*x^2-3)^(1/2)*EllipticF(1/6*(18-6*33^(1/2))^(1/2)*x
,1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4+3\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 3*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

3.58 $\int \frac{1}{\sqrt{-3+2x^2+2x^4}} \, dx$

Optimal. Leaf size=143

$$\frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-3}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-3}}\right),\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{2x^4+2x^2-3}}$$

```
[Out] (Sqrt[(3 - (1 - Sqrt[7])*x^2)/(3 - (1 + Sqrt[7])*x^2)]*Sqrt[-3 + (1 + Sqrt[
7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 + (1 + Sqrt[7])*x^2]]
, (7 + Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3 - (1 + Sqrt[7])*x^2)^(-1)]*Sq
rt[-3 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.0233424, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-3}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-3}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 2*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (1 - Sqrt[7])*x^2)/(3 - (1 + Sqrt[7])*x^2)]*Sqrt[-3 + (1 + Sqrt[
7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 + (1 + Sqrt[7])*x^2]]
, (7 + Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3 - (1 + Sqrt[7])*x^2)^(-1)]*Sq
rt[-3 + 2*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{-3+\left(1+\sqrt{7}\right)x^2}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-3+\left(1+\sqrt{7}\right)x^2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{-3+2x^2+2x^4}}$$

Mathematica [C] time = 0.0515208, size = 83, normalized size = 0.58

$$-\frac{i\sqrt{-2x^4-2x^2+3}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right),-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}\sqrt{2x^4+2x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 - 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x],
 -4/3 - Sqrt[7]/3])/(Sqrt[-1 + Sqrt[7]]*Sqrt[-3 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.18, size = 84, normalized size = 0.6

$$3\,\frac{\sqrt{1-\left(-1/3\,\sqrt{7}+1/3\right)x^2}\sqrt{1-\left(1/3\,\sqrt{7}+1/3\right)x^2}\text{EllipticF}\left(1/3\,\sqrt{3-3\,\sqrt{7}}x,i/6\sqrt{6}+i/6\sqrt{42}\right)}{\sqrt{3-3\,\sqrt{7}}\sqrt{2\,x^4+2\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+2*x^2-3)^(1/2),x)
```

```
[Out] 3/(3-3*7^(1/2))^(1/2)*(1-(-1/3*7^(1/2)+1/3)*x^2)^(1/2)*(1-(1/3*7^(1/2)+1/3)
*x^2)^(1/2)/(2*x^4+2*x^2-3)^(1/2)*EllipticF(1/3*(3-3*7^(1/2))^(1/2)*x,1/6*I
*6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+2\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+2x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 2*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

3.59 $\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2-1}\sqrt{2x^2+3}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{x^2-1}}\right),\frac{3}{5}\right)}{\sqrt{5}\sqrt{2x^4+x^2-3}}$$

```
[Out] (Sqrt[-1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5/3]*x)/Sqrt[-1 + x^
2]], 3/5])/(Sqrt[5]*Sqrt[-3 + x^2 + 2*x^4])
```

Rubi [A] time = 0.0074047, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1097}

$$\frac{\sqrt{x^2-1}\sqrt{2x^2+3}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{x^2-1}}\right)|\frac{3}{5}\right)}{\sqrt{5}\sqrt{2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5/3]*x)/Sqrt[-1 + x^
2]], 3/5])/(Sqrt[5]*Sqrt[-3 + x^2 + 2*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx = \frac{\sqrt{-1+x^2}\sqrt{3+2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{-1+x^2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\sqrt{-3+x^2+2x^4}}$$

Mathematica [C] time = 0.0223485, size = 63, normalized size = 1.

$$-\frac{i\sqrt{1-x^2}\sqrt{2x^2+3}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], -3/2]
)/(Sqrt[2]*Sqrt[-3 + x^2 + 2*x^4])
```

Maple [C] time = 0.067, size = 51, normalized size = 0.8

$$-\frac{i}{6}\sqrt{6}\text{EllipticF}\left(\frac{i}{3}x\sqrt{6},\frac{i}{2}\sqrt{6}\right)\sqrt{6\,x^2+9}\sqrt{-x^2+1}\frac{1}{\sqrt{2\,x^4+x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+x^2-3)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(2*x^4+x^2-3)^(1/2)*EllipticF
(1/3*I*x*6^(1/2),1/2*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(2*x^4 + x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(2*x^4 + x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+x**2-3)**(1/2),x)

[Out] Integral(1/sqrt(2*x**4 + x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(2*x^4 + x^2 - 3), x)
```

3.60 $\int \frac{1}{\sqrt{-3+2x^4}} \, dx$

Optimal. Leaf size=112

$$\frac{\sqrt{\sqrt{6}x^2-3}\sqrt{\frac{\sqrt{6}x^2+3}{3-\sqrt{6}x^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-3}}\right),\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}x^2}}\sqrt{2x^4-3}}$$

```
[Out] (Sqrt[-3 + Sqrt[6]*x^2]*Sqrt[(3 + Sqrt[6]*x^2)/(3 - Sqrt[6]*x^2)]*EllipticF
[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-3 + Sqrt[6]*x^2]], 1/2])/(6^(3/4)*Sqrt[(3
 - Sqrt[6]*x^2)^(-1)]*Sqrt[-3 + 2*x^4])
```

Rubi [A] time = 0.0154549, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {223}

$$\frac{\sqrt{\sqrt{6}x^2-3}\sqrt{\frac{\sqrt{6}x^2+3}{3-\sqrt{6}x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-3}}\right)|\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}x^2}}\sqrt{2x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 + Sqrt[6]*x^2]*Sqrt[(3 + Sqrt[6]*x^2)/(3 - Sqrt[6]*x^2)]*EllipticF
[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-3 + Sqrt[6]*x^2]], 1/2])/(6^(3/4)*Sqrt[(3
 - Sqrt[6]*x^2)^(-1)]*Sqrt[-3 + 2*x^4])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[(a - q*x^2)/(a + q*x^2)]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqr
t[(a + q*x^2)/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[a + b*x^4]*Sqrt[a/(a + q*x^2)]),
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+2x^4}}\,dx = \frac{\sqrt{-3+\sqrt{6}x^2}\sqrt{\frac{3+\sqrt{6}x^2}{3-\sqrt{6}x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{-3+\sqrt{6}x^2}}\right)|\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}x^2}}\sqrt{-3+2x^4}}$$

Mathematica [A] time = 0.0244002, size = 40, normalized size = 0.36

$$\frac{\sqrt{3-2x^4}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{\sqrt[4]{6}\sqrt{2x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^4]*EllipticF[ArcSin[(2/3)^(1/4)*x], -1])/(6^(1/4)*Sqrt[-3 + 2
*x^4])
```

Maple [C] time = 0.173, size = 56, normalized size = 0.5

$$\frac{1}{3\sqrt{-3\sqrt{6}}}\sqrt{9+3x^2\sqrt{6}}\sqrt{9-3x^2\sqrt{6}}\,\text{EllipticF}\left(\frac{x\sqrt{-3\sqrt{6}}}{3},i\right)\frac{1}{\sqrt{2x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-3)^(1/2),x)
```

```
[Out] 1/3/(-3*6^(1/2))^(1/2)*(9+3*x^2*6^(1/2))^(1/2)*(9-3*x^2*6^(1/2))^(1/2)/(2*x
^4-3)^(1/2)*EllipticF(1/3*x*(-3*6^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 3), x)
```

Sympy [C] time = 0.62534, size = 34, normalized size = 0.3

$$-\frac{\sqrt{3}ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-3)**(1/2),x)
```

```
[Out] -sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4/3)/(12*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(2*x^4 - 3), x)
```

3.61 $\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x^2+1}\sqrt{2x^2-3}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{5}x}{\sqrt{2x^2-3}}\right),\frac{2}{5}\right)}{\sqrt{5}\sqrt{2x^4-x^2-3}}$$

```
[Out] (Sqrt[1 + x^2]*Sqrt[-3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-3 + 2*x^
2]], 2/5])/(Sqrt[5]*Sqrt[-3 - x^2 + 2*x^4])
```

Rubi [A] time = 0.0068685, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2+1}\sqrt{2x^2-3}F\left(\sin^{-1}\left(\frac{\sqrt{5}x}{\sqrt{2x^2-3}}\right)|\frac{2}{5}\right)}{\sqrt{5}\sqrt{2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[-3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-3 + 2*x^
2]], 2/5])/(Sqrt[5]*Sqrt[-3 - x^2 + 2*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx=\frac{\sqrt{1+x^2}\sqrt{-3+2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{5}x}{\sqrt{-3+2x^2}}\right)|\frac{2}{5}\right)}{\sqrt{5}\sqrt{-3-x^2+2x^4}}$$

Mathematica [A] time = 0.0230434, size = 51, normalized size = 0.78

$$\frac{\sqrt{3-2x^2}\sqrt{x^2+1}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right),-\frac{3}{2}\right)}{\sqrt{4x^4-2x^2-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], -3/2])/Sqrt[-
6 - 2*x^2 + 4*x^4]
```

Maple [C] time = 0.052, size = 45, normalized size = 0.7

$$-\frac{i}{3}\text{EllipticF}\left(ix,\frac{i}{3}\sqrt{6}\right)\sqrt{x^2+1}\sqrt{-6\,x^2+9}\frac{1}{\sqrt{2\,x^4-x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-x^2-3)^(1/2),x)
```

```
[Out] -1/3*I*(x^2+1)^(1/2)*(-6*x^2+9)^(1/2)/(2*x^4-x^2-3)^(1/2)*EllipticF(I*x,1/3
*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 - 3), x)
```

3.62 $\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{-\left(1-\sqrt{7}\right)x^2-3}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+3}{\left(1-\sqrt{7}\right)x^2+3}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(1-\sqrt{7}\right)x^2-3}}\right),\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+3}}\sqrt{2x^4-2x^2-3}}$$

```
[Out] (Sqrt[-3 - (1 - Sqrt[7])*x^2]*Sqrt[(3 + (1 + Sqrt[7])*x^2)/(3 + (1 - Sqrt[7
])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 - (1 - Sqrt[7])*x^2]]
, (7 - Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3 + (1 - Sqrt[7])*x^2)^(-1)]*Sq
rt[-3 - 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.0212946, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(1-\sqrt{7}\right)x^2-3}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+3}{\left(1-\sqrt{7}\right)x^2+3}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(1-\sqrt{7}\right)x^2-3}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+3}}\sqrt{2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 2*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 - (1 - Sqrt[7])*x^2]*Sqrt[(3 + (1 + Sqrt[7])*x^2)/(3 + (1 - Sqrt[7
])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 - (1 - Sqrt[7])*x^2]]
, (7 - Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3 + (1 - Sqrt[7])*x^2)^(-1)]*Sq
rt[-3 - 2*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = \frac{\sqrt{-3-\left(1-\sqrt{7}\right)x^2}\sqrt{\frac{3+\left(1+\sqrt{7}\right)x^2}{3+\left(1-\sqrt{7}\right)x^2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-3-\left(1-\sqrt{7}\right)x^2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3+\left(1-\sqrt{7}\right)x^2}}\sqrt{-3-2x^2+2x^4}}$$

Mathematica [C] time = 0.045838, size = 81, normalized size = 0.54

$$-\frac{i\sqrt{-2x^4+2x^2+3}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{\sqrt{7}-1}}x\right),\frac{1}{3}\left(\sqrt{7}-4\right)\right)}{\sqrt{1+\sqrt{7}}\sqrt{2x^4-2x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x]
, (-4 + Sqrt[7])/3])/(Sqrt[1 + Sqrt[7]]*Sqrt[-3 - 2*x^2 + 2*x^4])
```

Maple [C] time = 0.185, size = 84, normalized size = 0.6

$$3\,\frac{\sqrt{1-\left(-1/3-1/3\sqrt{7}\right)x^2}\sqrt{1-\left(-1/3+1/3\sqrt{7}\right)x^2}\,\text{EllipticF}\left(1/3\sqrt{-3-3\sqrt{7}}x,\,i/6\sqrt{42}-i/6\sqrt{6}\right)}{\sqrt{-3-3\sqrt{7}}\sqrt{2x^4-2x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-2*x^2-3)^(1/2),x)
```

```
[Out] 3/(-3-3*7^(1/2))^(1/2)*(1-(-1/3-1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3+1/3*7^(1/2
))*x^2)^(1/2)/(2*x^4-2*x^2-3)^(1/2)*EllipticF(1/3*(-3-3*7^(1/2))^(1/2)*x,1/
6*I*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - 2x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2x^4 - 2x^2 - 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - 2x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 2*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - 2x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

3.63 $\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{-\left(3-\sqrt{33}\right)x^2-6}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+6}{\left(3-\sqrt{33}\right)x^2+6}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(3-\sqrt{33}\right)x^2-6}}\right),\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+6}}\sqrt{2x^4-3x^2-3}}$$

```
[Out] (Sqrt[-6 - (3 - Sqrt[33])*x^2]*Sqrt[(6 + (3 + Sqrt[33])*x^2)/(6 + (3 - Sqrt
[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 - (3 - Sqrt[33])*
x^2]], (11 - Sqrt[33])/22])/(2*3^(3/4)*11^(1/4)*Sqrt[(6 + (3 - Sqrt[33])*x^
2)^(-1)]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

Rubi [A] time = 0.0235416, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(3-\sqrt{33}\right)x^2-6}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+6}{\left(3-\sqrt{33}\right)x^2+6}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(3-\sqrt{33}\right)x^2-6}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+6}}\sqrt{2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 3*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-6 - (3 - Sqrt[33])*x^2]*Sqrt[(6 + (3 + Sqrt[33])*x^2)/(6 + (3 - Sqrt
[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 - (3 - Sqrt[33])*
x^2]], (11 - Sqrt[33])/22])/(2*3^(3/4)*11^(1/4)*Sqrt[(6 + (3 - Sqrt[33])*x^
2)^(-1)]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx = \frac{\sqrt{-6-\left(3-\sqrt{33}\right)x^2}\sqrt{\frac{6+\left(3+\sqrt{33}\right)x^2}{6+\left(3-\sqrt{33}\right)x^2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-6-\left(3-\sqrt{33}\right)x^2}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\ 3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6+\left(3-\sqrt{33}\right)x^2}}\sqrt{-3-3x^2+2x^4}}$$

Mathematica [C] time = 0.0565178, size = 78, normalized size = 0.51

$$-\frac{i\sqrt{-4x^4+6x^2+6}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{\sqrt{33}-3}}\right),\frac{1}{4}\left(\sqrt{33}-7\right)\right)}{\sqrt{3+\sqrt{33}}\sqrt{2x^4-3x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 - 3*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[6 + 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + Sqrt[33]]
], (-7 + Sqrt[33])/4])/(Sqrt[3 + Sqrt[33]]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

Maple [C] time = 0.185, size = 84, normalized size = 0.6

$$6\,\frac{\sqrt{1-\left(-1/2-1/6\,\sqrt{33}\right)x^2}\sqrt{1-\left(-1/2+1/6\,\sqrt{33}\right)x^2}\text{EllipticF}\left(1/6\,\sqrt{-18-6\,\sqrt{33}}x,i/4\sqrt{22}-i/4\sqrt{6}\right)}{\sqrt{-18-6\,\sqrt{33}}\sqrt{2\,x^4-3\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-3*x^2-3)^(1/2),x)
```

```
[Out] 6/(-18-6*33^(1/2))^(1/2)*(1-(-1/2-1/6*33^(1/2))*x^2)^(1/2)*(1-(-1/2+1/6*33^
(1/2))*x^2)^(1/2)/(2*x^4-3*x^2-3)^(1/2)*EllipticF(1/6*(-18-6*33^(1/2))^(1/2
)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-3\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 3*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

3.64 $\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx$

Optimal. Leaf size=155

$$\frac{\sqrt{-\left(2-\sqrt{10}\right)x^2-3}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+3}{\left(2-\sqrt{10}\right)x^2+3}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(2-\sqrt{10}\right)x^2-3}}\right),\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+3}}\sqrt{2x^4-4x^2-3}}$$

```
[Out] (Sqrt[-3 - (2 - Sqrt[10])*x^2]*Sqrt[(3 + (2 + Sqrt[10])*x^2)/(3 + (2 - Sqrt
[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 - (2 - Sqrt[10])*x
^2]], (5 - Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3 + (2 - Sqrt[10])
*x^2)^(-1)]*Sqrt[-3 - 4*x^2 + 2*x^4])
```

Rubi [A] time = 0.0236497, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1098}

$$\frac{\sqrt{-\left(2-\sqrt{10}\right)x^2-3}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+3}{\left(2-\sqrt{10}\right)x^2+3}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(2-\sqrt{10}\right)x^2-3}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+3}}\sqrt{2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 4*x^2 + 2*x^4],x]
```


```
[Out] (Sqrt[-3 - (2 - Sqrt[10])*x^2]*Sqrt[(3 + (2 + Sqrt[10])*x^2)/(3 + (2 - Sqrt
[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 - (2 - Sqrt[10])*x
^2]], (5 - Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3 + (2 - Sqrt[10])
*x^2)^(-1)]*Sqrt[-3 - 4*x^2 + 2*x^4])
```


Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = \frac{\sqrt{-3-\left(2-\sqrt{10}\right)x^2}\sqrt{\frac{3+\left(2+\sqrt{10}\right)x^2}{3+\left(2-\sqrt{10}\right)x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-3-\left(2-\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3+\left(2-\sqrt{10}\right)x^2}}\sqrt{-3-4x^2+2x^4}}$$

Mathematica [C] time = 0.0652713, size = 83, normalized size = 0.54

$$-\frac{i\sqrt{-2x^4+4x^2+3}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{\sqrt{10}-2}}x\right),\frac{2\sqrt{10}}{3}-\frac{7}{3}\right)}{\sqrt{2+\sqrt{10}}\sqrt{2x^4-4x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x
], -7/3 + (2*Sqrt[10])/3])/(Sqrt[2 + Sqrt[10]]*Sqrt[-3 - 4*x^2 + 2*x^4])
```

Maple [C] time = 0.183, size = 84, normalized size = 0.5

$$3\,\frac{\sqrt{1-\left(-2/3-1/3\,\sqrt{10}\right)x^2}\sqrt{1-\left(-2/3+1/3\,\sqrt{10}\right)x^2}\,\text{EllipticF}\left(1/3\,\sqrt{-6-3\,\sqrt{10}}x,\,i/3\sqrt{15}-i/3\sqrt{6}\right)}{\sqrt{-6-3\,\sqrt{10}}\sqrt{2\,x^4-4\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-4*x^2-3)^(1/2),x)
```

```
[Out] 3/(-6-3*10^(1/2))^(1/2)*(1-(-2/3-1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3+1/3*10^(
1/2))*x^2)^(1/2)/(2*x^4-4*x^2-3)^(1/2)*EllipticF(1/3*(-6-3*10^(1/2))^(1/2)*
x,1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - 4\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4 - 4\,x^2 - 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - 4x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 4*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - 4\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

3.65 $\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2-3}\sqrt{2x^2+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{7}x}{\sqrt{x^2-3}}\right),\frac{1}{7}\right)}{\sqrt{7}\sqrt{2x^4-5x^2-3}}$$

```
[Out] (Sqrt[-3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-3 + x^2]
], 1/7])/(Sqrt[7]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Rubi [A] time = 0.0064966, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1097}

$$\frac{\sqrt{x^2-3}\sqrt{2x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{7}x}{\sqrt{x^2-3}}\right)|\frac{1}{7}\right)}{\sqrt{7}\sqrt{2x^4-5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-3 + x^2]
], 1/7])/(Sqrt[7]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Rule 1097

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[-2*a - (b - q)*x^2]*Sqrt[(2*a + (b + q)*x^2)/q]
*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)])/(2*Sq
rt[-a]*Sqrt[a + b*x^2 + c*x^4]), x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx=\frac{\sqrt{-3+x^2}\sqrt{1+2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{7}x}{\sqrt{-3+x^2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\sqrt{-3-5x^2+2x^4}}$$

Mathematica [C] time = 0.0249575, size = 65, normalized size = 1.03

$$-\frac{i\sqrt{1-\frac{x^2}{3}}\sqrt{2x^2+1}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\right),-\frac{1}{6}\right)}{\sqrt{2}\sqrt{2x^4-5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 5*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/3]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/6]
)/(Sqrt[2]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Maple [C] time = 0.052, size = 53, normalized size = 0.8

$$-\frac{i}{6}\sqrt{2}\text{EllipticF}\left(ix\sqrt{2},\frac{i}{6}\sqrt{6}\right)\sqrt{2\,x^2+1}\sqrt{-3\,x^2+9}\frac{1}{\sqrt{2\,x^4-5\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-5*x^2-3)^(1/2),x)
```

```
[Out] -1/6*I*2^(1/2)*(2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-5*x^2-3)^(1/2)*Ellip
ticF(I*x*2^(1/2),1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4 - 5\,x^2 - 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - 5x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 5*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - 5\,x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

3.66 $\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\tan^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] time = 0.007413, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1100}

$$\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rule 1100

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b - q)*x^2)*Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]*EllipticF[ArcTan[Rt[(b - q)/(2*a), 2]*x], (-2*q)/(b - q)])/(2
*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx=\frac{\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

Mathematica [C] time = 0.0252434, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{x^2+1}\sqrt{3x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{9x^4+15x^2+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])
/Sqrt[6 + 15*x^2 + 9*x^4]
```

Maple [A] time = 0.053, size = 44, normalized size = 0.9

$$-\frac{i}{2}\sqrt{x^2+1}\sqrt{6\,x^2+4}\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{3\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/
2*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

3.67 $\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3x^4+4x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[2 + 4*x^2 + 3*x^
4])
```

Rubi [A] time = 0.0212167, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3x^4+4x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[2 + 4*x^2 + 3*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2+4x^2+3x^4}}$$

Mathematica [C] time = 0.0860561, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-2-i\sqrt{2}}}x\right),\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{3x^4+4x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[3]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[2 + 4*x^2 + 3*x^4])
```

Maple [C] time = 0.764, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1+i/2\sqrt{2}\right)x^2}\sqrt{1-\left(-1-i/2\sqrt{2}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-4+2\,i\sqrt{2}},1/3\,\sqrt{3+6\,i\sqrt{2}}\right)}{\sqrt{-4+2\,i\sqrt{2}}\sqrt{3\,x^4+4\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+4*x^2+2)^(1/2),x)
```

```
[Out] 2/(-4+2*I*2^(1/2))^(1/2)*(1-(-1+1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1-1/2*I*2^(1
/2))*x^2)^(1/2)/(3*x^4+4*x^2+2)^(1/2)*EllipticF(1/2*x*(-4+2*I*2^(1/2))^(1/2
),1/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+4\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 4*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

3.68 $\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+3x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 + 3*x^2 + 3*x^
4])
```

Rubi [A] time = 0.0160884, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^2 + 3*x^4],x]
```


```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 + 3*x^2 + 3*x^
4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2+3x^2+3x^4}}$$

Mathematica [C] time = 0.116151, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-3-i\sqrt{15}}}x\right),\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{3x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(-3 + I*Sqrt[15]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-3 - I*Sqrt[15])]*x], (-3 - I*Sqrt[15])/(-3
 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[2 + 3*x^2 + 3*
x^4])
```

Maple [C] time = 0.825, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-3/4+i/4\sqrt{15}\right)x^2}\sqrt{1-\left(-3/4-i/4\sqrt{15}\right)x^2}\,\text{EllipticF}\left(1/2\,x\sqrt{-3+i\sqrt{15}},1/2\,\sqrt{-1+i\sqrt{15}}\right)}{\sqrt{-3+i\sqrt{15}}\sqrt{3\,x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 2/(-3+I*15^(1/2))^(1/2)*(1-(-3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*I*
15^(1/2))*x^2)^(1/2)/(3*x^4+3*x^2+2)^(1/2)*EllipticF(1/2*x*(-3+I*15^(1/2))^
(1/2),1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{3\,x^4 + 3\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 + 3x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

3.69 $\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+2x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[2 + 2*x^2 + 3*x
^4])
```

Rubi [A] time = 0.0159053, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*x^2 + 3*x^4],x]
```


```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[2 + 2*x^2 + 3*x
^4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2+2x^2+3x^4}}$$

Mathematica [C] time = 0.080682, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{3x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{-1+i\sqrt{5}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-1-i\sqrt{5}}}x\right),\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{3x^4+2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[3]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[2 + 2*x^2 + 3*x^4])
```

Maple [C] time = 0.76, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1/2+i/2\sqrt{5}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{5}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-2+2\,i\sqrt{5}},1/3\,\sqrt{-6+3\,i\sqrt{5}}\right)}{\sqrt{-2+2\,i\sqrt{5}}\sqrt{3\,x^4+2\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2*x^2+2)^(1/2),x)
```

```
[Out] 2/(-2+2*I*5^(1/2))^(1/2)*(1-(-1/2+1/2*I*5^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
5^(1/2))*x^2)^(1/2)/(3*x^4+2*x^2+2)^(1/2)*EllipticF(1/2*x*(-2+2*I*5^(1/2))^
(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+2\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 2*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

3.70 $\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 + x^2 + 3*x^4]
)
```

Rubi [A] time = 0.0138713, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 + x^2 + 3*x^4]
)
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2+x^2+3x^4}}$$

Mathematica [C] time = 0.0841275, size = 142, normalized size = 1.61

$$-\frac{i\sqrt{1-\frac{6x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+i\sqrt{23}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-1-i\sqrt{23}}}x\right),\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{3x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(-1 + I*Sqrt[23]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-1 - I*Sqrt[23])]*x], (-1 - I*Sqrt[23])/(-1
 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[2 + x^2 + 3*x^
4])
```

Maple [C] time = 0.758, size = 85, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1/4+i/4\sqrt{23}\right)x^2}\sqrt{1-\left(-1/4-i/4\sqrt{23}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-1+i\sqrt{23}},1/6\,\sqrt{-33+3\,i\sqrt{23}}\right)}{\sqrt{-1+i\sqrt{23}}\sqrt{3\,x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+x^2+2)^(1/2),x)
```

```
[Out] 2/(-1+I*23^(1/2))^(1/2)*(1-(-1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-1/4-1/4*I*
23^(1/2))*x^2)^(1/2)/(3*x^4+x^2+2)^(1/2)*EllipticF(1/2*x*(-1+I*23^(1/2))^(1
/2),1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 + 2), x)
```

3.71 $\int \frac{1}{\sqrt{2+3x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{3x^4+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[2 + 3*x^4])
```

Rubi [A] time = 0.0072882, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {220}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{3x^4+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[2 + 3*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{2+3x^4}}$$

Mathematica [C] time = 0.025312, size = 25, normalized size = 0.35

$$-\sqrt[4]{-\frac{1}{6}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{3}{2}}x\right),-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 3*x^4],x]
```

```
[Out] -((-1/6)^(1/4)*EllipticF[I*ArcSinh[(-3/2)^(1/4)*x], -1])
```

Maple [C] time = 0.196, size = 66, normalized size = 0.9

$$\frac{\sqrt{2}}{4\sqrt{i\sqrt{6}}}\sqrt{4-2i\sqrt{6}x^2}\sqrt{4+2i\sqrt{6}x^2}\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{i\sqrt{6}}}{2},i\right)\frac{1}{\sqrt{3x^4+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/(I*6^(1/2))^(1/2)*(4-2*I*6^(1/2)*x^2)^(1/2)*(4+2*I*6^(1/2)*x^2)
^(1/2)/(3*x^4+2)^(1/2)*EllipticF(1/2*x*2^(1/2)*(I*6^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 2), x)
```

Sympy [C] time = 0.684426, size = 36, normalized size = 0.5

$$\frac{\sqrt{2}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{3x^4e^{i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2)**(1/2),x)
```

```
[Out] sqrt(2)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(I*pi)/2)/(8
*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2), x)
```

3.72 $\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - x^2 + 3*x^4]
)
```

Rubi [A] time = 0.0154063, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - x^2 + 3*x^4]
)
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2-x^2+3x^4}}$$

Mathematica [C] time = 0.0783183, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{1-i\sqrt{23}}}x\right),\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{3x^4-x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(1 + I*Sqrt[23])]
*EllipticF[I*ArcSinh[Sqrt[-6/(1 - I*Sqrt[23])]*x], (1 - I*Sqrt[23])/(1 + I*
Sqrt[23])])/(Sqrt[6]*Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[2 - x^2 + 3*x^4])
```

Maple [C] time = 0.786, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1/4+i/4\sqrt{23}\right)x^2}\sqrt{1-\left(1/4-i/4\sqrt{23}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{i\sqrt{23}+1},1/6\,\sqrt{-33-3\,i\sqrt{23}}\right)}{\sqrt{i\sqrt{23}+1}\sqrt{3\,x^4-x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-x^2+2)^(1/2),x)
```

```
[Out] 2/(I*23^(1/2)+1)^(1/2)*(1-(1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4-1/4*I*23^
(1/2))*x^2)^(1/2)/(3*x^4-x^2+2)^(1/2)*EllipticF(1/2*x*(I*23^(1/2)+1)^(1/2),
1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 + 2), x)
```

3.73 $\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-2x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[2 - 2*x^2 + 3*x
^4])
```

Rubi [A] time = 0.0151346, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 2*x^2 + 3*x^4],x]
```



```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[2 - 2*x^2 + 3*x
^4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2-2x^2+3x^4}}$$

Mathematica [C] time = 0.0764541, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{1-i\sqrt{5}}}x\right),\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{3x^4-2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-3/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[3]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[2 - 2*x^2 + 3*x^4])
```

Maple [C] time = 0.79, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1/2+i/2\sqrt{5}\right)x^2}\sqrt{1-\left(1/2-i/2\sqrt{5}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{2+2\,i\sqrt{5}},1/3\,\sqrt{-6-3\,i\sqrt{5}}\right)}{\sqrt{2+2\,i\sqrt{5}}\sqrt{3\,x^4-2\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-2*x^2+2)^(1/2),x)
```

```
[Out] 2/(2+2*I*5^(1/2))^(1/2)*(1-(1/2+1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2-1/2*I*5^(
1/2))*x^2)^(1/2)/(3*x^4-2*x^2+2)^(1/2)*EllipticF(1/2*x*(2+2*I*5^(1/2))^(1/2
),1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-2\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 2*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

3.74 $\int \frac{1}{\sqrt{2-3x^2+3x^4}} \, dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-3x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 - 3*x^2 + 3*x^
4])
```

Rubi [A] time = 0.0156303, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 - 3*x^2 + 3*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2-3x^2+3x^4}}$$

Mathematica [C] time = 0.107259, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{3-i\sqrt{15}}}x\right),\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{3x^4-3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(3 + I*Sqrt[15])]
*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[15])]*x], (3 - I*Sqrt[15])/(3 + I*
Sqrt[15])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[2 - 3*x^2 + 3*x^4])
```

Maple [C] time = 0.801, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(3/4+i/4\sqrt{15}\right)x^2}\sqrt{1-\left(3/4-i/4\sqrt{15}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{i\sqrt{15}+3},1/2\,\sqrt{-1-i\sqrt{15}}\right)}{\sqrt{i\sqrt{15}+3}\sqrt{3\,x^4-3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-3*x^2+2)^(1/2),x)
```

```
[Out] 2/(I*15^(1/2)+3)^(1/2)*(1-(3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4-1/4*I*15^
(1/2))*x^2)^(1/2)/(3*x^4-3*x^2+2)^(1/2)*EllipticF(1/2*x*(I*15^(1/2)+3)^(1/2
),1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

3.75 $\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3x^4-4x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[2 - 4*x^2 + 3*x^
4])
```

Rubi [A] time = 0.0115275, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3x^4-4x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 4*x^2 + 3*x^4],x]
```


```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[2 - 4*x^2 + 3*x^
4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2-4x^2+3x^4}}$$

Mathematica [C] time = 0.0812713, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{2-i\sqrt{2}}}x\right),\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{3x^4-4x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-3/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[3]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[2 - 4*x^2 + 3*x^4])
```

Maple [C] time = 0.755, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1+i/2\sqrt{2}\right)x^2}\sqrt{1-\left(1-i/2\sqrt{2}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{4+2\,i\sqrt{2}},1/3\,\sqrt{3-6\,i\sqrt{2}}\right)}{\sqrt{4+2\,i\sqrt{2}}\sqrt{3\,x^4-4\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-4*x^2+2)^(1/2),x)
```

```
[Out] 2/(4+2*I*2^(1/2))^(1/2)*(1-(1+1/2*I*2^(1/2))*x^2)^(1/2)*(1-(1-1/2*I*2^(1/2)
)*x^2)^(1/2)/(3*x^4-4*x^2+2)^(1/2)*EllipticF(1/2*x*(4+2*I*2^(1/2))^(1/2),1/
3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4 - 4\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 - 4x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 4*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 - 4\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

3.76 $\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-5x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-5x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 5*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - 5*x^2 +
3*x^4])
```

Rubi [A] time = 0.0151634, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1096}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-5x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 5*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 5*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - 5*x^2 +
3*x^4])
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-5x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2-5x^2+3x^4}}$$

Mathematica [A] time = 0.022652, size = 53, normalized size = 0.58

$$\frac{\sqrt{2-3x^2}\sqrt{1-x^2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{9x^4-15x^2+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/Sqrt[6
- 15*x^2 + 9*x^4]
```

Maple [A] time = 0.047, size = 42, normalized size = 0.5

$$\frac{1}{2}\sqrt{-x^2+1}\sqrt{-6x^2+4}\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{3x^4-5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(3*x^4-5*x^2+2)^(1/2)*EllipticF(x,1/2*6
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

3.77 $\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-6x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-6x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 6*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[2 - 6*x^2 + 3*x^
4])
```

Rubi [A] time = 0.0163397, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1096}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-6x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-6x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 6*x^2 + 3*x^4],x]
```


```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 6*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[2 - 6*x^2 + 3*x^
4])
```


Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-6x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2-6x^2+3x^4}}$$

Mathematica [A] time = 0.0752576, size = 85, normalized size = 0.94

$$\frac{\sqrt{-3x^2-\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right),2-\sqrt{3}\right)}{\sqrt{6}\sqrt{3x^4-6x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 6*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[2 - 6*x^2 + 3*x^4])
```

Maple [A] time = 0.218, size = 82, normalized size = 0.9

$$2\,\frac{\sqrt{1-\left(1/2\,\sqrt{3}+3/2\right)x^2}\sqrt{1-\left(3/2-1/2\,\sqrt{3}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{6+2\,\sqrt{3}},1/2\,\sqrt{6}-1/2\,\sqrt{2}\right)}{\sqrt{6+2\,\sqrt{3}}\sqrt{3\,x^4-6\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-6*x^2+2)^(1/2),x)
```

```
[Out] 2/(6+2*3^(1/2))^(1/2)*(1-(1/2*3^(1/2)+3/2)*x^2)^(1/2)*(1-(3/2-1/2*3^(1/2))*
x^2)^(1/2)/(3*x^4-6*x^2+2)^(1/2)*EllipticF(1/2*x*(6+2*3^(1/2))^(1/2),1/2*6^
(1/2)-1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-6\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-6\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-6x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-6*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 6*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-6\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

3.78 $\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\frac{(9-\sqrt{57})x^2+6}{(9+\sqrt{57})x^2+6}}\left(\left(9+\sqrt{57}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}x\right),\frac{1}{4}\left(3\sqrt{57}-19\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{2x^4+9x^2+3}}$$

```
[Out] (Sqrt[(6 + (9 - Sqrt[57])*x^2)/(6 + (9 + Sqrt[57])*x^2)]*(6 + (9 + Sqrt[57]
)*x^2)*EllipticF[ArcTan[Sqrt[(9 + Sqrt[57])/6]*x], (-19 + 3*Sqrt[57])/4])/(
Sqrt[6*(9 + Sqrt[57])]*Sqrt[3 + 9*x^2 + 2*x^4])
```

Rubi [A] time = 0.091197, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\sqrt{\frac{(9-\sqrt{57})x^2+6}{(9+\sqrt{57})x^2+6}}\left(\left(9+\sqrt{57}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}x\right)|\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{2x^4+9x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 9*x^2 + 2*x^4],x]
```


```
[Out] (Sqrt[(6 + (9 - Sqrt[57])*x^2)/(6 + (9 + Sqrt[57])*x^2)]*(6 + (9 + Sqrt[57]
)*x^2)*EllipticF[ArcTan[Sqrt[(9 + Sqrt[57])/6]*x], (-19 + 3*Sqrt[57])/4])/(
Sqrt[6*(9 + Sqrt[57])]*Sqrt[3 + 9*x^2 + 2*x^4])
```


Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx = \frac{\sqrt{\frac{6+(9-\sqrt{57})x^2}{6+(9+\sqrt{57})x^2}}\left(6+\left(9+\sqrt{57}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}x\right)\middle|\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{3+9x^2+2x^4}}$$

Mathematica [C] time = 0.0721278, size = 97, normalized size = 0.88

$$-\frac{i\sqrt{\frac{-4x^2+\sqrt{57}-9}{\sqrt{57}-9}}\sqrt{4x^2+\sqrt{57}+9}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{9+\sqrt{57}}}\right),\frac{23}{4}+\frac{3\sqrt{57}}{4}\right)}{2\sqrt{2x^4+9x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 9*x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[(-9 + Sqrt[57] - 4*x^2)/(-9 + Sqrt[57])]*Sqrt[9 + Sqrt[57] + 4
*x^2]*EllipticF[I*ArcSinh[(2*x)/Sqrt[9 + Sqrt[57]]], 23/4 + (3*Sqrt[57])/4]
)/Sqrt[3 + 9*x^2 + 2*x^4]
```

Maple [A] time = 0.236, size = 82, normalized size = 0.8

$$6\,\frac{\sqrt{1-\left(-3/2+1/6\sqrt{57}\right)x^2}\sqrt{1-\left(-3/2-1/6\sqrt{57}\right)x^2}\,\text{EllipticF}\left(1/6\,x\sqrt{-54+6\sqrt{57}},3/4\sqrt{6}+1/4\sqrt{38}\right)}{\sqrt{-54+6\sqrt{57}}\sqrt{2x^4+9x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+9*x^2+3)^(1/2),x)
```

```
[Out] 6/(-54+6*57^(1/2))^(1/2)*(1-(-3/2+1/6*57^(1/2))*x^2)^(1/2)*(1-(-3/2-1/6*57^
(1/2))*x^2)^(1/2)/(2*x^4+9*x^2+3)^(1/2)*EllipticF(1/6*x*(-54+6*57^(1/2))^(1
/2),3/4*6^(1/2)+1/4*38^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+9x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+9\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+9x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+9*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 9*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+9\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

3.79 $\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\frac{(4-\sqrt{10})x^2+3}{(4+\sqrt{10})x^2+3}}\left(\left(4+\sqrt{10}\right)x^2+3\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right),-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{2x^4+8x^2+3}}$$

```
[Out] (Sqrt[(3 + (4 - Sqrt[10])*x^2)/(3 + (4 + Sqrt[10])*x^2)]*(3 + (4 + Sqrt[10]
)*x^2)*EllipticF[ArcTan[Sqrt[(4 + Sqrt[10])/3]*x], (-2*(5 - 2*Sqrt[10]))/3]
)/(Sqrt[3*(4 + Sqrt[10])]*Sqrt[3 + 8*x^2 + 2*x^4])
```

Rubi [A] time = 0.0757186, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\sqrt{\frac{(4-\sqrt{10})x^2+3}{(4+\sqrt{10})x^2+3}}\left(\left(4+\sqrt{10}\right)x^2+3\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right)\middle|-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{2x^4+8x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 8*x^2 + 2*x^4],x]
```


```
[Out] (Sqrt[(3 + (4 - Sqrt[10])*x^2)/(3 + (4 + Sqrt[10])*x^2)]*(3 + (4 + Sqrt[10]
)*x^2)*EllipticF[ArcTan[Sqrt[(4 + Sqrt[10])/3]*x], (-2*(5 - 2*Sqrt[10]))/3]
)/(Sqrt[3*(4 + Sqrt[10])]*Sqrt[3 + 8*x^2 + 2*x^4])
```


Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3+\left(4-\sqrt{10}\right)x^2}{3+\left(4+\sqrt{10}\right)x^2}}\left(3+\left(4+\sqrt{10}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right)|-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{3+8x^2+2x^4}}$$

Mathematica [C] time = 0.0794726, size = 98, normalized size = 0.89

$$-\frac{i\sqrt{\frac{-2x^2+\sqrt{10}-4}{\sqrt{10}-4}}\sqrt{2x^2+\sqrt{10}+4}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{4+\sqrt{10}}}x\right),\frac{13}{3}+\frac{4\sqrt{10}}{3}\right)}{\sqrt{4x^4+16x^2+6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 8*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[(-4 + Sqrt[10] - 2*x^2)/(-4 + Sqrt[10])]*Sqrt[4 + Sqrt[10] + 2*x
^2]*EllipticF[I*ArcSinh[Sqrt[2/(4 + Sqrt[10])]*x], 13/3 + (4*Sqrt[10])/3])/
Sqrt[6 + 16*x^2 + 4*x^4]
```

Maple [A] time = 0.231, size = 82, normalized size = 0.8

$$3\,\frac{\sqrt{1-\left(-4/3+1/3\sqrt{10}\right)x^2}\sqrt{1-\left(-4/3-1/3\sqrt{10}\right)x^2}\,\text{EllipticF}\left(1/3\,x\sqrt{-12+3\sqrt{10}},2/3\sqrt{6}+1/3\sqrt{15}\right)}{\sqrt{-12+3\sqrt{10}}\sqrt{2\,x^4+8\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+8*x^2+3)^(1/2),x)
```

```
[Out] 3/(-12+3*10^(1/2))^(1/2)*(1-(-4/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-4/3-1/3*10^
(1/2))*x^2)^(1/2)/(2*x^4+8*x^2+3)^(1/2)*EllipticF(1/3*x*(-12+3*10^(1/2))^(1
/2),2/3*6^(1/2)+1/3*15^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+8\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+8\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^4+8x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+8*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 8*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4+8\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

3.80 $\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{\frac{x^2+3}{2x^2+1}}\left(2x^2+1\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{2}x\right),\frac{5}{6}\right)}{\sqrt{6}\sqrt{2x^4+7x^2+3}}$$

```
[Out] (Sqrt[(3 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 5/6])
/(Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Rubi [A] time = 0.0074505, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\sqrt{\frac{x^2+3}{2x^2+1}}\left(2x^2+1\right)F\left(\tan^{-1}\left(\sqrt{2}x\right)|\frac{5}{6}\right)}{\sqrt{6}\sqrt{2x^4+7x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 5/6])
/(Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx=\frac{\sqrt{\frac{3+x^2}{1+2x^2}}\left(1+2x^2\right)F\left(\tan^{-1}\left(\sqrt{2}x\right)|\frac{5}{6}\right)}{\sqrt{6}\sqrt{3+7x^2+2x^4}}$$

Mathematica [C] time = 0.0280144, size = 61, normalized size = 1.02

$$-\frac{i\sqrt{x^2+3}\sqrt{2x^2+1}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\right),\frac{1}{6}\right)}{\sqrt{6}\sqrt{2x^4+7x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 7*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/6])/(
Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Maple [C] time = 0.054, size = 50, normalized size = 0.8

$$-\frac{i}{3}\sqrt{3}\text{EllipticF}\left(\frac{i}{3}\sqrt{3}x,\sqrt{6}\right)\sqrt{3x^2+9}\sqrt{2x^2+1}\frac{1}{\sqrt{2x^4+7x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+7*x^2+3)^(1/2),x)
```

```
[Out] -1/3*I*3^(1/2)*(3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+7*x^2+3)^(1/2)*Ellipt
icF(1/3*I*3^(1/2)*x,6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^4+7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+7\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 7*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

3.81 $\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{\frac{(3-\sqrt{3})x^2+3}{(3+\sqrt{3})x^2+3}}\left(\left(3+\sqrt{3}\right)x^2+3\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right),\sqrt{3}-1\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{2x^4+6x^2+3}}$$

```
[Out] (Sqrt[(3 + (3 - Sqrt[3])*x^2)/(3 + (3 + Sqrt[3])*x^2)]*(3 + (3 + Sqrt[3])*x
^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[3])/3]*x], -1 + Sqrt[3]])/(Sqrt[3*(3 +
Sqrt[3])]*Sqrt[3 + 6*x^2 + 2*x^4])
```

Rubi [A] time = 0.0642674, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\sqrt{\frac{(3-\sqrt{3})x^2+3}{(3+\sqrt{3})x^2+3}}\left(\left(3+\sqrt{3}\right)x^2+3\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right)|-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{2x^4+6x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + (3 - Sqrt[3])*x^2)/(3 + (3 + Sqrt[3])*x^2)]*(3 + (3 + Sqrt[3])*x
^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[3])/3]*x], -1 + Sqrt[3]])/(Sqrt[3*(3 +
Sqrt[3])]*Sqrt[3 + 6*x^2 + 2*x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3+(3-\sqrt{3})x^2}{3+(3+\sqrt{3})x^2}}\left(3+\left(3+\sqrt{3}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right)\middle|-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{3+6x^2+2x^4}}$$

Mathematica [C] time = 0.0747967, size = 90, normalized size = 0.87

$$-\frac{i\sqrt{\frac{-2x^2+\sqrt{3}-3}{\sqrt{3}-3}}\sqrt{2x^2+\sqrt{3}+3}\,\textbf{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-\frac{1}{\sqrt{3}}}x\right),2+\sqrt{3}\right)}{\sqrt{4x^4+12x^2+6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 6*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[(-3 + Sqrt[3] - 2*x^2)/(-3 + Sqrt[3])]*Sqrt[3 + Sqrt[3] + 2*x^2]
*EllipticF[I*ArcSinh[Sqrt[1 - 1/Sqrt[3]]*x], 2 + Sqrt[3]])/Sqrt[6 + 12*x^2
+ 4*x^4]
```

Maple [A] time = 0.229, size = 82, normalized size = 0.8

$$3\,\frac{\sqrt{1-\left(-1+1/3\sqrt{3}\right)x^2}\sqrt{1-\left(-1-1/3\sqrt{3}\right)x^2}\,\mathrm{EllipticF}\left(1/3\,x\sqrt{-9+3\sqrt{3}},1/2\sqrt{6}+1/2\sqrt{2}\right)}{\sqrt{-9+3\sqrt{3}}\sqrt{2\,x^4+6\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+6*x^2+3)^(1/2),x)
```

```
[Out] 3/(-9+3*3^(1/2))^(1/2)*(1-(-1+1/3*3^(1/2))*x^2)^(1/2)*(1-(-1-1/3*3^(1/2))*x
^2)^(1/2)/(2*x^4+6*x^2+3)^(1/2)*EllipticF(1/3*x*(-9+3*3^(1/2))^(1/2),1/2*6^
(1/2)+1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+6\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 6*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

3.82 $\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{(x^2+1)\sqrt{\frac{2x^2+3}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{3}\right)}{\sqrt{3}\sqrt{2x^4+5x^2+3}}$$

```
[Out] ((1 + x^2)*Sqrt[(3 + 2*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*
Sqrt[3 + 5*x^2 + 2*x^4])
```

Rubi [A] time = 0.006334, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{(x^2+1)\sqrt{\frac{2x^2+3}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{3}\right)}{\sqrt{3}\sqrt{2x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(3 + 2*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*
Sqrt[3 + 5*x^2 + 2*x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx=\frac{(1+x^2)\sqrt{\frac{3+2x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{3}\right)}{\sqrt{3}\sqrt{3+5x^2+2x^4}}$$

Mathematica [C] time = 0.0268087, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{x^2+1}\sqrt{2x^2+3}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}x\right),\frac{3}{2}\right)}{\sqrt{4x^4+10x^2+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], 3/2])
/Sqrt[6 + 10*x^2 + 4*x^4]
```

Maple [C] time = 0.055, size = 50, normalized size = 1.

$$-\frac{i}{6}\sqrt{6}\sqrt{6\,x^2+9}\sqrt{x^2+1}\text{EllipticF}\left(\frac{i}{3}x\sqrt{6},\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{2\,x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(2*x^4+5*x^2+3)^(1/2)*Elliptic
F(1/3*I*x*6^(1/2),1/2*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+5\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

3.83 $\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4+4x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 + 4*x^2 + 2*x^
4])
```

Rubi [A] time = 0.0163867, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4+4x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 + 4*x^2 + 2*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3+4x^2+2x^4}}$$

Mathematica [C] time = 0.0949952, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-2-i\sqrt{2}}}x\right),\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{2x^4+4x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[2]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[3 + 4*x^2 + 2*x^4])
```

Maple [C] time = 0.737, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(-2/3+i/3\sqrt{2}\right)x^2}\sqrt{1-\left(-2/3-i/3\sqrt{2}\right)x^2}\text{EllipticF}\left(1/3\,x\sqrt{-6+3\,i\sqrt{2}},1/3\,\sqrt{3+6\,i\sqrt{2}}\right)}{\sqrt{-6+3\,i\sqrt{2}}\sqrt{2\,x^4+4\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+4*x^2+3)^(1/2),x)
```

```
[Out] 3/(-6+3*I*2^(1/2))^(1/2)*(1-(-2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3*I*
2^(1/2))*x^2)^(1/2)/(2*x^4+4*x^2+3)^(1/2)*EllipticF(1/3*x*(-6+3*I*2^(1/2))^
(1/2),1/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+4\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 4*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

3.84 $\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+3x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 + 3*x^2 + 2*x^
4])
```

Rubi [A] time = 0.0099577, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 + 3*x^2 + 2*x^
4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3+3x^2+2x^4}}$$

Mathematica [C] time = 0.102204, size = 142, normalized size = 1.54

$$-\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-3-i\sqrt{15}}}x\right),\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{2x^4+3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(-3 + I*Sqrt[1
5])]*EllipticF[I*ArcSinh[2*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*x], (-3 - I*Sqrt[1
5])/(-3 + I*Sqrt[15])])/(Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[3 + 3*x^2 + 2*x
^4])
```

Maple [C] time = 0.745, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(-1/2+i/6\sqrt{15}\right)x^2}\sqrt{1-\left(-1/2-i/6\sqrt{15}\right)x^2}\text{EllipticF}\left(1/6\,x\sqrt{-18+6\,i\sqrt{15}},1/2\,\sqrt{-1+i\sqrt{15}}\right)}{\sqrt{-18+6\,i\sqrt{15}}\sqrt{2\,x^4+3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+3*x^2+3)^(1/2),x)
```

```
[Out] 6/(-18+6*I*15^(1/2))^(1/2)*(1-(-1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6
*I*15^(1/2))*x^2)^(1/2)/(2*x^4+3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*I*15^(
1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+3\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 3*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

3.85 $\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+2x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[3 + 2*x^2 + 2*x
^4])
```

Rubi [A] time = 0.0095349, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+2x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 2*x^2 + 2*x^4],x]
```


```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[3 + 2*x^2 + 2*x
^4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3+2x^2+2x^4}}$$

Mathematica [C] time = 0.0848206, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-1-i\sqrt{5}}}x\right),\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{2x^4+2x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[2]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[3 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.778, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(-1/3+i/3\sqrt{5}\right)x^2}\sqrt{1-\left(-1/3-i/3\sqrt{5}\right)x^2}\text{EllipticF}\left(1/3\,x\sqrt{-3+3\,i\sqrt{5}},1/3\,\sqrt{-6+3\,i\sqrt{5}}\right)}{\sqrt{-3+3\,i\sqrt{5}}\sqrt{2\,x^4+2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+2*x^2+3)^(1/2),x)
```

```
[Out] 3/(-3+3*I*5^(1/2))^(1/2)*(1-(-1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*I*
5^(1/2))*x^2)^(1/2)/(2*x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*(-3+3*I*5^(1/2))^
(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+2\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 2*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

3.86 $\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 + x^2 + 2*x^4]
)
```

Rubi [A] time = 0.0119184, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 + x^2 + 2*x^4]
)
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3+x^2+2x^4}}$$

Mathematica [C] time = 0.0717328, size = 140, normalized size = 1.59

$$-\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-1-i\sqrt{23}}}x\right),\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{2x^4+x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(-1 + I*Sqrt[2
3])]*EllipticF[I*ArcSinh[2*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*x], (-1 - I*Sqrt[2
3])/(-1 + I*Sqrt[23])])/(Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[3 + x^2 + 2*x^4
])
```

Maple [C] time = 0.752, size = 85, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(-1/6+i/6\sqrt{23}\right)x^2}\sqrt{1-\left(-1/6-i/6\sqrt{23}\right)x^2}\text{EllipticF}\left(1/6\,x\sqrt{-6+6\,i\sqrt{23}},1/6\,\sqrt{-33+3\,i\sqrt{23}}\right)}{\sqrt{-6+6\,i\sqrt{23}}\sqrt{2\,x^4+x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+x^2+3)^(1/2),x)
```

```
[Out] 6/(-6+6*I*23^(1/2))^(1/2)*(1-(-1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*
I*23^(1/2))*x^2)^(1/2)/(2*x^4+x^2+3)^(1/2)*EllipticF(1/6*x*(-6+6*I*23^(1/2)
)^(1/2),1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 + 3), x)
```

3.87 $\int \frac{1}{\sqrt{3+2x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{2x^4+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[3 + 2*x^4])
```

Rubi [A] time = 0.0073546, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {220}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{2x^4+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[3 + 2*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+2x^4}}\,dx=\frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{3+2x^4}}$$

Mathematica [C] time = 0.0244129, size = 25, normalized size = 0.35

$$-\sqrt[4]{-\frac{1}{6}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{2}{3}}x\right),-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 2*x^4],x]
```

```
[Out] -((-1/6)^(1/4)*EllipticF[I*ArcSinh[(-2/3)^(1/4)*x], -1])
```

Maple [C] time = 0.193, size = 66, normalized size = 0.9

$$\frac{\sqrt{3}}{9\sqrt{i\sqrt{6}}}\sqrt{9-3\,i\sqrt{6}x^2}\sqrt{9+3\,i\sqrt{6}x^2}\text{EllipticF}\left(\frac{x\sqrt{3}\sqrt{i\sqrt{6}}}{3},i\right)\frac{1}{\sqrt{2\,x^4+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+3)^(1/2),x)
```

```
[Out] 1/9*3^(1/2)/(I*6^(1/2))^(1/2)*(9-3*I*6^(1/2)*x^2)^(1/2)*(9+3*I*6^(1/2)*x^2)
^(1/2)/(2*x^4+3)^(1/2)*EllipticF(1/3*x*3^(1/2)*(I*6^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 3), x)
```

Sympy [C] time = 0.690768, size = 36, normalized size = 0.5

$$\frac{\sqrt{3}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+3)**(1/2),x)
```

```
[Out] sqrt(3)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(I*pi)/3)/(1
2*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3), x)
```

3.88 $\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - x^2 + 2*x^4]
)
```

Rubi [A] time = 0.0096487, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - x^2 + 2*x^4]
)
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3-x^2+2x^4}}$$

Mathematica [C] time = 0.0682245, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{1-i\sqrt{23}}}x\right),\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{2x^4-x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(1 + I*Sqrt[23]
)]*EllipticF[I*ArcSinh[2*Sqrt[-(1 - I*Sqrt[23])^(-1)]*x], (1 - I*Sqrt[23])/
(1 + I*Sqrt[23])])/(Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[3 - x^2 + 2*x^4])
```

Maple [C] time = 0.737, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(1/6+i/6\sqrt{23}\right)x^2}\sqrt{1-\left(1/6-i/6\sqrt{23}\right)x^2}\text{EllipticF}\left(1/6\,x\sqrt{6+6\,i\sqrt{23}},1/6\,\sqrt{-33-3\,i\sqrt{23}}\right)}{\sqrt{6+6\,i\sqrt{23}}\sqrt{2\,x^4-x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-x^2+3)^(1/2),x)
```

```
[Out] 6/(6+6*I*23^(1/2))^(1/2)*(1-(1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*I*2
3^(1/2))*x^2)^(1/2)/(2*x^4-x^2+3)^(1/2)*EllipticF(1/6*x*(6+6*I*23^(1/2))^(1
/2),1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 + 3), x)
```

3.89 $\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-2x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[3 - 2*x^2 + 2*x
^4])
```

Rubi [A] time = 0.0096703, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-2x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^2 + 2*x^4],x]
```



```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[3 - 2*x^2 + 2*x
^4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3-2x^2+2x^4}}$$

Mathematica [C] time = 0.082065, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{1-i\sqrt{5}}}x\right),\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{2x^4-2x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-2/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[2]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[3 - 2*x^2 + 2*x^4])
```

Maple [C] time = 0.748, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(1/3+i/3\sqrt{5}\right)x^2}\sqrt{1-\left(1/3-i/3\sqrt{5}\right)x^2}\text{EllipticF}\left(1/3\,x\sqrt{3+3\,i\sqrt{5}},1/3\,\sqrt{-6-3\,i\sqrt{5}}\right)}{\sqrt{3+3\,i\sqrt{5}}\sqrt{2\,x^4-2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-2*x^2+3)^(1/2),x)
```

```
[Out] 3/(3+3*I*5^(1/2))^(1/2)*(1-(1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3-1/3*I*5^(
1/2))*x^2)^(1/2)/(2*x^4-2*x^2+3)^(1/2)*EllipticF(1/3*x*(3+3*I*5^(1/2))^(1/2
),1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4 - 2\,x^2 + 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - 2x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 2*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - 2\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

3.90 $\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-3x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 - 3*x^2 + 2*x^
4])
```

Rubi [A] time = 0.0092666, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 3*x^2 + 2*x^4],x]
```


```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 - 3*x^2 + 2*x^
4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3-3x^2+2x^4}}$$

Mathematica [C] time = 0.104115, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}x\right),\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{2x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 3*x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(3 + I*Sqrt[15]
)]*EllipticF[I*ArcSinh[2*Sqrt[-(3 - I*Sqrt[15])^(-1)]*x], (3 - I*Sqrt[15])/
(3 + I*Sqrt[15])])/(Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[3 - 3*x^2 + 2*x^4])
```

Maple [C] time = 0.74, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(1/2+i/6\sqrt{15}\right)x^2}\sqrt{1-\left(1/2-i/6\sqrt{15}\right)x^2}\text{EllipticF}\left(1/6\,x\sqrt{18+6\,i\sqrt{15}},1/2\,\sqrt{-1-i\sqrt{15}}\right)}{\sqrt{18+6\,i\sqrt{15}}\sqrt{2\,x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-3*x^2+3)^(1/2),x)
```

```
[Out] 6/(18+6*I*15^(1/2))^(1/2)*(1-(1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*I*
15^(1/2))*x^2)^(1/2)/(2*x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*(18+6*I*15^(1/2)
)^(1/2),1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-3\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 3*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

3.91 $\int \frac{1}{\sqrt{3-4x^2+2x^4}} \, dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4-4x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 - 4*x^2 + 2*x^
4])
```

Rubi [A] time = 0.0095326, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4-4x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*x^2 + 2*x^4],x]
```


```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 - 4*x^2 + 2*x^
4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3-4x^2+2x^4}}$$

Mathematica [C] time = 0.0856035, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{2-i\sqrt{2}}}x\right),\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{2x^4-4x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-2/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[2]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[3 - 4*x^2 + 2*x^4])
```

Maple [C] time = 0.74, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(2/3+i/3\sqrt{2}\right)x^2}\sqrt{1-\left(2/3-i/3\sqrt{2}\right)x^2}\text{EllipticF}\left(1/3\,x\sqrt{6+3\,i\sqrt{2}},1/3\,\sqrt{3-6\,i\sqrt{2}}\right)}{\sqrt{6+3\,i\sqrt{2}}\sqrt{2\,x^4-4\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-4*x^2+3)^(1/2),x)
```

```
[Out] 3/(6+3*I*2^(1/2))^(1/2)*(1-(2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*I*2^(
1/2))*x^2)^(1/2)/(2*x^4-4*x^2+3)^(1/2)*EllipticF(1/3*x*(6+3*I*2^(1/2))^(1/2
),1/3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-4\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 4*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

3.92 $\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-5x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-5x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 5*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - 5*x^2 +
2*x^4])
```

Rubi [A] time = 0.0091571, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1096}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-5x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 5*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 5*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - 5*x^2 +
2*x^4])
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-5x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3-5x^2+2x^4}}$$

Mathematica [A] time = 0.0257186, size = 53, normalized size = 0.58

$$\frac{\sqrt{3-2x^2}\sqrt{1-x^2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right),\frac{3}{2}\right)}{\sqrt{4x^4-10x^2+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], 3/2])/Sqrt[6
- 10*x^2 + 4*x^4]
```

Maple [A] time = 0.048, size = 42, normalized size = 0.5

$$\frac{1}{3}\sqrt{-x^2+1}\sqrt{-6\,x^2+9}\text{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)\frac{1}{\sqrt{2\,x^4-5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-5*x^2+3)^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(-6*x^2+9)^(1/2)/(2*x^4-5*x^2+3)^(1/2)*EllipticF(x,1/3*6
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-5\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 5*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

3.93 $\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-6x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 6*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[3 - 6*x^2 + 2*x^
4])
```

Rubi [A] time = 0.0095625, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1096}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-6x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 6*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 6*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[3 - 6*x^2 + 2*x^
4])
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-6x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3-6x^2+2x^4}}$$

Mathematica [A] time = 0.0748645, size = 81, normalized size = 0.9

$$\frac{\sqrt{-2x^2-\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+3}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{1+\frac{1}{\sqrt{3}}}x\right),2-\sqrt{3}\right)}{\sqrt{6}\sqrt{2x^4-6x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[3 - 6*x^2 + 2*x^4])
```

Maple [A] time = 0.202, size = 82, normalized size = 0.9

$$3\,\frac{\sqrt{1-\left(1+1/3\,\sqrt{3}\right)x^2}\sqrt{1-\left(1-1/3\,\sqrt{3}\right)x^2}\text{EllipticF}\left(1/3\,x\sqrt{9+3\,\sqrt{3}},1/2\,\sqrt{6}-1/2\,\sqrt{2}\right)}{\sqrt{9+3\,\sqrt{3}}\sqrt{2\,x^4-6\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-6*x^2+3)^(1/2),x)
```

```
[Out] 3/(9+3*3^(1/2))^(1/2)*(1-(1+1/3*3^(1/2))*x^2)^(1/2)*(1-(1-1/3*3^(1/2))*x^2)
^(1/2)/(2*x^4-6*x^2+3)^(1/2)*EllipticF(1/3*x*(9+3*3^(1/2))^(1/2),1/2*6^(1/2
)-1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-6\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^4-6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 6*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

3.94 $\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-7x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-7x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 7*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (12 + 7*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - 7*x^2 +
2*x^4])
```

Rubi [A] time = 0.013894, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1096}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-7x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-7x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 7*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 7*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (12 + 7*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - 7*x^2 +
2*x^4])
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-7x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3-7x^2+2x^4}}$$

Mathematica [A] time = 0.0227614, size = 58, normalized size = 0.63

$$\frac{\sqrt{1-2x^2}\sqrt{1-\frac{x^2}{3}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),\frac{1}{6}\right)}{\sqrt{2}\sqrt{2x^4-7x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2/3]*EllipticF[ArcSin[Sqrt[2]*x], 1/6])/(Sqrt[2
]*Sqrt[3 - 7*x^2 + 2*x^4])
```

Maple [A] time = 0.051, size = 49, normalized size = 0.5

$$\frac{\sqrt{2}}{6}\sqrt{-2\,x^2+1}\sqrt{-3\,x^2+9}\,\text{EllipticF}\left(x\sqrt{2},\frac{\sqrt{6}}{6}\right)\frac{1}{\sqrt{2\,x^4-7\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-7*x^2+3)^(1/2),x)
```

```
[Out] 1/6*2^(1/2)*(-2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-7*x^2+3)^(1/2)*Ellipti
cF(x*2^(1/2),1/6*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-7\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 7*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

3.95 $\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{\text{EllipticF}\left(\cos^{-1}\left(\frac{x}{\sqrt{3}}\right),\frac{6}{5}\right)}{\sqrt{5}}$$

```
[Out] -(EllipticF[ArcCos[x/Sqrt[3]], 6/5]/Sqrt[5])
```

Rubi [A] time = 0.0119673, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-\frac{F\left(\cos^{-1}\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 7*x^2 - 2*x^4],x]
```

```
[Out] -(EllipticF[ArcCos[x/Sqrt[3]], 6/5]/Sqrt[5])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{12-4x^2}\sqrt{-2+4x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}}$$

Mathematica [B] time = 0.0226898, size = 58, normalized size = 3.05

$$\frac{\sqrt{1-2x^2}\sqrt{1-\frac{x^2}{3}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{2}x\right),\frac{1}{6}\right)}{\sqrt{2}\sqrt{-2x^4+7x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 7*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2/3]*EllipticF[ArcSin[Sqrt[2]*x], 1/6])/(Sqrt[2
]*Sqrt[-3 + 7*x^2 - 2*x^4])
```

Maple [A] time = 0.049, size = 48, normalized size = 2.5

$$\frac{\sqrt{3}}{3}\sqrt{-3\,x^2+9}\sqrt{-2\,x^2+1}\text{EllipticF}\left(\frac{x\sqrt{3}}{3},\sqrt{6}\right)\frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+7*x^2-3)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(-2*x^4+7*x^2-3)^(1/2)*Ellipt
icF(1/3*x*3^(1/2),6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+7\,x^2-3}}{2\,x^4-7\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 7*x^2 - 3)/(2*x^4 - 7*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+7x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+7*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 7*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

3.96 $\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\text{EllipticF}\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

```
[Out] -(EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3])/2]/(Sqrt[2]*3^(1
/4)))
```

Rubi [A] time = 0.0607096, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 6*x^2 - 2*x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3])/2]/(Sqrt[2]*3^(1
/4)))
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6+2\sqrt{3}-4x^2}\sqrt{-6+2\sqrt{3}+4x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

Mathematica [A] time = 0.0272034, size = 81, normalized size = 1.84

$$\frac{\sqrt{-2x^2-\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+3}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{1+\frac{1}{\sqrt{3}}}x\right),2-\sqrt{3}\right)}{\sqrt{6}\sqrt{-2x^4+6x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 6*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[-3 + 6*x^2 - 2*x^4])
```

Maple [A] time = 0.173, size = 82, normalized size = 1.9

$$3\,\frac{\sqrt{1-\left(1-1/3\,\sqrt{3}\right)x^2}\sqrt{1-\left(1+1/3\,\sqrt{3}\right)x^2}\text{EllipticF}\left(1/3\,x\sqrt{9-3\,\sqrt{3}},1/2\,\sqrt{6}+1/2\,\sqrt{2}\right)}{\sqrt{9-3\,\sqrt{3}}\sqrt{-2\,x^4+6\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+6*x^2-3)^(1/2),x)
```

```
[Out] 3/(9-3*3^(1/2))^(1/2)*(1-(1-1/3*3^(1/2))*x^2)^(1/2)*(1-(1+1/3*3^(1/2))*x^2)
^(1/2)/(-2*x^4+6*x^2-3)^(1/2)*EllipticF(1/3*x*(9-3*3^(1/2))^(1/2),1/2*6^(1/
2)+1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+6\,x^2-3}}{2\,x^4-6\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 6*x^2 - 3)/(2*x^4 - 6*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+6x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+6*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 6*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

3.97 $\int \frac{1}{\sqrt{-3+5x^2-2x^4}} dx$

Optimal. Leaf size=14

$$-\text{EllipticF}\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}x\right),3\right)$$

```
[Out] -EllipticF[ArcCos[Sqrt[2/3]*x], 3]
```

Rubi [A] time = 0.0116204, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-F\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}x\right)\middle|3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 5*x^2 - 2*x^4],x]
```

```
[Out] -EllipticF[ArcCos[Sqrt[2/3]*x], 3]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6-4x^2}\sqrt{-4+4x^2}}\,dx$$
$$= -F\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}x\right)\middle|3\right)$$

Mathematica [B] time = 0.0246362, size = 53, normalized size = 3.79

$$\frac{\sqrt{3-2x^2}\sqrt{1-x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right),\frac{3}{2}\right)}{\sqrt{-4x^4+10x^2-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], 3/2])/Sqrt[-6
 + 10*x^2 - 4*x^4]
```

Maple [A] time = 0.051, size = 50, normalized size = 3.6

$$\frac{\sqrt{6}}{6}\sqrt{-6x^2+9}\sqrt{-x^2+1}\,\text{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{-2x^4+5x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+5*x^2-3)^(1/2),x)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(-2*x^4+5*x^2-3)^(1/2)*Elliptic
F(1/3*x*6^(1/2),1/2*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+5x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+5\,x^2-3}}{2\,x^4-5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 5*x^2 - 3)/(2*x^4 - 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+5x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

3.98 $\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4+4x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 + 4*x^2 - 2*x
^4])
```

Rubi [A] time = 0.010028, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 4*x^2 - 2*x^4],x]
```


```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 + 4*x^2 - 2*x
^4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3+4x^2-2x^4}}$$

Mathematica [C] time = 0.0520562, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{2-i\sqrt{2}}}x\right),\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{-2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-2/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[2]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[-3 + 4*x^2 - 2*x^4])
```

Maple [C] time = 0.669, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(2/3-i/3\sqrt{2}\right)x^2}\sqrt{1-\left(2/3+i/3\sqrt{2}\right)x^2}\text{EllipticF}\left(1/3\,\sqrt{6-3\,i\sqrt{2}}x,1/3\,\sqrt{3+6\,i\sqrt{2}}\right)}{\sqrt{6-3\,i\sqrt{2}}\sqrt{-2\,x^4+4\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+4*x^2-3)^(1/2),x)
```

```
[Out] 3/(6-3*I*2^(1/2))^(1/2)*(1-(2/3-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3+1/3*I*2^(
1/2))*x^2)^(1/2)/(-2*x^4+4*x^2-3)^(1/2)*EllipticF(1/3*(6-3*I*2^(1/2))^(1/2)
*x,1/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+4\,x^2-3}}{2\,x^4-4\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 4*x^2 - 3)/(2*x^4 - 4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 4*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 - 3), x)
```

3.99 $\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+3x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-3 + 3*x^2 - 2*x
^4])
```

Rubi [A] time = 0.0102385, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 3*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-3 + 3*x^2 - 2*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3+3x^2-2x^4}}$$

Mathematica [C] time = 0.099026, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}x\right),\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 3*x^2 - 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(3 + I*Sqrt[15]
)]*EllipticF[I*ArcSinh[2*Sqrt[-(3 - I*Sqrt[15])^(-1)]*x], (3 - I*Sqrt[15])/
(3 + I*Sqrt[15])])/(Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[-3 + 3*x^2 - 2*x^4])
```

Maple [C] time = 0.667, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(1/2-i/6\sqrt{15}\right)x^2}\sqrt{1-\left(1/2+i/6\sqrt{15}\right)x^2}\text{EllipticF}\left(1/6\,\sqrt{18-6\,i\sqrt{15}}x,1/2\,\sqrt{-1+i\sqrt{15}}\right)}{\sqrt{18-6\,i\sqrt{15}}\sqrt{-2\,x^4+3\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+3*x^2-3)^(1/2),x)
```

```
[Out] 6/(18-6*I*15^(1/2))^(1/2)*(1-(1/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1-(1/2+1/6*I*
15^(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2-3)^(1/2)*EllipticF(1/6*(18-6*I*15^(1/2))
^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+3\,x^2-3}}{2\,x^4-3\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 3*x^2 - 3)/(2*x^4 - 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 3*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 - 3), x)
```

3.100 $\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+2x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-3 + 2*x^2 - 2*
x^4])
```

Rubi [A] time = 0.0096004, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-3 + 2*x^2 - 2*
x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3+2x^2-2x^4}}$$

Mathematica [C] time = 0.072151, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{1-i\sqrt{5}}}x\right),\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-2/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[2]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[-3 + 2*x^2 - 2*x^4])
```

Maple [C] time = 0.667, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(1/3-i/3\sqrt{5}\right)x^2}\sqrt{1-\left(1/3+i/3\sqrt{5}\right)x^2}\text{EllipticF}\left(1/3\,\sqrt{3-3\,i\sqrt{5}}x,1/3\,\sqrt{-6+3\,i\sqrt{5}}\right)}{\sqrt{3-3\,i\sqrt{5}}\sqrt{-2\,x^4+2\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+2*x^2-3)^(1/2),x)
```

```
[Out] 3/(3-3*I*5^(1/2))^(1/2)*(1-(1/3-1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3+1/3*I*5^(
1/2))*x^2)^(1/2)/(-2*x^4+2*x^2-3)^(1/2)*EllipticF(1/3*(3-3*I*5^(1/2))^(1/2)
*x,1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+2\,x^2-3}}{2\,x^4-2\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 2*x^2 - 3)/(2*x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+2x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 2*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 - 3), x)
```

3.101 $\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-3 + x^2 - 2*x^4
])
```

Rubi [A] time = 0.00898, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-3 + x^2 - 2*x^4
])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3+x^2-2x^4}}$$

Mathematica [C] time = 0.0668921, size = 140, normalized size = 1.59

$$-\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{1-i\sqrt{23}}}x\right),\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + x^2 - 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(1 + I*Sqrt[23]
)]*EllipticF[I*ArcSinh[2*Sqrt[-(1 - I*Sqrt[23])^(-1)]*x], (1 - I*Sqrt[23])/
(1 + I*Sqrt[23])])/(Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[-3 + x^2 - 2*x^4])
```

Maple [C] time = 0.671, size = 85, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(1/6-i/6\sqrt{23}\right)x^2}\sqrt{1-\left(1/6+i/6\sqrt{23}\right)x^2}\text{EllipticF}\left(1/6\sqrt{6-6\,i\sqrt{23}}x,1/6\sqrt{-33+3\,i\sqrt{23}}\right)}{\sqrt{6-6\,i\sqrt{23}}\sqrt{-2\,x^4+x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+x^2-3)^(1/2),x)
```

```
[Out] 6/(6-6*I*23^(1/2))^(1/2)*(1-(1/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1/6+1/6*I*2
3^(1/2))*x^2)^(1/2)/(-2*x^4+x^2-3)^(1/2)*EllipticF(1/6*(6-6*I*23^(1/2))^(1/
2)*x,1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+x^2-3}}{2\,x^4-x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + x^2 - 3)/(2*x^4 - x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 - 3), x)
```

3.102 $\int \frac{1}{\sqrt{-3-2x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-3 - 2*x^4])
```

Rubi [A] time = 0.0070373, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {220}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-3 - 2*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx=\frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-3-2x^4}}$$

Mathematica [C] time = 0.0276189, size = 47, normalized size = 0.65

$$-\frac{\sqrt[4]{-\frac{1}{6}}\sqrt{2x^4+3}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{2}{3}}x\right),-1\right)}{\sqrt{-2x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 2*x^4],x]
```

```
[Out] -(((-1/6)^(1/4)*Sqrt[3 + 2*x^4]*EllipticF[I*ArcSinh[(-2/3)^(1/4)*x], -1])/S
qrt[-3 - 2*x^4])
```

Maple [C] time = 0.168, size = 66, normalized size = 0.9

$$\frac{\sqrt{3}}{9\sqrt{-i\sqrt{6}}}\sqrt{9+3\,i\sqrt{6}x^2}\sqrt{9-3\,i\sqrt{6}x^2}\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{-i\sqrt{6}}x}{3},i\right)\frac{1}{\sqrt{-2\,x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-3)^(1/2),x)
```

```
[Out] 1/9*3^(1/2)/(-I*6^(1/2))^(1/2)*(9+3*I*6^(1/2)*x^2)^(1/2)*(9-3*I*6^(1/2)*x^2
)^(1/2)/(-2*x^4-3)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*6^(1/2))^(1/2)*x,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4-3}}{2\,x^4+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 3)/(2*x^4 + 3), x)
```

Sympy [C] time = 0.602676, size = 39, normalized size = 0.54

$$-\frac{\sqrt{3}ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-3)**(1/2),x)
```

```
[Out] -sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(I*pi)/3)
/(12*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3), x)
```

3.103 $\int \frac{1}{\sqrt{-3-x^2-2x^4}} \, dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-3 - x^2 - 2*x^4
])
```

Rubi [A] time = 0.0100277, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - x^2 - 2*x^4],x]
```


```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-3 - x^2 - 2*x^4
])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3-x^2-2x^4}}$$

Mathematica [C] time = 0.0633103, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-1-i\sqrt{23}}}x\right),\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{-2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - x^2 - 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(-1 + I*Sqrt[2
3])]*EllipticF[I*ArcSinh[2*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*x], (-1 - I*Sqrt[2
3])/(-1 + I*Sqrt[23])])/(Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[-3 - x^2 - 2*x^
4])
```

Maple [C] time = 0.711, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(-1/6-i/6\sqrt{23}\right)x^2}\sqrt{1-\left(-1/6+i/6\sqrt{23}\right)x^2}\text{EllipticF}\left(1/6\sqrt{-6-6\,i\sqrt{23}}x,1/6\sqrt{-33-3\,i\sqrt{23}}\right)}{\sqrt{-6-6\,i\sqrt{23}}\sqrt{-2\,x^4-x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-x^2-3)^(1/2),x)
```

```
[Out] 6/(-6-6*I*23^(1/2))^(1/2)*(1-(-1/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-(-1/6+1/6*
I*23^(1/2))*x^2)^(1/2)/(-2*x^4-x^2-3)^(1/2)*EllipticF(1/6*(-6-6*I*23^(1/2))
^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 - x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4 - x^2 - 3}}{2\,x^4 + x^2 + 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - x^2 - 3)/(2*x^4 + x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 - x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 - x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 - 3), x)
```

3.104 $\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-2x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-3 - 2*x^2 - 2*
x^4])
```

Rubi [A] time = 0.010235, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-3 - 2*x^2 - 2*
x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3-2x^2-2x^4}}$$

Mathematica [C] time = 0.0753406, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-1-i\sqrt{5}}}x\right),\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{-2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[2]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[-3 - 2*x^2 - 2*x^4])
```

Maple [C] time = 0.662, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(-1/3-i/3\sqrt{5}\right)x^2}\sqrt{1-\left(-1/3+i/3\sqrt{5}\right)x^2}\text{EllipticF}\left(1/3\sqrt{-3-3\,i\sqrt{5}}x,1/3\sqrt{-6-3\,i\sqrt{5}}\right)}{\sqrt{-3-3\,i\sqrt{5}}\sqrt{-2\,x^4-2\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-2*x^2-3)^(1/2),x)
```

```
[Out] 3/(-3-3*I*5^(1/2))^(1/2)*(1-(-1/3-1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3+1/3*I*
5^(1/2))*x^2)^(1/2)/(-2*x^4-2*x^2-3)^(1/2)*EllipticF(1/3*(-3-3*I*5^(1/2))^(
1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-2\,x^2-3}}{2\,x^4+2\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 2*x^2 - 3)/(2*x^4 + 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-2x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 2*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 - 3), x)
```

3.105 $\int \frac{1}{\sqrt{-3-3x^2-2x^4}} dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-3x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-3 - 3*x^2 - 2*x
^4])
```

Rubi [A] time = 0.009903, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 3*x^2 - 2*x^4],x]
```


```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-3 - 3*x^2 - 2*x
^4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3-3x^2-2x^4}}$$

Mathematica [C] time = 0.099948, size = 142, normalized size = 1.54

$$-\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-3-i\sqrt{15}}}x\right),\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 3*x^2 - 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(-3 + I*Sqrt[1
5])]*EllipticF[I*ArcSinh[2*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*x], (-3 - I*Sqrt[1
5])/(-3 + I*Sqrt[15])])/(Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[-3 - 3*x^2 - 2*
x^4])
```

Maple [C] time = 0.663, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(-1/2-i/6\sqrt{15}\right)x^2}\sqrt{1-\left(-1/2+i/6\sqrt{15}\right)x^2}\text{EllipticF}\left(1/6\sqrt{-18-6\,i\sqrt{15}}x,1/2\sqrt{-1-i\sqrt{15}}\right)}{\sqrt{-18-6\,i\sqrt{15}}\sqrt{-2\,x^4-3\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-3*x^2-3)^(1/2),x)
```

```
[Out] 6/(-18-6*I*15^(1/2))^(1/2)*(1-(-1/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1-(-1/2+1/6
*I*15^(1/2))*x^2)^(1/2)/(-2*x^4-3*x^2-3)^(1/2)*EllipticF(1/6*(-18-6*I*15^(1
/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-3\,x^2-3}}{2\,x^4+3\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 3*x^2 - 3)/(2*x^4 + 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 3*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 - 3), x)
```

3.106 $\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-4x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 - 4*x^2 - 2*x
^4])
```

Rubi [A] time = 0.0093564, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 - 4*x^2 - 2*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3-4x^2-2x^4}}$$

Mathematica [C] time = 0.0846889, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-2-i\sqrt{2}}}x\right),\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[2]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[-3 - 4*x^2 - 2*x^4])
```

Maple [C] time = 0.67, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(-2/3-i/3\sqrt{2}\right)x^2}\sqrt{1-\left(-2/3+i/3\sqrt{2}\right)x^2}\text{EllipticF}\left(1/3\sqrt{-6-3\,i\sqrt{2}}x,1/3\sqrt{3-6\,i\sqrt{2}}\right)}{\sqrt{-6-3\,i\sqrt{2}}\sqrt{-2\,x^4-4\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-4*x^2-3)^(1/2),x)
```

```
[Out] 3/(-6-3*I*2^(1/2))^(1/2)*(1-(-2/3-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3+1/3*I*
2^(1/2))*x^2)^(1/2)/(-2*x^4-4*x^2-3)^(1/2)*EllipticF(1/3*(-6-3*I*2^(1/2))^(
1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-4\,x^2-3}}{2\,x^4+4\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 4*x^2 - 3)/(2*x^4 + 4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 4*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 - 3), x)
```

3.107 $\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{2x^2+3}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}\sqrt{\frac{2x^2+3}{x^2+1}}}$$

```
[Out] (Sqrt[3 + 2*x^2]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*Sqrt[-1 - x^2]*Sqrt[(3
 + 2*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0161595, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 418}

$$\frac{\sqrt{2x^2+3}F\left(\tan^{-1}(x)|\frac{1}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}\sqrt{\frac{2x^2+3}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 + 2*x^2]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*Sqrt[-1 - x^2]*Sqrt[(3
 + 2*x^2)/(1 + x^2)])
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-4-4x^2}\sqrt{6+4x^2}}\,dx$$
$$= \frac{\sqrt{3+2x^2}F\left(\tan^{-1}(x)|\frac{1}{3}\right)}{\sqrt{3}\sqrt{-1-x^2}\sqrt{\frac{3+2x^2}{1+x^2}}}$$

Mathematica [C] time = 0.0243654, size = 63, normalized size = 1.19

$$-\frac{i\sqrt{x^2+1}\sqrt{2x^2+3}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}x\right),\frac{3}{2}\right)}{\sqrt{2}\sqrt{-2x^4-5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 5*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], 3/2])
/(Sqrt[2]*Sqrt[-3 - 5*x^2 - 2*x^4])
```

Maple [C] time = 0.053, size = 44, normalized size = 0.8

$$-\frac{i}{3}\sqrt{x^2+1}\sqrt{6\,x^2+9}\text{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)\frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-5*x^2-3)^(1/2),x)
```

```
[Out] -1/3*I*(x^2+1)^(1/2)*(6*x^2+9)^(1/2)/(-2*x^4-5*x^2-3)^(1/2)*EllipticF(I*x,1
/3*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-5\,x^2-3}}{2\,x^4+5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 5*x^2 - 3)/(2*x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-5x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 5*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 - 3), x)
```

3.108 $\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx$

Optimal. Leaf size=42

$$-\frac{\text{EllipticF}\left(\cos^{-1}\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

```
[Out] -(EllipticF[ArcCos[Sqrt[3/(3 + Sqrt[3])]*x], (1 + Sqrt[3])/2]/(Sqrt[2]*3^(1
/4)))
```

Rubi [A] time = 0.0520854, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 6*x^2 - 3*x^4],x]
```



```
[Out] -(EllipticF[ArcCos[Sqrt[3/(3 + Sqrt[3])]*x], (1 + Sqrt[3])/2]/(Sqrt[2]*3^(1
/4)))
```


Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{6+2\sqrt{3}-6x^2}\sqrt{-6+2\sqrt{3}+6x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

Mathematica [B] time = 0.082354, size = 85, normalized size = 2.02

$$\frac{\sqrt{-3x^2-\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right),2-\sqrt{3}\right)}{\sqrt{6}\sqrt{-3x^4+6x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 6*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[-2 + 6*x^2 - 3*x^4])
```

Maple [A] time = 0.175, size = 82, normalized size = 2.

$$2\,\frac{\sqrt{1-\left(3/2-1/2\,\sqrt{3}\right)x^2}\sqrt{1-\left(1/2\,\sqrt{3}+3/2\right)x^2}\text{EllipticF}\left(1/2\,\sqrt{6-2\,\sqrt{3}}x,1/2\,\sqrt{6}+1/2\,\sqrt{2}\right)}{\sqrt{6-2\,\sqrt{3}}\sqrt{-3\,x^4+6\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+6*x^2-2)^(1/2),x)
```

```
[Out] 2/(6-2*3^(1/2))^(1/2)*(1-(3/2-1/2*3^(1/2))*x^2)^(1/2)*(1-(1/2*3^(1/2)+3/2)*
x^2)^(1/2)/(-3*x^4+6*x^2-2)^(1/2)*EllipticF(1/2*(6-2*3^(1/2))^(1/2)*x,1/2*6
^(1/2)+1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+6\,x^2-2}}{3\,x^4-6\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 6*x^2 - 2)/(3*x^4 - 6*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+6x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+6*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 6*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

3.109 $\int \frac{1}{\sqrt{-2+5x^2-3x^4}} dx$

Optimal. Leaf size=6

$$-\text{EllipticF}\left(\cos^{-1}(x), 3\right)$$

```
[Out] -EllipticF[ArcCos[x], 3]
```

Rubi [A] time = 0.0104999, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-F\left(\cos^{-1}(x)\middle|3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x^2 - 3*x^4],x]
```

```
[Out] -EllipticF[ArcCos[x], 3]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+5x^2-3x^4}} dx &= \left(2\sqrt{3}\right) \int \frac{1}{\sqrt{6-6x^2}\sqrt{-4+6x^2}} dx\\ &= -F\left(\cos^{-1}(x)\middle|3\right)\end{aligned}$$

Mathematica [B] time = 0.0264863, size = 53, normalized size = 8.83

$$\frac{\sqrt{2-3x^2}\sqrt{1-x^2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{-9x^4+15x^2-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/Sqrt[-6
 + 15*x^2 - 9*x^4]
```

Maple [A] time = 0.049, size = 42, normalized size = 7.

$$\frac{1}{2}\sqrt{-x^2+1}\sqrt{-6\,x^2+4}\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+5*x^2-2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(-3*x^4+5*x^2-2)^(1/2)*EllipticF(x,1/2*
6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+5\,x^2-2}}{3\,x^4-5\,x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 5*x^2 - 2)/(3*x^4 - 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

3.110 $\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3x^4+4x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-2 + 4*x^2 - 3*x
^4])
```

Rubi [A] time = 0.0095612, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-2 + 4*x^2 - 3*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2+4x^2-3x^4}}$$

Mathematica [C] time = 0.0840316, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{2-i\sqrt{2}}}x\right),\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{-3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-3/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[3]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[-2 + 4*x^2 - 3*x^4])
```

Maple [C] time = 0.701, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1-i/2\sqrt{2}\right)x^2}\sqrt{1-\left(1+i/2\sqrt{2}\right)x^2}\text{EllipticF}\left(1/2\sqrt{4-2\,i\sqrt{2}}x,1/3\sqrt{3+6\,i\sqrt{2}}\right)}{\sqrt{4-2\,i\sqrt{2}}\sqrt{-3\,x^4+4\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+4*x^2-2)^(1/2),x)
```

```
[Out] 2/(4-2*I*2^(1/2))^(1/2)*(1-(1-1/2*I*2^(1/2))*x^2)^(1/2)*(1-(1+1/2*I*2^(1/2)
)*x^2)^(1/2)/(-3*x^4+4*x^2-2)^(1/2)*EllipticF(1/2*(4-2*I*2^(1/2))^(1/2)*x,1
/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+4\,x^2-2}}{3\,x^4-4\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 4*x^2 - 2)/(3*x^4 - 4*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 4*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 - 2), x)
```

3.111 $\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+3x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 + 3*x^2 - 3*x
^4])
```

Rubi [A] time = 0.0094841, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 3*x^2 - 3*x^4],x]
```


```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 + 3*x^2 - 3*x
^4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2+3x^2-3x^4}}$$

Mathematica [C] time = 0.111374, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{3-i\sqrt{15}}}x\right),\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 3*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(3 + I*Sqrt[15])]
*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[15])]*x], (3 - I*Sqrt[15])/(3 + I*
Sqrt[15])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[-2 + 3*x^2 - 3*x^4])
```

Maple [C] time = 0.693, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(3/4-i/4\sqrt{15}\right)x^2}\sqrt{1-\left(3/4+i/4\sqrt{15}\right)x^2}\text{EllipticF}\left(1/2\sqrt{3-i\sqrt{15}}x,1/2\sqrt{-1+i\sqrt{15}}\right)}{\sqrt{3-i\sqrt{15}}\sqrt{-3x^4+3x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+3*x^2-2)^(1/2),x)
```

```
[Out] 2/(3-I*15^(1/2))^(1/2)*(1-(3/4-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4+1/4*I*15^
(1/2))*x^2)^(1/2)/(-3*x^4+3*x^2-2)^(1/2)*EllipticF(1/2*(3-I*15^(1/2))^(1/2)
*x,1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+3\,x^2-2}}{3\,x^4-3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 3*x^2 - 2)/(3*x^4 - 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 3*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 - 2), x)
```

3.112 $\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+2x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-2 + 2*x^2 - 3*
x^4])
```

Rubi [A] time = 0.0139037, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-2 + 2*x^2 - 3*
x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2+2x^2-3x^4}}$$

Mathematica [C] time = 0.0809243, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{1-i\sqrt{5}}}x\right),\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-3/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[3]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[-2 + 2*x^2 - 3*x^4])
```

Maple [C] time = 0.697, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1/2-i/2\sqrt{5}\right)x^2}\sqrt{1-\left(1/2+i/2\sqrt{5}\right)x^2}\text{EllipticF}\left(1/2\sqrt{2-2\,i\sqrt{5}}x,1/3\sqrt{-6+3\,i\sqrt{5}}\right)}{\sqrt{2-2\,i\sqrt{5}}\sqrt{-3\,x^4+2\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+2*x^2-2)^(1/2),x)
```

```
[Out] 2/(2-2*I*5^(1/2))^(1/2)*(1-(1/2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2+1/2*I*5^(
1/2))*x^2)^(1/2)/(-3*x^4+2*x^2-2)^(1/2)*EllipticF(1/2*(2-2*I*5^(1/2))^(1/2)
*x,1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+2\,x^2-2}}{3\,x^4-2\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 2*x^2 - 2)/(3*x^4 - 2*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 2*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 - 2), x)
```

3.113 $\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-2 + x^2 - 3*x^4
])
```

Rubi [A] time = 0.0093471, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-2 + x^2 - 3*x^4
])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2+x^2-3x^4}}$$

Mathematica [C] time = 0.081277, size = 142, normalized size = 1.61

$$-\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{1-i\sqrt{23}}}x\right),\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(1 + I*Sqrt[23])]
*EllipticF[I*ArcSinh[Sqrt[-6/(1 - I*Sqrt[23])]*x], (1 - I*Sqrt[23])/(1 + I*
Sqrt[23])])/(Sqrt[6]*Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[-2 + x^2 - 3*x^4])
```

Maple [C] time = 0.693, size = 85, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1/4-i/4\sqrt{23}\right)x^2}\sqrt{1-\left(1/4+i/4\sqrt{23}\right)x^2}\text{EllipticF}\left(1/2\sqrt{1-i\sqrt{23}}x,1/6\sqrt{-33+3\,i\sqrt{23}}\right)}{\sqrt{1-i\sqrt{23}}\sqrt{-3\,x^4+x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+x^2-2)^(1/2),x)
```

```
[Out] 2/(1-I*23^(1/2))^(1/2)*(1-(1/4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4+1/4*I*23^
(1/2))*x^2)^(1/2)/(-3*x^4+x^2-2)^(1/2)*EllipticF(1/2*(1-I*23^(1/2))^(1/2)*x
,1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+x^2-2}}{3\,x^4-x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + x^2 - 2)/(3*x^4 - x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 - 2), x)
```

3.114 $\int \frac{1}{\sqrt{-2-3x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-3x^4-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-2 - 3*x^4])
```

Rubi [A] time = 0.0079274, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {220}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-3x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-2 - 3*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3x^4}}\,dx=\frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-2-3x^4}}$$

Mathematica [C] time = 0.0275166, size = 47, normalized size = 0.65

$$-\frac{\sqrt[4]{-\frac{1}{6}}\sqrt{3x^4+2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{3}{2}}x\right),-1\right)}{\sqrt{-3x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 3*x^4],x]
```

```
[Out] -(((-1/6)^(1/4)*Sqrt[2 + 3*x^4]*EllipticF[I*ArcSinh[(-3/2)^(1/4)*x], -1])/S
qrt[-2 - 3*x^4])
```

Maple [C] time = 0.174, size = 66, normalized size = 0.9

$$\frac{\sqrt{2}}{4\sqrt{-i\sqrt{6}}}\sqrt{4+2\,i\sqrt{6}x^2}\sqrt{4-2\,i\sqrt{6}x^2}\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{-i\sqrt{6}}}{2},i\right)\frac{1}{\sqrt{-3\,x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-2)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/(-I*6^(1/2))^(1/2)*(4+2*I*6^(1/2)*x^2)^(1/2)*(4-2*I*6^(1/2)*x^2
)^(1/2)/(-3*x^4-2)^(1/2)*EllipticF(1/2*x*2^(1/2)*(-I*6^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-2}}{3\,x^4+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 2)/(3*x^4 + 2), x)
```

Sympy [C] time = 0.613024, size = 39, normalized size = 0.54

$$-\frac{\sqrt{2}ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{3x^4e^{i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-2)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(I*pi)/2)
/(8*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2), x)
```

3.115 $\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-2 - x^2 - 3*x^4
])
```

Rubi [A] time = 0.0097881, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - x^2 - 3*x^4],x]
```


```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-2 - x^2 - 3*x^4
])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2-x^2-3x^4}}$$

Mathematica [C] time = 0.0795968, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{6x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+i\sqrt{23}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-1-i\sqrt{23}}}x\right),\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{-3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(-1 + I*Sqrt[23]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-1 - I*Sqrt[23])]*x], (-1 - I*Sqrt[23])/(-1
 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[-2 - x^2 - 3*x
^4])
```

Maple [C] time = 0.696, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1/4-i/4\sqrt{23}\right)x^2}\sqrt{1-\left(-1/4+i/4\sqrt{23}\right)x^2}\text{EllipticF}\left(1/2\sqrt{-1-i\sqrt{23}}x,1/6\sqrt{-33-3\,i\sqrt{23}}\right)}{\sqrt{-1-i\sqrt{23}}\sqrt{-3\,x^4-x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-x^2-2)^(1/2),x)
```

```
[Out] 2/(-1-I*23^(1/2))^(1/2)*(1-(-1/4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-1/4+1/4*I*
23^(1/2))*x^2)^(1/2)/(-3*x^4-x^2-2)^(1/2)*EllipticF(1/2*(-1-I*23^(1/2))^(1/
2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 - x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4 - x^2 - 2}}{3\,x^4 + x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - x^2 - 2)/(3*x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4 - x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 - x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 - 2), x)
```

3.116 $\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-2x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-2 - 2*x^2 - 3*
x^4])
```

Rubi [A] time = 0.0102529, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 2*x^2 - 3*x^4],x]
```


```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-2 - 2*x^2 - 3*
x^4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2-2x^2-3x^4}}$$

Mathematica [C] time = 0.0817626, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{3x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{-1+i\sqrt{5}}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-1-i\sqrt{5}}}x\right),\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{-3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[3]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[-2 - 2*x^2 - 3*x^4])
```

Maple [C] time = 0.442, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1/2-i/2\sqrt{5}\right)x^2}\sqrt{1-\left(-1/2+i/2\sqrt{5}\right)x^2}\,\text{EllipticF}\left(1/2\sqrt{-2-2\,i\sqrt{5}}x,1/3\sqrt{-6-3\,i\sqrt{5}}\right)}{\sqrt{-2-2\,i\sqrt{5}}\sqrt{-3\,x^4-2\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-2*x^2-2)^(1/2),x)
```

```
[Out] 2/(-2-2*I*5^(1/2))^(1/2)*(1-(-1/2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(-1/2+1/2*I*
5^(1/2))*x^2)^(1/2)/(-3*x^4-2*x^2-2)^(1/2)*EllipticF(1/2*(-2-2*I*5^(1/2))^(
1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-3\,x^4-2\,x^2-2}}{3\,x^4+2\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 2*x^2 - 2)/(3*x^4 + 2*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 2*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 - 2), x)
```

3.117 $\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-3x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 - 3*x^2 - 3*x
^4])
```

Rubi [A] time = 0.0095006, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 3*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 - 3*x^2 - 3*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2-3x^2-3x^4}}$$

Mathematica [C] time = 0.115442, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-3-i\sqrt{15}}}x\right),\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 3*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(-3 + I*Sqrt[15]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-3 - I*Sqrt[15])]*x], (-3 - I*Sqrt[15])/(-3
 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[-2 - 3*x^2 - 3
*x^4])
```

Maple [C] time = 0.699, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-3/4-i/4\sqrt{15}\right)x^2}\sqrt{1-\left(-3/4+i/4\sqrt{15}\right)x^2}\text{EllipticF}\left(1/2\,\sqrt{-3-i\sqrt{15}}x,1/2\,\sqrt{-1-i\sqrt{15}}\right)}{\sqrt{-3-i\sqrt{15}}\sqrt{-3\,x^4-3\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-3*x^2-2)^(1/2),x)
```

```
[Out] 2/(-3-I*15^(1/2))^(1/2)*(1-(-3/4-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/4+1/4*I*
15^(1/2))*x^2)^(1/2)/(-3*x^4-3*x^2-2)^(1/2)*EllipticF(1/2*(-3-I*15^(1/2))^(
1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-3\,x^2-2}}{3\,x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 3*x^2 - 2)/(3*x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 3*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 - 2), x)
```

3.118 $\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right),\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3x^4-4x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-2 - 4*x^2 - 3*x
^4])
```

Rubi [A] time = 0.0094641, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-2 - 4*x^2 - 3*x
^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2-4x^2-3x^4}}$$

Mathematica [C] time = 0.0870506, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-2-i\sqrt{2}}}x\right),\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[3]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[-2 - 4*x^2 - 3*x^4])
```

Maple [C] time = 0.445, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1-i/2\sqrt{2}\right)x^2}\sqrt{1-\left(-1+i/2\sqrt{2}\right)x^2}\text{EllipticF}\left(1/2\sqrt{-4-2\,i\sqrt{2}}x,1/3\sqrt{3-6\,i\sqrt{2}}\right)}{\sqrt{-4-2\,i\sqrt{2}}\sqrt{-3\,x^4-4\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-4*x^2-2)^(1/2),x)
```

```
[Out] 2/(-4-2*I*2^(1/2))^(1/2)*(1-(-1-1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1+1/2*I*2^(1
/2))*x^2)^(1/2)/(-3*x^4-4*x^2-2)^(1/2)*EllipticF(1/2*(-4-2*I*2^(1/2))^(1/2)
*x,1/3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-4\,x^2-2}}{3\,x^4+4\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 4*x^2 - 2)/(3*x^4 + 4*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3x^4-4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 4*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 - 2), x)
```

3.119 $\int \frac{1}{\sqrt{-2-5x^2-3x^4}} dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{-3x^2-2}\text{EllipticF}\left(\tan^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] -((Sqrt[-2 - 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt
[(2 + 3*x^2)/(1 + x^2)]))
```

Rubi [A] time = 0.0167469, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 418}

$$-\frac{\sqrt{-3x^2-2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 5*x^2 - 3*x^4],x]
```

```
[Out] -((Sqrt[-2 - 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt
[(2 + 3*x^2)/(1 + x^2)]))
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-4-6x^2}\sqrt{6+6x^2}}\,dx$$
$$= -\frac{\sqrt{-2-3x^2}F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Mathematica [C] time = 0.024037, size = 63, normalized size = 1.21

$$-\frac{i\sqrt{x^2+1}\sqrt{3x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{3}\sqrt{-3x^4-5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])
/(Sqrt[3]*Sqrt[-2 - 5*x^2 - 3*x^4])
```

Maple [A] time = 0.054, size = 50, normalized size = 1.

$$-\frac{i}{6}\sqrt{6}\sqrt{6x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{i}{2}x\sqrt{6},\frac{\sqrt{6}}{3}\right)\frac{1}{\sqrt{-3x^4-5x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-5*x^2-2)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-5*x^2-2)^(1/2)*Ellipti
cF(1/2*I*x*6^(1/2),1/3*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-5\,x^2-2}}{3\,x^4+5\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 5*x^2 - 2)/(3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 5*x**2 - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 - 2), x)
```

3.120 $\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{10}x^2+2\right)\sqrt{\frac{5x^4+5x^2+2}{\left(\sqrt{10}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{\frac{5}{2}}x\right),\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{5x^4+5x^2+2}}$$

```
[Out] ((2 + Sqrt[10]*x^2)*Sqrt[(2 + 5*x^2 + 5*x^4)/(2 + Sqrt[10]*x^2)^2]*Elliptic
F[2*ArcTan[(5/2)^(1/4)*x], (4 - Sqrt[10])/8])/(2*10^(1/4)*Sqrt[2 + 5*x^2 +
5*x^4])
```

Rubi [A] time = 0.0258335, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{10}x^2+2\right)\sqrt{\frac{5x^4+5x^2+2}{\left(\sqrt{10}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{5}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{5x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 5*x^4],x]
```

```
[Out] ((2 + Sqrt[10]*x^2)*Sqrt[(2 + 5*x^2 + 5*x^4)/(2 + Sqrt[10]*x^2)^2]*Elliptic
F[2*ArcTan[(5/2)^(1/4)*x], (4 - Sqrt[10])/8])/(2*10^(1/4)*Sqrt[2 + 5*x^2 +
5*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx = \frac{\left(2+\sqrt{10}x^2\right)\sqrt{\frac{2+5x^2+5x^4}{\left(2+\sqrt{10}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{5}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{2+5x^2+5x^4}}$$

Mathematica [C] time = 0.114411, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{10x^2}{-5-i\sqrt{15}}}\sqrt{1-\frac{10x^2}{-5+i\sqrt{15}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{10}{-5-i\sqrt{15}}}x\right),\frac{-5-i\sqrt{15}}{-5+i\sqrt{15}}\right)}{\sqrt{10}\sqrt{-\frac{1}{-5-i\sqrt{15}}}\sqrt{5x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 5*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (10*x^2)/(-5 - I*Sqrt[15])]*Sqrt[1 - (10*x^2)/(-5 + I*Sqrt[1
5])]*EllipticF[I*ArcSinh[Sqrt[-10/(-5 - I*Sqrt[15])]*x], (-5 - I*Sqrt[15])/
(-5 + I*Sqrt[15])])/(Sqrt[10]*Sqrt[-(-5 - I*Sqrt[15])^(-1)]*Sqrt[2 + 5*x^2
+ 5*x^4])
```

Maple [C] time = 0.749, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-5/4+i/4\sqrt{15}\right)x^2}\sqrt{1-\left(-5/4-i/4\sqrt{15}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-5+i\sqrt{15}},1/2\,\sqrt{1+i\sqrt{15}}\right)}{\sqrt{-5+i\sqrt{15}}\sqrt{5\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+I*15^(1/2))^(1/2)*(1-(-5/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*I*
15^(1/2))*x^2)^(1/2)/(5*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+I*15^(1/2))^
(1/2),1/2*(1+I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^4 + 5\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{5\,x^4 + 5\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^4 + 5x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(5*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^4 + 5\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

3.121 $\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{4x^4+5x^2+2}}$$

```
[Out] ((1 + Sqrt[2]*x^2)*Sqrt[(2 + 5*x^2 + 4*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (8 - 5*Sqrt[2])/16])/(2*2^(3/4)*Sqrt[2 + 5*x^2 + 4*x^4
])
```

Rubi [A] time = 0.0189061, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{4x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 4*x^4],x]
```

```
[Out] ((1 + Sqrt[2]*x^2)*Sqrt[(2 + 5*x^2 + 4*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (8 - 5*Sqrt[2])/16])/(2*2^(3/4)*Sqrt[2 + 5*x^2 + 4*x^4
])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx = \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{2+5x^2+4x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\ 2^{3/4}\sqrt{2+5x^2+4x^4}}$$

Mathematica [C] time = 0.0882328, size = 147, normalized size = 1.63

$$-\frac{i\sqrt{1-\frac{8x^2}{-5-i\sqrt{7}}}\sqrt{1-\frac{8x^2}{-5+i\sqrt{7}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{-\frac{2}{-5-i\sqrt{7}}}x\right),\frac{-5-i\sqrt{7}}{-5+i\sqrt{7}}\right)}{2\sqrt{2}\sqrt{-\frac{1}{-5-i\sqrt{7}}}\sqrt{4x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 4*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (8*x^2)/(-5 - I*Sqrt[7])]*Sqrt[1 - (8*x^2)/(-5 + I*Sqrt[7]
)]*EllipticF[I*ArcSinh[2*Sqrt[-2/(-5 - I*Sqrt[7])]*x], (-5 - I*Sqrt[7])/(-5
 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-5 - I*Sqrt[7])^(-1)]*Sqrt[2 + 5*x^2 + 4*x^
4])
```

Maple [C] time = 0.743, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-5/4+i/4\sqrt{7}\right)x^2}\sqrt{1-\left(-5/4-i/4\sqrt{7}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-5+i\sqrt{7}},1/4\,\sqrt{9+5\,i\sqrt{7}}\right)}{\sqrt{-5+i\sqrt{7}}\sqrt{4\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+I*7^(1/2))^(1/2)*(1-(-5/4+1/4*I*7^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*I*7^
(1/2))*x^2)^(1/2)/(4*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+I*7^(1/2))^(1/2
),1/4*(9+5*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{4\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

3.122 $\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\tan^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] time = 0.0072609, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1100}

$$\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rule 1100

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b - q)*x^2)*Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]*EllipticF[ArcTan[Rt[(b - q)/(2*a), 2]*x], (-2*q)/(b - q)])/(2
*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx=\frac{\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

Mathematica [C] time = 0.020094, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{x^2+1}\sqrt{3x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{9x^4+15x^2+6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])
/Sqrt[6 + 15*x^2 + 9*x^4]
```

Maple [A] time = 0.046, size = 44, normalized size = 0.9

$$-\frac{i}{2}\sqrt{x^2+1}\sqrt{6\,x^2+4}\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{3\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/
2*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

3.123 $\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{\frac{x^2+2}{2x^2+1}}\left(2x^2+1\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{2}x\right),\frac{3}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

```
[Out] (Sqrt[(2 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 3/4])
/(2*Sqrt[2 + 5*x^2 + 2*x^4])
```

Rubi [A] time = 0.0071399, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1099}

$$\frac{\sqrt{\frac{x^2+2}{2x^2+1}}\left(2x^2+1\right)F\left(\tan^{-1}\left(\sqrt{2}x\right)|\frac{3}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(2 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 3/4])
/(2*Sqrt[2 + 5*x^2 + 2*x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx=\frac{\sqrt{\frac{2+x^2}{1+2x^2}}\left(1+2x^2\right)F\left(\tan^{-1}\left(\sqrt{2}x\right)|\frac{3}{4}\right)}{2\sqrt{2+5x^2+2x^4}}$$

Mathematica [C] time = 0.0224686, size = 58, normalized size = 1.

$$-\frac{i\sqrt{x^2+2}\sqrt{2x^2+1}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\right),\frac{1}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[2 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/4])
/Sqrt[2 + 5*x^2 + 2*x^4]
```

Maple [C] time = 0.053, size = 48, normalized size = 0.8

$$-\frac{i}{2}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},2\right)\sqrt{2x^2+4}\sqrt{2x^2+1}\frac{1}{\sqrt{2x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+5*x^2+2)^(1/2)*Ellipt
icF(1/2*I*x*2^(1/2),2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

3.124 $\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{\frac{\left(5-\sqrt{17}\right)x^2+4}{\left(5+\sqrt{17}\right)x^2+4}}\left(\left(5+\sqrt{17}\right)x^2+4\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}x\right),\frac{1}{4}\left(5\sqrt{17}-17\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{x^4+5x^2+2}}$$

```
[Out] (Sqrt[(4 + (5 - Sqrt[17])*x^2)/(4 + (5 + Sqrt[17])*x^2)]*(4 + (5 + Sqrt[17]
)*x^2)*EllipticF[ArcTan[(Sqrt[5 + Sqrt[17]]*x)/2], (-17 + 5*Sqrt[17])/4])/(
2*Sqrt[5 + Sqrt[17]]*Sqrt[2 + 5*x^2 + x^4])
```

Rubi [A] time = 0.0420829, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1099}

$$\frac{\sqrt{\frac{\left(5-\sqrt{17}\right)x^2+4}{\left(5+\sqrt{17}\right)x^2+4}}\left(\left(5+\sqrt{17}\right)x^2+4\right)F\left(\tan^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}x\right)|\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[(4 + (5 - Sqrt[17])*x^2)/(4 + (5 + Sqrt[17])*x^2)]*(4 + (5 + Sqrt[17]
)*x^2)*EllipticF[ArcTan[(Sqrt[5 + Sqrt[17]]*x)/2], (-17 + 5*Sqrt[17])/4])/(
2*Sqrt[5 + Sqrt[17]]*Sqrt[2 + 5*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx = \frac{\sqrt{\frac{4+\left(5-\sqrt{17}\right)x^2}{4+\left(5+\sqrt{17}\right)x^2}}\left(4+\left(5+\sqrt{17}\right)x^2\right)F\left(\tan^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}x\right)|\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{2+5x^2+x^4}}$$

Mathematica [C] time = 0.0818699, size = 103, normalized size = 0.95

$$-\frac{i\sqrt{2x^2-\sqrt{17}+5}\sqrt{2x^2+\sqrt{17}+5}\,\mathbf{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{17}}}x\right),\frac{21}{4}+\frac{5\sqrt{17}}{4}\right)}{\sqrt{2\left(5-\sqrt{17}\right)}\sqrt{x^4+5x^2+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[5 - Sqrt[17] + 2*x^2]*Sqrt[5 + Sqrt[17] + 2*x^2]*EllipticF[I*Arc
Sinh[Sqrt[2/(5 + Sqrt[17])]*x], 21/4 + (5*Sqrt[17])/4])/(Sqrt[2*(5 - Sqrt[1
7])]*Sqrt[2 + 5*x^2 + x^4])
```

Maple [A] time = 0.226, size = 76, normalized size = 0.7

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\,\sqrt{17}\right)x^2}\sqrt{1-\left(-5/4-1/4\,\sqrt{17}\right)x^2}\,\mathrm{EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{17}},5/4\,\sqrt{2}+1/4\,\sqrt{34}\right)}{\sqrt{-5+\sqrt{17}}\sqrt{x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+17^(1/2))^(1/2)*(1-(-5/4+1/4*17^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*17^(1/
2))*x^2)^(1/2)/(x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+17^(1/2))^(1/2),5/4*
2^(1/2)+1/4*34^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 5*x^2 + 2), x)
```

3.125 $\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{\sqrt{33}-5}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right),\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[2/(5 + Sqrt[33])]*x], (-29 -
5*Sqrt[33])/4]
```

Rubi [A] time = 0.0983492, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{33}-5}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right)|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[2/(5 + Sqrt[33])]*x], (-29 -
5*Sqrt[33])/4]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx = 2\int \frac{1}{\sqrt{5+\sqrt{33}-2x^2}\sqrt{-5+\sqrt{33}+2x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{33}}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right)|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

Mathematica [C] time = 0.0619445, size = 55, normalized size = 1.15

$$-i\sqrt{\frac{2}{5+\sqrt{33}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{\sqrt{33}-5}}x\right),\frac{5\sqrt{33}}{4}-\frac{29}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[2/(-5 + Sqrt[33])]*x],
 -29/4 + (5*Sqrt[33])/4]
```

Maple [B] time = 0.241, size = 80, normalized size = 1.7

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\,\sqrt{33}\right)x^2}\sqrt{1-\left(-5/4-1/4\,\sqrt{33}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{33}},5/4\,i\sqrt{2}+i/4\sqrt{66}\right)}{\sqrt{-5+\sqrt{33}}\sqrt{-x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+33^(1/2))^(1/2)*(1-(-5/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*33^(1/
2))*x^2)^(1/2)/(-x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+33^(1/2))^(1/2),5/4
*I*2^(1/2)+1/4*I*66^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+5x^2+2}}{x^4-5x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 5*x^2 + 2)/(x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

3.126 $\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{41}-5}}\text{EllipticF}\left(\sin^{-1}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right),\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[41])]*EllipticF[ArcSin[(2*x)/Sqrt[5 + Sqrt[41]]], (-33 -
5*Sqrt[41])/8]
```

Rubi [A] time = 0.062061, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{41}-5}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[41])]*EllipticF[ArcSin[(2*x)/Sqrt[5 + Sqrt[41]]], (-33 -
5*Sqrt[41])/8]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{5+\sqrt{41}-4x^2}\sqrt{-5+\sqrt{41}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{41}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

Mathematica [C] time = 0.0464176, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{5+\sqrt{41}}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{\sqrt{41}-5}}\right),\frac{5\sqrt{41}}{8}-\frac{33}{8}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[41])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-5 + Sqrt[41]]],
 -33/8 + (5*Sqrt[41])/8]
```

Maple [B] time = 0.228, size = 76, normalized size = 1.7

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\sqrt{41}\right)x^2}\sqrt{1-\left(-5/4-1/4\sqrt{41}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{41}},5/4\,i+i/4\sqrt{41}\right)}{\sqrt{-5+\sqrt{41}}\sqrt{-2\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+41^(1/2))^(1/2)*(1-(-5/4+1/4*41^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*41^(1/
2))*x^2)^(1/2)/(-2*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+41^(1/2))^(1/2),5
/4*I+1/4*I*41^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+5\,x^2+2}}{2\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 5*x^2 + 2)/(2*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

3.127 $\int \frac{1}{\sqrt{2+5x^2-3x^4}} \, dx$

Optimal. Leaf size=10

$$\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-6\right)$$

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Rubi [A] time = 0.011636, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{12-6x^2}\sqrt{2+6x^2}}\,dx$$
$$= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -6\right)$$

Mathematica [C] time = 0.023036, size = 65, normalized size = 6.5

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{3x^2+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{3}x\right),-\frac{1}{6}\right)}{\sqrt{3}\sqrt{-3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6]
)/(Sqrt[3]*Sqrt[2 + 5*x^2 - 3*x^4])
```

Maple [B] time = 0.046, size = 51, normalized size = 5.1

$$\frac{\sqrt{2}}{2}\sqrt{-2\,x^2+4}\sqrt{3\,x^2+1}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{6}\right)\frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*Ellipti
cF(1/2*x*2^(1/2),I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+5\,x^2+2}}{3\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 5*x^2 + 2)/(3*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

3.128 $\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{\sqrt{57}-5}}\text{EllipticF}\left(\sin^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right),\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[57])]*EllipticF[ArcSin[2*Sqrt[2/(5 + Sqrt[57])]*x], (-41
- 5*Sqrt[57])/16]
```

Rubi [A] time = 0.103751, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{57}-5}}F\left(\sin^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right)|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 4*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[57])]*EllipticF[ArcSin[2*Sqrt[2/(5 + Sqrt[57])]*x], (-41
- 5*Sqrt[57])/16]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx = 4\int \frac{1}{\sqrt{5+\sqrt{57}-8x^2}\sqrt{-5+\sqrt{57}+8x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{57}}}F\left(\sin^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right)|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

Mathematica [C] time = 0.0620758, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{57}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{\frac{2}{\sqrt{57}-5}}x\right),\frac{1}{16}\left(5\sqrt{57}-41\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 4*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[57])]*EllipticF[I*ArcSinh[2*Sqrt[2/(-5 + Sqrt[57])]*x
], (-41 + 5*Sqrt[57])/16]
```

Maple [B] time = 0.246, size = 80, normalized size = 1.6

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\sqrt{57}\right)x^2}\sqrt{1-\left(-5/4-1/4\sqrt{57}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{57}},5/8\,i\sqrt{2}+i/8\sqrt{114}\right)}{\sqrt{-5+\sqrt{57}}\sqrt{-4\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+57^(1/2))^(1/2)*(1-(-5/4+1/4*57^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*57^(1/
2))*x^2)^(1/2)/(-4*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+57^(1/2))^(1/2),5
/8*I*2^(1/2)+1/8*I*114^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-4\,x^4+5\,x^2+2}}{4\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*x^4 + 5*x^2 + 2)/(4*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-4x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-4*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

3.129 $\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{\sqrt{65}-5}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{10}{5+\sqrt{65}}}\,x\right),\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[65])]*EllipticF[ArcSin[Sqrt[10/(5 + Sqrt[65])]*x], (-9 -
Sqrt[65])/4]
```

Rubi [A] time = 0.10425, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{65}-5}}\,F\left(\sin^{-1}\left(\sqrt{\frac{10}{5+\sqrt{65}}}\,x\right)\middle|\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 5*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[65])]*EllipticF[ArcSin[Sqrt[10/(5 + Sqrt[65])]*x], (-9 -
Sqrt[65])/4]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx = \left(2\sqrt{5}\right)\int \frac{1}{\sqrt{5+\sqrt{65}-10x^2}\sqrt{-5+\sqrt{65}+10x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{65}}}F\left(\sin^{-1}\left(\sqrt{\frac{10}{5+\sqrt{65}}}x\right)\middle|\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

Mathematica [C] time = 0.0575499, size = 52, normalized size = 1.08

$$-i\sqrt{\frac{2}{5+\sqrt{65}}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{65}}x\right),\frac{1}{4}\left(\sqrt{65}-9\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 5*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[65])]*EllipticF[I*ArcSinh[(Sqrt[5 + Sqrt[65]]*x)/2],
(-9 + Sqrt[65])/4]
```

Maple [B] time = 0.24, size = 80, normalized size = 1.7

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\sqrt{65}\right)x^2}\sqrt{1-\left(-5/4-1/4\sqrt{65}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{65}},i/4\sqrt{10}+i/4\sqrt{26}\right)}{\sqrt{-5+\sqrt{65}}\sqrt{-5\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+65^(1/2))^(1/2)*(1-(-5/4+1/4*65^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*65^(1/
2))*x^2)^(1/2)/(-5*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+65^(1/2))^(1/2),1
/4*I*10^(1/2)+1/4*I*26^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-5\,x^4+5\,x^2+2}}{5\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-5*x^4 + 5*x^2 + 2)/(5*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-5x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-5*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-5\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

3.130 $\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{\sqrt{73}-5}}\,\text{EllipticF}\left(\sin^{-1}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right),\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[73])]*EllipticF[ArcSin[2*Sqrt[3/(5 + Sqrt[73])]*x], (-49
- 5*Sqrt[73])/24]
```

Rubi [A] time = 0.0782031, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{73}-5}}F\left(\sin^{-1}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 6*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[73])]*EllipticF[ArcSin[2*Sqrt[3/(5 + Sqrt[73])]*x], (-49
- 5*Sqrt[73])/24]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx = \left(2\sqrt{6}\right)\int \frac{1}{\sqrt{5+\sqrt{73}-12x^2}\sqrt{-5+\sqrt{73}+12x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{73}}}F\left(\sin^{-1}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

Mathematica [C] time = 0.0497594, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{73}}}\text{EllipticF}\left(i\sinh^{-1}\left(2\sqrt{\frac{3}{\sqrt{73}-5}}x\right),\frac{1}{24}\left(5\sqrt{73}-49\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 6*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[73])]*EllipticF[I*ArcSinh[2*Sqrt[3/(-5 + Sqrt[73])]*x
], (-49 + 5*Sqrt[73])/24]
```

Maple [B] time = 0.243, size = 80, normalized size = 1.6

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\sqrt{73}\right)x^2}\sqrt{1-\left(-5/4-1/4\sqrt{73}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{73}},\frac{5\,i}{12}\sqrt{3}+i/12\sqrt{219}\right)}{\sqrt{-5+\sqrt{73}}\sqrt{-6\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+73^(1/2))^(1/2)*(1-(-5/4+1/4*73^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*73^(1/
2))*x^2)^(1/2)/(-6*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+73^(1/2))^(1/2),5
/12*I*3^(1/2)+1/12*I*219^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-6\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-6\,x^4+5\,x^2+2}}{6\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-6*x^4 + 5*x^2 + 2)/(6*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-6x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-6*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-6\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

3.131 $\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-\frac{7}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -7/2]/Sqrt[2]
```

Rubi [A] time = 0.011929, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}(x)|-\frac{7}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 7*x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -7/2]/Sqrt[2]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx = \left(2\sqrt{7}\right)\int \frac{1}{\sqrt{14-14x^2}\sqrt{4+14x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}(x)\middle|-\frac{7}{2}\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.024769, size = 65, normalized size = 5.42

$$-\frac{i\sqrt{1-x^2}\sqrt{7x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{7}{2}}x\right),-\frac{2}{7}\right)}{\sqrt{7}\sqrt{-7x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 7*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 7*x^2]*EllipticF[I*ArcSinh[Sqrt[7/2]*x], -2/7]
)/(Sqrt[7]*Sqrt[2 + 5*x^2 - 7*x^4])
```

Maple [B] time = 0.053, size = 43, normalized size = 3.6

$$\frac{\text{EllipticF}\left(x,\frac{i}{2}\sqrt{14}\right)}{2}\sqrt{-x^2+1}\sqrt{14\,x^2+4}\frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-7*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(14*x^2+4)^(1/2)/(-7*x^4+5*x^2+2)^(1/2)*EllipticF(x,1/2*
I*14^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-7\,x^4+5\,x^2+2}}{7\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-7*x^4 + 5*x^2 + 2)/(7*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-7x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-7*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

3.132 $\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{89}-5}}\text{EllipticF}\left(\sin^{-1}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right),\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[89])]*EllipticF[ArcSin[(4*x)/Sqrt[5 + Sqrt[89]]], (-57 -
5*Sqrt[89])/32]
```

Rubi [A] time = 0.0696016, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{89}-5}}F\left(\sin^{-1}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 8*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[89])]*EllipticF[ArcSin[(4*x)/Sqrt[5 + Sqrt[89]]], (-57 -
5*Sqrt[89])/32]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx = \left(4\sqrt{2}\right)\int \frac{1}{\sqrt{5+\sqrt{89}-16x^2}\sqrt{-5+\sqrt{89}+16x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{89}}}F\left(\sin^{-1}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

Mathematica [C] time = 0.0428417, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{5+\sqrt{89}}}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{4x}{\sqrt{\sqrt{89}-5}}\right),\frac{1}{32}\left(5\sqrt{89}-57\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 8*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[89])]*EllipticF[I*ArcSinh[(4*x)/Sqrt[-5 + Sqrt[89]]],
 (-57 + 5*Sqrt[89])/32]
```

Maple [B] time = 0.229, size = 76, normalized size = 1.7

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\sqrt{89}\right)x^2}\sqrt{1-\left(-5/4-1/4\sqrt{89}\right)x^2}\text{EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{89}},5/8\,i+i/8\sqrt{89}\right)}{\sqrt{-5+\sqrt{89}}\sqrt{-8\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-8*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+89^(1/2))^(1/2)*(1-(-5/4+1/4*89^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*89^(1/
2))*x^2)^(1/2)/(-8*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+89^(1/2))^(1/2),5
/8*I+1/8*I*89^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-8\,x^4+5\,x^2+2}}{8\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-8*x^4 + 5*x^2 + 2)/(8*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-8x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-8*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

3.133 $\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{\sqrt{97}-5}}\text{EllipticF}\left(\sin^{-1}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right),\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[97])]*EllipticF[ArcSin[3*Sqrt[2/(5 + Sqrt[97])]*x], (-61
- 5*Sqrt[97])/36]
```

Rubi [A] time = 0.0785738, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\sqrt{\frac{2}{\sqrt{97}-5}}F\left(\sin^{-1}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right)|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 9*x^4],x]
```


```
[Out] Sqrt[2/(-5 + Sqrt[97])]*EllipticF[ArcSin[3*Sqrt[2/(5 + Sqrt[97])]*x], (-61
- 5*Sqrt[97])/36]
```


Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx = 6\int \frac{1}{\sqrt{5+\sqrt{97}-18x^2}\sqrt{-5+\sqrt{97}+18x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{97}}}F\left(\sin^{-1}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right)\middle|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

Mathematica [C] time = 0.0472162, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{97}}}\text{EllipticF}\left(i\sinh^{-1}\left(3\sqrt{\frac{2}{\sqrt{97}-5}}x\right),\frac{1}{36}\left(5\sqrt{97}-61\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 9*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[97])]*EllipticF[I*ArcSinh[3*Sqrt[2/(-5 + Sqrt[97])]*x
], (-61 + 5*Sqrt[97])/36]
```

Maple [B] time = 0.237, size = 80, normalized size = 1.6

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\sqrt{97}\right)x^2}\sqrt{1-\left(-5/4-1/4\sqrt{97}\right)x^2}\,\text{EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{97}},\frac{5i}{12}\sqrt{2}+i/12\sqrt{194}\right)}{\sqrt{-5+\sqrt{97}}\sqrt{-9\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+97^(1/2))^(1/2)*(1-(-5/4+1/4*97^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*97^(1/
2))*x^2)^(1/2)/(-9*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+97^(1/2))^(1/2),5
/12*I*2^(1/2)+1/12*I*194^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-9\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-9\,x^4+5\,x^2+2}}{9\,x^4-5\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-9*x^4 + 5*x^2 + 2)/(9*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-9x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-9*x**4 + 5*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-9\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

3.134 $\int x^2 \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=17

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

Rubi [A] time = 0.0058893, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(bx^2+cx^4\right) dx &= \int \left(bx^4+cx^6\right) dx \\ &= \frac{bx^5}{5}+\frac{cx^7}{7} \end{aligned}$$

Mathematica [A] time = 0.0017171, size = 17, normalized size = 1.

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

Maple [A] time = 0.043, size = 14, normalized size = 0.8

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2),x)
```

```
[Out] 1/5*b*x^5+1/7*c*x^7
```

Maxima [A] time = 0.986232, size = 18, normalized size = 1.06

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

Fricas [A] time = 1.08648, size = 31, normalized size = 1.82

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c + 1/5*x^5*b
```

Sympy [A] time = 0.066037, size = 12, normalized size = 0.71

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2),x)
```

```
[Out] b*x**5/5 + c*x**7/7
```

Giac [A] time = 1.24423, size = 18, normalized size = 1.06

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

3.135 $\int x (bx^2 + cx^4) \, dx$

Optimal. Leaf size=17

$$\frac{bx^4}{4} + \frac{cx^6}{6}$$

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

Rubi [A] time = 0.0056222, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x (bx^2 + cx^4) \, dx &= \int (bx^3 + cx^5) \, dx \\ &= \frac{bx^4}{4} + \frac{cx^6}{6} \end{aligned}$$

Mathematica [A] time = 0.0012668, size = 17, normalized size = 1.

$$\frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

Maple [A] time = 0.04, size = 14, normalized size = 0.8

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2),x)
```

```
[Out] 1/4*b*x^4+1/6*c*x^6
```

Maxima [A] time = 0.944011, size = 18, normalized size = 1.06

$$\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

Fricas [A] time = 1.00956, size = 31, normalized size = 1.82

$$\frac{1}{6}x^6c+\frac{1}{4}x^4b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c + 1/4*x^4*b
```

Sympy [A] time = 0.064077, size = 12, normalized size = 0.71

$$\frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2),x)
```

```
[Out] b*x**4/4 + c*x**6/6
```

Giac [A] time = 1.29331, size = 18, normalized size = 1.06

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

3.136 $\int (bx^2 + cx^4) \, dx$

Optimal. Leaf size=17

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

```
[Out] (b*x^3)/3 + (c*x^5)/5
```

Rubi [A] time = 0.0030681, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[b*x^2 + c*x^4,x]
```

```
[Out] (b*x^3)/3 + (c*x^5)/5
```

Rubi steps

$$\int (bx^2 + cx^4) \, dx = \frac{bx^3}{3} + \frac{cx^5}{5}$$

Mathematica [A] time = 0.000033, size = 17, normalized size = 1.

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x^2 + c*x^4,x]
```

```
[Out] (b*x^3)/3 + (c*x^5)/5
```

Maple [A] time = 0.042, size = 14, normalized size = 0.8

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^2,x)
```

```
[Out] 1/3*b*x^3+1/5*c*x^5
```

Maxima [A] time = 0.955986, size = 18, normalized size = 1.06

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

Fricas [A] time = 1.09512, size = 31, normalized size = 1.82

$$\frac{1}{5}x^5c + \frac{1}{3}x^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c + 1/3*x^3*b
```

Sympy [A] time = 0.074365, size = 12, normalized size = 0.71

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**2,x)
```

```
[Out] b*x**3/3 + c*x**5/5
```

Giac [A] time = 1.21641, size = 18, normalized size = 1.06

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

3.137 $\int \frac{bx^2+cx^4}{x} dx$

Optimal. Leaf size=17

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

Rubi [A] time = 0.0049654, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2 + cx^4}{x} dx &= \int \left(bx + cx^3\right) dx\\ &= \frac{bx^2}{2} + \frac{cx^4}{4}\end{aligned}$$

Mathematica [A] time = 0.0012194, size = 17, normalized size = 1.

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

Maple [A] time = 0.042, size = 14, normalized size = 0.8

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x,x)
```

```
[Out] 1/2*b*x^2+1/4*c*x^4
```

Maxima [A] time = 0.963044, size = 18, normalized size = 1.06

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

Fricas [A] time = 1.20785, size = 31, normalized size = 1.82

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

Sympy [A] time = 0.058903, size = 12, normalized size = 0.71

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x,x)
```

```
[Out] b*x**2/2 + c*x**4/4
```

Giac [A] time = 1.26469, size = 18, normalized size = 1.06

$$\frac{1}{4}\,cx^4 + \frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

3.138 $\int \frac{bx^2+cx^4}{x^2} \, dx$

Optimal. Leaf size=12

$$bx + \frac{cx^3}{3}$$

```
[Out] b*x + (c*x^3)/3
```

Rubi [A] time = 0.0044172, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^2,x]
```

```
[Out] b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2 + cx^4}{x^2} \, dx &= \int \left(b + cx^2\right) \, dx \\ &= bx + \frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.0003584, size = 12, normalized size = 1.

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^2,x]
```

```
[Out] b*x + (c*x^3)/3
```

Maple [A] time = 0.041, size = 11, normalized size = 0.9

$$bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^2,x)
```

```
[Out] b*x+1/3*c*x^3
```

Maxima [A] time = 0.96898, size = 14, normalized size = 1.17

$$\frac{1}{3}cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + b*x
```

Fricas [A] time = 1.11098, size = 23, normalized size = 1.92

$$\frac{1}{3}cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*c*x^3 + b*x
```

Sympy [A] time = 0.062623, size = 8, normalized size = 0.67

$$bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**2,x)
```

```
[Out] b*x + c*x**3/3
```

Giac [A] time = 1.25198, size = 14, normalized size = 1.17

$$\frac{1}{3}\,cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x
```

3.139 $\int \frac{bx^2+cx^4}{x^3} \, dx$

Optimal. Leaf size=13

$$b\log(x) + \frac{cx^2}{2}$$

```
[Out] (c*x^2)/2 + b*Log[x]
```

Rubi [A] time = 0.0053259, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$b\log(x) + \frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^3,x]
```

```
[Out] (c*x^2)/2 + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2 + cx^4}{x^3} \, dx &= \int \left(\frac{b}{x} + cx\right) dx \\ &= \frac{cx^2}{2} + b\log(x) \end{aligned}$$

Mathematica [A] time = 0.0011614, size = 13, normalized size = 1.

$$b\log(x) + \frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^3,x]
```

```
[Out] (c*x^2)/2 + b*Log[x]
```

Maple [A] time = 0.043, size = 12, normalized size = 0.9

$$\frac{cx^2}{2} + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^3,x)
```

```
[Out] 1/2*c*x^2+b*ln(x)
```

Maxima [A] time = 0.97259, size = 19, normalized size = 1.46

$$\frac{1}{2}cx^2 + \frac{1}{2}b\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2 + 1/2*b*log(x^2)
```

Fricas [A] time = 1.27428, size = 30, normalized size = 2.31

$$\frac{1}{2}cx^2 + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*c*x^2 + b*log(x)
```

Sympy [A] time = 0.088606, size = 10, normalized size = 0.77

$$b\log(x) + \frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**3,x)
```

```
[Out] b*log(x) + c*x**2/2
```

Giac [A] time = 1.15788, size = 19, normalized size = 1.46

$$\frac{1}{2}\,cx^2 + \frac{1}{2}\,b\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*c*x^2 + 1/2*b*log(x^2)
```

3.140 $\int \frac{bx^2+cx^4}{x^4} dx$

Optimal. Leaf size=10

$$cx - \frac{b}{x}$$

```
[Out] -(b/x) + c*x
```

Rubi [A] time = 0.0052681, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$cx - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -(b/x) + c*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2 + cx^4}{x^4} dx &= \int \left(c + \frac{b}{x^2}\right) dx \\ &= -\frac{b}{x} + cx\end{aligned}$$

Mathematica [A] time = 0.0008532, size = 10, normalized size = 1.

$$cx - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -(b/x) + c*x
```

Maple [A] time = 0.045, size = 11, normalized size = 1.1

$$-\frac{b}{x}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^4,x)
```

```
[Out] -b/x+c*x
```

Maxima [A] time = 1.00607, size = 14, normalized size = 1.4

$$cx-\frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] c*x - b/x
```

Fricas [A] time = 1.23142, size = 20, normalized size = 2.

$$\frac{cx^2-b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] (c*x^2 - b)/x
```

Sympy [A] time = 0.253475, size = 5, normalized size = 0.5

$$-\frac{b}{x} + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**4,x)
```

```
[Out] -b/x + c*x
```

Giac [A] time = 1.12271, size = 14, normalized size = 1.4

$$cx - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="giac")
```

```
[Out] c*x - b/x
```

3.141 $\int \frac{bx^2+cx^4}{x^5}\,dx$

Optimal. Leaf size=13

$$c\log(x) - \frac{b}{2x^2}$$

```
[Out] -b/(2*x^2) + c*Log[x]
```

Rubi [A] time = 0.00563, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$c\log(x) - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -b/(2*x^2) + c*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^5}\,dx &= \int \left(\frac{b}{x^3}+\frac{c}{x}\right)dx\\ &= -\frac{b}{2x^2}+c\log(x)\end{aligned}$$

Mathematica [A] time = 0.0023213, size = 13, normalized size = 1.

$$c\log(x) - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -b/(2*x^2) + c*Log[x]
```

Maple [A] time = 0.046, size = 12, normalized size = 0.9

$$-\frac{b}{2\,x^2}+c\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^5,x)
```

```
[Out] -1/2/x^2*b+c*ln(x)
```

Maxima [A] time = 1.03018, size = 19, normalized size = 1.46

$$\frac{1}{2}\,c\log\left(x^2\right)-\frac{b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2) - 1/2*b/x^2
```

Fricas [A] time = 1.25098, size = 41, normalized size = 3.15

$$\frac{2\,cx^2\log\left(x\right)-b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(2*c*x^2*log(x) - b)/x^2
```

Sympy [A] time = 0.290268, size = 10, normalized size = 0.77

$$-\frac{b}{2x^2} + c\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**5,x)
```

```
[Out] -b/(2*x**2) + c*log(x)
```

Giac [A] time = 1.25866, size = 27, normalized size = 2.08

$$\frac{1}{2}\,c\log\left(x^2\right) - \frac{cx^2 + b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2) - 1/2*(c*x^2 + b)/x^2
```

3.142 $\int \frac{bx^2+cx^4}{x^6} dx$

Optimal. Leaf size=15

$$-\frac{b}{3x^3}-\frac{c}{x}$$

[Out] -b/(3*x^3) - c/x

Rubi [A] time = 0.0056849, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)/x^6,x]

[Out] -b/(3*x^3) - c/x

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^6}\,dx &= \int \left(\frac{b}{x^4}+\frac{c}{x^2}\right)dx\\ &= -\frac{b}{3x^3}-\frac{c}{x}\end{aligned}$$

Mathematica [A] time = 0.0020695, size = 15, normalized size = 1.

$$-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -b/(3*x^3) - c/x
```

Maple [A] time = 0.046, size = 14, normalized size = 0.9

$$-\frac{b}{3x^3}-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^6,x)
```

```
[Out] -1/3*b/x^3-c/x
```

Maxima [A] time = 0.974695, size = 18, normalized size = 1.2

$$-\frac{3\,cx^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/3*(3*c*x^2 + b)/x^3
```

Fricas [A] time = 1.21469, size = 32, normalized size = 2.13

$$-\frac{3\,cx^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/3*(3*c*x^2 + b)/x^3
```

Sympy [A] time = 0.298958, size = 14, normalized size = 0.93

$$-\frac{b + 3cx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**6,x)
```

```
[Out] -(b + 3*c*x**2)/(3*x**3)
```

Giac [A] time = 1.30789, size = 18, normalized size = 1.2

$$-\frac{3\,cx^2 + b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="giac")
```

```
[Out] -1/3*(3*c*x^2 + b)/x^3
```

3.143 $\int \frac{bx^2+cx^4}{x^7}\,dx$

Optimal. Leaf size=17

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

```
[Out] -b/(4*x^4) - c/(2*x^2)
```

Rubi [A] time = 0.0060832, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -b/(4*x^4) - c/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^7}\,dx &= \int \left(\frac{b}{x^5}+\frac{c}{x^3}\right)dx\\ &= -\frac{b}{4x^4}-\frac{c}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.002192, size = 17, normalized size = 1.

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -b/(4*x^4) - c/(2*x^2)
```

Maple [A] time = 0.046, size = 14, normalized size = 0.8

$$-\frac{b}{4\,x^4}-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^7,x)
```

```
[Out] -1/4*b/x^4-1/2*c/x^2
```

Maxima [A] time = 0.980625, size = 18, normalized size = 1.06

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

Fricas [A] time = 1.26226, size = 32, normalized size = 1.88

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

Sympy [A] time = 0.331721, size = 14, normalized size = 0.82

$$-\frac{b+2cx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**7,x)
```

```
[Out] -(b + 2*c*x**2)/(4*x**4)
```

Giac [A] time = 1.30271, size = 18, normalized size = 1.06

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="giac")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

3.144 $\int \frac{bx^2+cx^4}{x^8} \, dx$

Optimal. Leaf size=17

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

```
[Out] -b/(5*x^5) - c/(3*x^3)
```

Rubi [A] time = 0.0056135, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^8,x]

[Out] -b/(5*x^5) - c/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2+cx^4}{x^8} \, dx &= \int \left(\frac{b}{x^6}+\frac{c}{x^4}\right) \, dx \\ &= -\frac{b}{5x^5}-\frac{c}{3x^3} \end{aligned}$$

Mathematica [A] time = 0.0022654, size = 17, normalized size = 1.

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -b/(5*x^5) - c/(3*x^3)
```

Maple [A] time = 0.046, size = 14, normalized size = 0.8

$$-\frac{b}{5\,x^5}-\frac{c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^8,x)
```

```
[Out] -1/5*b/x^5-1/3*c/x^3
```

Maxima [A] time = 0.993127, size = 20, normalized size = 1.18

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

Fricas [A] time = 1.22487, size = 36, normalized size = 2.12

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

Sympy [A] time = 0.349806, size = 15, normalized size = 0.88

$$-\frac{3b + 5cx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**8,x)
```

```
[Out] -(3*b + 5*c*x**2)/(15*x**5)
```

Giac [A] time = 1.26271, size = 20, normalized size = 1.18

$$-\frac{5\,cx^2 + 3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="giac")
```

```
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

3.145 $\int \left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=30

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rubi [A] time = 0.0131603, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 270}

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(bx^2+cx^4\right)^2\,dx &= \int x^4\left(b+cx^2\right)^2\,dx\\ &= \int \left(b^2x^4+2bcx^6+c^2x^8\right)dx\\ &= \frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0012875, size = 30, normalized size = 1.

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A] time = 0.043, size = 25, normalized size = 0.8

$$\frac{b^2x^5}{5}+\frac{2\,bcx^7}{7}+\frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2,x)
```

```
[Out] 1/5*b^2*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A] time = 0.970388, size = 32, normalized size = 1.07

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

Fricas [A] time = 1.06599, size = 55, normalized size = 1.83

$$\frac{1}{9}x^9c^2 + \frac{2}{7}x^7cb + \frac{1}{5}x^5b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2
```

Sympy [A] time = 0.065676, size = 26, normalized size = 0.87

$$\frac{b^2x^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2,x)
```

```
[Out] b**2*x**5/5 + 2*b*c*x**7/7 + c**2*x**9/9
```

Giac [A] time = 1.27854, size = 32, normalized size = 1.07

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

3.146 $\int \frac{(bx^2+cx^4)^2}{x}\,dx$

Optimal. Leaf size=30

$$\frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

```
[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Rubi [A] time = 0.0253981, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x,x]
```

```
[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^2}{x}\,dx &= \int x^3\left(b+cx^2\right)^2\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int x(b+cx)^2\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(b^2x+2bcx^2+c^2x^3\right)\,dx, x, x^2\right)\\
&= \frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}
\end{aligned}
$$

Mathematica [A] time = 0.0008755, size = 30, normalized size = 1.

$$\frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x,x]
```

```
[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Maple [A] time = 0.043, size = 25, normalized size = 0.8

$$\frac{b^2x^4}{4}+\frac{bcx^6}{3}+\frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x,x)
```

```
[Out] 1/4*b^2*x^4+1/3*b*c*x^6+1/8*c^2*x^8
```

Maxima [A] time = 0.984127, size = 32, normalized size = 1.07

$$\frac{1}{8}\,c^2x^8+\frac{1}{3}\,bcx^6+\frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

Fricas [A] time = 1.25871, size = 55, normalized size = 1.83

$$\frac{1}{8}\,c^2x^8 + \frac{1}{3}\,bcx^6 + \frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

Sympy [A] time = 0.077763, size = 24, normalized size = 0.8

$$\frac{b^2x^4}{4} + \frac{bcx^6}{3} + \frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x,x)
```

```
[Out] b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8
```

Giac [A] time = 1.26989, size = 32, normalized size = 1.07

$$\frac{1}{8}\,c^2x^8 + \frac{1}{3}\,bcx^6 + \frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

3.147 $\int \frac{(bx^2+cx^4)^2}{x^2} \, dx$

Optimal. Leaf size=30

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

```
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rubi [A] time = 0.0172618, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^2}\,dx = \int x^2\left(b+cx^2\right)^2\,dx$$
$$= \int \left(b^2x^2+2bcx^4+c^2x^6\right)\,dx$$
$$= \frac{b^2x^3}{3}+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Mathematica [A] time = 0.000866, size = 30, normalized size = 1.

$$\frac{b^2x^3}{3}+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Maple [A] time = 0.04, size = 25, normalized size = 0.8

$$\frac{b^2x^3}{3}+\frac{2\,bcx^5}{5}+\frac{c^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^2,x)
```

```
[Out] 1/3*b^2*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

Maxima [A] time = 0.985917, size = 32, normalized size = 1.07

$$\frac{1}{7}\,c^2x^7+\frac{2}{5}\,bcx^5+\frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

Fricas [A] time = 1.20079, size = 55, normalized size = 1.83

$$\frac{1}{7}c^2x^7 + \frac{2}{5}bcx^5 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

Sympy [A] time = 0.070299, size = 26, normalized size = 0.87

$$\frac{b^2x^3}{3} + \frac{2bcx^5}{5} + \frac{c^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**2,x)
```

```
[Out] b**2*x**3/3 + 2*b*c*x**5/5 + c**2*x**7/7
```

Giac [A] time = 1.2573, size = 32, normalized size = 1.07

$$\frac{1}{7}c^2x^7 + \frac{2}{5}bcx^5 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

3.148 $\int \frac{(bx^2+cx^4)^2}{x^3}\,dx$

Optimal. Leaf size=16

$$\frac{(b+cx^2)^3}{6c}$$

```
[Out] (b + c*x^2)^3/(6*c)
```

Rubi [A] time = 0.0086475, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$\frac{(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] (b + c*x^2)^3/(6*c)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^3}\,dx = \int x\left(b+cx^2\right)^2\,dx$$
$$= \frac{\left(b+cx^2\right)^3}{6c}$$

Mathematica [A] time = 0.0023698, size = 16, normalized size = 1.

$$\frac{\left(b+cx^2\right)^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] (b + c*x^2)^3/(6*c)
```

Maple [A] time = 0.041, size = 25, normalized size = 1.6

$$\frac{c^2x^6}{6}+\frac{bcx^4}{2}+\frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^3,x)
```

```
[Out] 1/6*c^2*x^6+1/2*b*c*x^4+1/2*b^2*x^2
```

Maxima [A] time = 0.983297, size = 32, normalized size = 2.

$$\frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\,b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

Fricas [A] time = 1.20554, size = 55, normalized size = 3.44

$$\frac{1}{6}c^2x^6 + \frac{1}{2}bcx^4 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

Sympy [B] time = 0.067287, size = 24, normalized size = 1.5

$$\frac{b^2x^2}{2} + \frac{bcx^4}{2} + \frac{c^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**3,x)
```

```
[Out] b**2*x**2/2 + b*c*x**4/2 + c**2*x**6/6
```

Giac [A] time = 1.31295, size = 32, normalized size = 2.

$$\frac{1}{6}c^2x^6 + \frac{1}{2}bcx^4 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

3.149 $\int \frac{(bx^2+cx^4)^2}{x^4}\,dx$

Optimal. Leaf size=25

$$b^2x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

```
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rubi [A] time = 0.0131654, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 194}

$$b^2x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^4}\,dx = \int (b+cx^2)^2\,dx$$
$$= \int (b^2+2bcx^2+c^2x^4)\,dx$$
$$= b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

Mathematica [A] time = 0.0014582, size = 25, normalized size = 1.

$$b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Maple [A] time = 0.043, size = 22, normalized size = 0.9

$$b^2x + \frac{2\,bc x^3}{3} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^4,x)
```

```
[Out] b^2*x+2/3*b*c*x^3+1/5*c^2*x^5
```

Maxima [A] time = 0.989572, size = 28, normalized size = 1.12

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

Fricas [A] time = 1.15821, size = 47, normalized size = 1.88

$$\frac{1}{5}c^2x^5 + \frac{2}{3}bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

Sympy [A] time = 0.065741, size = 22, normalized size = 0.88

$$b^2x + \frac{2bcx^3}{3} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**4,x)
```

```
[Out] b**2*x + 2*b*c*x**3/3 + c**2*x**5/5
```

Giac [A] time = 1.19576, size = 28, normalized size = 1.12

$$\frac{1}{5}c^2x^5 + \frac{2}{3}bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

3.150 $\int \frac{(bx^2+cx^4)^2}{x^5} \, dx$

Optimal. Leaf size=23

$$b^2 \log(x) + bcx^2 + \frac{c^2x^4}{4}$$

```
[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

Rubi [A] time = 0.0186775, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$b^2 \log(x) + bcx^2 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^5}\,dx=\int \frac{\left(b+cx^2\right)^2}{x}\,dx$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{(b+cx)^2}{x}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \left(2bc+\frac{b^2}{x}+c^2x\right)dx,x,x^2\right)$$
$$=bcx^2+\frac{c^2x^4}{4}+b^2\log(x)$$

Mathematica [A] time = 0.0008704, size = 23, normalized size = 1.

$$b^2\log(x)+bcx^2+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

Maple [A] time = 0.043, size = 22, normalized size = 1.

$$bcx^2+\frac{c^2x^4}{4}+b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^5,x)
```

```
[Out] b*c*x^2+1/4*c^2*x^4+b^2*ln(x)
```

Maxima [A] time = 0.961073, size = 32, normalized size = 1.39

$$\frac{1}{4}\,c^2x^4+bcx^2+\frac{1}{2}\,b^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

Fricas [A] time = 1.27176, size = 49, normalized size = 2.13

$$\frac{1}{4}c^2x^4 + bcx^2 + b^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + b^2*log(x)
```

Sympy [A] time = 0.267114, size = 20, normalized size = 0.87

$$b^2\log(x) + bcx^2 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**5,x)
```

```
[Out] b**2*log(x) + b*c*x**2 + c**2*x**4/4
```

Giac [A] time = 1.26192, size = 32, normalized size = 1.39

$$\frac{1}{4}c^2x^4 + bcx^2 + \frac{1}{2}b^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

3.151 $\int \frac{(bx^2+cx^4)^2}{x^6} dx$

Optimal. Leaf size=24

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

```
[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Rubi [A] time = 0.0167236, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^6,x]

[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^6}\,dx = \int \frac{(b+cx^2)^2}{x^2}\,dx$$
$$= \int \left(2bc+\frac{b^2}{x^2}+c^2x^2\right)dx$$
$$= -\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

Mathematica [A] time = 0.0006806, size = 24, normalized size = 1.

$$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Maple [A] time = 0.046, size = 23, normalized size = 1.

$$-\frac{b^2}{x}+2\,bcx+\frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^6,x)
```

```
[Out] -b^2/x+2*b*c*x+1/3*c^2*x^3
```

Maxima [A] time = 0.963049, size = 30, normalized size = 1.25

$$\frac{1}{3}\,c^2x^3+2\,bcx-\frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

Fricas [A] time = 1.21534, size = 50, normalized size = 2.08

$$\frac{c^2x^4 + 6\,bcx^2 - 3\,b^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/3*(c^2*x^4 + 6*b*c*x^2 - 3*b^2)/x
```

Sympy [A] time = 0.265041, size = 19, normalized size = 0.79

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**6,x)
```

```
[Out] -b**2/x + 2*b*c*x + c**2*x**3/3
```

Giac [A] time = 1.25199, size = 30, normalized size = 1.25

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

3.152 $\int \frac{\left(bx^2+cx^4\right)^2}{x^7}\,dx$

Optimal. Leaf size=27

$$-\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}$$

```
[Out] -b^2/(2*x^2) + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Rubi [A] time = 0.0211402, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -b^2/(2*x^2) + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^2}{x^7}\,dx &= \int \frac{\left(b+cx^2\right)^2}{x^3}\,dx\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(b+cx)^2}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(c^2+\frac{b^2}{x^2}+\frac{2bc}{x}\right)dx,x,x^2\right)\\ &= -\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)\end{aligned}$$

Mathematica [A] time = 0.0009015, size = 27, normalized size = 1.

$$-\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -b^2/(2*x^2) + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Maple [A] time = 0.048, size = 24, normalized size = 0.9

$$-\frac{b^2}{2\,x^2}+\frac{c^2x^2}{2}+2\,bc\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^7,x)
```

```
[Out] -1/2*b^2/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Maxima [A] time = 0.992043, size = 32, normalized size = 1.19

$$\frac{1}{2}\,c^2x^2+bc\log\left(x^2\right)-\frac{b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/2*b^2/x^2
```

Fricas [A] time = 1.24135, size = 59, normalized size = 2.19

$$\frac{c^2x^4 + 4\,bcx^2\log\left(x\right) - b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^4 + 4*b*c*x^2*log(x) - b^2)/x^2
```

Sympy [A] time = 0.321896, size = 24, normalized size = 0.89

$$-\frac{b^2}{2x^2} + 2bc\log\left(x\right) + \frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**7,x)
```

```
[Out] -b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2
```

Giac [A] time = 1.23333, size = 43, normalized size = 1.59

$$\frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{2\,bcx^2 + b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/2*(2*b*c*x^2 + b^2)/x^2
```

3.153 $\int \frac{(bx^2+cx^4)^2}{x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

```
[Out] -b^2/(3*x^3) - (2*b*c)/x + c^2*x
```

Rubi [A] time = 0.0180535, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -b^2/(3*x^3) - (2*b*c)/x + c^2*x
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^8}\,dx = \int \frac{(b+cx^2)^2}{x^4}\,dx$$
$$= \int \left(c^2+\frac{b^2}{x^4}+\frac{2bc}{x^2}\right) dx$$
$$= -\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

Mathematica [A] time = 0.0008095, size = 23, normalized size = 1.

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -b^2/(3*x^3) - (2*b*c)/x + c^2*x
```

Maple [A] time = 0.046, size = 22, normalized size = 1.

$$-\frac{b^2}{3\,x^3}-2\,\frac{bc}{x}+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^8,x)
```

```
[Out] -1/3*b^2/x^3-2*b*c/x+c^2*x
```

Maxima [A] time = 1.00879, size = 30, normalized size = 1.3

$$c^2x-\frac{6\,bcx^2+b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

Fricas [A] time = 1.22724, size = 53, normalized size = 2.3

$$\frac{3\,c^2x^4 - 6\,bcx^2 - b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^2*x^4 - 6*b*c*x^2 - b^2)/x^3
```

Sympy [A] time = 0.311306, size = 20, normalized size = 0.87

$$c^2x - \frac{b^2 + 6bcx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**8,x)
```

```
[Out] c**2*x - (b**2 + 6*b*c*x**2)/(3*x**3)
```

Giac [A] time = 1.27357, size = 30, normalized size = 1.3

$$c^2x - \frac{6\,bcx^2 + b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="giac")
```

```
[Out] c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

3.154 $\int \frac{(bx^2+cx^4)^2}{x^9} dx$

Optimal. Leaf size=24

$$-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

```
[Out] -b^2/(4*x^4) - (b*c)/x^2 + c^2*Log[x]
```

Rubi [A] time = 0.0185294, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^9,x]

[Out] -b^2/(4*x^4) - (b*c)/x^2 + c^2*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^2}{x^9}\,dx &= \int \frac{\left(b+cx^2\right)^2}{x^5}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^2}{x^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2bc}{x^2}+\frac{c^2}{x}\right)dx,x,x^2\right)\\ &= -\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0007747, size = 24, normalized size = 1.

$$-\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -b^2/(4*x^4) - (b*c)/x^2 + c^2*Log[x]
```

Maple [A] time = 0.047, size = 23, normalized size = 1.

$$-\frac{b^2}{4\,x^4}-\frac{bc}{x^2}+c^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^9,x)
```

```
[Out] -1/4*b^2/x^4-b*c/x^2+c^2*ln(x)
```

Maxima [A] time = 0.982372, size = 35, normalized size = 1.46

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{4\,bcx^2+b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2) - 1/4*(4*b*c*x^2 + b^2)/x^4
```

Fricas [A] time = 1.20648, size = 62, normalized size = 2.58

$$\frac{4\,c^2x^4\log\left(x\right)-4\,bcx^2-b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(4*c^2*x^4*log(x) - 4*b*c*x^2 - b^2)/x^4
```

Sympy [A] time = 0.356161, size = 22, normalized size = 0.92

$$c^2\log\left(x\right)-\frac{b^2+4bcx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**9,x)
```

```
[Out] c**2*log(x) - (b**2 + 4*b*c*x**2)/(4*x**4)
```

Giac [A] time = 1.32315, size = 46, normalized size = 1.92

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{3\,c^2x^4+4\,bcx^2+b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2) - 1/4*(3*c^2*x^4 + 4*b*c*x^2 + b^2)/x^4
```

3.155 $\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=28

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

```
[Out] -b^2/(5*x^5) - (2*b*c)/(3*x^3) - c^2/x
```

Rubi [A] time = 0.0164468, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -b^2/(5*x^5) - (2*b*c)/(3*x^3) - c^2/x
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx = \int \frac{(b+cx^2)^2}{x^6}\,dx$$
$$= \int \left(\frac{b^2}{x^6}+\frac{2bc}{x^4}+\frac{c^2}{x^2}\right)dx$$
$$= -\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Mathematica [A] time = 0.0008079, size = 28, normalized size = 1.

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -b^2/(5*x^5) - (2*b*c)/(3*x^3) - c^2/x
```

Maple [A] time = 0.049, size = 25, normalized size = 0.9

$$-\frac{b^2}{5\,x^5}-\frac{2\,bc}{3\,x^3}-\frac{c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^10,x)
```

```
[Out] -1/5*b^2/x^5-2/3*b*c/x^3-c^2/x
```

Maxima [A] time = 0.977015, size = 35, normalized size = 1.25

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

Fricas [A] time = 1.19844, size = 61, normalized size = 2.18

$$-\frac{15\,c^2x^4 + 10\,bcx^2 + 3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

Sympy [A] time = 0.358796, size = 27, normalized size = 0.96

$$-\frac{3b^2 + 10bcx^2 + 15c^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**10,x)
```

```
[Out] -(3*b**2 + 10*b*c*x**2 + 15*c**2*x**4)/(15*x**5)
```

Giac [A] time = 1.32373, size = 35, normalized size = 1.25

$$-\frac{15\,c^2x^4 + 10\,bcx^2 + 3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

3.156 $\int \frac{(bx^2+cx^4)^2}{x^{11}} \, dx$

Optimal. Leaf size=19

$$-\frac{(b+cx^2)^3}{6bx^6}$$

```
[Out] -(b + c*x^2)^3/(6*b*x^6)
```

Rubi [A] time = 0.009546, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$-\frac{(b+cx^2)^3}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -(b + c*x^2)^3/(6*b*x^6)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^{11}}\,dx = \int \frac{\left(b+cx^2\right)^2}{x^7}\,dx$$
$$= -\frac{\left(b+cx^2\right)^3}{6bx^6}$$

Mathematica [A] time = 0.0008909, size = 30, normalized size = 1.58

$$-\frac{b^2}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -b^2/(6*x^6) - (b*c)/(2*x^4) - c^2/(2*x^2)
```

Maple [A] time = 0.049, size = 25, normalized size = 1.3

$$-\frac{bc}{2\,x^4}-\frac{c^2}{2\,x^2}-\frac{b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^11,x)
```

```
[Out] -1/2*b*c/x^4-1/2*c^2/x^2-1/6*b^2/x^6
```

Maxima [A] time = 0.995802, size = 32, normalized size = 1.68

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

Fricas [A] time = 1.13117, size = 54, normalized size = 2.84

$$-\frac{3\,c^2x^4 + 3\,bcx^2 + b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

Sympy [A] time = 0.385201, size = 26, normalized size = 1.37

$$-\frac{b^2 + 3bcx^2 + 3c^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**11,x)
```

```
[Out] -(b**2 + 3*b*c*x**2 + 3*c**2*x**4)/(6*x**6)
```

Giac [A] time = 1.29452, size = 32, normalized size = 1.68

$$-\frac{3\,c^2x^4 + 3\,bcx^2 + b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

3.157 $\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=30

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

```
[Out] -b^2/(7*x^7) - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Rubi [A] time = 0.0159052, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^12,x]

[Out] -b^2/(7*x^7) - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2 + cx^4)^2}{x^{12}}\,dx = \int \frac{(b + cx^2)^2}{x^8}\,dx$$
$$= \int \left(\frac{b^2}{x^8} + \frac{2bc}{x^6} + \frac{c^2}{x^4}\right) dx$$
$$= -\frac{b^2}{7x^7} - \frac{2bc}{5x^5} - \frac{c^2}{3x^3}$$

Mathematica [A] time = 0.0008539, size = 30, normalized size = 1.

$$-\frac{b^2}{7x^7} - \frac{2bc}{5x^5} - \frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -b^2/(7*x^7) - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Maple [A] time = 0.047, size = 25, normalized size = 0.8

$$-\frac{b^2}{7\,x^7} - \frac{2\,bc}{5\,x^5} - \frac{c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^12,x)
```

```
[Out] -1/7*b^2/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Maxima [A] time = 1.08368, size = 35, normalized size = 1.17

$$-\frac{35\,c^2x^4 + 42\,bcx^2 + 15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

Fricas [A] time = 1.26561, size = 63, normalized size = 2.1

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

Sympy [A] time = 0.429421, size = 27, normalized size = 0.9

$$-\frac{15b^2+42bcx^2+35c^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**12,x)
```

```
[Out] -(15*b**2 + 42*b*c*x**2 + 35*c**2*x**4)/(105*x**7)
```

Giac [A] time = 1.27573, size = 35, normalized size = 1.17

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

3.158 $\int \frac{(bx^2+cx^4)^3}{x^2} dx$

Optimal. Leaf size=43

$$\frac{3}{7}b^2cx^7 + \frac{b^3x^5}{5} + \frac{1}{3}bc^2x^9 + \frac{c^3x^{11}}{11}$$

```
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rubi [A] time = 0.023521, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$\frac{3}{7}b^2cx^7 + \frac{b^3x^5}{5} + \frac{1}{3}bc^2x^9 + \frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^2}\,dx = \int x^4\left(b+cx^2\right)^3\,dx$$
$$= \int \left(b^3x^4+3b^2cx^6+3bc^2x^8+c^3x^{10}\right)dx$$
$$= \frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Mathematica [A] time = 0.0019193, size = 43, normalized size = 1.

$$\frac{3}{7}b^2cx^7+\frac{b^3x^5}{5}+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Maple [A] time = 0.043, size = 36, normalized size = 0.8

$$\frac{b^3x^5}{5}+\frac{3\,b^2cx^7}{7}+\frac{bc^2x^9}{3}+\frac{c^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^2,x)
```

```
[Out] 1/5*b^3*x^5+3/7*b^2*c*x^7+1/3*b*c^2*x^9+1/11*c^3*x^11
```

Maxima [A] time = 1.01646, size = 47, normalized size = 1.09

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

Fricas [A] time = 1.18463, size = 82, normalized size = 1.91

$$\frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,b^2cx^7 + \frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

Sympy [A] time = 0.072205, size = 37, normalized size = 0.86

$$\frac{b^3x^5}{5} + \frac{3b^2cx^7}{7} + \frac{bc^2x^9}{3} + \frac{c^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**2,x)
```

```
[Out] b**3*x**5/5 + 3*b**2*c*x**7/7 + b*c**2*x**9/3 + c**3*x**11/11
```

Giac [A] time = 1.237, size = 47, normalized size = 1.09

$$\frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,b^2cx^7 + \frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

3.159 $\int \frac{(bx^2+cx^4)^3}{x^3} dx$

Optimal. Leaf size=34

$$\frac{(b+cx^2)^5}{10c^2}-\frac{b(b+cx^2)^4}{8c^2}$$

```
[Out] -(b*(b + c*x^2)^4)/(8*c^2) + (b + c*x^2)^5/(10*c^2)
```

Rubi [A] time = 0.0387291, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{(b+cx^2)^5}{10c^2}-\frac{b(b+cx^2)^4}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] -(b*(b + c*x^2)^4)/(8*c^2) + (b + c*x^2)^5/(10*c^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^3}\,dx &= \int x^3\left(b+cx^2\right)^3\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int x(b+cx)^3\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b(b+cx)^3}{c}+\frac{(b+cx)^4}{c}\right)dx, x, x^2\right)\\
&= -\frac{b\left(b+cx^2\right)^4}{8c^2}+\frac{\left(b+cx^2\right)^5}{10c^2}
\end{aligned}$$

Mathematica [A] time = 0.001832, size = 43, normalized size = 1.26

$$\frac{1}{2}b^2cx^6+\frac{b^3x^4}{4}+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] (b^3*x^4)/4 + (b^2*c*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10
```

Maple [A] time = 0.044, size = 36, normalized size = 1.1

$$\frac{c^3x^{10}}{10}+\frac{3\,bc^2x^8}{8}+\frac{b^2cx^6}{2}+\frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^3,x)
```

```
[Out] 1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*b^2*c*x^6+1/4*b^3*x^4
```

Maxima [A] time = 0.994273, size = 47, normalized size = 1.38

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

Fricas [A] time = 1.24893, size = 82, normalized size = 2.41

$$\frac{1}{10}c^3x^{10} + \frac{3}{8}bc^2x^8 + \frac{1}{2}b^2cx^6 + \frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

Sympy [A] time = 0.07257, size = 37, normalized size = 1.09

$$\frac{b^3x^4}{4} + \frac{b^2cx^6}{2} + \frac{3bc^2x^8}{8} + \frac{c^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**3,x)
```

```
[Out] b**3*x**4/4 + b**2*c*x**6/2 + 3*b*c**2*x**8/8 + c**3*x**10/10
```

Giac [A] time = 1.28466, size = 47, normalized size = 1.38

$$\frac{1}{10}c^3x^{10} + \frac{3}{8}bc^2x^8 + \frac{1}{2}b^2cx^6 + \frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

3.160 $\int \frac{(bx^2+cx^4)^3}{x^4}\,dx$

Optimal. Leaf size=43

$$\frac{3}{5}b^2cx^5+\frac{b^3x^3}{3}+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

```
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rubi [A] time = 0.0208536, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$\frac{3}{5}b^2cx^5+\frac{b^3x^3}{3}+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^4}\,dx = \int x^2\left(b+cx^2\right)^3\,dx$$
$$= \int \left(b^3x^2+3b^2cx^4+3bc^2x^6+c^3x^8\right)dx$$
$$= \frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Mathematica [A] time = 0.0019144, size = 43, normalized size = 1.

$$\frac{3}{5}b^2cx^5+\frac{b^3x^3}{3}+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Maple [A] time = 0.043, size = 36, normalized size = 0.8

$$\frac{b^3x^3}{3}+\frac{3\,b^2cx^5}{5}+\frac{3\,bc^2x^7}{7}+\frac{c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^4,x)
```

```
[Out] 1/3*b^3*x^3+3/5*b^2*c*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

Maxima [A] time = 1.05931, size = 47, normalized size = 1.09

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

Fricas [A] time = 1.199, size = 80, normalized size = 1.86

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

Sympy [A] time = 0.074509, size = 39, normalized size = 0.91

$$\frac{b^3x^3}{3}+\frac{3b^2cx^5}{5}+\frac{3bc^2x^7}{7}+\frac{c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**4,x)
```

```
[Out] b**3*x**3/3 + 3*b**2*c*x**5/5 + 3*b*c**2*x**7/7 + c**3*x**9/9
```

Giac [A] time = 1.29417, size = 47, normalized size = 1.09

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

3.161 $\int \frac{(bx^2+cx^4)^3}{x^5}\,dx$

Optimal. Leaf size=16

$$\frac{(b+cx^2)^4}{8c}$$

```
[Out] (b + c*x^2)^4/(8*c)
```

Rubi [A] time = 0.0088485, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$\frac{(b+cx^2)^4}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^5,x]
```

```
[Out] (b + c*x^2)^4/(8*c)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^5}\,dx = \int x\left(b+cx^2\right)^3\,dx$$
$$= \frac{(b+cx^2)^4}{8c}$$

Mathematica [A] time = 0.0023785, size = 16, normalized size = 1.

$$\frac{(b+cx^2)^4}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^5,x]
```

```
[Out] (b + c*x^2)^4/(8*c)
```

Maple [B] time = 0.042, size = 36, normalized size = 2.3

$$\frac{c^3x^8}{8} + \frac{bc^2x^6}{2} + \frac{3\,b^2cx^4}{4} + \frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^5,x)
```

```
[Out] 1/8*c^3*x^8+1/2*b*c^2*x^6+3/4*b^2*c*x^4+1/2*b^3*x^2
```

Maxima [B] time = 0.971654, size = 47, normalized size = 2.94

$$\frac{1}{8}\,c^3x^8 + \frac{1}{2}\,bc^2x^6 + \frac{3}{4}\,b^2cx^4 + \frac{1}{2}\,b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

Fricas [B] time = 1.15507, size = 80, normalized size = 5.

$$\frac{1}{8}c^3x^8 + \frac{1}{2}bc^2x^6 + \frac{3}{4}b^2cx^4 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

Sympy [B] time = 0.075562, size = 37, normalized size = 2.31

$$\frac{b^3x^2}{2} + \frac{3b^2cx^4}{4} + \frac{bc^2x^6}{2} + \frac{c^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**5,x)
```

```
[Out] b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8
```

Giac [B] time = 1.20831, size = 47, normalized size = 2.94

$$\frac{1}{8}c^3x^8 + \frac{1}{2}bc^2x^6 + \frac{3}{4}b^2cx^4 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

3.162 $\int \frac{(bx^2+cx^4)^3}{x^6}\,dx$

Optimal. Leaf size=35

$$b^2cx^3 + b^3x + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

```
[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Rubi [A] time = 0.0172534, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 194}

$$b^2cx^3 + b^3x + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^6,x]
```

```
[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^6}\,dx = \int (b+cx^2)^3\,dx$$
$$= \int (b^3+3b^2cx^2+3bc^2x^4+c^3x^6)\,dx$$
$$= b^3x+b^2cx^3+\frac{3}{5}bc^2x^5+\frac{c^3x^7}{7}$$

Mathematica [A] time = 0.0010966, size = 35, normalized size = 1.

$$b^2cx^3+b^3x+\frac{3}{5}bc^2x^5+\frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^6,x]
```

```
[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Maple [A] time = 0.041, size = 32, normalized size = 0.9

$$b^3x+b^2cx^3+\frac{3\,bc^2x^5}{5}+\frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^6,x)
```

```
[Out] b^3*x+b^2*c*x^3+3/5*b*c^2*x^5+1/7*c^3*x^7
```

Maxima [A] time = 1.06005, size = 42, normalized size = 1.2

$$\frac{1}{7}\,c^3x^7+\frac{3}{5}\,bc^2x^5+b^2cx^3+b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

Fricas [A] time = 1.41902, size = 66, normalized size = 1.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

Sympy [A] time = 0.079728, size = 32, normalized size = 0.91

$$b^3x + b^2cx^3 + \frac{3bc^2x^5}{5} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**6,x)
```

```
[Out] b**3*x + b**2*c*x**3 + 3*b*c**2*x**5/5 + c**3*x**7/7
```

Giac [A] time = 1.29702, size = 42, normalized size = 1.2

$$\frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

3.163 $\int \frac{(bx^2+cx^4)^3}{x^7} \, dx$

Optimal. Leaf size=39

$$\frac{3}{2}b^2cx^2 + b^3\log(x) + \frac{3}{4}bc^2x^4 + \frac{c^3x^6}{6}$$

```
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Rubi [A] time = 0.0259172, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{3}{2}b^2cx^2 + b^3\log(x) + \frac{3}{4}bc^2x^4 + \frac{c^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^7,x]
```

```
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^3}{x^7}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(3b^2c+\frac{b^3}{x}+3bc^2x+c^3x^2\right)dx, x, x^2\right)\\ &= \frac{3}{2}b^2cx^2+\frac{3}{4}bc^2x^4+\frac{c^3x^6}{6}+b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0044091, size = 39, normalized size = 1.

$$\frac{3}{2}b^2cx^2+b^3\log(x)+\frac{3}{4}bc^2x^4+\frac{c^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^7,x]
```

```
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Maple [A] time = 0.043, size = 34, normalized size = 0.9

$$\frac{3\,b^2cx^2}{2}+\frac{3\,bc^2x^4}{4}+\frac{c^3x^6}{6}+b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^7,x)
```

```
[Out] 3/2*b^2*c*x^2+3/4*b*c^2*x^4+1/6*c^3*x^6+b^3*ln(x)
```

Maxima [A] time = 0.98711, size = 49, normalized size = 1.26

$$\frac{1}{6}\,c^3x^6+\frac{3}{4}\,bc^2x^4+\frac{3}{2}\,b^2cx^2+\frac{1}{2}\,b^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

Fricas [A] time = 1.43224, size = 78, normalized size = 2.

$$\frac{1}{6}c^3x^6 + \frac{3}{4}bc^2x^4 + \frac{3}{2}b^2cx^2 + b^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + b^3*log(x)
```

Sympy [A] time = 0.298766, size = 37, normalized size = 0.95

$$b^3\log(x) + \frac{3b^2cx^2}{2} + \frac{3bc^2x^4}{4} + \frac{c^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**7,x)
```

```
[Out] b**3*log(x) + 3*b**2*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6
```

Giac [A] time = 1.28543, size = 49, normalized size = 1.26

$$\frac{1}{6}c^3x^6 + \frac{3}{4}bc^2x^4 + \frac{3}{2}b^2cx^2 + \frac{1}{2}b^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

3.164 $\int \frac{(bx^2+cx^4)^3}{x^8}\,dx$

Optimal. Leaf size=34

$$3b^2cx-\frac{b^3}{x}+bc^2x^3+\frac{c^3x^5}{5}$$

```
[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Rubi [A] time = 0.0194427, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$3b^2cx-\frac{b^3}{x}+bc^2x^3+\frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^8,x]
```

```
[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2 + cx^4)^3}{x^8} dx = \int \frac{(b + cx^2)^3}{x^2} dx$$
$$= \int \left(3b^2c + \frac{b^3}{x^2} + 3bc^2x^2 + c^3x^4\right) dx$$
$$= -\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

Mathematica [A] time = 0.0040092, size = 34, normalized size = 1.

$$3b^2cx - \frac{b^3}{x} + bc^2x^3 + \frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^8,x]
```

```
[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Maple [A] time = 0.048, size = 33, normalized size = 1.

$$-\frac{b^3}{x} + 3\,b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^8,x)
```

```
[Out] -b^3/x+3*b^2*c*x+b*c^2*x^3+1/5*c^3*x^5
```

Maxima [A] time = 1.0081, size = 43, normalized size = 1.26

$$\frac{1}{5}\,c^3x^5 + bc^2x^3 + 3\,b^2cx - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

Fricas [A] time = 1.41913, size = 73, normalized size = 2.15

$$\frac{c^3x^6 + 5\,bc^2x^4 + 15\,b^2cx^2 - 5\,b^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/5*(c^3*x^6 + 5*b*c^2*x^4 + 15*b^2*c*x^2 - 5*b^3)/x
```

Sympy [A] time = 0.293493, size = 29, normalized size = 0.85

$$-\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**8,x)
```

```
[Out] -b**3/x + 3*b**2*c*x + b*c**2*x**3 + c**3*x**5/5
```

Giac [A] time = 1.27456, size = 43, normalized size = 1.26

$$\frac{1}{5}\,c^3x^5 + bc^2x^3 + 3\,b^2cx - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="giac")
```

```
[Out] 1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

3.165 $\int \frac{(bx^2+cx^4)^3}{x^9} \, dx$

Optimal. Leaf size=40

$$3b^2c\log(x) - \frac{b^3}{2x^2} + \frac{3}{2}bc^2x^2 + \frac{c^3x^4}{4}$$

```
[Out] -b^3/(2*x^2) + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Rubi [A] time = 0.0291111, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$3b^2c\log(x) - \frac{b^3}{2x^2} + \frac{3}{2}bc^2x^2 + \frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^9,x]
```

```
[Out] -b^3/(2*x^2) + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^9}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x^3}\,dx \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(b+cx)^3}{x^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(3bc^2+\frac{b^3}{x^2}+\frac{3b^2c}{x}+c^3x\right)dx,x,x^2\right) \\
&= -\frac{b^3}{2x^2}+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}+3b^2c\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0063887, size = 40, normalized size = 1.

$$3b^2c\log(x)-\frac{b^3}{2x^2}+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^9,x]
```

```
[Out] -b^3/(2*x^2) + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Maple [A] time = 0.046, size = 35, normalized size = 0.9

$$-\frac{b^3}{2\,x^2}+\frac{3\,bc^2x^2}{2}+\frac{c^3x^4}{4}+3\,b^2c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^9,x)
```

```
[Out] -1/2*b^3/x^2+3/2*b*c^2*x^2+1/4*c^3*x^4+3*b^2*c*ln(x)
```

Maxima [A] time = 1.01006, size = 49, normalized size = 1.22

$$\frac{1}{4}\,c^3x^4+\frac{3}{2}\,bc^2x^2+\frac{3}{2}\,b^2c\log\left(x^2\right)-\frac{b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*b^3/x^2
```

Fricas [A] time = 1.50259, size = 85, normalized size = 2.12

$$\frac{c^3x^6 + 6\,bc^2x^4 + 12\,b^2cx^2\log\left(x\right) - 2\,b^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*x^6 + 6*b*c^2*x^4 + 12*b^2*c*x^2*log(x) - 2*b^3)/x^2
```

Sympy [A] time = 0.326276, size = 37, normalized size = 0.92

$$-\frac{b^3}{2x^2} + 3b^2c\log\left(x\right) + \frac{3bc^2x^2}{2} + \frac{c^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**9,x)
```

```
[Out] -b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4
```

Giac [A] time = 1.28012, size = 62, normalized size = 1.55

$$\frac{1}{4}\,c^3x^4 + \frac{3}{2}\,bc^2x^2 + \frac{3}{2}\,b^2c\log\left(x^2\right) - \frac{3\,b^2cx^2 + b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*(3*b^2*c*x^2 + b^3)/
x^2
```

3.166 $\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx$

Optimal. Leaf size=37

$$-\frac{3b^2c}{x}-\frac{b^3}{3x^3}+3bc^2x+\frac{c^3x^3}{3}$$

```
[Out] -b^3/(3*x^3) - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Rubi [A] time = 0.0193349, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{3b^2c}{x}-\frac{b^3}{3x^3}+3bc^2x+\frac{c^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^10,x]
```

```
[Out] -b^3/(3*x^3) - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx = \int \frac{(b+cx^2)^3}{x^4}\,dx$$
$$= \int \left(3bc^2+\frac{b^3}{x^4}+\frac{3b^2c}{x^2}+c^3x^2\right)dx$$
$$= -\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

Mathematica [A] time = 0.003986, size = 37, normalized size = 1.

$$-\frac{3b^2c}{x}-\frac{b^3}{3x^3}+3bc^2x+\frac{c^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^10,x]
```

```
[Out] -b^3/(3*x^3) - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Maple [A] time = 0.048, size = 34, normalized size = 0.9

$$-\frac{b^3}{3\,x^3}-3\,\frac{b^2c}{x}+3\,bc^2x+\frac{c^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^10,x)
```

```
[Out] -1/3*b^3/x^3-3*b^2*c/x+3*b*c^2*x+1/3*c^3*x^3
```

Maxima [A] time = 0.954365, size = 46, normalized size = 1.24

$$\frac{1}{3}\,c^3x^3+3\,bc^2x-\frac{9\,b^2cx^2+b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

Fricas [A] time = 1.43342, size = 72, normalized size = 1.95

$$\frac{c^3x^6 + 9\,bc^2x^4 - 9\,b^2cx^2 - b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/3*(c^3*x^6 + 9*b*c^2*x^4 - 9*b^2*c*x^2 - b^3)/x^3
```

Sympy [A] time = 0.323275, size = 34, normalized size = 0.92

$$3bc^2x + \frac{c^3x^3}{3} - \frac{b^3 + 9b^2cx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**10,x)
```

```
[Out] 3*b*c**2*x + c**3*x**3/3 - (b**3 + 9*b**2*c*x**2)/(3*x**3)
```

Giac [A] time = 1.30085, size = 46, normalized size = 1.24

$$\frac{1}{3}\,c^3x^3 + 3\,bc^2x - \frac{9\,b^2cx^2 + b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

3.167 $\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx$

Optimal. Leaf size=40

$$-\frac{3b^2c}{2x^2}-\frac{b^3}{4x^4}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

```
[Out] -b^3/(4*x^4) - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Rubi [A] time = 0.0256056, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{3b^2c}{2x^2}-\frac{b^3}{4x^4}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^11,x]
```

```
[Out] -b^3/(4*x^4) - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^{11}}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x^5}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+cx)^3}{x^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(c^3+\frac{b^3}{x^3}+\frac{3b^2c}{x^2}+\frac{3bc^2}{x}\right)dx, x, x^2\right)\\
&= -\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+\frac{c^3x^2}{2}+3bc^2\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0042707, size = 40, normalized size = 1.

$$-\frac{3b^2c}{2x^2}-\frac{b^3}{4x^4}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^11,x]
```

```
[Out] -b^3/(4*x^4) - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Maple [A] time = 0.05, size = 35, normalized size = 0.9

$$-\frac{b^3}{4\,x^4}-\frac{3\,b^2c}{2\,x^2}+\frac{c^3x^2}{2}+3\,bc^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^11,x)
```

```
[Out] -1/4*b^3/x^4-3/2*b^2*c/x^2+1/2*c^3*x^2+3*b*c^2*ln(x)
```

Maxima [A] time = 0.934611, size = 50, normalized size = 1.25

$$\frac{1}{2}\,c^3x^2+\frac{3}{2}\,bc^2\log\left(x^2\right)-\frac{6\,b^2cx^2+b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(6*b^2*c*x^2 + b^3)/x^4
```

Fricas [A] time = 1.42323, size = 85, normalized size = 2.12

$$\frac{2\,c^3x^6 + 12\,bc^2x^4\log\left(x\right) - 6\,b^2cx^2 - b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*x^6 + 12*b*c^2*x^4*log(x) - 6*b^2*c*x^2 - b^3)/x^4
```

Sympy [A] time = 0.369361, size = 36, normalized size = 0.9

$$3bc^2\log\left(x\right) + \frac{c^3x^2}{2} - \frac{b^3 + 6b^2cx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**11,x)
```

```
[Out] 3*b*c**2*log(x) + c**3*x**2/2 - (b**3 + 6*b**2*c*x**2)/(4*x**4)
```

Giac [A] time = 1.23102, size = 62, normalized size = 1.55

$$\frac{1}{2}\,c^3x^2 + \frac{3}{2}\,bc^2\log\left(x^2\right) - \frac{9\,bc^2x^4 + 6\,b^2cx^2 + b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(9*b*c^2*x^4 + 6*b^2*c*x^2 + b^3)/x^
4
```

3.168 $\int \frac{(bx^2+cx^4)^3}{x^{12}} \, dx$

Optimal. Leaf size=34

$$-\frac{b^2c}{x^3}-\frac{b^3}{5x^5}-\frac{3bc^2}{x}+c^3x$$

```
[Out] -b^3/(5*x^5) - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Rubi [A] time = 0.0202517, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{b^2c}{x^3}-\frac{b^3}{5x^5}-\frac{3bc^2}{x}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^12,x]
```

```
[Out] -b^3/(5*x^5) - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx = \int \frac{(b+cx^2)^3}{x^6}\,dx$$
$$= \int \left(c^3+\frac{b^3}{x^6}+\frac{3b^2c}{x^4}+\frac{3bc^2}{x^2}\right)dx$$
$$= -\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

Mathematica [A] time = 0.0053235, size = 34, normalized size = 1.

$$-\frac{b^2c}{x^3}-\frac{b^3}{5x^5}-\frac{3bc^2}{x}+c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^12,x]
```

```
[Out] -b^3/(5*x^5) - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Maple [A] time = 0.048, size = 33, normalized size = 1.

$$-\frac{b^3}{5\,x^5}-\frac{b^2c}{x^3}-3\,\frac{bc^2}{x}+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^12,x)
```

```
[Out] -1/5*b^3/x^5-b^2*c/x^3-3*b*c^2/x+c^3*x
```

Maxima [A] time = 0.962633, size = 45, normalized size = 1.32

$$c^3x-\frac{15\,bc^2x^4+5\,b^2cx^2+b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

Fricas [A] time = 1.46828, size = 76, normalized size = 2.24

$$\frac{5\,c^3x^6-15\,bc^2x^4-5\,b^2cx^2-b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/5*(5*c^3*x^6 - 15*b*c^2*x^4 - 5*b^2*c*x^2 - b^3)/x^5
```

Sympy [A] time = 0.382389, size = 32, normalized size = 0.94

$$c^3x-\frac{b^3+5b^2cx^2+15bc^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**12,x)
```

```
[Out] c**3*x - (b**3 + 5*b**2*c*x**2 + 15*b*c**2*x**4)/(5*x**5)
```

Giac [A] time = 1.26849, size = 45, normalized size = 1.32

$$c^3x-\frac{15\,bc^2x^4+5\,b^2cx^2+b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="giac")
```

```
[Out] c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

3.169 $\int \frac{(bx^2+cx^4)^3}{x^{13}} \, dx$

Optimal. Leaf size=39

$$-\frac{3b^2c}{4x^4}-\frac{b^3}{6x^6}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

```
[Out] -b^3/(6*x^6) - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Rubi [A] time = 0.0262622, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{3b^2c}{4x^4}-\frac{b^3}{6x^6}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^13,x]
```

```
[Out] -b^3/(6*x^6) - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^3}{x^{13}}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x^7}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+cx)^3}{x^4}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3b^2c}{x^3}+\frac{3bc^2}{x^2}+\frac{c^3}{x}\right)dx, x, x^2\right)\\
&= -\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0041102, size = 39, normalized size = 1.

$$
-\frac{3b^2c}{4x^4}-\frac{b^3}{6x^6}-\frac{3bc^2}{2x^2}+c^3\log(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^13,x]
```

```
[Out] -b^3/(6*x^6) - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Maple [A] time = 0.048, size = 34, normalized size = 0.9

$$
-\frac{b^3}{6\,x^6}-\frac{3\,b^2c}{4\,x^4}-\frac{3\,bc^2}{2\,x^2}+c^3\ln(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^13,x)
```

```
[Out] -1/6*b^3/x^6-3/4*b^2*c/x^4-3/2*b*c^2/x^2+c^3*ln(x)
```

Maxima [A] time = 0.966241, size = 53, normalized size = 1.36

$$
\frac{1}{2}\,c^3\log\left(x^2\right)-\frac{18\,bc^2x^4+9\,b^2cx^2+2\,b^3}{12\,x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(x^2) - 1/12*(18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/x^6
```

Fricas [A] time = 1.48488, size = 90, normalized size = 2.31

$$\frac{12\,c^3x^6\log\left(x\right)-18\,bc^2x^4-9\,b^2cx^2-2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^3*x^6*log(x) - 18*b*c^2*x^4 - 9*b^2*c*x^2 - 2*b^3)/x^6
```

Sympy [A] time = 0.424689, size = 36, normalized size = 0.92

$$c^3\log\left(x\right)-\frac{2b^3+9b^2cx^2+18bc^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**13,x)
```

```
[Out] c**3*log(x) - (2*b**3 + 9*b**2*c*x**2 + 18*b*c**2*x**4)/(12*x**6)
```

Giac [A] time = 1.14453, size = 63, normalized size = 1.62

$$\frac{1}{2}\,c^3\log\left(x^2\right)-\frac{11\,c^3x^6+18\,bc^2x^4+9\,b^2cx^2+2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*c^3*log(x^2) - 1/12*(11*c^3*x^6 + 18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/x
^6
```

3.170 $\int \frac{(bx^2+cx^4)^3}{x^{14}} dx$

Optimal. Leaf size=39

$$-\frac{3b^2c}{5x^5}-\frac{b^3}{7x^7}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

```
[Out] -b^3/(7*x^7) - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Rubi [A] time = 0.0213671, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{3b^2c}{5x^5}-\frac{b^3}{7x^7}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^14,x]
```

```
[Out] -b^3/(7*x^7) - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx = \int \frac{(b+cx^2)^3}{x^8}\,dx$$
$$= \int \left(\frac{b^3}{x^8}+\frac{3b^2c}{x^6}+\frac{3bc^2}{x^4}+\frac{c^3}{x^2}\right)dx$$
$$= -\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Mathematica [A] time = 0.0038905, size = 39, normalized size = 1.

$$-\frac{3b^2c}{5x^5}-\frac{b^3}{7x^7}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^14,x]
```

```
[Out] -b^3/(7*x^7) - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Maple [A] time = 0.049, size = 36, normalized size = 0.9

$$-\frac{b^3}{7\,x^7}-\frac{3\,b^2c}{5\,x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^14,x)
```

```
[Out] -1/7*b^3/x^7-3/5*b^2*c/x^5-b*c^2/x^3-c^3/x
```

Maxima [A] time = 0.973394, size = 50, normalized size = 1.28

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

Fricas [A] time = 1.46214, size = 84, normalized size = 2.15

$$-\frac{35\,c^3x^6 + 35\,bc^2x^4 + 21\,b^2cx^2 + 5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

Sympy [A] time = 0.425772, size = 39, normalized size = 1.

$$-\frac{5b^3 + 21b^2cx^2 + 35bc^2x^4 + 35c^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**14,x)
```

```
[Out] -(5*b**3 + 21*b**2*c*x**2 + 35*b*c**2*x**4 + 35*c**3*x**6)/(35*x**7)
```

Giac [A] time = 1.28919, size = 50, normalized size = 1.28

$$-\frac{35\,c^3x^6 + 35\,bc^2x^4 + 21\,b^2cx^2 + 5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="giac")
```

```
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

3.171 $\int \frac{(bx^2+cx^4)^3}{x^{15}}\,dx$

Optimal. Leaf size=19

$$-\frac{(b+cx^2)^4}{8bx^8}$$

```
[Out] -(b + c*x^2)^4/(8*b*x^8)
```

Rubi [A] time = 0.0097317, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$-\frac{(b+cx^2)^4}{8bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^15,x]
```

```
[Out] -(b + c*x^2)^4/(8*b*x^8)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^{15}}\,dx = \int \frac{\left(b+cx^2\right)^3}{x^9}\,dx$$
$$= -\frac{\left(b+cx^2\right)^4}{8bx^8}$$

Mathematica [B] time = 0.006791, size = 43, normalized size = 2.26

$$-\frac{b^2c}{2x^6}-\frac{b^3}{8x^8}-\frac{3bc^2}{4x^4}-\frac{c^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^15,x]
```

```
[Out] -b^3/(8*x^8) - (b^2*c)/(2*x^6) - (3*b*c^2)/(4*x^4) - c^3/(2*x^2)
```

Maple [B] time = 0.048, size = 36, normalized size = 1.9

$$-\frac{3\,bc^2}{4\,x^4}-\frac{c^3}{2\,x^2}-\frac{b^3}{8\,x^8}-\frac{b^2c}{2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^15,x)
```

```
[Out] -3/4*b*c^2/x^4-1/2*c^3/x^2-1/8*b^3/x^8-1/2*b^2*c/x^6
```

Maxima [B] time = 0.973116, size = 47, normalized size = 2.47

$$-\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

Fricas [B] time = 1.41029, size = 76, normalized size = 4.

$$-\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

Sympy [B] time = 0.494945, size = 37, normalized size = 1.95

$$-\frac{b^3+4b^2cx^2+6bc^2x^4+4c^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**15,x)
```

```
[Out] -(b**3 + 4*b**2*c*x**2 + 6*b*c**2*x**4 + 4*c**3*x**6)/(8*x**8)
```

Giac [B] time = 1.27809, size = 47, normalized size = 2.47

$$-\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="giac")
```

```
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

3.172 $\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx$

Optimal. Leaf size=43

$$-\frac{3b^2c}{7x^7}-\frac{b^3}{9x^9}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

```
[Out] -b^3/(9*x^9) - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

Rubi [A] time = 0.0213949, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 270}

$$-\frac{3b^2c}{7x^7}-\frac{b^3}{9x^9}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^16,x]
```

```
[Out] -b^3/(9*x^9) - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx = \int \frac{(b+cx^2)^3}{x^{10}}\,dx$$
$$= \int \left(\frac{b^3}{x^{10}} + \frac{3b^2c}{x^8} + \frac{3bc^2}{x^6} + \frac{c^3}{x^4}\right) dx$$
$$= -\frac{b^3}{9x^9} - \frac{3b^2c}{7x^7} - \frac{3bc^2}{5x^5} - \frac{c^3}{3x^3}$$

Mathematica [A] time = 0.0044876, size = 43, normalized size = 1.

$$-\frac{3b^2c}{7x^7} - \frac{b^3}{9x^9} - \frac{3bc^2}{5x^5} - \frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^16,x]
```

```
[Out] -b^3/(9*x^9) - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

Maple [A] time = 0.049, size = 36, normalized size = 0.8

$$-\frac{b^3}{9\,x^9} - \frac{3\,b^2c}{7\,x^7} - \frac{3\,bc^2}{5\,x^5} - \frac{c^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^16,x)
```

```
[Out] -1/9*b^3/x^9-3/7*b^2*c/x^7-3/5*b*c^2/x^5-1/3*c^3/x^3
```

Maxima [A] time = 0.971687, size = 50, normalized size = 1.16

$$-\frac{105\,c^3x^6 + 189\,bc^2x^4 + 135\,b^2cx^2 + 35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

Fricas [A] time = 1.45717, size = 90, normalized size = 2.09

$$-\frac{105\,c^3x^6 + 189\,bc^2x^4 + 135\,b^2cx^2 + 35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

Sympy [A] time = 0.487765, size = 39, normalized size = 0.91

$$-\frac{35b^3 + 135b^2cx^2 + 189bc^2x^4 + 105c^3x^6}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**16,x)
```

```
[Out] -(35*b**3 + 135*b**2*c*x**2 + 189*b*c**2*x**4 + 105*c**3*x**6)/(315*x**9)
```

Giac [A] time = 1.25488, size = 50, normalized size = 1.16

$$-\frac{105\,c^3x^6 + 189\,bc^2x^4 + 135\,b^2cx^2 + 35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="giac")
```

```
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

3.173 $\int \frac{(bx^2+cx^4)^3}{x^{17}}\,dx$

Optimal. Leaf size=40

$$\frac{c(b+cx^2)^4}{40b^2x^8}-\frac{(b+cx^2)^4}{10bx^{10}}$$

```
[Out] -(b + c*x^2)^4/(10*b*x^10) + (c*(b + c*x^2)^4)/(40*b^2*x^8)
```

Rubi [A] time = 0.0232954, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 266, 45, 37}

$$\frac{c(b+cx^2)^4}{40b^2x^8}-\frac{(b+cx^2)^4}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^17,x]
```

```
[Out] -(b + c*x^2)^4/(10*b*x^10) + (c*(b + c*x^2)^4)/(40*b^2*x^8)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
```

```
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^3}{x^{17}}\,dx &= \int \frac{\left(b+cx^2\right)^3}{x^{11}}\,dx\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(b+cx)^3}{x^6}\,dx,x,x^2\right)\\ &= -\frac{\left(b+cx^2\right)^4}{10bx^{10}}-\frac{c\,\mathrm{Subst}\left(\int \frac{(b+cx)^3}{x^5}\,dx,x,x^2\right)}{10b}\\ &= -\frac{\left(b+cx^2\right)^4}{10bx^{10}}+\frac{c\left(b+cx^2\right)^4}{40b^2x^8}\end{aligned}$$

Mathematica [A] time = 0.0049599, size = 43, normalized size = 1.08

$$-\frac{3b^2c}{8x^8}-\frac{b^3}{10x^{10}}-\frac{bc^2}{2x^6}-\frac{c^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^17,x]
```

```
[Out] -b^3/(10*x^10) - (3*b^2*c)/(8*x^8) - (b*c^2)/(2*x^6) - c^3/(4*x^4)
```

Maple [A] time = 0.051, size = 36, normalized size = 0.9

$$-\frac{b^3}{10\,x^{10}}-\frac{c^3}{4\,x^4}-\frac{3\,b^2c}{8\,x^8}-\frac{bc^2}{2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^17,x)
```

```
[Out] -1/10*b^3/x^10-1/4*c^3/x^4-3/8*b^2*c/x^8-1/2*b*c^2/x^6
```

Maxima [A] time = 0.960143, size = 50, normalized size = 1.25

$$-\frac{10\,c^3x^6+20\,bc^2x^4+15\,b^2cx^2+4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

Fricas [A] time = 1.40091, size = 85, normalized size = 2.12

$$-\frac{10\,c^3x^6+20\,bc^2x^4+15\,b^2cx^2+4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

Sympy [A] time = 0.481648, size = 39, normalized size = 0.98

$$-\frac{4b^3+15b^2cx^2+20bc^2x^4+10c^3x^6}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**17,x)
```

```
[Out] -(4*b**3 + 15*b**2*c*x**2 + 20*b*c**2*x**4 + 10*c**3*x**6)/(40*x**10)
```

Giac [A] time = 1.21098, size = 50, normalized size = 1.25

$$-\frac{10\,c^3x^6 + 20\,bc^2x^4 + 15\,b^2cx^2 + 4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

3.174 $\int \frac{x^{10}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=68

$$\frac{b^2x^3}{3c^3}-\frac{b^3x}{c^4}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Rubi [A] time = 0.0382052, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 302, 205}

$$\frac{b^2x^3}{3c^3}-\frac{b^3x}{c^4}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{10}}{bx^2+cx^4}\,dx &= \int \frac{x^8}{b+cx^2}\,dx \\
&= \int \left(-\frac{b^3}{c^4}+\frac{b^2x^2}{c^3}-\frac{bx^4}{c^2}+\frac{x^6}{c}+\frac{b^4}{c^4\left(b+cx^2\right)}\right)dx \\
&= -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^4\int \frac{1}{b+cx^2}\,dx}{c^4} \\
&= -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0264946, size = 68, normalized size = 1.

$$
\frac{b^2x^3}{3c^3}-\frac{b^3x}{c^4}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Maple [A] time = 0.046, size = 60, normalized size = 0.9

$$
\frac{x^7}{7\,c}-\frac{bx^5}{5\,c^2}+\frac{b^2x^3}{3\,c^3}-\frac{b^3x}{c^4}+\frac{b^4}{c^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2),x)
```

```
[Out] 1/7*x^7/c-1/5*b*x^5/c^2+1/3*b^2*x^3/c^3-b^3*x/c^4+b^4/c^4/(b*c)^(1/2)*arcta
n(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45701, size = 336, normalized size = 4.94

$$\left[\frac{30\,c^3x^7 - 42\,bc^2x^5 + 70\,b^2cx^3 + 105\,b^3\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right) - 210\,b^3x}{210\,c^4}, \frac{15\,c^3x^7 - 21\,bc^2x^5 + 35\,b^2cx^3 + 105\,b^3\sqrt{\frac{b}{c}}\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^3*x^7 - 42*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*sqrt(-b/c)*log((
c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) - 210*b^3*x)/c^4, 1/105*(15*c^3*
x^7 - 21*b*c^2*x^5 + 35*b^2*c*x^3 + 105*b^3*sqrt(b/c)*arctan(c*x*sqrt(b/c)/
b) - 105*b^3*x)/c^4]
```

Sympy [A] time = 0.38582, size = 107, normalized size = 1.57

$$-\frac{b^3x}{c^4} + \frac{b^2x^3}{3c^3} - \frac{bx^5}{5c^2} - \frac{\sqrt{-\frac{b^7}{c^9}}\log\left(x - \frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2} + \frac{\sqrt{-\frac{b^7}{c^9}}\log\left(x + \frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2} + \frac{x^7}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*x/c**4 + b**2*x**3/(3*c**3) - b*x**5/(5*c**2) - sqrt(-b**7/c**9)*log(
x - c**4*sqrt(-b**7/c**9)/b**3)/2 + sqrt(-b**7/c**9)*log(x + c**4*sqrt(-b**
```

```
7/c**9)/b**3)/2 + x**7/(7*c)
```

Giac [A] time = 1.22828, size = 88, normalized size = 1.29

$$\frac{b^4 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{15\,c^6x^7 - 21\,bc^5x^5 + 35\,b^2c^4x^3 - 105\,b^3c^3x}{105\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] b^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*c^6*x^7 - 21*b*c^5*x^
5 + 35*b^2*c^4*x^3 - 105*b^3*c^3*x)/c^7
```

3.175 $\int \frac{x^9}{bx^2+cx^4} \, dx$

Optimal. Leaf size=53

$$\frac{b^2x^2}{2c^3} - \frac{b^3 \log \left(b + cx^2\right)}{2c^4} - \frac{bx^4}{4c^2} + \frac{x^6}{6c}$$

```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*x^2])/(2*c
^4)
```

Rubi [A] time = 0.0452063, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^2x^2}{2c^3} - \frac{b^3 \log \left(b + cx^2\right)}{2c^4} - \frac{bx^4}{4c^2} + \frac{x^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4),x]
```


```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*x^2])/(2*c
^4)
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{bx^2+cx^4}\,dx &= \int \frac{x^7}{b+cx^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{b+cx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^2}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}-\frac{b^3}{c^3(b+cx)}\right)dx, x, x^2\right)\\ &= \frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}-\frac{b^3\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A] time = 0.0059786, size = 53, normalized size = 1.

$$\frac{b^2x^2}{2c^3}-\frac{b^3\log\left(b+cx^2\right)}{2c^4}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*x^2])/(2*c
^4)
```

Maple [A] time = 0.045, size = 46, normalized size = 0.9

$$\frac{b^2x^2}{2\,c^3}-\frac{bx^4}{4\,c^2}+\frac{x^6}{6\,c}-\frac{b^3\ln\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*b^2*x^2/c^3-1/4*b*x^4/c^2+1/6*x^6/c-1/2*b^3*ln(c*x^2+b)/c^4
```

Maxima [A] time = 1.03044, size = 62, normalized size = 1.17

$$-\frac{b^3 \log\left(cx^2 + b\right)}{2\,c^4} + \frac{2\,c^2x^6 - 3\,bcx^4 + 6\,b^2x^2}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(c*x^2 + b)/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2*x^2)/c^3
```

Fricas [A] time = 1.48985, size = 99, normalized size = 1.87

$$\frac{2\,c^3x^6 - 3\,bc^2x^4 + 6\,b^2cx^2 - 6\,b^3 \log\left(cx^2 + b\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*c^3*x^6 - 3*b*c^2*x^4 + 6*b^2*c*x^2 - 6*b^3*log(c*x^2 + b))/c^4
```

Sympy [A] time = 0.332092, size = 44, normalized size = 0.83

$$-\frac{b^3 \log\left(b + cx^2\right)}{2c^4} + \frac{b^2x^2}{2c^3} - \frac{bx^4}{4c^2} + \frac{x^6}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*log(b + c*x**2)/(2*c**4) + b**2*x**2/(2*c**3) - b*x**4/(4*c**2) + x**
6/(6*c)
```

Giac [A] time = 1.28292, size = 63, normalized size = 1.19

$$-\frac{b^3 \log\left(\left|cx^2 + b\right|\right)}{2\,c^4} + \frac{2\,c^2x^6 - 3\,bcx^4 + 6\,b^2x^2}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(abs(c*x^2 + b))/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2*x^2)
/c^3
```

3.176 $\int \frac{x^8}{bx^2+cx^4}\,dx$

Optimal. Leaf size=55

$$\frac{b^2x}{c^3} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}} - \frac{bx^3}{3c^2} + \frac{x^5}{5c}$$

```
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/c^(7/2)
```

Rubi [A] time = 0.0326585, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 302, 205}

$$\frac{b^2x}{c^3} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}} - \frac{bx^3}{3c^2} + \frac{x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/c^(7/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8}{bx^2+cx^4}\,dx = \int \frac{x^6}{b+cx^2}\,dx$$
$$= \int \left(\frac{b^2}{c^3}-\frac{bx^2}{c^2}+\frac{x^4}{c}-\frac{b^3}{c^3\left(b+cx^2\right)}\right)dx$$
$$= \frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}-\frac{b^3\int \frac{1}{b+cx^2}\,dx}{c^3}$$
$$= \frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}$$

Mathematica [A] time = 0.0265003, size = 55, normalized size = 1.

$$\frac{b^2x}{c^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/c^(7/2)
```

Maple [A] time = 0.048, size = 49, normalized size = 0.9

$$\frac{x^5}{5\,c}-\frac{bx^3}{3\,c^2}+\frac{b^2x}{c^3}-\frac{b^3}{c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2),x)
```

```
[Out] 1/5*x^5/c-1/3*b*x^3/c^2+b^2*x/c^3-b^3/c^3/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46424, size = 278, normalized size = 5.05

$$\left[\frac{6\,c^{2}x^{5}-10\,bcx^{3}+15\,b^{2}\sqrt{-\frac{b}{c}}\log\left(\frac{cx^{2}-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^{2}+b}\right)+30\,b^{2}x}{30\,c^{3}},\frac{3\,c^{2}x^{5}-5\,bcx^{3}-15\,b^{2}\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)+15\,b^{2}x}{15\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*c^2*x^5 - 10*b*c*x^3 + 15*b^2*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-
b/c) - b)/(c*x^2 + b)) + 30*b^2*x)/c^3, 1/15*(3*c^2*x^5 - 5*b*c*x^3 - 15*b^
2*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 15*b^2*x)/c^3]
```

Sympy [A] time = 0.363509, size = 95, normalized size = 1.73

$$\frac{b^{2}x}{c^{3}}-\frac{bx^{3}}{3c^{2}}+\frac{\sqrt{-\frac{b^{5}}{c^{7}}}\log\left(x-\frac{c^{3}\sqrt{-\frac{b^{5}}{c^{7}}}}{b^{2}}\right)}{2}-\frac{\sqrt{-\frac{b^{5}}{c^{7}}}\log\left(x+\frac{c^{3}\sqrt{-\frac{b^{5}}{c^{7}}}}{b^{2}}\right)}{2}+\frac{x^{5}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2),x)
```

```
[Out] b**2*x/c**3 - b*x**3/(3*c**2) + sqrt(-b**5/c**7)*log(x - c**3*sqrt(-b**5/c*
*7)/b**2)/2 - sqrt(-b**5/c**7)*log(x + c**3*sqrt(-b**5/c**7)/b**2)/2 + x**5
```

```
/(5*c)
```

Giac [A] time = 1.27465, size = 74, normalized size = 1.35

$$-\frac{b^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3\,c^4x^5 - 5\,bc^3x^3 + 15\,b^2c^2x}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*c^4*x^5 - 5*b*c^3*x^3
+ 15*b^2*c^2*x)/c^5
```

3.177 $\int \frac{x^7}{bx^2+cx^4}\,dx$

Optimal. Leaf size=40

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Rubi [A] time = 0.0337389, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{bx^2+cx^4}\,dx &= \int \frac{x^5}{b+cx^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{b+cx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{b^2}{c^2(b+cx)}\right)dx, x, x^2\right)\\ &= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b^2\log\left(b+cx^2\right)}{2c^3}\end{aligned}$$

Mathematica [A] time = 0.0051693, size = 40, normalized size = 1.

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Maple [A] time = 0.045, size = 35, normalized size = 0.9

$$-\frac{bx^2}{2\,c^2}+\frac{x^4}{4\,c}+\frac{b^2\ln\left(cx^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2),x)
```

```
[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/2*b^2*ln(c*x^2+b)/c^3
```

Maxima [A] time = 1.00612, size = 46, normalized size = 1.15

$$\frac{b^2\log\left(cx^2+b\right)}{2\,c^3}+\frac{cx^4-2\,bx^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(c*x^2 + b)/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

Fricas [A] time = 1.42159, size = 73, normalized size = 1.82

$$\frac{c^2x^4 - 2\,bcx^2 + 2\,b^2\log\left(cx^2 + b\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*x^4 - 2*b*c*x^2 + 2*b^2*log(c*x^2 + b))/c^3
```

Sympy [A] time = 0.357193, size = 32, normalized size = 0.8

$$\frac{b^2\log\left(b + cx^2\right)}{2c^3} - \frac{bx^2}{2c^2} + \frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2),x)
```

```
[Out] b**2*log(b + c*x**2)/(2*c**3) - b*x**2/(2*c**2) + x**4/(4*c)
```

Giac [A] time = 1.28906, size = 47, normalized size = 1.18

$$\frac{b^2\log\left(\left|cx^2 + b\right|\right)}{2\,c^3} + \frac{cx^4 - 2\,bx^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(abs(c*x^2 + b))/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

3.178 $\int \frac{x^6}{bx^2+cx^4}\,dx$

Optimal. Leaf size=42

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

```
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

Rubi [A] time = 0.0266642, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 302, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{bx^2+cx^4}\,dx = \int \frac{x^4}{b+cx^2}\,dx$$
$$= \int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right) dx$$
$$= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{b^2\int \frac{1}{b+cx^2}\,dx}{c^2}$$
$$= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}$$

Mathematica [A] time = 0.0185414, size = 42, normalized size = 1.

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

Maple [A] time = 0.045, size = 38, normalized size = 0.9

$$\frac{x^3}{3\,c}-\frac{bx}{c^2}+\frac{b^2}{c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2),x)
```

```
[Out] 1/3*x^3/c-b*x/c^2+b^2/c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54459, size = 217, normalized size = 5.17

$$\left[\frac{2\,cx^3 + 3\,b\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right) - 6\,bx}{6\,c^2}, \frac{cx^3 + 3\,b\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right) - 3\,bx}{3\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*x^3 + 3*b*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 +
b)) - 6*b*x)/c^2, 1/3*(c*x^3 + 3*b*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*b*
x)/c^2]
```

Sympy [B] time = 0.385021, size = 80, normalized size = 1.9

$$-\frac{bx}{c^2} - \frac{\sqrt{-\frac{b^3}{c^5}}\log\left(x - \frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2} + \frac{\sqrt{-\frac{b^3}{c^5}}\log\left(x + \frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2} + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2),x)
```

```
[Out] -b*x/c**2 - sqrt(-b**3/c**5)*log(x - c**2*sqrt(-b**3/c**5)/b)/2 + sqrt(-b**
3/c**5)*log(x + c**2*sqrt(-b**3/c**5)/b)/2 + x**3/(3*c)
```

Giac [A] time = 1.21578, size = 54, normalized size = 1.29

$$\frac{b^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^2} + \frac{c^2x^3 - 3\,bcx}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

3.179 $\int \frac{x^5}{bx^2+cx^4}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2c} - \frac{b\log\left(b+cx^2\right)}{2c^2}$$

```
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.0266021, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{x^2}{2c} - \frac{b\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{bx^2+cx^4}\,dx &= \int \frac{x^3}{b+cx^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{b+cx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{c}-\frac{b}{c(b+cx)}\right)dx,x,x^2\right)\\ &= \frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.0046211, size = 27, normalized size = 1.

$$\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Maple [A] time = 0.045, size = 24, normalized size = 0.9

$$\frac{x^2}{2\,c}-\frac{b\ln\left(cx^2+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*x^2/c-1/2*b*ln(c*x^2+b)/c^2
```

Maxima [A] time = 0.989689, size = 31, normalized size = 1.15

$$\frac{x^2}{2\,c}-\frac{b\log\left(cx^2+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - 1/2*b*log(c*x^2 + b)/c^2
```

Fricas [A] time = 1.44295, size = 49, normalized size = 1.81

$$\frac{cx^2 - b\log\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^2 - b*log(c*x^2 + b))/c^2
```

Sympy [A] time = 0.31091, size = 20, normalized size = 0.74

$$-\frac{b\log\left(b + cx^2\right)}{2c^2} + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2),x)
```

```
[Out] -b*log(b + c*x**2)/(2*c**2) + x**2/(2*c)
```

Giac [A] time = 1.29538, size = 32, normalized size = 1.19

$$\frac{x^2}{2\,c} - \frac{b\log\left(\left|cx^2 + b\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/2*b*log(abs(c*x^2 + b))/c^2
```

3.180 $\int \frac{x^4}{bx^2+cx^4} \, dx$

Optimal. Leaf size=31

$$\frac{x}{c} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}$$

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Rubi [A] time = 0.0181689, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 321, 205}

$$\frac{x}{c} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4),x]
```

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{bx^2+cx^4}\,dx = \int \frac{x^2}{b+cx^2}\,dx$$
$$= \frac{x}{c} - \frac{b\int \frac{1}{b+cx^2}\,dx}{c}$$
$$= \frac{x}{c} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.008255, size = 31, normalized size = 1.

$$\frac{x}{c} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4),x]
```

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Maple [A] time = 0.043, size = 27, normalized size = 0.9

$$\frac{x}{c} - \frac{b}{c}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2),x)
```

```
[Out] x/c-b/c/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51991, size = 165, normalized size = 5.32

$$\left[\frac{\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+2\,x}{2\,c},\,-\frac{\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-x}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 2*x)/c,
-(sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - x)/c]
```

Sympy [B] time = 0.312609, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{b}{c^3}}\log\left(-c\sqrt{-\frac{b}{c^3}}+x\right)}{2}-\frac{\sqrt{-\frac{b}{c^3}}\log\left(c\sqrt{-\frac{b}{c^3}}+x\right)}{2}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-b/c**3)*log(-c*sqrt(-b/c**3) + x)/2 - sqrt(-b/c**3)*log(c*sqrt(-b/c**
3) + x)/2 + x/c
```

Giac [A] time = 1.24588, size = 35, normalized size = 1.13

$$-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) + x/c
```

3.181 $\int \frac{x^3}{bx^2+cx^4} dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^2)}{2c}$$

```
[Out] Log[b + c*x^2]/(2*c)
```

Rubi [A] time = 0.0089441, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 260}

$$\frac{\log(b+cx^2)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4),x]
```

```
[Out] Log[b + c*x^2]/(2*c)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{bx^2+cx^4}\,dx &= \int \frac{x}{b+cx^2}\,dx\\ &= \frac{\log(b+cx^2)}{2c}\end{aligned}$$

Mathematica [A] time = 0.0023321, size = 15, normalized size = 1.

$$\frac{\log \left(b+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4),x]
```

```
[Out] Log[b + c*x^2]/(2*c)
```

Maple [A] time = 0.042, size = 14, normalized size = 0.9

$$\frac{\ln \left(cx^2+b\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*ln(c*x^2+b)/c
```

Maxima [A] time = 1.00259, size = 18, normalized size = 1.2

$$\frac{\log \left(cx^2+b\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b)/c
```

Fricas [A] time = 1.43033, size = 30, normalized size = 2.

$$\frac{\log \left(cx^2+b\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b)/c
```

Sympy [A] time = 0.113073, size = 10, normalized size = 0.67

$$\frac{\log\left(b+cx^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2),x)
```

```
[Out] log(b + c*x**2)/(2*c)
```

Giac [A] time = 1.20426, size = 19, normalized size = 1.27

$$\frac{\log\left(\left|cx^2+b\right|\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^2 + b))/c
```

3.182 $\int \frac{x^2}{bx^2+cx^4}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Rubi [A] time = 0.0117432, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{bx^2 + cx^4}\,dx = \int \frac{1}{b + cx^2}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

Mathematica [A] time = 0.0042586, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Maple [A] time = 0.044, size = 16, normalized size = 0.7

$$\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2),x)
```

```
[Out] 1/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43387, size = 151, normalized size = 6.29

$$\left[-\frac{\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right)}{2\,bc},\ \frac{\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b))/(b*c), sqrt(
b*c)*arctan(sqrt(b*c)*x/b)/(b*c)]
```

Sympy [B] time = 0.136575, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{bc}}\log\left(-b\sqrt{-\frac{1}{bc}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{bc}}\log\left(b\sqrt{-\frac{1}{bc}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-1/(b*c))*log(-b*sqrt(-1/(b*c)) + x)/2 + sqrt(-1/(b*c))*log(b*sqrt(-1
/(b*c)) + x)/2
```

Giac [A] time = 1.28577, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] arctan(c*x/sqrt(b*c))/sqrt(b*c)
```

3.183 $\int \frac{x}{bx^2+cx^4}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^2\right)}{2b}$$

```
[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

Rubi [A] time = 0.0160743, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1584, 266, 36, 29, 31}

$$\frac{\log(x)}{b}-\frac{\log\left(b+cx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{bx^2+cx^4}\,dx &= \int \frac{1}{x\left(b+cx^2\right)}\,dx \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x(b+cx)}\,dx, x, x^2\right) \\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2b} - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{b+cx}\,dx, x, x^2\right)}{2b} \\
&= \frac{\log(x)}{b} - \frac{\log\left(b+cx^2\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.005064, size = 22, normalized size = 1.

$$\frac{\log(x)}{b} - \frac{\log\left(b+cx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

Maple [A] time = 0.046, size = 21, normalized size = 1.

$$\frac{\ln(x)}{b} - \frac{\ln\left(cx^2+b\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2),x)
```

```
[Out] ln(x)/b-1/2*ln(c*x^2+b)/b
```

Maxima [A] time = 1.02275, size = 31, normalized size = 1.41

$$-\frac{\log\left(cx^2+b\right)}{2\,b}+\frac{\log\left(x^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(c*x^2 + b)/b + 1/2*log(x^2)/b
```

Fricas [A] time = 1.43001, size = 49, normalized size = 2.23

$$-\frac{\log\left(cx^2+b\right)-2\,\log\left(x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(log(c*x^2 + b) - 2*log(x))/b
```

Sympy [A] time = 0.203062, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{b}-\frac{\log\left(\frac{b}{c}+x^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2),x)
```

```
[Out] log(x)/b - log(b/c + x**2)/(2*b)
```

Giac [A] time = 1.17321, size = 30, normalized size = 1.36

$$-\frac{\log\left(\left|cx^2+b\right|\right)}{2\,b}+\frac{\log\left(\left|x\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(c*x^2 + b))/b + log(abs(x))/b
```

3.184 $\int \frac{1}{bx^2+cx^4} \, dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

```
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Rubi [A] time = 0.0142081, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 325, 205}

$$-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-1),x]
```

```
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{bx^2+cx^4}\,dx &= \int \frac{1}{x^2\left(b+cx^2\right)}\,dx\\ &= -\frac{1}{bx}-\frac{c\int \frac{1}{b+cx^2}\,dx}{b}\\ &= -\frac{1}{bx}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0123175, size = 34, normalized size = 1.

$$-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-1),x]
```

```
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Maple [A] time = 0.046, size = 30, normalized size = 0.9

$$-\frac{1}{bx}-\frac{c}{b}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2),x)
```

```
[Out] -1/b/x-c/b/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50907, size = 173, normalized size = 5.09

$$\left[\frac{x\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-2}{2\,bx}, -\frac{x\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+1}{bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 2)/(b*
x), -(x*sqrt(c/b)*arctan(x*sqrt(c/b)) + 1)/(b*x)]
```

Sympy [B] time = 0.376627, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{c}{b^3}}\log\left(-\frac{b^2\sqrt{-\frac{c}{b^3}}}{c}+x\right)}{2}-\frac{\sqrt{-\frac{c}{b^3}}\log\left(\frac{b^2\sqrt{-\frac{c}{b^3}}}{c}+x\right)}{2}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c/b**3)*log(-b**2*sqrt(-c/b**3)/c + x)/2 - sqrt(-c/b**3)*log(b**2*sqr
t(-c/b**3)/c + x)/2 - 1/(b*x)
```

Giac [A] time = 1.21555, size = 39, normalized size = 1.15

$$-\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - 1/(b*x)
```

3.185 $\int \frac{1}{x(bx^2+cx^4)}\,dx$

Optimal. Leaf size=35

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

```
[Out] -1/(2*b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Rubi [A] time = 0.0275042, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(bx^2+cx^4\right)}\,dx &= \int \frac{1}{x^3\left(b+cx^2\right)}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(b+cx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{bx^2}-\frac{c}{b^2x}+\frac{c^2}{b^2(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{2bx^2}-\frac{c\log(x)}{b^2}+\frac{c\log\left(b+cx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0068712, size = 35, normalized size = 1.

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Maple [A] time = 0.047, size = 32, normalized size = 0.9

$$-\frac{1}{2\,bx^2}-\frac{c\ln(x)}{b^2}+\frac{c\ln\left(cx^2+b\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2),x)
```

```
[Out] -1/2/b/x^2-c*ln(x)/b^2+1/2*c*ln(c*x^2+b)/b^2
```

Maxima [A] time = 0.955562, size = 45, normalized size = 1.29

$$\frac{c\log\left(cx^2+b\right)}{2\,b^2}-\frac{c\log\left(x^2\right)}{2\,b^2}-\frac{1}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(c*x^2 + b)/b^2 - 1/2*c*log(x^2)/b^2 - 1/2/(b*x^2)
```

Fricas [A] time = 1.50013, size = 80, normalized size = 2.29

$$\frac{cx^2 \log\left(cx^2 + b\right) - 2\,cx^2 \log\left(x\right) - b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^2*log(c*x^2 + b) - 2*c*x^2*log(x) - b)/(b^2*x^2)
```

Sympy [A] time = 0.468961, size = 31, normalized size = 0.89

$$-\frac{1}{2bx^2} - \frac{c\log\left(x\right)}{b^2} + \frac{c\log\left(\frac{b}{c} + x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2),x)
```

```
[Out] -1/(2*b*x**2) - c*log(x)/b**2 + c*log(b/c + x**2)/(2*b**2)
```

Giac [A] time = 1.22322, size = 58, normalized size = 1.66

$$-\frac{c\log\left(x^2\right)}{2\,b^2} + \frac{c\log\left(\left|cx^2 + b\right|\right)}{2\,b^2} + \frac{cx^2 - b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*c*log(x^2)/b^2 + 1/2*c*log(abs(c*x^2 + b))/b^2 + 1/2*(c*x^2 - b)/(b^2*
x^2)
```

3.186 $\int \frac{1}{x^2(bx^2+cx^4)}\,dx$

Optimal. Leaf size=43

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{c}{b^2x}-\frac{1}{3bx^3}$$

```
[Out] -1/(3*b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(5/2)
```

Rubi [A] time = 0.0226364, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 325, 205}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{c}{b^2x}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(5/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^4\left(b+cx^2\right)}\,dx$$
$$= -\frac{1}{3bx^3} - \frac{c\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{b}$$
$$= -\frac{1}{3bx^3} + \frac{c}{b^2x} + \frac{c^2\int \frac{1}{b+cx^2}\,dx}{b^2}$$
$$= -\frac{1}{3bx^3} + \frac{c}{b^2x} + \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.0200254, size = 43, normalized size = 1.

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}} + \frac{c}{b^2x} - \frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(5/2)
```

Maple [A] time = 0.05, size = 39, normalized size = 0.9

$$-\frac{1}{3\,bx^3} + \frac{c}{b^2x} + \frac{c^2}{b^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2),x)
```

```
[Out] -1/3/b/x^3+c/b^2/x+c^2/b^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39671, size = 234, normalized size = 5.44

$$\left[\frac{3\,cx^{3}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^{2}+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)+6\,cx^{2}-2\,b}{6\,b^{2}x^{3}},\ \frac{3\,cx^{3}\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+3\,cx^{2}-b}{3\,b^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*c*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) +
6*c*x^2 - 2*b)/(b^2*x^3), 1/3*(3*c*x^3*sqrt(c/b)*arctan(x*sqrt(c/b)) + 3*c*
x^2 - b)/(b^2*x^3)]
```

Sympy [B] time = 0.402925, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{c^{3}}{b^{5}}}\log\left(-\frac{b^{3}\sqrt{-\frac{c^{3}}{b^{5}}}}{c^{2}}+x\right)}{2}+\frac{\sqrt{-\frac{c^{3}}{b^{5}}}\log\left(\frac{b^{3}\sqrt{-\frac{c^{3}}{b^{5}}}}{c^{2}}+x\right)}{2}+\frac{-b+3cx^{2}}{3b^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-c**3/b**5)*log(-b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + sqrt(-c**3/b**5)
*log(b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + (-b + 3*c*x**2)/(3*b**2*x**3)
```

Giac [A] time = 1.27426, size = 54, normalized size = 1.26

$$\frac{c^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^2} + \frac{3\,cx^2 - b}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/3*(3*c*x^2 - b)/(b^2*x^3)
```

3.187 $\int \frac{1}{x^3(bx^2+cx^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{c^2\log\left(b+cx^2\right)}{2b^3}+\frac{c^2\log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

```
[Out] -1/(4*b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^2])/(2*b
^3)
```

Rubi [A] time = 0.0350715, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{c^2\log\left(b+cx^2\right)}{2b^3}+\frac{c^2\log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^2])/(2*b
^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(bx^2+cx^4\right)}\,dx &= \int \frac{1}{x^5\left(b+cx^2\right)}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(b+cx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{bx^3}-\frac{c}{b^2x^2}+\frac{c^2}{b^3x}-\frac{c^3}{b^3(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{4bx^4}+\frac{c}{2b^2x^2}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(b+cx^2\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.0070187, size = 49, normalized size = 1.

$$-\frac{c^2\log\left(b+cx^2\right)}{2b^3}+\frac{c^2\log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^2])/(2*b
^3)
```

Maple [A] time = 0.049, size = 44, normalized size = 0.9

$$-\frac{1}{4\,bx^4}+\frac{c}{2\,b^2x^2}+\frac{c^2\ln(x)}{b^3}-\frac{c^2\ln\left(cx^2+b\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2),x)
```

```
[Out] -1/4/b/x^4+1/2*c/b^2/x^2+c^2*ln(x)/b^3-1/2*c^2*ln(c*x^2+b)/b^3
```

Maxima [A] time = 0.97574, size = 63, normalized size = 1.29

$$-\frac{c^2\log\left(cx^2+b\right)}{2\,b^3}+\frac{c^2\log\left(x^2\right)}{2\,b^3}+\frac{2\,cx^2-b}{4\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*log(c*x^2 + b)/b^3 + 1/2*c^2*log(x^2)/b^3 + 1/4*(2*c*x^2 - b)/(b^2
*x^4)
```

Fricas [A] time = 1.49403, size = 108, normalized size = 2.2

$$-\frac{2\,c^2x^4\log\left(cx^2+b\right)-4\,c^2x^4\log\left(x\right)-2\,bcx^2+b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*c^2*x^4*log(c*x^2 + b) - 4*c^2*x^4*log(x) - 2*b*c*x^2 + b^2)/(b^3*x
^4)
```

Sympy [A] time = 0.530072, size = 42, normalized size = 0.86

$$\frac{-b+2cx^2}{4b^2x^4}+\frac{c^2\log\left(x\right)}{b^3}-\frac{c^2\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2),x)
```

```
[Out] (-b + 2*c*x**2)/(4*b**2*x**4) + c**2*log(x)/b**3 - c**2*log(b/c + x**2)/(2*
b**3)
```

Giac [A] time = 1.2567, size = 77, normalized size = 1.57

$$\frac{c^2 \log\left(x^2\right)}{2\, b^3} - \frac{c^2 \log\left(\left|cx^2 + b\right|\right)}{2\, b^3} - \frac{3\, c^2 x^4 - 2\, bcx^2 + b^2}{4\, b^3 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2)/b^3 - 1/2*c^2*log(abs(c*x^2 + b))/b^3 - 1/4*(3*c^2*x^4 - 2
*b*c*x^2 + b^2)/(b^3*x^4)
```

3.188 $\int \frac{1}{x^4(bx^2+cx^4)}\,dx$

Optimal. Leaf size=58

$$-\frac{c^2}{b^3x}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

```
[Out] -1/(5*b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/b^(7/2)
```

Rubi [A] time = 0.0362493, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 325, 205}

$$-\frac{c^2}{b^3x}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(5*b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/b^(7/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(bx^2+cx^4\right)}\,dx &= \int \frac{1}{x^6\left(b+cx^2\right)}\,dx\\ &= -\frac{1}{5bx^5}-\frac{c\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{b}\\ &= -\frac{1}{5bx^5}+\frac{c}{3b^2x^3}+\frac{c^2\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{b^2}\\ &= -\frac{1}{5bx^5}+\frac{c}{3b^2x^3}-\frac{c^2}{b^3x}-\frac{c^3\int \frac{1}{b+cx^2}\,dx}{b^3}\\ &= -\frac{1}{5bx^5}+\frac{c}{3b^2x^3}-\frac{c^2}{b^3x}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0267664, size = 58, normalized size = 1.

$$-\frac{c^2}{b^3x}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(5*b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/b^(7/2)
```

Maple [A] time = 0.05, size = 52, normalized size = 0.9

$$-\frac{1}{5\,bx^5}-\frac{c^2}{b^3x}+\frac{c}{3\,b^2x^3}-\frac{c^3}{b^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2),x)
```

```
[Out] -1/5/b/x^5-c^2/b^3/x+1/3*c/b^2/x^3-c^3/b^3/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46194, size = 296, normalized size = 5.1

$$\left[\frac{15\,c^{2}x^{5}\sqrt{-\frac{c}{b}}\log\left(\frac{cx^{2}-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)-30\,c^{2}x^{4}+10\,bcx^{2}-6\,b^{2}}{30\,b^{3}x^{5}},\,-\frac{15\,c^{2}x^{5}\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+15\,c^{2}x^{4}-5\,bcx^{2}+3\,b^{2}}{15\,b^{3}x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*c^2*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)
) - 30*c^2*x^4 + 10*b*c*x^2 - 6*b^2)/(b^3*x^5), -1/15*(15*c^2*x^5*sqrt(c/b)
*arctan(x*sqrt(c/b)) + 15*c^2*x^4 - 5*b*c*x^2 + 3*b^2)/(b^3*x^5)]
```

Sympy [B] time = 0.549863, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{c^{5}}{b^{7}}}\log\left(-\frac{b^{4}\sqrt{-\frac{c^{5}}{b^{7}}}}{c^{3}}+x\right)}{2}-\frac{\sqrt{-\frac{c^{5}}{b^{7}}}\log\left(\frac{b^{4}\sqrt{-\frac{c^{5}}{b^{7}}}}{c^{3}}+x\right)}{2}-\frac{3b^{2}-5bcx^{2}+15c^{2}x^{4}}{15b^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c**5/b**7)*log(-b**4*sqrt(-c**5/b**7)/c**3 + x)/2 - sqrt(-c**5/b**7)*
log(b**4*sqrt(-c**5/b**7)/c**3 + x)/2 - (3*b**2 - 5*b*c*x**2 + 15*c**2*x**4
)/(15*b**3*x**5)
```

Giac [A] time = 1.23844, size = 70, normalized size = 1.21

$$-\frac{c^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^3} - \frac{15\,c^2x^4 - 5\,bcx^2 + 3\,b^2}{15\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/15*(15*c^2*x^4 - 5*b*c*x^2 +
 3*b^2)/(b^3*x^5)
```

3.189 $\int \frac{1}{x^5(bx^2+cx^4)}\,dx$

Optimal. Leaf size=63

$$-\frac{c^2}{2b^3x^2}+\frac{c^3\log\left(b+cx^2\right)}{2b^4}-\frac{c^3\log(x)}{b^4}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

```
[Out] -1/(6*b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4 + (c^3*Lo
g[b + c*x^2])/(2*b^4)
```

Rubi [A] time = 0.0410101, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{c^2}{2b^3x^2}+\frac{c^3\log\left(b+cx^2\right)}{2b^4}-\frac{c^3\log(x)}{b^4}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(6*b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4 + (c^3*Lo
g[b + c*x^2])/(2*b^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(bx^2+cx^4\right)}\,dx &= \int \frac{1}{x^7\left(b+cx^2\right)}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4(b+cx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{bx^4}-\frac{c}{b^2x^3}+\frac{c^2}{b^3x^2}-\frac{c^3}{b^4x}+\frac{c^4}{b^4(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{6bx^6}+\frac{c}{4b^2x^4}-\frac{c^2}{2b^3x^2}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(b+cx^2\right)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.0074705, size = 63, normalized size = 1.

$$-\frac{c^2}{2b^3x^2}+\frac{c^3\log\left(b+cx^2\right)}{2b^4}-\frac{c^3\log(x)}{b^4}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(6*b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4 + (c^3*Lo
g[b + c*x^2])/(2*b^4)
```

Maple [A] time = 0.05, size = 56, normalized size = 0.9

$$-\frac{1}{6\,bx^6}+\frac{c}{4\,b^2x^4}-\frac{c^2}{2\,b^3x^2}-\frac{c^3\ln(x)}{b^4}+\frac{c^3\ln\left(cx^2+b\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2),x)
```

```
[Out] -1/6/b/x^6+1/4*c/b^2/x^4-1/2*c^2/b^3/x^2-c^3*ln(x)/b^4+1/2*c^3*ln(c*x^2+b)/
b^4
```

Maxima [A] time = 1.02829, size = 78, normalized size = 1.24

$$\frac{c^3 \log\left(cx^2+b\right)}{2\,b^4}-\frac{c^3 \log\left(x^2\right)}{2\,b^4}-\frac{6\,c^2x^4-3\,bcx^2+2\,b^2}{12\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(c*x^2 + b)/b^4 - 1/2*c^3*log(x^2)/b^4 - 1/12*(6*c^2*x^4 - 3*b*c
*x^2 + 2*b^2)/(b^3*x^6)
```

Fricas [A] time = 1.43942, size = 134, normalized size = 2.13

$$\frac{6\,c^3x^6\log\left(cx^2+b\right)-12\,c^3x^6\log\left(x\right)-6\,bc^2x^4+3\,b^2cx^2-2\,b^3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*c^3*x^6*log(c*x^2 + b) - 12*c^3*x^6*log(x) - 6*b*c^2*x^4 + 3*b^2*c*
x^2 - 2*b^3)/(b^4*x^6)
```

Sympy [A] time = 0.629941, size = 56, normalized size = 0.89

$$-\frac{2b^2-3bcx^2+6c^2x^4}{12b^3x^6}-\frac{c^3\log\left(x\right)}{b^4}+\frac{c^3\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2),x)
```

```
[Out] -(2*b**2 - 3*b*c*x**2 + 6*c**2*x**4)/(12*b**3*x**6) - c**3*log(x)/b**4 + c*
*3*log(b/c + x**2)/(2*b**4)
```

Giac [A] time = 1.28618, size = 95, normalized size = 1.51

$$-\frac{c^3 \log\left(x^2\right)}{2\,b^4} + \frac{c^3 \log\left(\left|cx^2 + b\right|\right)}{2\,b^4} + \frac{11\,c^3x^6 - 6\,bc^2x^4 + 3\,b^2cx^2 - 2\,b^3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*c^3*log(x^2)/b^4 + 1/2*c^3*log(abs(c*x^2 + b))/b^4 + 1/12*(11*c^3*x^6
- 6*b*c^2*x^4 + 3*b^2*c*x^2 - 2*b^3)/(b^4*x^6)
```

3.190 $\int \frac{x^{12}}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=79

$$\frac{7b^2x}{2c^4} - \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}} - \frac{7bx^3}{6c^3} - \frac{x^7}{2c\left(b+cx^2\right)} + \frac{7x^5}{10c^2}$$

```
[Out] (7*b^2*x)/(2*c^4) - (7*b*x^3)/(6*c^3) + (7*x^5)/(10*c^2) - x^7/(2*c*(b + c*
x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rubi [A] time = 0.0387618, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 302, 205}

$$\frac{7b^2x}{2c^4} - \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}} - \frac{7bx^3}{6c^3} - \frac{x^7}{2c\left(b+cx^2\right)} + \frac{7x^5}{10c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (7*b^2*x)/(2*c^4) - (7*b*x^3)/(6*c^3) + (7*x^5)/(10*c^2) - x^7/(2*c*(b + c*
x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{12}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^8}{\left(b+cx^2\right)^2}\,dx\\ &= -\frac{x^7}{2c\left(b+cx^2\right)}+\frac{7\int \frac{x^6}{b+cx^2}\,dx}{2c}\\ &= -\frac{x^7}{2c\left(b+cx^2\right)}+\frac{7\int \left(\frac{b^2}{c^3}-\frac{bx^2}{c^2}+\frac{x^4}{c}-\frac{b^3}{c^3\left(b+cx^2\right)}\right)dx}{2c}\\ &= \frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}+\frac{7x^5}{10c^2}-\frac{x^7}{2c\left(b+cx^2\right)}-\frac{\left(7b^3\right)\int \frac{1}{b+cx^2}\,dx}{2c^4}\\ &= \frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}+\frac{7x^5}{10c^2}-\frac{x^7}{2c\left(b+cx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.0524963, size = 71, normalized size = 0.9

$$\frac{x\left(\frac{15b^3}{b+cx^2}+90b^2-20bcx^2+6c^2x^4\right)}{30c^4}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(90*b^2 - 20*b*c*x^2 + 6*c^2*x^4 + (15*b^3)/(b + c*x^2)))/(30*c^4) - (7*
b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Maple [A] time = 0.052, size = 68, normalized size = 0.9

$$\frac{x^5}{5\,c^2}-\frac{2\,bx^3}{3\,c^3}+3\,\frac{b^2x}{c^4}+\frac{b^3x}{2\,c^4\left(cx^2+b\right)}-\frac{7\,b^3}{2\,c^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/5*x^5/c^2-2/3*b*x^3/c^3+3*b^2*x/c^4+1/2/c^4*b^3*x/(c*x^2+b)-7/2/c^4*b^3/(
b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47675, size = 409, normalized size = 5.18

$$\left[\frac{12\,c^3x^7-28\,bc^2x^5+140\,b^2cx^3+210\,b^3x+105\left(b^2cx^2+b^3\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{60\left(c^5x^2+bc^4\right)},\frac{6\,c^3x^7-14\,bc^2x^5+70\,b^2cx^3+105}{30\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^3*x^7 - 28*b*c^2*x^5 + 140*b^2*c*x^3 + 210*b^3*x + 105*(b^2*c*x
^2 + b^3)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^5*
x^2 + b*c^4), 1/30*(6*c^3*x^7 - 14*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*x - 1
```

```
05*(b^2*c*x^2 + b^3)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^5*x^2 + b*c^4)]
```

Sympy [A] time = 0.482897, size = 124, normalized size = 1.57

$$\frac{b^3x}{2bc^4+2c^5x^2}+\frac{3b^2x}{c^4}-\frac{2bx^3}{3c^3}+\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}-\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}+\frac{x^5}{5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3*x/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*x/c**4 - 2*b*x**3/(3*c**3) + 7*sqr
t(-b**5/c**9)*log(x - c**4*sqrt(-b**5/c**9)/b**2)/4 - 7*sqrt(-b**5/c**9)*lo
g(x + c**4*sqrt(-b**5/c**9)/b**2)/4 + x**5/(5*c**2)
```

Giac [A] time = 1.21212, size = 99, normalized size = 1.25

$$-\frac{7\,b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^4}+\frac{b^3x}{2\left(cx^2+b\right)c^4}+\frac{3\,c^8x^5-10\,bc^7x^3+45\,b^2c^6x}{15\,c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*b^3*x/((c*x^2 + b)*c^4
) + 1/15*(3*c^8*x^5 - 10*b*c^7*x^3 + 45*b^2*c^6*x)/c^10
```

3.191 $\int \frac{x^{11}}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=57

$$\frac{b^3}{2c^4(b+cx^2)} + \frac{3b^2\log(b+cx^2)}{2c^4} - \frac{bx^2}{c^3} + \frac{x^4}{4c^2}$$

```
[Out] -((b*x^2)/c^3) + x^4/(4*c^2) + b^3/(2*c^4*(b + c*x^2)) + (3*b^2*Log[b + c*x
^2])/(2*c^4)
```

Rubi [A] time = 0.0513479, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^3}{2c^4(b+cx^2)} + \frac{3b^2\log(b+cx^2)}{2c^4} - \frac{bx^2}{c^3} + \frac{x^4}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*x^2)/c^3) + x^4/(4*c^2) + b^3/(2*c^4*(b + c*x^2)) + (3*b^2*Log[b + c*x
^2])/(2*c^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^7}{\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(b+cx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{2b}{c^3}+\frac{x}{c^2}-\frac{b^3}{c^3(b+cx)^2}+\frac{3b^2}{c^3(b+cx)}\right)dx, x, x^2\right)\\ &= -\frac{bx^2}{c^3}+\frac{x^4}{4c^2}+\frac{b^3}{2c^4\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A] time = 0.016942, size = 49, normalized size = 0.86

$$\frac{\frac{2b^3}{b+cx^2}+6b^2\log\left(b+cx^2\right)-4bcx^2+c^2x^4}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-4*b*c*x^2 + c^2*x^4 + (2*b^3)/(b + c*x^2) + 6*b^2*Log[b + c*x^2])/(4*c^4)
```

Maple [A] time = 0.053, size = 52, normalized size = 0.9

$$-\frac{bx^2}{c^3}+\frac{x^4}{4\,c^2}+\frac{b^3}{2\,c^4\left(cx^2+b\right)}+\frac{3\,b^2\ln\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2)^2,x)
```

```
[Out] -b*x^2/c^3+1/4*x^4/c^2+1/2*b^3/c^4/(c*x^2+b)+3/2*b^2*ln(c*x^2+b)/c^4
```

Maxima [A] time = 0.974718, size = 73, normalized size = 1.28

$$\frac{b^3}{2\left(c^5x^2 + bc^4\right)} + \frac{3\,b^2 \log\left(cx^2 + b\right)}{2\,c^4} + \frac{cx^4 - 4\,bx^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3/(c^5*x^2 + b*c^4) + 3/2*b^2*log(c*x^2 + b)/c^4 + 1/4*(c*x^4 - 4*b*x
^2)/c^3
```

Fricas [A] time = 1.44286, size = 143, normalized size = 2.51

$$\frac{c^3x^6 - 3\,bc^2x^4 - 4\,b^2cx^2 + 2\,b^3 + 6\left(b^2cx^2 + b^3\right)\log\left(cx^2 + b\right)}{4\left(c^5x^2 + bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*x^6 - 3*b*c^2*x^4 - 4*b^2*c*x^2 + 2*b^3 + 6*(b^2*c*x^2 + b^3)*log(
c*x^2 + b))/(c^5*x^2 + b*c^4)
```

Sympy [A] time = 0.436322, size = 53, normalized size = 0.93

$$\frac{b^3}{2bc^4 + 2c^5x^2} + \frac{3b^2 \log\left(b + cx^2\right)}{2c^4} - \frac{bx^2}{c^3} + \frac{x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*log(b + c*x**2)/(2*c**4) - b*x**2/c*
*3 + x**4/(4*c**2)
```

Giac [A] time = 1.25606, size = 90, normalized size = 1.58

$$\frac{3\,b^2\log\left(\left|cx^2+b\right|\right)}{2\,c^4}+\frac{c^2x^4-4\,bcx^2}{4\,c^4}-\frac{3\,b^2cx^2+2\,b^3}{2\left(cx^2+b\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(abs(c*x^2 + b))/c^4 + 1/4*(c^2*x^4 - 4*b*c*x^2)/c^4 - 1/2*(3*b^
2*c*x^2 + 2*b^3)/((c*x^2 + b)*c^4)
```

3.192 $\int \frac{x^{10}}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=66

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{5bx}{2c^3}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5x^3}{6c^2}$$

```
[Out] (-5*b*x)/(2*c^3) + (5*x^3)/(6*c^2) - x^5/(2*c*(b + c*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rubi [A] time = 0.0347578, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 302, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{5bx}{2c^3}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5x^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-5*b*x)/(2*c^3) + (5*x^3)/(6*c^2) - x^5/(2*c*(b + c*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{10}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^6}{\left(b+cx^2\right)^2}\,dx \\
&= -\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5\int \frac{x^4}{b+cx^2}\,dx}{2c} \\
&= -\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right)dx}{2c} \\
&= -\frac{5bx}{2c^3}+\frac{5x^3}{6c^2}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{\left(5b^2\right)\int \frac{1}{b+cx^2}\,dx}{2c^3} \\
&= -\frac{5bx}{2c^3}+\frac{5x^3}{6c^2}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0414106, size = 60, normalized size = 0.91

$$
\frac{x\left(-\frac{3b^2}{b+cx^2}-12b+2cx^2\right)}{6c^3}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(-12*b + 2*c*x^2 - (3*b^2)/(b + c*x^2)))/(6*c^3) + (5*b^(3/2)*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Maple [A] time = 0.052, size = 57, normalized size = 0.9

$$\frac{x^3}{3\,c^2} - 2\,\frac{bx}{c^3} - \frac{b^2x}{2\,c^3\left(cx^2+b\right)} + \frac{5\,b^2}{2\,c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/3*x^3/c^2-2*b*x/c^3-1/2/c^3*b^2*x/(c*x^2+b)+5/2/c^3*b^2/(b*c)^(1/2)*arcta
n(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54293, size = 348, normalized size = 5.27

$$\left[\frac{4\,c^2x^5 - 20\,bcx^3 - 30\,b^2x + 15\left(bcx^2+b^2\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{12\left(c^4x^2+bc^3\right)}, \frac{2\,c^2x^5 - 10\,bcx^3 - 15\,b^2x + 15\left(bcx^2+b^2\right)\sqrt{\frac{b}{c}}\,\mathrm{arctan}}{6\left(c^4x^2+bc^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*c^2*x^5 - 20*b*c*x^3 - 30*b^2*x + 15*(b*c*x^2 + b^2)*sqrt(-b/c)*lo
g((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^4*x^2 + b*c^3), 1/6*(2*c^
2*x^5 - 10*b*c*x^3 - 15*b^2*x + 15*(b*c*x^2 + b^2)*sqrt(b/c)*arctan(c*x*sqr
```

```
t(b/c)/b))/(c^4*x^2 + b*c^3)]
```

Sympy [A] time = 0.489909, size = 107, normalized size = 1.62

$$-\frac{b^2x}{2bc^3+2c^4x^2}-\frac{2bx}{c^3}-\frac{5\sqrt{-\frac{b^3}{c^7}}\log\left(x-\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{5\sqrt{-\frac{b^3}{c^7}}\log\left(x+\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{x^3}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2)**2,x)
```

```
[Out] -b**2*x/(2*b*c**3 + 2*c**4*x**2) - 2*b*x/c**3 - 5*sqrt(-b**3/c**7)*log(x -
c**3*sqrt(-b**3/c**7)/b)/4 + 5*sqrt(-b**3/c**7)*log(x + c**3*sqrt(-b**3/c**
7)/b)/4 + x**3/(3*c**2)
```

Giac [A] time = 1.2252, size = 82, normalized size = 1.24

$$\frac{5\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^3}-\frac{b^2x}{2\left(cx^2+b\right)c^3}+\frac{c^4x^3-6\,bc^3x}{3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*b^2*x/((c*x^2 + b)*c^3)
 + 1/3*(c^4*x^3 - 6*b*c^3*x)/c^6
```

3.193 $\int \frac{x^9}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{b^2}{2c^3\left(b+cx^2\right)}-\frac{b\log\left(b+cx^2\right)}{c^3}+\frac{x^2}{2c^2}$$

```
[Out] x^2/(2*c^2) - b^2/(2*c^3*(b + c*x^2)) - (b*Log[b + c*x^2])/c^3
```

Rubi [A] time = 0.0370274, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{2c^3\left(b+cx^2\right)}-\frac{b\log\left(b+cx^2\right)}{c^3}+\frac{x^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^2/(2*c^2) - b^2/(2*c^3*(b + c*x^2)) - (b*Log[b + c*x^2])/c^3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^5}{\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(b+cx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{c^2}+\frac{b^2}{c^2(b+cx)^2}-\frac{2b}{c^2(b+cx)}\right)dx, x, x^2\right)\\ &= \frac{x^2}{2c^2}-\frac{b^2}{2c^3\left(b+cx^2\right)}-\frac{b\log\left(b+cx^2\right)}{c^3}\end{aligned}$$

Mathematica [A] time = 0.0153954, size = 38, normalized size = 0.86

$$\frac{-\frac{b^2}{b+cx^2}-2b\log\left(b+cx^2\right)+cx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (c*x^2 - b^2/(b + c*x^2) - 2*b*Log[b + c*x^2])/(2*c^3)
```

Maple [A] time = 0.05, size = 41, normalized size = 0.9

$$\frac{x^2}{2\,c^2}-\frac{b^2}{2\,c^3\left(cx^2+b\right)}-\frac{b\ln\left(cx^2+b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^2/c^2-1/2*b^2/c^3/(c*x^2+b)-b*ln(c*x^2+b)/c^3
```

Maxima [A] time = 0.983227, size = 58, normalized size = 1.32

$$-\frac{b^2}{2\left(c^4x^2+bc^3\right)}+\frac{x^2}{2\,c^2}-\frac{b\log\left(cx^2+b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2/(c^4*x^2 + b*c^3) + 1/2*x^2/c^2 - b*log(c*x^2 + b)/c^3
```

Fricas [A] time = 1.41205, size = 113, normalized size = 2.57

$$\frac{c^2x^4+bcx^2-b^2-2\left(bcx^2+b^2\right)\log\left(cx^2+b\right)}{2\left(c^4x^2+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^4 + b*c*x^2 - b^2 - 2*(b*c*x^2 + b^2)*log(c*x^2 + b))/(c^4*x^2 +
 b*c^3)
```

Sympy [A] time = 0.457414, size = 39, normalized size = 0.89

$$-\frac{b^2}{2bc^3+2c^4x^2}-\frac{b\log\left(b+cx^2\right)}{c^3}+\frac{x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**2,x)
```

```
[Out] -b**2/(2*b*c**3 + 2*c**4*x**2) - b*log(b + c*x**2)/c**3 + x**2/(2*c**2)
```

Giac [A] time = 1.23267, size = 66, normalized size = 1.5

$$\frac{x^2}{2\,c^2} - \frac{b\log\left(\left|cx^2 + b\right|\right)}{c^3} + \frac{2\,bcx^2 + b^2}{2\,\left(cx^2 + b\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/c^2 - b*log(abs(c*x^2 + b))/c^3 + 1/2*(2*b*c*x^2 + b^2)/((c*x^2 + b
)*c^3)
```

3.194 $\int \frac{x^8}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}-\frac{x^3}{2c\left(b+cx^2\right)}+\frac{3x}{2c^2}$$

```
[Out] (3*x)/(2*c^2) - x^3/(2*c*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[
b]])/(2*c^(5/2))
```

Rubi [A] time = 0.0244315, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 321, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}-\frac{x^3}{2c\left(b+cx^2\right)}+\frac{3x}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (3*x)/(2*c^2) - x^3/(2*c*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[
b]])/(2*c^(5/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^4}{\left(b+cx^2\right)^2}\,dx \\
&= -\frac{x^3}{2c\left(b+cx^2\right)}+\frac{3\int \frac{x^2}{b+cx^2}\,dx}{2c} \\
&= \frac{3x}{2c^2}-\frac{x^3}{2c\left(b+cx^2\right)}-\frac{(3b)\int \frac{1}{b+cx^2}\,dx}{2c^2} \\
&= \frac{3x}{2c^2}-\frac{x^3}{2c\left(b+cx^2\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0315579, size = 51, normalized size = 0.93

$$\frac{bx}{2c^2\left(b+cx^2\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/c^2 + (b*x)/(2*c^2*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])
/(2*c^(5/2))
```

Maple [A] time = 0.05, size = 43, normalized size = 0.8

$$\frac{x}{c^2}+\frac{bx}{2\,c^2\left(cx^2+b\right)}-\frac{3\,b}{2\,c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2)^2,x)
```

```
[Out] x/c^2+1/2/c^2*b*x/(c*x^2+b)-3/2/c^2*b/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53923, size = 285, normalized size = 5.18

$$\left[\frac{4\,cx^3+3\left(cx^2+b\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+6\,bx}{4\left(c^3x^2+bc^2\right)},\ \frac{2\,cx^3-3\left(cx^2+b\right)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)+3\,bx}{2\left(c^3x^2+bc^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*x^3 + 3*(c*x^2 + b)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)
/(c*x^2 + b)) + 6*b*x)/(c^3*x^2 + b*c^2), 1/2*(2*c*x^3 - 3*(c*x^2 + b)*sqrt
(b/c)*arctan(c*x*sqrt(b/c)/b) + 3*b*x)/(c^3*x^2 + b*c^2)]
```

Sympy [A] time = 0.464708, size = 83, normalized size = 1.51

$$\frac{bx}{2bc^2 + 2c^3x^2} + \frac{3\sqrt{-\frac{b}{c^5}}\log\left(-c^2\sqrt{-\frac{b}{c^5}} + x\right)}{4} - \frac{3\sqrt{-\frac{b}{c^5}}\log\left(c^2\sqrt{-\frac{b}{c^5}} + x\right)}{4} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2)**2,x)
```

```
[Out] b*x/(2*b*c**2 + 2*c**3*x**2) + 3*sqrt(-b/c**5)*log(-c**2*sqrt(-b/c**5) + x)
/4 - 3*sqrt(-b/c**5)*log(c**2*sqrt(-b/c**5) + x)/4 + x/c**2
```

Giac [A] time = 1.2387, size = 57, normalized size = 1.04

$$-\frac{3\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^2} + \frac{bx}{2\left(cx^2 + b\right)c^2} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*b*x/((c*x^2 + b)*c^2) +
x/c^2
```

3.195 $\int \frac{x^7}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=33

$$\frac{b}{2c^2(b+cx^2)}+\frac{\log(b+cx^2)}{2c^2}$$

```
[Out] b/(2*c^2*(b + c*x^2)) + Log[b + c*x^2]/(2*c^2)
```

Rubi [A] time = 0.0319761, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b}{2c^2(b+cx^2)}+\frac{\log(b+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^2,x]
```

```
[Out] b/(2*c^2*(b + c*x^2)) + Log[b + c*x^2]/(2*c^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^3}{\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(b+cx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{b}{c(b+cx)^2}+\frac{1}{c(b+cx)}\right)dx, x, x^2\right)\\ &= \frac{b}{2c^2\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.007953, size = 27, normalized size = 0.82

$$\frac{\frac{b}{b+cx^2}+\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b/(b + c*x^2) + Log[b + c*x^2])/(2*c^2)
```

Maple [A] time = 0.051, size = 30, normalized size = 0.9

$$\frac{b}{2\,c^2\left(cx^2+b\right)}+\frac{\ln\left(cx^2+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*b/c^2/(c*x^2+b)+1/2*ln(c*x^2+b)/c^2
```

Maxima [A] time = 0.965196, size = 43, normalized size = 1.3

$$\frac{b}{2\left(c^3x^2+bc^2\right)}+\frac{\log\left(cx^2+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b/(c^3*x^2 + b*c^2) + 1/2*log(c*x^2 + b)/c^2
```

Fricas [A] time = 1.49813, size = 76, normalized size = 2.3

$$\frac{\left(cx^2+b\right)\log\left(cx^2+b\right)+b}{2\left(c^3x^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((c*x^2 + b)*log(c*x^2 + b) + b)/(c^3*x^2 + b*c^2)
```

Sympy [A] time = 0.36137, size = 29, normalized size = 0.88

$$\frac{b}{2bc^2+2c^3x^2}+\frac{\log\left(b+cx^2\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**2,x)
```

```
[Out] b/(2*b*c**2 + 2*c**3*x**2) + log(b + c*x**2)/(2*c**2)
```

Giac [A] time = 1.20305, size = 43, normalized size = 1.3

$$-\frac{x^2}{2\left(cx^2+b\right)c}+\frac{\log\left(\left|cx^2+b\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^2/((c*x^2 + b)*c) + 1/2*log(abs(c*x^2 + b))/c^2
```

3.196 $\int \frac{x^6}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}}-\frac{x}{2c\left(b+cx^2\right)}$$

```
[Out] -x/(2*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

Rubi [A] time = 0.0188261, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}}-\frac{x}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -x/(2*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{x^2}{\left(b+cx^2\right)^2}\,dx\\ &= -\frac{x}{2c\left(b+cx^2\right)}+\frac{\int \frac{1}{b+cx^2}\,dx}{2c}\\ &= -\frac{x}{2c\left(b+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.021083, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}}-\frac{x}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -x/(2*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

Maple [A] time = 0.051, size = 36, normalized size = 0.8

$$-\frac{x}{2\,c\left(cx^2+b\right)}+\frac{1}{2\,c}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x/c/(c*x^2+b)+1/2/c/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53569, size = 263, normalized size = 5.84

$$\left[-\frac{2\,bcx + \left(cx^2 + b\right)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{4\left(bc^3x^2 + b^2c^2\right)}, -\frac{bcx - \left(cx^2 + b\right)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{2\left(bc^3x^2 + b^2c^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b*c*x + (c*x^2 + b)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c
*x^2 + b)))/(b*c^3*x^2 + b^2*c^2), -1/2*(b*c*x - (c*x^2 + b)*sqrt(b*c)*arct
an(sqrt(b*c)*x/b))/(b*c^3*x^2 + b^2*c^2)]
```

Sympy [B] time = 0.421139, size = 78, normalized size = 1.73

$$-\frac{x}{2bc + 2c^2x^2} - \frac{\sqrt{-\frac{1}{bc^3}}\log\left(-bc\sqrt{-\frac{1}{bc^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{bc^3}}\log\left(bc\sqrt{-\frac{1}{bc^3}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**2,x)
```

```
[Out] -x/(2*b*c + 2*c**2*x**2) - sqrt(-1/(b*c**3))*log(-b*c*sqrt(-1/(b*c**3)) + x
)/4 + sqrt(-1/(b*c**3))*log(b*c*sqrt(-1/(b*c**3)) + x)/4
```

Giac [A] time = 1.21564, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c} - \frac{x}{2\left(cx^2 + b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) - 1/2*x/((c*x^2 + b)*c)
```

3.197 $\int \frac{x^5}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2c(b+cx^2)}$$

[Out] -1/(2*c*(b + c*x^2))

Rubi [A] time = 0.0093297, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$-\frac{1}{2c(b+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^5/(b*x^2 + c*x^4)^2,x]

[Out] -1/(2*c*(b + c*x^2))

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x}{\left(b+cx^2\right)^2}\,dx$$
$$= -\frac{1}{2c\left(b+cx^2\right)}$$

Mathematica [A] time = 0.0020596, size = 16, normalized size = 1.

$$-\frac{1}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/(2*c*(b + c*x^2))
```

Maple [A] time = 0.042, size = 15, normalized size = 0.9

$$-\frac{1}{2\,c\left(cx^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/c/(c*x^2+b)
```

Maxima [A] time = 0.970984, size = 20, normalized size = 1.25

$$-\frac{1}{2\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(c^2*x^2 + b*c)
```

Fricas [A] time = 1.40502, size = 30, normalized size = 1.88

$$-\frac{1}{2\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2/(c^2*x^2 + b*c)
```

Sympy [A] time = 0.337265, size = 15, normalized size = 0.94

$$-\frac{1}{2bc+2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**2,x)
```

```
[Out] -1/(2*b*c + 2*c**2*x**2)
```

Giac [A] time = 1.26203, size = 19, normalized size = 1.19

$$-\frac{1}{2\left(cx^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2/((c*x^2 + b)*c)
```

3.198 $\int \frac{x^4}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\left(b+cx^2\right)}$$

```
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

Rubi [A] time = 0.0173041, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{\left(b+cx^2\right)^2}\,dx$$
$$= \frac{x}{2b\left(b+cx^2\right)} + \frac{\int \frac{1}{b+cx^2}\,dx}{2b}$$
$$= \frac{x}{2b\left(b+cx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}$$

Mathematica [A] time = 0.0247956, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}} + \frac{x}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

Maple [A] time = 0.048, size = 36, normalized size = 0.8

$$\frac{x}{2\,b\left(cx^2+b\right)} + \frac{1}{2\,b}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x/b/(c*x^2+b)+1/2/b/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48401, size = 261, normalized size = 5.8

$$\left[\frac{2\,bcx-\left(cx^2+b\right)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{4\left(b^2c^2x^2+b^3c\right)},\frac{bcx+\left(cx^2+b\right)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{2\left(b^2c^2x^2+b^3c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c*x - (c*x^2 + b)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*
x^2 + b)))/(b^2*c^2*x^2 + b^3*c), 1/2*(b*c*x + (c*x^2 + b)*sqrt(b*c)*arctan
(sqrt(b*c)*x/b))/(b^2*c^2*x^2 + b^3*c)]
```

Sympy [B] time = 0.390433, size = 78, normalized size = 1.73

$$\frac{x}{2b^2+2bcx^2}-\frac{\sqrt{-\frac{1}{b^3c}}\log\left(-b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{b^3c}}\log\left(b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**2,x)
```

```
[Out] x/(2*b**2 + 2*b*c*x**2) - sqrt(-1/(b**3*c))*log(-b**2*sqrt(-1/(b**3*c)) + x
)/4 + sqrt(-1/(b**3*c))*log(b**2*sqrt(-1/(b**3*c)) + x)/4
```

Giac [A] time = 1.26283, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}b} + \frac{x}{2\left(cx^2 + b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) + 1/2*x/((c*x^2 + b)*b)
```

3.199 $\int \frac{x^3}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=38

$$-\frac{\log(b+cx^2)}{2b^2}+\frac{\log(x)}{b^2}+\frac{1}{2b(b+cx^2)}$$

```
[Out] 1/(2*b*(b + c*x^2)) + Log[x]/b^2 - Log[b + c*x^2]/(2*b^2)
```

Rubi [A] time = 0.0347204, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{\log(b+cx^2)}{2b^2}+\frac{\log(x)}{b^2}+\frac{1}{2b(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4)^2,x]
```

```
[Out] 1/(2*b*(b + c*x^2)) + Log[x]/b^2 - Log[b + c*x^2]/(2*b^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(bx^2+cx^4\right)^2}\,dx&=\int \frac{1}{x\left(b+cx^2\right)^2}\,dx\\&=\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(b+cx)^2}\,dx,x,x^2\right)\\&=\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^2x}-\frac{c}{b(b+cx)^2}-\frac{c}{b^2(b+cx)}\right)dx,x,x^2\right)\\&=\frac{1}{2b\left(b+cx^2\right)}+\frac{\log(x)}{b^2}-\frac{\log\left(b+cx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0127739, size = 33, normalized size = 0.87

$$\frac{\frac{b}{b+cx^2}-\log\left(b+cx^2\right)+2\log(x)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b/(b + c*x^2) + 2*Log[x] - Log[b + c*x^2])/(2*b^2)
```

Maple [A] time = 0.054, size = 35, normalized size = 0.9

$$\frac{1}{2\,b\left(cx^2+b\right)}+\frac{\ln(x)}{b^2}-\frac{\ln\left(cx^2+b\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b/(c*x^2+b)+ln(x)/b^2-1/2*ln(c*x^2+b)/b^2
```

Maxima [A] time = 0.965884, size = 50, normalized size = 1.32

$$\frac{1}{2\left(bcx^2+b^2\right)}-\frac{\log\left(cx^2+b\right)}{2\,b^2}+\frac{\log\left(x^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(b*c*x^2 + b^2) - 1/2*log(c*x^2 + b)/b^2 + 1/2*log(x^2)/b^2
```

Fricas [A] time = 1.4848, size = 108, normalized size = 2.84

$$-\frac{\left(cx^2+b\right)\log\left(cx^2+b\right)-2\left(cx^2+b\right)\log\left(x\right)-b}{2\left(b^2cx^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((c*x^2 + b)*log(c*x^2 + b) - 2*(c*x^2 + b)*log(x) - b)/(b^2*c*x^2 + b
^3)
```

Sympy [A] time = 0.492261, size = 34, normalized size = 0.89

$$\frac{1}{2b^2+2bcx^2}+\frac{\log\left(x\right)}{b^2}-\frac{\log\left(\frac{b}{c}+x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b**2 + 2*b*c*x**2) + log(x)/b**2 - log(b/c + x**2)/(2*b**2)
```

Giac [A] time = 1.20935, size = 49, normalized size = 1.29

$$-\frac{\log\left(\left|cx^2+b\right|\right)}{2\,b^2}+\frac{\log\left(\left|x\right|\right)}{b^2}+\frac{1}{2\left(cx^2+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*log(abs(c*x^2 + b))/b^2 + log(abs(x))/b^2 + 1/2/((c*x^2 + b)*b)
```

3.200 $\int \frac{x^2}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}$$

```
[Out] -3/(2*b^2*x) + 1/(2*b*x*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b
]])/(2*b^(5/2))
```

Rubi [A] time = 0.025463, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$-\frac{3\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^2,x]
```



```
[Out] -3/(2*b^2*x) + 1/(2*b*x*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b
]])/(2*b^(5/2))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^2\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2bx\left(b+cx^2\right)}+\frac{3\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{2b}\\ &= -\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}-\frac{(3c)\int \frac{1}{b+cx^2}\,dx}{2b^2}\\ &= -\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.036817, size = 54, normalized size = 0.95

$$-\frac{cx}{2b^2\left(b+cx^2\right)}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{1}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(1/(b^2*x)) - (c*x)/(2*b^2*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(2*b^(5/2))
```

Maple [A] time = 0.053, size = 46, normalized size = 0.8

$$-\frac{1}{b^{2}x}-\frac{cx}{2\,b^{2}\left(cx^{2}+b\right)}-\frac{3\,c}{2\,b^{2}}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/b^2/x-1/2*c/b^2*x/(c*x^2+b)-3/2*c/b^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49902, size = 288, normalized size = 5.05

$$\left[-\frac{6\,cx^{2}-3\left(cx^{3}+bx\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^{2}-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)+4\,b}{4\left(b^{2}cx^{3}+b^{3}x\right)},-\frac{3\,cx^{2}+3\left(cx^{3}+bx\right)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+2\,b}{2\left(b^{2}cx^{3}+b^{3}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*c*x^2 - 3*(c*x^3 + b*x)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) -
 b)/(c*x^2 + b)) + 4*b)/(b^2*c*x^3 + b^3*x), -1/2*(3*c*x^2 + 3*(c*x^3 + b*x
)*sqrt(c/b)*arctan(x*sqrt(c/b)) + 2*b)/(b^2*c*x^3 + b^3*x)]
```

Sympy [A] time = 0.525042, size = 90, normalized size = 1.58

$$\frac{3\sqrt{-\frac{c}{b^5}}\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}-\frac{3\sqrt{-\frac{c}{b^5}}\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}-\frac{2b+3cx^2}{2b^3x+2b^2cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 3*sqrt(-c/b**5)*log(-b**3*sqrt(-c/b**5)/c + x)/4 - 3*sqrt(-c/b**5)*log(b**3
*sqrt(-c/b**5)/c + x)/4 - (2*b + 3*c*x**2)/(2*b**3*x + 2*b**2*c*x**3)
```

Giac [A] time = 1.26842, size = 63, normalized size = 1.11

$$-\frac{3\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^2}-\frac{3\,cx^2+2\,b}{2\,(cx^3+bx)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/2*(3*c*x^2 + 2*b)/((c*x^3
+ b*x)*b^2)
```

3.201 $\int \frac{x}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=49

$$-\frac{c}{2b^2\left(b+cx^2\right)}+\frac{c\log\left(b+cx^2\right)}{b^3}-\frac{2c\log(x)}{b^3}-\frac{1}{2b^2x^2}$$

```
[Out] -1/(2*b^2*x^2) - c/(2*b^2*(b + c*x^2)) - (2*c*Log[x])/b^3 + (c*Log[b + c*x^
2])/b^3
```

Rubi [A] time = 0.0405369, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 266, 44}

$$-\frac{c}{2b^2\left(b+cx^2\right)}+\frac{c\log\left(b+cx^2\right)}{b^3}-\frac{2c\log(x)}{b^3}-\frac{1}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/(2*b^2*x^2) - c/(2*b^2*(b + c*x^2)) - (2*c*Log[x])/b^3 + (c*Log[b + c*x^
2])/b^3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^3\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^2x^2}-\frac{2c}{b^3x}+\frac{c^2}{b^2(b+cx)^2}+\frac{2c^2}{b^3(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{1}{2b^2x^2}-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{2c\log(x)}{b^3}+\frac{c\log\left(b+cx^2\right)}{b^3}
\end{aligned}
$$

Mathematica [A] time = 0.0363019, size = 41, normalized size = 0.84

$$
-\frac{b\left(\frac{c}{b+cx^2}+\frac{1}{x^2}\right)-2c\log\left(b+cx^2\right)+4c\log(x)}{2b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*(x^(-2) + c/(b + c*x^2)) + 4*c*Log[x] - 2*c*Log[b + c*x^2])/(2*b^3)
```

Maple [A] time = 0.057, size = 46, normalized size = 0.9

$$
-\frac{1}{2\,b^2x^2}-\frac{c}{2\,b^2\left(cx^2+b\right)}-2\,\frac{c\ln(x)}{b^3}+\frac{c\ln\left(cx^2+b\right)}{b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2/x^2-1/2*c/b^2/(c*x^2+b)-2*c*ln(x)/b^3+c*ln(c*x^2+b)/b^3
```

Maxima [A] time = 0.962003, size = 70, normalized size = 1.43

$$-\frac{2\,cx^2+b}{2\left(b^2cx^4+b^3x^2\right)}+\frac{c\log\left(cx^2+b\right)}{b^3}-\frac{c\log\left(x^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2) + c*log(c*x^2 + b)/b^3 - c*log(x^2
)/b^3
```

Fricas [A] time = 1.5285, size = 157, normalized size = 3.2

$$-\frac{2\,bcx^2+b^2-2\left(c^2x^4+bcx^2\right)\log\left(cx^2+b\right)+4\left(c^2x^4+bcx^2\right)\log\left(x\right)}{2\left(b^3cx^4+b^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*x^2 + b^2 - 2*(c^2*x^4 + b*c*x^2)*log(c*x^2 + b) + 4*(c^2*x^4 +
 b*c*x^2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

Sympy [A] time = 0.577194, size = 49, normalized size = 1.

$$-\frac{b+2cx^2}{2b^3x^2+2b^2cx^4}-\frac{2c\log\left(x\right)}{b^3}+\frac{c\log\left(\frac{b}{c}+x^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**2,x)
```

```
[Out] -(b + 2*c*x**2)/(2*b**3*x**2 + 2*b**2*c*x**4) - 2*c*log(x)/b**3 + c*log(b/c
 + x**2)/b**3
```

Giac [A] time = 1.31746, size = 68, normalized size = 1.39

$$\frac{c \log \left(\left|cx^{2}+b\right|\right)}{b^{3}}-\frac{2 c \log \left(\left|x\right|\right)}{b^{3}}-\frac{2 cx^{2}+b}{2 \left(cx^{4}+bx^{2}\right) b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] c*log(abs(c*x^2 + b))/b^3 - 2*c*log(abs(x))/b^3 - 1/2*(2*c*x^2 + b)/((c*x^4
 + b*x^2)*b^2)
```

3.202 $\int \frac{1}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=68

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5c}{2b^3x}-\frac{5}{6b^2x^3}+\frac{1}{2bx^3\left(b+cx^2\right)}$$

```
[Out] -5/(6*b^2*x^3) + (5*c)/(2*b^3*x) + 1/(2*b*x^3*(b + c*x^2)) + (5*c^(3/2)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rubi [A] time = 0.029305, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1593, 290, 325, 205}

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5c}{2b^3x}-\frac{5}{6b^2x^3}+\frac{1}{2bx^3\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] -5/(6*b^2*x^3) + (5*c)/(2*b^3*x) + 1/(2*b*x^3*(b + c*x^2)) + (5*c^(3/2)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^4\left(b+cx^2\right)^2}\,dx\\
&= \frac{1}{2bx^3\left(b+cx^2\right)}+\frac{5\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{2b}\\
&= -\frac{5}{6b^2x^3}+\frac{1}{2bx^3\left(b+cx^2\right)}-\frac{(5c)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{2b^2}\\
&= -\frac{5}{6b^2x^3}+\frac{5c}{2b^3x}+\frac{1}{2bx^3\left(b+cx^2\right)}+\frac{\left(5c^2\right)\int \frac{1}{b+cx^2}\,dx}{2b^3}\\
&= -\frac{5}{6b^2x^3}+\frac{5c}{2b^3x}+\frac{1}{2bx^3\left(b+cx^2\right)}+\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0383423, size = 67, normalized size = 0.99

$$\frac{c^2x}{2b^3\left(b+cx^2\right)}+\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{2c}{b^3x}-\frac{1}{3b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] -1/(3*b^2*x^3) + (2*c)/(b^3*x) + (c^2*x)/(2*b^3*(b + c*x^2)) + (5*c^(3/2)*A
rcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Maple [A] time = 0.055, size = 59, normalized size = 0.9

$$-\frac{1}{3\,b^2x^3}+2\,\frac{c}{b^3x}+\frac{c^2x}{2\,b^3\left(cx^2+b\right)}+\frac{5\,c^2}{2\,b^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/3/b^2/x^3+2*c/b^3/x+1/2*c^2/b^3*x/(c*x^2+b)+5/2*c^2/b^3/(b*c)^(1/2)*arct
an(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58413, size = 359, normalized size = 5.28

$$\left[\frac{30\,c^2x^4+20\,bcx^2+15\left(c^2x^5+bcx^3\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-4\,b^2}{12\left(b^3cx^5+b^4x^3\right)},\ \frac{15\,c^2x^4+10\,bcx^2+15\left(c^2x^5+bcx^3\right)\sqrt{\frac{c}{b}}\arctan\left(x\right.}{6\left(b^3cx^5+b^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*c^2*x^4 + 20*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(-c/b)*log((c*x
^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 4*b^2)/(b^3*c*x^5 + b^4*x^3), 1/6
*(15*c^2*x^4 + 10*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(c/b)*arctan(x*sqrt(
c/b)) - 2*b^2)/(b^3*c*x^5 + b^4*x^3)]
```

Sympy [A] time = 0.575817, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4}+\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4}+\frac{-2b^2+10bcx^2+15c^2x^4}{6b^4x^3+6b^3cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**2,x)
```

```
[Out] -5*sqrt(-c**3/b**7)*log(-b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + 5*sqrt(-c**3/b
**7)*log(b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + (-2*b**2 + 10*b*c*x**2 + 15*c*
*2*x**4)/(6*b**4*x**3 + 6*b**3*c*x**5)
```

Giac [A] time = 1.23951, size = 80, normalized size = 1.18

$$\frac{5\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}b^3}+\frac{c^2x}{2\left(cx^2+b\right)b^3}+\frac{6\,cx^2-b}{3\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 5/2*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/2*c^2*x/((c*x^2 + b)*b^3)
 + 1/3*(6*c*x^2 - b)/(b^3*x^3)
```

3.203 $\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{c^2}{2b^3\left(b+cx^2\right)}-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}+\frac{3c^2\log(x)}{b^4}+\frac{c}{b^3x^2}-\frac{1}{4b^2x^4}$$

```
[Out] -1/(4*b^2*x^4) + c/(b^3*x^2) + c^2/(2*b^3*(b + c*x^2)) + (3*c^2*Log[x])/b^4
 - (3*c^2*Log[b + c*x^2])/(2*b^4)
```

Rubi [A] time = 0.054881, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$\frac{c^2}{2b^3\left(b+cx^2\right)}-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}+\frac{3c^2\log(x)}{b^4}+\frac{c}{b^3x^2}-\frac{1}{4b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/(4*b^2*x^4) + c/(b^3*x^2) + c^2/(2*b^3*(b + c*x^2)) + (3*c^2*Log[x])/b^4
 - (3*c^2*Log[b + c*x^2])/(2*b^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^5\left(b+cx^2\right)^2}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(b+cx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^2x^3}-\frac{2c}{b^3x^2}+\frac{3c^2}{b^4x}-\frac{c^3}{b^3(b+cx)^2}-\frac{3c^3}{b^4(b+cx)}\right)dx, x, x^2\right)\\
&= -\frac{1}{4b^2x^4}+\frac{c}{b^3x^2}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{3c^2\log(x)}{b^4}-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0504022, size = 57, normalized size = 0.86

$$\frac{b\left(\frac{2c^2}{b+cx^2}-\frac{b}{x^4}+\frac{4c}{x^2}\right)-6c^2\log\left(b+cx^2\right)+12c^2\log(x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (b*(-(b/x^4) + (4*c)/x^2 + (2*c^2)/(b + c*x^2)) + 12*c^2*Log[x] - 6*c^2*Log
[b + c*x^2])/(4*b^4)
```

Maple [A] time = 0.056, size = 61, normalized size = 0.9

$$-\frac{1}{4\,b^2x^4}+\frac{c}{b^3x^2}+\frac{c^2}{2\,b^3\left(cx^2+b\right)}+3\,\frac{c^2\ln(x)}{b^4}-\frac{3\,c^2\ln\left(cx^2+b\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/4/b^2/x^4+c/b^3/x^2+1/2*c^2/b^3/(c*x^2+b)+3*c^2*ln(x)/b^4-3/2*c^2*ln(c*x
^2+b)/b^4
```

Maxima [A] time = 0.973278, size = 95, normalized size = 1.44

$$\frac{6\,c^2x^4+3\,bcx^2-b^2}{4\left(b^3cx^6+b^4x^4\right)}-\frac{3\,c^2\log\left(cx^2+b\right)}{2\,b^4}+\frac{3\,c^2\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*c^2*x^4 + 3*b*c*x^2 - b^2)/(b^3*c*x^6 + b^4*x^4) - 3/2*c^2*log(c*x^2
 + b)/b^4 + 3/2*c^2*log(x^2)/b^4
```

Fricas [A] time = 1.50927, size = 184, normalized size = 2.79

$$\frac{6\,bc^2x^4+3\,b^2cx^2-b^3-6\left(c^3x^6+bc^2x^4\right)\log\left(cx^2+b\right)+12\left(c^3x^6+bc^2x^4\right)\log\left(x\right)}{4\left(b^4cx^6+b^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*c^2*x^4 + 3*b^2*c*x^2 - b^3 - 6*(c^3*x^6 + b*c^2*x^4)*log(c*x^2 +
b) + 12*(c^3*x^6 + b*c^2*x^4)*log(x))/(b^4*c*x^6 + b^5*x^4)
```

Sympy [A] time = 0.812859, size = 68, normalized size = 1.03

$$\frac{-b^2+3bcx^2+6c^2x^4}{4b^4x^4+4b^3cx^6}+\frac{3c^2\log\left(x\right)}{b^4}-\frac{3c^2\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-b**2 + 3*b*c*x**2 + 6*c**2*x**4)/(4*b**4*x**4 + 4*b**3*c*x**6) + 3*c**2*l
og(x)/b**4 - 3*c**2*log(b/c + x**2)/(2*b**4)
```

Giac [A] time = 1.2248, size = 116, normalized size = 1.76

$$\frac{3\,c^2\log\left(x^2\right)}{2\,b^4}-\frac{3\,c^2\log\left(\left|cx^2+b\right|\right)}{2\,b^4}+\frac{3\,c^3x^2+4\,bc^2}{2\left(cx^2+b\right)b^4}-\frac{9\,c^2x^4-4\,bcx^2+b^2}{4\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*c^2*log(x^2)/b^4 - 3/2*c^2*log(abs(c*x^2 + b))/b^4 + 1/2*(3*c^3*x^2 + 4
*b*c^2)/((c*x^2 + b)*b^4) - 1/4*(9*c^2*x^4 - 4*b*c*x^2 + b^2)/(b^4*x^4)
```

3.204 $\int \frac{1}{x^2(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{7c^2}{2b^4x}-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}+\frac{7c}{6b^3x^3}-\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}$$

```
[Out] -7/(10*b^2*x^5) + (7*c)/(6*b^3*x^3) - (7*c^2)/(2*b^4*x) + 1/(2*b*x^5*(b + c
*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rubi [A] time = 0.0454493, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$-\frac{7c^2}{2b^4x}-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}+\frac{7c}{6b^3x^3}-\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -7/(10*b^2*x^5) + (7*c)/(6*b^3*x^3) - (7*c^2)/(2*b^4*x) + 1/(2*b*x^5*(b + c
*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2 \left(bx^2+cx^4\right)^2} \, dx &= \int \frac{1}{x^6 \left(b+cx^2\right)^2} \, dx \\
&= \frac{1}{2bx^5 \left(b+cx^2\right)} + \frac{7 \int \frac{1}{x^6\left(b+cx^2\right)} \, dx}{2b} \\
&= -\frac{7}{10b^2x^5} + \frac{1}{2bx^5 \left(b+cx^2\right)} - \frac{(7c) \int \frac{1}{x^4\left(b+cx^2\right)} \, dx}{2b^2} \\
&= -\frac{7}{10b^2x^5} + \frac{7c}{6b^3x^3} + \frac{1}{2bx^5 \left(b+cx^2\right)} + \frac{\left(7c^2\right) \int \frac{1}{x^2\left(b+cx^2\right)} \, dx}{2b^3} \\
&= -\frac{7}{10b^2x^5} + \frac{7c}{6b^3x^3} - \frac{7c^2}{2b^4x} + \frac{1}{2bx^5 \left(b+cx^2\right)} - \frac{\left(7c^3\right) \int \frac{1}{b+cx^2} \, dx}{2b^4} \\
&= -\frac{7}{10b^2x^5} + \frac{7c}{6b^3x^3} - \frac{7c^2}{2b^4x} + \frac{1}{2bx^5 \left(b+cx^2\right)} - \frac{7c^{5/2} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0441998, size = 80, normalized size = 0.99

$$
-\frac{c^3 x}{2b^4 \left(b+cx^2\right)} - \frac{3c^2}{b^4 x} - \frac{7c^{5/2} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}} + \frac{2c}{3b^3x^3} - \frac{1}{5b^2x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/(5*b^2*x^5) + (2*c)/(3*b^3*x^3) - (3*c^2)/(b^4*x) - (c^3*x)/(2*b^4*(b +
c*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Maple [A] time = 0.055, size = 70, normalized size = 0.9

$$-\frac{1}{5\,b^2x^5}-3\,\frac{c^2}{b^4x}+\frac{2\,c}{3\,b^3x^3}-\frac{c^3x}{2\,b^4\left(cx^2+b\right)}-\frac{7\,c^3}{2\,b^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/5/b^2/x^5-3*c^2/b^4/x+2/3*c/b^3/x^3-1/2*c^3/b^4*x/(c*x^2+b)-7/2*c^3/b^4/
(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48718, size = 423, normalized size = 5.22

$$\left[-\frac{210\,c^3x^6+140\,bc^2x^4-28\,b^2cx^2+12\,b^3-105\left(c^3x^7+bc^2x^5\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{60\left(b^4cx^7+b^5x^5\right)},-\frac{105\,c^3x^6+70\,bc^2x^4-14\,b^2cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(210*c^3*x^6 + 140*b*c^2*x^4 - 28*b^2*c*x^2 + 12*b^3 - 105*(c^3*x^7
+ b*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b
^4*c*x^7 + b^5*x^5), -1/30*(105*c^3*x^6 + 70*b*c^2*x^4 - 14*b^2*c*x^2 + 6*b
^3 + 105*(c^3*x^7 + b*c^2*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^4*c*x^7 +
b^5*x^5)]
```

Sympy [A] time = 0.848313, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{c^5}{b^9}}\log\left(-\frac{b^5\sqrt{-\frac{c^5}{b^9}}}{c^3}+x\right)}{4}-\frac{7\sqrt{-\frac{c^5}{b^9}}\log\left(\frac{b^5\sqrt{-\frac{c^5}{b^9}}}{c^3}+x\right)}{4}-\frac{6b^3-14b^2cx^2+70bc^2x^4+105c^3x^6}{30b^5x^5+30b^4cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 7*sqrt(-c**5/b**9)*log(-b**5*sqrt(-c**5/b**9)/c**3 + x)/4 - 7*sqrt(-c**5/b*
*9)*log(b**5*sqrt(-c**5/b**9)/c**3 + x)/4 - (6*b**3 - 14*b**2*c*x**2 + 70*b
*c**2*x**4 + 105*c**3*x**6)/(30*b**5*x**5 + 30*b**4*c*x**7)
```

Giac [A] time = 1.25199, size = 95, normalized size = 1.17

$$-\frac{7\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^4}-\frac{c^3x}{2\left(cx^2+b\right)b^4}-\frac{45\,c^2x^4-10\,bcx^2+3\,b^2}{15\,b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/2*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/2*c^3*x/((c*x^2 + b)*b^4
) - 1/15*(45*c^2*x^4 - 10*b*c*x^2 + 3*b^2)/(b^4*x^5)
```

3.205 $\int \frac{x^{14}}{\left(bx^2+cx^4\right)^3} dx$

Optimal. Leaf size=85

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{7x^5}{8c^2\left(b+cx^2\right)}-\frac{35bx}{8c^4}-\frac{x^7}{4c\left(b+cx^2\right)^2}+\frac{35x^3}{24c^3}$$

```
[Out] (-35*b*x)/(8*c^4) + (35*x^3)/(24*c^3) - x^7/(4*c*(b + c*x^2)^2) - (7*x^5)/(
8*c^2*(b + c*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rubi [A] time = 0.0424448, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 302, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{7x^5}{8c^2\left(b+cx^2\right)}-\frac{35bx}{8c^4}-\frac{x^7}{4c\left(b+cx^2\right)^2}+\frac{35x^3}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-35*b*x)/(8*c^4) + (35*x^3)/(24*c^3) - x^7/(4*c*(b + c*x^2)^2) - (7*x^5)/(
8*c^2*(b + c*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^8}{\left(b+cx^2\right)^3}\,dx\\
&= -\frac{x^7}{4c\left(b+cx^2\right)^2}+\frac{7\int \frac{x^6}{\left(b+cx^2\right)^2}\,dx}{4c}\\
&= -\frac{x^7}{4c\left(b+cx^2\right)^2}-\frac{7x^5}{8c^2\left(b+cx^2\right)}+\frac{35\int \frac{x^4}{b+cx^2}\,dx}{8c^2}\\
&= -\frac{x^7}{4c\left(b+cx^2\right)^2}-\frac{7x^5}{8c^2\left(b+cx^2\right)}+\frac{35\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2\left(b+cx^2\right)}\right)dx}{8c^2}\\
&= -\frac{35bx}{8c^4}+\frac{35x^3}{24c^3}-\frac{x^7}{4c\left(b+cx^2\right)^2}-\frac{7x^5}{8c^2\left(b+cx^2\right)}+\frac{\left(35b^2\right)\int \frac{1}{b+cx^2}\,dx}{8c^4}\\
&= -\frac{35bx}{8c^4}+\frac{35x^3}{24c^3}-\frac{x^7}{4c\left(b+cx^2\right)^2}-\frac{7x^5}{8c^2\left(b+cx^2\right)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0470508, size = 77, normalized size = 0.91

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{175b^2cx^3+105b^3x+56bc^2x^5-8c^3x^7}{24c^4\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(105*b^3*x + 175*b^2*c*x^3 + 56*b*c^2*x^5 - 8*c^3*x^7)/(24*c^4*(b + c*x^2)
^2) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Maple [A] time = 0.051, size = 77, normalized size = 0.9

$$\frac{x^3}{3c^3} - 3\frac{bx}{c^4} - \frac{13b^2x^3}{8c^3(cx^2+b)^2} - \frac{11b^3x}{8c^4(cx^2+b)^2} + \frac{35b^2}{8c^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/3*x^3/c^3-3*b*x/c^4-13/8/c^3*b^2/(c*x^2+b)^2*x^3-11/8/c^4*b^3/(c*x^2+b)^2
*x+35/8/c^4*b^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53735, size = 493, normalized size = 5.8

$$\left[\frac{16c^3x^7 - 112bc^2x^5 - 350b^2cx^3 - 210b^3x + 105(bc^2x^4 + 2b^2cx^2 + b^3)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{48(c^6x^4 + 2bc^5x^2 + b^2c^4)}, \frac{8c^3x^7 - 56bc^2x^5 - 17}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^3*x^7 - 112*b*c^2*x^5 - 350*b^2*c*x^3 - 210*b^3*x + 105*(b*c^2*
x^4 + 2*b^2*c*x^2 + b^3)*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x
^2 + b)))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4), 1/24*(8*c^3*x^7 - 56*b*c^2*x^5
 - 175*b^2*c*x^3 - 105*b^3*x + 105*(b*c^2*x^4 + 2*b^2*c*x^2 + b^3)*sqrt(b/c
)*arctan(c*x*sqrt(b/c)/b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

Sympy [A] time = 0.636313, size = 131, normalized size = 1.54

$$-\frac{3bx}{c^4}-\frac{35\sqrt{-\frac{b^3}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^3}{c^9}}}{b}\right)}{16}+\frac{35\sqrt{-\frac{b^3}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^3}{c^9}}}{b}\right)}{16}-\frac{11b^3x+13b^2cx^3}{8b^2c^4+16bc^5x^2+8c^6x^4}+\frac{x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*b*x/c**4 - 35*sqrt(-b**3/c**9)*log(x - c**4*sqrt(-b**3/c**9)/b)/16 + 35*
sqrt(-b**3/c**9)*log(x + c**4*sqrt(-b**3/c**9)/b)/16 - (11*b**3*x + 13*b**2
*c*x**3)/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c**6*x**4) + x**3/(3*c**3)
```

Giac [A] time = 1.22774, size = 99, normalized size = 1.16

$$\frac{35\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^4}-\frac{13\,b^2cx^3+11\,b^3x}{8\left(cx^2+b\right)^2c^4}+\frac{c^6x^3-9\,bc^5x}{3\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*b^2*c*x^3 + 11*b^3
*x)/((c*x^2 + b)^2*c^4) + 1/3*(c^6*x^3 - 9*b*c^5*x)/c^9
```

3.206 $\int \frac{x^{13}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=65

$$\frac{b^3}{4c^4\left(b+cx^2\right)^2}-\frac{3b^2}{2c^4\left(b+cx^2\right)}-\frac{3b\log\left(b+cx^2\right)}{2c^4}+\frac{x^2}{2c^3}$$

```
[Out] x^2/(2*c^3) + b^3/(4*c^4*(b + c*x^2)^2) - (3*b^2)/(2*c^4*(b + c*x^2)) - (3*
b*Log[b + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.0541817, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$\frac{b^3}{4c^4\left(b+cx^2\right)^2}-\frac{3b^2}{2c^4\left(b+cx^2\right)}-\frac{3b\log\left(b+cx^2\right)}{2c^4}+\frac{x^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^2/(2*c^3) + b^3/(4*c^4*(b + c*x^2)^2) - (3*b^2)/(2*c^4*(b + c*x^2)) - (3*
b*Log[b + c*x^2])/(2*c^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{13}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^7}{\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(b+cx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{c^3}-\frac{b^3}{c^3(b+cx)^3}+\frac{3b^2}{c^3(b+cx)^2}-\frac{3b}{c^3(b+cx)}\right)dx, x, x^2\right)\\
&= \frac{x^2}{2c^3}+\frac{b^3}{4c^4\left(b+cx^2\right)^2}-\frac{3b^2}{2c^4\left(b+cx^2\right)}-\frac{3b\log\left(b+cx^2\right)}{2c^4}
\end{aligned}
$$

Mathematica [A] time = 0.0612385, size = 48, normalized size = 0.74

$$
-\frac{\frac{b^2\left(5b+6cx^2\right)}{\left(b+cx^2\right)^2}+6b\log\left(b+cx^2\right)-2cx^2}{4c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(-2*c*x^2 + (b^2*(5*b + 6*c*x^2))/(b + c*x^2)^2 + 6*b*Log[b + c*x^2])/(4*c
^4)
```

Maple [A] time = 0.052, size = 58, normalized size = 0.9

$$
\frac{x^2}{2\,c^3}+\frac{b^3}{4\,c^4\left(cx^2+b\right)^2}-\frac{3\,b^2}{2\,c^4\left(cx^2+b\right)}-\frac{3\,b\ln\left(cx^2+b\right)}{2\,c^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/2*x^2/c^3+1/4*b^3/c^4/(c*x^2+b)^2-3/2*b^2/c^4/(c*x^2+b)-3/2*b*ln(c*x^2+b)
/c^4
```

Maxima [A] time = 0.984314, size = 89, normalized size = 1.37

$$-\frac{6\,b^2cx^2+5\,b^3}{4\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}+\frac{x^2}{2\,c^3}-\frac{3\,b\log\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*c*x^2 + 5*b^3)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 1/2*x^2/c^3
- 3/2*b*log(c*x^2 + b)/c^4
```

Fricas [A] time = 1.50194, size = 186, normalized size = 2.86

$$\frac{2\,c^3x^6+4\,bc^2x^4-4\,b^2cx^2-5\,b^3-6\left(bc^2x^4+2\,b^2cx^2+b^3\right)\log\left(cx^2+b\right)}{4\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*x^6 + 4*b*c^2*x^4 - 4*b^2*c*x^2 - 5*b^3 - 6*(b*c^2*x^4 + 2*b^2*c
*x^2 + b^3)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

Sympy [A] time = 0.619005, size = 66, normalized size = 1.02

$$-\frac{3b\log\left(b+cx^2\right)}{2c^4}-\frac{5b^3+6b^2cx^2}{4b^2c^4+8bc^5x^2+4c^6x^4}+\frac{x^2}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*b*log(b + c*x**2)/(2*c**4) - (5*b**3 + 6*b**2*c*x**2)/(4*b**2*c**4 + 8*b
*c**5*x**2 + 4*c**6*x**4) + x**2/(2*c**3)
```

<u>Giac</u> [A] time = 1.19791, size = 84, normalized size = 1.29

$$\frac{x^2}{2\,c^3} - \frac{3\,b\log\left(\left|cx^2 + b\right|\right)}{2\,c^4} + \frac{9\,bc^2x^4 + 12\,b^2cx^2 + 4\,b^3}{4\left(cx^2 + b\right)^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/c^3 - 3/2*b*log(abs(c*x^2 + b))/c^4 + 1/4*(9*b*c^2*x^4 + 12*b^2*c*x
^2 + 4*b^3)/((c*x^2 + b)^2*c^4)
```

3.207 $\int \frac{x^{12}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=74

$$-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}-\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{15x}{8c^3}$$

```
[Out] (15*x)/(8*c^3) - x^5/(4*c*(b + c*x^2)^2) - (5*x^3)/(8*c^2*(b + c*x^2)) - (1
5*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Rubi [A] time = 0.0324637, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 321, 205}

$$-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}-\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{15x}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (15*x)/(8*c^3) - x^5/(4*c*(b + c*x^2)^2) - (5*x^3)/(8*c^2*(b + c*x^2)) - (1
5*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{12}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^6}{\left(b+cx^2\right)^3}\,dx\\
&= -\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{5\int \frac{x^4}{\left(b+cx^2\right)^2}\,dx}{4c}\\
&= -\frac{x^5}{4c\left(b+cx^2\right)^2}-\frac{5x^3}{8c^2\left(b+cx^2\right)}+\frac{15\int \frac{x^2}{b+cx^2}\,dx}{8c^2}\\
&= \frac{15x}{8c^3}-\frac{x^5}{4c\left(b+cx^2\right)^2}-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{(15b)\int \frac{1}{b+cx^2}\,dx}{8c^3}\\
&= \frac{15x}{8c^3}-\frac{x^5}{4c\left(b+cx^2\right)^2}-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0460064, size = 66, normalized size = 0.89

$$\frac{15b^2x+25bcx^3+8c^2x^5}{8c^3\left(b+cx^2\right)^2}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (15*b^2*x + 25*b*c*x^3 + 8*c^2*x^5)/(8*c^3*(b + c*x^2)^2) - (15*Sqrt[b]*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Maple [A] time = 0.052, size = 63, normalized size = 0.9

$$\frac{x}{c^3}+\frac{9\,bx^3}{8\,c^2\left(cx^2+b\right)^2}+\frac{7\,b^2x}{8\,c^3\left(cx^2+b\right)^2}-\frac{15\,b}{8\,c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(c*x^4+b*x^2)^3,x)
```

```
[Out] x/c^3+9/8/c^2*b/(c*x^2+b)^2*x^3+7/8/c^3*b^2/(c*x^2+b)^2*x-15/8/c^3*b/(b*c)^
(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44713, size = 425, normalized size = 5.74

$$\left[\frac{16\,c^2x^5+50\,bcx^3+30\,b^2x+15\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{16\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)},\frac{8\,c^2x^5+25\,bcx^3+15\,b^2x-15\left(c^2x^4+2\,b\right.}{8\left(c^5x^4+2\,bc^4x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*c^2*x^5 + 50*b*c*x^3 + 30*b^2*x + 15*(c^2*x^4 + 2*b*c*x^2 + b^2)*
sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^5*x^4 + 2*b*
c^4*x^2 + b^2*c^3), 1/8*(8*c^2*x^5 + 25*b*c*x^3 + 15*b^2*x - 15*(c^2*x^4 +
2*b*c*x^2 + b^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^5*x^4 + 2*b*c^4*x^2
+ b^2*c^3)]
```

Sympy [A] time = 0.601758, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{b}{c^7}}\log\left(-c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{c^7}}\log\left(c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16}+\frac{7b^2x+9bcx^3}{8b^2c^3+16bc^4x^2+8c^5x^4}+\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(c*x**4+b*x**2)**3,x)
```

```
[Out] 15*sqrt(-b/c**7)*log(-c**3*sqrt(-b/c**7) + x)/16 - 15*sqrt(-b/c**7)*log(c**
3*sqrt(-b/c**7) + x)/16 + (7*b**2*x + 9*b*c*x**3)/(8*b**2*c**3 + 16*b*c**4*
x**2 + 8*c**5*x**4) + x/c**3
```

Giac [A] time = 1.35001, size = 73, normalized size = 0.99

$$-\frac{15\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^3}+\frac{x}{c^3}+\frac{9\,bcx^3+7\,b^2x}{8\,\left(cx^2+b\right)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + x/c^3 + 1/8*(9*b*c*x^3 + 7*
b^2*x)/((c*x^2 + b)^2*c^3)
```

3.208 $\int \frac{x^{11}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

```
[Out] -b^2/(4*c^3*(b + c*x^2)^2) + b/(c^3*(b + c*x^2)) + Log[b + c*x^2]/(2*c^3)
```

Rubi [A] time = 0.0447711, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 43}

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -b^2/(4*c^3*(b + c*x^2)^2) + b/(c^3*(b + c*x^2)) + Log[b + c*x^2]/(2*c^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^5}{\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(b+cx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^2}{c^2(b+cx)^3}-\frac{2b}{c^2(b+cx)^2}+\frac{1}{c^2(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}
\end{aligned}$$

Mathematica [A] time = 0.015503, size = 39, normalized size = 0.8

$$\frac{\frac{b\left(3b+4cx^2\right)}{\left(b+cx^2\right)^2}+2\log\left(b+cx^2\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(3*b + 4*c*x^2))/(b + c*x^2)^2 + 2*Log[b + c*x^2])/(4*c^3)
```

Maple [A] time = 0.051, size = 46, normalized size = 0.9

$$-\frac{b^2}{4\,c^3\left(cx^2+b\right)^2}+\frac{b}{c^3\left(cx^2+b\right)}+\frac{\ln\left(cx^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4*b^2/c^3/(c*x^2+b)^2+b/c^3/(c*x^2+b)+1/2*ln(c*x^2+b)/c^3
```

Maxima [A] time = 1.02124, size = 74, normalized size = 1.51

$$\frac{4\,bcx^2+3\,b^2}{4\,\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)}+\frac{\log\left(cx^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*b*c*x^2 + 3*b^2)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) + 1/2*log(c*x^2 +
 b)/c^3
```

Fricas [A] time = 1.45995, size = 143, normalized size = 2.92

$$\frac{4\,bcx^2+3\,b^2+2\,\left(c^2x^4+2\,bcx^2+b^2\right)\log\left(cx^2+b\right)}{4\,\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*c*x^2 + 3*b^2 + 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2 + b))/(c^5
*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [A] time = 0.50805, size = 53, normalized size = 1.08

$$\frac{3b^2+4bcx^2}{4b^2c^3+8bc^4x^2+4c^5x^4}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2)**3,x)
```

```
[Out] (3*b**2 + 4*b*c*x**2)/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4) + log(b +
 c*x**2)/(2*c**3)
```

Giac [A] time = 1.2059, size = 57, normalized size = 1.16

$$\frac{\log\left(\left|cx^2+b\right|\right)}{2\,c^3}-\frac{3\,cx^4+2\,bx^2}{4\,\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^2 + b))/c^3 - 1/4*(3*c*x^4 + 2*b*x^2)/((c*x^2 + b)^2*c^2)
```

3.209 $\int \frac{x^{10}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{3x}{8c^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{5/2}}-\frac{x^3}{4c\left(b+cx^2\right)^2}$$

```
[Out] -x^3/(4*c*(b + c*x^2)^2) - (3*x)/(8*c^2*(b + c*x^2)) + (3*ArcTan[(Sqrt[c]*x
)/Sqrt[b]])/(8*Sqrt[b]*c^(5/2))
```

Rubi [A] time = 0.0255314, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 288, 205}

$$-\frac{3x}{8c^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{5/2}}-\frac{x^3}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^3/(4*c*(b + c*x^2)^2) - (3*x)/(8*c^2*(b + c*x^2)) + (3*ArcTan[(Sqrt[c]*x
)/Sqrt[b]])/(8*Sqrt[b]*c^(5/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx &= \int \frac{x^4}{(b+cx^2)^3}\,dx \\
&= -\frac{x^3}{4c\left(b+cx^2\right)^2} + \frac{3\int \frac{x^2}{(b+cx^2)^2}\,dx}{4c} \\
&= -\frac{x^3}{4c\left(b+cx^2\right)^2} - \frac{3x}{8c^2\left(b+cx^2\right)} + \frac{3\int \frac{1}{b+cx^2}\,dx}{8c^2} \\
&= -\frac{x^3}{4c\left(b+cx^2\right)^2} - \frac{3x}{8c^2\left(b+cx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0426801, size = 55, normalized size = 0.86

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{5/2}} - \frac{3bx+5cx^3}{8c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(3*b*x + 5*c*x^3)/(8*c^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
(8*Sqrt[b]*c^(5/2))
```

Maple [A] time = 0.048, size = 47, normalized size = 0.7

$$\frac{1}{\left(cx^2+b\right)^2}\left(-\frac{5x^3}{8c} - \frac{3bx}{8c^2}\right) + \frac{3}{8c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2)^3,x)
```

```
[Out] (-5/8*x^3/c-3/8*b*x/c^2)/(c*x^2+b)^2+3/8/c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59582, size = 404, normalized size = 6.31

$$\left[-\frac{10\,bc^2x^3+6\,b^2cx+3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{16\left(bc^5x^4+2\,b^2c^4x^2+b^3c^3\right)},-\frac{5\,bc^2x^3+3\,b^2cx-3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{bc}\,\mathrm{arc}}{8\left(bc^5x^4+2\,b^2c^4x^2+b^3c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*b*c^2*x^3 + 6*b^2*c*x + 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)
*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b*c^5*x^4 + 2*b^2*c^4*x^2
+ b^3*c^3), -1/8*(5*b*c^2*x^3 + 3*b^2*c*x - 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*s
qrt(b*c)*arctan(sqrt(b*c)*x/b))/(b*c^5*x^4 + 2*b^2*c^4*x^2 + b^3*c^3)]
```

Sympy [A] time = 0.532172, size = 109, normalized size = 1.7

$$-\frac{3\sqrt{-\frac{1}{bc^5}}\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{bc^5}}\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16}-\frac{3bx+5cx^3}{8b^2c^2+16bc^3x^2+8c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b*c**5))*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/16 + 3*sqrt(-1/(b*c
**5))*log(b*c**2*sqrt(-1/(b*c**5)) + x)/16 - (3*b*x + 5*c*x**3)/(8*b**2*c**
2 + 16*b*c**3*x**2 + 8*c**4*x**4)
```

Giac [A] time = 1.28926, size = 61, normalized size = 0.95

$$\frac{3\,\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^2} - \frac{5\,cx^3 + 3\,bx}{8\,\left(cx^2 + b\right)^2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) - 1/8*(5*c*x^3 + 3*b*x)/((c*x^2 +
 b)^2*c^2)
```

3.210 $\int \frac{x^9}{\left(bx^2+cx^4\right)^3} dx$

Optimal. Leaf size=19

$$\frac{x^4}{4b\left(b+cx^2\right)^2}$$

```
[Out] x^4/(4*b*(b + c*x^2)^2)
```

Rubi [A] time = 0.0110066, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$\frac{x^4}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^4/(4*b*(b + c*x^2)^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^9}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^3}{\left(b+cx^2\right)^3}\,dx$$
$$= \frac{x^4}{4b\left(b+cx^2\right)^2}$$

Mathematica [A] time = 0.0067747, size = 24, normalized size = 1.26

$$-\frac{b+2cx^2}{4c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b + 2*c*x^2)/(4*c^2*(b + c*x^2)^2)
```

Maple [A] time = 0.049, size = 31, normalized size = 1.6

$$-\frac{1}{2\,c^2\left(cx^2+b\right)}+\frac{b}{4\,c^2\left(cx^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/2/c^2/(c*x^2+b)+1/4*b/c^2/(c*x^2+b)^2
```

Maxima [B] time = 1.00143, size = 49, normalized size = 2.58

$$-\frac{2\,cx^2+b}{4\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Fricas [B] time = 1.47332, size = 73, normalized size = 3.84

$$-\frac{2\,cx^2+b}{4\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [B] time = 0.535455, size = 36, normalized size = 1.89

$$-\frac{b+2cx^2}{4b^2c^2+8bc^3x^2+4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**3,x)
```

```
[Out] -(b + 2*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

Giac [A] time = 1.28051, size = 30, normalized size = 1.58

$$-\frac{2\,cx^2+b}{4\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*c*x^2 + b)/((c*x^2 + b)^2*c^2)
```

3.211 $\int \frac{x^8}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}+\frac{x}{8bc\left(b+cx^2\right)}-\frac{x}{4c\left(b+cx^2\right)^2}$$

```
[Out] -x/(4*c*(b + c*x^2)^2) + x/(8*b*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]
]/(8*b^(3/2)*c^(3/2))
```

Rubi [A] time = 0.0249015, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}+\frac{x}{8bc\left(b+cx^2\right)}-\frac{x}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x/(4*c*(b + c*x^2)^2) + x/(8*b*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]
]/(8*b^(3/2)*c^(3/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^2}{\left(b+cx^2\right)^3}\,dx\\ &= -\frac{x}{4c\left(b+cx^2\right)^2}+\frac{\int \frac{1}{\left(b+cx^2\right)^2}\,dx}{4c}\\ &= -\frac{x}{4c\left(b+cx^2\right)^2}+\frac{x}{8bc\left(b+cx^2\right)}+\frac{\int \frac{1}{b+cx^2}\,dx}{8bc}\\ &= -\frac{x}{4c\left(b+cx^2\right)^2}+\frac{x}{8bc\left(b+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0297943, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{b}\sqrt{c}x\left(cx^2-b\right)}{\left(b+cx^2\right)^2}+\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*Sqrt[c]*x*(-b + c*x^2))/(b + c*x^2)^2 + ArcTan[(Sqrt[c]*x)/Sqrt[b
]])/(8*b^(3/2)*c^(3/2))
```

Maple [A] time = 0.051, size = 49, normalized size = 0.8

$$\frac{1}{\left(cx^2+b\right)^2}\left(\frac{x^3}{8\,b}-\frac{x}{8\,c}\right)+\frac{1}{8\,bc}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2)^3,x)
```

```
[Out] (1/8/b*x^3-1/8*x/c)/(c*x^2+b)^2+1/8/b/c/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49934, size = 394, normalized size = 6.06

$$\left[\frac{2\,bc^2x^3-2\,b^2cx-\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{16\left(b^2c^4x^4+2\,b^3c^3x^2+b^4c^2\right)},\ \frac{bc^2x^3-b^2cx+\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{8\left(b^2c^4x^4+2\,b^3c^3x^2+b^4c^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^3 - 2*b^2*c*x - (c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)*log
((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b^2*c^4*x^4 + 2*b^3*c^3*x^2 +
b^4*c^2), 1/8*(b*c^2*x^3 - b^2*c*x + (c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(b*c)*
arctan(sqrt(b*c)*x/b))/(b^2*c^4*x^4 + 2*b^3*c^3*x^2 + b^4*c^2)]
```

Sympy [B] time = 0.531297, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{b^{3}c^{3}}}\log\left(-b^{2}c\sqrt{-\frac{1}{b^{3}c^{3}}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^{3}c^{3}}}\log\left(b^{2}c\sqrt{-\frac{1}{b^{3}c^{3}}}+x\right)}{16}+\frac{-bx+cx^{3}}{8b^{3}c+16b^{2}c^{2}x^{2}+8bc^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**3*c**3))*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + sqrt(-1/(b
**3*c**3))*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + (-b*x + c*x**3)/(8*b**
3*c + 16*b**2*c**2*x**2 + 8*b*c**3*x**4)
```

Giac [A] time = 1.1987, size = 68, normalized size = 1.05

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}bc}+\frac{cx^{3}-bx}{8\left(cx^{2}+b\right)^{2}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) + 1/8*(c*x^3 - b*x)/((c*x^2 + b)^
2*b*c)
```

3.212 $\int \frac{x^7}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=16

$$-\frac{1}{4c(b+cx^2)^2}$$

```
[Out] -1/(4*c*(b + c*x^2)^2)
```

Rubi [A] time = 0.0094585, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$-\frac{1}{4c(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(4*c*(b + c*x^2)^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^7}{(bx^2+cx^4)^3}\,dx = \int \frac{x}{(b+cx^2)^3}\,dx$$
$$= -\frac{1}{4c(b+cx^2)^2}$$

Mathematica [A] time = 0.0024142, size = 16, normalized size = 1.

$$-\frac{1}{4c(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(4*c*(b + c*x^2)^2)
```

Maple [A] time = 0.043, size = 15, normalized size = 0.9

$$-\frac{1}{4\,c\,(cx^2+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4/c/(c*x^2+b)^2
```

Maxima [A] time = 0.986563, size = 35, normalized size = 2.19

$$-\frac{1}{4\,(c^3x^4+2\,bc^2x^2+b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

Fricas [A] time = 1.49308, size = 51, normalized size = 3.19

$$-\frac{1}{4\left(c^3x^4 + 2\,bc^2x^2 + b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

Sympy [A] time = 0.426289, size = 27, normalized size = 1.69

$$-\frac{1}{4b^2c + 8bc^2x^2 + 4c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**3,x)
```

```
[Out] -1/(4*b**2*c + 8*b*c**2*x**2 + 4*c**3*x**4)
```

Giac [A] time = 1.19693, size = 19, normalized size = 1.19

$$-\frac{1}{4\left(cx^2 + b\right)^2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4/((c*x^2 + b)^2*c)
```

3.213 $\int \frac{x^6}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=62

$$\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{x}{4b\left(b+cx^2\right)^2}$$

```
[Out] x/(4*b*(b + c*x^2)^2) + (3*x)/(8*b^2*(b + c*x^2)) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[b]])/(8*b^(5/2)*Sqrt[c])
```

Rubi [A] time = 0.0234265, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 199, 205}

$$\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{x}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^3,x]
```


```
[Out] x/(4*b*(b + c*x^2)^2) + (3*x)/(8*b^2*(b + c*x^2)) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[b]])/(8*b^(5/2)*Sqrt[c])
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{\left(b+cx^2\right)^3}\,dx \\
&= \frac{x}{4b\left(b+cx^2\right)^2}+\frac{3\int \frac{1}{\left(b+cx^2\right)^2}\,dx}{4b} \\
&= \frac{x}{4b\left(b+cx^2\right)^2}+\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{3\int \frac{1}{b+cx^2}\,dx}{8b^2} \\
&= \frac{x}{4b\left(b+cx^2\right)^2}+\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.0338992, size = 55, normalized size = 0.89

$$\frac{5bx+3cx^3}{8b^2\left(b+cx^2\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*b*x + 3*c*x^3)/(8*b^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(
8*b^(5/2)*Sqrt[c])
```

Maple [A] time = 0.046, size = 51, normalized size = 0.8

$$\frac{x}{4\,b\left(cx^2+b\right)^2}+\frac{3\,x}{8\,b^2\left(cx^2+b\right)}+\frac{3}{8\,b^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x/b/(c*x^2+b)^2+3/8*x/b^2/(c*x^2+b)+3/8/b^2/(b*c)^(1/2)*arctan(x*c/(b*c
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.518, size = 401, normalized size = 6.47

$$\left[\frac{6\,bc^2x^3+10\,b^2cx-3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{16\left(b^3c^3x^4+2\,b^4c^2x^2+b^5c\right)},\ \frac{3\,bc^2x^3+5\,b^2cx+3\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{bc}\,\mathrm{arcta}}{8\left(b^3c^3x^4+2\,b^4c^2x^2+b^5c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*b*c^2*x^3 + 10*b^2*c*x - 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)*
log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b^3*c^3*x^4 + 2*b^4*c^2*x^2
 + b^5*c), 1/8*(3*b*c^2*x^3 + 5*b^2*c*x + 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqr
t(b*c)*arctan(sqrt(b*c)*x/b))/(b^3*c^3*x^4 + 2*b^4*c^2*x^2 + b^5*c)]
```

Sympy [A] time = 0.517811, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(-b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{16}+\frac{5bx+3cx^3}{8b^4+16b^3cx^2+8b^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b**5*c))*log(-b**3*sqrt(-1/(b**5*c)) + x)/16 + 3*sqrt(-1/(b**5*
c))*log(b**3*sqrt(-1/(b**5*c)) + x)/16 + (5*b*x + 3*c*x**3)/(8*b**4 + 16*b*
*3*c*x**2 + 8*b**2*c**2*x**4)
```

Giac [A] time = 1.30779, size = 61, normalized size = 0.98

$$\frac{3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8 \sqrt{bc} b^2} + \frac{3 cx^3 + 5 bx}{8 \left(cx^2 + b\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/8*(3*c*x^3 + 5*b*x)/((c*x^2 +
 b)^2*b^2)
```

3.214 $\int \frac{x^5}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=54

$$\frac{1}{2b^2(b+cx^2)} - \frac{\log(b+cx^2)}{2b^3} + \frac{\log(x)}{b^3} + \frac{1}{4b(b+cx^2)^2}$$

```
[Out] 1/(4*b*(b + c*x^2)^2) + 1/(2*b^2*(b + c*x^2)) + Log[x]/b^3 - Log[b + c*x^2]
/(2*b^3)
```

Rubi [A] time = 0.0441454, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$\frac{1}{2b^2(b+cx^2)} - \frac{\log(b+cx^2)}{2b^3} + \frac{\log(x)}{b^3} + \frac{1}{4b(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^3,x]
```

```
[Out] 1/(4*b*(b + c*x^2)^2) + 1/(2*b^2*(b + c*x^2)) + Log[x]/b^3 - Log[b + c*x^2]
/(2*b^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(b+cx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^3x}-\frac{c}{b(b+cx)^3}-\frac{c}{b^2(b+cx)^2}-\frac{c}{b^3(b+cx)}\right)dx,x,x^2\right)\\
&= \frac{1}{4b\left(b+cx^2\right)^2}+\frac{1}{2b^2\left(b+cx^2\right)}+\frac{\log(x)}{b^3}-\frac{\log\left(b+cx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0310812, size = 43, normalized size = 0.8

$$\frac{\frac{b\left(3b+2cx^2\right)}{\left(b+cx^2\right)^2}-2\log\left(b+cx^2\right)+4\log(x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(3*b + 2*c*x^2))/(b + c*x^2)^2 + 4*Log[x] - 2*Log[b + c*x^2])/(4*b^3)
```

Maple [A] time = 0.056, size = 49, normalized size = 0.9

$$\frac{1}{4\,b\left(cx^2+b\right)^2}+\frac{1}{2\,b^2\left(cx^2+b\right)}+\frac{\ln(x)}{b^3}-\frac{\ln\left(cx^2+b\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4/b/(c*x^2+b)^2+1/2/b^2/(c*x^2+b)+ln(x)/b^3-1/2*ln(c*x^2+b)/b^3
```

Maxima [A] time = 0.985488, size = 81, normalized size = 1.5

$$\frac{2\,cx^2+3\,b}{4\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)}-\frac{\log\left(cx^2+b\right)}{2\,b^3}+\frac{\log\left(x^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*c*x^2 + 3*b)/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) - 1/2*log(c*x^2 + b)/
b^3 + 1/2*log(x^2)/b^3
```

Fricas [A] time = 1.46932, size = 196, normalized size = 3.63

$$\frac{2\,bcx^2+3\,b^2-2\left(c^2x^4+2\,bcx^2+b^2\right)\log\left(cx^2+b\right)+4\left(c^2x^4+2\,bcx^2+b^2\right)\log\left(x\right)}{4\left(b^3c^2x^4+2\,b^4cx^2+b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^2 + 3*b^2 - 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2 + b) + 4*(
c^2*x^4 + 2*b*c*x^2 + b^2)*log(x))/(b^3*c^2*x^4 + 2*b^4*c*x^2 + b^5)
```

Sympy [A] time = 0.62134, size = 56, normalized size = 1.04

$$\frac{3b+2cx^2}{4b^4+8b^3cx^2+4b^2c^2x^4}+\frac{\log\left(x\right)}{b^3}-\frac{\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**3,x)
```

```
[Out] (3*b + 2*c*x**2)/(4*b**4 + 8*b**3*c*x**2 + 4*b**2*c**2*x**4) + log(x)/b**3
- log(b/c + x**2)/(2*b**3)
```

Giac [A] time = 1.31074, size = 80, normalized size = 1.48

$$\frac{\log\left(x^2\right)}{2\,b^3}-\frac{\log\left(\left|cx^2+b\right|\right)}{2\,b^3}+\frac{3\,c^2x^4+8\,bcx^2+6\,b^2}{4\,\left(cx^2+b\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/b^3 - 1/2*log(abs(c*x^2 + b))/b^3 + 1/4*(3*c^2*x^4 + 8*b*c*x^2
 + 6*b^2)/((c*x^2 + b)^2*b^3)
```

3.215 $\int \frac{x^4}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=76

$$\frac{5}{8b^2x\left(b+cx^2\right)}-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{15}{8b^3x}+\frac{1}{4bx\left(b+cx^2\right)^2}$$

```
[Out] -15/(8*b^3*x) + 1/(4*b*x*(b + c*x^2)^2) + 5/(8*b^2*x*(b + c*x^2)) - (15*Sqr
t[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Rubi [A] time = 0.0356755, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$\frac{5}{8b^2x\left(b+cx^2\right)}-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{15}{8b^3x}+\frac{1}{4bx\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -15/(8*b^3*x) + 1/(4*b*x*(b + c*x^2)^2) + 5/(8*b^2*x*(b + c*x^2)) - (15*Sqr
t[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^2\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{4bx\left(b+cx^2\right)^2}+\frac{5\int \frac{1}{x^2\left(b+cx^2\right)^2}\,dx}{4b}\\
&= \frac{1}{4bx\left(b+cx^2\right)^2}+\frac{5}{8b^2x\left(b+cx^2\right)}+\frac{15\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{8b^2}\\
&= -\frac{15}{8b^3x}+\frac{1}{4bx\left(b+cx^2\right)^2}+\frac{5}{8b^2x\left(b+cx^2\right)}-\frac{(15c)\int \frac{1}{b+cx^2}\,dx}{8b^3}\\
&= -\frac{15}{8b^3x}+\frac{1}{4bx\left(b+cx^2\right)^2}+\frac{5}{8b^2x\left(b+cx^2\right)}-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0413318, size = 68, normalized size = 0.89

$$-\frac{8b^2+25bcx^2+15c^2x^4}{8b^3x\left(b+cx^2\right)^2}-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(8*b^2 + 25*b*c*x^2 + 15*c^2*x^4)/(8*b^3*x*(b + c*x^2)^2) - (15*Sqrt[c]*Ar
cTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Maple [A] time = 0.055, size = 66, normalized size = 0.9

$$-\frac{1}{b^3x}-\frac{7\,c^2x^3}{8\,b^3\left(cx^2+b\right)^2}-\frac{9\,cx}{8\,b^2\left(cx^2+b\right)^2}-\frac{15\,c}{8\,b^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/b^3/x-7/8/b^3*c^2/(c*x^2+b)^2*x^3-9/8/b^2*c/(c*x^2+b)^2*x-15/8/b^3*c/(b*
c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53436, size = 428, normalized size = 5.63

$$\left[-\frac{30\,c^2x^4+50\,bcx^2-15\left(c^2x^5+2\,bcx^3+b^2x\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+16\,b^2}{16\left(b^3c^2x^5+2\,b^4cx^3+b^5x\right)},-\frac{15\,c^2x^4+25\,bcx^2+15\left(c^2x^5+2\,bcx^3\right.}{8\left(b^3c^2x^5+2\,b^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*c^2*x^4 + 50*b*c*x^2 - 15*(c^2*x^5 + 2*b*c*x^3 + b^2*x)*sqrt(-c/
b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) + 16*b^2)/(b^3*c^2*x^5 +
 2*b^4*c*x^3 + b^5*x), -1/8*(15*c^2*x^4 + 25*b*c*x^2 + 15*(c^2*x^5 + 2*b*c*
x^3 + b^2*x)*sqrt(c/b)*arctan(x*sqrt(c/b)) + 8*b^2)/(b^3*c^2*x^5 + 2*b^4*c*
x^3 + b^5*x)]
```

Sympy [A] time = 0.782732, size = 114, normalized size = 1.5

$$\frac{15\sqrt{-\frac{c}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}}{c}+x\right)}{16}-\frac{15\sqrt{-\frac{c}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}}{c}+x\right)}{16}-\frac{8b^2+25bcx^2+15c^2x^4}{8b^5x+16b^4cx^3+8b^3c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**3,x)
```

```
[Out] 15*sqrt(-c/b**7)*log(-b**4*sqrt(-c/b**7)/c + x)/16 - 15*sqrt(-c/b**7)*log(b
**4*sqrt(-c/b**7)/c + x)/16 - (8*b**2 + 25*b*c*x**2 + 15*c**2*x**4)/(8*b**5
*x + 16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

Giac [A] time = 1.2728, size = 77, normalized size = 1.01

$$-\frac{15\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^3}-\frac{7\,c^2x^3+9\,bcx}{8\,\left(cx^2+b\right)^2b^3}-\frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/8*(7*c^2*x^3 + 9*b*c*x)/(
(c*x^2 + b)^2*b^3) - 1/(b^3*x)
```

3.216 $\int \frac{x^3}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=67

$$-\frac{c}{b^3(b+cx^2)}-\frac{c}{4b^2(b+cx^2)^2}+\frac{3c\log(b+cx^2)}{2b^4}-\frac{3c\log(x)}{b^4}-\frac{1}{2b^3x^2}$$

```
[Out] -1/(2*b^3*x^2) - c/(4*b^2*(b + c*x^2)^2) - c/(b^3*(b + c*x^2)) - (3*c*Log[x
])/b^4 + (3*c*Log[b + c*x^2])/(2*b^4)
```

Rubi [A] time = 0.0605118, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{c}{b^3(b+cx^2)}-\frac{c}{4b^2(b+cx^2)^2}+\frac{3c\log(b+cx^2)}{2b^4}-\frac{3c\log(x)}{b^4}-\frac{1}{2b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(2*b^3*x^2) - c/(4*b^2*(b + c*x^2)^2) - c/(b^3*(b + c*x^2)) - (3*c*Log[x
])/b^4 + (3*c*Log[b + c*x^2])/(2*b^4)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^3\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(b+cx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^3x^2}-\frac{3c}{b^4x}+\frac{c^2}{b^2(b+cx)^3}+\frac{2c^2}{b^3(b+cx)^2}+\frac{3c^2}{b^4(b+cx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2b^3x^2}-\frac{c}{4b^2\left(b+cx^2\right)^2}-\frac{c}{b^3\left(b+cx^2\right)}-\frac{3c\log(x)}{b^4}+\frac{3c\log\left(b+cx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.0567536, size = 59, normalized size = 0.88

$$-\frac{\frac{b\left(2b^2+9bcx^2+6c^2x^4\right)}{x^2\left(b+cx^2\right)^2}-6c\log\left(b+cx^2\right)+12c\log(x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b*(2*b^2 + 9*b*c*x^2 + 6*c^2*x^4))/(x^2*(b + c*x^2)^2) + 12*c*Log[x] - 6
*c*Log[b + c*x^2])/(4*b^4)
```

Maple [A] time = 0.06, size = 62, normalized size = 0.9

$$-\frac{1}{2\,b^3x^2}-\frac{c}{4\,b^2\left(cx^2+b\right)^2}-\frac{c}{b^3\left(cx^2+b\right)}-3\,\frac{c\ln(x)}{b^4}+\frac{3\,c\ln\left(cx^2+b\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/2/b^3/x^2-1/4*c/b^2/(c*x^2+b)^2-c/b^3/(c*x^2+b)-3*c*ln(x)/b^4+3/2*c*ln(c
*x^2+b)/b^4
```

Maxima [A] time = 0.988503, size = 104, normalized size = 1.55

$$-\frac{6\,c^2x^4 + 9\,bcx^2 + 2\,b^2}{4\left(b^3c^2x^6 + 2\,b^4cx^4 + b^5x^2\right)} + \frac{3\,c\log\left(cx^2 + b\right)}{2\,b^4} - \frac{3\,c\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*c^2*x^4 + 9*b*c*x^2 + 2*b^2)/(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)
+ 3/2*c*log(c*x^2 + b)/b^4 - 3/2*c*log(x^2)/b^4
```

Fricas [A] time = 1.62646, size = 247, normalized size = 3.69

$$-\frac{6\,bc^2x^4 + 9\,b^2cx^2 + 2\,b^3 - 6\left(c^3x^6 + 2\,bc^2x^4 + b^2cx^2\right)\log\left(cx^2 + b\right) + 12\left(c^3x^6 + 2\,bc^2x^4 + b^2cx^2\right)\log\left(x\right)}{4\left(b^4c^2x^6 + 2\,b^5cx^4 + b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3 - 6*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c*
x^2)*log(c*x^2 + b) + 12*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c*x^2)*log(x))/(b^4*c
^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)
```

Sympy [A] time = 0.980319, size = 78, normalized size = 1.16

$$-\frac{2b^2 + 9bcx^2 + 6c^2x^4}{4b^5x^2 + 8b^4cx^4 + 4b^3c^2x^6} - \frac{3c\log\left(x\right)}{b^4} + \frac{3c\log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**3,x)
```

```
[Out] -(2*b**2 + 9*b*c*x**2 + 6*c**2*x**4)/(4*b**5*x**2 + 8*b**4*c*x**4 + 4*b**3*
c**2*x**6) - 3*c*log(x)/b**4 + 3*c*log(b/c + x**2)/(2*b**4)
```

Giac [A] time = 1.2833, size = 89, normalized size = 1.33

$$\frac{3\,c\log\left(\left|cx^2+b\right|\right)}{2\,b^4}-\frac{3\,c\log\left(|x|\right)}{b^4}-\frac{6\,bc^2x^4+9\,b^2cx^2+2\,b^3}{4\,\left(cx^2+b\right)^2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/2*c*log(abs(c*x^2 + b))/b^4 - 3*c*log(abs(x))/b^4 - 1/4*(6*b*c^2*x^4 + 9*
b^2*c*x^2 + 2*b^3)/((c*x^2 + b)^2*b^4*x^2)
```

3.217 $\int \frac{x^2}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=87

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{35c}{8b^4x}-\frac{35}{24b^3x^3}+\frac{1}{4bx^3\left(b+cx^2\right)^2}$$

```
[Out] -35/(24*b^3*x^3) + (35*c)/(8*b^4*x) + 1/(4*b*x^3*(b + c*x^2)^2) + 7/(8*b^2*
x^3*(b + c*x^2)) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rubi [A] time = 0.0418952, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1584, 290, 325, 205}

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{35c}{8b^4x}-\frac{35}{24b^3x^3}+\frac{1}{4bx^3\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -35/(24*b^3*x^3) + (35*c)/(8*b^4*x) + 1/(4*b*x^3*(b + c*x^2)^2) + 7/(8*b^2*
x^3*(b + c*x^2)) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^4\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7\int \frac{1}{x^4\left(b+cx^2\right)^2}\,dx}{4b} \\
&= \frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{35\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{8b^2} \\
&= -\frac{35}{24b^3x^3}+\frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7}{8b^2x^3\left(b+cx^2\right)}-\frac{(35c)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{8b^3} \\
&= -\frac{35}{24b^3x^3}+\frac{35c}{8b^4x}+\frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{\left(35c^2\right)\int \frac{1}{b+cx^2}\,dx}{8b^4} \\
&= -\frac{35}{24b^3x^3}+\frac{35c}{8b^4x}+\frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0435254, size = 79, normalized size = 0.91

$$\frac{56b^2cx^2-8b^3+175bc^2x^4+105c^3x^6}{24b^4x^3\left(b+cx^2\right)^2}+\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-8*b^3 + 56*b^2*c*x^2 + 175*b*c^2*x^4 + 105*c^3*x^6)/(24*b^4*x^3*(b + c*x^
2)^2) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Maple [A] time = 0.056, size = 79, normalized size = 0.9

$$-\frac{1}{3\,b^3x^3}+3\,\frac{c}{b^4x}+\frac{11\,c^3x^3}{8\,b^4\left(cx^2+b\right)^2}+\frac{13\,c^2x}{8\,b^3\left(cx^2+b\right)^2}+\frac{35\,c^2}{8\,b^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/3/b^3/x^3+3*c/b^4/x+11/8/b^4*c^3/(c*x^2+b)^2*x^3+13/8/b^3*c^2/(c*x^2+b)^
2*x+35/8/b^4*c^2/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80281, size = 504, normalized size = 5.79

$$\left[\frac{210\,c^3x^6+350\,bc^2x^4+112\,b^2cx^2-16\,b^3+105\left(c^3x^7+2\,bc^2x^5+b^2cx^3\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{48\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)},\frac{105\,c^3x^6+175\,bc^2x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(210*c^3*x^6 + 350*b*c^2*x^4 + 112*b^2*c*x^2 - 16*b^3 + 105*(c^3*x^7
+ 2*b*c^2*x^5 + b^2*c*x^3)*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c
*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3), 1/24*(105*c^3*x^6 + 175*
b*c^2*x^4 + 56*b^2*c*x^2 - 8*b^3 + 105*(c^3*x^7 + 2*b*c^2*x^5 + b^2*c*x^3)*
sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

Sympy [A] time = 0.927996, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{c^3}{b^9}}\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2}+x\right)}{16}+\frac{35\sqrt{-\frac{c^3}{b^9}}\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2}+x\right)}{16}+\frac{-8b^3+56b^2cx^2+175bc^2x^4+105c^3x^6}{24b^6x^3+48b^5cx^5+24b^4c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**3,x)
```

```
[Out] -35*sqrt(-c**3/b**9)*log(-b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + 35*sqrt(-c**
3/b**9)*log(b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + (-8*b**3 + 56*b**2*c*x**2
+ 175*b*c**2*x**4 + 105*c**3*x**6)/(24*b**6*x**3 + 48*b**5*c*x**5 + 24*b**4
*c**2*x**7)
```

Giac [A] time = 1.22641, size = 96, normalized size = 1.1

$$\frac{35\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^4}+\frac{11\,c^3x^3+13\,bc^2x}{8\left(cx^2+b\right)^2b^4}+\frac{9\,cx^2-b}{3\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/8*(11*c^3*x^3 + 13*b*c^2
*x)/((c*x^2 + b)^2*b^4) + 1/3*(9*c*x^2 - b)/(b^4*x^3)
```

3.218 $\int \frac{x}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=86

$$\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}-\frac{3c^2\log\left(b+cx^2\right)}{b^5}+\frac{6c^2\log(x)}{b^5}+\frac{3c}{2b^4x^2}-\frac{1}{4b^3x^4}$$

```
[Out] -1/(4*b^3*x^4) + (3*c)/(2*b^4*x^2) + c^2/(4*b^3*(b + c*x^2)^2) + (3*c^2)/(2
*b^4*(b + c*x^2)) + (6*c^2*Log[x])/b^5 - (3*c^2*Log[b + c*x^2])/b^5
```

Rubi [A] time = 0.0718563, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 266, 44}

$$\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}-\frac{3c^2\log\left(b+cx^2\right)}{b^5}+\frac{6c^2\log(x)}{b^5}+\frac{3c}{2b^4x^2}-\frac{1}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(4*b^3*x^4) + (3*c)/(2*b^4*x^2) + c^2/(4*b^3*(b + c*x^2)^2) + (3*c^2)/(2
*b^4*(b + c*x^2)) + (6*c^2*Log[x])/b^5 - (3*c^2*Log[b + c*x^2])/b^5
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^5\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(b+cx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^3x^3}-\frac{3c}{b^4x^2}+\frac{6c^2}{b^5x}-\frac{c^3}{b^3(b+cx)^3}-\frac{3c^3}{b^4(b+cx)^2}-\frac{6c^3}{b^5(b+cx)}\right)dx, x, x^2\right)\\
&= -\frac{1}{4b^3x^4}+\frac{3c}{2b^4x^2}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}+\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{6c^2\log(x)}{b^5}-\frac{3c^2\log\left(b+cx^2\right)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.0456188, size = 74, normalized size = 0.86

$$\frac{\frac{b\left(4b^2cx^2-b^3+18bc^2x^4+12c^3x^6\right)}{x^4\left(b+cx^2\right)^2}-12c^2\log\left(b+cx^2\right)+24c^2\log(x)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(-b^3 + 4*b^2*c*x^2 + 18*b*c^2*x^4 + 12*c^3*x^6))/(x^4*(b + c*x^2)^2) +
 24*c^2*Log[x] - 12*c^2*Log[b + c*x^2])/(4*b^5)
```

Maple [A] time = 0.056, size = 79, normalized size = 0.9

$$-\frac{1}{4\,b^3x^4}+\frac{3\,c}{2\,b^4x^2}+\frac{c^2}{4\,b^3\left(cx^2+b\right)^2}+\frac{3\,c^2}{2\,b^4\left(cx^2+b\right)}+6\,\frac{c^2\ln(x)}{b^5}-3\,\frac{c^2\ln\left(cx^2+b\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4/b^3/x^4+3/2*c/b^4/x^2+1/4*c^2/b^3/(c*x^2+b)^2+3/2*c^2/b^4/(c*x^2+b)+6*
c^2*ln(x)/b^5-3*c^2*ln(c*x^2+b)/b^5
```

Maxima [A] time = 0.978365, size = 124, normalized size = 1.44

$$\frac{12\,c^3x^6 + 18\,bc^2x^4 + 4\,b^2cx^2 - b^3}{4\,\left(b^4c^2x^8 + 2\,b^5cx^6 + b^6x^4\right)} - \frac{3\,c^2\log\left(cx^2 + b\right)}{b^5} + \frac{3\,c^2\log\left(x^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/(b^4*c^2*x^8 + 2*b^5*c*
x^6 + b^6*x^4) - 3*c^2*log(c*x^2 + b)/b^5 + 3*c^2*log(x^2)/b^5
```

Fricas [A] time = 1.50158, size = 274, normalized size = 3.19

$$\frac{12\,bc^3x^6 + 18\,b^2c^2x^4 + 4\,b^3cx^2 - b^4 - 12\,\left(c^4x^8 + 2\,bc^3x^6 + b^2c^2x^4\right)\log\left(cx^2 + b\right) + 24\,\left(c^4x^8 + 2\,bc^3x^6 + b^2c^2x^4\right)\log\left(x\right)}{4\,\left(b^5c^2x^8 + 2\,b^6cx^6 + b^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*b*c^3*x^6 + 18*b^2*c^2*x^4 + 4*b^3*c*x^2 - b^4 - 12*(c^4*x^8 + 2*b*
c^3*x^6 + b^2*c^2*x^4)*log(c*x^2 + b) + 24*(c^4*x^8 + 2*b*c^3*x^6 + b^2*c^2
*x^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x^4)
```

Sympy [A] time = 1.2709, size = 90, normalized size = 1.05

$$\frac{-b^3 + 4b^2cx^2 + 18bc^2x^4 + 12c^3x^6}{4b^6x^4 + 8b^5cx^6 + 4b^4c^2x^8} + \frac{6c^2\log\left(x\right)}{b^5} - \frac{3c^2\log\left(\frac{b}{c} + x^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-b**3 + 4*b**2*c*x**2 + 18*b*c**2*x**4 + 12*c**3*x**6)/(4*b**6*x**4 + 8*b*
*5*c*x**6 + 4*b**4*c**2*x**8) + 6*c**2*log(x)/b**5 - 3*c**2*log(b/c + x**2)
/b**5
```

Giac [A] time = 1.28543, size = 107, normalized size = 1.24

$$-\frac{3\,c^2\log\left(\left|cx^2+b\right|\right)}{b^5}+\frac{6\,c^2\log\left(|x|\right)}{b^5}+\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2-b^3}{4\left(cx^4+bx^2\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -3*c^2*log(abs(c*x^2 + b))/b^5 + 6*c^2*log(abs(x))/b^5 + 1/4*(12*c^3*x^6 +
18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/((c*x^4 + b*x^2)^2*b^4)
```

3.219 $\int \frac{1}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{63c^2}{8b^5x}-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}+\frac{21c}{8b^4x^3}+\frac{9}{8b^2x^5\left(b+cx^2\right)}-\frac{63}{40b^3x^5}+\frac{1}{4bx^5\left(b+cx^2\right)^2}$$

```
[Out] -63/(40*b^3*x^5) + (21*c)/(8*b^4*x^3) - (63*c^2)/(8*b^5*x) + 1/(4*b*x^5*(b
+ c*x^2)^2) + 9/(8*b^2*x^5*(b + c*x^2)) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(8*b^(11/2))
```

Rubi [A] time = 0.0491003, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1593, 290, 325, 205}

$$-\frac{63c^2}{8b^5x}-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}+\frac{21c}{8b^4x^3}+\frac{9}{8b^2x^5\left(b+cx^2\right)}-\frac{63}{40b^3x^5}+\frac{1}{4bx^5\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-3),x]
```


```
[Out] -63/(40*b^3*x^5) + (21*c)/(8*b^4*x^3) - (63*c^2)/(8*b^5*x) + 1/(4*b*x^5*(b
+ c*x^2)^2) + 9/(8*b^2*x^5*(b + c*x^2)) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(8*b^(11/2))
```


Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^6\left(b+cx^2\right)^3}\,dx\\
&= \frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9\int \frac{1}{x^6\left(b+cx^2\right)^2}\,dx}{4b}\\
&= \frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9}{8b^2x^5\left(b+cx^2\right)}+\frac{63\int \frac{1}{x^6\left(b+cx^2\right)}\,dx}{8b^2}\\
&= -\frac{63}{40b^3x^5}+\frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9}{8b^2x^5\left(b+cx^2\right)}-\frac{(63c)\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{8b^3}\\
&= -\frac{63}{40b^3x^5}+\frac{21c}{8b^4x^3}+\frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9}{8b^2x^5\left(b+cx^2\right)}+\frac{\left(63c^2\right)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{8b^4}\\
&= -\frac{63}{40b^3x^5}+\frac{21c}{8b^4x^3}-\frac{63c^2}{8b^5x}+\frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9}{8b^2x^5\left(b+cx^2\right)}-\frac{\left(63c^3\right)\int \frac{1}{b+cx^2}\,dx}{8b^5}\\
&= -\frac{63}{40b^3x^5}+\frac{21c}{8b^4x^3}-\frac{63c^2}{8b^5x}+\frac{1}{4bx^5\left(b+cx^2\right)^2}+\frac{9}{8b^2x^5\left(b+cx^2\right)}-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}
\end{aligned}
$$

Mathematica [A] time = 0.053974, size = 90, normalized size = 0.9

$$-\frac{168b^2c^2x^4-24b^3cx^2+8b^4+525bc^3x^6+315c^4x^8}{40b^5x^5\left(b+cx^2\right)^2}-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-3),x]
```

```
[Out] -(8*b^4 - 24*b^3*c*x^2 + 168*b^2*c^2*x^4 + 525*b*c^3*x^6 + 315*c^4*x^8)/(40
*b^5*x^5*(b + c*x^2)^2) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(11
/2))
```

Maple [A] time = 0.056, size = 89, normalized size = 0.9

$$-\frac{1}{5\,b^3x^5}-6\,\frac{c^2}{b^5x}+\frac{c}{b^4x^3}-\frac{15\,c^4x^3}{8\,b^5\left(cx^2+b\right)^2}-\frac{17\,c^3x}{8\,b^4\left(cx^2+b\right)^2}-\frac{63\,c^3}{8\,b^5}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/5/b^3/x^5-6*c^2/b^5/x+c/b^4/x^3-15/8/b^5*c^4/(c*x^2+b)^2*x^3-17/8/b^4*c^
3/(c*x^2+b)^2*x-63/8/b^5*c^3/(b*c)^(1/2)*arctan(x*c/(b*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55846, size = 560, normalized size = 5.6

$$\left[-\frac{630\,c^4x^8+1050\,bc^3x^6+336\,b^2c^2x^4-48\,b^3cx^2+16\,b^4-315\,\left(c^4x^9+2\,bc^3x^7+b^2c^2x^5\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{80\,\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)},-\frac{315\,c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(630*c^4*x^8 + 1050*b*c^3*x^6 + 336*b^2*c^2*x^4 - 48*b^3*c*x^2 + 16*
b^4 - 315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b
*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5), -1/
40*(315*c^4*x^8 + 525*b*c^3*x^6 + 168*b^2*c^2*x^4 - 24*b^3*c*x^2 + 8*b^4 +
315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b
^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)]
```

Sympy [A] time = 1.75648, size = 150, normalized size = 1.5

$$\frac{63\sqrt{-\frac{c^5}{b^{11}}}\log\left(-\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3}+x\right)}{16}-\frac{63\sqrt{-\frac{c^5}{b^{11}}}\log\left(\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3}+x\right)}{16}-\frac{8b^4-24b^3cx^2+168b^2c^2x^4+525bc^3x^6+315c^4x^8}{40b^7x^5+80b^6cx^7+40b^5c^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**3,x)
```

```
[Out] 63*sqrt(-c**5/b**11)*log(-b**6*sqrt(-c**5/b**11)/c**3 + x)/16 - 63*sqrt(-c*
*5/b**11)*log(b**6*sqrt(-c**5/b**11)/c**3 + x)/16 - (8*b**4 - 24*b**3*c*x**
2 + 168*b**2*c**2*x**4 + 525*b*c**3*x**6 + 315*c**4*x**8)/(40*b**7*x**5 + 8
0*b**6*c*x**7 + 40*b**5*c**2*x**9)
```

Giac [A] time = 1.30243, size = 108, normalized size = 1.08

$$-\frac{63\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^5}-\frac{15\,c^4x^3+17\,bc^3x}{8\,\left(cx^2+b\right)^2b^5}-\frac{30\,c^2x^4-5\,bcx^2+b^2}{5\,b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -63/8*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) - 1/8*(15*c^4*x^3 + 17*b*c^
3*x)/((c*x^2 + b)^2*b^5) - 1/5*(30*c^2*x^4 - 5*b*c*x^2 + b^2)/(b^5*x^5)
```

3.220 $\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=95

$$-\frac{2c^3}{b^5\left(b+cx^2\right)}-\frac{c^3}{4b^4\left(b+cx^2\right)^2}-\frac{3c^2}{b^5x^2}+\frac{5c^3\log\left(b+cx^2\right)}{b^6}-\frac{10c^3\log(x)}{b^6}+\frac{3c}{4b^4x^4}-\frac{1}{6b^3x^6}$$

```
[Out] -1/(6*b^3*x^6) + (3*c)/(4*b^4*x^4) - (3*c^2)/(b^5*x^2) - c^3/(4*b^4*(b + c*
x^2)^2) - (2*c^3)/(b^5*(b + c*x^2)) - (10*c^3*Log[x])/b^6 + (5*c^3*Log[b +
c*x^2])/b^6
```

Rubi [A] time = 0.0847774, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1584, 266, 44}

$$-\frac{2c^3}{b^5\left(b+cx^2\right)}-\frac{c^3}{4b^4\left(b+cx^2\right)^2}-\frac{3c^2}{b^5x^2}+\frac{5c^3\log\left(b+cx^2\right)}{b^6}-\frac{10c^3\log(x)}{b^6}+\frac{3c}{4b^4x^4}-\frac{1}{6b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/(6*b^3*x^6) + (3*c)/(4*b^4*x^4) - (3*c^2)/(b^5*x^2) - c^3/(4*b^4*(b + c*
x^2)^2) - (2*c^3)/(b^5*(b + c*x^2)) - (10*c^3*Log[x])/b^6 + (5*c^3*Log[b +
c*x^2])/b^6
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^7\left(b+cx^2\right)^3}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4(b+cx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^3x^4}-\frac{3c}{b^4x^3}+\frac{6c^2}{b^5x^2}-\frac{10c^3}{b^6x}+\frac{c^4}{b^4(b+cx)^3}+\frac{4c^4}{b^5(b+cx)^2}+\frac{10c^4}{b^6(b+cx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{6b^3x^6}+\frac{3c}{4b^4x^4}-\frac{3c^2}{b^5x^2}-\frac{c^3}{4b^4\left(b+cx^2\right)^2}-\frac{2c^3}{b^5\left(b+cx^2\right)}-\frac{10c^3\log(x)}{b^6}+\frac{5c^3\log\left(b+cx^2\right)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.0728489, size = 85, normalized size = 0.89

$$-\frac{\frac{b\left(20b^2c^2x^4-5b^3cx^2+2b^4+90bc^3x^6+60c^4x^8\right)}{x^6\left(b+cx^2\right)^2}-60c^3\log\left(b+cx^2\right)+120c^3\log(x)}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -((b*(2*b^4 - 5*b^3*c*x^2 + 20*b^2*c^2*x^4 + 90*b*c^3*x^6 + 60*c^4*x^8))/(x
^6*(b + c*x^2)^2) + 120*c^3*Log[x] - 60*c^3*Log[b + c*x^2])/(12*b^6)
```

Maple [A] time = 0.058, size = 90, normalized size = 1.

$$-\frac{1}{6\,b^3x^6}+\frac{3\,c}{4\,b^4x^4}-3\,\frac{c^2}{b^5x^2}-\frac{c^3}{4\,b^4\left(cx^2+b\right)^2}-2\,\frac{c^3}{b^5\left(cx^2+b\right)}-10\,\frac{c^3\ln(x)}{b^6}+5\,\frac{c^3\ln\left(cx^2+b\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/6/b^3/x^6+3/4*c/b^4/x^4-3*c^2/b^5/x^2-1/4*c^3/b^4/(c*x^2+b)^2-2*c^3/b^5/
(c*x^2+b)-10*c^3*ln(x)/b^6+5*c^3*ln(c*x^2+b)/b^6
```

Maxima [A] time = 1.03071, size = 139, normalized size = 1.46

$$-\frac{60\,c^4x^8+90\,bc^3x^6+20\,b^2c^2x^4-5\,b^3cx^2+2\,b^4}{12\left(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6\right)}+\frac{5\,c^3\log\left(cx^2+b\right)}{b^6}-\frac{5\,c^3\log\left(x^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(60*c^4*x^8 + 90*b*c^3*x^6 + 20*b^2*c^2*x^4 - 5*b^3*c*x^2 + 2*b^4)/(b
^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6) + 5*c^3*log(c*x^2 + b)/b^6 - 5*c^3*log
(x^2)/b^6
```

Fricas [A] time = 1.56263, size = 308, normalized size = 3.24

$$-\frac{60\,bc^4x^8+90\,b^2c^3x^6+20\,b^3c^2x^4-5\,b^4cx^2+2\,b^5-60\left(c^5x^{10}+2\,bc^4x^8+b^2c^3x^6\right)\log\left(cx^2+b\right)+120\left(c^5x^{10}+2\,bc^4x^8+\right.}{12\left(b^6c^2x^{10}+2\,b^7cx^8+b^8x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(60*b*c^4*x^8 + 90*b^2*c^3*x^6 + 20*b^3*c^2*x^4 - 5*b^4*c*x^2 + 2*b^5
 - 60*(c^5*x^10 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(c*x^2 + b) + 120*(c^5*x^10
 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(x))/(b^6*c^2*x^10 + 2*b^7*c*x^8 + b^8*x^6
)
```

Sympy [A] time = 2.05117, size = 104, normalized size = 1.09

$$-\frac{2b^4-5b^3cx^2+20b^2c^2x^4+90bc^3x^6+60c^4x^8}{12b^7x^6+24b^6cx^8+12b^5c^2x^{10}}-\frac{10c^3\log\left(x\right)}{b^6}+\frac{5c^3\log\left(\frac{b}{c}+x^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**3,x)
```

```
[Out] -(2*b**4 - 5*b**3*c*x**2 + 20*b**2*c**2*x**4 + 90*b*c**3*x**6 + 60*c**4*x**
8)/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) - 10*c**3*log(x)/b*
*6 + 5*c**3*log(b/c + x**2)/b**6
```

Giac [A] time = 1.30037, size = 149, normalized size = 1.57

$$-\frac{5\,c^3\log\left(x^2\right)}{b^6}+\frac{5\,c^3\log\left(\left|cx^2+b\right|\right)}{b^6}-\frac{30\,c^5x^4+68\,bc^4x^2+39\,b^2c^3}{4\left(cx^2+b\right)^2b^6}+\frac{110\,c^3x^6-36\,bc^2x^4+9\,b^2cx^2-2\,b^3}{12\,b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -5*c^3*log(x^2)/b^6 + 5*c^3*log(abs(c*x^2 + b))/b^6 - 1/4*(30*c^5*x^4 + 68*
b*c^4*x^2 + 39*b^2*c^3)/((c*x^2 + b)^2*b^6) + 1/12*(110*c^3*x^6 - 36*b*c^2*
x^4 + 9*b^2*c*x^2 - 2*b^3)/(b^6*x^6)
```

3.221 $\int x^5\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=119

$$\frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}$$

```
[Out] (5*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - (5*b*(b*x^2 + c*x^4)^
(3/2))/(48*c^2) + (x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*b^4*ArcTanh[(Sqrt[
c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rubi [A] time = 0.130758, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 670, 640, 612, 620, 206}

$$\frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (5*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - (5*b*(b*x^2 + c*x^4)^
(3/2))/(48*c^2) + (x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*b^4*ArcTanh[(Sqrt[
c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
```

```
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5\sqrt{bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{bx+cx^2}\,dx,x,x^2\right)$$
$$= \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c} - \frac{(5b)\text{Subst}\left(\int x\sqrt{bx+cx^2}\,dx,x,x^2\right)}{16c}$$
$$= -\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c} + \frac{\left(5b^2\right)\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx,x,x^2\right)}{32c^2}$$
$$= \frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3} - \frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c} - \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,\right.}{256c^3}$$
$$= \frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3} - \frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c} - \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\right.}{128c^3}$$
$$= \frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3} - \frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c} - \frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}$$

Mathematica [A] time = 0.0757847, size = 114, normalized size = 0.96

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}\left(-10b^2cx^2+15b^3+8bc^2x^4+48c^3x^6\right)-15b^4\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{384c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(15*b^3 - 10*b^2*c*x^2 + 8*b*
c^2*x^4 + 48*c^3*x^6) - 15*b^4*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(384*c^
(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.05, size = 124, normalized size = 1.

$$\frac{1}{384\,x}\sqrt{cx^4+bx^2}\left(48\,x^5\left(cx^2+b\right)^{3/2}c^{5/2}-40\,c^{3/2}\left(cx^2+b\right)^{3/2}x^3b+30\,\sqrt{c}\left(cx^2+b\right)^{3/2}xb^2-15\,\sqrt{c}\sqrt{cx^2+b}xb^3-15\,\ln\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/384*(c*x^4+b*x^2)^(1/2)*(48*x^5*(c*x^2+b)^(3/2)*c^(5/2)-40*c^(3/2)*(c*x^2
+b)^(3/2)*x^3*b+30*c^(1/2)*(c*x^2+b)^(3/2)*x*b^2-15*c^(1/2)*(c*x^2+b)^(1/2)
*x*b^3-15*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^4)/x/(c*x^2+b)^(1/2)/c^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69643, size = 427, normalized size = 3.59

$$\left[\frac{15\,b^4\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\left(48\,c^4x^6+8\,bc^3x^4-10\,b^2c^2x^2+15\,b^3c\right)\sqrt{cx^4+bx^2}}{768\,c^4}, \frac{15\,b^4\sqrt{-c}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*b^4*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
2*(48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^4 + b*x^2
))/c^4, 1/384*(15*b^4*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 +
 b)) + (48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^4 +
b*x^2))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.32193, size = 136, normalized size = 1.14

$$\frac{1}{384}\left(2\left(4\left(6x^2\operatorname{sgn}(x)+\frac{b\operatorname{sgn}(x)}{c}\right)x^2-\frac{5b^2\operatorname{sgn}(x)}{c^2}\right)x^2+\frac{15b^3\operatorname{sgn}(x)}{c^3}\right)\sqrt{cx^2+b}\,x+\frac{5b^4\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\operatorname{sgn}(x)}{128c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*x^2*sgn(x) + b*sgn(x)/c)*x^2 - 5*b^2*sgn(x)/c^2)*x^2 + 15*b^
3*sgn(x)/c^3)*sqrt(c*x^2 + b)*x + 5/128*b^4*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + b)))*sgn(x)/c^(7/2) - 5/256*b^4*log(abs(b))*sgn(x)/c^(7/2)
```

3.222 $\int x^3\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=91

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}$$

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c^2) + (b*x^2 + c*x^4)^(3/2)/(6*
c) + (b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(5/2))
```

Rubi [A] time = 0.0996667, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 640, 612, 620, 206}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c^2) + (b*x^2 + c*x^4)^(3/2)/(6*
c) + (b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(5/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x\sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\left(bx^2+cx^4\right)^{3/2}}{6c}-\frac{b\,\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^3\,\text{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{32c^2}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^3\,\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^2}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0626423, size = 103, normalized size = 1.13

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}\left(-3b^2+2bcx^2+8c^2x^4\right)+3b^3\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{48c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-3*b^2 + 2*b*c*x^2 + 8*c^2*x
^4) + 3*b^3*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(48*c^(5/2)*Sqrt[x^2*(b +
c*x^2)])
```

Maple [A] time = 0.049, size = 104, normalized size = 1.1

$$\frac{1}{48\,x}\sqrt{cx^4+bx^2}\left(8\,x^3\left(cx^2+b\right)^{3/2}c^{3/2}-6\,\left(cx^2+b\right)^{3/2}\sqrt{c}xb+3\,\sqrt{cx^2+b}\sqrt{c}xb^2+3\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^3\right)\frac{1}{\sqrt{cx^2+b}}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(1/2)*(8*x^3*(c*x^2+b)^(3/2)*c^(3/2)-6*(c*x^2+b)^(3/2)*c
^(1/2)*x*b+3*(c*x^2+b)^(1/2)*c^(1/2)*x*b^2+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*
b^3)/x/(c*x^2+b)^(1/2)/c^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58959, size = 371, normalized size = 4.08

$$\left[\frac{3\,b^3\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\left(8\,c^3x^4+2\,bc^2x^2-3\,b^2c\right)\sqrt{cx^4+bx^2}}{96\,c^3},-\frac{3\,b^3\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*b^3*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^3, -1/48*(3*b^3*
sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (8*c^3*x^4 + 2*
b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{x^2 \left(b + cx^2\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.32884, size = 115, normalized size = 1.26

$$\frac{1}{48} \left(2 \left(4 x^2 \mathrm{sgn}\left(x\right) + \frac{b \mathrm{sgn}\left(x\right)}{c}\right) x^2 - \frac{3 b^2 \mathrm{sgn}\left(x\right)}{c^2}\right) \sqrt{cx^2 + b} x - \frac{b^3 \log\left(\left|-\sqrt{c} x + \sqrt{cx^2 + b}\right|\right) \mathrm{sgn}\left(x\right)}{16 c^{\frac{5}{2}}} + \frac{b^3 \log\left(|b|\right) \mathrm{sgn}\left(x\right)}{32 c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sgn(x) + b*sgn(x)/c)*x^2 - 3*b^2*sgn(x)/c^2)*sqrt(c*x^2 + b)
*x - 1/16*b^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(5/2) + 1/32*
b^3*log(abs(b))*sgn(x)/c^(5/2)
```

3.223 $\int x\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=68

$$\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

```
[Out] ((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt
[b*x^2 + c*x^4]])/(8*c^(3/2))
```

Rubi [A] time = 0.0638366, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2013, 612, 620, 206}

$$\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt
[b*x^2 + c*x^4]])/(8*c^(3/2))
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{8c} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c}\\
&= \frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{8c} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c}\\
&= \frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{8c} - \frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0501411, size = 90, normalized size = 1.32

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}\left(b+2cx^2\right)-b^2\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{8c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(b + 2*c*x^2) - b^2*Log[c*x +
 Sqrt[c]*Sqrt[b + c*x^2]]))/(8*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.049, size = 84, normalized size = 1.2

$$\frac{1}{8x}\sqrt{cx^4+bx^2}\left(2x\left(cx^2+b\right)^{3/2}\sqrt{c}-\sqrt{c}\sqrt{cx^2+b}xb-\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^2\right)\frac{1}{\sqrt{cx^2+b}}c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(2*x*(c*x^2+b)^(3/2)*c^(1/2)-c^(1/2)*(c*x^2+b)^(1/2
)*x*b-ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2)/x/(c*x^2+b)^(1/2)/c^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61851, size = 315, normalized size = 4.63

$$\left[\frac{b^2\sqrt{c}\log\left(-2\,cx^2 - b + 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right) + 2\,\sqrt{cx^4 + bx^2}\left(2\,c^2x^2 + bc\right)}{16\,c^2}, \frac{b^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right) + \sqrt{cx^4 + bx^2}\left(2\,c^2\right.}{8\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(b^2*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sq
rt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c))/c^2, 1/8*(b^2*sqrt(-c)*arctan(sqrt(c*x
^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c))/
c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2\left(b + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.25786, size = 93, normalized size = 1.37

$$\frac{1}{8}\sqrt{cx^2+b}\left(2x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x+\frac{b^2\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{8c^{\frac{3}{2}}}-\frac{b^2\log(|b|)\,\mathrm{sgn}(x)}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^2 + b)*(2*x^2*sgn(x) + b*sgn(x)/c)*x + 1/8*b^2*log(abs(-sqrt(c
)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(3/2) - 1/16*b^2*log(abs(b))*sgn(x)/c^(3/2
)
```

3.224 $\int \frac{\sqrt{bx^2+cx^4}}{x} \, dx$

Optimal. Leaf size=55

$$\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] Sqrt[b*x^2 + c*x^4]/2 + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*S
qrt[c])
```

Rubi [A] time = 0.0714512, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 664, 620, 206}

$$\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x,x]
```



```
[Out] Sqrt[b*x^2 + c*x^4]/2 + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*S
qrt[c])
```


Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0253418, size = 64, normalized size = 1.16

$$\frac{1}{2}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{b\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)}{\sqrt{c}x\sqrt{b+cx^2}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(1 + (b*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]])/(Sqrt[c]
*x*Sqrt[b + c*x^2])))/2
```

Maple [A] time = 0.046, size = 64, normalized size = 1.2

$$\frac{1}{2\,x}\sqrt{cx^4+bx^2}\left(x\sqrt{cx^2+b}\sqrt{c}+b\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\right)\frac{1}{\sqrt{cx^2+b}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(1/2)+b*ln(x*c^(1/2)+(c*x^2+b)
^(1/2)))/x/(c*x^2+b)^(1/2)/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60958, size = 261, normalized size = 4.75

$$\left[\frac{b\sqrt{c}\log\left(-2\,cx^2 - b - 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right) + 2\,\sqrt{cx^4 + bx^2}c}{4\,c}, -\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right) - \sqrt{cx^4 + bx^2}c}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c, -1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(
c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*c)/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x, x)
```

Giac [A] time = 1.17933, size = 70, normalized size = 1.27

$$\frac{b\log\left(|b|\right)\operatorname{sgn}\left(x\right)}{4\sqrt{c}}+\frac{1}{2}\left(\sqrt{cx^2+b}x-\frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{\sqrt{c}}\right)\operatorname{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(b))*sgn(x)/sqrt(c) + 1/2*(sqrt(c*x^2 + b)*x - b*log(abs(-sqrt
(c)*x + sqrt(c*x^2 + b)))/sqrt(c))*sgn(x)
```

3.225 $\int \frac{\sqrt{bx^2+cx^4}}{x^3} dx$

Optimal. Leaf size=52

$$\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{\sqrt{bx^2+cx^4}}{x^2}$$

```
[Out] -(Sqrt[b*x^2 + c*x^4]/x^2) + Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]]
```

Rubi [A] time = 0.0770799, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 662, 620, 206}

$$\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{\sqrt{bx^2+cx^4}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/x^2) + Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{bx^2+cx^4}}{x^2} + \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{bx^2+cx^4}}{x^2} + c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{\sqrt{bx^2+cx^4}}{x^2} + \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.093011, size = 60, normalized size = 1.15

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\frac{\sqrt{c}x\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{cx^2}{b}+1}}-1\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-1 + (Sqrt[c]*x*ArcSinh[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt
[b]*Sqrt[1 + (c*x^2)/b])))/x^2
```

Maple [A] time = 0.049, size = 86, normalized size = 1.7

$$-\frac{1}{b x^{2}}\sqrt{c x^{4}+b x^{2}}\left(-\sqrt{c x^{2}+b}\,c^{\frac{3}{2}} x^{2}+\left(c x^{2}+b\right)^{\frac{3}{2}}\sqrt{c}-\ln\left(x\sqrt{c}+\sqrt{c x^{2}+b}\right) x b c\right)\frac{1}{\sqrt{c x^{2}+b}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^3,x)
```

```
[Out] -(c*x^4+b*x^2)^(1/2)*(-(c*x^2+b)^(1/2)*c^(3/2)*x^2+(c*x^2+b)^(3/2)*c^(1/2)-
ln(x*c^(1/2)+(c*x^2+b)^(1/2))*x*b*c)/x^2/(c*x^2+b)^(1/2)/b/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5805, size = 261, normalized size = 5.02

$$\left[\frac{\sqrt{c}x^{2}\log\left(-2\,c x^{2}-b-2\,\sqrt{c x^{4}+b x^{2}}\sqrt{c}\right)-2\,\sqrt{c x^{4}+b x^{2}}}{2\,x^{2}},\,-\frac{\sqrt{-c}x^{2}\arctan\left(\frac{\sqrt{c x^{4}+b x^{2}}\sqrt{-c}}{c x^{2}+b}\right)+\sqrt{c x^{4}+b x^{2}}}{x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqr
t(c*x^4 + b*x^2))/x^2, -(sqrt(-c)*x^2*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(
c*x^2 + b)) + sqrt(c*x^4 + b*x^2))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**3, x)
```

Giac [A] time = 1.25569, size = 82, normalized size = 1.58

$$-\frac{1}{2}\,\sqrt{c}\log\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2\right)\operatorname{sgn}\left(x\right) + \frac{2\,b\sqrt{c}\operatorname{sgn}\left(x\right)}{\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2*b*sqrt(c)*sgn(
x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)
```

3.226 $\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx$

Optimal. Leaf size=25

$$-\frac{\left(bx^2+cx^4\right)^{3/2}}{3bx^6}$$

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(3*b*x^6)
```

Rubi [A] time = 0.0393856, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{\left(bx^2+cx^4\right)^{3/2}}{3bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(3*b*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx=-\frac{\left(bx^2+cx^4\right)^{3/2}}{3bx^6}$$

Mathematica [A] time = 0.0100465, size = 25, normalized size = 1.

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}}{3bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -(x^2*(b + c*x^2))^(3/2)/(3*b*x^6)
```

Maple [A] time = 0.045, size = 29, normalized size = 1.2

$$-\frac{cx^2+b}{3\,bx^4}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^5,x)
```

```
[Out] -1/3/x^4*(c*x^2+b)/b*(c*x^4+b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5287, size = 62, normalized size = 2.48

$$-\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{3\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(b*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**5, x)
```

Giac [B] time = 1.34027, size = 85, normalized size = 3.4

$$\frac{2\left(3\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^4 c^{\frac{3}{2}} \operatorname{sgn}(x) + b^2 c^{\frac{3}{2}} \operatorname{sgn}(x)\right)}{3\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*c^(3/2)*sgn(x) + b^2*c^(3/2)*sgn(x))
/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

3.227 $\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx$

Optimal. Leaf size=52

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(5*b*x^8) + (2*c*(b*x^2 + c*x^4)^(3/2))/(15*b^2*x^6)
```

Rubi [A] time = 0.0831615, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^7,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(5*b*x^8) + (2*c*(b*x^2 + c*x^4)^(3/2))/(15*b^2*x^6)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8} - \frac{(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{5b}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}$$

Mathematica [A] time = 0.0112408, size = 35, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(2cx^2-3b\right)}{15b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^7,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-3*b + 2*c*x^2))/(15*b^2*x^8)
```

Maple [A] time = 0.046, size = 39, normalized size = 0.8

$$-\frac{\left(cx^2+b\right)\left(-2\,cx^2+3\,b\right)}{15\,b^2x^6}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^7,x)
```

```
[Out] -1/15*(c*x^2+b)*(-2*c*x^2+3*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54929, size = 89, normalized size = 1.71

$$\frac{\left(2\,c^2x^4-bcx^2-3\,b^2\right)\sqrt{cx^4+bx^2}}{15\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/15*(2*c^2*x^4 - b*c*x^2 - 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**7, x)
```

Giac [B] time = 1.31997, size = 162, normalized size = 3.12

$$\frac{4\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6c^{\frac{5}{2}}\operatorname{sgn}(x)+5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4bc^{\frac{5}{2}}\operatorname{sgn}(x)+5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{5}{2}}\operatorname{sgn}(x)-b^3c^{\frac{5}{2}}\operatorname{sgn}(x)\right)}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^6*c^(5/2)*sgn(x) + 5*(sqrt(c)*x - sq
rt(c*x^2 + b))^4*b*c^(5/2)*sgn(x) + 5*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^2*c
```

```
^(5/2)*sgn(x) - b^3*c^(5/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

3.228 $\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=80

$$-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}+\frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(7*b*x^10) + (4*c*(b*x^2 + c*x^4)^(3/2))/(35*b^2*x^8
) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^6)
```

Rubi [A] time = 0.119807, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}+\frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(7*b*x^10) + (4*c*(b*x^2 + c*x^4)^(3/2))/(35*b^2*x^8
) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^6)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}} - \frac{(4c)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{7b}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}} + \frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8} + \frac{\left(8c^2\right)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{35b^2}$$
$$= -\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}} + \frac{4c\left(bx^2+cx^4\right)^{3/2}}{35b^2x^8} - \frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}$$

Mathematica [A] time = 0.0119416, size = 46, normalized size = 0.57

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(15b^2-12bcx^2+8c^2x^4\right)}{105b^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -((x^2*(b + c*x^2))^(3/2)*(15*b^2 - 12*b*c*x^2 + 8*c^2*x^4))/(105*b^3*x^10)
```

Maple [A] time = 0.046, size = 50, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(8\,c^2x^4-12\,bcx^2+15\,b^2\right)}{105\,x^8b^3}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^9,x)
```

```
[Out] -1/105*(c*x^2+b)*(8*c^2*x^4-12*b*c*x^2+15*b^2)*(c*x^4+b*x^2)^(1/2)/x^8/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54625, size = 117, normalized size = 1.46

$$-\frac{\left(8\,c^{3}x^{6}-4\,bc^{2}x^{4}+3\,b^{2}cx^{2}+15\,b^{3}\right)\sqrt{cx^{4}+bx^{2}}}{105\,b^{3}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/105*(8*c^3*x^6 - 4*b*c^2*x^4 + 3*b^2*c*x^2 + 15*b^3)*sqrt(c*x^4 + b*x^2)
/(b^3*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^{2}\left(b+cx^{2}\right)}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**9, x)
```

Giac [B] time = 1.28066, size = 200, normalized size = 2.5

$$\frac{16\left(70\left(\sqrt{c}x-\sqrt{cx^{2}+b}\right)^{8}c^{\frac{7}{2}}\operatorname{sgn}(x)+35\left(\sqrt{c}x-\sqrt{cx^{2}+b}\right)^{6}bc^{\frac{7}{2}}\operatorname{sgn}(x)+21\left(\sqrt{c}x-\sqrt{cx^{2}+b}\right)^{4}b^{2}c^{\frac{7}{2}}\operatorname{sgn}(x)-7\left(\sqrt{c}x-\sqrt{\right.\right.}{105\left(\left(\sqrt{c}x-\sqrt{cx^{2}+b}\right)^{2}-b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(7/2)*sgn(x) + 35*(sqrt(c)*x -
 sqrt(c*x^2 + b))^6*b*c^(7/2)*sgn(x) + 21*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b
^2*c^(7/2)*sgn(x) - 7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^3*c^(7/2)*sgn(x) +
b^4*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

3.229 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(9*b*x^12) + (2*c*(b*x^2 + c*x^4)^(3/2))/(21*b^2*x^1
0) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^8) + (16*c^3*(b*x^2 + c*x^4)^
(3/2))/(315*b^4*x^6)
```

Rubi [A] time = 0.163684, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^11,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(9*b*x^12) + (2*c*(b*x^2 + c*x^4)^(3/2))/(21*b^2*x^1
0) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^8) + (16*c^3*(b*x^2 + c*x^4)^
(3/2))/(315*b^4*x^6)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
```

```
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} - \frac{(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx}{3b} \\
&= -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} + \frac{\left(8c^2\right)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{21b^2} \\
&= -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} - \frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8} - \frac{\left(16c^3\right)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{105b^3} \\
&= -\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}} - \frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8} + \frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}
\end{aligned}$$

Mathematica [A] time = 0.0143128, size = 57, normalized size = 0.53

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(30b^2cx^2-35b^3-24bc^2x^4+16c^3x^6\right)}{315b^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^11,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-35*b^3 + 30*b^2*c*x^2 - 24*b*c^2*x^4 + 16*c^3*x^
6))/(315*b^4*x^12)
```

Maple [A] time = 0.046, size = 61, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(-16\,c^3x^6+24\,bc^2x^4-30\,b^2cx^2+35\,b^3\right)}{315\,x^{10}b^4}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^11,x)
```

```
[Out] -1/315*(c*x^2+b)*(-16*c^3*x^6+24*b*c^2*x^4-30*b^2*c*x^2+35*b^3)*(c*x^4+b*x^
2)^(1/2)/x^10/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58623, size = 140, normalized size = 1.3

$$\frac{\left(16\,c^4x^8-8\,bc^3x^6+6\,b^2c^2x^4-5\,b^3cx^2-35\,b^4\right)\sqrt{cx^4+bx^2}}{315\,b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/315*(16*c^4*x^8 - 8*b*c^3*x^6 + 6*b^2*c^2*x^4 - 5*b^3*c*x^2 - 35*b^4)*sqr
t(c*x^4 + b*x^2)/(b^4*x^10)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**11, x)
```

<u>Giac</u> [A] time = 1.3714, size = 240, normalized size = 2.22

$$\frac{32\left(315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}c^{\frac{9}{2}}\operatorname{sgn}(x)+189\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}bc^{\frac{9}{2}}\operatorname{sgn}(x)+84\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}b^2c^{\frac{9}{2}}\operatorname{sgn}(x)-36\left(\sqrt{\right.}\right.}{315\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(9/2)*sgn(x) + 189*(sqrt(c)*
x - sqrt(c*x^2 + b))^8*b*c^(9/2)*sgn(x) + 84*(sqrt(c)*x - sqrt(c*x^2 + b))^
6*b^2*c^(9/2)*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^3*c^(9/2)*sgn(x
) + 9*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^4*c^(9/2)*sgn(x) - b^5*c^(9/2)*sgn(
x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

3.230 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

Optimal. Leaf size=136

$$-\frac{128c^4\left(bx^2+cx^4\right)^{3/2}}{3465b^5x^6}+\frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(11*b*x^14) + (8*c*(b*x^2 + c*x^4)^(3/2))/(99*b^2*x^
12) - (16*c^2*(b*x^2 + c*x^4)^(3/2))/(231*b^3*x^10) + (64*c^3*(b*x^2 + c*x^
4)^(3/2))/(1155*b^4*x^8) - (128*c^4*(b*x^2 + c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rubi [A] time = 0.21401, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{128c^4\left(bx^2+cx^4\right)^{3/2}}{3465b^5x^6}+\frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^13,x]
```


```
[Out] -(b*x^2 + c*x^4)^(3/2)/(11*b*x^14) + (8*c*(b*x^2 + c*x^4)^(3/2))/(99*b^2*x^
12) - (16*c^2*(b*x^2 + c*x^4)^(3/2))/(231*b^3*x^10) + (64*c^3*(b*x^2 + c*x^
4)^(3/2))/(1155*b^4*x^8) - (128*c^4*(b*x^2 + c*x^4)^(3/2))/(3465*b^5*x^6)
```


Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
```

```
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}-\frac{(8c)\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx}{11b}\\ &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}+\frac{\left(16c^2\right)\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx}{33b^2}\\ &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}-\frac{\left(64c^3\right)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{231b^3}\\ &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8}+\frac{\left(128c^4\right)\int \frac{\sqrt{bx^2+}}{x^5}}{1155b^4}\\ &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8}-\frac{128c^4\left(bx^2+cx\right.}{3465b^5x^6}\end{aligned}$$

Mathematica [A] time = 0.0152996, size = 68, normalized size = 0.5

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(240b^2c^2x^4-280b^3cx^2+315b^4-192bc^3x^6+128c^4x^8\right)}{3465b^5x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

```
[Out] -((x^2*(b + c*x^2))^(3/2)*(315*b^4 - 280*b^3*c*x^2 + 240*b^2*c^2*x^4 - 192*
b*c^3*x^6 + 128*c^4*x^8))/(3465*b^5*x^14)
```

Maple [A] time = 0.046, size = 72, normalized size = 0.5

$$-\frac{\left(cx^2+b\right)\left(128\,c^4x^8-192\,c^3x^6b+240\,c^2x^4b^2-280\,cx^2b^3+315\,b^4\right)}{3465\,x^{12}b^5}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^13,x)
```

```
[Out] -1/3465*(c*x^2+b)*(128*c^4*x^8-192*b*c^3*x^6+240*b^2*c^2*x^4-280*b^3*c*x^2+
315*b^4)*(c*x^4+b*x^2)^(1/2)/x^12/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67381, size = 174, normalized size = 1.28

$$-\frac{\left(128\,c^5x^{10}-64\,bc^4x^8+48\,b^2c^3x^6-40\,b^3c^2x^4+35\,b^4cx^2+315\,b^5\right)\sqrt{cx^4+bx^2}}{3465\,b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/3465*(128*c^5*x^10 - 64*b*c^4*x^8 + 48*b^2*c^3*x^6 - 40*b^3*c^2*x^4 + 35
*b^4*c*x^2 + 315*b^5)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**13, x)
```

Giac [A] time = 1.35179, size = 278, normalized size = 2.04

$$\frac{256\left(1386\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}c^{\frac{11}{2}}\operatorname{sgn}(x)+924\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{11}{2}}\operatorname{sgn}(x)+330\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{11}{2}}\operatorname{sgn}(x)-\cdots\right.}{3465\left(\left(\sqrt{c}x\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] 256/3465*(1386*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(11/2)*sgn(x) + 924*(sqrt
(c)*x - sqrt(c*x^2 + b))^10*b*c^(11/2)*sgn(x) + 330*(sqrt(c)*x - sqrt(c*x^2
 + b))^8*b^2*c^(11/2)*sgn(x) - 165*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^3*c^(1
1/2)*sgn(x) + 55*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(11/2)*sgn(x) - 11*(
sqrt(c)*x - sqrt(c*x^2 + b))^2*b^5*c^(11/2)*sgn(x) + b^6*c^(11/2)*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

3.231 $\int x^4\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=78

$$\frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}$$

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - (4*b*(b*x^2 + c*x^4)^(3/2))/(
35*c^2*x) + (x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rubi [A] time = 0.0942747, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - (4*b*(b*x^2 + c*x^4)^(3/2))/(
35*c^2*x) + (x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int x^4\sqrt{bx^2+cx^4}\,dx = \frac{x\left(bx^2+cx^4\right)^{3/2}}{7c} - \frac{(4b)\int x^2\sqrt{bx^2+cx^4}\,dx}{7c}$$
$$= -\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x} + \frac{x\left(bx^2+cx^4\right)^{3/2}}{7c} + \frac{(8b^2)\int\sqrt{bx^2+cx^4}\,dx}{35c^2}$$
$$= \frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3} - \frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x} + \frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}$$

Mathematica [A] time = 0.0226631, size = 46, normalized size = 0.59

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(8b^2-12bcx^2+15c^2x^4\right)}{105c^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(8*b^2 - 12*b*c*x^2 + 15*c^2*x^4))/(105*c^3*x^3)
```

Maple [A] time = 0.046, size = 50, normalized size = 0.6

$$\frac{\left(cx^2+b\right)\left(15\,c^2x^4-12\,bcx^2+8\,b^2\right)}{105\,c^3x}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/105*(c*x^2+b)*(15*c^2*x^4-12*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(1/2)/c^3/x
```

Maxima [A] time = 1.00055, size = 62, normalized size = 0.79

$$\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^2+b}}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)/c^3
```

Fricas [A] time = 1.59245, size = 113, normalized size = 1.45

$$\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^4+bx^2}}{105\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2)/
(c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.2429, size = 76, normalized size = 0.97

$$-\frac{8\,b^{\frac{7}{2}}\operatorname{sgn}(x)}{105\,c^3}+\frac{\left(15\left(cx^2+b\right)^{\frac{7}{2}}-42\left(cx^2+b\right)^{\frac{5}{2}}b+35\left(cx^2+b\right)^{\frac{3}{2}}b^2\right)\operatorname{sgn}(x)}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -8/105*b^(7/2)*sgn(x)/c^3 + 1/105*(15*(c*x^2 + b)^(7/2) - 42*(c*x^2 + b)^(5
/2)*b + 35*(c*x^2 + b)^(3/2)*b^2)*sgn(x)/c^3
```

3.232 $\int x^2\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=52

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}$$

```
[Out] (-2*b*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (b*x^2 + c*x^4)^(3/2)/(5*c*x)
```

Rubi [A] time = 0.0486033, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (b*x^2 + c*x^4)^(3/2)/(5*c*x)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{bx^2+cx^4}\,dx = \frac{\left(bx^2+cx^4\right)^{3/2}}{5cx} - \frac{(2b)\int \sqrt{bx^2+cx^4}\,dx}{5c}$$
$$= -\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3} + \frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}$$

Mathematica [A] time = 0.01787, size = 35, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(3cx^2-2b\right)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-2*b + 3*c*x^2))/(15*c^2*x^3)
```

Maple [A] time = 0.047, size = 39, normalized size = 0.8

$$-\frac{\left(cx^2+b\right)\left(-3\,cx^2+2\,b\right)}{15\,c^2x}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(-3*c*x^2+2*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

Maxima [A] time = 0.997549, size = 46, normalized size = 0.88

$$\frac{\left(3\,c^2x^4+bcx^2-2\,b^2\right)\sqrt{cx^2+b}}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)/c^2
```

Fricas [A] time = 1.5388, size = 86, normalized size = 1.65

$$\frac{\left(3\,c^2x^4 + bcx^2 - 2\,b^2\right)\sqrt{cx^4 + bx^2}}{15\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{x^2\left(b + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.26036, size = 57, normalized size = 1.1

$$\frac{2\,b^{\frac{5}{2}}\operatorname{sgn}\left(x\right)}{15\,c^2} + \frac{\left(3\left(cx^2 + b\right)^{\frac{5}{2}} - 5\left(cx^2 + b\right)^{\frac{3}{2}}b\right)\operatorname{sgn}\left(x\right)}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*b^(5/2)*sgn(x)/c^2 + 1/15*(3*(c*x^2 + b)^(5/2) - 5*(c*x^2 + b)^(3/2)*b
)*sgn(x)/c^2
```

3.233 $\int \sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=25

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

```
[Out] (b*x^2 + c*x^4)^(3/2)/(3*c*x^3)
```

Rubi [A] time = 0.0058113, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4],x]

[Out] (b*x^2 + c*x^4)^(3/2)/(3*c*x^3)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \sqrt{bx^2+cx^4}\,dx=\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Mathematica [A] time = 0.0053323, size = 25, normalized size = 1.

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x^2*(b + c*x^2))^(3/2)/(3*c*x^3)
```

Maple [A] time = 0.044, size = 29, normalized size = 1.2

$$\frac{cx^2+b}{3\,cx}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/3*(c*x^2+b)/c/x*(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 0.984779, size = 19, normalized size = 0.76

$$\frac{\left(cx^2+b\right)^{\frac{3}{2}}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^2 + b)^(3/2)/c
```

Fricas [A] time = 1.51192, size = 58, normalized size = 2.32

$$\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{3\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + c*x**4), x)
```

Giac [A] time = 1.30426, size = 36, normalized size = 1.44

$$\frac{\left(cx^2 + b\right)^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,c} - \frac{b^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(c*x^2 + b)^(3/2)*sgn(x)/c - 1/3*b^(3/2)*sgn(x)/c
```

3.234 $\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rubi [A] time = 0.0491527, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx &= \frac{\sqrt{bx^2+cx^4}}{x}+b\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{\sqrt{bx^2+cx^4}}{x}-b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0351292, size = 60, normalized size = 1.2

$$\frac{x\left(-\sqrt{b}\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)+b+cx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

```
[Out] (x*(b + c*x^2 - Sqrt[b]*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/
Sqrt[x^2*(b + c*x^2)]
```

Maple [A] time = 0.047, size = 65, normalized size = 1.3

$$-\frac{1}{x}\sqrt{cx^4+bx^2}\left(\sqrt{b}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)-\sqrt{cx^2+b}\right)\frac{1}{\sqrt{cx^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^2,x)
```

```
[Out] -(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)-(c*x^2+b)
^(1/2))/x/(c*x^2+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^2, x)
```

Fricas [A] time = 1.53665, size = 262, normalized size = 5.24

$$\left[\frac{\sqrt{b}x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}}{2\,x}, \frac{\sqrt{-b}x\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) +
 2*sqrt(c*x^4 + b*x^2))/x, (sqrt(-b)*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/
(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**2, x)
```

Giac [A] time = 1.2643, size = 92, normalized size = 1.84

$$\left(\frac{b\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{cx^2+b}\right)\operatorname{sgn}(x)-\frac{\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\sqrt{b}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) + sqrt(c*x^2 + b))*sgn(x) - (b
*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))*sgn(x)/sqrt(-b)
```

3.235 $\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx$

Optimal. Leaf size=56

$$-\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*x^3) - (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])
/(2*Sqrt[b])
```

Rubi [A] time = 0.0521658, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^4,x]
```


```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*x^3) - (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])
/(2*Sqrt[b])
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{2x^3}+\frac{1}{2}c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= -\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0417727, size = 63, normalized size = 1.12

$$-\frac{cx^2\sqrt{\frac{cx^2}{b}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)+b+cx^2}{2x\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -(b + c*x^2 + c*x^2*Sqrt[1 + (c*x^2)/b]*ArcTanh[Sqrt[1 + (c*x^2)/b]])/(2*x*
Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.049, size = 85, normalized size = 1.5

$$-\frac{1}{2\,bx^3}\sqrt{cx^4+bx^2}\left(\sqrt{b}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^2c-\sqrt{cx^2+b}x^2c+\left(cx^2+b\right)^{\frac{3}{2}}\right)\frac{1}{\sqrt{cx^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2*c
-(c*x^2+b)^(1/2)*x^2*c+(c*x^2+b)^(3/2))/x^3/(c*x^2+b)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^4, x)
```

Fricas [A] time = 1.61267, size = 300, normalized size = 5.36

$$\left[\frac{\sqrt{b}cx^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) - 2\,\sqrt{cx^4+bx^2}b}{4\,bx^3}, \frac{\sqrt{-b}cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) - \sqrt{cx^4+bx^2}b}{2\,bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b*x^3), 1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c*x^
4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*b)/(b*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**4, x)
```

Giac [A] time = 1.33085, size = 61, normalized size = 1.09

$$\frac{1}{2}c\left(\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{\sqrt{cx^2+b}}{cx^2}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*c*(arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - sqrt(c*x^2 + b)/(c*x^2))
*sgn(x)
```

3.236 $\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=84

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}-\frac{\sqrt{bx^2+cx^4}}{4x^5}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*x^5) - (c*Sqrt[b*x^2 + c*x^4])/(8*b*x^3) + (c^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(3/2))
```

Rubi [A] time = 0.0980381, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2008, 206}

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}-\frac{\sqrt{bx^2+cx^4}}{4x^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*x^5) - (c*Sqrt[b*x^2 + c*x^4])/(8*b*x^3) + (c^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(3/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
```

```
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}+\frac{1}{4}c\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}-\frac{c^2\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0152151, size = 46, normalized size = 0.55

$$-\frac{c^2\left(x^2\left(b+cx^2\right)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx^2}{b}+1\right)}{3b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -(c^2*(x^2*(b + c*x^2))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x^2)/b]
)/(3*b^3*x^3)
```

Maple [A] time = 0.048, size = 106, normalized size = 1.3

$$\frac{1}{8\,b^{2}x^{5}}\sqrt{cx^{4}+bx^{2}}\left(\sqrt{b}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^{2}+b}+b}{x}\right)x^{4}c^{2}-\sqrt{cx^{2}+b}x^{4}c^{2}+\left(cx^{2}+b\right)^{\frac{3}{2}}x^{2}c-2\,\left(cx^{2}+b\right)^{3/2}b\right)\frac{1}{\sqrt{cx^{2}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^6,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^4*c^
2-(c*x^2+b)^(1/2)*x^4*c^2+(c*x^2+b)^(3/2)*x^2*c-2*(c*x^2+b)^(3/2)*b)/x^5/(c
*x^2+b)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{4}+bx^{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^6, x)
```

Fricas [A] time = 1.62325, size = 356, normalized size = 4.24

$$\left[\frac{\sqrt{b}c^{2}x^{5}\log\left(-\frac{cx^{3}+2\,bx+2\,\sqrt{cx^{4}+bx^{2}}\sqrt{b}}{x^{3}}\right)-2\,\sqrt{cx^{4}+bx^{2}}\left(bcx^{2}+2\,b^{2}\right)}{16\,b^{2}x^{5}},-\frac{\sqrt{-b}c^{2}x^{5}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\sqrt{-b}}{cx^{3}+bx}\right)+\sqrt{cx^{4}+bx^{2}}\left(bcx^{2}+\right.}{8\,b^{2}x^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))
/x^3) - 2*sqrt(c*x^4 + b*x^2)*(b*c*x^2 + 2*b^2))/(b^2*x^5), -1/8*(sqrt(-b)*
c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
```

```
*x^2)*(b*c*x^2 + 2*b^2))/(b^2*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**6, x)
```

Giac [A] time = 1.30026, size = 86, normalized size = 1.02

$$-\frac{1}{8}\, c^2 \left(\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} + \frac{\left(cx^2 + b\right)^{\frac{3}{2}} + \sqrt{cx^2 + b}b}{bc^2x^4} \right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/8*c^2*(arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) + ((c*x^2 + b)^(3/2
) + sqrt(c*x^2 + b)*b)/(b*c^2*x^4))*sgn(x)
```

3.237 $\int \frac{\sqrt{bx^2+cx^4}}{x^8} \, dx$

Optimal. Leaf size=112

$$\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3} - \frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}} - \frac{c\sqrt{bx^2+cx^4}}{24bx^5} - \frac{\sqrt{bx^2+cx^4}}{6x^7}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(6*x^7) - (c*Sqrt[b*x^2 + c*x^4])/(24*b*x^5) + (c^2*Sq
rt[b*x^2 + c*x^4])/(16*b^2*x^3) - (c^3*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]])/(16*b^(5/2))
```

Rubi [A] time = 0.144887, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2008, 206}

$$\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3} - \frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}} - \frac{c\sqrt{bx^2+cx^4}}{24bx^5} - \frac{\sqrt{bx^2+cx^4}}{6x^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(6*x^7) - (c*Sqrt[b*x^2 + c*x^4])/(24*b*x^5) + (c^2*Sq
rt[b*x^2 + c*x^4])/(16*b^2*x^3) - (c^3*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]])/(16*b^(5/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{6x^7}+\frac{1}{6}c\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}-\frac{c^2\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{8b}\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}+\frac{c^3\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{16b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{16b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0141793, size = 46, normalized size = 0.41

$$
\frac{c^3\left(x^2\left(b+cx^2\right)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{cx^2}{b}+1\right)}{3b^4x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] (c^3*(x^2*(b + c*x^2))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (c*x^2)/b])
/(3*b^4*x^3)
```

Maple [A] time = 0.051, size = 128, normalized size = 1.1

$$-\frac{1}{48 x^7 b^3}\sqrt{cx^4+bx^2}\left(3\sqrt{b}\ln\left(2\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^6c^3-3\sqrt{cx^2+b}x^6c^3+3\left(cx^2+b\right)^{3/2}x^4c^2-6\left(cx^2+b\right)^{3/2}x^2bc+8\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^8,x)
```

```
[Out] -1/48*(c*x^4+b*x^2)^(1/2)*(3*b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^
6*c^3-3*(c*x^2+b)^(1/2)*x^6*c^3+3*(c*x^2+b)^(3/2)*x^4*c^2-6*(c*x^2+b)^(3/2)
*x^2*b*c+8*(c*x^2+b)^(3/2)*b^2)/x^7/(c*x^2+b)^(1/2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^8, x)
```

Fricas [A] time = 1.67624, size = 410, normalized size = 3.66

$$\left[\frac{3\sqrt{b}c^3x^7\log\left(-\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\left(3bc^2x^4-2b^2cx^2-8b^3\right)\sqrt{cx^4+bx^2}}{96b^3x^7}, \frac{3\sqrt{-b}c^3x^7\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\left(3bc^2\right.}{48b^3x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(b)*c^3*x^7*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*(3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^3*x
^7), 1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 +
b*x)) + (3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^3*x^7)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**8,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**8, x)
```

Giac [A] time = 1.34729, size = 111, normalized size = 0.99

$$\frac{1}{48}\,c^3\left(\frac{3\,\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{3\left(cx^2+b\right)^{\frac{5}{2}}-8\left(cx^2+b\right)^{\frac{3}{2}}b-3\sqrt{cx^2+b}b^2}{b^2c^3x^6}\right)\mathrm{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/48*c^3*(3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(c*x^2 + b
)^(5/2) - 8*(c*x^2 + b)^(3/2)*b - 3*sqrt(c*x^2 + b)*b^2)/(b^2*c^3*x^6))*sgn
(x)
```

3.238 $\int x^3 \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=124

$$\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}$$

```
[Out] (3*b^3*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - (b*(b + 2*c*x^2)*(b*x
^2 + c*x^4)^(3/2))/(32*c^2) + (b*x^2 + c*x^4)^(5/2)/(10*c) - (3*b^5*ArcTanh
[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(7/2))
```

Rubi [A] time = 0.132281, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 640, 612, 620, 206}

$$\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b^3*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - (b*(b + 2*c*x^2)*(b*x
^2 + c*x^4)^(3/2))/(32*c^2) + (b*x^2 + c*x^4)^(5/2)/(10*c) - (3*b^5*ArcTanh
[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(7/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{b\,\text{Subst}\left(\int\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}+\frac{\left(3b^3\right)\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{64c^2}\\
&= \frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{\left(3b^5\right)\text{Subst}\left(\int\cdots\right.}{512\cdots}\\
&= \frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{\left(3b^5\right)\text{Subst}\left(\int\cdots\right.}{25\cdots}\\
&= \frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{c}x\cdots}{\sqrt{bx^2\cdots}}\right.}{256c^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.101127, size = 126, normalized size = 1.02

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(8b^2c^2x^4-10b^3cx^2+15b^4+176bc^3x^6+128c^4x^8\right)-15b^{9/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{1280c^{7/2}x\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(15*b^4 - 10*b^3*c*x^
2 + 8*b^2*c^2*x^4 + 176*b*c^3*x^6 + 128*c^4*x^8) - 15*b^(9/2)*ArcSinh[(Sqrt
[c]*x)/Sqrt[b]]))/(1280*c^(7/2)*x*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.052, size = 142, normalized size = 1.2

$$\frac{1}{1280\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(128\,x^5\left(cx^2+b\right)^{5/2}c^{5/2}-80\left(cx^2+b\right)^{5/2}c^{3/2}x^3b+40\left(cx^2+b\right)^{5/2}\sqrt{c}xb^2-10\left(cx^2+b\right)^{3/2}\sqrt{c}xb^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(128*x^5*(c*x^2+b)^(5/2)*c^(5/2)-80*(c*x^2+b)^(5
/2)*c^(3/2)*x^3*b+40*(c*x^2+b)^(5/2)*c^(1/2)*x*b^2-10*(c*x^2+b)^(3/2)*c^(1/
2)*x*b^3-15*(c*x^2+b)^(1/2)*c^(1/2)*x*b^4-15*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*
b^5)/x^3/(c*x^2+b)^(3/2)/c^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64961, size = 481, normalized size = 3.88

$$\left[\frac{15\,b^5\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\left(128\,c^5x^8+176\,bc^4x^6+8\,b^2c^3x^4-10\,b^3c^2x^2+15\,b^4c\right)\sqrt{cx^4+bx^2}}{2560\,c^4},\frac{15\,b^5\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(15*b^5*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 - 10*b^3*c^2*x^2 + 15*b^4*c
)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*b^5*sqrt(-c)*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-c)/(c*x^2 + b)) + (128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 -
10*b^3*c^2*x^2 + 15*b^4*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.20441, size = 155, normalized size = 1.25

$$\frac{3\, b^5 \log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + b}\right|\right) \mathrm{sgn}\left(x\right)}{256\, c^{\frac{7}{2}}} - \frac{3\, b^5 \log\left(|b|\right) \mathrm{sgn}\left(x\right)}{512\, c^{\frac{7}{2}}} + \frac{1}{1280} \left(2 \left(4 \left(2 \left(8\, cx^2 \mathrm{sgn}\left(x\right) + 11\, b \mathrm{sgn}\left(x\right)\right) x^2 + \frac{b^2 \mathrm{sgn}\left(x\right)}{c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/256*b^5*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(7/2) - 3/512*b^5
*log(abs(b))*sgn(x)/c^(7/2) + 1/1280*(2*(4*(2*(8*c*x^2*sgn(x) + 11*b*sgn(x)
)*x^2 + b^2*sgn(x)/c)*x^2 - 5*b^3*sgn(x)/c^2)*x^2 + 15*b^4*sgn(x)/c^3)*sqrt
(c*x^2 + b)*x
```

3.239 $\int x \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=101

$$-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}$$

```
[Out] (-3*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^2) + ((b + 2*c*x^2)*(b*x^
2 + c*x^4)^(3/2))/(16*c) + (3*b^4*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]
])/(128*c^(5/2))
```

Rubi [A] time = 0.0922218, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2013, 612, 620, 206}

$$-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^2) + ((b + 2*c*x^2)*(b*x^
2 + c*x^4)^(3/2))/(16*c) + (3*b^4*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]
])/(128*c^(5/2))
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}-\frac{\left(3b^2\right)\operatorname{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{32c}\\
&= -\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3b^4\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{256c^2}\\
&= -\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3b^4\right)\operatorname{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^2}\\
&= -\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.101972, size = 115, normalized size = 1.14

$$
\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(2b^2cx^2-3b^3+24bc^2x^4+16c^3x^6\right)+3b^{7/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{128c^{5/2}x\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(-3*b^3 + 2*b^2*c*x^2
 + 24*b*c^2*x^4 + 16*c^3*x^6) + 3*b^(7/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(1
28*c^(5/2)*x*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.048, size = 122, normalized size = 1.2

$$\frac{1}{128\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(16\,x^3\left(cx^2+b\right)^{5/2}c^{3/2}-8\,\sqrt{c}\left(cx^2+b\right)^{5/2}xb+2\,\sqrt{c}\left(cx^2+b\right)^{3/2}xb^2+3\,\sqrt{c}\sqrt{cx^2+b}xb^3+3\,\ln\left(x\sqrt{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/128*(c*x^4+b*x^2)^(3/2)*(16*x^3*(c*x^2+b)^(5/2)*c^(3/2)-8*c^(1/2)*(c*x^2+
b)^(5/2)*x*b+2*c^(1/2)*(c*x^2+b)^(3/2)*x*b^2+3*c^(1/2)*(c*x^2+b)^(1/2)*x*b^
3+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^4)/x^3/(c*x^2+b)^(3/2)/c^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34129, size = 423, normalized size = 4.19

$$\left[\frac{3\,b^4\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\left(16\,c^4x^6+24\,bc^3x^4+2\,b^2c^2x^2-3\,b^3c\right)\sqrt{cx^4+bx^2}}{256\,c^3},-\frac{3\,b^4\sqrt{-c}\arctan\left(\frac{\sqrt{cx}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*b^4*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*(16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 + b*x^2))
/c^3, -1/128*(3*b^4*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b
)) - (16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 + b*x
```

```
^2))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.28781, size = 134, normalized size = 1.33

$$-\frac{3\,b^4\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\operatorname{sgn}(x)}{128\,c^{\frac{5}{2}}}+\frac{3\,b^4\log\left(|b|\right)\operatorname{sgn}(x)}{256\,c^{\frac{5}{2}}}+\frac{1}{128}\left(2\left(4\left(2\,cx^2\operatorname{sgn}(x)+3\,b\operatorname{sgn}(x)\right)x^2+\frac{b^2\operatorname{sgn}(x)}{c}\right)x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -3/128*b^4*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(5/2) + 3/256*b^
4*log(abs(b))*sgn(x)/c^(5/2) + 1/128*(2*(4*(2*c*x^2*sgn(x) + 3*b*sgn(x))*x^
2 + b^2*sgn(x)/c)*x^2 - 3*b^3*sgn(x)/c^2)*sqrt(c*x^2 + b)*x
```

3.240 $\int \frac{(bx^2+cx^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=88

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}$$

```
[Out] (b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + (b*x^2 + c*x^4)^(3/2)/6 - (b
^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

Rubi [A] time = 0.0991041, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 664, 612, 620, 206}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + (b*x^2 + c*x^4)^(3/2)/6 - (b
^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{6}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{4}b\operatorname{Subst}\left(\int \sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{32c}\\
&= \frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c}\\
&= \frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.079886, size = 104, normalized size = 1.18

$$
\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}\left(3b^2+14bcx^2+8c^2x^4\right)-3b^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{48c^{3/2}x\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*(3*b^2 + 14*b*c*x^2 +
 8*c^2*x^4) - 3*b^(5/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(48*c^(3/2)*x*Sqrt[1
 + (c*x^2)/b])
```

Maple [A] time = 0.049, size = 102, normalized size = 1.2

$$\frac{1}{48\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(8\,x\left(cx^2+b\right)^{5/2}\sqrt{c}-2\left(cx^2+b\right)^{3/2}\sqrt{c}xb-3\sqrt{cx^2+b}\sqrt{c}xb^2-3\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^3\right)\left(cx^2+b\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(8*x*(c*x^2+b)^(5/2)*c^(1/2)-2*(c*x^2+b)^(3/2)*c^(
1/2)*x*b-3*(c*x^2+b)^(1/2)*c^(1/2)*x*b^2-3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^
3)/x^3/(c*x^2+b)^(3/2)/c^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36909, size = 373, normalized size = 4.24

$$\left[\frac{3\,b^3\sqrt{c}\log\left(-2\,cx^2-b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\left(8\,c^3x^4+14\,bc^2x^2+3\,b^2c\right)\sqrt{cx^4+bx^2}}{96\,c^2},\frac{3\,b^3\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\left(\cdots\right.}{4\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(3*b^3*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^2, 1/48*(3*b^3*
sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*c^3*x^4 + 14
*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x, x)
```

Giac [A] time = 1.2987, size = 113, normalized size = 1.28

$$\frac{b^3\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\operatorname{sgn}(x)}{16\,c^{\frac{3}{2}}}-\frac{b^3\log\left(|b|\right)\operatorname{sgn}(x)}{32\,c^{\frac{3}{2}}}+\frac{1}{48}\left(2\left(4\,cx^2\operatorname{sgn}(x)+7\,b\operatorname{sgn}(x)\right)x^2+\frac{3\,b^2\operatorname{sgn}(x)}{c}\right)\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/16*b^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(3/2) - 1/32*b^3*l
og(abs(b))*sgn(x)/c^(3/2) + 1/48*(2*(4*c*x^2*sgn(x) + 7*b*sgn(x))*x^2 + 3*b
^2*sgn(x)/c)*sqrt(c*x^2 + b)*x
```

3.241 $\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=80

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}+\frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}$$

```
[Out] (3*b*Sqrt[b*x^2 + c*x^4])/8 + (b*x^2 + c*x^4)^(3/2)/(4*x^2) + (3*b^2*ArcTan
h[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rubi [A] time = 0.0999827, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 664, 620, 206}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}+\frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*b*Sqrt[b*x^2 + c*x^4])/8 + (b*x^2 + c*x^4)^(3/2)/(4*x^2) + (3*b^2*ArcTan
h[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^2}\,dx, x, x^2\right)\\ &= \frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\ &= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{1}{16}\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{1}{8}\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^2}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.111658, size = 71, normalized size = 0.89

$$\frac{1}{8}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{3b^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}}+5b+2cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(5*b + 2*c*x^2 + (3*b^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt
[b]])/(Sqrt[c]*x*Sqrt[1 + (c*x^2)/b])))/8
```

Maple [A] time = 0.048, size = 84, normalized size = 1.1

$$\frac{1}{8x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2x\left(cx^2+b\right)^{3/2}\sqrt{c}+3\sqrt{c}\sqrt{cx^2+b}xb+3\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^2\right)\left(cx^2+b\right)^{-\frac{3}{2}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^3,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(3/2)*(2*x*(c*x^2+b)^(3/2)*c^(1/2)+3*c^(1/2)*(c*x^2+b)^(1
/2)*x*b+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2)/x^3/(c*x^2+b)^(3/2)/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37256, size = 321, normalized size = 4.01

$$\left[\frac{3b^2\sqrt{c}\log\left(-2cx^2-b-2\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\sqrt{cx^4+bx^2}\left(2c^2x^2+5bc\right)}{16c},-\frac{3b^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}(}{8c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^2*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 5*b*c))/c, -1/8*(3*b^2*sqrt(-c)*arctan(sqr
t(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 5
*b*c))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**3, x)
```

Giac [A] time = 1.29204, size = 92, normalized size = 1.15

$$-\frac{3\,b^2\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\operatorname{sgn}(x)}{8\sqrt{c}}+\frac{3\,b^2\log\left(|b|\right)\operatorname{sgn}(x)}{16\sqrt{c}}+\frac{1}{8}\left(2\,cx^2\operatorname{sgn}(x)+5\,b\operatorname{sgn}(x)\right)\sqrt{cx^2+b}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -3/8*b^2*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/sqrt(c) + 3/16*b^2*l
og(abs(b))*sgn(x)/sqrt(c) + 1/8*(2*c*x^2*sgn(x) + 5*b*sgn(x))*sqrt(c*x^2 +
b)*x
```

3.242 $\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=76

$$-\frac{(bx^2+cx^4)^{3/2}}{x^4}+\frac{3}{2}c\sqrt{bx^2+cx^4}+\frac{3}{2}b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/2 - (b*x^2 + c*x^4)^(3/2)/x^4 + (3*b*Sqrt[c]*ArcT
anh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rubi [A] time = 0.110827, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 662, 664, 620, 206}

$$-\frac{(bx^2+cx^4)^{3/2}}{x^4}+\frac{3}{2}c\sqrt{bx^2+cx^4}+\frac{3}{2}b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/2 - (b*x^2 + c*x^4)^(3/2)/x^4 + (3*b*Sqrt[c]*ArcT
anh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^5}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{1}{2}(3c)\,\mathrm{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4}-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{1}{4}(3bc)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4}-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{1}{2}(3bc)\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4}-\frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}+\frac{3}{2}b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.0137465, size = 54, normalized size = 0.71

$$-\frac{b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx^2}{b}\right)}{x^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] -((b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/b)]
)/(x^2*Sqrt[1 + (c*x^2)/b]))
```

Maple [A] time = 0.048, size = 107, normalized size = 1.4

$$-\frac{1}{2\,bx^4}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-2\left(cx^2+b\right)^{3/2}c^{3/2}x^2+2\left(cx^2+b\right)^{5/2}\sqrt{c}-3\sqrt{cx^2+b}c^{3/2}x^2b-3\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)xb^2c\right)\left(cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^5,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(3/2)*(-2*(c*x^2+b)^(3/2)*c^(3/2)*x^2+2*(c*x^2+b)^(5/2)*
c^(1/2)-3*(c*x^2+b)^(1/2)*c^(3/2)*x^2*b-3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*x*b
^2*c)/x^4/(c*x^2+b)^(3/2)/b/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30058, size = 315, normalized size = 4.14

$$\left[\frac{3\,b\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(cx^2-2\,b\right)}{4\,x^2},-\frac{3\,b\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}\left(c\right.}{2\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b))/x^2, -1/2*(3*b*sqrt(-c)*x^2*arctan(sqrt
(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b))/
x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**5, x)
```

Giac [A] time = 1.31144, size = 107, normalized size = 1.41

$$\frac{1}{2}\sqrt{cx^2+b}cx\mathrm{sgn}\left(x\right)-\frac{3}{4}b\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}\left(x\right)+\frac{2\,b^2\sqrt{c}\mathrm{sgn}\left(x\right)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*c*x*sgn(x) - 3/4*b*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2
+ b))^2)*sgn(x) + 2*b^2*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b
)
```

3.243 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^7} dx$

Optimal. Leaf size=75

$$c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6}$$

```
[Out] -((c*Sqrt[b*x^2 + c*x^4])/x^2) - (b*x^2 + c*x^4)^(3/2)/(3*x^6) + c^(3/2)*Ar
cTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rubi [A] time = 0.103917, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 662, 620, 206}

$$c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -((c*Sqrt[b*x^2 + c*x^4])/x^2) - (b*x^2 + c*x^4)^(3/2)/(3*x^6) + c^(3/2)*Ar
cTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^4}\,dx,x,x^2\right)\\ &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6}+\frac{1}{2}c\operatorname{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx,x,x^2\right)\\ &= -\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6}+\frac{1}{2}c^2\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\ &= -\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6}+c^2\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= -\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^6}+c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.0176505, size = 56, normalized size = 0.75

$$-\frac{b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{cx^2}{b}\right)}{3x^4\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -(b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((c*x^2)/b)]
)/(3*x^4*Sqrt[1 + (c*x^2)/b])
```

Maple [B] time = 0.054, size = 129, normalized size = 1.7

$$\frac{1}{3\,b^2x^6}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2\,c^{5/2}\left(cx^2+b\right)^{3/2}x^4+3\,c^{5/2}\sqrt{cx^2+b}x^4b-2\,c^{3/2}\left(cx^2+b\right)^{5/2}x^2+3\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)x^3b^2c^2-\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^7,x)
```

```
[Out] 1/3*(c*x^4+b*x^2)^(3/2)*(2*c^(5/2)*(c*x^2+b)^(3/2)*x^4+3*c^(5/2)*(c*x^2+b)^
(1/2)*x^4*b-2*c^(3/2)*(c*x^2+b)^(5/2)*x^2+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*x
^3*b^2*c^2-(c*x^2+b)^(5/2)*b*c^(1/2))/x^6/(c*x^2+b)^(3/2)/b^2/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29729, size = 312, normalized size = 4.16

$$\left[\frac{3\,c^{\frac{3}{2}}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\left(4\,cx^2+b\right)}{6\,x^4},-\frac{3\,\sqrt{-c}cx^4\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\left(4\,c\right.}{3\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/6*(3*c^(3/2)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*s
qrt(c*x^4 + b*x^2)*(4*c*x^2 + b))/x^4, -1/3*(3*sqrt(-c)*c*x^4*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*(4*c*x^2 + b))/x^
```

4]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**7, x)
```

Giac [A] time = 1.79774, size = 165, normalized size = 2.2

$$-\frac{1}{2}\,c^{\frac{3}{2}}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\operatorname{sgn}(x)+\frac{4\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4bc^{\frac{3}{2}}\operatorname{sgn}(x)-3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{3}{2}}\operatorname{sgn}(x)+2\,b^3c^{\frac{3}{2}}\operatorname{sgn}(x)\right)}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 4/3*(3*(sqrt(c)*
x - sqrt(c*x^2 + b))^4*b*c^(3/2)*sgn(x) - 3*(sqrt(c)*x - sqrt(c*x^2 + b))^2
*b^2*c^(3/2)*sgn(x) + 2*b^3*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^
2 - b)^3
```

3.244 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=25

$$-\frac{\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}$$

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(5*b*x^10)
```

Rubi [A] time = 0.0459559, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(5*b*x^10)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^9}\,dx=-\frac{\left(bx^2+cx^4\right)^{5/2}}{5bx^{10}}$$

Mathematica [A] time = 0.0141597, size = 25, normalized size = 1.

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}}{5bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -(x^2*(b + c*x^2))^(5/2)/(5*b*x^10)
```

Maple [A] time = 0.047, size = 29, normalized size = 1.2

$$-\frac{cx^2+b}{5\,x^8b}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^9,x)
```

```
[Out] -1/5/x^8*(c*x^2+b)/b*(c*x^4+b*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26809, size = 84, normalized size = 3.36

$$-\frac{\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{cx^4+bx^2}}{5\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(b*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**9, x)
```

Giac [B] time = 1.26731, size = 124, normalized size = 4.96

$$\frac{2\left(5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8 c^{\frac{5}{2}}\operatorname{sgn}(x)+10\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4 b^2 c^{\frac{5}{2}}\operatorname{sgn}(x)+b^4 c^{\frac{5}{2}}\operatorname{sgn}(x)\right)}{5\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(5/2)*sgn(x) + 10*(sqrt(c)*x - sqr
t(c*x^2 + b))^4*b^2*c^(5/2)*sgn(x) + b^4*c^(5/2)*sgn(x))/((sqrt(c)*x - sqrt
(c*x^2 + b))^2 - b)^5
```

3.245 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}} dx$

Optimal. Leaf size=52

$$\frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}} - \frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(7*b*x^12) + (2*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*x^1
0)
```

Rubi [A] time = 0.0930483, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}} - \frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(7*b*x^12) + (2*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*x^1
0)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx = -\frac{\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}} - \frac{(2c)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^9}\,dx}{7b}$$
$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}} + \frac{2c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}$$

Mathematica [A] time = 0.0146587, size = 35, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(2cx^2-5b\right)}{35b^2x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*b + 2*c*x^2))/(35*b^2*x^12)
```

Maple [A] time = 0.046, size = 39, normalized size = 0.8

$$-\frac{\left(cx^2+b\right)\left(-2\,cx^2+5\,b\right)}{35\,x^{10}b^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^11,x)
```

```
[Out] -1/35*(c*x^2+b)*(-2*c*x^2+5*b)*(c*x^4+b*x^2)^(3/2)/x^10/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36337, size = 111, normalized size = 2.13

$$\frac{\left(2\,c^3x^6 - bc^2x^4 - 8\,b^2cx^2 - 5\,b^3\right)\sqrt{cx^4 + bx^2}}{35\,b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/35*(2*c^3*x^6 - b*c^2*x^4 - 8*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b*x^2)/(b^2
*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**11,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**11, x)
```

Giac [B] time = 1.31217, size = 240, normalized size = 4.62

$$\frac{4\left(35\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^{10} c^{\frac{7}{2}}\operatorname{sgn}(x) + 35\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^{8} bc^{\frac{7}{2}}\operatorname{sgn}(x) + 70\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^{6} b^2c^{\frac{7}{2}}\operatorname{sgn}(x) + 14\left(\sqrt{c}x -\right.\right.}{35\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] 4/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(7/2)*sgn(x) + 35*(sqrt(c)*x -
sqrt(c*x^2 + b))^8*b*c^(7/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^
2*c^(7/2)*sgn(x) + 14*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^3*c^(7/2)*sgn(x) +
7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^4*c^(7/2)*sgn(x) - b^5*c^(7/2)*sgn(x))/
((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

3.246 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{13}} \, dx$

Optimal. Leaf size=80

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}$$

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(9*b*x^14) + (4*c*(b*x^2 + c*x^4)^(5/2))/(63*b^2*x^1
2) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(315*b^3*x^10)
```

Rubi [A] time = 0.140599, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(9*b*x^14) + (4*c*(b*x^2 + c*x^4)^(5/2))/(63*b^2*x^1
2) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(315*b^3*x^10)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}-\frac{(4c)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx}{9b}\\ &= -\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}+\frac{\left(8c^2\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^9}\,dx}{63b^2}\\ &= -\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}\end{aligned}$$

Mathematica [A] time = 0.0150695, size = 46, normalized size = 0.57

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(35b^2-20bcx^2+8c^2x^4\right)}{315b^3x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -((x^2*(b + c*x^2))^(5/2)*(35*b^2 - 20*b*c*x^2 + 8*c^2*x^4))/(315*b^3*x^14)
```

Maple [A] time = 0.046, size = 50, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(8\,c^2x^4-20\,bcx^2+35\,b^2\right)}{315\,x^{12}b^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^13,x)
```

```
[Out] -1/315*(c*x^2+b)*(8*c^2*x^4-20*b*c*x^2+35*b^2)*(c*x^4+b*x^2)^(3/2)/x^12/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3423, size = 142, normalized size = 1.78

$$-\frac{\left(8\,c^4x^8-4\,bc^3x^6+3\,b^2c^2x^4+50\,b^3cx^2+35\,b^4\right)\sqrt{cx^4+bx^2}}{315\,b^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/315*(8*c^4*x^8 - 4*b*c^3*x^6 + 3*b^2*c^2*x^4 + 50*b^3*c*x^2 + 35*b^4)*sq
rt(c*x^4 + b*x^2)/(b^3*x^10)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**13,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**13, x)
```

Giac [B] time = 1.34965, size = 278, normalized size = 3.48

$$\frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}c^{\frac{9}{2}}\operatorname{sgn}(x)+315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{9}{2}}\operatorname{sgn}(x)+441\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{9}{2}}\operatorname{sgn}(x)+126\right.}{315\left(\left(\sqrt{c}x-\sqrt{c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(9/2)*sgn(x) + 315*(sqrt(c)*
x - sqrt(c*x^2 + b))^10*b*c^(9/2)*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + b)
)^8*b^2*c^(9/2)*sgn(x) + 126*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^3*c^(9/2)*sg
n(x) + 36*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(9/2)*sgn(x) - 9*(sqrt(c)*x
 - sqrt(c*x^2 + b))^2*b^5*c^(9/2)*sgn(x) + b^6*c^(9/2)*sgn(x))/((sqrt(c)*x
- sqrt(c*x^2 + b))^2 - b)^9
```

3.247 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3(bx^2+cx^4)^{5/2}}{1155b^4x^{10}}-\frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}}+\frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}}-\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}}$$

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(11*b*x^16) + (2*c*(b*x^2 + c*x^4)^(5/2))/(33*b^2*x^
14) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(231*b^3*x^12) + (16*c^3*(b*x^2 + c*x^4
)^(5/2))/(1155*b^4*x^10)
```

Rubi [A] time = 0.183943, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16c^3(bx^2+cx^4)^{5/2}}{1155b^4x^{10}}-\frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}}+\frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}}-\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(11*b*x^16) + (2*c*(b*x^2 + c*x^4)^(5/2))/(33*b^2*x^
14) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(231*b^3*x^12) + (16*c^3*(b*x^2 + c*x^4
)^(5/2))/(1155*b^4*x^10)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
```

```
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}-\frac{(6c)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx}{11b}\\
&= -\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}}{33b^2x^{14}}+\frac{\left(8c^2\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx}{33b^2}\\
&= -\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}}{33b^2x^{14}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{231b^3x^{12}}-\frac{\left(16c^3\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{9}}\,dx}{231b^3}\\
&= -\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}}{33b^2x^{14}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{231b^3x^{12}}+\frac{16c^3\left(bx^2+cx^4\right)^{5/2}}{1155b^4x^{10}}
\end{aligned}$$

Mathematica [A] time = 0.0176025, size = 57, normalized size = 0.53

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(70b^2cx^2-105b^3-40bc^2x^4+16c^3x^6\right)}{1155b^4x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-105*b^3 + 70*b^2*c*x^2 - 40*b*c^2*x^4 + 16*c^3*x
^6))/(1155*b^4*x^16)
```

Maple [A] time = 0.045, size = 61, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(-16\,c^3x^6+40\,bc^2x^4-70\,b^2cx^2+105\,b^3\right)}{1155\,x^{14}b^4}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^15,x)
```

```
[Out] -1/1155*(c*x^2+b)*(-16*c^3*x^6+40*b*c^2*x^4-70*b^2*c*x^2+105*b^3)*(c*x^4+b*
x^2)^(3/2)/x^14/b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47126, size = 169, normalized size = 1.56

$$\frac{\left(16\,c^5x^{10} - 8\,bc^4x^8 + 6\,b^2c^3x^6 - 5\,b^3c^2x^4 - 140\,b^4cx^2 - 105\,b^5\right)\sqrt{cx^4 + bx^2}}{1155\,b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] 1/1155*(16*c^5*x^10 - 8*b*c^4*x^8 + 6*b^2*c^3*x^6 - 5*b^3*c^2*x^4 - 140*b^4
*c*x^2 - 105*b^5)*sqrt(c*x^4 + b*x^2)/(b^4*x^12)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**15,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**15, x)
```

Giac [B] time = 1.26963, size = 319, normalized size = 2.95

$$\frac{32\left(1155\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}c^{\frac{11}{2}}\operatorname{sgn}(x)+2079\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}bc^{\frac{11}{2}}\operatorname{sgn}(x)+2541\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}b^2c^{\frac{11}{2}}\operatorname{sgn}(x)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] 32/1155*(1155*(sqrt(c)*x - sqrt(c*x^2 + b))^14*c^(11/2)*sgn(x) + 2079*(sqrt
(c)*x - sqrt(c*x^2 + b))^12*b*c^(11/2)*sgn(x) + 2541*(sqrt(c)*x - sqrt(c*x^
2 + b))^10*b^2*c^(11/2)*sgn(x) + 825*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b^3*c^
(11/2)*sgn(x) + 165*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^4*c^(11/2)*sgn(x) - 5
5*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^5*c^(11/2)*sgn(x) + 11*(sqrt(c)*x - sqr
t(c*x^2 + b))^2*b^6*c^(11/2)*sgn(x) - b^7*c^(11/2)*sgn(x))/((sqrt(c)*x - sq
rt(c*x^2 + b))^2 - b)^11
```

3.248 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=136

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}}{15015b^5x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}}+\frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(13*b*x^18) + (8*c*(b*x^2 + c*x^4)^(5/2))/(143*b^2*x
^16) - (16*c^2*(b*x^2 + c*x^4)^(5/2))/(429*b^3*x^14) + (64*c^3*(b*x^2 + c*x
^4)^(5/2))/(3003*b^4*x^12) - (128*c^4*(b*x^2 + c*x^4)^(5/2))/(15015*b^5*x^1
0)
```

Rubi [A] time = 0.262011, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}}{15015b^5x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}}+\frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(13*b*x^18) + (8*c*(b*x^2 + c*x^4)^(5/2))/(143*b^2*x
^16) - (16*c^2*(b*x^2 + c*x^4)^(5/2))/(429*b^3*x^14) + (64*c^3*(b*x^2 + c*x
^4)^(5/2))/(3003*b^4*x^12) - (128*c^4*(b*x^2 + c*x^4)^(5/2))/(15015*b^5*x^1
0)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
```

```
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{17}}\,dx = -\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} - \frac{(8c)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx}{13b}$$

$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} + \frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} + \frac{\left(48c^2\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx}{143b^2}$$

$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} + \frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} - \frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}} - \frac{\left(64c^3\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx}{429b^3}$$

$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} + \frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} - \frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}} + \frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}} + \frac{\left(128c^4\right)\int \frac{\left(bx^2+\right.}{}}{3003b^4}$$

$$= -\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} + \frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} - \frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}} + \frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}} - \frac{128c^4\left(bx^2+cx\right.}{15015b^5x^{10}}$$

Mathematica [A] time = 0.0186833, size = 68, normalized size = 0.5

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(560b^2c^2x^4-840b^3cx^2+1155b^4-320bc^3x^6+128c^4x^8\right)}{15015b^5x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] -((x^2*(b + c*x^2))^(5/2)*(1155*b^4 - 840*b^3*c*x^2 + 560*b^2*c^2*x^4 - 320
*b*c^3*x^6 + 128*c^4*x^8))/(15015*b^5*x^18)
```

Maple [A] time = 0.045, size = 72, normalized size = 0.5

$$-\frac{\left(cx^2+b\right)\left(128\,c^4x^8-320\,c^3x^6b+560\,c^2x^4b^2-840\,cx^2b^3+1155\,b^4\right)}{15015\,x^{16}b^5}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^17,x)
```

```
[Out] -1/15015*(c*x^2+b)*(128*c^4*x^8-320*b*c^3*x^6+560*b^2*c^2*x^4-840*b^3*c*x^2
+1155*b^4)*(c*x^4+b*x^2)^(3/2)/x^16/b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6755, size = 204, normalized size = 1.5

$$-\frac{\left(128\,c^{6}x^{12}-64\,bc^{5}x^{10}+48\,b^{2}c^{4}x^{8}-40\,b^{3}c^{3}x^{6}+35\,b^{4}c^{2}x^{4}+1470\,b^{5}cx^{2}+1155\,b^{6}\right)\sqrt{cx^{4}+bx^{2}}}{15015\,b^{5}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/15015*(128*c^6*x^12 - 64*b*c^5*x^10 + 48*b^2*c^4*x^8 - 40*b^3*c^3*x^6 +
35*b^4*c^2*x^4 + 1470*b^5*c*x^2 + 1155*b^6)*sqrt(c*x^4 + b*x^2)/(b^5*x^14)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**17,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**17, x)
```

Giac [B] time = 1.26971, size = 356, normalized size = 2.62

$$\frac{256\left(6006\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}c^{\frac{13}{2}}\operatorname{sgn}(x)+12012\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}bc^{\frac{13}{2}}\operatorname{sgn}(x)+13728\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}b^2c^{\frac{13}{2}}\operatorname{sgn}(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] 256/15015*(6006*(sqrt(c)*x - sqrt(c*x^2 + b))^16*c^(13/2)*sgn(x) + 12012*(s
qrt(c)*x - sqrt(c*x^2 + b))^14*b*c^(13/2)*sgn(x) + 13728*(sqrt(c)*x - sqrt(
c*x^2 + b))^12*b^2*c^(13/2)*sgn(x) + 4719*(sqrt(c)*x - sqrt(c*x^2 + b))^10*
b^3*c^(13/2)*sgn(x) + 715*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b^4*c^(13/2)*sgn(
x) - 286*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^5*c^(13/2)*sgn(x) + 78*(sqrt(c)*
x - sqrt(c*x^2 + b))^4*b^6*c^(13/2)*sgn(x) - 13*(sqrt(c)*x - sqrt(c*x^2 + b
))^2*b^7*c^(13/2)*sgn(x) + b^8*c^(13/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 +
b))^2 - b)^13
```

3.249 $\int x^6 \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=134

$$\frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}$$

```
[Out] (128*b^4*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (64*b^3*(b*x^2 + c*x^4)^(
5/2))/(3003*c^4*x^3) + (16*b^2*(b*x^2 + c*x^4)^(5/2))/(429*c^3*x) - (8*b*x*
(b*x^2 + c*x^4)^(5/2))/(143*c^2) + (x^3*(b*x^2 + c*x^4)^(5/2))/(13*c)
```

Rubi [A] time = 0.252684, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (128*b^4*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (64*b^3*(b*x^2 + c*x^4)^(
5/2))/(3003*c^4*x^3) + (16*b^2*(b*x^2 + c*x^4)^(5/2))/(429*c^3*x) - (8*b*x*
(b*x^2 + c*x^4)^(5/2))/(143*c^2) + (x^3*(b*x^2 + c*x^4)^(5/2))/(13*c)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
```

```
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^6 \left(bx^2+cx^4\right)^{3/2} dx = \frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c} - \frac{(8b)\int x^4\left(bx^2+cx^4\right)^{3/2} dx}{13c}$$

$$= -\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2} + \frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c} + \frac{\left(48b^2\right)\int x^2\left(bx^2+cx^4\right)^{3/2} dx}{143c^2}$$

$$= \frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x} - \frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2} + \frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c} - \frac{\left(64b^3\right)\int \left(bx^2+cx^4\right)^{3/2} dx}{429c^3}$$

$$= -\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3} + \frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x} - \frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2} + \frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c} + \frac{\left(128b^4\right)}{}$$

$$= \frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5} - \frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3} + \frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x} - \frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2} + \frac{x^3\left(bx\right.}{}$$

Mathematica [A] time = 0.0382531, size = 75, normalized size = 0.56

$$\frac{x\left(b+cx^2\right)^3\left(560b^2c^2x^4-320b^3cx^2+128b^4-840bc^3x^6+1155c^4x^8\right)}{15015c^5\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(128*b^4 - 320*b^3*c*x^2 + 560*b^2*c^2*x^4 - 840*b*c^3*x^6
 + 1155*c^4*x^8))/(15015*c^5*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.046, size = 72, normalized size = 0.5

$$\frac{\left(cx^{2}+b\right)\left(1155\,x^{8}c^{4}-840\,bx^{6}c^{3}+560\,b^{2}x^{4}c^{2}-320\,b^{3}x^{2}c+128\,b^{4}\right)}{15015\,c^{5}x^{3}}\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/15015*(c*x^2+b)*(1155*c^4*x^8-840*b*c^3*x^6+560*b^2*c^2*x^4-320*b^3*c*x^2
+128*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

Maxima [A] time = 1.04213, size = 107, normalized size = 0.8

$$\frac{\left(1155\,c^{6}x^{12}+1470\,bc^{5}x^{10}+35\,b^{2}c^{4}x^{8}-40\,b^{3}c^{3}x^{6}+48\,b^{4}c^{2}x^{4}-64\,b^{5}cx^{2}+128\,b^{6}\right)\sqrt{cx^{2}+b}}{15015\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6
+ 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^2 + b)/c^5
```

Fricas [A] time = 1.38608, size = 198, normalized size = 1.48

$$\frac{\left(1155\,c^{6}x^{12}+1470\,bc^{5}x^{10}+35\,b^{2}c^{4}x^{8}-40\,b^{3}c^{3}x^{6}+48\,b^{4}c^{2}x^{4}-64\,b^{5}cx^{2}+128\,b^{6}\right)\sqrt{cx^{4}+bx^{2}}}{15015\,c^{5}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6
+ 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^4 + b*x^2)/(c^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.22254, size = 240, normalized size = 1.79

$$-\frac{128\, b^{\frac{13}{2}} \operatorname{sgn}(x)}{15015\, c^5} + \frac{\frac{13\left(315\left(cx^2+b\right)^{\frac{11}{2}} - 1540\left(cx^2+b\right)^{\frac{9}{2}} b + 2970\left(cx^2+b\right)^{\frac{7}{2}} b^2 - 2772\left(cx^2+b\right)^{\frac{5}{2}} b^3 + 1155\left(cx^2+b\right)^{\frac{3}{2}} b^4\right) b \operatorname{sgn}(x)}{c^4} + \frac{5\left(693\left(cx^2+b\right)^{\frac{13}{2}} - 4095\left(cx^2 \cdots\right.\right.}{\cdots}}{45045\, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -128/15015*b^(13/2)*sgn(x)/c^5 + 1/45045*(13*(315*(c*x^2 + b)^(11/2) - 1540
*(c*x^2 + b)^(9/2)*b + 2970*(c*x^2 + b)^(7/2)*b^2 - 2772*(c*x^2 + b)^(5/2)*
b^3 + 1155*(c*x^2 + b)^(3/2)*b^4)*b*sgn(x)/c^4 + 5*(693*(c*x^2 + b)^(13/2)
- 4095*(c*x^2 + b)^(11/2)*b + 10010*(c*x^2 + b)^(9/2)*b^2 - 12870*(c*x^2 +
b)^(7/2)*b^3 + 9009*(c*x^2 + b)^(5/2)*b^4 - 3003*(c*x^2 + b)^(3/2)*b^5)*sgn
(x)/c^4)/c
```

3.250 $\int x^4 \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=106

$$-\frac{16b^3\left(bx^2+cx^4\right)^{5/2}}{1155c^4x^5}+\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}$$

```
[Out] (-16*b^3*(b*x^2 + c*x^4)^(5/2))/(1155*c^4*x^5) + (8*b^2*(b*x^2 + c*x^4)^(5/
2))/(231*c^3*x^3) - (2*b*(b*x^2 + c*x^4)^(5/2))/(33*c^2*x) + (x*(b*x^2 + c*
x^4)^(5/2))/(11*c)
```

Rubi [A] time = 0.195626, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{16b^3\left(bx^2+cx^4\right)^{5/2}}{1155c^4x^5}+\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-16*b^3*(b*x^2 + c*x^4)^(5/2))/(1155*c^4*x^5) + (8*b^2*(b*x^2 + c*x^4)^(5/
2))/(231*c^3*x^3) - (2*b*(b*x^2 + c*x^4)^(5/2))/(33*c^2*x) + (x*(b*x^2 + c*
x^4)^(5/2))/(11*c)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
```

```
j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{(6b)\int x^2\left(bx^2+cx^4\right)^{3/2}\,dx}{11c}\\ &= -\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}+\frac{\left(8b^2\right)\int\left(bx^2+cx^4\right)^{3/2}\,dx}{33c^2}\\ &= \frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{\left(16b^3\right)\int\frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}\,dx}{231c^3}\\ &= -\frac{16b^3\left(bx^2+cx^4\right)^{5/2}}{1155c^4x^5}+\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}\end{aligned}$$

Mathematica [A] time = 0.0320802, size = 64, normalized size = 0.6

$$\frac{x\left(b+cx^2\right)^3\left(40b^2cx^2-16b^3-70bc^2x^4+105c^3x^6\right)}{1155c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(-16*b^3 + 40*b^2*c*x^2 - 70*b*c^2*x^4 + 105*c^3*x^6))/(11
55*c^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.047, size = 61, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(-105\,c^3x^6+70\,bc^2x^4-40\,b^2cx^2+16\,b^3\right)}{1155\,c^4x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/1155*(c*x^2+b)*(-105*c^3*x^6+70*b*c^2*x^4-40*b^2*c*x^2+16*b^3)*(c*x^4+b*
x^2)^(3/2)/c^4/x^3
```

Maxima [A] time = 1.04, size = 92, normalized size = 0.87

$$\frac{\left(105\,c^{5}x^{10}+140\,bc^{4}x^{8}+5\,b^{2}c^{3}x^{6}-6\,b^{3}c^{2}x^{4}+8\,b^{4}cx^{2}-16\,b^{5}\right)\sqrt{cx^{2}+b}}{1155\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)/c^4
```

Fricas [A] time = 1.28476, size = 165, normalized size = 1.56

$$\frac{\left(105\,c^{5}x^{10}+140\,bc^{4}x^{8}+5\,b^{2}c^{3}x^{6}-6\,b^{3}c^{2}x^{4}+8\,b^{4}cx^{2}-16\,b^{5}\right)\sqrt{cx^{4}+bx^{2}}}{1155\,c^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{4}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.17888, size = 201, normalized size = 1.9

$$\frac{16\,b^{\frac{11}{2}}\operatorname{sgn}(x)}{1155\,c^{4}}+\frac{\frac{11\left(35\left(cx^{2}+b\right)^{\frac{9}{2}}-135\left(cx^{2}+b\right)^{\frac{7}{2}}b+189\left(cx^{2}+b\right)^{\frac{5}{2}}b^{2}-105\left(cx^{2}+b\right)^{\frac{3}{2}}b^{3}\right)b\operatorname{sgn}(x)}{c^{3}}+\frac{\left(315\left(cx^{2}+b\right)^{\frac{11}{2}}-1540\left(cx^{2}+b\right)^{\frac{9}{2}}b+2970\left(cx^{2}+b\right)^{\frac{7}{2}}b^{2}-277\right.}{c^{3}}}{3465\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 16/1155*b^(11/2)*sgn(x)/c^4 + 1/3465*(11*(35*(c*x^2 + b)^(9/2) - 135*(c*x^2
 + b)^(7/2)*b + 189*(c*x^2 + b)^(5/2)*b^2 - 105*(c*x^2 + b)^(3/2)*b^3)*b*sg
n(x)/c^3 + (315*(c*x^2 + b)^(11/2) - 1540*(c*x^2 + b)^(9/2)*b + 2970*(c*x^2
 + b)^(7/2)*b^2 - 2772*(c*x^2 + b)^(5/2)*b^3 + 1155*(c*x^2 + b)^(3/2)*b^4)*
sgn(x)/c^3)/c
```

3.251 $\int x^2 \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=80

$$\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - (4*b*(b*x^2 + c*x^4)^(5/2))/(
63*c^2*x^3) + (b*x^2 + c*x^4)^(5/2)/(9*c*x)
```

Rubi [A] time = 0.111375, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - (4*b*(b*x^2 + c*x^4)^(5/2))/(
63*c^2*x^3) + (b*x^2 + c*x^4)^(5/2)/(9*c*x)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^2 \left(bx^2+cx^4\right)^{3/2} dx = \frac{\left(bx^2+cx^4\right)^{5/2}}{9cx} - \frac{(4b)\int \left(bx^2+cx^4\right)^{3/2} dx}{9c}$$
$$= -\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3} + \frac{\left(bx^2+cx^4\right)^{5/2}}{9cx} + \frac{\left(8b^2\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^2} dx}{63c^2}$$
$$= \frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5} - \frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3} + \frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

Mathematica [A] time = 0.025244, size = 53, normalized size = 0.66

$$\frac{x\left(b+cx^2\right)^3\left(8b^2-20bcx^2+35c^2x^4\right)}{315c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(8*b^2 - 20*b*c*x^2 + 35*c^2*x^4))/(315*c^3*Sqrt[x^2*(b +
c*x^2)])
```

Maple [A] time = 0.046, size = 50, normalized size = 0.6

$$\frac{\left(cx^2+b\right)\left(35\,c^2x^4-20\,bcx^2+8\,b^2\right)}{315\,c^3x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/315*(c*x^2+b)*(35*c^2*x^4-20*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(3/2)/c^3/x^3
```

Maxima [A] time = 1.04395, size = 77, normalized size = 0.96

$$\frac{\left(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4\right)\sqrt{cx^2+b}}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)/c^3
```

Fricas [A] time = 1.3391, size = 136, normalized size = 1.7

$$\frac{\left(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4\right)\sqrt{cx^4+bx^2}}{315\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^4 + b*x^2)/(c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.15817, size = 163, normalized size = 2.04

$$-\frac{8\,b^{\frac{9}{2}}\operatorname{sgn}\left(x\right)}{315\,c^{3}}+\frac{\frac{3\left(15\left(cx^{2}+b\right)^{\frac{7}{2}}-42\left(cx^{2}+b\right)^{\frac{5}{2}}b+35\left(cx^{2}+b\right)^{\frac{3}{2}}b^{2}\right)b\operatorname{sgn}(x)}{c^{2}}+\frac{\left(35\left(cx^{2}+b\right)^{\frac{9}{2}}-135\left(cx^{2}+b\right)^{\frac{7}{2}}b+189\left(cx^{2}+b\right)^{\frac{5}{2}}b^{2}-105\left(cx^{2}+b\right)^{\frac{3}{2}}b^{3}\right)\operatorname{sgn}(x)}{c^{2}}}{315\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/315*b^(9/2)*sgn(x)/c^3 + 1/315*(3*(15*(c*x^2 + b)^(7/2) - 42*(c*x^2 + b)
^(5/2)*b + 35*(c*x^2 + b)^(3/2)*b^2)*b*sgn(x)/c^2 + (35*(c*x^2 + b)^(9/2) -
 135*(c*x^2 + b)^(7/2)*b + 189*(c*x^2 + b)^(5/2)*b^2 - 105*(c*x^2 + b)^(3/2
)*b^3)*sgn(x)/c^2)/c
```

3.252 $\int \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=52

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}$$

```
[Out] (-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*c*x^3)
```

Rubi [A] time = 0.0540039, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*c*x^3)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \left(bx^2+cx^4\right)^{3/2} dx = \frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3} - \frac{(2b)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^2} dx}{7c}$$
$$= -\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5} + \frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}$$

Mathematica [A] time = 0.0191722, size = 42, normalized size = 0.81

$$\frac{x\left(b+cx^2\right)^3\left(5cx^2-2b\right)}{35c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(-2*b + 5*c*x^2))/(35*c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.046, size = 39, normalized size = 0.8

$$-\frac{\left(cx^2+b\right)\left(-5\,cx^2+2\,b\right)}{35\,c^2x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-5*c*x^2+2*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

Maxima [A] time = 1.04154, size = 61, normalized size = 1.17

$$\frac{\left(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3\right)\sqrt{cx^2+b}}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)/c^2
```

Fricas [A] time = 1.31199, size = 108, normalized size = 2.08

$$\frac{\left(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3\right)\sqrt{cx^4+bx^2}}{35\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^4 + b*x^2)/(c^2
*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((b*x**2 + c*x**4)**(3/2), x)
```

Giac [B] time = 1.15115, size = 126, normalized size = 2.42

$$\frac{2\,b^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}{35\,c^2}+\frac{\frac{7\left(3\left(cx^2+b\right)^{\frac{5}{2}}-5\left(cx^2+b\right)^{\frac{3}{2}}b\right)b\,\mathrm{sgn}(x)}{c}+\frac{\left(15\left(cx^2+b\right)^{\frac{7}{2}}-42\left(cx^2+b\right)^{\frac{5}{2}}b+35\left(cx^2+b\right)^{\frac{3}{2}}b^2\right)\mathrm{sgn}(x)}{c}}{105\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/35*b^(7/2)*sgn(x)/c^2 + 1/105*(7*(3*(c*x^2 + b)^(5/2) - 5*(c*x^2 + b)^(3/
2)*b)*b*sgn(x)/c + (15*(c*x^2 + b)^(7/2) - 42*(c*x^2 + b)^(5/2)*b + 35*(c*x
^2 + b)^(3/2)*b^2)*sgn(x)/c)/c
```

3.253 $\int \frac{(bx^2+cx^4)^{3/2}}{x^2} dx$

Optimal. Leaf size=25

$$\frac{(bx^2 + cx^4)^{5/2}}{5cx^5}$$

```
[Out] (b*x^2 + c*x^4)^(5/2)/(5*c*x^5)
```

Rubi [A] time = 0.0475792, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{(bx^2 + cx^4)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^2,x]

[Out] (b*x^2 + c*x^4)^(5/2)/(5*c*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^2} dx = \frac{(bx^2 + cx^4)^{5/2}}{5cx^5}$$

Mathematica [A] time = 0.0090662, size = 25, normalized size = 1.

$$\frac{(x^2 (b + cx^2))^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (x^2*(b + c*x^2))^(5/2)/(5*c*x^5)
```

Maple [A] time = 0.044, size = 29, normalized size = 1.2

$$\frac{cx^2+b}{5\,cx^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^2,x)
```

```
[Out] 1/5*(c*x^2+b)/c/x^3*(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 1.02987, size = 43, normalized size = 1.72

$$\frac{\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{cx^2+b}}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)/c
```

Fricas [A] time = 1.3784, size = 80, normalized size = 3.2

$$\frac{\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{cx^4+bx^2}}{5\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**2, x)
```

Giac [B] time = 1.13411, size = 78, normalized size = 3.12

$$-\frac{b^{\frac{5}{2}}\operatorname{sgn}(x)}{5\,c}+\frac{5\left(cx^2+b\right)^{\frac{3}{2}}b\operatorname{sgn}(x)+\left(3\left(cx^2+b\right)^{\frac{5}{2}}-5\left(cx^2+b\right)^{\frac{3}{2}}b\right)\operatorname{sgn}(x)}{15\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/5*b^(5/2)*sgn(x)/c + 1/15*(5*(c*x^2 + b)^(3/2)*b*sgn(x) + (3*(c*x^2 + b)
^(5/2) - 5*(c*x^2 + b)^(3/2)*b)*sgn(x))/c
```

3.254 $\int \frac{(bx^2+cx^4)^{3/2}}{x^4} dx$

Optimal. Leaf size=73

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$

```
[Out] (b*Sqrt[b*x^2 + c*x^4])/x + (b*x^2 + c*x^4)^(3/2)/(3*x^3) - b^(3/2)*ArcTanh
[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rubi [A] time = 0.110463, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```


```
[Out] (b*Sqrt[b*x^2 + c*x^4])/x + (b*x^2 + c*x^4)^(3/2)/(3*x^3) - b^(3/2)*ArcTanh
[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```


Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^4}\,dx &= \frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}+b\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx \\
&= \frac{b\sqrt{bx^2+cx^4}}{x}+\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}+b^2\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{b\sqrt{bx^2+cx^4}}{x}+\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}-b^2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right) \\
&= \frac{b\sqrt{bx^2+cx^4}}{x}+\frac{\left(bx^2+cx^4\right)^{3/2}}{3x^3}-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0531942, size = 76, normalized size = 1.04

$$\frac{x\left(-3b^{3/2}\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)+4b^2+5bcx^2+c^2x^4\right)}{3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (x*(4*b^2 + 5*b*c*x^2 + c^2*x^4 - 3*b^(3/2)*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b
+ c*x^2]/Sqrt[b]]))/(3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.046, size = 78, normalized size = 1.1

$$-\frac{1}{3\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(3\,b^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)-\left(cx^2+b\right)^{\frac{3}{2}}-3\,\sqrt{cx^2+b}b\right)\left(cx^2+b\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^4,x)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)-(c*
x^2+b)^(3/2)-3*(c*x^2+b)^(1/2)*b)/x^3/(c*x^2+b)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^4, x)
```

Fricas [A] time = 1.40754, size = 313, normalized size = 4.29

$$\left[\frac{3\,b^{\frac{3}{2}}x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(cx^2+4\,b\right)}{6\,x},\ \frac{3\,\sqrt{-b}bx\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\left(cx^2+4\,b\right)}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^(3/2)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)
 + 2*sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))/x, 1/3*(3*sqrt(-b)*b*x*arctan(sqrt(
c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))
/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**4, x)
```

Giac [A] time = 1.157, size = 119, normalized size = 1.63

$$\frac{1}{3}\left(\frac{3\,b^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\left(cx^2+b\right)^{\frac{3}{2}}+3\,\sqrt{cx^2+b}b\right)\operatorname{sgn}\left(x\right)-\frac{\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\,\sqrt{-b}b^{\frac{3}{2}}\right)\operatorname{sgn}\left(x\right)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) + (c*x^2 + b)^(3/2) +
3*sqrt(c*x^2 + b)*b)*sgn(x) - 1/3*(3*b^2*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt(
-b)*b^(3/2))*sgn(x)/sqrt(-b)
```

3.255 $\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{(bx^2+cx^4)^{3/2}}{2x^5}+\frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{3}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/(2*x) - (b*x^2 + c*x^4)^(3/2)/(2*x^5) - (3*Sqrt[b
]*c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

Rubi [A] time = 0.118345, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2021, 2008, 206}

$$-\frac{(bx^2+cx^4)^{3/2}}{2x^5}+\frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{3}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```


```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/(2*x) - (b*x^2 + c*x^4)^(3/2)/(2*x^5) - (3*Sqrt[b
]*c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^6}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}+\frac{1}{2}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx \\
&= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}+\frac{1}{2}(3bc)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}-\frac{1}{2}(3bc)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right) \\
&= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{\left(bx^2+cx^4\right)^{3/2}}{2x^5}-\frac{3}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0170345, size = 44, normalized size = 0.56

$$
\frac{c\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^2x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (c*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c*x^2)/b])/(
5*b^2*x^5)
```

Maple [A] time = 0.047, size = 102, normalized size = 1.3

$$-\frac{1}{2\,b x^5}\left(c x^4+b x^2\right)^{\frac{3}{2}}\left(3\,b^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{c x^2+b}+b}{x}\right)x^2c-\left(c x^2+b\right)^{\frac{3}{2}}x^2c+\left(c x^2+b\right)^{\frac{5}{2}}-3\,\sqrt{c x^2+b}\,x^2bc\right)\left(c x^2+b\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2
*c-(c*x^2+b)^(3/2)*x^2*c+(c*x^2+b)^(5/2)-3*(c*x^2+b)^(1/2)*x^2*b*c)/x^5/(c*
x^2+b)^(3/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c x^4+b x^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^6, x)
```

Fricas [A] time = 1.43626, size = 327, normalized size = 4.14

$$\left[\frac{3\,\sqrt{b}c x^3\log\left(-\frac{c x^3+2\,b x-2\,\sqrt{c x^4+b x^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{c x^4+b x^2}\left(2\,c x^2-b\right)}{4\,x^3},\ \frac{3\,\sqrt{-b}c x^3\arctan\left(\frac{\sqrt{c x^4+b x^2}\sqrt{-b}}{c x^3+b x}\right)+\sqrt{c x^4+b x^2}\left(2\,c x^2-b\right)}{2\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/4*(3*sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/
x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b))/x^3, 1/2*(3*sqrt(-b)*c*x^3*arct
an(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(2*c*x
```

```
^2 - b))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**6, x)
```

Giac [A] time = 1.20255, size = 80, normalized size = 1.01

$$\frac{1}{2}\left(\frac{3\,b\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\sqrt{cx^2+b}-\frac{\sqrt{cx^2+b}b}{cx^2}\right)\mathrm{csgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(3*b*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(c*x^2 + b) - sq
rt(c*x^2 + b)*b/(c*x^2))*c*sgn(x)
```

3.256 $\int \frac{(bx^2+cx^4)^{3/2}}{x^8} dx$

Optimal. Leaf size=81

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{(bx^2+cx^4)^{3/2}}{4x^7}$$

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(8*x^3) - (b*x^2 + c*x^4)^(3/2)/(4*x^7) - (3*c^2
*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rubi [A] time = 0.114552, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{(bx^2+cx^4)^{3/2}}{4x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(8*x^3) - (b*x^2 + c*x^4)^(3/2)/(4*x^7) - (3*c^2
*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}+\frac{1}{4}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx\\
&= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}+\frac{1}{8}\left(3c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}-\frac{1}{8}\left(3c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.0501643, size = 80, normalized size = 0.99

$$-\frac{2b^2+3c^2x^4\sqrt{\frac{cx^2}{b}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)+7bcx^2+5c^2x^4}{8x^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -(2*b^2 + 7*b*c*x^2 + 5*c^2*x^4 + 3*c^2*x^4*Sqrt[1 + (c*x^2)/b]*ArcTanh[Sqr
t[1 + (c*x^2)/b]])/(8*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.048, size = 125, normalized size = 1.5

$$-\frac{1}{8\,b^2x^7}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-\left(cx^2+b\right)^{\frac{3}{2}}x^4c^2+3\,b^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4c^2+\left(cx^2+b\right)^{\frac{5}{2}}x^2c-3\,\sqrt{cx^2+b}x^4bc^2+2\,(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^8,x)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)*(-(c*x^2+b)^(3/2)*x^4*c^2+3*b^(3/2)*ln(2*(b^(1/2)*
(c*x^2+b)^(1/2)+b)/x)*x^4*c^2+(c*x^2+b)^(5/2)*x^2*c-3*(c*x^2+b)^(1/2)*x^4*b
*c^2+2*(c*x^2+b)^(5/2)*b)/x^7/(c*x^2+b)^(3/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^8, x)
```

Fricas [A] time = 1.26905, size = 360, normalized size = 4.44

$$\left[\frac{3\sqrt{b}c^2x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) - 2\,\sqrt{cx^4+bx^2}\left(5\,bcx^2+2\,b^2\right)}{16\,bx^5}, \frac{3\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) - \sqrt{cx^4+bx^2}\left(5\,bc\right.}{8\,bx^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5), 1/8*(3*sqrt(-
b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4
+ b*x^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**8, x)
```

Giac [A] time = 1.23184, size = 85, normalized size = 1.05

$$\frac{1}{8}c^2\left(\frac{3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{5\left(cx^2+b\right)^{\frac{3}{2}}-3\sqrt{cx^2+b}b}{c^2x^4}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*c^2*(3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - (5*(c*x^2 + b)^(3/2)
 - 3*sqrt(c*x^2 + b)*b)/(c^2*x^4))*sgn(x)
```

3.257 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=109

$$\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}$$

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(8*x^5) - (c^2*Sqrt[b*x^2 + c*x^4])/(16*b*x^3) - (
b*x^2 + c*x^4)^(3/2)/(6*x^9) + (c^3*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(16*b^(3/2))
```

Rubi [A] time = 0.164261, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2008, 206}

$$\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(8*x^5) - (c^2*Sqrt[b*x^2 + c*x^4])/(16*b*x^3) - (
b*x^2 + c*x^4)^(3/2)/(6*x^9) + (c^3*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(16*b^(3/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{10}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{1}{2}c\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{1}{8}c^2\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}-\frac{c^3\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{16b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{c^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{16b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{6x^9}+\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0193717, size = 46, normalized size = 0.42

$$\frac{c^3\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] (c^3*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (c*x^2)/b])
/(5*b^4*x^5)
```

Maple [A] time = 0.05, size = 145, normalized size = 1.3

$$\frac{1}{48\,x^9b^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-\left(cx^2+b\right)^{\frac{3}{2}}x^6c^3+3\,b^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^6c^3+\left(cx^2+b\right)^{\frac{5}{2}}x^4c^2-3\,\sqrt{cx^2+b}x^6bc^3+2\,\left(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^10,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(-(c*x^2+b)^(3/2)*x^6*c^3+3*b^(3/2)*ln(2*(b^(1/2)*
(c*x^2+b)^(1/2)+b)/x)*x^6*c^3+(c*x^2+b)^(5/2)*x^4*c^2-3*(c*x^2+b)^(1/2)*x^6
*b*c^3+2*(c*x^2+b)^(5/2)*x^2*b*c-8*(c*x^2+b)^(5/2)*b^2)/x^9/(c*x^2+b)^(3/2)
/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^10, x)
```

Fricas [A] time = 1.29058, size = 414, normalized size = 3.8

$$\left[\frac{3\,\sqrt{b}c^3x^7\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\left(3\,bc^2x^4+14\,b^2cx^2+8\,b^3\right)\sqrt{cx^4+bx^2}}{96\,b^2x^7},-\frac{3\,\sqrt{-b}c^3x^7\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\left(3\right.}{48\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(b)*c^3*x^7*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) - 2*(3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^2*
x^7), -1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3
+ b*x)) + (3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^2*x^
7)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**10,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**10, x)
```

Giac [A] time = 1.18825, size = 111, normalized size = 1.02

$$-\frac{1}{48}\,c^3\left(\frac{3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{3\left(cx^2+b\right)^{\frac{5}{2}}+8\left(cx^2+b\right)^{\frac{3}{2}}b-3\sqrt{cx^2+b}b^2}{bc^3x^6}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/48*c^3*(3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(c*x^2 + b)
^(5/2) + 8*(c*x^2 + b)^(3/2)*b - 3*sqrt(c*x^2 + b)*b^2)/(b*c^3*x^6))*sgn(x)
```

3.258 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{12}} \, dx$

Optimal. Leaf size=137

$$\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}$$

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(16*x^7) - (c^2*Sqrt[b*x^2 + c*x^4])/(64*b*x^5) +
(3*c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - (b*x^2 + c*x^4)^(3/2)/(8*x^11)
- (3*c^4*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2))
```

Rubi [A] time = 0.203488, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2008, 206}

$$\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(16*x^7) - (c^2*Sqrt[b*x^2 + c*x^4])/(64*b*x^5) +
(3*c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - (b*x^2 + c*x^4)^(3/2)/(8*x^11)
- (3*c^4*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{12}}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}+\frac{1}{8}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}+\frac{1}{16}c^2\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{\left(3c^3\right)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{64b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}+\frac{\left(3c^4\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{128b^2}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{\left(3c^4\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\right.}{128b^2}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0172812, size = 46, normalized size = 0.34

$$-\frac{c^4\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

```
[Out] -(c^4*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + (c*x^2)/b]
)/(5*b^5*x^5)
```

Maple [A] time = 0.055, size = 165, normalized size = 1.2

$$-\frac{1}{128\,b^4x^{11}}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(3\,b^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^8c^4-\left(cx^2+b\right)^{\frac{3}{2}}x^8c^4+\left(cx^2+b\right)^{\frac{5}{2}}x^6c^3-3\,\sqrt{cx^2+b}x^8bc^4+2\,(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^12,x)
```

```
[Out] -1/128*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x
^8*c^4-(c*x^2+b)^(3/2)*x^8*c^4+(c*x^2+b)^(5/2)*x^6*c^3-3*(c*x^2+b)^(1/2)*x^
8*b*c^4+2*(c*x^2+b)^(5/2)*x^4*b*c^2-8*(c*x^2+b)^(5/2)*x^2*b^2*c+16*(c*x^2+b
)^(5/2)*b^3)/x^11/(c*x^2+b)^(3/2)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^12, x)
```

Fricas [A] time = 1.44683, size = 462, normalized size = 3.37

$$\left[\frac{3\,\sqrt{b}c^4x^9\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\left(3\,bc^3x^6-2\,b^2c^2x^4-24\,b^3cx^2-16\,b^4\right)\sqrt{cx^4+bx^2}}{256\,b^3x^9},\frac{3\,\sqrt{-b}c^4x^9\arctan\left(\frac{\sqrt{cx^4+bx^2}}{cx^3+bx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] [1/256*(3*sqrt(b)*c^4*x^9*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(
b))/x^3) + 2*(3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 16*b^4)*sqrt(c*x
^4 + b*x^2))/(b^3*x^9), 1/128*(3*sqrt(-b)*c^4*x^9*arctan(sqrt(c*x^4 + b*x^2
)*sqrt(-b)/(c*x^3 + b*x)) + (3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 1
6*b^4)*sqrt(c*x^4 + b*x^2))/(b^3*x^9)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**12,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**12, x)
```

Giac [A] time = 1.34524, size = 130, normalized size = 0.95

$$\frac{1}{128}c^4\left(\frac{3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{3\left(cx^2+b\right)^{\frac{7}{2}}-11\left(cx^2+b\right)^{\frac{5}{2}}b-11\left(cx^2+b\right)^{\frac{3}{2}}b^2+3\sqrt{cx^2+b}b^3}{b^2c^4x^8}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/128*c^4*(3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(c*x^2 +
b)^(7/2) - 11*(c*x^2 + b)^(5/2)*b - 11*(c*x^2 + b)^(3/2)*b^2 + 3*sqrt(c*x^2
 + b)*b^3)/(b^2*c^4*x^8))*sgn(x)
```

3.259 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{14}} \, dx$

Optimal. Leaf size=165

$$-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}+\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}$$

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(80*x^9) - (c^2*Sqrt[b*x^2 + c*x^4])/(160*b*x^7)
 + (c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^5) - (3*c^4*Sqrt[b*x^2 + c*x^4])/(2
56*b^3*x^3) - (b*x^2 + c*x^4)^(3/2)/(10*x^13) + (3*c^5*ArcTanh[(Sqrt[b]*x)/
Sqrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```

Rubi [A] time = 0.254928, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2008, 206}

$$-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}+\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```



```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(80*x^9) - (c^2*Sqrt[b*x^2 + c*x^4])/(160*b*x^7)
 + (c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^5) - (3*c^4*Sqrt[b*x^2 + c*x^4])/(2
56*b^3*x^3) - (b*x^2 + c*x^4)^(3/2)/(10*x^13) + (3*c^5*ArcTanh[(Sqrt[b]*x)/
Sqrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{14}}\,dx = -\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}} + \frac{1}{10}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^{10}}\,dx$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}} + \frac{1}{80}\left(3c^2\right)\int \frac{1}{x^6\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} - \frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}} - \frac{c^3\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx}{32b}$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} + \frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}} + \frac{\left(3c^4\right)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{128b^2}$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} + \frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}} - \frac{\left(3c^5\right)\int \frac{}{\sqrt{}}}{25}$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} + \frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}} + \frac{\left(3c^5\right)\text{Sub}}{}$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} + \frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}} + \frac{3c^5\tanh}{25}$$

Mathematica [C] time = 0.0180447, size = 46, normalized size = 0.28

$$\frac{c^5\left(x^2\left(b+cx^2\right)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{cx^2}{b}+1\right)}{5b^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

```
[Out] (c^5*(x^2*(b + c*x^2))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, 1 + (c*x^2)/b])
/(5*b^6*x^5)
```

Maple [A] time = 0.071, size = 186, normalized size = 1.1

$$\frac{1}{1280\,x^{13}b^5}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(15\,b^{3/2}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^{10}c^5-5\,\left(cx^2+b\right)^{3/2}x^{10}c^5+5\,\left(cx^2+b\right)^{5/2}x^8c^4-15\,\sqrt{cx^2+b}x^1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^14,x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(15*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*
x^10*c^5-5*(c*x^2+b)^(3/2)*x^10*c^5+5*(c*x^2+b)^(5/2)*x^8*c^4-15*(c*x^2+b)^
(1/2)*x^10*b*c^5+10*(c*x^2+b)^(5/2)*x^6*b*c^3-40*(c*x^2+b)^(5/2)*x^4*b^2*c^
2+80*(c*x^2+b)^(5/2)*x^2*b^3*c-128*(c*x^2+b)^(5/2)*b^4)/x^13/(c*x^2+b)^(3/2
)/b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^14, x)
```

Fricas [A] time = 1.48118, size = 528, normalized size = 3.2

$$\left[\frac{15\sqrt{b}c^5x^{11}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\left(15\,bc^4x^8-10\,b^2c^3x^6+8\,b^3c^2x^4+176\,b^4cx^2+128\,b^5\right)\sqrt{cx^4+bx^2}}{2560\,b^4x^{11}},-\frac{15\sqrt{-b}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*sqrt(b)*c^5*x^11*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sq
rt(b))/x^3) - 2*(15*b*c^4*x^8 - 10*b^2*c^3*x^6 + 8*b^3*c^2*x^4 + 176*b^4*c*
x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))/(b^4*x^11), -1/1280*(15*sqrt(-b)*c^5*x^
11*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*b*c^4*x^8 - 10*
b^2*c^3*x^6 + 8*b^3*c^2*x^4 + 176*b^4*c*x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))
/(b^4*x^11)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**14,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**14, x)
```

Giac [A] time = 1.2302, size = 149, normalized size = 0.9

$$-\frac{1}{1280}\,c^5\left(\frac{15\,\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}+\frac{15\left(cx^2+b\right)^{\frac{9}{2}}-70\left(cx^2+b\right)^{\frac{7}{2}}b+128\left(cx^2+b\right)^{\frac{5}{2}}b^2+70\left(cx^2+b\right)^{\frac{3}{2}}b^3-15\sqrt{cx^2+b}b^4}{b^3c^5x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/1280*c^5*(15*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^3) + (15*(c*x^
2 + b)^(9/2) - 70*(c*x^2 + b)^(7/2)*b + 128*(c*x^2 + b)^(5/2)*b^2 + 70*(c*x
^2 + b)^(3/2)*b^3 - 15*sqrt(c*x^2 + b)*b^4)/(b^3*c^5*x^10))*sgn(x)
```

3.260 $\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=114

$$\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}$$

```
[Out] (5*b^2*Sqrt[b*x^2 + c*x^4])/(16*c^3) - (5*b*x^2*Sqrt[b*x^2 + c*x^4])/(24*c^
2) + (x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (5*b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*
x^2 + c*x^4]])/(16*c^(7/2))
```

Rubi [A] time = 0.12934, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 670, 640, 620, 206}

$$\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (5*b^2*Sqrt[b*x^2 + c*x^4])/(16*c^3) - (5*b*x^2*Sqrt[b*x^2 + c*x^4])/(24*c^
2) + (x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (5*b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*
x^2 + c*x^4]])/(16*c^(7/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
```

```
+ 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= \frac{x^4\sqrt{bx^2+cx^4}}{6c}-\frac{(5b)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{12c}\\
&= -\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}+\frac{\left(5b^2\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{16c^2}\\
&= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}-\frac{\left(5b^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{32c^3}\\
&= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}-\frac{\left(5b^3\right)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^3}\\
&= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0528215, size = 100, normalized size = 0.88

$$\frac{x\left(\sqrt{c}x\left(5b^2cx^2+15b^3-2bc^2x^4+8c^3x^6\right)-15b^3\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)\right)}{48c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(15*b^3 + 5*b^2*c*x^2 - 2*b*c^2*x^4 + 8*c^3*x^6) - 15*b^3*Sqr
t[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]]))/(48*c^(7/2)*Sqrt[x^2*(b
 + c*x^2)])
```

Maple [A] time = 0.051, size = 105, normalized size = 0.9

$$\frac{x}{48}\sqrt{cx^2+b}\left(8\,x^5\sqrt{cx^2+b}c^{7/2}-10\,c^{5/2}\sqrt{cx^2+b}x^3b+15\,c^{3/2}\sqrt{cx^2+b}xb^2-15\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^3c\right)\frac{1}{\sqrt{cx^4+bx^2}}c^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*x*(c*x^2+b)^(1/2)*(8*x^5*(c*x^2+b)^(1/2)*c^(7/2)-10*c^(5/2)*(c*x^2+b)^
(1/2)*x^3*b+15*c^(3/2)*(c*x^2+b)^(1/2)*x*b^2-15*ln(x*c^(1/2)+(c*x^2+b)^(1/2
))*b^3*c)/(c*x^4+b*x^2)^(1/2)/c^(9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34681, size = 378, normalized size = 3.32

$$\left[\frac{15\,b^3\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\left(8\,c^3x^4-10\,bc^2x^2+15\,b^2c\right)\sqrt{cx^4+bx^2}}{96\,c^4},\ \frac{15\,b^3\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*b^3*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2))/c^4, 1/48*(15*b
^3*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*c^3*x^4 -
 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sqrt(c*x^4 + b*x^2), x)
```

3.261 $\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=86

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}$$

```
[Out] (-3*b*Sqrt[b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[b*x^2 + c*x^4])/(4*c) + (3*b
^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/2))
```

Rubi [A] time = 0.103347, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 670, 640, 620, 206}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (-3*b*Sqrt[b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[b*x^2 + c*x^4])/(4*c) + (3*b
^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/2))
```


Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= \frac{x^2\sqrt{bx^2+cx^4}}{4c}-\frac{(3b)\operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{8c}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}+\frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{16c^2}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}+\frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^2}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0369363, size = 89, normalized size = 1.03

$$\frac{x\left(\sqrt{c}x\left(-3b^2-bcx^2+2c^2x^4\right)+3b^2\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)\right)}{8c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(-3*b^2 - b*c*x^2 + 2*c^2*x^4) + 3*b^2*Sqrt[b + c*x^2]*ArcTan
h[(Sqrt[c]*x)/Sqrt[b + c*x^2]]))/(8*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.046, size = 85, normalized size = 1.

$$\frac{x}{8}\sqrt{cx^2+b}\left(2x^3\sqrt{cx^2+b}c^{5/2}-3\sqrt{cx^2+b}c^{3/2}xb+3\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^2c\right)\frac{1}{\sqrt{cx^4+bx^2}}c^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/8*x*(c*x^2+b)^(1/2)*(2*x^3*(c*x^2+b)^(1/2)*c^(5/2)-3*(c*x^2+b)^(1/2)*c^(3
/2)*x*b+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2*c)/(c*x^4+b*x^2)^(1/2)/c^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38589, size = 327, normalized size = 3.8

$$\left[\frac{3b^2\sqrt{c}\log\left(-2cx^2-b-2\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\sqrt{cx^4+bx^2}\left(2c^2x^2-3bc\right)}{16c^3}, -\frac{3b^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx}}{8c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^2*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 - 3*b*c))/c^3, -1/8*(3*b^2*sqrt(-c)*arctan(s
qrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 -
 3*b*c))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(c*x^4 + b*x^2), x)
```

3.262 $\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(2*c^(3/2))
```

Rubi [A] time = 0.0831443, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 640, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(2*c^(3/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{4c}\\
&= \frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c}\\
&= \frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0278462, size = 73, normalized size = 1.26

$$
\frac{x\left(\sqrt{c}x\left(b+cx^2\right)-b\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2) - b*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b +
c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.048, size = 64, normalized size = 1.1

$$
\frac{x}{2}\sqrt{cx^2+b}\left(x\sqrt{cx^2+b}c^{\frac{3}{2}}-b\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)c\right)\frac{1}{\sqrt{cx^4+bx^2}}c^{-\frac{5}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(3/2)-b*ln(x*c^(1/2)+(c*x^2+b)^(
1/2))*c)/(c*x^4+b*x^2)^(1/2)/c^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37527, size = 265, normalized size = 4.57

$$\left[\frac{b\sqrt{c}\log\left(-2\,cx^2 - b + 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right) + 2\,\sqrt{cx^4 + bx^2}c}{4\,c^2}, \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right) + \sqrt{cx^4 + bx^2}c}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/
(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.24843, size = 80, normalized size = 1.38

$$\frac{b\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(3/2)
+ 1/2*sqrt(c*x^4 + b*x^2)/c
```

3.263 $\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rubi [A] time = 0.0541314, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2013, 620, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= \operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Mathematica [A] time = 0.0138622, size = 52, normalized size = 1.68

$$\frac{x\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]*Sqrt[x^2*
(b + c*x^2)])
```

Maple [A] time = 0.046, size = 44, normalized size = 1.4

$$x\sqrt{cx^2+b}\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\frac{1}{\sqrt{cx^4+bx^2}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)*ln(x*c^(1/2)+(c*x^2+b)^(1/2))/c^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.295, size = 174, normalized size = 5.61

$$\left[\frac{\log\left(-2\,cx^2 - b - 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c), -sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.18633, size = 53, normalized size = 1.71

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)\sqrt{c} - b\right|\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/sqrt(c)
```

3.264 $\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rubi [A] time = 0.0405952, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx=-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Mathematica [A] time = 0.0089309, size = 23, normalized size = 1.

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

Maple [A] time = 0.046, size = 26, normalized size = 1.1

$$-\frac{cx^2+b}{b}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -(c*x^2+b)/b/(c*x^4+b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.23003, size = 41, normalized size = 1.78

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.1543, size = 19, normalized size = 0.83

$$-\frac{\sqrt{c+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(c + b/x^2)/b
```

3.265 $\int \frac{1}{x^3\sqrt{bx^2+cx^4}} dx$

Optimal. Leaf size=52

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2} - \frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Rubi [A] time = 0.0825069, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2} - \frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{3bx^4} - \frac{(2c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{3bx^4} + \frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

Mathematica [A] time = 0.0153207, size = 35, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(2cx^2-b\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

Maple [A] time = 0.048, size = 37, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(-2\,cx^2+b\right)}{3\,b^2x^2}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-2*c*x^2+b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30803, size = 66, normalized size = 1.27

$$\frac{\sqrt{cx^4 + bx^2}\left(2\,cx^2 - b\right)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.14449, size = 36, normalized size = 0.69

$$-\frac{\left(c + \frac{b}{x^2}\right)^{\frac{3}{2}} - 3\,\sqrt{c + \frac{b}{x^2}}c}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*((c + b/x^2)^(3/2) - 3*sqrt(c + b/x^2)*c)/b^2
```

3.266 $\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(5*b*x^6) + (4*c*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^4) - (
8*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rubi [A] time = 0.125382, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(5*b*x^6) + (4*c*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^4) - (
8*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{5bx^6} - \frac{(4c)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{5b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{5bx^6} + \frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4} + \frac{\left(8c^2\right)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{5bx^6} + \frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4} - \frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}$$

Mathematica [A] time = 0.0140421, size = 46, normalized size = 0.57

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3b^2-4bcx^2+8c^2x^4\right)}{15b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4))/(15*b^3*x^6)
```

Maple [A] time = 0.044, size = 50, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(8\,c^2x^4-4\,bcx^2+3\,b^2\right)}{15\,b^3x^4}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(8*c^2*x^4-4*b*c*x^2+3*b^2)/x^4/b^3/(c*x^4+b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29769, size = 93, normalized size = 1.16

$$-\frac{\left(8\,c^2x^4-4\,bcx^2+3\,b^2\right)\sqrt{cx^4+bx^2}}{15\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*c^2*x^4 - 4*b*c*x^2 + 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.14868, size = 58, normalized size = 0.72

$$-\frac{3\left(c+\frac{b}{x^2}\right)^{\frac{5}{2}}-10\left(c+\frac{b}{x^2}\right)^{\frac{3}{2}}c+15\sqrt{c+\frac{b}{x^2}}c^2}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(3*(c + b/x^2)^(5/2) - 10*(c + b/x^2)^(3/2)*c + 15*sqrt(c + b/x^2)*c^
2)/b^3
```

3.267 $\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(7*b*x^8) + (6*c*Sqrt[b*x^2 + c*x^4])/(35*b^2*x^6) - (
8*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^4) + (16*c^3*Sqrt[b*x^2 + c*x^4])/(35*
b^4*x^2)
```

Rubi [A] time = 0.169381, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(7*b*x^8) + (6*c*Sqrt[b*x^2 + c*x^4])/(35*b^2*x^6) - (
8*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^4) + (16*c^3*Sqrt[b*x^2 + c*x^4])/(35*
b^4*x^2)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(6c)\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{7b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{\left(24c^2\right)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{35b^2}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4} - \frac{\left(16c^3\right)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{35b^3}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4} + \frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}$$

Mathematica [A] time = 0.0147405, size = 57, normalized size = 0.53

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(6b^2cx^2-5b^3-8bc^2x^4+16c^3x^6\right)}{35b^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-5*b^3 + 6*b^2*c*x^2 - 8*b*c^2*x^4 + 16*c^3*x^6))/(
35*b^4*x^8)
```

Maple [A] time = 0.045, size = 61, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(-16\,c^3x^6+8\,bc^2x^4-6\,b^2cx^2+5\,b^3\right)}{35\,b^4x^6}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-16*c^3*x^6+8*b*c^2*x^4-6*b^2*c*x^2+5*b^3)/x^6/b^4/(c*x^4+
b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32417, size = 115, normalized size = 1.06

$$\frac{\left(16\,c^3x^6 - 8\,bc^2x^4 + 6\,b^2cx^2 - 5\,b^3\right)\sqrt{cx^4 + bx^2}}{35\,b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*c^3*x^6 - 8*b*c^2*x^4 + 6*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b*x^2)/(
b^4*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.17345, size = 77, normalized size = 0.71

$$-\frac{5\left(c + \frac{b}{x^2}\right)^{\frac{7}{2}} - 21\left(c + \frac{b}{x^2}\right)^{\frac{5}{2}}c + 35\left(c + \frac{b}{x^2}\right)^{\frac{3}{2}}c^2 - 35\sqrt{c + \frac{b}{x^2}}c^3}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35*(5*(c + b/x^2)^(7/2) - 21*(c + b/x^2)^(5/2)*c + 35*(c + b/x^2)^(3/2)*
c^2 - 35*sqrt(c + b/x^2)*c^3)/b^4
```

3.268 $\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=50

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Rubi [A] time = 0.078702, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\sqrt{bx^2+cx^4}}{3c} - \frac{(2b)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}$$
$$= -\frac{2b\sqrt{bx^2+cx^4}}{3c^2x} + \frac{x\sqrt{bx^2+cx^4}}{3c}$$

Mathematica [A] time = 0.0179125, size = 34, normalized size = 0.68

$$\frac{(cx^2-2b)\sqrt{x^2(b+cx^2)}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

Maple [A] time = 0.043, size = 37, normalized size = 0.7

$$-\frac{(cx^2+b)(-cx^2+2\,b)\,x}{3\,c^2}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-c*x^2+2*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 1.03021, size = 46, normalized size = 0.92

$$\frac{c^2x^4-bcx^2-2\,b^2}{3\,\sqrt{cx^2+b}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

Fricas [A] time = 1.25613, size = 63, normalized size = 1.26

$$\frac{\sqrt{cx^4 + bx^2}\left(cx^2 - 2\,b\right)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2), x)
```

3.269 $\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rubi [A] time = 0.0168739, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^2 + c*x^4],x]

[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{bx^2+cx^4}}{cx}$$

Mathematica [A] time = 0.0052852, size = 22, normalized size = 1.

$$\frac{\sqrt{x^2\left(b+cx^2\right)}}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[x^2*(b + c*x^2)]/(c*x)
```

Maple [A] time = 0.043, size = 26, normalized size = 1.2

$$\frac{x\left(cx^{2}+b\right)}{c}\frac{1}{\sqrt{cx^{4}+bx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] (c*x^2+b)/c*x/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 1.01831, size = 18, normalized size = 0.82

$$\frac{\sqrt{cx^{2}+b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + b)/c
```

Fricas [A] time = 1.0958, size = 36, normalized size = 1.64

$$\frac{\sqrt{cx^{4}+bx^{2}}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2 \left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.13133, size = 42, normalized size = 1.91

$$-\frac{2\sqrt{b}}{\left(\sqrt{c + \frac{b}{x^2}} - \frac{\sqrt{b}}{x}\right)^2 - c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b)/((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)
```

3.270 $\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rubi [A] time = 0.0089355, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^2 + c*x^4],x]

[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx = -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.009542, size = 52, normalized size = 1.73

$$-\frac{x\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -((x*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[x^2*(b
 + c*x^2)]))
```

Maple [B] time = 0.043, size = 50, normalized size = 1.7

$$-x\sqrt{cx^2+b}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\frac{1}{\sqrt{cx^4+bx^2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1
/2)+b)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2), x)
```

Fricas [A] time = 1.42514, size = 186, normalized size = 6.2

$$\left[\frac{\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)/sqrt(b), sqr
t(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*x**2 + c*x**4), x)
```

Giac [A] time = 1.16131, size = 62, normalized size = 2.07

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(c*x^2 + b)/sqrt(-b)
)/(sqrt(-b)*sgn(x))
```

3.271 $\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(2*b^(3/2))
```

Rubi [A] time = 0.055138, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(2*b^(3/2))
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}-\frac{c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.062066, size = 68, normalized size = 1.15

$$\frac{c\sqrt{x^2\left(b+cx^2\right)}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)}{2\sqrt{\frac{cx^2}{b}+1}}-\frac{b}{2cx^2}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (c*Sqrt[x^2*(b + c*x^2)]*(-b/(2*c*x^2) + ArcTanh[Sqrt[1 + (c*x^2)/b]]/(2*Sq
rt[1 + (c*x^2)/b])))/(b^2*x)
```

Maple [A] time = 0.044, size = 73, normalized size = 1.2

$$-\frac{1}{2x}\sqrt{cx^2+b}\left(-c\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^2b+\sqrt{cx^2+b}b^{\frac{3}{2}}\right)\frac{1}{\sqrt{cx^4+bx^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(-c*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2*b+(c*x^2
+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

Fricas [A] time = 1.66414, size = 306, normalized size = 5.19

$$\left[\frac{\sqrt{b}cx^3 \log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) - 2\,\sqrt{cx^4 + bx^2}b}{4\,b^2x^3}, -\frac{\sqrt{-b}cx^3 \arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) + \sqrt{cx^4 + bx^2}b}{2\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.272 $\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^3) - (3
*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

Rubi [A] time = 0.0974708, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^3) - (3
*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(3c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b} \\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} + \frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3} + \frac{\left(3c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2} \\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} + \frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3} - \frac{\left(3c^2\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^2} \\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} + \frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3} - \frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.0124084, size = 44, normalized size = 0.51

$$-\frac{c^2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{cx^2}{b}+1\right)}{b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -((c^2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (c*x^2)/b])
/(b^3*x))
```

Maple [A] time = 0.046, size = 94, normalized size = 1.1

$$-\frac{1}{8\,x^3}\sqrt{cx^2+b}\left(3\,\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4bc^2-3\,\sqrt{cx^2+b}b^{3/2}x^2c+2\,\sqrt{cx^2+b}b^{5/2}\right)\frac{1}{\sqrt{cx^4+bx^2}}b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b)^(1/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^4*b*c^2-3*(c*
x^2+b)^(1/2)*b^(3/2)*x^2*c+2*(c*x^2+b)^(1/2)*b^(5/2))/x^3/(c*x^4+b*x^2)^(1/
2)/b^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

Fricas [A] time = 1.62939, size = 366, normalized size = 4.21

$$\left[\frac{3\sqrt{b}c^2x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(3\,bcx^2-2\,b^2\right)}{16\,b^3x^5}, \frac{3\,\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\left(3\,b\right.}{8\,b^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5), 1/8*(3*sqrt
(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^
4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.273 $\int \frac{x^9}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=109

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}-\frac{x^6}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^6/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(8*c^3) + (5*x^
2*Sqrt[b*x^2 + c*x^4])/(4*c^2) + (15*b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(8*c^(7/2))
```

Rubi [A] time = 0.128751, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}-\frac{x^6}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^6/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(8*c^3) + (5*x^
2*Sqrt[b*x^2 + c*x^4])/(4*c^2) + (15*b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(8*c^(7/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
```

```
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^9}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} + \frac{5\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}$$
$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} - \frac{(15b)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c^2}$$
$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^3}$$
$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^3}$$
$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}$$

Mathematica [A] time = 0.0512658, size = 88, normalized size = 0.81

$$\frac{x\left(\sqrt{c}x\left(-15b^2-5bcx^2+2c^2x^4\right)+15b^{5/2}\sqrt{\frac{cx^2}{b}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{8c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(-15*b^2 - 5*b*c*x^2 + 2*c^2*x^4) + 15*b^(5/2)*Sqrt[1 + (c*x^
2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]]))/(8*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.051, size = 87, normalized size = 0.8

$$\frac{x^3\left(cx^2+b\right)}{8}\left(2\,x^5c^{7/2}-5\,c^{5/2}x^3b-15\,c^{3/2}xb^2+15\,\sqrt{cx^2+b}\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^2c\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}c^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/8*x^3*(c*x^2+b)*(2*x^5*c^(7/2)-5*c^(5/2)*x^3*b-15*c^(3/2)*x*b^2+15*(c*x^2
+b)^(1/2)*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2*c)/(c*x^4+b*x^2)^(3/2)/c^(9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60523, size = 451, normalized size = 4.14

$$\left[\frac{15\left(b^{2}cx^{2}+b^{3}\right)\sqrt{c}\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)+2\left(2\,c^{3}x^{4}-5\,bc^{2}x^{2}-15\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}}{16\left(c^{5}x^{2}+bc^{4}\right)},-\frac{15\left(b^{2}cx^{2}+b^{3}\right)\sqrt{-c}\,\mathrm{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(b^2*c*x^2 + b^3)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2
)*sqrt(c)) + 2*(2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^4 + b*x^2))/(c
^5*x^2 + b*c^4), -1/8*(15*(b^2*c*x^2 + b^3)*sqrt(-c)*arctan(sqrt(c*x^4 + b*
x^2)*sqrt(-c)/(c*x^2 + b)) - (2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^
4 + b*x^2))/(c^5*x^2 + b*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{9}}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**9/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{(cx^4 + bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^9/(c*x^4 + b*x^2)^(3/2), x)
```

3.274 $\int \frac{x^7}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}-\frac{x^4}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^4/(c*Sqrt[b*x^2 + c*x^4])) + (3*Sqrt[b*x^2 + c*x^4])/(2*c^2) - (3*b*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*c^(5/2))
```

Rubi [A] time = 0.108931, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 668, 640, 620, 206}

$$\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}-\frac{x^4}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^4/(c*Sqrt[b*x^2 + c*x^4])) + (3*Sqrt[b*x^2 + c*x^4])/(2*c^2) - (3*b*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*c^(5/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}}+\frac{3\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{2c}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{4c^2}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^2}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0418383, size = 76, normalized size = 0.94

$$
\frac{x\left(\sqrt{c}x\left(3b+cx^2\right)-3b^{3/2}\sqrt{\frac{cx^2}{b}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)\right)}{2c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(3*b + c*x^2) - 3*b^(3/2)*Sqrt[1 + (c*x^2)/b]*ArcSinh[(Sqrt[c
]*x)/Sqrt[b]]))/(2*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.048, size = 73, normalized size = 0.9

$$\frac{x^3\left(cx^2+b\right)}{2}\left(x^3c^{\frac{5}{2}}+3\,c^{3/2}xb-3\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\sqrt{cx^2+b}bc\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}c^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2*x^3*(c*x^2+b)*(x^3*c^(5/2)+3*c^(3/2)*x*b-3*ln(x*c^(1/2)+(c*x^2+b)^(1/2)
)*(c*x^2+b)^(1/2)*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29421, size = 389, normalized size = 4.8

$$\left[\frac{3\left(bcx^2+b^2\right)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(c^2x^2+3\,bc\right)}{4\left(c^4x^2+bc^3\right)},\frac{3\left(bcx^2+b^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right.}{2\left(c^4x^2+bc\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b*c*x^2 + b^2)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sq
rt(c)) + 2*sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(c^4*x^2 + b*c^3), 1/2*(3
*(b*c*x^2 + b^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))
+ sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(c^4*x^2 + b*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**7/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^4 + b*x^2)^(3/2), x)
```

3.275 $\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^2/(c*Sqrt[b*x^2 + c*x^4])) + ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
/c^(3/2)
```

Rubi [A] time = 0.0920113, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 652, 620, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^2/(c*Sqrt[b*x^2 + c*x^4])) + ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
/c^(3/2)
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\ &= -\frac{x^2}{c\sqrt{bx^2+cx^4}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{2c}\\ &= -\frac{x^2}{c\sqrt{bx^2+cx^4}}+\frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{c}\\ &= -\frac{x^2}{c\sqrt{bx^2+cx^4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0783588, size = 66, normalized size = 1.2

$$\frac{\sqrt{b}x\sqrt{\frac{cx^2}{b}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)-\sqrt{c}x^2}{c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[c]*x^2) + Sqrt[b]*x*Sqrt[1 + (c*x^2)/b]*ArcSinh[(Sqrt[c]*x)/Sqrt[b]
])/(c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.045, size = 62, normalized size = 1.1

$$x^{3}\left(c x^{2}+b\right)\left(-x c^{\frac{3}{2}}+\ln\left(x\sqrt{c}+\sqrt{c x^{2}+b}\right) c\sqrt{c x^{2}+b}\right)\left(c x^{4}+b x^{2}\right)^{-\frac{3}{2}} c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b)*(-x*c^(3/2)+ln(x*c^(1/2)+(c*x^2+b)^(1/2))*c*(c*x^2+b)^(1/2))/
(c*x^4+b*x^2)^(3/2)/c^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.17722, size = 325, normalized size = 5.91

$$\left[\frac{\left(c x^{2}+b\right)\sqrt{c}\log\left(-2\, c x^{2}-b-2\,\sqrt{c x^{4}+b x^{2}}\sqrt{c}\right)-2\,\sqrt{c x^{4}+b x^{2}} c}{2\,\left(c^{3} x^{2}+b c^{2}\right)}, -\frac{\left(c x^{2}+b\right)\sqrt{-c}\arctan\left(\frac{\sqrt{c x^{4}+b x^{2}}\sqrt{-c}}{c x^{2}+b}\right)+\sqrt{c x^{4}+b x^{2}} c}{c^{3} x^{2}+b c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*x^2 + b)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
 - 2*sqrt(c*x^4 + b*x^2)*c)/(c^3*x^2 + b*c^2), -((c*x^2 + b)*sqrt(-c)*arcta
n(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/(c^3*x
^2 + b*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.15527, size = 55, normalized size = 1.

$$-\frac{\arctan\left(\frac{\sqrt{c+\frac{b}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{1}{\sqrt{c+\frac{b}{x^2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(c + b/x^2)/sqrt(-c))/(sqrt(-c)*c) - 1/(sqrt(c + b/x^2)*c)
```

3.276 $\int \frac{x^3}{(bx^2+cx^4)^{3/2}} dx$

Optimal. Leaf size=22

$$\frac{x^2}{b\sqrt{bx^2 + cx^4}}$$

```
[Out] x^2/(b*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.0553208, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{x^2}{b\sqrt{bx^2 + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4)^(3/2),x]

[Out] x^2/(b*Sqrt[b*x^2 + c*x^4])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(bx^2 + cx^4\right)^{3/2}} dx = \frac{x^2}{b\sqrt{bx^2 + cx^4}}$$

Mathematica [A] time = 0.0075433, size = 22, normalized size = 1.

$$\frac{x^2}{b\sqrt{x^2\left(b + cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^2/(b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.043, size = 28, normalized size = 1.3

$$\frac{\left(cx^2+b\right)x^4}{b}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] (c*x^2+b)*x^4/b/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 0.986302, size = 27, normalized size = 1.23

$$\frac{x^2}{\sqrt{cx^4+bx^2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(c*x^4 + b*x^2)*b)
```

Fricas [A] time = 1.25365, size = 50, normalized size = 2.27

$$\frac{\sqrt{cx^4+bx^2}}{bcx^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)/(b*c*x^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.20895, size = 47, normalized size = 2.14

$$\frac{1}{\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}+b\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) + b)*sqrt(c))
```

3.277 $\int \frac{x}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

```
[Out] -((b + 2*c*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rubi [A] time = 0.0464225, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2013, 613}

$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b + 2*c*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

Mathematica [A] time = 0.0093953, size = 29, normalized size = 1.04

$$\frac{-b-2cx^2}{b^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-b - 2*c*x^2)/(b^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.044, size = 37, normalized size = 1.3

$$-\frac{x^2\left(cx^2+b\right)\left(2\,cx^2+b\right)}{b^2}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -x^2*(c*x^2+b)*(2*c*x^2+b)/b^2/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 1.0129, size = 55, normalized size = 1.96

$$-\frac{2\,cx^2}{\sqrt{cx^4+bx^2}b^2}-\frac{1}{\sqrt{cx^4+bx^2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) - 1/(sqrt(c*x^4 + b*x^2)*b)
```

Fricas [A] time = 1.21007, size = 78, normalized size = 2.79

$$-\frac{\sqrt{cx^4+bx^2}\left(2\,cx^2+b\right)}{b^2cx^4+b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2)*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.208, size = 38, normalized size = 1.36

$$-\frac{\frac{2\,cx^2}{b^2}+\frac{1}{b}}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(2*c*x^2/b^2 + 1/b)/sqrt(c*x^4 + b*x^2)
```

3.278 $\int \frac{1}{x(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{1}{bx^2\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^2*Sqrt[b*x^2 + c*x^4]) - (4*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^4) + (8*c*
Sqrt[b*x^2 + c*x^4])/(3*b^3*x^2)
```

Rubi [A] time = 0.129287, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{1}{bx^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^2*Sqrt[b*x^2 + c*x^4]) - (4*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^4) + (8*c*
Sqrt[b*x^2 + c*x^4])/(3*b^3*x^2)
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^2\sqrt{bx^2+cx^4}}+\frac{4\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{b}\\ &= \frac{1}{bx^2\sqrt{bx^2+cx^4}}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}-\frac{(8c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b^2}\\ &= \frac{1}{bx^2\sqrt{bx^2+cx^4}}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}\end{aligned}$$

Mathematica [A] time = 0.0119983, size = 48, normalized size = 0.65

$$-\frac{\left(b+cx^2\right)\left(b^2-4bcx^2-8c^2x^4\right)}{3b^3\left(x^2\left(b+cx^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((b + c*x^2)*(b^2 - 4*b*c*x^2 - 8*c^2*x^4))/(3*b^3*(x^2*(b + c*x^2))^(3/2)
)
```

Maple [A] time = 0.046, size = 45, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(-8\,c^2x^4-4\,bcx^2+b^2\right)}{3\,b^3}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-8*c^2*x^4-4*b*c*x^2+b^2)/b^3/(c*x^4+b*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27117, size = 104, normalized size = 1.41

$$\frac{\left(8\,c^{2}x^{4}+4\,bcx^{2}-b^{2}\right)\sqrt{cx^{4}+bx^{2}}}{3\left(b^{3}cx^{6}+b^{4}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(8*c^2*x^4 + 4*b*c*x^2 - b^2)*sqrt(c*x^4 + b*x^2)/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x), x)
```

3.279 $\int \frac{1}{x^3 \left(bx^2+cx^4\right)^{3/2}} \, dx$

Optimal. Leaf size=102

$$-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{1}{bx^4\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^4*Sqrt[b*x^2 + c*x^4]) - (6*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^6) + (8*c*
Sqrt[b*x^2 + c*x^4])/(5*b^3*x^4) - (16*c^2*Sqrt[b*x^2 + c*x^4])/(5*b^4*x^2)
```

Rubi [A] time = 0.184875, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{1}{bx^4\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^4*Sqrt[b*x^2 + c*x^4]) - (6*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^6) + (8*c*
Sqrt[b*x^2 + c*x^4])/(5*b^3*x^4) - (16*c^2*Sqrt[b*x^2 + c*x^4])/(5*b^4*x^2)
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^4\sqrt{bx^2+cx^4}}+\frac{6\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{b}\\ &= \frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}-\frac{(24c)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{5b^2}\\ &= \frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}+\frac{\left(16c^2\right)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{5b^3}\\ &= \frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}\end{aligned}$$

Mathematica [A] time = 0.0122275, size = 57, normalized size = 0.56

$$\frac{2b^2cx^2-b^3-8bc^2x^4-16c^3x^6}{5b^4x^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-b^3 + 2*b^2*c*x^2 - 8*b*c^2*x^4 - 16*c^3*x^6)/(5*b^4*x^4*Sqrt[x^2*(b + c*
x^2)])
```

Maple [A] time = 0.046, size = 59, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(16\,c^3x^6+8\,bc^2x^4-2\,b^2cx^2+b^3\right)}{5\,b^4x^2}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/5*(c*x^2+b)*(16*c^3*x^6+8*b*c^2*x^4-2*b^2*c*x^2+b^3)/x^2/b^4/(c*x^4+b*x^
2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34104, size = 128, normalized size = 1.25

$$-\frac{\left(16\,c^3x^6+8\,bc^2x^4-2\,b^2cx^2+b^3\right)\sqrt{cx^4+bx^2}}{5\left(b^4cx^8+b^5x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(16*c^3*x^6 + 8*b*c^2*x^4 - 2*b^2*c*x^2 + b^3)*sqrt(c*x^4 + b*x^2)/(b^
4*c*x^8 + b^5*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2)^{\frac{3}{2}} x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^3), x)
```

3.280 $\int \frac{1}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{1}{bx^6\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^6*Sqrt[b*x^2 + c*x^4]) - (8*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^8) + (48*c
*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^6) - (64*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^4*x
^4) + (128*c^3*Sqrt[b*x^2 + c*x^4])/(35*b^5*x^2)
```

Rubi [A] time = 0.233326, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{1}{bx^6\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^6*Sqrt[b*x^2 + c*x^4]) - (8*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^8) + (48*c
*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^6) - (64*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^4*x
^4) + (128*c^3*Sqrt[b*x^2 + c*x^4])/(35*b^5*x^2)
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^6\sqrt{bx^2+cx^4}}+\frac{8\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= \frac{1}{bx^6\sqrt{bx^2+cx^4}}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}-\frac{(48c)\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{7b^2}\\
&= \frac{1}{bx^6\sqrt{bx^2+cx^4}}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}+\frac{\left(192c^2\right)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{35b^3}\\
&= \frac{1}{bx^6\sqrt{bx^2+cx^4}}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}-\frac{\left(128c^3\right)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{35b^4}\\
&= \frac{1}{bx^6\sqrt{bx^2+cx^4}}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}
\end{aligned}$$

Mathematica [A] time = 0.0137668, size = 68, normalized size = 0.52

$$\frac{-16b^2c^2x^4+8b^3cx^2-5b^4+64bc^3x^6+128c^4x^8}{35b^5x^6\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-5*b^4 + 8*b^3*c*x^2 - 16*b^2*c^2*x^4 + 64*b*c^3*x^6 + 128*c^4*x^8)/(35*b^
5*x^6*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.047, size = 72, normalized size = 0.6

$$-\frac{\left(c x^{2}+b\right)\left(-128 c^{4} x^{8}-64 c^{3} x^{6} b+16 c^{2} x^{4} b^{2}-8 c x^{2} b^{3}+5 b^{4}\right)}{35 x^{4} b^{5}}\left(c x^{4}+b x^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-128*c^4*x^8-64*b*c^3*x^6+16*b^2*c^2*x^4-8*b^3*c*x^2+5*b^4
)/x^4/b^5/(c*x^4+b*x^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36518, size = 158, normalized size = 1.22

$$\frac{\left(128 c^{4} x^{8}+64 b c^{3} x^{6}-16 b^{2} c^{2} x^{4}+8 b^{3} c x^{2}-5 b^{4}\right) \sqrt{c x^{4}+b x^{2}}}{35\left(b^{5} c x^{10}+b^{6} x^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(128*c^4*x^8 + 64*b*c^3*x^6 - 16*b^2*c^2*x^4 + 8*b^3*c*x^2 - 5*b^4)*sq
rt(c*x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^5), x)
```

3.281 $\int \frac{x^6}{\left(bx^2+cx^4\right)^{3/2}} dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{bx^2+cx^4}}{c^2x}-\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^3/(c*Sqrt[b*x^2 + c*x^4])) + (2*Sqrt[b*x^2 + c*x^4])/(c^2*x)
```

Rubi [A] time = 0.067865, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 1588}

$$\frac{2\sqrt{bx^2+cx^4}}{c^2x}-\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^3/(c*Sqrt[b*x^2 + c*x^4])) + (2*Sqrt[b*x^2 + c*x^4])/(c^2*x)
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^3}{c\sqrt{bx^2+cx^4}} + \frac{2\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{c}$$
$$= -\frac{x^3}{c\sqrt{bx^2+cx^4}} + \frac{2\sqrt{bx^2+cx^4}}{c^2x}$$

Mathematica [A] time = 0.0152588, size = 29, normalized size = 0.62

$$\frac{x\left(2b+cx^2\right)}{c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*b + c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.046, size = 37, normalized size = 0.8

$$\frac{\left(cx^2+b\right)\left(cx^2+2\,b\right)x^3}{c^2}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] (c*x^2+b)*(c*x^2+2*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 0.99026, size = 30, normalized size = 0.64

$$\frac{cx^2+2\,b}{\sqrt{cx^2+b}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (c*x^2 + 2*b)/(sqrt(c*x^2 + b)*c^2)
```

Fricas [A] time = 1.24328, size = 74, normalized size = 1.57

$$\frac{\sqrt{cx^4 + bx^2}\left(cx^2 + 2\,b\right)}{c^3x^3 + bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)*(c*x^2 + 2*b)/(c^3*x^3 + b*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.16965, size = 70, normalized size = 1.49

$$-\frac{2\sqrt{b}}{\left(\left(\sqrt{c + \frac{b}{x^2}} - \frac{\sqrt{b}}{x}\right)^2 - c\right)c} + \frac{b}{\sqrt{c + \frac{b}{x^2}}c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b)/(((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)*c) + b/(sqrt(c + b/x^2)*c
^2*x)
```

3.282 $\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x/(c*Sqrt[b*x^2 + c*x^4]))
```

Rubi [A] time = 0.0194372, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x/(c*Sqrt[b*x^2 + c*x^4]))
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x}{c\sqrt{bx^2+cx^4}}$$

Mathematica [A] time = 0.0047655, size = 21, normalized size = 1.

$$-\frac{x}{c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x/(c*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A] time = 0.044, size = 29, normalized size = 1.4

$$-\frac{x^3\left(cx^2+b\right)}{c}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -(c*x^2+b)/c*x^3/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 1.00746, size = 19, normalized size = 0.9

$$-\frac{1}{\sqrt{cx^2+b}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(c*x^2 + b)*c)
```

Fricas [A] time = 1.28105, size = 54, normalized size = 2.57

$$-\frac{\sqrt{cx^4+bx^2}}{c^2x^3+bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2)/(c^2*x^3 + b*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A] time = 1.16458, size = 23, normalized size = 1.1

$$-\frac{1}{\sqrt{c+\frac{b}{x^2}}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(c + b/x^2)*c*x)
```

3.283 $\int \frac{x^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

```
[Out] x/(b*Sqrt[b*x^2 + c*x^4]) - ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/b^(3/2
)
```

Rubi [A] time = 0.0618186, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2023, 2008, 206}

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x/(b*Sqrt[b*x^2 + c*x^4]) - ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/b^(3/2
)
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{x}{b\sqrt{bx^2+cx^4}}+\frac{\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{b}\\ &= \frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{b}\\ &= \frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0091179, size = 38, normalized size = 0.75

$$\frac{x\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{b}+1\right)}{b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/b])/(b*Sqrt[x^2*(b + c*x^2)]
)
```

Maple [A] time = 0.046, size = 67, normalized size = 1.3

$$-x^3\left(cx^2+b\right)\left(\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)b\sqrt{cx^2+b}-b^{\frac{3}{2}}\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -x^3*(c*x^2+b)*(ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*(c*x^2+b)^(1/2)-b^(3/
2))/(c*x^4+b*x^2)^(3/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [A] time = 1.32026, size = 348, normalized size = 6.82

$$\left[\frac{\left(cx^3 + bx\right)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\,\sqrt{cx^4 + bx^2}b}{2\left(b^2cx^3 + b^3x\right)}, \frac{\left(cx^3 + bx\right)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right) + \sqrt{cx^4 + bx^2}b}{b^2cx^3 + b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*x^3 + b*x)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqr
t(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*c*x^3 + b^3*x), ((c*x^3 + b*x)*s
qrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*b)/(b^2*c*x^3 + b^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.284 $\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(2*b^2*x^3) + (3*c*Ar
cTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rubi [A] time = 0.0646457, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2006, 2025, 2008, 206}

$$-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] 1/(b*x*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(2*b^2*x^3) + (3*c*Ar
cTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
```

```
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx\sqrt{bx^2+cx^4}}+\frac{3\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{b}\\ &= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}-\frac{(3c)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}\\ &= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{(3c)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b^2}\\ &= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.0081719, size = 40, normalized size = 0.49

$$-\frac{cx\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{b}+1\right)}{b^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] -((c*x*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c*x^2)/b])/(b^2*Sqrt[x^2*(b + c
*x^2)]))
```

Maple [A] time = 0.046, size = 77, normalized size = 1.

$$-\frac{x\left(cx^{2}+b\right)}{2}\left(3\,b^{3/2}x^{2}c-3\,\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^{2}+b}+b}{x}\right)\sqrt{cx^{2}+b}x^{2}bc+b^{\frac{5}{2}}\right)\left(cx^{4}+bx^{2}\right)^{-\frac{3}{2}}b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2*x*(c*x^2+b)*(3*b^(3/2)*x^2*c-3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*(c*
x^2+b)^(1/2)*x^2*b*c+b^(5/2))/(c*x^4+b*x^2)^(3/2)/b^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(-3/2), x)
```

Fricas [A] time = 1.41731, size = 425, normalized size = 5.25

$$\left[\frac{3\left(c^{2}x^{5}+bcx^{3}\right)\sqrt{b}\log\left(-\frac{cx^{3}+2\,bx+2\,\sqrt{cx^{4}+bx^{2}}\sqrt{b}}{x^{3}}\right)-2\,\sqrt{cx^{4}+bx^{2}}\left(3\,bcx^{2}+b^{2}\right)}{4\left(b^{3}cx^{5}+b^{4}x^{3}\right)},-\frac{3\left(c^{2}x^{5}+bcx^{3}\right)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\sqrt{-b}}{cx^{3}+bx}\right.}{2\left(b^{3}cx^{5}+b^{4}x^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(c^2*x^5 + b*c*x^3)*sqrt(b)*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*
x^2)*sqrt(b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 + b^2))/(b^3*c*x^5 +
```

```
b^4*x^3), -1/2*(3*(c^2*x^5 + b*c*x^3)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*s
qrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 + b^2))/(b^3*c*x^5
+ b^4*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((b*x**2 + c*x**4)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.285 $\int \frac{1}{x^2(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^3*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(4*b^2*x^5) + (15*c
*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) - (15*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2
+ c*x^4]])/(8*b^(7/2))
```

Rubi [A] time = 0.153449, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2023, 2025, 2008, 206}

$$-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^3*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(4*b^2*x^5) + (15*c
*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) - (15*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2
+ c*x^4]])/(8*b^(7/2))
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^3\sqrt{bx^2+cx^4}}+\frac{5\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}-\frac{(15c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b^2}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}+\frac{\left(15c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^3}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{\left(15c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^3}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0111332, size = 41, normalized size = 0.38

$$
\frac{c^2x\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{cx^2}{b}+1\right)}{b^3\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (c^2*x*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (c*x^2)/b])/(b^3*Sqrt[x^2*(b + c
*x^2)])
```

Maple [A] time = 0.046, size = 94, normalized size = 0.9

$$-\frac{cx^2+b}{8x}\left(15\sqrt{cx^2+b}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4bc^2-15\,b^{3/2}x^4c^2-5\,b^{5/2}x^2c+2\,b^{7/2}\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/8/x*(c*x^2+b)*(15*(c*x^2+b)^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^
4*b*c^2-15*b^(3/2)*x^4*c^2-5*b^(5/2)*x^2*c+2*b^(7/2))/(c*x^4+b*x^2)^(3/2)/b
^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

Fricas [A] time = 1.34802, size = 485, normalized size = 4.45

$$\left[\frac{15\left(c^3x^7+bc^2x^5\right)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\left(15\,bc^2x^4+5\,b^2cx^2-2\,b^3\right)\sqrt{cx^4+bx^2}}{16\left(b^4cx^7+b^5x^5\right)},\ \frac{15\left(c^3x^7+bc^2x^5\right)\sqrt{-b}\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4
+ b*x^2)*sqrt(b))/x^3) + 2*(15*b*c^2*x^4 + 5*b^2*c*x^2 - 2*b^3)*sqrt(c*x^4
+ b*x^2))/(b^4*c*x^7 + b^5*x^5), 1/8*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(-b)*arc
tan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*b*c^2*x^4 + 5*b^2*c*x
^2 - 2*b^3)*sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

3.286 $\int \frac{x^3}{\sqrt{3x^2-4x^4}} \, dx$

Optimal. Leaf size=34

$$-\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)$$

```
[Out] -Sqrt[3*x^2 - 4*x^4]/8 - (3*ArcSin[1 - (8*x^2)/3])/32
```

Rubi [A] time = 0.0582664, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 640, 619, 216}

$$-\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

```
[Out] -Sqrt[3*x^2 - 4*x^4]/8 - (3*ArcSin[1 - (8*x^2)/3])/32
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{3x-4x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{8}\sqrt{3x^2-4x^4}+\frac{3}{16}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3x-4x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{1}{32}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 3-8x^2\right)\\ &= -\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0188481, size = 57, normalized size = 1.68

$$\frac{x\left(8x^3+3\sqrt{4x^2-3}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-3}}\right)-6x\right)}{16\sqrt{3x^2-4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

```
[Out] (x*(-6*x + 8*x^3 + 3*Sqrt[-3 + 4*x^2]*ArcTanh[(2*x)/Sqrt[-3 + 4*x^2]]))/(16
*Sqrt[3*x^2 - 4*x^4])
```

Maple [A] time = 0.046, size = 48, normalized size = 1.4

$$\frac{x}{16}\sqrt{-4\,x^2+3}\left(-2\,x\sqrt{-4\,x^2+3}+3\,\arcsin\left(2/3\,x\sqrt{3}\right)\right)\frac{1}{\sqrt{-4\,x^4+3\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^4+3*x^2)^(1/2),x)
```

```
[Out] 1/16*x*(-4*x^2+3)^(1/2)*(-2*x*(-4*x^2+3)^(1/2)+3*arcsin(2/3*x*3^(1/2)))/(-4
*x^4+3*x^2)^(1/2)
```

Maxima [A] time = 1.44845, size = 35, normalized size = 1.03

$$-\frac{1}{8}\sqrt{-4x^4+3x^2}-\frac{3}{32}\arcsin\left(-\frac{8}{3}x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) - 3/32*arcsin(-8/3*x^2 + 1)
```

Fricas [A] time = 1.25666, size = 96, normalized size = 2.82

$$-\frac{1}{8}\sqrt{-4x^4+3x^2}-\frac{3}{16}\arctan\left(\frac{\sqrt{-4x^4+3x^2}}{2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) - 3/16*arctan(1/2*sqrt(-4*x^4 + 3*x^2)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{-x^2\left(4x^2-3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(4*x**2 - 3)), x)
```

Giac [A] time = 1.15154, size = 35, normalized size = 1.03

$$-\frac{1}{8}\sqrt{-4x^4+3x^2}+\frac{3}{32}\arcsin\left(\frac{8}{3}x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) + 3/32*arcsin(8/3*x^2 - 1)
```

3.287 $\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{8}\sqrt{-4x^4-3x^2}-\frac{3}{32}\sin^{-1}\left(\frac{8x^2}{3}+1\right)$$

```
[Out] -Sqrt[-3*x^2 - 4*x^4]/8 - (3*ArcSin[1 + (8*x^2)/3])/32
```

Rubi [A] time = 0.0572119, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 640, 619, 216}

$$-\frac{1}{8}\sqrt{-4x^4-3x^2}-\frac{3}{32}\sin^{-1}\left(\frac{8x^2}{3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

```
[Out] -Sqrt[-3*x^2 - 4*x^4]/8 - (3*ArcSin[1 + (8*x^2)/3])/32
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{-3x-4x^2}}\,dx, x, x^2\right)\\
&= -\frac{1}{8}\sqrt{-3x^2-4x^4}-\frac{3}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{-3x-4x^2}}\,dx, x, x^2\right)\\
&= -\frac{1}{8}\sqrt{-3x^2-4x^4}+\frac{1}{32}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, -3-8x^2\right)\\
&= -\frac{1}{8}\sqrt{-3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1+\frac{8x^2}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.0169786, size = 52, normalized size = 1.53

$$\frac{x\left(8x^3-3\sqrt{4x^2+3}\sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+6x\right)}{16\sqrt{-x^2\left(4x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

```
[Out] (x*(6*x + 8*x^3 - 3*Sqrt[3 + 4*x^2]*ArcSinh[(2*x)/Sqrt[3]]))/(16*Sqrt[-(x^2
*(3 + 4*x^2))])
```

Maple [B] time = 0.047, size = 54, normalized size = 1.6

$$-\frac{x}{16}\sqrt{-4\,x^2-3}\left(2\,x\sqrt{-4\,x^2-3}+3\,\arctan\left(2\,\frac{x}{\sqrt{-4\,x^2-3}}\right)\right)\frac{1}{\sqrt{-4\,x^4-3\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^4-3*x^2)^(1/2),x)
```

```
[Out] -1/16*x*(-4*x^2-3)^(1/2)*(2*x*(-4*x^2-3)^(1/2)+3*arctan(2*x/(-4*x^2-3)^(1/2
)))/(-4*x^4-3*x^2)^(1/2)
```

Maxima [A] time = 1.47273, size = 35, normalized size = 1.03

$$-\frac{1}{8}\sqrt{-4x^4-3x^2}+\frac{3}{32}\arcsin\left(-\frac{8}{3}x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^4 - 3*x^2) + 3/32*arcsin(-8/3*x^2 - 1)
```

Fricas [C] time = 1.1557, size = 158, normalized size = 4.65

$$-\frac{1}{8}\sqrt{-4x^2-3}x-\frac{3}{32}i\log\left(-\frac{8x+4i\sqrt{-4x^2-3}}{x}\right)+\frac{3}{32}i\log\left(-\frac{8x-4i\sqrt{-4x^2-3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 - 3)*x - 3/32*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 3))/x) + 3/3
2*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 3))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{-x^2\left(4x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**4-3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(4*x**2 + 3)), x)
```

Giac [A] time = 1.1691, size = 36, normalized size = 1.06

$$-\frac{1}{8}\sqrt{4x^4+3x^2}i-\frac{3}{32}\arcsin\left(\frac{8}{3}x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(4*x^4 + 3*x^2)*i - 3/32*arcsin(8/3*x^2 + 1)
```

3.288 $\int \frac{x^3}{\sqrt{3x^2+4x^4}} \, dx$

Optimal. Leaf size=45

$$\frac{1}{8}\sqrt{4x^4+3x^2}-\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)$$

```
[Out] Sqrt[3*x^2 + 4*x^4]/8 - (3*ArcTanh[(2*x^2)/Sqrt[3*x^2 + 4*x^4]])/16
```

Rubi [A] time = 0.0561752, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 640, 620, 206}

$$\frac{1}{8}\sqrt{4x^4+3x^2}-\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[3*x^2 + 4*x^4],x]
```

```
[Out] Sqrt[3*x^2 + 4*x^4]/8 - (3*ArcTanh[(2*x^2)/Sqrt[3*x^2 + 4*x^4]])/16
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{3x+4x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{8}\sqrt{3x^2+4x^4}-\frac{3}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{3x+4x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{8}\sqrt{3x^2+4x^4}-\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x^2}{\sqrt{3x^2+4x^4}}\right)\\
&= \frac{1}{8}\sqrt{3x^2+4x^4}-\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{3x^2+4x^4}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0097691, size = 51, normalized size = 1.13

$$
\frac{x\left(8x^3-3\sqrt{4x^2+3}\sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+6x\right)}{16\sqrt{x^2\left(4x^2+3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[3*x^2 + 4*x^4],x]
```

```
[Out] (x*(6*x + 8*x^3 - 3*Sqrt[3 + 4*x^2]*ArcSinh[(2*x)/Sqrt[3]]))/(16*Sqrt[x^2*(
3 + 4*x^2)])
```

Maple [A] time = 0.046, size = 48, normalized size = 1.1

$$
-\frac{x}{16}\sqrt{4\,x^2+3}\left(-2\,x\sqrt{4\,x^2+3}+3\,\text{Arcsinh}\left(2/3\,x\sqrt{3}\right)\right)\frac{1}{\sqrt{4\,x^4+3\,x^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4+3*x^2)^(1/2),x)
```

```
[Out] -1/16*x*(4*x^2+3)^(1/2)*(-2*x*(4*x^2+3)^(1/2)+3*arcsinh(2/3*x*3^(1/2)))/(4*
x^4+3*x^2)^(1/2)
```

Maxima [A] time = 1.45947, size = 55, normalized size = 1.22

$$\frac{1}{8}\sqrt{4x^4+3x^2}-\frac{3}{32}\log\left(8x^2+4\sqrt{4x^4+3x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) - 3/32*log(8*x^2 + 4*sqrt(4*x^4 + 3*x^2) + 3)
```

Fricas [A] time = 1.23217, size = 95, normalized size = 2.11

$$\frac{1}{8}\sqrt{4x^4+3x^2}+\frac{3}{16}\log\left(-\frac{2x^2-\sqrt{4x^4+3x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) + 3/16*log(-(2*x^2 - sqrt(4*x^4 + 3*x^2))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{x^2\left(4x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(4*x**2 + 3)), x)
```

Giac [A] time = 1.1693, size = 55, normalized size = 1.22

$$\frac{1}{8}\sqrt{4x^4+3x^2}+\frac{3}{32}\log\left(8x^2-4\sqrt{4x^4+3x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) + 3/32*log(8*x^2 - 4*sqrt(4*x^4 + 3*x^2) + 3)
```

3.289 $\int \frac{x^3}{\sqrt{-3x^2+4x^4}} \, dx$

Optimal. Leaf size=45

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)$$

```
[Out] Sqrt[-3*x^2 + 4*x^4]/8 + (3*ArcTanh[(2*x^2)/Sqrt[-3*x^2 + 4*x^4]])/16
```

Rubi [A] time = 0.0567401, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 640, 620, 206}

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-3*x^2 + 4*x^4],x]
```

```
[Out] Sqrt[-3*x^2 + 4*x^4]/8 + (3*ArcTanh[(2*x^2)/Sqrt[-3*x^2 + 4*x^4]])/16
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{-3x+4x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{-3x+4x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x^2}{\sqrt{-3x^2+4x^4}}\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{-3x^2+4x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.0123398, size = 57, normalized size = 1.27

$$\frac{x\left(8x^3+3\sqrt{4x^2-3}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-3}}\right)-6x\right)}{16\sqrt{x^2\left(4x^2-3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-3*x^2 + 4*x^4],x]
```

```
[Out] (x*(-6*x + 8*x^3 + 3*Sqrt[-3 + 4*x^2]*ArcTanh[(2*x)/Sqrt[-3 + 4*x^2]]))/(16
*Sqrt[x^2*(-3 + 4*x^2)])
```

Maple [A] time = 0.046, size = 60, normalized size = 1.3

$$\frac{x}{32}\sqrt{4\,x^2-3}\left(3\,\ln\left(\sqrt{4}x+\sqrt{4\,x^2-3}\right)\sqrt{4}+4\,x\sqrt{4\,x^2-3}\right)\frac{1}{\sqrt{4\,x^4-3\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4-3*x^2)^(1/2),x)
```

[Out] 1/32*x*(4*x^2-3)^(1/2)*(3*ln(4^(1/2)*x+(4*x^2-3)^(1/2))*4^(1/2)+4*x*(4*x^2-3)^(1/2))/(4*x^4-3*x^2)^(1/2)

Maxima [A] time = 1.44159, size = 55, normalized size = 1.22

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{32}\log\left(8x^2+4\sqrt{4x^4-3x^2}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="maxima")

[Out] 1/8*sqrt(4*x^4 - 3*x^2) + 3/32*log(8*x^2 + 4*sqrt(4*x^4 - 3*x^2) - 3)

Fricas [A] time = 1.2476, size = 95, normalized size = 2.11

$$\frac{1}{8}\sqrt{4x^4-3x^2}-\frac{3}{16}\log\left(-\frac{2x^2-\sqrt{4x^4-3x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/8*sqrt(4*x^4 - 3*x^2) - 3/16*log(-(2*x^2 - sqrt(4*x^4 - 3*x^2))/x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{x^2\left(4x^2-3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(4*x**4-3*x**2)**(1/2),x)

[Out] Integral(x**3/sqrt(x**2*(4*x**2 - 3)), x)

Giac [A] time = 1.1701, size = 57, normalized size = 1.27

$$\frac{1}{8}\sqrt{4x^4-3x^2}-\frac{3}{32}\log\left(\left|-8x^2+4\sqrt{4x^4-3x^2}+3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^4 - 3*x^2) - 3/32*log(abs(-8*x^2 + 4*sqrt(4*x^4 - 3*x^2) + 3))
```

3.290 $\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

```
[Out] Sqrt[a*x^2 + b*x^4]/(2*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^2 + b*x^4]])/
(2*b^(3/2))
```

Rubi [A] time = 0.0827262, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2018, 640, 620, 206}

$$\frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^4]/(2*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^2 + b*x^4]])/
(2*b^(3/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{ax+bx^2}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{ax+bx^2}}\,dx, x, x^2\right)}{4b}\\ &= \frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^2+bx^4}}\right)}{2b}\\ &= \frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0325601, size = 73, normalized size = 1.26

$$\frac{x\left(\sqrt{b}x\left(a+bx^2\right)-a\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)}{2b^{3/2}\sqrt{x^2\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^4],x]
```

```
[Out] (x*(Sqrt[b]*x*(a + b*x^2) - a*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]]))/(2*b^(3/2)*Sqrt[x^2*(a + b*x^2)])
```

Maple [A] time = 0.047, size = 64, normalized size = 1.1

$$\frac{x}{2}\sqrt{bx^2+a}\left(x\sqrt{bx^2+a}b^{\frac{3}{2}}-a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b\right)\frac{1}{\sqrt{bx^4+ax^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)*(x*(b*x^2+a)^(1/2)*b^(3/2)-a*ln(x*b^(1/2)+(b*x^2+a)^(
1/2))*b)/(b*x^4+a*x^2)^(1/2)/b^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30028, size = 265, normalized size = 4.57

$$\left[\frac{a\sqrt{b}\log\left(-2\,bx^2 - a + 2\,\sqrt{bx^4 + ax^2}\sqrt{b}\right) + 2\,\sqrt{bx^4 + ax^2}b}{4\,b^2}, \frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx^4+ax^2}\sqrt{-b}}{bx^2+a}\right) + \sqrt{bx^4 + ax^2}b}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*log(-2*b*x^2 - a + 2*sqrt(b*x^4 + a*x^2)*sqrt(b)) + 2*sqrt(
b*x^4 + a*x^2)*b)/b^2, 1/2*(a*sqrt(-b)*arctan(sqrt(b*x^4 + a*x^2)*sqrt(-b)/
(b*x^2 + a)) + sqrt(b*x^4 + a*x^2)*b)/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2\left(a + bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x**2)), x)
```

Giac [A] time = 1.22151, size = 80, normalized size = 1.38

$$\frac{a \log\left(\left|-2\left(\sqrt{b}x^2 - \sqrt{bx^4 + ax^2}\right)\sqrt{b} - a\right|\right)}{4\,b^{\frac{3}{2}}} + \frac{\sqrt{bx^4 + ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*a*log(abs(-2*(sqrt(b)*x^2 - sqrt(b*x^4 + a*x^2))*sqrt(b) - a))/b^(3/2)
+ 1/2*sqrt(b*x^4 + a*x^2)/b
```

3.291 $\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

```
[Out] -Sqrt[a*x^2 - b*x^4]/(2*b) + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a*x^2 - b*x^4]])/
(2*b^(3/2))
```

Rubi [A] time = 0.0816661, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2018, 640, 620, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 - b*x^4],x]
```

```
[Out] -Sqrt[a*x^2 - b*x^4]/(2*b) + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a*x^2 - b*x^4]])/
(2*b^(3/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{ax-bx^2}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{ax-bx^2}}\,dx, x, x^2\right)}{4b}\\ &= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^2-bx^4}}\right)}{2b}\\ &= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0406761, size = 77, normalized size = 1.28

$$\frac{x\left(\sqrt{b}x\left(bx^2-a\right)+a\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)\right)}{2b^{3/2}\sqrt{x^2\left(a-bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 - b*x^4],x]
```

```
[Out] (x*(Sqrt[b]*x*(-a + b*x^2) + a*Sqrt[a - b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a -
b*x^2]]))/(2*b^(3/2)*Sqrt[x^2*(a - b*x^2)])
```

Maple [A] time = 0.049, size = 67, normalized size = 1.1

$$-\frac{x}{2}\sqrt{-bx^2+a}\left(x\sqrt{-bx^2+a}b^{\frac{3}{2}}-a\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2+a}}\right)b\right)\frac{1}{\sqrt{-bx^4+ax^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^4+a*x^2)^(1/2),x)
```

```
[Out] -1/2*x*(-b*x^2+a)^(1/2)*(x*(-b*x^2+a)^(1/2)*b^(3/2)-a*arctan(b^(1/2)*x/(-b*
x^2+a)^(1/2))*b)/(-b*x^4+a*x^2)^(1/2)/b^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26835, size = 271, normalized size = 4.52

$$\left[-\frac{a\sqrt{-b}\log\left(2\,bx^2 - a - 2\,\sqrt{-bx^4 + ax^2}\sqrt{-b}\right) + 2\,\sqrt{-bx^4 + ax^2}b}{4\,b^2}, -\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{-bx^4+ax^2}\sqrt{b}}{bx^2-a}\right) + \sqrt{-bx^4 + ax^2}b}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*sqrt(-b)*log(2*b*x^2 - a - 2*sqrt(-b*x^4 + a*x^2)*sqrt(-b)) + 2*sq
rt(-b*x^4 + a*x^2)*b)/b^2, -1/2*(a*sqrt(b)*arctan(sqrt(-b*x^4 + a*x^2)*sqrt
(b)/(b*x^2 - a)) + sqrt(-b*x^4 + a*x^2)*b)/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-x^2\left(-a + bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**4+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(-a + b*x**2)), x)
```

Giac [A] time = 1.19304, size = 92, normalized size = 1.53

$$-\frac{a\log\left(\left|2\left(\sqrt{-b}x^{2}-\sqrt{-bx^{4}+ax^{2}}\right)\sqrt{-b}+a\right|\right)}{4\sqrt{-b}b}-\frac{\sqrt{-bx^{4}+ax^{2}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*a*log(abs(2*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a*x^2))*sqrt(-b) + a))/(sqrt
(-b)*b) - 1/2*sqrt(-b*x^4 + a*x^2)/b
```

3.292 $\int x^{7/2}\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

```
[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

Rubi [A] time = 0.0047629, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(bx^2+cx^4\right)\,dx &= \int \left(bx^{11/2}+cx^{15/2}\right)\,dx\\ &= \frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.0057258, size = 21, normalized size = 1.

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

Maple [A] time = 0.042, size = 16, normalized size = 0.8

$$\frac{26\,c x^{2}+34\,b}{221}x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2),x)
```

```
[Out] 2/221*x^(13/2)*(13*c*x^2+17*b)
```

Maxima [A] time = 0.972262, size = 18, normalized size = 0.86

$$\frac{2}{17}\,c x^{\frac{17}{2}}+\frac{2}{13}\,b x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

Fricas [A] time = 1.24512, size = 50, normalized size = 2.38

$$\frac{2}{221}\left(13\,c x^{8}+17\,b x^{6}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/221*(13*c*x^8 + 17*b*x^6)*sqrt(x)
```

Sympy [A] time = 14.1793, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{13}{2}}}{13} + \frac{2cx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(13/2)/13 + 2*c*x**(17/2)/17
```

Giac [A] time = 1.14206, size = 18, normalized size = 0.86

$$\frac{2}{17}cx^{\frac{17}{2}} + \frac{2}{13}bx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

3.293 $\int x^{5/2}\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

```
[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rubi [A] time = 0.0053335, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4),x]

[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(bx^2+cx^4\right)\,dx&=\int\left(bx^{9/2}+cx^{13/2}\right)\,dx\\&=\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.0047712, size = 21, normalized size = 1.

$$\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Maple [A] time = 0.042, size = 16, normalized size = 0.8

$$\frac{22\,cx^2+30\,b}{165}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2),x)
```

```
[Out] 2/165*x^(11/2)*(11*c*x^2+15*b)
```

Maxima [A] time = 0.998516, size = 18, normalized size = 0.86

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

Fricas [A] time = 1.23168, size = 50, normalized size = 2.38

$$\frac{2}{165}\left(11\,cx^7+15\,bx^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/165*(11*c*x^7 + 15*b*x^5)*sqrt(x)
```

Sympy [A] time = 7.91652, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{11}{2}}}{11} + \frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

Giac [A] time = 1.13242, size = 18, normalized size = 0.86

$$\frac{2}{15}\,cx^{\frac{15}{2}} + \frac{2}{11}\,bx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

3.294 $\int x^{3/2} \left(bx^2 + cx^4\right) dx$

Optimal. Leaf size=21

$$\frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

```
[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rubi [A] time = 0.0049, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2} \left(bx^2 + cx^4\right) dx &= \int \left(bx^{7/2} + cx^{11/2}\right) dx\\ &= \frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0047868, size = 21, normalized size = 1.

$$\frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Maple [A] time = 0.044, size = 16, normalized size = 0.8

$$\frac{18\,c x^2 + 26\,b}{117} x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2),x)
```

```
[Out] 2/117*x^(9/2)*(9*c*x^2+13*b)
```

Maxima [A] time = 0.969289, size = 18, normalized size = 0.86

$$\frac{2}{13}\,c x^{\frac{13}{2}} + \frac{2}{9}\,b x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

Fricas [A] time = 1.2052, size = 49, normalized size = 2.33

$$\frac{2}{117}\left(9\,c x^6 + 13\,b x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/117*(9*c*x^6 + 13*b*x^4)*sqrt(x)
```

Sympy [A] time = 3.25703, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{9}{2}}}{9} + \frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

Giac [A] time = 1.11468, size = 18, normalized size = 0.86

$$\frac{2}{13}\,cx^{\frac{13}{2}} + \frac{2}{9}\,bx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

3.295 $\int \sqrt{x}\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

```
[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rubi [A] time = 0.0047368, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(bx^2+cx^4\right)\,dx &= \int \left(bx^{5/2}+cx^{9/2}\right)\,dx\\ &= \frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0077376, size = 21, normalized size = 1.

$$\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Maple [A] time = 0.044, size = 16, normalized size = 0.8

$$\frac{14\,c x^{2}+22\,b}{77}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2),x)
```

```
[Out] 2/77*x^(7/2)*(7*c*x^2+11*b)
```

Maxima [A] time = 0.965287, size = 18, normalized size = 0.86

$$\frac{2}{11}\,c x^{\frac{11}{2}}+\frac{2}{7}\,b x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

Fricas [A] time = 1.25043, size = 47, normalized size = 2.24

$$\frac{2}{77}\left(7\,c x^{5}+11\,b x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*c*x^5 + 11*b*x^3)*sqrt(x)
```

Sympy [A] time = 1.6907, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

Giac [A] time = 1.11819, size = 18, normalized size = 0.86

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

3.296 $\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

```
[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rubi [A] time = 0.0050601, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx &= \int \left(bx^{3/2}+cx^{7/2}\right)dx\\ &= \frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0054225, size = 21, normalized size = 1.

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Maple [A] time = 0.043, size = 16, normalized size = 0.8

$$\frac{10\,c x^{2}+18\,b}{45}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(1/2),x)
```

```
[Out] 2/45*x^(5/2)*(5*c*x^2+9*b)
```

Maxima [A] time = 0.958717, size = 18, normalized size = 0.86

$$\frac{2}{9}\,c x^{\frac{9}{2}}+\frac{2}{5}\,b x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

Fricas [A] time = 1.28677, size = 46, normalized size = 2.19

$$\frac{2}{45}\left(5\,c x^{4}+9\,b x^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*c*x^4 + 9*b*x^2)*sqrt(x)
```

Sympy [A] time = 0.762273, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

Giac [A] time = 1.10424, size = 18, normalized size = 0.86

$$\frac{2}{9}\,cx^{\frac{9}{2}} + \frac{2}{5}\,bx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

3.297 $\int \frac{bx^2+cx^4}{x^{3/2}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2}$$

```
[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rubi [A] time = 0.0051502, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2 + cx^4}{x^{3/2}}\,dx &= \int \left(b\sqrt{x} + cx^{5/2}\right)dx\\ &= \frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0044157, size = 21, normalized size = 1.

$$\frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Maple [A] time = 0.043, size = 16, normalized size = 0.8

$$\frac{6\,c x^{2}+14\,b}{21} x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(3/2),x)
```

```
[Out] 2/21*x^(3/2)*(3*c*x^2+7*b)
```

Maxima [A] time = 0.953166, size = 18, normalized size = 0.86

$$\frac{2}{7}\, c x^{\frac{7}{2}} + \frac{2}{3}\, b x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

Fricas [A] time = 1.29651, size = 43, normalized size = 2.05

$$\frac{2}{21}\left(3\, c x^{3} + 7\, b x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*c*x^3 + 7*b*x)*sqrt(x)
```

Sympy [A] time = 0.915969, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{3}{2}}}{3} + \frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(3/2),x)
```

```
[Out] 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

Giac [A] time = 1.12433, size = 18, normalized size = 0.86

$$\frac{2}{7}\,cx^{\frac{7}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

3.298 $\int \frac{bx^2+cx^4}{x^{5/2}} \, dx$

Optimal. Leaf size=19

$$2b\sqrt{x} + \frac{2}{5}cx^{5/2}$$

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rubi [A] time = 0.0047181, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$2b\sqrt{x} + \frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2+cx^4}{x^{5/2}} \, dx &= \int \left(\frac{b}{\sqrt{x}} + cx^{3/2}\right) dx \\ &= 2b\sqrt{x} + \frac{2}{5}cx^{5/2} \end{aligned}$$

Mathematica [A] time = 0.0047264, size = 19, normalized size = 1.

$$2b\sqrt{x} + \frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Maple [A] time = 0.043, size = 15, normalized size = 0.8

$$\frac{2\,c x^2+10\,b}{5}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(5/2),x)
```

```
[Out] 2/5*x^(1/2)*(c*x^2+5*b)
```

Maxima [A] time = 0.960773, size = 18, normalized size = 0.95

$$\frac{2}{5}\,c x^{\frac{5}{2}}+2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x)
```

Fricas [A] time = 1.20248, size = 36, normalized size = 1.89

$$\frac{2}{5}\left(c x^2+5\,b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/5*(c*x^2 + 5*b)*sqrt(x)
```

Sympy [A] time = 1.1772, size = 17, normalized size = 0.89

$$2b\sqrt{x} + \frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(5/2),x)
```

```
[Out] 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

Giac [A] time = 1.11644, size = 18, normalized size = 0.95

$$\frac{2}{5} cx^{\frac{5}{2}} + 2 b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x)
```

3.299 $\int \frac{bx^2+cx^4}{x^{7/2}} dx$

Optimal. Leaf size=19

$$\frac{2}{3}cx^{3/2} - \frac{2b}{\sqrt{x}}$$

```
[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rubi [A] time = 0.0050894, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{3}cx^{3/2} - \frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^{7/2}}\,dx &= \int \left(\frac{b}{x^{3/2}} + c\sqrt{x}\right) dx\\ &= -\frac{2b}{\sqrt{x}} + \frac{2}{3}cx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0060615, size = 19, normalized size = 1.

$$\frac{2}{3}cx^{3/2} - \frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Maple [A] time = 0.045, size = 16, normalized size = 0.8

$$-\frac{-2\,c x^2+6\,b}{3}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(7/2),x)
```

```
[Out] -2/3/x^(1/2)*(-c*x^2+3*b)
```

Maxima [A] time = 0.980051, size = 18, normalized size = 0.95

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*c*x^(3/2) - 2*b/sqrt(x)
```

Fricas [A] time = 1.37157, size = 36, normalized size = 1.89

$$\frac{2\left(cx^2-3\,b\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*x^2 - 3*b)/sqrt(x)
```

Sympy [A] time = 2.09811, size = 17, normalized size = 0.89

$$-\frac{2b}{\sqrt{x}} + \frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(7/2),x)
```

```
[Out] -2*b/sqrt(x) + 2*c*x**(3/2)/3
```

Giac [A] time = 1.18016, size = 18, normalized size = 0.95

$$\frac{2}{3}cx^{\frac{3}{2}} - \frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*c*x^(3/2) - 2*b/sqrt(x)
```

3.300 $\int x^{7/2} \left(bx^2 + cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{17}b^2x^{17/2} + \frac{4}{21}bcx^{21/2} + \frac{2}{25}c^2x^{25/2}$$

```
[Out] (2*b^2*x^(17/2))/17 + (4*b*c*x^(21/2))/21 + (2*c^2*x^(25/2))/25
```

Rubi [A] time = 0.0160419, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{17}b^2x^{17/2} + \frac{4}{21}bcx^{21/2} + \frac{2}{25}c^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(17/2))/17 + (4*b*c*x^(21/2))/21 + (2*c^2*x^(25/2))/25
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx=\int x^{15/2}\left(b+cx^2\right)^2\,dx$$
$$=\int\left(b^2x^{15/2}+2bcx^{19/2}+c^2x^{23/2}\right)dx$$
$$=\frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}$$

Mathematica [A] time = 0.008169, size = 30, normalized size = 0.83

$$\frac{2x^{17/2}\left(525b^2+850bcx^2+357c^2x^4\right)}{8925}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(17/2)*(525*b^2 + 850*b*c*x^2 + 357*c^2*x^4))/8925
```

Maple [A] time = 0.045, size = 27, normalized size = 0.8

$$\frac{714\,c^2x^4+1700\,bcx^2+1050\,b^2}{8925}x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/8925*x^(17/2)*(357*c^2*x^4+850*b*c*x^2+525*b^2)
```

Maxima [A] time = 0.963226, size = 32, normalized size = 0.89

$$\frac{2}{25}\,c^2x^{\frac{25}{2}}+\frac{4}{21}\,bcx^{\frac{21}{2}}+\frac{2}{17}\,b^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

Fricas [A] time = 1.2389, size = 81, normalized size = 2.25

$$\frac{2}{8925}\left(357\,c^2x^{12}+850\,bcx^{10}+525\,b^2x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/8925*(357*c^2*x^12 + 850*b*c*x^10 + 525*b^2*x^8)*sqrt(x)
```

Sympy [A] time = 41.1167, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{17}{2}}}{17}+\frac{4bcx^{\frac{21}{2}}}{21}+\frac{2c^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(17/2)/17 + 4*b*c*x**(21/2)/21 + 2*c**2*x**(25/2)/25
```

Giac [A] time = 1.15506, size = 32, normalized size = 0.89

$$\frac{2}{25}\,c^2x^{\frac{25}{2}}+\frac{4}{21}\,bcx^{\frac{21}{2}}+\frac{2}{17}\,b^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

3.301 $\int x^{5/2} \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

```
[Out] (2*b^2*x^(15/2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rubi [A] time = 0.0179989, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(15/2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx=\int x^{13/2}\left(b+cx^2\right)^2\,dx$$
$$=\int\left(b^2x^{13/2}+2bcx^{17/2}+c^2x^{21/2}\right)dx$$
$$=\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Mathematica [A] time = 0.0077621, size = 30, normalized size = 0.83

$$\frac{2x^{15/2}\left(437b^2+690bcx^2+285c^2x^4\right)}{6555}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(15/2)*(437*b^2 + 690*b*c*x^2 + 285*c^2*x^4))/6555
```

Maple [A] time = 0.045, size = 27, normalized size = 0.8

$$\frac{570\,c^2x^4+1380\,bcx^2+874\,b^2}{6555}x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/6555*x^(15/2)*(285*c^2*x^4+690*b*c*x^2+437*b^2)
```

Maxima [A] time = 0.969372, size = 32, normalized size = 0.89

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

Fricas [A] time = 1.26604, size = 80, normalized size = 2.22

$$\frac{2}{6555}\left(285\,c^2x^{11}+690\,bcx^9+437\,b^2x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/6555*(285*c^2*x^11 + 690*b*c*x^9 + 437*b^2*x^7)*sqrt(x)
```

Sympy [A] time = 24.1289, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{15}{2}}}{15}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(15/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

Giac [A] time = 1.12866, size = 32, normalized size = 0.89

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

3.302 $\int x^{3/2} \left(bx^2 + cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{13}b^2x^{13/2} + \frac{4}{17}bcx^{17/2} + \frac{2}{21}c^2x^{21/2}$$

```
[Out] (2*b^2*x^(13/2))/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rubi [A] time = 0.0149835, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{13}b^2x^{13/2} + \frac{4}{17}bcx^{17/2} + \frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(13/2))/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx=\int x^{11/2}\left(b+cx^2\right)^2\,dx$$
$$=\int\left(b^2x^{11/2}+2bcx^{15/2}+c^2x^{19/2}\right)dx$$
$$=\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Mathematica [A] time = 0.0082043, size = 30, normalized size = 0.83

$$\frac{2x^{13/2}\left(357b^2+546bcx^2+221c^2x^4\right)}{4641}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(13/2)*(357*b^2 + 546*b*c*x^2 + 221*c^2*x^4))/4641
```

Maple [A] time = 0.047, size = 27, normalized size = 0.8

$$\frac{442\,c^2x^4+1092\,bcx^2+714\,b^2}{4641}x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/4641*x^(13/2)*(221*c^2*x^4+546*b*c*x^2+357*b^2)
```

Maxima [A] time = 0.980056, size = 32, normalized size = 0.89

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

Fricas [A] time = 1.24746, size = 80, normalized size = 2.22

$$\frac{2}{4641}\left(221\,c^2x^{10}+546\,bcx^8+357\,b^2x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/4641*(221*c^2*x^10 + 546*b*c*x^8 + 357*b^2*x^6)*sqrt(x)
```

Sympy [A] time = 13.1359, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(13/2)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

Giac [A] time = 1.11707, size = 32, normalized size = 0.89

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

3.303 $\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

```
[Out] (2*b^2*x^(11/2))/11 + (4*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rubi [A] time = 0.0139267, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(11/2))/11 + (4*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx &= \int x^{9/2}\left(b+cx^2\right)^2\,dx\\
&= \int \left(b^2x^{9/2}+2bcx^{13/2}+c^2x^{17/2}\right)dx\\
&= \frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}
\end{aligned}
$$

Mathematica [A] time = 0.0079015, size = 30, normalized size = 0.83

$$
\frac{2x^{11/2}\left(285b^2+418bcx^2+165c^2x^4\right)}{3135}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(11/2)*(285*b^2 + 418*b*c*x^2 + 165*c^2*x^4))/3135
```

Maple [A] time = 0.046, size = 27, normalized size = 0.8

$$
\frac{330\,c^2x^4+836\,bcx^2+570\,b^2}{3135}x^{\frac{11}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/3135*x^(11/2)*(165*c^2*x^4+418*b*c*x^2+285*b^2)
```

Maxima [A] time = 0.989824, size = 32, normalized size = 0.89

$$
\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)

Fricas [A] time = 1.24831, size = 78, normalized size = 2.17

$$\frac{2}{3135}\left(165\,c^2x^9+418\,bcx^7+285\,b^2x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")

[Out] 2/3135*(165*c^2*x^9 + 418*b*c*x^7 + 285*b^2*x^5)*sqrt(x)

Sympy [A] time = 3.47159, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{11}{2}}}{11}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)*(c*x**4+b*x**2)**2,x)

[Out] 2*b**2*x**(11/2)/11 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2)/19

Giac [A] time = 1.11144, size = 32, normalized size = 0.89

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")

[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)

3.304 $\int \frac{(bx^2+cx^4)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{9}b^2x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

```
[Out] (2*b^2*x^(9/2))/9 + (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rubi [A] time = 0.0143971, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{9}b^2x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] (2*b^2*x^(9/2))/9 + (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \int x^{7/2}\left(b+cx^2\right)^2\,dx$$
$$= \int \left(b^2x^{7/2}+2bcx^{11/2}+c^2x^{15/2}\right)dx$$
$$= \frac{2}{9}b^2x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Mathematica [A] time = 0.0081585, size = 30, normalized size = 0.83

$$\frac{2x^{9/2}\left(221b^2+306bcx^2+117c^2x^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] (2*x^(9/2)*(221*b^2 + 306*b*c*x^2 + 117*c^2*x^4))/1989
```

Maple [A] time = 0.046, size = 27, normalized size = 0.8

$$\frac{234\,c^2x^4+612\,bcx^2+442\,b^2}{1989}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(1/2),x)
```

```
[Out] 2/1989*x^(9/2)*(117*c^2*x^4+306*b*c*x^2+221*b^2)
```

Maxima [A] time = 1.00119, size = 32, normalized size = 0.89

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

Fricas [A] time = 1.21497, size = 78, normalized size = 2.17

$$\frac{2}{1989}\left(117\,c^2x^8 + 306\,bcx^6 + 221\,b^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*c^2*x^8 + 306*b*c*x^6 + 221*b^2*x^4)*sqrt(x)
```

Sympy [A] time = 5.49748, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{9}{2}}}{9} + \frac{4bcx^{\frac{13}{2}}}{13} + \frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] 2*b**2*x**(9/2)/9 + 4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

Giac [A] time = 1.1709, size = 32, normalized size = 0.89

$$\frac{2}{17}\,c^2x^{\frac{17}{2}} + \frac{4}{13}\,bcx^{\frac{13}{2}} + \frac{2}{9}\,b^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

3.305 $\int \frac{(bx^2+cx^4)^2}{x^{3/2}} dx$

Optimal. Leaf size=36

$$\frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

```
[Out] (2*b^2*x^(7/2))/7 + (4*b*c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rubi [A] time = 0.0157798, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*b^2*x^(7/2))/7 + (4*b*c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^{3/2}}\,dx &= \int x^{5/2}\left(b+cx^2\right)^2\,dx\\ &= \int \left(b^2x^{5/2}+2bcx^{9/2}+c^2x^{13/2}\right)dx\\ &= \frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.0080964, size = 30, normalized size = 0.83

$$\frac{2x^{7/2}\left(165b^2+210bcx^2+77c^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*x^(7/2)*(165*b^2 + 210*b*c*x^2 + 77*c^2*x^4))/1155
```

Maple [A] time = 0.046, size = 27, normalized size = 0.8

$$\frac{154\,c^2x^4+420\,bcx^2+330\,b^2}{1155}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(3/2),x)
```

```
[Out] 2/1155*x^(7/2)*(77*c^2*x^4+210*b*c*x^2+165*b^2)
```

Maxima [A] time = 0.985145, size = 32, normalized size = 0.89

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\,b^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="maxima")
```

[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)

Fricas [A] time = 1.22613, size = 77, normalized size = 2.14

$$\frac{2}{1155}\left(77\,c^2x^7+210\,bcx^5+165\,b^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="fricas")

[Out] 2/1155*(77*c^2*x^7 + 210*b*c*x^5 + 165*b^2*x^3)*sqrt(x)

Sympy [A] time = 6.09704, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2)**2/x**(3/2),x)

[Out] 2*b**2*x**(7/2)/7 + 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15

Giac [A] time = 1.18949, size = 32, normalized size = 0.89

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\,b^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="giac")

[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)

3.306 $\int \frac{(bx^2+cx^4)^2}{x^{5/2}} dx$

Optimal. Leaf size=36

$$\frac{2}{5}b^2x^{5/2} + \frac{4}{9}bcx^{9/2} + \frac{2}{13}c^2x^{13/2}$$

```
[Out] (2*b^2*x^(5/2))/5 + (4*b*c*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rubi [A] time = 0.0150283, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{5}b^2x^{5/2} + \frac{4}{9}bcx^{9/2} + \frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (2*b^2*x^(5/2))/5 + (4*b*c*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx = \int x^{3/2}\left(b+cx^2\right)^2\,dx$$
$$= \int \left(b^2x^{3/2}+2bcx^{7/2}+c^2x^{11/2}\right)dx$$
$$= \frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Mathematica [A] time = 0.0079996, size = 30, normalized size = 0.83

$$\frac{2}{585}x^{5/2}\left(117b^2+130bcx^2+45c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (2*x^(5/2)*(117*b^2 + 130*b*c*x^2 + 45*c^2*x^4))/585
```

Maple [A] time = 0.046, size = 27, normalized size = 0.8

$$\frac{90\,c^2x^4+260\,bcx^2+234\,b^2}{585}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(5/2),x)
```

```
[Out] 2/585*x^(5/2)*(45*c^2*x^4+130*b*c*x^2+117*b^2)
```

Maxima [A] time = 0.989313, size = 32, normalized size = 0.89

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,b^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

Fricas [A] time = 1.26497, size = 76, normalized size = 2.11

$$\frac{2}{585}\left(45\,c^2x^6+130\,bcx^4+117\,b^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*c^2*x^6 + 130*b*c*x^4 + 117*b^2*x^2)*sqrt(x)
```

Sympy [A] time = 7.10723, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4bcx^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(5/2),x)
```

```
[Out] 2*b**2*x**(5/2)/5 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

Giac [A] time = 1.16087, size = 32, normalized size = 0.89

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,b^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

3.307 $\int \frac{(bx^2+cx^4)^2}{x^{7/2}} dx$

Optimal. Leaf size=36

$$\frac{2}{3}b^2x^{3/2} + \frac{4}{7}bcx^{7/2} + \frac{2}{11}c^2x^{11/2}$$

```
[Out] (2*b^2*x^(3/2))/3 + (4*b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rubi [A] time = 0.0162743, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{3}b^2x^{3/2} + \frac{4}{7}bcx^{7/2} + \frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*b^2*x^(3/2))/3 + (4*b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx = \int \sqrt{x}\left(b+cx^2\right)^2\,dx$$
$$= \int \left(b^2\sqrt{x}+2bcx^{5/2}+c^2x^{9/2}\right)\,dx$$
$$= \frac{2}{3}b^2x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

Mathematica [A] time = 0.0079873, size = 30, normalized size = 0.83

$$\frac{2}{231}x^{3/2}\left(77b^2+66bcx^2+21c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*x^(3/2)*(77*b^2 + 66*b*c*x^2 + 21*c^2*x^4))/231
```

Maple [A] time = 0.045, size = 27, normalized size = 0.8

$$\frac{42\,c^2x^4+132\,bcx^2+154\,b^2}{231}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(7/2),x)
```

```
[Out] 2/231*x^(3/2)*(21*c^2*x^4+66*b*c*x^2+77*b^2)
```

Maxima [A] time = 0.990744, size = 32, normalized size = 0.89

$$\frac{2}{11}\,c^2x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\,b^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

Fricas [A] time = 1.24509, size = 70, normalized size = 1.94

$$\frac{2}{231}\left(21\,c^2x^5 + 66\,bcx^3 + 77\,b^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*c^2*x^5 + 66*b*c*x^3 + 77*b^2*x)*sqrt(x)
```

Sympy [A] time = 10.0807, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{3}{2}}}{3} + \frac{4bcx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(7/2),x)
```

```
[Out] 2*b**2*x**(3/2)/3 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

Giac [A] time = 1.14193, size = 32, normalized size = 0.89

$$\frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\,b^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

3.308 $\int x^{7/2} \left(bx^2 + cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{6}{25}b^2cx^{25/2} + \frac{2}{21}b^3x^{21/2} + \frac{6}{29}bc^2x^{29/2} + \frac{2}{33}c^3x^{33/2}$$

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c^
3*x^(33/2))/33
```

Rubi [A] time = 0.0212143, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{25}b^2cx^{25/2} + \frac{2}{21}b^3x^{21/2} + \frac{6}{29}bc^2x^{29/2} + \frac{2}{33}c^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c^
3*x^(33/2))/33
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx=\int x^{19/2}\left(b+cx^2\right)^3\,dx$$
$$=\int\left(b^3x^{19/2}+3b^2cx^{23/2}+3bc^2x^{27/2}+c^3x^{31/2}\right)dx$$
$$=\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

Mathematica [A] time = 0.0138796, size = 51, normalized size = 1.

$$\frac{6}{25}b^2cx^{25/2}+\frac{2}{21}b^3x^{21/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c^
3*x^(33/2))/33
```

Maple [A] time = 0.047, size = 38, normalized size = 0.8

$$\frac{10150\,c^3x^6+34650\,bc^2x^4+40194\,b^2cx^2+15950\,b^3}{167475}x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/167475*x^(21/2)*(5075*c^3*x^6+17325*b*c^2*x^4+20097*b^2*c*x^2+7975*b^3)
```

Maxima [A] time = 0.968687, size = 47, normalized size = 0.92

$$\frac{2}{33}\,c^3x^{\frac{33}{2}}+\frac{6}{29}\,bc^2x^{\frac{29}{2}}+\frac{6}{25}\,b^2cx^{\frac{25}{2}}+\frac{2}{21}\,b^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2/21*b^3*x^
(21/2)
```

Fricas [A] time = 1.26853, size = 119, normalized size = 2.33

$$\frac{2}{167475}\left(5075\,c^3x^{16}+17325\,bc^2x^{14}+20097\,b^2cx^{12}+7975\,b^3x^{10}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/167475*(5075*c^3*x^16 + 17325*b*c^2*x^14 + 20097*b^2*c*x^12 + 7975*b^3*x^
10)*sqrt(x)
```

Sympy [A] time = 112.55, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{21}{2}}}{21}+\frac{6b^2cx^{\frac{25}{2}}}{25}+\frac{6bc^2x^{\frac{29}{2}}}{29}+\frac{2c^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(21/2)/21 + 6*b**2*c*x**(25/2)/25 + 6*b*c**2*x**(29/2)/29 + 2*c**
3*x**(33/2)/33
```

Giac [A] time = 1.15837, size = 47, normalized size = 0.92

$$\frac{2}{33}\,c^3x^{\frac{33}{2}}+\frac{6}{29}\,bc^2x^{\frac{29}{2}}+\frac{6}{25}\,b^2cx^{\frac{25}{2}}+\frac{2}{21}\,b^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2/21*b^3*x^
(21/2)
```

3.309 $\int x^{5/2} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{6}{23}b^2cx^{23/2}+\frac{2}{19}b^3x^{19/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3
*x^(31/2))/31
```

Rubi [A] time = 0.0204168, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{23}b^2cx^{23/2}+\frac{2}{19}b^3x^{19/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3
*x^(31/2))/31
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx &= \int x^{17/2}\left(b+cx^2\right)^3\,dx \\
&= \int\left(b^3x^{17/2}+3b^2cx^{21/2}+3bc^2x^{25/2}+c^3x^{29/2}\right)dx \\
&= \frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}
\end{aligned}
$$

Mathematica [A] time = 0.0126824, size = 51, normalized size = 1.

$$\frac{6}{23}b^2cx^{23/2}+\frac{2}{19}b^3x^{19/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3
*x^(31/2))/31
```

Maple [A] time = 0.047, size = 38, normalized size = 0.8

$$\frac{7866\,c^3x^6+27094\,bc^2x^4+31806\,b^2cx^2+12834\,b^3}{121923}x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/121923*x^(19/2)*(3933*c^3*x^6+13547*b*c^2*x^4+15903*b^2*c*x^2+6417*b^3)
```

Maxima [A] time = 1.06052, size = 47, normalized size = 0.92

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/19*b^3*x^(
19/2)
```

Fricas [A] time = 1.21276, size = 117, normalized size = 2.29

$$\frac{2}{121923}\left(3933\,c^3x^{15}+13547\,bc^2x^{13}+15903\,b^2cx^{11}+6417\,b^3x^9\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/121923*(3933*c^3*x^15 + 13547*b*c^2*x^13 + 15903*b^2*c*x^11 + 6417*b^3*x^
9)*sqrt(x)
```

Sympy [A] time = 70.7733, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{19}{2}}}{19}+\frac{6b^2cx^{\frac{23}{2}}}{23}+\frac{2bc^2x^{\frac{27}{2}}}{9}+\frac{2c^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(19/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/9 + 2*c**3
*x**(31/2)/31
```

Giac [A] time = 1.12223, size = 47, normalized size = 0.92

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/19*b^3*x^(
19/2)
```

3.310 $\int x^{3/2} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{7}b^2cx^{21/2}+\frac{2}{17}b^3x^{17/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3
*x^(29/2))/29
```

Rubi [A] time = 0.019643, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{7}b^2cx^{21/2}+\frac{2}{17}b^3x^{17/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3
*x^(29/2))/29
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^{3/2}\left(bx^2+cx^4\right)^3 dx = \int x^{15/2}\left(b+cx^2\right)^3 dx$$
$$= \int \left(b^3x^{15/2}+3b^2cx^{19/2}+3bc^2x^{23/2}+c^3x^{27/2}\right) dx$$
$$= \frac{2}{17}b^3x^{17/2}+\frac{2}{7}b^2cx^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Mathematica [A] time = 0.0114702, size = 51, normalized size = 1.

$$\frac{2}{7}b^2cx^{21/2}+\frac{2}{17}b^3x^{17/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3
*x^(29/2))/29
```

Maple [A] time = 0.046, size = 38, normalized size = 0.8

$$\frac{5950\,c^3x^6+20706\,bc^2x^4+24650\,b^2cx^2+10150\,b^3}{86275}x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/86275*x^(17/2)*(2975*c^3*x^6+10353*b*c^2*x^4+12325*b^2*c*x^2+5075*b^3)
```

Maxima [A] time = 0.989323, size = 47, normalized size = 0.92

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/17*b^3*x^(
17/2)
```

Fricas [A] time = 1.21739, size = 116, normalized size = 2.27

$$\frac{2}{86275}\left(2975\,c^3x^{14} + 10353\,bc^2x^{12} + 12325\,b^2cx^{10} + 5075\,b^3x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/86275*(2975*c^3*x^14 + 10353*b*c^2*x^12 + 12325*b^2*c*x^10 + 5075*b^3*x^8
)*sqrt(x)
```

Sympy [A] time = 41.7851, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{17}{2}}}{17} + \frac{2b^2cx^{\frac{21}{2}}}{7} + \frac{6bc^2x^{\frac{25}{2}}}{25} + \frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(17/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/25 + 2*c**3
*x**(29/2)/29
```

Giac [A] time = 1.15085, size = 47, normalized size = 0.92

$$\frac{2}{29}\,c^3x^{\frac{29}{2}} + \frac{6}{25}\,bc^2x^{\frac{25}{2}} + \frac{2}{7}\,b^2cx^{\frac{21}{2}} + \frac{2}{17}\,b^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/17*b^3*x^(
17/2)
```

3.311 $\int \sqrt{x} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{6}{19}b^2cx^{19/2}+\frac{2}{15}b^3x^{15/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

```
[Out] (2*b^3*x^(15/2))/15 + (6*b^2*c*x^(19/2))/19 + (6*b*c^2*x^(23/2))/23 + (2*c^
3*x^(27/2))/27
```

Rubi [A] time = 0.0191347, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{19}b^2cx^{19/2}+\frac{2}{15}b^3x^{15/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(15/2))/15 + (6*b^2*c*x^(19/2))/19 + (6*b*c^2*x^(23/2))/23 + (2*c^
3*x^(27/2))/27
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\left(bx^2+cx^4\right)^3\,dx &= \int x^{13/2}\left(b+cx^2\right)^3\,dx \\
&= \int \left(b^3x^{13/2}+3b^2cx^{17/2}+3bc^2x^{21/2}+c^3x^{25/2}\right)dx \\
&= \frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}
\end{aligned}
$$

Mathematica [A] time = 0.0100261, size = 41, normalized size = 0.8

$$\frac{2x^{15/2}\left(9315b^2cx^2+3933b^3+7695bc^2x^4+2185c^3x^6\right)}{58995}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(15/2)*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^6))/58
995
```

Maple [A] time = 0.047, size = 38, normalized size = 0.8

$$\frac{4370\,c^3x^6+15390\,bc^2x^4+18630\,b^2cx^2+7866\,b^3}{58995}x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/58995*x^(15/2)*(2185*c^3*x^6+7695*b*c^2*x^4+9315*b^2*c*x^2+3933*b^3)
```

Maxima [A] time = 0.984155, size = 47, normalized size = 0.92

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\,b^2cx^{\frac{19}{2}}+\frac{2}{15}\,b^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2/15*b^3*x^
(15/2)
```

Fricas [A] time = 1.29486, size = 112, normalized size = 2.2

$$\frac{2}{58995}\left(2185\,c^{3}x^{13} + 7695\,bc^{2}x^{11} + 9315\,b^{2}cx^{9} + 3933\,b^{3}x^{7}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/58995*(2185*c^3*x^13 + 7695*b*c^2*x^11 + 9315*b^2*c*x^9 + 3933*b^3*x^7)*s
qrt(x)
```

Sympy [A] time = 7.64515, size = 49, normalized size = 0.96

$$\frac{2b^{3}x^{\frac{15}{2}}}{15} + \frac{6b^{2}cx^{\frac{19}{2}}}{19} + \frac{6bc^{2}x^{\frac{23}{2}}}{23} + \frac{2c^{3}x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(15/2)/15 + 6*b**2*c*x**(19/2)/19 + 6*b*c**2*x**(23/2)/23 + 2*c**
3*x**(27/2)/27
```

Giac [A] time = 1.13674, size = 47, normalized size = 0.92

$$\frac{2}{27}\,c^{3}x^{\frac{27}{2}} + \frac{6}{23}\,bc^{2}x^{\frac{23}{2}} + \frac{6}{19}\,b^{2}cx^{\frac{19}{2}} + \frac{2}{15}\,b^{3}x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2/15*b^3*x^
(15/2)
```

3.312 $\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=51

$$\frac{6}{17}b^2cx^{17/2}+\frac{2}{13}b^3x^{13/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

```
[Out] (2*b^3*x^(13/2))/13 + (6*b^2*c*x^(17/2))/17 + (2*b*c^2*x^(21/2))/7 + (2*c^3
*x^(25/2))/25
```

Rubi [A] time = 0.0202165, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{17}b^2cx^{17/2}+\frac{2}{13}b^3x^{13/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*b^3*x^(13/2))/13 + (6*b^2*c*x^(17/2))/17 + (2*b*c^2*x^(21/2))/7 + (2*c^3
*x^(25/2))/25
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \int x^{11/2}\left(b+cx^2\right)^3\,dx$$
$$= \int \left(b^3x^{11/2}+3b^2cx^{15/2}+3bc^2x^{19/2}+c^3x^{23/2}\right)dx$$
$$= \frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Mathematica [A] time = 0.0112586, size = 41, normalized size = 0.8

$$\frac{2x^{13/2}\left(6825b^2cx^2+2975b^3+5525bc^2x^4+1547c^3x^6\right)}{38675}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*x^(13/2)*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^6))/38
675
```

Maple [A] time = 0.047, size = 38, normalized size = 0.8

$$\frac{3094\,c^3x^6+11050\,bc^2x^4+13650\,b^2cx^2+5950\,b^3}{38675}x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] 2/38675*x^(13/2)*(1547*c^3*x^6+5525*b*c^2*x^4+6825*b^2*c*x^2+2975*b^3)
```

Maxima [A] time = 1.00679, size = 47, normalized size = 0.92

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2)
```

Fricas [A] time = 1.25456, size = 112, normalized size = 2.2

$$\frac{2}{38675}\left(1547\,c^3x^{12}+5525\,bc^2x^{10}+6825\,b^2cx^8+2975\,b^3x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/38675*(1547*c^3*x^12 + 5525*b*c^2*x^10 + 6825*b^2*c*x^8 + 2975*b^3*x^6)*s
qrt(x)
```

Sympy [A] time = 18.7054, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{17}{2}}}{17}+\frac{2bc^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] 2*b**3*x**(13/2)/13 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/7 + 2*c**3
*x**(25/2)/25
```

Giac [A] time = 1.16078, size = 47, normalized size = 0.92

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2)
```

3.313 $\int \frac{\left(bx^2+cx^4\right)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{5}b^2cx^{15/2}+\frac{2}{11}b^3x^{11/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

```
[Out] (2*b^3*x^(11/2))/11 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2*x^(19/2))/19 + (2*c^3
*x^(23/2))/23
```

Rubi [A] time = 0.0200503, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{2}{5}b^2cx^{15/2}+\frac{2}{11}b^3x^{11/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*b^3*x^(11/2))/11 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2*x^(19/2))/19 + (2*c^3
*x^(23/2))/23
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx = \int x^{9/2}\left(b+cx^2\right)^3\,dx$$
$$= \int \left(b^3x^{9/2}+3b^2cx^{13/2}+3bc^2x^{17/2}+c^3x^{21/2}\right)dx$$
$$= \frac{2}{11}b^3x^{11/2}+\frac{2}{5}b^2cx^{15/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Mathematica [A] time = 0.0117037, size = 41, normalized size = 0.8

$$\frac{2x^{11/2}\left(4807b^2cx^2+2185b^3+3795bc^2x^4+1045c^3x^6\right)}{24035}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*x^(11/2)*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x^6))/24
035
```

Maple [A] time = 0.046, size = 38, normalized size = 0.8

$$\frac{2090\,c^3x^6+7590\,bc^2x^4+9614\,b^2cx^2+4370\,b^3}{24035}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(3/2),x)
```

```
[Out] 2/24035*x^(11/2)*(1045*c^3*x^6+3795*b*c^2*x^4+4807*b^2*c*x^2+2185*b^3)
```

Maxima [A] time = 0.991777, size = 47, normalized size = 0.92

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\,b^2cx^{\frac{15}{2}}+\frac{2}{11}\,b^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/11*b^3*x^(
11/2)
```

Fricas [A] time = 1.21423, size = 111, normalized size = 2.18

$$\frac{2}{24035}\left(1045\,c^3x^{11}+3795\,bc^2x^9+4807\,b^2cx^7+2185\,b^3x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/24035*(1045*c^3*x^11 + 3795*b*c^2*x^9 + 4807*b^2*c*x^7 + 2185*b^3*x^5)*sq
rt(x)
```

Sympy [A] time = 20.3286, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{2b^2cx^{\frac{15}{2}}}{5}+\frac{6bc^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(3/2),x)
```

```
[Out] 2*b**3*x**(11/2)/11 + 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19 + 2*c**3
*x**(23/2)/23
```

Giac [A] time = 1.1151, size = 47, normalized size = 0.92

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\,b^2cx^{\frac{15}{2}}+\frac{2}{11}\,b^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/11*b^3*x^(
11/2)
```

3.314 $\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=51

$$\frac{6}{13}b^2cx^{13/2}+\frac{2}{9}b^3x^{9/2}+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

```
[Out] (2*b^3*x^(9/2))/9 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*x^(17/2))/17 + (2*c^3*
x^(21/2))/21
```

Rubi [A] time = 0.0189882, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{13}b^2cx^{13/2}+\frac{2}{9}b^3x^{9/2}+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*b^3*x^(9/2))/9 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*x^(17/2))/17 + (2*c^3*
x^(21/2))/21
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx = \int x^{7/2}\left(b+cx^2\right)^3\,dx$$
$$= \int \left(b^3x^{7/2}+3b^2cx^{11/2}+3bc^2x^{15/2}+c^3x^{19/2}\right)dx$$
$$= \frac{2}{9}b^3x^{9/2}+\frac{6}{13}b^2cx^{13/2}+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

Mathematica [A] time = 0.0106717, size = 41, normalized size = 0.8

$$\frac{2x^{9/2}\left(3213b^2cx^2+1547b^3+2457bc^2x^4+663c^3x^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*x^(9/2)*(1547*b^3 + 3213*b^2*c*x^2 + 2457*b*c^2*x^4 + 663*c^3*x^6))/1392
3
```

Maple [A] time = 0.048, size = 38, normalized size = 0.8

$$\frac{1326\,c^3x^6+4914\,bc^2x^4+6426\,b^2cx^2+3094\,b^3}{13923}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(5/2),x)
```

```
[Out] 2/13923*x^(9/2)*(663*c^3*x^6+2457*b*c^2*x^4+3213*b^2*c*x^2+1547*b^3)
```

Maxima [A] time = 1.01459, size = 47, normalized size = 0.92

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2/9*b^3*x^(
9/2)
```

Fricas [A] time = 1.27837, size = 109, normalized size = 2.14

$$\frac{2}{13923}\left(663\,c^3x^{10} + 2457\,bc^2x^8 + 3213\,b^2cx^6 + 1547\,b^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*c^3*x^10 + 2457*b*c^2*x^8 + 3213*b^2*c*x^6 + 1547*b^3*x^4)*sqr
t(x)
```

Sympy [A] time = 22.7064, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{9}{2}}}{9} + \frac{6b^2cx^{\frac{13}{2}}}{13} + \frac{6bc^2x^{\frac{17}{2}}}{17} + \frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(5/2),x)
```

```
[Out] 2*b**3*x**(9/2)/9 + 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17 + 2*c**3*
x**(21/2)/21
```

Giac [A] time = 1.13714, size = 47, normalized size = 0.92

$$\frac{2}{21}\,c^3x^{\frac{21}{2}} + \frac{6}{17}\,bc^2x^{\frac{17}{2}} + \frac{6}{13}\,b^2cx^{\frac{13}{2}} + \frac{2}{9}\,b^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2/9*b^3*x^(
9/2)
```

3.315 $\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=51

$$\frac{6}{11}b^2cx^{11/2}+\frac{2}{7}b^3x^{7/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

```
[Out] (2*b^3*x^(7/2))/7 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*x^(15/2))/5 + (2*c^3*x
^(19/2))/19
```

Rubi [A] time = 0.0222626, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1584, 270}

$$\frac{6}{11}b^2cx^{11/2}+\frac{2}{7}b^3x^{7/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (2*b^3*x^(7/2))/7 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*x^(15/2))/5 + (2*c^3*x
^(19/2))/19
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx = \int x^{5/2}\left(b+cx^2\right)^3\,dx$$
$$= \int \left(b^3x^{5/2}+3b^2cx^{9/2}+3bc^2x^{13/2}+c^3x^{17/2}\right)dx$$
$$= \frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Mathematica [A] time = 0.0097158, size = 41, normalized size = 0.8

$$\frac{2x^{7/2}\left(1995b^2cx^2+1045b^3+1463bc^2x^4+385c^3x^6\right)}{7315}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (2*x^(7/2)*(1045*b^3 + 1995*b^2*c*x^2 + 1463*b*c^2*x^4 + 385*c^3*x^6))/7315
```

Maple [A] time = 0.046, size = 38, normalized size = 0.8

$$\frac{770\,c^3x^6+2926\,bc^2x^4+3990\,b^2cx^2+2090\,b^3}{7315}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(7/2),x)
```

```
[Out] 2/7315*x^(7/2)*(385*c^3*x^6+1463*b*c^2*x^4+1995*b^2*c*x^2+1045*b^3)
```

Maxima [A] time = 0.96881, size = 47, normalized size = 0.92

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,b^2cx^{\frac{11}{2}}+\frac{2}{7}\,b^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/7*b^3*x^(7
/2)
```

Fricas [A] time = 1.21656, size = 107, normalized size = 2.1

$$\frac{2}{7315}\left(385\,c^{3}x^{9}+1463\,bc^{2}x^{7}+1995\,b^{2}cx^{5}+1045\,b^{3}x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*c^3*x^9 + 1463*b*c^2*x^7 + 1995*b^2*c*x^5 + 1045*b^3*x^3)*sqrt(
x)
```

Sympy [A] time = 28.3509, size = 49, normalized size = 0.96

$$\frac{2b^{3}x^{\frac{7}{2}}}{7}+\frac{6b^{2}cx^{\frac{11}{2}}}{11}+\frac{2bc^{2}x^{\frac{15}{2}}}{5}+\frac{2c^{3}x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(7/2),x)
```

```
[Out] 2*b**3*x**(7/2)/7 + 6*b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 + 2*c**3*x
**(19/2)/19
```

Giac [A] time = 1.115, size = 47, normalized size = 0.92

$$\frac{2}{19}\,c^{3}x^{\frac{19}{2}}+\frac{2}{5}\,bc^{2}x^{\frac{15}{2}}+\frac{6}{11}\,b^{2}cx^{\frac{11}{2}}+\frac{2}{7}\,b^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="giac")
```



```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/7*b^3*x^(7
/2)
```

3.316 $\int \frac{x^{13/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=217

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{\sqrt{2}c^{11/4}}$$

```
[Out] (-2*b*x^(3/2))/(3*c^2) + (2*x^(7/2))/(7*c) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)*Log[Sqrt[b] - Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(7/4)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(1
1/4))
```

Rubi [A] time = 0.23226, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{\sqrt{2}c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*b*x^(3/2))/(3*c^2) + (2*x^(7/2))/(7*c) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)*Log[Sqrt[b] - Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(7/4)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(1
1/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{13/2}}{bx^2+cx^4}\,dx = \int \frac{x^{9/2}}{b+cx^2}\,dx$$
$$= \frac{2x^{7/2}}{7c} - \frac{b\int \frac{x^{5/2}}{b+cx^2}\,dx}{c}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^2\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^2}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} - \frac{b^2\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^3} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^3} + \frac{b^{7/4}\,\mathrm{S}}{}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} - \frac{b^{7/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} + \frac{b^{7/4}\,\mathrm{Su}}{}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} - \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} + \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} + \frac{b^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{2\sqrt{2}c^{11/4}}$$

Mathematica [A] time = 0.0448197, size = 89, normalized size = 0.41

$$\frac{2c^{3/4}x^{3/2}\left(3cx^2-7b\right)+21(-b)^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)+21b(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{21c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*c^(3/4)*x^(3/2)*(-7*b + 3*c*x^2) + 21*(-b)^(7/4)*ArcTan[(c^(1/4)*Sqrt[x]
)/(-b)^(1/4)] + 21*(-b)^(3/4)*b*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/(21*
c^(11/4))
```

Maple [A] time = 0.051, size = 158, normalized size = 0.7

$$\frac{2}{7c}x^{\frac{7}{2}} - \frac{2b}{3c^2}x^{\frac{3}{2}} + \frac{b^2\sqrt{2}}{4c^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}} + \frac{b^2\sqrt{2}}{2c^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/7*x^(7/2)/c-2/3*b*x^(3/2)/c^2+1/4*b^2/c^3/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)
^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1
/2)))+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34972, size = 429, normalized size = 1.98

$$-\frac{84\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^5c^3\sqrt{x}\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}-\sqrt{-b^7c^5\sqrt{-\frac{b^7}{c^{11}}}+b^{10}xc^3}\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}}{b^7}\right)-21\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\log\left(c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}}+b^5\sqrt{x}\right)+21\,c^2\left(-\frac{b^7}{c^{11}}\right.}{42\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/42*(84*c^2*(-b^7/c^11)^(1/4)*arctan(-(b^5*c^3*sqrt(x)*(-b^7/c^11)^(1/4)
- sqrt(-b^7*c^5*sqrt(-b^7/c^11) + b^10*x)*c^3*(-b^7/c^11)^(1/4))/b^7) - 21*
c^2*(-b^7/c^11)^(1/4)*log(c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) + 21*c^2*(-b
^7/c^11)^(1/4)*log(-c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) - 4*(3*c*x^3 - 7*b
*x)*sqrt(x))/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18599, size = 266, normalized size = 1.23

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}} b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^5}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}} b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^5}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}} b \log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{}\right.}{4c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/4*sqrt(2)*(b*c^3)^(3/4
)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*(b*c
^3)^(3/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 2/21*(3
*c^6*x^(7/2) - 7*b*c^5*x^(3/2))/c^7
```

3.317 $\int \frac{x^{11/2}}{bx^2+cx^4} dx$

Optimal. Leaf size=215

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(\right.}{\sqrt{2}c}$$

```
[Out] (-2*b*Sqrt[x])/c^2 + (2*x^(5/2))/(5*c) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) - (b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4)) + (b^(5/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rubi [A] time = 0.193761, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(\right.}{\sqrt{2}c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4),x]
```


```
[Out] (-2*b*Sqrt[x])/c^2 + (2*x^(5/2))/(5*c) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) - (b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4)) + (b^(5/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11/2}}{bx^2+cx^4}\,dx = \int \frac{x^{7/2}}{b+cx^2}\,dx$$
$$= \frac{2x^{5/2}}{5c} - \frac{b\int \frac{x^{3/2}}{b+cx^2}\,dx}{c}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^2\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{c^2}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2} + \frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}} + \frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}} - \ldots$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} - \frac{b^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} + \frac{b^{5/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} + \frac{b^{5/4}\,\mathrm{S}\ldots}{\ldots}$$
$$= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} - \frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}} + \frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}} - \frac{b^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\ldots\right)}{2\sqrt{2}c^{9/4}}$$

Mathematica [A] time = 0.0662873, size = 203, normalized size = 0.94

$$\frac{-5\sqrt{2}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+5\sqrt{2}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-10\sqrt{2}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+\ldots}{20c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-40*b*c^(1/4)*Sqrt[x] + 8*c^(5/4)*x^(5/2) - 10*Sqrt[2]*b^(5/4)*ArcTan[1 -
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 10*Sqrt[2]*b^(5/4)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)] - 5*Sqrt[2]*b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 5*Sqrt[2]*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(20*c^(9/4))
```

Maple [A] time = 0.051, size = 152, normalized size = 0.7

$$\frac{2}{5c}x^{\frac{5}{2}}-2\frac{b\sqrt{x}}{c^2}+\frac{b\sqrt{2}}{4c^2}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{b\sqrt{2}}{2c^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/5*x^(5/2)/c-2*b*x^(1/2)/c^2+1/4*b/c^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/
4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))
)+1/2*b/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*b
/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35495, size = 393, normalized size = 1.83

$$\frac{20\,c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc^{7}\sqrt{x}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{3}{4}}-\sqrt{c^{4}\sqrt{-\frac{b^{5}}{c^{9}}}+b^{2}x}\,c^{7}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{3}{4}}}{b^{5}}\right)+5\,c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}\log\left(c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}+b\sqrt{x}\right)-5\,c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}\log\left(-c^{2}\left(-\right.\right.}{10\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*c^2*(-b^5/c^9)^(1/4)*arctan(-(b*c^7*sqrt(x)*(-b^5/c^9)^(3/4) - sqr
t(c^4*sqrt(-b^5/c^9) + b^2*x)*c^7*(-b^5/c^9)^(3/4))/b^5) + 5*c^2*(-b^5/c^9)
^(1/4)*log(c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) - 5*c^2*(-b^5/c^9)^(1/4)*log(-
c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) + 4*(c*x^2 - 5*b)*sqrt(x))/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15436, size = 265, normalized size = 1.23

$$\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^{3}}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^{3}}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\right.}{4\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/4*sqrt(2)*(b*c^3)^(1/4
)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*(b*c
^3)^(1/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 2/5*(c^
4*x^(5/2) - 5*b*c^3*sqrt(x))/c^5
```

3.318 $\int \frac{x^{9/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=204

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(\cdots\right.}{\sqrt{2}c}$$

```
[Out] (2*x^(3/2))/(3*c) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(Sqrt[2]*c^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(Sqrt[2]*c^(7/4)) - (b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4)) + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4))
```

Rubi [A] time = 0.185877, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(\cdots\right.}{\sqrt{2}c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4),x]
```



```
[Out] (2*x^(3/2))/(3*c) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(Sqrt[2]*c^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(Sqrt[2]*c^(7/4)) - (b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4)) + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx = \int \frac{x^{5/2}}{b+cx^2}\,dx$$
$$= \frac{2x^{3/2}}{3c} - \frac{b\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c}$$
$$= \frac{2x^{3/2}}{3c} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$
$$= \frac{2x^{3/2}}{3c} + \frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}} - \frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}}$$
$$= \frac{2x^{3/2}}{3c} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2} - \frac{b^{3/4}\,\mathrm{Subst}\left(\int \cdots\right.}{2\cdots}$$
$$= \frac{2x^{3/2}}{3c} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}} + \frac{b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}} - \frac{b^{3/4}\,\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$
$$= \frac{2x^{3/2}}{3c} + \frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{7/4}} - \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{7/4}} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}} + \cdots$$

Mathematica [A] time = 0.0208927, size = 78, normalized size = 0.38

$$\frac{(-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{7/4}} - \frac{(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)}{c^{7/4}} + \frac{2x^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) + ((-b)^(3/4)*ArcTan[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/c^(7/
4) - ((-b)^(3/4)*ArcTanh[(c^(1/4)*Sqrt[x])/(-b)^(1/4)])/c^(7/4)
```

Maple [A] time = 0.051, size = 143, normalized size = 0.7

$$\frac{2}{3c}x^{\frac{3}{2}} - \frac{b\sqrt{2}}{4c^2}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}} - \frac{b\sqrt{2}}{2c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}} - \frac{b\sqrt{2}}{2c^2}\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/3*x^(3/2)/c-1/4*b/c^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/
2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2*b/c^2/(b/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*b/c^2/(b/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.28749, size = 377, normalized size = 1.85

$$\frac{12\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^2c^2\sqrt{x}\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}-\sqrt{-b^3c^3\sqrt{-\frac{b^3}{c^7}}+b^4xc^2}\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}}{b^3}\right)-3\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log\left(c^5\left(-\frac{b^3}{c^7}\right)^{\frac{3}{4}}+b^2\sqrt{x}\right)+3\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log\left(-c^5\left(-\right.\right.}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(12*c*(-b^3/c^7)^(1/4)*arctan(-(b^2*c^2*sqrt(x)*(-b^3/c^7)^(1/4) - sqrt
(-b^3*c^3*sqrt(-b^3/c^7) + b^4*x)*c^2*(-b^3/c^7)^(1/4))/b^3) - 3*c*(-b^3/c^
7)^(1/4)*log(c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) + 3*c*(-b^3/c^7)^(1/4)*log
(-c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) + 4*x^(3/2))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14407, size = 240, normalized size = 1.18

$$\frac{2\,x^{\frac{3}{2}}}{3\,c} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^{4}} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^{4}} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x \cdots\right.}{4\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/c - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)
^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/4*sqrt(2)*
(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/4*sq
rt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4
```

3.319 $\int \frac{x^{7/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}c^{5/4}}$$

```
[Out] (2*Sqrt[x])/c + (b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sq
rt[2]*c^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(S
qrt[2]*c^(5/4)) + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4)) - (b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4))
```

Rubi [A] time = 0.184984, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + (b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sq
rt[2]*c^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(S
qrt[2]*c^(5/4)) + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4)) - (b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx = \int \frac{x^{3/2}}{b+cx^2}\,dx$$
$$= \frac{2\sqrt{x}}{c} - \frac{b\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{c}$$
$$= \frac{2\sqrt{x}}{c} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$
$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$
$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/2}} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/2}} + \frac{\sqrt[4]{b}\,\mathrm{Subst}\left(\int -\right.}{2}$$
$$= \frac{2\sqrt{x}}{c} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}} - \frac{\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}} - \frac{\sqrt[4]{b}\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\right.}{\sqrt{2}c}$$
$$= \frac{2\sqrt{x}}{c} + \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{5/4}} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{5/4}} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}} - \frac{\sqrt[4]{b}}{}$$

Mathematica [A] time = 0.0354507, size = 189, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+2\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\sqrt{2}\sqrt[4]{b}\,t}{4c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (8*c^(1/4)*Sqrt[x] + 2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)] - 2*Sqrt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
 + Sqrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x] - Sqrt[2]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(4*c^(5/4))
```

Maple [A] time = 0.048, size = 140, normalized size = 0.7

$$2\,\frac{\sqrt{x}}{c}-\frac{\sqrt{2}}{4\,c}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)-\frac{\sqrt{2}}{2\,c}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{\sqrt{2}}{2\,c}\sqrt[4]{\frac{b}{c}}\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2*x^(1/2)/c-1/4/c*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/
c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2/c*(b/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/c*(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32815, size = 312, normalized size = 1.54

$$-\frac{4\,c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^2\sqrt{-\frac{b}{c^5}}+x}\,c^4\left(-\frac{b}{c^5}\right)^{\frac{3}{4}}-c^4\sqrt{x}\left(-\frac{b}{c^5}\right)^{\frac{3}{4}}}{b}\right)+c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(-c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*c*(-b/c^5)^(1/4)*arctan((sqrt(c^2*sqrt(-b/c^5) + x)*c^4*(-b/c^5)^(3
/4) - c^4*sqrt(x)*(-b/c^5)^(3/4))/b) + c*(-b/c^5)^(1/4)*log(c*(-b/c^5)^(1/4
) + sqrt(x)) - c*(-b/c^5)^(1/4)*log(-c*(-b/c^5)^(1/4) + sqrt(x)) - 4*sqrt(x
))/c
```

Sympy [A] time = 130.819, size = 177, normalized size = 0.88

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } b = 0 \wedge c = 0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } c = 0 \\ \frac{2\sqrt{x}}{c} & \text{for } b = 0 \\ \frac{\sqrt[4]{-1}\sqrt[4]{b}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c^8\left(\frac{1}{c}\right)^{\frac{27}{4}}} - \frac{\sqrt[4]{-1}\sqrt[4]{b}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2c^8\left(\frac{1}{c}\right)^{\frac{27}{4}}} + \frac{\sqrt[4]{-1}\sqrt[4]{b}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{c^8\left(\frac{1}{c}\right)^{\frac{27}{4}}} + \frac{2\sqrt{x}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(b, 0) & Eq(c, 0)), (2*x**(5/2)/(5*b), Eq(c, 0)),
 (2*sqrt(x)/c, Eq(b, 0)), ((-1)**(1/4)*b**(1/4)*log(-(-1)**(1/4)*b**(1/4)*(
1/c)**(1/4) + sqrt(x))/(2*c**8*(1/c)**(27/4)) - (-1)**(1/4)*b**(1/4)*log((-
1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*c**8*(1/c)**(27/4)) + (-1)**(
1/4)*b**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(c**8*(1/c)
**(27/4)) + 2*sqrt(x)/c, True))
```

Giac [A] time = 1.15255, size = 240, normalized size = 1.19

$$-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^2} - \frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^2} - \frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/c^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^2 - 1/4*sqrt(2)*(b*c^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^2 + 1/4*sqrt(2)*(b*c^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^2 + 2*sqrt(x)/c
```

3.320 $\int \frac{x^{5/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(3/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(3/4)) +
 Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1
/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(
2*Sqrt[2]*b^(1/4)*c^(3/4))
```

Rubi [A] time = 0.142039, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1584, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4),x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(3/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(3/4)) +
 Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1
/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(
2*Sqrt[2]*b^(1/4)*c^(3/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx = \int \frac{\sqrt{x}}{b+cx^2}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x,\right.}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}$$
$$= \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}$$

Mathematica [A] time = 0.0250279, size = 54, normalized size = 0.28

$$\frac{b\left(\tan^{-1}\left(\frac{b\sqrt[4]{c}\sqrt{x}}{(-b)^{5/4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b}}\right)\right)}{(-b)^{5/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(ArcTan[(b*c^(1/4)*Sqrt[x])/(-b)^(5/4)] + ArcTanh[(c^(1/4)*Sqrt[x])/(-b)
^(1/4)]))/((-b)^(5/4)*c^(3/4))
```

Maple [A] time = 0.049, size = 132, normalized size = 0.7

$$\frac{\sqrt{2}}{4\,c}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}}{2\,c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}}{2\,c}\arctan\left(\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4/c/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x
+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2/c/(b/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33029, size = 332, normalized size = 1.73

$$-2\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc\sqrt{-\frac{1}{bc^3}}+x}\,c\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}-c\sqrt{x}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -2*(-1/(b*c^3))^(1/4)*arctan(sqrt(-b*c*sqrt(-1/(b*c^3)) + x)*c*(-1/(b*c^3))
^(1/4) - c*sqrt(x)*(-1/(b*c^3))^(1/4)) + 1/2*(-1/(b*c^3))^(1/4)*log(b*c^2*(
-1/(b*c^3))^(3/4) + sqrt(x)) - 1/2*(-1/(b*c^3))^(1/4)*log(-b*c^2*(-1/(b*c^3
))^(3/4) + sqrt(x))
```

Sympy [A] time = 72.7057, size = 170, normalized size = 0.89

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } b = 0 \wedge c = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } c = 0 \\ -\frac{2}{c\sqrt{x}} & \text{for } b = 0 \\ -\frac{(-1)^{\frac{3}{4}} \log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2\sqrt[4]{b}c^2\left(\frac{1}{c}\right)^{\frac{5}{4}}} + \frac{(-1)^{\frac{3}{4}} \log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2\sqrt[4]{b}c^2\left(\frac{1}{c}\right)^{\frac{5}{4}}} + \frac{(-1)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{\sqrt[4]{b}c^2\left(\frac{1}{c}\right)^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(b, 0) & Eq(c, 0)), (2*x**(3/2)/(3*b), Eq(c, 0)),
 (-2/(c*sqrt(x)), Eq(b, 0)), (-(-1)**(3/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)*
*(1/4) + sqrt(x))/(2*b**(1/4)*c**2*(1/c)**(5/4)) + (-1)**(3/4)*log((-1)**(1
/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(1/4)*c**2*(1/c)**(5/4)) + (-1)*
*(3/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(1/4)*c**2*(1/
c)**(5/4)), True))
```

Giac [A] time = 1.13058, size = 246, normalized size = 1.28

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^3} + \frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^3} - \frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c^3) + 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/4*sqrt(2)*(b*c^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) + 1/4*sqrt(2
)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3)
```

3.321 $\int \frac{x^{3/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=192

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*c^(1/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*c^(1/4)) -
 Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(3
/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(
2*Sqrt[2]*b^(3/4)*c^(1/4))
```

Rubi [A] time = 0.138691, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1584, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4),x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*c^(1/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*c^(1/4)) -
 Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(3
/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(
2*Sqrt[2]*b^(3/4)*c^(1/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx = \int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\sqrt{c}} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x\right.}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}$$
$$= -\frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{3/4}\sqrt[4]{c}}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{2\sqrt{2}b^{3/4}}$$

Mathematica [A] time = 0.0373756, size = 146, normalized size = 0.76

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)] - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x] + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2
*Sqrt[2]*b^(3/4)*c^(1/4))
```

Maple [A] time = 0.05, size = 132, normalized size = 0.7

$$\frac{\sqrt{2}}{4b}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{\sqrt{2}}{2b}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{\sqrt{2}}{2b}\sqrt[4]{\frac{b}{c}}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*(b/c)^(1/4)/b*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x
-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2*(b/c)^(1/4)/b*2^(1/2)*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34049, size = 329, normalized size = 1.71

$$2\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{-\frac{1}{b^3c}}+x}b^2c\left(-\frac{1}{b^3c}\right)^{\frac{3}{4}}-b^2c\sqrt{x}\left(-\frac{1}{b^3c}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(b^3*c))^(1/4)*arctan(sqrt(b^2*sqrt(-1/(b^3*c)) + x)*b^2*c*(-1/(b^3*c
))^(3/4) - b^2*c*sqrt(x)*(-1/(b^3*c))^(3/4)) + 1/2*(-1/(b^3*c))^(1/4)*log(b
*(-1/(b^3*c))^(1/4) + sqrt(x)) - 1/2*(-1/(b^3*c))^(1/4)*log(-b*(-1/(b^3*c))
^(1/4) + sqrt(x))
```

Sympy [A] time = 35.3051, size = 170, normalized size = 0.89

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{3cx^{\frac{3}{2}}} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } c = 0 \\ -\frac{\sqrt[4]{-1}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{3}{4}}c^{24}\left(\frac{1}{c}\right)^{\frac{95}{4}}} + \frac{\sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{3}{4}}c^{24}\left(\frac{1}{c}\right)^{\frac{95}{4}}} - \frac{\sqrt[4]{-1}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{3}{4}}c^{24}\left(\frac{1}{c}\right)^{\frac{95}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(b, 0) & Eq(c, 0)), (-2/(3*c*x**(3/2)), Eq(b, 0)
), (2*sqrt(x)/b, Eq(c, 0)), (-(-1)**(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**
(1/4) + sqrt(x))/(2*b**(3/4)*c**24*(1/c)**(95/4)) + (-1)**(1/4)*log((-1)**(
1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(3/4)*c**24*(1/c)**(95/4)) - (-
1)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(3/4)*c**24
*(1/c)**(95/4)), True))
```

Giac [A] time = 1.17153, size = 246, normalized size = 1.28

$$\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c) + 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c) + 1/4*sqrt(2)*(b*c^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c) - 1/4*sqrt(2)*(b*c
^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c)
```

3.322 $\int \frac{\sqrt{x}}{bx^2+cx^4} dx$

Optimal. Leaf size=202

$$-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{5/4}}$$

```
[Out] -2/(b*Sqrt[x]) + (c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(S
qrt[2]*b^(5/4)) - (c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(
Sqrt[2]*b^(5/4)) - (c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)) + (c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4))
```

Rubi [A] time = 0.164215, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4),x]
```


```
[Out] -2/(b*Sqrt[x]) + (c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(S
qrt[2]*b^(5/4)) - (c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(
Sqrt[2]*b^(5/4)) - (c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)) + (c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx = \int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2}{b\sqrt{x}} - \frac{c\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b}$$

$$= -\frac{2}{b\sqrt{x}} - \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= -\frac{2}{b\sqrt{x}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= -\frac{2}{b\sqrt{x}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} - \frac{\sqrt[4]{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}\cdots}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\cdots}{\cdots}}\cdots\right)}{2\sqrt{2}b^{5\cdots}}$$

$$= -\frac{2}{b\sqrt{x}} - \frac{\sqrt[4]{c}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}} + \frac{\sqrt[4]{c}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{c}\,\text{Subst}\left(\int \frac{1}{-1-x^{\cdots}}\cdots\right)}{\sqrt{2}\cdots}$$

$$= -\frac{2}{b\sqrt{x}} + \frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{c}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}} + \frac{\sqrt[4]{c}\cdots}{\cdots}$$

Mathematica [C] time = 0.0058459, size = 27, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{cx^2}{b}\right)}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4),x]
```

[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -((c*x^2)/b)])/(b*Sqrt[x])

Maple [A] time = 0.051, size = 140, normalized size = 0.7

$$-\frac{\sqrt{2}}{4b}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{\sqrt{2}}{2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{\sqrt{2}}{2b}\arctan\left(\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(c*x^4+b*x^2),x)

[Out] -1/4/b/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(
x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2/b/(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/b/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
b/c)^(1/4)*x^(1/2)-1)-2/b/x^(1/2)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.47478, size = 343, normalized size = 1.7

$$\frac{4\,bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc\sqrt{x}\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}-\sqrt{-b^3c\sqrt{-\frac{c}{b^5}}+c^2xb}\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}}{c}\right)-bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}+c\sqrt{x}\right)+bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(-b^4\left(-\frac{c}{b^5}\right)\right.}{2\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")

```
[Out] 1/2*(4*b*x*(-c/b^5)^(1/4)*arctan(-(b*c*sqrt(x)*(-c/b^5)^(1/4) - sqrt(-b^3*c
*sqrt(-c/b^5) + c^2*x)*b*(-c/b^5)^(1/4))/c) - b*x*(-c/b^5)^(1/4)*log(b^4*(-
c/b^5)^(3/4) + c*sqrt(x)) + b*x*(-c/b^5)^(1/4)*log(-b^4*(-c/b^5)^(3/4) + c*
sqrt(x)) - 4*sqrt(x))/(b*x)
```

Sympy [A] time = 24.3453, size = 180, normalized size = 0.89

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{5cx^{\frac{5}{2}}} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } c = 0 \\ -\frac{2}{b\sqrt{x}} + \frac{(-1)^{\frac{3}{4}} \log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{5}{4}}c^{6}\left(\frac{1}{c}\right)^{\frac{25}{4}}} - \frac{(-1)^{\frac{3}{4}} \log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}} + \sqrt{x}\right)}{2b^{\frac{5}{4}}c^{6}\left(\frac{1}{c}\right)^{\frac{25}{4}}} - \frac{(-1)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{5}{4}}c^{6}\left(\frac{1}{c}\right)^{\frac{25}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(b, 0) & Eq(c, 0)), (-2/(5*c*x**(5/2)), Eq(b, 0)
), (-2/(b*sqrt(x)), Eq(c, 0)), (-2/(b*sqrt(x)) + (-1)**(3/4)*log(-(-1)**(1/
4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)*c**6*(1/c)**(25/4)) - (-1)*
*(3/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/4)*c**6*(1
/c)**(25/4)) - (-1)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4))
)/(b**(5/4)*c**6*(1/c)**(25/4)), True))
```

Giac [A] time = 1.19183, size = 257, normalized size = 1.27

$$-\frac{2}{b\sqrt{x}} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{2}c^{2}} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{2}c^{2}} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \cdots\right.}{4\,b^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -2/(b*sqrt(x)) - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) - 1/2*sqrt(2)*(b*c^3)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) +
 1/4*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b^2*c^2) - 1/4*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/(b^2*c^2)
```

3.323 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=204

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\cdot}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{7/4}}$$

```
[Out] -2/(3*b*x^(3/2)) + (c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/
(Sqrt[2]*b^(7/4)) - (c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(Sqrt[2]*b^(7/4)) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)) - (c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4))
```

Rubi [A] time = 0.167133, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\cdot}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```


```
[Out] -2/(3*b*x^(3/2)) + (c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/
(Sqrt[2]*b^(7/4)) - (c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(Sqrt[2]*b^(7/4)) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)) - (c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2}{3bx^{3/2}} - \frac{c\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{b}$$

$$= -\frac{2}{3bx^{3/2}} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= -\frac{2}{3bx^{3/2}} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}}$$

$$= -\frac{2}{3bx^{3/2}} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}} + \frac{c^{3/4}\,\text{Su}}{}$$

$$= -\frac{2}{3bx^{3/2}} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}} - \frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}} - \frac{c^{3/4}\,\text{Subst}}{}$$

$$= -\frac{2}{3bx^{3/2}} + \frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}} - \frac{c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\right.}{2\sqrt{2}b^{7/4}}$$

Mathematica [C] time = 0.0065507, size = 29, normalized size = 0.14

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{cx^2}{b}\right)}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^2)/b)])/(3*b*x^(3/2))
```

Maple [A] time = 0.053, size = 143, normalized size = 0.7

$$-\frac{2}{3b}x^{-\frac{3}{2}}-\frac{c\sqrt{2}}{4b^2}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)-\frac{c\sqrt{2}}{2b^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{c\sqrt{2}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)/x^(1/2),x)
```

```
[Out] -2/3/b/x^(3/2)-1/4*c/b^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1
/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2*c/b^2*(b/
c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*c/b^2*(b/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39365, size = 387, normalized size = 1.9

$$-\frac{12\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^5c\sqrt{x}\left(-\frac{c^3}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{-\frac{c^3}{b^7}}+c^2x}\,b^5\left(-\frac{c^3}{b^7}\right)^{\frac{3}{4}}}{c^3}\right)+3\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}+c\sqrt{x}\right)-3\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\log\left(\right.}{6\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*b*x^2*(-c^3/b^7)^(1/4)*arctan(-(b^5*c*sqrt(x)*(-c^3/b^7)^(3/4) - s
qrt(b^4*sqrt(-c^3/b^7) + c^2*x)*b^5*(-c^3/b^7)^(3/4))/c^3) + 3*b*x^2*(-c^3/
b^7)^(1/4)*log(b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) - 3*b*x^2*(-c^3/b^7)^(1/4)
*log(-b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) + 4*sqrt(x))/(b*x^2)
```

Sympy [A] time = 41.386, size = 184, normalized size = 0.9

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } c = 0 \\ -\frac{2}{7cx^{\frac{7}{2}}} & \text{for } b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} + \frac{\sqrt[4]{-1}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{7}{4}}c^2\left(\frac{1}{c}\right)^{\frac{11}{4}}} - \frac{\sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{7}{4}}c^2\left(\frac{1}{c}\right)^{\frac{11}{4}}} + \frac{\sqrt[4]{-1}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{7}{4}}c^2\left(\frac{1}{c}\right)^{\frac{11}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(b, 0) & Eq(c, 0)), (-2/(3*b*x**(3/2)), Eq(c, 0)
), (-2/(7*c*x**(7/2)), Eq(b, 0)), (-2/(3*b*x**(3/2)) + (-1)**(1/4)*log(-(-1
)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(7/4)*c**2*(1/c)**(11/4)) -
 (-1)**(1/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(7/4)*c
**2*(1/c)**(11/4)) + (-1)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**
(1/4)))/(b**(7/4)*c**2*(1/c)**(11/4)), True))
```

Giac [A] time = 1.13961, size = 240, normalized size = 1.18

$$-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^2} - \frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^2} - \frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^2 - 1/4*sqrt(2)*(b*c^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^2 + 1/4*sqrt(2)*(b*c^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^2 - 2/3/(b*x^(3/2)
)
```

3.324 $\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=215

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}-\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}}+\frac{c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{9/4}}$$

```
[Out] -2/(5*b*x^(5/2)) + (2*c)/(b^2*Sqrt[x]) - (c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4)) - (c^(5/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rubi [A] time = 0.187666, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}-\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}}+\frac{c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{\sqrt{2}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(5*b*x^(5/2)) + (2*c)/(b^2*Sqrt[x]) - (c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4)) - (c^(5/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2}{5bx^{5/2}} - \frac{c\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{b}$$

$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c^2\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^2}$$

$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} - \frac{c^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} + \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} + -$$

$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}} - \frac{c^{5/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}} + -$$

$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} - \frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}} + \frac{c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}} + \frac{c^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{}\right.}{2\sqrt{2}b^{9/4}}$$

Mathematica [C] time = 0.0069805, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{cx^2}{b}\right)}{5bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^2)/b)])/(5*b*x^(5/2))
```

Maple [A] time = 0.055, size = 152, normalized size = 0.7

$$-\frac{2}{5b}x^{-\frac{5}{2}}+2\,\frac{c}{b^2\sqrt{x}}+\frac{c\sqrt{2}}{4\,b^2}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{c\sqrt{2}}{2\,b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] -2/5/b/x^(5/2)+2*c/b^2/x^(1/2)+1/4*c/b^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1
/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)
))+1/2*c/b^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*
c/b^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36692, size = 433, normalized size = 2.01

$$-\frac{20\,b^2x^3\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^2c^4\sqrt{x}\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}-\sqrt{-b^5c^5\sqrt{-\frac{c^5}{b^9}}+c^8xb^2}\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}}{c^5}\right)-5\,b^2x^3\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}\log\left(b^7\left(-\frac{c^5}{b^9}\right)^{\frac{3}{4}}+c^4\sqrt{x}\right)+5\,b^2x^3\left(-\frac{c^5}{b^9}\right)}{10\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*x^3*(-c^5/b^9)^(1/4)*arctan(-(b^2*c^4*sqrt(x)*(-c^5/b^9)^(1/4
) - sqrt(-b^5*c^5*sqrt(-c^5/b^9) + c^8*x)*b^2*(-c^5/b^9)^(1/4))/c^5) - 5*b^
2*x^3*(-c^5/b^9)^(1/4)*log(b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) + 5*b^2*x^3*
(-c^5/b^9)^(1/4)*log(-b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) - 4*(5*c*x^2 - b)
*sqrt(x))/(b^2*x^3)
```

Sympy [A] time = 67.5526, size = 196, normalized size = 0.91

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } b = 0 \wedge \\ -\frac{2}{9cx^{\frac{9}{2}}} & \text{for } b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } c = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} + \frac{2c}{b^2\sqrt{x}} - \frac{(-1)^{\frac{3}{4}}c^6\left(\frac{1}{c}\right)^{\frac{19}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{9}{4}}} + \frac{(-1)^{\frac{3}{4}}c^6\left(\frac{1}{c}\right)^{\frac{19}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{9}{4}}} + \frac{(-1)^{\frac{3}{4}}c^6\left(\frac{1}{c}\right)^{\frac{19}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{9}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(b, 0) & Eq(c, 0)), (-2/(9*c*x**(9/2)), Eq(b, 0)
), (-2/(5*b*x**(5/2)), Eq(c, 0)), (-2/(5*b*x**(5/2)) + 2*c/(b**2*sqrt(x)) -
 (-1)**(3/4)*c**6*(1/c)**(19/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sq
rt(x))/(2*b**(9/4)) + (-1)**(3/4)*c**6*(1/c)**(19/4)*log((-1)**(1/4)*b**(1/
4)*(1/c)**(1/4) + sqrt(x))/(2*b**(9/4)) + (-1)**(3/4)*c**6*(1/c)**(19/4)*at
an((-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/b**(9/4), True))
```

Giac [A] time = 1.13163, size = 270, normalized size = 1.26

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^3c} + \frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^3c} - \frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^3c} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b^3*c) + 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) - 1/4*sqrt(2)*(b*c^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/4*sqrt(2
)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) +
 2/5*(5*c*x^2 - b)/(b^2*x^(5/2))
```

3.325 $\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=217

$$-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}-\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}+\frac{c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\cdots}\right.}{\sqrt{2}b^{11/\cdots}}$$

```
[Out] -2/(7*b*x^(7/2)) + (2*c)/(3*b^2*x^(3/2)) - (c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) + (c^(7/4)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) - (c^(7/4)*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4)) + (c^(7/4)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/
4))
```

Rubi [A] time = 0.182158, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}-\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}+\frac{c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\cdots}\right.}{\sqrt{2}b^{11/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```


```
[Out] -2/(7*b*x^(7/2)) + (2*c)/(3*b^2*x^(3/2)) - (c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) + (c^(7/4)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) - (c^(7/4)*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4)) + (c^(7/4)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/
4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2}{7bx^{7/2}} - \frac{c\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{b}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{c^2\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{b^2}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^2}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{c^2\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^{5/2}} + \frac{c^2\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^{5/2}}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^{5/2}} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{}\right.}{2b^{5/2}}$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} - \frac{c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}} + \frac{c^{7/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}} +$$

$$= -\frac{2}{7bx^{7/2}} + \frac{2c}{3b^2x^{3/2}} - \frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}} + \frac{c^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}} - \frac{c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\right.}{2\sqrt{2}b^{11/}}$$

Mathematica [C] time = 0.0061213, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\frac{cx^2}{b}\right)}{7bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 1, -3/4, -((c*x^2)/b)])/(7*b*x^(7/2))
```

Maple [A] time = 0.055, size = 158, normalized size = 0.7

$$-\frac{2}{7b}x^{-\frac{7}{2}}+\frac{2c}{3b^2}x^{-\frac{3}{2}}+\frac{c^2\sqrt{2}}{4b^3}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{c^2\sqrt{2}}{2b^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2),x)
```

```
[Out] -2/7/b/x^(7/2)+2/3*c/b^2/x^(3/2)+1/4*c^2/b^3*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c
)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(
1/2)))+1/2*c^2/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1
)+1/2*c^2/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47466, size = 441, normalized size = 2.03

$$\frac{84\,b^2x^4\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^8c^2\sqrt{x}\left(-\frac{c^7}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^6\sqrt{-\frac{c^7}{b^{11}}}+c^4x}\,b^8\left(-\frac{c^7}{b^{11}}\right)^{\frac{3}{4}}}{c^7}\right)+21\,b^2x^4\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}+c^2\sqrt{x}\right)-21\,b^2x^4\left(-\right.}{42\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*b^2*x^4*(-c^7/b^11)^(1/4)*arctan(-(b^8*c^2*sqrt(x)*(-c^7/b^11)^(3/
4) - sqrt(b^6*sqrt(-c^7/b^11) + c^4*x)*b^8*(-c^7/b^11)^(3/4))/c^7) + 21*b^2
*x^4*(-c^7/b^11)^(1/4)*log(b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(x)) - 21*b^2*x^
4*(-c^7/b^11)^(1/4)*log(-b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(x)) + 4*(7*c*x^2
- 3*b)*sqrt(x))/(b^2*x^4)
```

Sympy [A] time = 151.415, size = 197, normalized size = 0.91

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} & \text{for}\ b = 0 \wedge c = 0 \\ -\frac{2}{11cx^{\frac{11}{2}}} & \text{for}\ b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for}\ c = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} + \frac{2c}{3b^2x^{\frac{3}{2}}} - \frac{\sqrt[4]{-1}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{11}{4}}c^{42}\left(\frac{1}{c}\right)^{\frac{175}{4}}} + \frac{\sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{11}{4}}c^{42}\left(\frac{1}{c}\right)^{\frac{175}{4}}} - \frac{\sqrt[4]{-1}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{11}{4}}c^{42}\left(\frac{1}{c}\right)^{\frac{175}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(11/2), Eq(b, 0) & Eq(c, 0)), (-2/(11*c*x**(11/2)), Eq(b,
 0)), (-2/(7*b*x**(7/2)), Eq(c, 0)), (-2/(7*b*x**(7/2)) + 2*c/(3*b**2*x**(3
/2)) - (-1)**(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**
(11/4)*c**42*(1/c)**(175/4)) + (-1)**(1/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**
(1/4) + sqrt(x))/(2*b**(11/4)*c**42*(1/c)**(175/4)) - (-1)**(1/4)*atan((-1)
**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(11/4)*c**42*(1/c)**(175/4)),
True))
```

Giac [A] time = 1.16776, size = 259, normalized size = 1.19

$$\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\cdot}\right.}{4\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/b^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3 + 1/4*sqrt(2)*(b*c^3)^(1/4
)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/4*sqrt(2)*(b*c
^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 2/21*(7
*c*x^2 - 3*b)/(b^2*x^(7/2))
```

3.326 $\int \frac{1}{x^{7/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=230

$$-\frac{2c^2}{b^3\sqrt{x}}-\frac{c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}+\frac{c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}+\frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}}-\frac{c^{9/4}\text{ta}}{}$$

```
[Out] -2/(9*b*x^(9/2)) + (2*c)/(5*b^2*x^(5/2)) - (2*c^2)/(b^3*Sqrt[x]) + (c^(9/4)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) - (c^(9/
4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) - (c^(
9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]
*b^(13/4)) + (c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.219664, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2c^2}{b^3\sqrt{x}}-\frac{c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}+\frac{c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}+\frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}}-\frac{c^{9/4}\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```


```
[Out] -2/(9*b*x^(9/2)) + (2*c)/(5*b^2*x^(5/2)) - (2*c^2)/(b^3*Sqrt[x]) + (c^(9/4)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) - (c^(9/
4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) - (c^(
9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]
*b^(13/4)) + (c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*b^(13/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2}{9bx^{9/2}} - \frac{c\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{b}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} + \frac{c^2\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{b^2}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^3}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} + \frac{c^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3} - \frac{c^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{b^3}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^3} - \frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx\right.}{2b^3}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^{9/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}} + \frac{c^{9/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{2\sqrt{2}b^{13/4}}$$

$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} + \frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} - \frac{c^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} - \frac{c^{9/4}\log\left(\sqrt{b}\right.}{}$$

Mathematica [C] time = 0.0067398, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{9}{4},1;-\frac{5}{4};-\frac{cx^2}{b}\right)}{9bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*Hypergeometric2F1[-9/4, 1, -5/4, -((c*x^2)/b)])/(9*b*x^(9/2))
```

Maple [A] time = 0.055, size = 169, normalized size = 0.7

$$-\frac{2}{9\,b}x^{-\frac{9}{2}}-2\,\frac{c^2}{b^3\sqrt{x}}+\frac{2\,c}{5\,b^2}x^{-\frac{5}{2}}-\frac{c^2\sqrt{2}}{4\,b^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{c^2\sqrt{2}}{2\,b^3}\arctan\left(\sqrt{2}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2),x)
```

```
[Out] -2/9/b/x^(9/2)-2*c^2/b^3/x^(1/2)+2/5*c/b^2/x^(5/2)-1/4*c^2/b^3/(b/c)^(1/4)*
2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/
2)*2^(1/2)+(b/c)^(1/2)))-1/2*c^2/b^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/
c)^(1/4)*x^(1/2)+1)-1/2*c^2/b^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1
/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32848, size = 477, normalized size = 2.07

$$\frac{180\, b^3x^5\left(-\frac{c^9}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^3c^7\sqrt{x}\left(-\frac{c^9}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-b^7c^9\sqrt{-\frac{c^9}{b^{13}}}+c^{14}x}\,b^3\left(-\frac{c^9}{b^{13}}\right)^{\frac{1}{4}}}{c^9}\right)-45\, b^3x^5\left(-\frac{c^9}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^{10}\left(-\frac{c^9}{b^{13}}\right)^{\frac{3}{4}}+c^7\sqrt{x}\right)+45\, b^3x^5 \cdots}{90\, b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*x^5*(-c^9/b^13)^(1/4)*arctan(-(b^3*c^7*sqrt(x)*(-c^9/b^13)^(1
/4) - sqrt(-b^7*c^9*sqrt(-c^9/b^13) + c^14*x)*b^3*(-c^9/b^13)^(1/4))/c^9) -
 45*b^3*x^5*(-c^9/b^13)^(1/4)*log(b^10*(-c^9/b^13)^(3/4) + c^7*sqrt(x)) + 4
5*b^3*x^5*(-c^9/b^13)^(1/4)*log(-b^10*(-c^9/b^13)^(3/4) + c^7*sqrt(x)) - 4*
(45*c^2*x^4 - 9*b*c*x^2 + 5*b^2)*sqrt(x))/(b^3*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15294, size = 269, normalized size = 1.17

$$-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\, b^4}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\, b^4}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\, b^4} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/4*sqrt(2)*(b*c^3)^(3/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/4*sqrt(2)*(b*c^3)^(
3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 2/45*(45*c^2*x
^4 - 9*b*c*x^2 + 5*b^2)/(b^3*x^(9/2))
```

3.327 $\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=243

$$-\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\tan^{-1}}{4\sqrt{}}$$

```
[Out] (-9*b*Sqrt[x])/(2*c^3) + (9*x^(5/2))/(10*c^2) - x^(9/2)/(2*c*(b + c*x^2)) -
 (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13
/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]
*c^(13/4)) - (9*b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(8*Sqrt[2]*c^(13/4)) + (9*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rubi [A] time = 0.210042, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\tan^{-1}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-9*b*Sqrt[x])/(2*c^3) + (9*x^(5/2))/(10*c^2) - x^(9/2)/(2*c*(b + c*x^2)) -
 (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13
/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]
*c^(13/4)) - (9*b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(8*Sqrt[2]*c^(13/4)) + (9*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{11/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{9\int \frac{x^{7/2}}{b+cx^2}\,dx}{4c}$$

$$= \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} - \frac{(9b)\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^2\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c^3}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^3}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^3} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\right.}{4c^3}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^{7/2}} + \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}}\right.}{8c}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} - \frac{9b^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}} + \frac{9b^{5/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{8\sqrt{2}c^{13/4}}$$

$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c\left(b+cx^2\right)} - \frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}} + \frac{9b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}} - \frac{9b^{5/4}\log}{}$$

Mathematica [A] time = 0.210854, size = 220, normalized size = 0.91

$$\frac{\frac{8\sqrt[4]{c}\sqrt{x}\left(-45b^2-36bcx^2+4c^2x^4\right)}{b+cx^2} - 45\sqrt{2}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right) + 45\sqrt{2}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right) - 90\sqrt{}}{80c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((8*c^(1/4)*Sqrt[x]*(-45*b^2 - 36*b*c*x^2 + 4*c^2*x^4))/(b + c*x^2) - 90*Sq
rt[2]*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 90*Sqrt[2]*b^
```

```
(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 45*Sqrt[2]*b^(5/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 45*Sqrt[2]*b^(5/
4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(80*c^(13/4)
)
```

Maple [A] time = 0.058, size = 172, normalized size = 0.7

$$\frac{2}{5c^{2}}x^{\frac{5}{2}}-4\,\frac{b\sqrt{x}}{c^{3}}-\frac{b^{2}}{2\,c^{3}\left(cx^{2}+b\right)}\sqrt{x}+\frac{9\,b\sqrt{2}}{16\,c^{3}}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{9\,b\sqrt{2}}{8\,c^{3}}\sqrt[4]{\frac{b}{c}}\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/5*x^(5/2)/c^2-4*b*x^(1/2)/c^3-1/2/c^3*b^2*x^(1/2)/(c*x^2+b)+9/16/c^3*b*(b
/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(
1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+9/8/c^3*b*(b/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/8/c^3*b*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39966, size = 524, normalized size = 2.16

$$\frac{180\left(c^{4}x^{2}+bc^{3}\right)\left(-\frac{b^{5}}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc^{10}\sqrt{x}\left(-\frac{b^{5}}{c^{13}}\right)^{\frac{3}{4}}-\sqrt{c^{6}\sqrt{-\frac{b^{5}}{c^{13}}}+b^{2}xc^{10}}\left(-\frac{b^{5}}{c^{13}}\right)^{\frac{3}{4}}}{b^{5}}\right)+45\left(c^{4}x^{2}+bc^{3}\right)\left(-\frac{b^{5}}{c^{13}}\right)^{\frac{1}{4}}\log\left(9\,c^{3}\left(-\frac{b^{5}}{c^{13}}\right)^{\frac{1}{4}}+9\,b\right.}{40\left(c^{4}x^{2}+bc^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(180*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*arctan(-(b*c^10*sqrt(x)*(-b^5
/c^13)^(3/4) - sqrt(c^6*sqrt(-b^5/c^13) + b^2*x)*c^10*(-b^5/c^13)^(3/4))/b^
5) + 45*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(9*c^3*(-b^5/c^13)^(1/4) + 9
*b*sqrt(x)) - 45*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(-9*c^3*(-b^5/c^13)
^(1/4) + 9*b*sqrt(x)) + 4*(4*c^2*x^4 - 36*b*c*x^2 - 45*b^2)*sqrt(x))/(c^4*x
^2 + b*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17726, size = 292, normalized size = 1.2

$$\frac{9\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^4}+\frac{9\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^4}+\frac{9\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\right.}{16c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^4 + 9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 9/16*sqrt(2)*(b*c^3)^(1/
4)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 9/16*sqrt(2)*(b
*c^3)^(1/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2*b
^2*sqrt(x)/((c*x^2 + b)*c^3) + 2/5*(c^8*x^(5/2) - 10*b*c^7*sqrt(x))/c^10
```

3.328 $\int \frac{x^{17/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{11/4}}+\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{11/4}}-\frac{7b^{3/4}\tan^{-1}}{4\sqrt{}}$$

```
[Out] (7*x^(3/2))/(6*c^2) - x^(7/2)/(2*c*(b + c*x^2)) + (7*b^(3/4)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) - (7*b^(3/4)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) - (7*b^(3/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(11/
4)) + (7*b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(8*Sqrt[2]*c^(11/4))
```

Rubi [A] time = 0.182154, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{11/4}}+\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{11/4}}-\frac{7b^{3/4}\tan^{-1}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```



```
[Out] (7*x^(3/2))/(6*c^2) - x^(7/2)/(2*c*(b + c*x^2)) + (7*b^(3/4)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) - (7*b^(3/4)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) - (7*b^(3/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(11/
4)) + (7*b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(8*Sqrt[2]*c^(11/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{9/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{x^{7/2}}{2c\left(b+cx^2\right)} + \frac{7\int \frac{x^{5/2}}{b+cx^2}\,dx}{4c}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{(7b)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c^2}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{(7b)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^2}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} + \frac{(7b)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{5/2}} - \frac{(7b)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{5/2}}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^3} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \right.}{8c^3}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} - \frac{7b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{11/4}} + \frac{7b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{cx}\right.}{8\sqrt{2}c^{11/4}}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c\left(b+cx^2\right)} + \frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{11/4}} - \frac{7b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{11/4}} - \frac{7b^{3/4}\log\left(\sqrt{b}-\sqrt{\right.}}{8\sqrt{2}}$$

Mathematica [C] time = 0.0167356, size = 57, normalized size = 0.25

$$-\frac{2x^{3/2}\left(7\left(b+cx^2\right){}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)-7b-cx^2\right)}{3c^2\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*x^(3/2)*(-7*b - c*x^2 + 7*(b + c*x^2)*Hypergeometric2F1[3/4, 2, 7/4, -(
(c*x^2)/b)]))/(3*c^2*(b + c*x^2))
```

Maple [A] time = 0.061, size = 161, normalized size = 0.7

$$\frac{2}{3c^2}x^{\frac{3}{2}}+\frac{b}{2c^2\left(cx^2+b\right)}x^{\frac{3}{2}}-\frac{7b\sqrt{2}}{16c^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{7b\sqrt{2}}{8c^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/3*x^(3/2)/c^2+1/2*b/c^2*x^(3/2)/(c*x^2+b)-7/16*b/c^3/(b/c)^(1/4)*2^(1/2)*
ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/
2)+(b/c)^(1/2)))-7/8*b/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)-7/8*b/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-
1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47583, size = 556, normalized size = 2.42

$$\frac{84\left(c^3x^2+bc^2\right)\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{343b^2c^3\sqrt{x}\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}-\sqrt{-117649b^3c^5\sqrt{-\frac{b^3}{c^{11}}}+117649b^4xc^3}\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}}{343b^3}\right)-21\left(c^3x^2+bc^2\right)\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}\log\left(34\ldots\right.}{24\left(c^3x^2+bc^2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(84*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*arctan(-1/343*(343*b^2*c^3*sqr
t(x)*(-b^3/c^11)^(1/4) - sqrt(-117649*b^3*c^5*sqrt(-b^3/c^11) + 117649*b^4*
x)*c^3*(-b^3/c^11)^(1/4))/b^3) - 21*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*log
(343*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) + 21*(c^3*x^2 + b*c^2)*(-b^3/
c^11)^(1/4)*log(-343*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) + 4*(4*c*x^3
+ 7*b*x)*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19619, size = 265, normalized size = 1.15

$$\frac{bx^{\frac{3}{2}}}{2\left(cx^{2}+b\right)c^{2}}+\frac{2\,x^{\frac{3}{2}}}{3\,c^{2}}-\frac{7\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{5}}-\frac{7\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{5}}+\frac{7\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^(3/2)/((c*x^2 + b)*c^2) + 2/3*x^(3/2)/c^2 - 7/8*sqrt(2)*(b*c^3)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 -
7/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^5 + 7/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^5 - 7/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt
(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5
```

3.329 $\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=230

$$\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\tan^{-1}\left(\sqrt{}\right.}{4\sqrt{2}c}$$

```
[Out] (5*Sqrt[x])/(2*c^2) - x^(5/2)/(2*c*(b + c*x^2)) + (5*b^(1/4)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) - (5*b^(1/4)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) + (5*b^(1/4)*Log[
Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(9/4))
 - (5*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(
8*Sqrt[2]*c^(9/4))
```

Rubi [A] time = 0.181984, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\tan^{-1}\left(\sqrt{}\right.}{4\sqrt{2}c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```



```
[Out] (5*Sqrt[x])/(2*c^2) - x^(5/2)/(2*c*(b + c*x^2)) + (5*b^(1/4)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) - (5*b^(1/4)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) + (5*b^(1/4)*Log[
Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(9/4))
 - (5*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(
8*Sqrt[2]*c^(9/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^2}\,dx=\int \frac{x^{7/2}}{\left(b+cx^2\right)^2}\,dx$$

$$=-\frac{x^{5/2}}{2c\left(b+cx^2\right)}+\frac{5\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c}$$

$$=\frac{5\sqrt{x}}{2c^2}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}-\frac{(5b)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c^2}$$

$$=\frac{5\sqrt{x}}{2c^2}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}-\frac{(5b)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2c^2}$$

$$=\frac{5\sqrt{x}}{2c^2}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}-\frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4c^2}-\frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4c^2}$$

$$=\frac{5\sqrt{x}}{2c^2}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}-\frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8c^{5/2}}-\frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx\right.}{8c^{5/2}}$$

$$=\frac{5\sqrt{x}}{2c^2}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}+\frac{5\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}$$

$$=\frac{5\sqrt{x}}{2c^2}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{9/4}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\right.}{8\sqrt{2}c}$$

Mathematica [A] time = 0.109156, size = 221, normalized size = 0.96

$$\frac{\frac{32c^{5/4}x^{5/2}}{b+cx^2}+\frac{40b\sqrt[4]{c}\sqrt{x}}{b+cx^2}+5\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-5\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+10\sqrt{2}\sqrt[4]{b}\tan}{16c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((40*b*c^(1/4)*Sqrt[x])/(b + c*x^2) + (32*c^(5/4)*x^(5/2))/(b + c*x^2) + 10
*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 10*Sqrt[2]
*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 5*Sqrt[2]*b^(1/4)*
Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 5*Sqrt[2]*b^(1
```

```
/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(16*c^(9/4)
)
```

Maple [A] time = 0.061, size = 158, normalized size = 0.7

$$2\frac{\sqrt{x}}{c^2}+\frac{b}{2c^2\left(cx^2+b\right)}\sqrt{x}-\frac{5\sqrt{2}}{16c^2}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)-\frac{5\sqrt{2}}{8c^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2*x^(1/2)/c^2+1/2*b/c^2*x^(1/2)/(c*x^2+b)-5/16/c^2*(b/c)^(1/4)*2^(1/2)*ln((
x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(
b/c)^(1/2)))-5/8/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)
+1)-5/8/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36376, size = 443, normalized size = 1.93

$$-\frac{20\left(c^3x^2+bc^2\right)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^4\sqrt{-\frac{b}{c^9}}+xc^7\left(-\frac{b}{c^9}\right)^{\frac{3}{4}}}-c^7\sqrt{x}\left(-\frac{b}{c^9}\right)^{\frac{3}{4}}}{b}\right)+5\left(c^3x^2+bc^2\right)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\log\left(5c^2\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}+5\sqrt{x}\right)-5\left(c^3\right.}{8\left(c^3x^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*(c^3*x^2 + b*c^2)*(-b/c^9)^(1/4)*arctan((sqrt(c^4*sqrt(-b/c^9) + x
)*c^7*(-b/c^9)^(3/4) - c^7*sqrt(x)*(-b/c^9)^(3/4))/b) + 5*(c^3*x^2 + b*c^2)
*(-b/c^9)^(1/4)*log(5*c^2*(-b/c^9)^(1/4) + 5*sqrt(x)) - 5*(c^3*x^2 + b*c^2)
*(-b/c^9)^(1/4)*log(-5*c^2*(-b/c^9)^(1/4) + 5*sqrt(x)) - 4*(4*c*x^2 + 5*b)*
sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23465, size = 265, normalized size = 1.15

$$-\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^3}-\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^3}-\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\right.}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/c^3 - 5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3 - 5/16*sqrt(2)*(b*c^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 5/16*sqrt(2)*(b*c^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 1/2*b*sqrt(x
)/((c*x^2 + b)*c^2) + 2*sqrt(x)/c^2
```

3.330 $\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=218

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}}-$$

```
[Out] -x^(3/2)/(2*c*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4)) - (3*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4))
```

Rubi [A] time = 0.165619, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```


```
[Out] -x^(3/2)/(2*c*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4)) - (3*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{5/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} - \frac{3\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{3/2}} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{3/2}}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^2} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^2} + \frac{3\,\mathrm{Subs}}{}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} + \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{3\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{3\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{x^{3/2}}{2c\left(b+cx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

Mathematica [C] time = 0.016464, size = 43, normalized size = 0.2

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{b}-\frac{1}{b+cx^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*(-(b + c*x^2)^(-1) + Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)
]/b))/c
```

Maple [A] time = 0.056, size = 149, normalized size = 0.7

$$-\frac{1}{2c\left(cx^2+b\right)}x^{\frac{3}{2}}+\frac{3\sqrt{2}}{16c^2}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{3\sqrt{2}}{8c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x^(3/2)/c/(c*x^2+b)+3/16/c^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(
1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+3/8/
c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/8/c^2/(b/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41057, size = 447, normalized size = 2.05

$$-\frac{12\left(c^2x^2+bc\right)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc^3\sqrt{-\frac{1}{bc^7}}+xc^2}\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}-c^2\sqrt{x}\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\right)-3\left(c^2x^2+bc\right)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\log\left(bc^5\left(-\frac{1}{bc^7}\right)^{\frac{3}{4}}\right.}{8\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*arctan(sqrt(-b*c^3*sqrt(-1/(b*c
^7)) + x)*c^2*(-1/(b*c^7))^(1/4) - c^2*sqrt(x)*(-1/(b*c^7))^(1/4)) - 3*(c^2
*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(b*c^5*(-1/(b*c^7))^(3/4) + sqrt(x)) + 3*
(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(-b*c^5*(-1/(b*c^7))^(3/4) + sqrt(x))
 + 4*x^(3/2))/(c^2*x^2 + b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18814, size = 269, normalized size = 1.23

$$-\frac{x^{\frac{3}{2}}}{2\left(cx^2+b\right)c}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^4}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^4}-\frac{3\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\cdots\right.}{16\,b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^(3/2)/((c*x^2 + b)*c) + 3/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) + 3/8*sqrt(2)*(b*c^
3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))
/(b*c^4) - 3/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/(b*c^4) + 3/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)
^(1/4) + x + sqrt(b/c))/(b*c^4)
```

3.331 $\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=218

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{3/4}c^{5/4}}-\frac{}{2c}$$

```
[Out] -Sqrt[x]/(2*c*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(4*Sqrt[2]*b^(3/4)*c^(5/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
/(4*Sqrt[2]*b^(3/4)*c^(5/4)) - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4))
```

Rubi [A] time = 0.159824, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{3/4}c^{5/4}}-\frac{}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -Sqrt[x]/(2*c*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(4*Sqrt[2]*b^(3/4)*c^(5/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
/(4*Sqrt[2]*b^(3/4)*c^(5/4)) - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{x^{3/2}}{\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} + \frac{\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4c}$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c}$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{b}c} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{b}c}$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{b}c^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{b}c^{3/2}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}} + \frac{\text{Subst}\left(\int \frac{}{-1}\right.}{4}$$

$$= -\frac{\sqrt{x}}{2c\left(b+cx^2\right)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{5/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{5/4}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}} + \frac{\text{l}}{}$$

Mathematica [A] time = 0.100777, size = 198, normalized size = 0.91

$$\frac{-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}} + \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}} - \frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}} + \frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}} - \frac{8\sqrt[4]{c}\sqrt{x}}{b+cx^2}}{16c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-8*c^(1/4)*Sqrt[x])/(b + c*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)])/b^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/b^(3/4) - (Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/b^(3/4) + (Sqrt[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x])/b^(3/4))/(16*c^(5/4))
```

Maple [A] time = 0.056, size = 158, normalized size = 0.7

$$-\frac{1}{2c\left(cx^2+b\right)}\sqrt{x}+\frac{\sqrt{2}}{16bc}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{\sqrt{2}}{8bc}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x^(1/2)/c/(c*x^2+b)+1/16/c*(b/c)^(1/4)/b*2^(1/2)*ln((x+(b/c)^(1/4)*x^(
1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/8/
c*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/8/c*(b/c)^(
1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58069, size = 467, normalized size = 2.14

$$\frac{4\left(c^2x^2+bc\right)\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2c^2\sqrt{-\frac{1}{b^3c^5}}+xb^2c^4\left(-\frac{1}{b^3c^5}\right)^{\frac{3}{4}}}-b^2c^4\sqrt{x}\left(-\frac{1}{b^3c^5}\right)^{\frac{3}{4}}\right)+\left(c^2x^2+bc\right)\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}\log\left(bc\left(-\frac{1}{b^3c^5}\right.\right.}{8\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*arctan(sqrt(b^2*c^2*sqrt(-1/(b^
3*c^5)) + x)*b^2*c^4*(-1/(b^3*c^5))^(3/4) - b^2*c^4*sqrt(x)*(-1/(b^3*c^5))^
(3/4)) + (c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(b*c*(-1/(b^3*c^5))^(1/4)
+ sqrt(x)) - (c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(-b*c*(-1/(b^3*c^5))^(
1/4) + sqrt(x)) - 4*sqrt(x))/(c^2*x^2 + b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16663, size = 269, normalized size = 1.23

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c^2) + 1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) + 1/16*sqrt(2)*(b*c^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2) - 1/16*sqrt
(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2)
 - 1/2*sqrt(x)/((c*x^2 + b)*c)
```

3.332 $\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{5/4}c^{3/4}}+\frac{x \cdots}{2b\left(b \cdots\right.}$$

```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(
4*Sqrt[2]*b^(5/4)*c^(3/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(4*Sqrt[2]*b^(5/4)*c^(3/4)) + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rubi [A] time = 0.16321, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{5/4}c^{3/4}}+\frac{x \cdots}{2b\left(b \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(
4*Sqrt[2]*b^(5/4)*c^(3/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(4*Sqrt[2]*b^(5/4)*c^(3/4)) + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx$$

$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b}$$

$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b\sqrt{c}}$$

$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc} + \ldots$$

$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} \ldots\right.}{4\sqrt{2}\ldots}$$

$$= \frac{x^{3/2}}{2b\left(b+cx^2\right)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{3/4}} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}} - \frac{\log \ldots}{\ldots}$$

Mathematica [C] time = 0.0059255, size = 29, normalized size = 0.13

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((c*x^2)/b)])/(3*b^2)
```

Maple [A] time = 0.054, size = 158, normalized size = 0.7

$$\frac{1}{2\,b\left(cx^{2}+b\right)}x^{\frac{3}{2}}+\frac{\sqrt{2}}{16\,bc}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}}{8\,bc}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^(3/2)/b/(c*x^2+b)+1/16/b/c/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/8/b
/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/8/b/c/(b/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62311, size = 467, normalized size = 2.14

$$-\frac{4\left(bcx^{2}+b^{2}\right)\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b^{3}c\sqrt{-\frac{1}{b^{5}c^{3}}}+xbc}\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{1}{4}}-bc\sqrt{x}\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{1}{4}}\right)-\left(bcx^{2}+b^{2}\right)\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{1}{4}}\log\left(b^{4}c^{2}\left(-\frac{1}{b^{5}c^{3}}\right.\right.}{8\left(bcx^{2}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*arctan(sqrt(-b^3*c*sqrt(-1/(b^
5*c^3)) + x)*b*c*(-1/(b^5*c^3))^(1/4) - b*c*sqrt(x)*(-1/(b^5*c^3))^(1/4)) -
 (b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(b^4*c^2*(-1/(b^5*c^3))^(3/4) + sq
rt(x)) + (b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(-b^4*c^2*(-1/(b^5*c^3))^(
3/4) + sqrt(x)) - 4*x^(3/2))/(b*c*x^2 + b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16447, size = 269, normalized size = 1.23

$$\frac{x^{\frac{3}{2}}}{2\left(cx^2+b\right)b}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^3}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)\right.}{16\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^(3/2)/((c*x^2 + b)*b) + 1/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/8*sqrt(2)*(b*c
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4)
)/(b^2*c^3) - 1/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) +
x + sqrt(b/c))/(b^2*c^3) + 1/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3)
```

3.333 $\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

```
[Out] Sqrt[x]/(2*b*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) - (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*Log[Sqrt[b] + Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rubi [A] time = 0.169269, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] Sqrt[x]/(2*b*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) - (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*Log[Sqrt[b] + Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{\sqrt{x}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} + \frac{3\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b}$$

$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}}$$

$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{3/2}\sqrt{c}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{3/2}\sqrt{c}} - \frac{3\,\text{Subs}}{}$$

$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} - \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{x}}{2b\left(b+cx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.111976, size = 199, normalized size = 0.91

$$\frac{\frac{8b^{3/4}\sqrt{x}}{b+cx^2} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{c}}}{16b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((8*b^(3/4)*Sqrt[x])/(b + c*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)])/c^(1/4) - (3*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/c^(1/4) + (3*Sqrt[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/c^(1/4))/(16*b^(7/4))
```

Maple [A] time = 0.055, size = 149, normalized size = 0.7

$$\frac{1}{2b\left(cx^2+b\right)}\sqrt{x}+\frac{3\sqrt{2}}{16b^2}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{3\sqrt{2}}{8b^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^(1/2)/b/(c*x^2+b)+3/16/b^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+3/8/b
^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/8/b^2*(b/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5859, size = 441, normalized size = 2.02

$$\frac{12\left(bcx^2+b^2\right)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^4\sqrt{-\frac{1}{b^7c}}+x}\,b^5c\left(-\frac{1}{b^7c}\right)^{\frac{3}{4}}-b^5c\sqrt{x}\left(-\frac{1}{b^7c}\right)^{\frac{3}{4}}\right)+3\left(bcx^2+b^2\right)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}+\right.}{8\left(bcx^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(12*(b*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*arctan(sqrt(b^4*sqrt(-1/(b^7*c))
 + x)*b^5*c*(-1/(b^7*c))^(3/4) - b^5*c*sqrt(x)*(-1/(b^7*c))^(3/4)) + 3*(b*c
*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(b^2*(-1/(b^7*c))^(1/4) + sqrt(x)) - 3*(b
*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(-b^2*(-1/(b^7*c))^(1/4) + sqrt(x)) + 4
*sqrt(x))/(b*c*x^2 + b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1578, size = 269, normalized size = 1.23

$$\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\right.}{16\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b^2*c) + 3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 3/16*sqrt(2)*(b*c^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 3/16*sqrt
(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c)
 + 1/2*sqrt(x)/((c*x^2 + b)*b)
```

3.334 $\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{5\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{c}\tan^{-1}}{4\sqrt{}}$$

```
[Out] -5/(2*b^2*Sqrt[x]) + 1/(2*b*Sqrt[x]*(b + c*x^2)) + (5*c^(1/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*c^(1/4)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*c^(1/4)*Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)
) + (5*c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(8*Sqrt[2]*b^(9/4))
```

Rubi [A] time = 0.190383, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{c}\tan^{-1}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```


```
[Out] -5/(2*b^2*Sqrt[x]) + 1/(2*b*Sqrt[x]*(b + c*x^2)) + (5*c^(1/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*c^(1/4)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*c^(1/4)*Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)
) + (5*c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(8*Sqrt[2]*b^(9/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{3/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} + \frac{5\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{(5c)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{(5c)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} + \frac{\left(5\sqrt{c}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^2} - \frac{\left(5\sqrt{c}\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x,\right.}{4b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^2} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x,\right.}{8b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} - \frac{5\sqrt[4]{c}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}} + \frac{5\sqrt[4]{c}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{8\sqrt{2}b^{9/4}}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\left(b+cx^2\right)} + \frac{5\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{c}\log\left(\sqrt{b}-\right.}{}$$

Mathematica [C] time = 0.0066771, size = 27, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-\frac{cx^2}{b}\right)}{b^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 2, 3/4, -((c*x^2)/b)])/(b^2*Sqrt[x])
```

Maple [A] time = 0.06, size = 158, normalized size = 0.7

$$-2\frac{1}{b^2\sqrt{x}}-\frac{c}{2\,b^2\left(cx^2+b\right)}x^{\frac{3}{2}}-\frac{5\sqrt{2}}{16\,b^2}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{5\sqrt{2}}{8\,b^2}\arctan\left(\sqrt{2}\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -2/b^2/x^(1/2)-1/2*c/b^2*x^(3/2)/(c*x^2+b)-5/16/b^2/(b/c)^(1/4)*2^(1/2)*ln(
(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+
(b/c)^(1/2)))-5/8/b^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)+1)-5/8/b^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56759, size = 512, normalized size = 2.23

$$\frac{20\left(b^2cx^3+b^3x\right)\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,b^2c\sqrt{x}\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}-\sqrt{-15625\,b^5c\sqrt{-\frac{c}{b^9}}+15625\,c^2x}b^2\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}}{125\,c}\right)-5\left(b^2cx^3+b^3x\right)\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\log\left(125\,b^7\right.}{8\left(b^2cx^3+b^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(20*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*arctan(-1/125*(125*b^2*c*sqrt(x)
*(-c/b^9)^(1/4) - sqrt(-15625*b^5*c*sqrt(-c/b^9) + 15625*c^2*x)*b^2*(-c/b^9
```

```
)^(1/4))/c) - 5*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*log(125*b^7*(-c/b^9)^(3/
4) + 125*c*sqrt(x)) + 5*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*log(-125*b^7*(-c
/b^9)^(3/4) + 125*c*sqrt(x)) - 4*(5*c*x^2 + 4*b)*sqrt(x))/(b^2*c*x^3 + b^3*
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18879, size = 284, normalized size = 1.23

$$-\frac{5\,cx^2+4\,b}{2\left(cx^{\frac{5}{2}}+b\sqrt{x}\right)b^2}-\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^2}-\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^2}+\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*c*x^2 + 4*b)/((c*x^(5/2) + b*sqrt(x))*b^2) - 5/8*sqrt(2)*(b*c^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*
c^2) - 5/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 5/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 5/16*sqrt(2)*(b*c^3)^(3/4
)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2)
```

3.335 $\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=230

$$\frac{7c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}}-\frac{7c^{3/4}\tan^{-1}}{4\sqrt{}}$$

```
[Out] -7/(6*b^2*x^(3/2)) + 1/(2*b*x^(3/2)*(b + c*x^2)) + (7*c^(3/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4)) - (7*c^(3/4)*ArcTan[
1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4)) + (7*c^(3/4)*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11
/4)) - (7*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(8*Sqrt[2]*b^(11/4))
```

Rubi [A] time = 0.184947, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}}-\frac{7c^{3/4}\tan^{-1}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -7/(6*b^2*x^(3/2)) + 1/(2*b*x^(3/2)*(b + c*x^2)) + (7*c^(3/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4)) - (7*c^(3/4)*ArcTan[
1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4)) + (7*c^(3/4)*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11
/4)) - (7*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(8*Sqrt[2]*b^(11/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{5/2}\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2bx^{3/2}\left(b+cx^2\right)} + \frac{7\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{4b}$$
$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} - \frac{(7c)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^2}$$
$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} - \frac{(7c)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2b^2}$$
$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} - \frac{(7c)\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^{5/2}} - \frac{(7c)\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^{5/2}}$$
$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} - \frac{\left(7\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^{5/2}} - \frac{\left(7\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}\right.}{8b^{5/2}}$$
$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} + \frac{7c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}} - \frac{7c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{8\sqrt{2}b^{11/4}}$$
$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}\left(b+cx^2\right)} + \frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}} - \frac{7c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}} + \frac{7c^{3/4}\log\left(\sqrt{\right.}}{}$$

Mathematica [C] time = 0.0069592, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-\frac{cx^2}{b}\right)}{3b^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 2, 1/4, -((c*x^2)/b)])/(3*b^2*x^(3/2))
```

Maple [A] time = 0.058, size = 161, normalized size = 0.7

$$-\frac{2}{3\,b^{2}}x^{-\frac{3}{2}}-\frac{c}{2\,b^{2}\left(c x^{2}+b\right)}\sqrt{x}-\frac{7\,c\sqrt{2}}{16\,b^{3}}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)-\frac{7\,c\sqrt{2}}{8\,b^{3}}\sqrt[4]{\frac{b}{c}}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -2/3/b^2/x^(3/2)-1/2*c/b^2*x^(1/2)/(c*x^2+b)-7/16*c/b^3*(b/c)^(1/4)*2^(1/2)
*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1
/2)+(b/c)^(1/2)))-7/8*c/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*
x^(1/2)+1)-7/8*c/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)
-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49951, size = 518, normalized size = 2.25

$$-\frac{84\left(b^{2} c x^{4}+b^{3} x^{2}\right)\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{8} c\sqrt{x}\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^{6}\sqrt{-\frac{c^{3}}{b^{11}}}+c^{2} x}\, b^{8}\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{3}{4}}}{c^{3}}\right)+21\left(b^{2} c x^{4}+b^{3} x^{2}\right)\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{1}{4}}\log\left(7\,b^{3}\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{1}{4}}\right.}{24\left(b^{2} c x^{4}+b^{3} x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(84*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*arctan(-(b^8*c*sqrt(x)*(-
c^3/b^11)^(3/4) - sqrt(b^6*sqrt(-c^3/b^11) + c^2*x)*b^8*(-c^3/b^11)^(3/4))/
c^3) + 21*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*log(7*b^3*(-c^3/b^11)^(1/
4) + 7*c*sqrt(x)) - 21*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*log(-7*b^3*(
-c^3/b^11)^(1/4) + 7*c*sqrt(x)) + 4*(7*c*x^2 + 4*b)*sqrt(x))/(b^2*c*x^4 + b
^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18647, size = 265, normalized size = 1.15

$$-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3}-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3}-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\right.}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^3 - 7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3 - 7/16*sqrt(2)*(b*c^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 7/16*sqrt(2)*(b*c^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/2*c*sqrt(x
)/((c*x^2 + b)*b^2) - 2/3/(b^2*x^(3/2))
```

3.336 $\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=243

$$\frac{9c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}}+\frac{9c^{5/4}\tan^{-1}}{4\sqrt{}}$$

```
[Out] -9/(10*b^2*x^(5/2)) + (9*c)/(2*b^3*Sqrt[x]) + 1/(2*b*x^(5/2)*(b + c*x^2)) -
 (9*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13
/4)) + (9*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]
*b^(13/4)) + (9*c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(8*Sqrt[2]*b^(13/4)) - (9*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.217297, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}}+\frac{9c^{5/4}\tan^{-1}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```


```
[Out] -9/(10*b^2*x^(5/2)) + (9*c)/(2*b^3*Sqrt[x]) + 1/(2*b*x^(5/2)*(b + c*x^2)) -
 (9*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13
/4)) + (9*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]
*b^(13/4)) + (9*c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(8*Sqrt[2]*b^(13/4)) - (9*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{9\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{(9c)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{\left(9c^2\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^3}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{\left(9c^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^3}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{\left(9c^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^3} + \frac{\left(9c^{3/2}\right)\text{Subst}\left(\int \ldots\right.}{4b\ldots}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{(9c)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^3} + \frac{(9c)\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{9c^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}} - \frac{9c^{5/4}\log\left(\sqrt{b}+\sqrt{\ldots}\right.}{8\sqrt{2}\ldots}$$

$$= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} + \frac{9c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} + \ldots$$

Mathematica [C] time = 0.0072908, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-\frac{cx^2}{b}\right)}{5b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 2, -1/4, -((c*x^2)/b)])/(5*b^2*x^(5/2))
```

Maple [A] time = 0.06, size = 172, normalized size = 0.7

$$-\frac{2}{5b^2}x^{-\frac{5}{2}}+4\frac{c}{b^3\sqrt{x}}+\frac{c^2}{2b^3(cx^2+b)}x^{\frac{3}{2}}+\frac{9c\sqrt{2}}{16b^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{9c\sqrt{2}}{8b^3}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -2/5/b^2/x^(5/2)+4*c/b^3/x^(1/2)+1/2*c^2/b^3*x^(3/2)/(c*x^2+b)+9/16*c/b^3/(
b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^
(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+9/8*c/b^3/(b/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/8*c/b^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43196, size = 605, normalized size = 2.49

$$-\frac{180\left(b^3cx^5+b^4x^3\right)\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729b^3c^4\sqrt{x}\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-531441b^7c^5\sqrt{-\frac{c^5}{b^{13}}}+531441c^8xb^3}\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}}{729c^5}\right)-45\left(b^3cx^5+b^4x^3\right)\left(-\frac{c^5}{b^{13}}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(180*(b^3*c*x^5 + b^4*x^3)*(-c^5/b^13)^(1/4)*arctan(-1/729*(729*b^3*c
^4*sqrt(x)*(-c^5/b^13)^(1/4) - sqrt(-531441*b^7*c^5*sqrt(-c^5/b^13) + 53144
1*c^8*x)*b^3*(-c^5/b^13)^(1/4))/c^5) - 45*(b^3*c*x^5 + b^4*x^3)*(-c^5/b^13)
^(1/4)*log(729*b^10*(-c^5/b^13)^(3/4) + 729*c^4*sqrt(x)) + 45*(b^3*c*x^5 +
b^4*x^3)*(-c^5/b^13)^(1/4)*log(-729*b^10*(-c^5/b^13)^(3/4) + 729*c^4*sqrt(x
)) - 4*(45*c^2*x^4 + 36*b*c*x^2 - 4*b^2)*sqrt(x))/(b^3*c*x^5 + b^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15871, size = 297, normalized size = 1.22

$$\frac{c^2 x^{\frac{3}{2}}}{2\left(cx^2+b\right)b^3} + \frac{9\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^4c} + \frac{9\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^4c} - \frac{9\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{\ldots}\right.}{16b^4\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^(3/2)/((c*x^2 + b)*b^3) + 9/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 9/8*sqrt(2)*
(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b^4*c) - 9/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4)
+ x + sqrt(b/c))/(b^4*c) + 9/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 2/5*(10*c*x^2 - b)/(b^3*x^(5/2))
```

3.337 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=243

$$-\frac{11c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}+\frac{11c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}-\frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}}+\ldots$$

```
[Out] -11/(14*b^2*x^(7/2)) + (11*c)/(6*b^3*x^(3/2)) + 1/(2*b*x^(7/2)*(b + c*x^2))
 - (11*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^
(15/4)) + (11*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqr
t[2]*b^(15/4)) - (11*c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4)) + (11*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rubi [A] time = 0.207384, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{11c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}+\frac{11c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}-\frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -11/(14*b^2*x^(7/2)) + (11*c)/(6*b^3*x^(3/2)) + 1/(2*b*x^(7/2)*(b + c*x^2))
 - (11*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^
(15/4)) + (11*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqr
t[2]*b^(15/4)) - (11*c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4)) + (11*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{11\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{(11c)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^3}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^3}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{7/2}} + \frac{\left(11c^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^{3/2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{7/2}} + \frac{\left(11c^{3/2}\right)\text{S}}{}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{11c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}} + \frac{11c^{7/4}\log\left(\sqrt{\right.}}{}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}} + \frac{11c^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{}}{\sqrt[4]{b}}\right.}{4\sqrt{2}b^{15/4}}$$

Mathematica [C] time = 0.0064712, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{7}{4},2;-\frac{3}{4};-\frac{cx^2}{b}\right)}{7b^2x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 2, -3/4, -((c*x^2)/b)])/(7*b^2*x^(7/2))
```

Maple [A] time = 0.062, size = 178, normalized size = 0.7

$$-\frac{2}{7b^2}x^{-\frac{7}{2}}+\frac{4c}{3b^3}x^{-\frac{3}{2}}+\frac{c^2}{2b^3\left(cx^2+b\right)}\sqrt{x}+\frac{11c^2\sqrt{2}}{16b^4}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{11c^2\sqrt{}}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^2/x^(1/2),x)
```

```
[Out] -2/7/b^2/x^(7/2)+4/3*c/b^3/x^(3/2)+1/2*c^2/b^3*x^(1/2)/(c*x^2+b)+11/16*c^2/
b^4*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(
b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+11/8*c^2/b^4*(b/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+11/8*c^2/b^4*(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43512, size = 564, normalized size = 2.32

$$\frac{924\left(b^3cx^6+b^4x^4\right)\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{11}c^2\sqrt{x}\left(-\frac{c^7}{b^{15}}\right)^{\frac{3}{4}}-\sqrt{b^8\sqrt{-\frac{c^7}{b^{15}}}+c^4x}\,b^{11}\left(-\frac{c^7}{b^{15}}\right)^{\frac{3}{4}}}{c^7}\right)+231\left(b^3cx^6+b^4x^4\right)\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}\log\left(11b^4\left(-\right.\right.}{168\left(b^3cx^6+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/168*(924*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*arctan(-(b^11*c^2*sqrt(x
)*(-c^7/b^15)^(3/4) - sqrt(b^8*sqrt(-c^7/b^15) + c^4*x)*b^11*(-c^7/b^15)^(3
/4))/c^7) + 231*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*log(11*b^4*(-c^7/b^
15)^(1/4) + 11*c^2*sqrt(x)) - 231*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*l
og(-11*b^4*(-c^7/b^15)^(1/4) + 11*c^2*sqrt(x)) + 4*(77*c^2*x^4 + 44*b*c*x^2
 - 12*b^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13407, size = 286, normalized size = 1.18

$$\frac{11\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{11\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{11\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\right.}{16\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/b^4 + 11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 11/16*sqrt(2)*(b*c^3)^
(1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 11/16*sqrt(2
)*(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 1
/2*c^2*sqrt(x)/((c*x^2 + b)*b^3) + 2/21*(14*c*x^2 - 3*b)/(b^3*x^(7/2))
```

3.338 $\int \frac{1}{x^{3/2}(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=258

$$-\frac{13c^2}{2b^4\sqrt{x}}-\frac{13c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}}$$

```
[Out] -13/(18*b^2*x^(9/2)) + (13*c)/(10*b^3*x^(5/2)) - (13*c^2)/(2*b^4*Sqrt[x]) +
 1/(2*b*x^(9/2)*(b + c*x^2)) + (13*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt
[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(9/4)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4)) + (13*c^(9/4
)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^
(17/4))
```

Rubi [A] time = 0.234889, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{13c^2}{2b^4\sqrt{x}}-\frac{13c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```


```
[Out] -13/(18*b^2*x^(9/2)) + (13*c)/(10*b^3*x^(5/2)) - (13*c^2)/(2*b^4*Sqrt[x]) +
 1/(2*b*x^(9/2)*(b + c*x^2)) + (13*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt
[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(9/4)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4)) + (13*c^(9/4
)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^
(17/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{13\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{4b}$$
$$= -\frac{13}{18b^2x^{9/2}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{(13c)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{4b^2}$$
$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{\left(13c^2\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^3}$$
$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{\left(13c^3\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^4}$$
$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{\left(13c^3\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^4}$$
$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{\left(13c^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^4} - \frac{(1}{}$$
$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{\left(13c^2\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^4} -$$
$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{13c^{9/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}} + \frac{13}{}$$
$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}} - \frac{13c^{9/4}\tan^{-1}}{4\sqrt{2}}$$

Mathematica [C] time = 0.0067058, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{9}{4},2;-\frac{5}{4};-\frac{cx^2}{b}\right)}{9b^2x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-9/4, 2, -5/4, -((c*x^2)/b)])/(9*b^2*x^(9/2))
```

Maple [A] time = 0.06, size = 189, normalized size = 0.7

$$-\frac{2}{9\,b^{2}}x^{-\frac{9}{2}}-6\,\frac{c^{2}}{b^{4}\sqrt{x}}+\frac{4\,c}{5\,b^{3}}x^{-\frac{5}{2}}-\frac{c^{3}}{2\,b^{4}\left(cx^{2}+b\right)}x^{\frac{3}{2}}-\frac{13\,c^{2}\sqrt{2}}{16\,b^{4}}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -2/9/b^2/x^(9/2)-6*c^2/b^4/x^(1/2)+4/5*c/b^3/x^(5/2)-1/2*c^3/b^4*x^(3/2)/(c
*x^2+b)-13/16*c^2/b^4/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)
+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-13/8*c^2/b^4/(b/
c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-13/8*c^2/b^4/(b/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42345, size = 648, normalized size = 2.51

$$2340\left(b^{4}cx^{7}+b^{5}x^{5}\right)\left(-\frac{c^{9}}{b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{2197\,b^{4}c^{7}\sqrt{x}\left(-\frac{c^{9}}{b^{17}}\right)^{\frac{1}{4}}-\sqrt{-4826809\,b^{9}c^{9}\sqrt{-\frac{c^{9}}{b^{17}}}+4826809\,c^{14}xb^{4}}\left(-\frac{c^{9}}{b^{17}}\right)^{\frac{1}{4}}}{2197\,c^{9}}\right)-585\left(b^{4}cx^{7}+b^{5}x^{5}\right)\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(2340*(b^4*c*x^7 + b^5*x^5)*(-c^9/b^17)^(1/4)*arctan(-1/2197*(2197*b^
4*c^7*sqrt(x)*(-c^9/b^17)^(1/4) - sqrt(-4826809*b^9*c^9*sqrt(-c^9/b^17) + 4
826809*c^14*x)*b^4*(-c^9/b^17)^(1/4))/c^9) - 585*(b^4*c*x^7 + b^5*x^5)*(-c^
9/b^17)^(1/4)*log(2197*b^13*(-c^9/b^17)^(3/4) + 2197*c^7*sqrt(x)) + 585*(b^
4*c*x^7 + b^5*x^5)*(-c^9/b^17)^(1/4)*log(-2197*b^13*(-c^9/b^17)^(3/4) + 219
7*c^7*sqrt(x)) - 4*(585*c^3*x^6 + 468*b*c^2*x^4 - 52*b^2*c*x^2 + 20*b^3)*sq
rt(x))/(b^4*c*x^7 + b^5*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15583, size = 296, normalized size = 1.15

$$-\frac{c^3x^{\frac{3}{2}}}{2\left(cx^2+b\right)b^4}-\frac{13\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^5}-\frac{13\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^5}+\frac{13\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*c^3*x^(3/2)/((c*x^2 + b)*b^4) - 13/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*
sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 - 13/8*sqrt(2)*(
b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1
/4))/b^5 + 13/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
```

```
+ sqrt(b/c))/b^5 - 13/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(
1/4) + x + sqrt(b/c))/b^5 - 2/45*(135*c^2*x^4 - 18*b*c*x^2 + 5*b^2)/(b^4*x^
(9/2))
```

3.339 $\int \frac{x^{23/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=251

$$-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}+\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{32\sqrt{2}c^{13/4}}$$

```
[Out] (45*Sqrt[x])/(16*c^3) - x^(9/2)/(4*c*(b + c*x^2)^2) - (9*x^(5/2))/(16*c^2*(
b + c*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*c^(13/4)) + (45*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4))
```

Rubi [A] time = 0.210625, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}+\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{32\sqrt{2}c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```



```
[Out] (45*Sqrt[x])/(16*c^3) - x^(9/2)/(4*c*(b + c*x^2)^2) - (9*x^(5/2))/(16*c^2*(
b + c*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*c^(13/4)) + (45*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{23/2}}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^{11/2}}{\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}+\frac{9\int \frac{x^{7/2}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$=-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}+\frac{45\int \frac{x^{3/2}}{b+cx^2}\,dx}{32c^2}$$

$$=\frac{45\sqrt{x}}{16c^3}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}-\frac{(45b)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32c^3}$$

$$=\frac{45\sqrt{x}}{16c^3}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}-\frac{(45b)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16c^3}$$

$$=\frac{45\sqrt{x}}{16c^3}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}-\frac{\left(45\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32c^3}-\frac{\left(45\sqrt{b}\right)\text{Subst}\left(\right.}{}$$

$$=\frac{45\sqrt{x}}{16c^3}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}-\frac{\left(45\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64c^{7/2}}-\frac{\left(45\sqrt{b}\right)\text{Sul}}{}$$

$$=\frac{45\sqrt{x}}{16c^3}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}+\frac{45\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\log\left(\sqrt{b}\right.}{64}$$

$$=\frac{45\sqrt{x}}{16c^3}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{13/4}}$$

Mathematica [A] time = 0.261666, size = 220, normalized size = 0.88

$$\frac{\frac{8\sqrt[4]{c}\sqrt{x}\left(45b^2+81bcx^2+32c^2x^4\right)}{\left(b+cx^2\right)^2}+45\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-45\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+90\sqrt{2}}{128c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((8*c^(1/4)*Sqrt[x]*(45*b^2 + 81*b*c*x^2 + 32*c^2*x^4))/(b + c*x^2)^2 + 90*
Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 90*Sqrt[2]*
b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 45*Sqrt[2]*b^(1/4)*
Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 45*Sqrt[2]*b^(
1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(128*c^(13
/4))
```

Maple [A] time = 0.06, size = 178, normalized size = 0.7

$$2\frac{\sqrt{x}}{c^3}+\frac{17\,b}{16\,c^2\left(cx^2+b\right)^2}x^{\frac{5}{2}}+\frac{13\,b^2}{16\,c^3\left(cx^2+b\right)^2}\sqrt{x}-\frac{45\,\sqrt{2}}{128\,c^3}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)-\frac{4}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*x^(1/2)/c^3+17/16/c^2*b/(c*x^2+b)^2*x^(5/2)+13/16/c^3*b^2/(c*x^2+b)^2*x^(
1/2)-45/128/c^3*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)
^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-45/64/c^3*(b/c)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/c^3*(b/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38763, size = 579, normalized size = 2.31

$$-\frac{180\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^6\sqrt{-\frac{b}{c^{13}}}+x}c^{10}\left(-\frac{b}{c^{13}}\right)^{\frac{3}{4}}-c^{10}\sqrt{x}\left(-\frac{b}{c^{13}}\right)^{\frac{3}{4}}}{b}\right)+45\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\mathrm{lo}}{64\left(c^5x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(180*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*arctan((sqrt(c
^6*sqrt(-b/c^13) + x)*c^10*(-b/c^13)^(3/4) - c^10*sqrt(x)*(-b/c^13)^(3/4))/
b) + 45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*log(45*c^3*(-b/c^
13)^(1/4) + 45*sqrt(x)) - 45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1
/4)*log(-45*c^3*(-b/c^13)^(1/4) + 45*sqrt(x)) - 4*(32*c^2*x^4 + 81*b*c*x^2
+ 45*b^2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22288, size = 281, normalized size = 1.12

$$-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^4}-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^4}-\frac{45\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\right.}{128\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/c^4 - 45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 - 45/128*sqrt(2)*(b*c^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 45/128*sqrt(2)
*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 2*sq
rt(x)/c^3 + 1/16*(17*b*c*x^(5/2) + 13*b^2*sqrt(x))/((c*x^2 + b)^2*c^3)
```

3.340 $\int \frac{x^{21/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}+\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}+$$

```
[Out] -x^(7/2)/(4*c*(b + c*x^2)^2) - (7*x^(3/2))/(16*c^2*(b + c*x^2)) - (21*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/4)*c^(11/4)) + (
21*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/4)*c^(11
/4)) + (21*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*
Sqrt[2]*b^(1/4)*c^(11/4)) - (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(11/4))
```

Rubi [A] time = 0.202912, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}+\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -x^(7/2)/(4*c*(b + c*x^2)^2) - (7*x^(3/2))/(16*c^2*(b + c*x^2)) - (21*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/4)*c^(11/4)) + (
21*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/4)*c^(11
/4)) + (21*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*
Sqrt[2]*b^(1/4)*c^(11/4)) - (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(11/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{21/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{9/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} + \frac{7\int \frac{x^{5/2}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32c^2}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^2}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} - \frac{21\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32c^{5/2}} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{\right.}}{32c^{5/2}}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64c^3} + \frac{21\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+\right.}{64c^3}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} + \frac{21\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}} - \frac{21\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}$$

$$= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2} - \frac{7x^{3/2}}{16c^2\left(b+cx^2\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}} + \frac{21\log\left(\sqrt{b}\right.}{6}$$

Mathematica [C] time = 0.0202231, size = 66, normalized size = 0.28

$$\frac{2x^{3/2}\left(7\left(b+cx^2\right)^2 {}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)-b\left(7b+5cx^2\right)\right)}{5bc^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*(-(b*(7*b + 5*c*x^2)) + 7*(b + c*x^2)^2*Hypergeometric2F1[3/4, 3
, 7/4, -((c*x^2)/b)]))/(5*b*c^2*(b + c*x^2)^2)
```

Maple [A] time = 0.058, size = 161, normalized size = 0.7

$$2\frac{1}{\left(cx^2+b\right)^2}\left(-\frac{11\,x^{7/2}}{32\,c}-\frac{7\,bx^{3/2}}{32\,c^2}\right)+\frac{21\,\sqrt{2}}{128\,c^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{21\,\sqrt{2}}{64\,c^3}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(-11/32*x^(7/2)/c-7/32*b*x^(3/2)/c^2)/(c*x^2+b)^2+21/128/c^3/(b/c)^(1/4)*
2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/
2)*2^(1/2)+(b/c)^(1/2)))+21/64/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)+1)+21/64/c^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*
x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33587, size = 585, normalized size = 2.45

$$-\frac{84\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc^{5}\sqrt{-\frac{1}{bc^{11}}}+xc^{3}}\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}-c^{3}\sqrt{x}\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\right)-21\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}\cdots\right.}{64\left(c^{4}x^{4}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(84*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*arctan(sqrt
(-b*c^5*sqrt(-1/(b*c^11)) + x)*c^3*(-1/(b*c^11))^(1/4) - c^3*sqrt(x)*(-1/(b
*c^11))^(1/4)) - 21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*l
og(b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) + 21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c
^2)*(-1/(b*c^11))^(1/4)*log(-b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) + 4*(11*c
*x^3 + 7*b*x)*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18798, size = 282, normalized size = 1.18

$$-\frac{11\,cx^{\frac{7}{2}}+7\,bx^{\frac{3}{2}}}{16\left(cx^{2}+b\right)^{2}c^{2}}+\frac{21\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^{5}}+\frac{21\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^{5}}-\frac{21\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\,\mathrm{l}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*c*x^(7/2) + 7*b*x^(3/2))/((c*x^2 + b)^2*c^2) + 21/64*sqrt(2)*(b*c
^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))
/(b*c^5) + 21/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(
1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) - 21/128*sqrt(2)*(b*c^3)^(3/4)*log(s
qrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5) + 21/128*sqrt(2)*(b*c^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

3.341 $\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{3/4}c^{9/4}}$$

```
[Out] -x^(5/2)/(4*c*(b + c*x^2)^2) - (5*Sqrt[x])/(16*c^2*(b + c*x^2)) - (5*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(9/4)) + (5*
ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(9/4))
 - (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[
2]*b^(3/4)*c^(9/4)) + (5*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(9/4))
```

Rubi [A] time = 0.189325, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{3/4}c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^(5/2)/(4*c*(b + c*x^2)^2) - (5*Sqrt[x])/(16*c^2*(b + c*x^2)) - (5*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(9/4)) + (5*
ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(9/4))
 - (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[
2]*b^(3/4)*c^(9/4)) + (5*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(9/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{x^{7/2}}{\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} + \frac{5\int \frac{x^{3/2}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32c^2}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^2}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32\sqrt{b}c^2} + \frac{5\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{32\sqrt{b}c^2}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{b}c^{5/2}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\right.}{64\sqrt{b}c^{5/2}}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} - \frac{5\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{9/4}} + \frac{5\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{64\sqrt{2}b^{3/4}c^{9/4}}$$

$$= -\frac{x^{5/2}}{4c\left(b+cx^2\right)^2} - \frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{9/4}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{9/4}} - \frac{5\log\left(\sqrt{b}-\right.}{64}$$

Mathematica [A] time = 0.111162, size = 242, normalized size = 1.01

$$\frac{-\frac{15\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}+\frac{15\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}-\frac{30\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}}+\frac{30\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}}-\frac{256c^{5/4}x^{5/2}}{\left(b+cx^2\right)^2}+\frac{40\sqrt[4]{c}\ldots}{b+c\ldots}}{384c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-160*b*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 - (256*c^(5/4)*x^(5/2))/(b + c*x^2)
^2 + (40*c^(1/4)*Sqrt[x])/(b + c*x^2) - (30*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) + (30*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/b^(3/4) - (15*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) + (15*Sqrt[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4))/(384*c^(9/4))
```

Maple [A] time = 0.06, size = 170, normalized size = 0.7

$$2\frac{1}{\left(cx^2+b\right)^2}\left(-\frac{9x^{5/2}}{32c}-\frac{5b\sqrt{x}}{32c^2}\right)+\frac{5\sqrt{2}}{128bc^2}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{5\sqrt{2}}{64bc^2}\sqrt[4]{\frac{b}{c}}\operatorname{arcta}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(-9/32*x^(5/2)/c-5/32*b*x^(1/2)/c^2)/(c*x^2+b)^2+5/128/c^2*(b/c)^(1/4)/b*
2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/
2)*2^(1/2)+(b/c)^(1/2)))+5/64/c^2*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)+1)+5/64/c^2*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4
)*x^(1/2)-1)
```

Maple [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40115, size = 599, normalized size = 2.51

$$\frac{20\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)\left(-\frac{1}{b^3c^9}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2c^4\sqrt{-\frac{1}{b^3c^9}}+x}\,b^2c^7\left(-\frac{1}{b^3c^9}\right)^{\frac{3}{4}}-b^2c^7\sqrt{x}\left(-\frac{1}{b^3c^9}\right)^{\frac{3}{4}}\right)+5\left(c^4x^4+2\,bc^3x^2+b\right.}{64\left(c^4x^4+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*arctan(sqrt
(b^2*c^4*sqrt(-1/(b^3*c^9)) + x)*b^2*c^7*(-1/(b^3*c^9))^(3/4) - b^2*c^7*sqr
t(x)*(-1/(b^3*c^9))^(3/4)) + 5*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c
^9))^(1/4)*log(b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt(x)) - 5*(c^4*x^4 + 2*b*c^3
*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*log(-b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt
(x)) - 4*(9*c*x^2 + 5*b)*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14332, size = 282, normalized size = 1.18

$$\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^3}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^3}+\frac{5\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{}\right.}{128\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b*c^3) + 5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) + 5/128*sqrt(2)*(b*c
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) - 5/128*
sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*
c^3) - 1/16*(9*c*x^(5/2) + 5*b*sqrt(x))/((c*x^2 + b)^2*c^2)
```

3.342 $\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=242

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{5/4}c^{7/4}}$$

```
[Out] -x^(3/2)/(4*c*(b + c*x^2)^2) + (3*x^(3/2))/(16*b*c*(b + c*x^2)) - (3*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(7/4)) + (3*
ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(7/4))
 + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[
2]*b^(5/4)*c^(7/4)) - (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(7/4))
```

Rubi [A] time = 0.188428, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{5/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -x^(3/2)/(4*c*(b + c*x^2)^2) + (3*x^(3/2))/(16*b*c*(b + c*x^2)) - (3*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(7/4)) + (3*
ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(7/4))
 + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[
2]*b^(5/4)*c^(7/4)) - (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(7/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^{5/2}}{\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}+\frac{3\int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx}{8c}$$

$$=-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}+\frac{3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32bc}$$

$$=-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16bc}$$

$$=-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}-\frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32bc^{3/2}}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32bc^{3/2}}$$

$$=-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64bc^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,d\right.}{64bc^2}$$

$$=-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}+\frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{64\sqrt{2}b^{5/4}c^{7/4}}$$

$$=-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{7/4}}+\frac{3\log\left(\sqrt{b}-\sqrt{\right.}}{64\sqrt{}}$$

Mathematica [C] time = 0.0154096, size = 45, normalized size = 0.19

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{b^2}-\frac{1}{\left(b+cx^2\right)^2}\right)}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*(-(b + c*x^2)^(-2) + Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)
]/b^2))/(5*c)
```

Maple [A] time = 0.061, size = 169, normalized size = 0.7

$$2\,\frac{1}{\left(cx^{2}+b\right)^{2}}\left(\frac{3\,x^{7/2}}{32\,b}-1/32\,\frac{x^{3/2}}{c}\right)+\frac{3\,\sqrt{2}}{128\,bc^{2}}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{3\,\sqrt{2}}{64\,bc^{2}}\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(3/32/b*x^(7/2)-1/32*x^(3/2)/c)/(c*x^2+b)^2+3/128/b/c^2/(b/c)^(1/4)*2^(1/
2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^
(1/2)+(b/c)^(1/2)))+3/64/b/c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/
4)*x^(1/2)+1)+3/64/b/c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3718, size = 613, normalized size = 2.53

$$-\frac{12\left(bc^{3}x^{4}+2\,b^{2}c^{2}x^{2}+b^{3}c\right)\left(-\frac{1}{b^{5}c^{7}}\right)^{\frac{1}{4}}\text{arctan}\left(\sqrt{-b^{3}c^{3}\sqrt{-\frac{1}{b^{5}c^{7}}}+xbc^{2}}\left(-\frac{1}{b^{5}c^{7}}\right)^{\frac{1}{4}}-bc^{2}\sqrt{x}\left(-\frac{1}{b^{5}c^{7}}\right)^{\frac{1}{4}}\right)-3\left(bc^{3}x^{4}+2\,b^{2}c^{2}x^{2}\right.}{64\left(bc^{3}x^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)*arctan(s
qrt(-b^3*c^3*sqrt(-1/(b^5*c^7)) + x)*b*c^2*(-1/(b^5*c^7))^(1/4) - b*c^2*sqr
t(x)*(-1/(b^5*c^7))^(1/4)) - 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5
*c^7))^(1/4)*log(b^4*c^5*(-1/(b^5*c^7))^(3/4) + sqrt(x)) + 3*(b*c^3*x^4 + 2
*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)*log(-b^4*c^5*(-1/(b^5*c^7))^(3/4
) + sqrt(x)) - 4*(3*c*x^3 - b*x)*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*
c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16774, size = 286, normalized size = 1.18

$$\frac{3\,cx^{\frac{7}{2}} - bx^{\frac{3}{2}}}{16\left(cx^2+b\right)^2bc} + \frac{3\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^4} + \frac{3\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^4} - \frac{3\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\right.}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*c*x^(7/2) - b*x^(3/2))/((c*x^2 + b)^2*b*c) + 3/64*sqrt(2)*(b*c^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2
*c^4) + 3/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^4) - 3/128*sqrt(2)*(b*c^3)^(3/4)*log(sqrt
(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4) + 3/128*sqrt(2)*(b*c^3)^
(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4)
```

3.343 $\int \frac{x^{15/2}}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=242

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{7/4}c^{5/4}}$$

```
[Out] -Sqrt[x]/(4*c*(b + c*x^2)^2) + Sqrt[x]/(16*b*c*(b + c*x^2)) - (3*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(5/4)) + (3*ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(5/4)) - (
3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b
^(7/4)*c^(5/4)) + (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(64*Sqrt[2]*b^(7/4)*c^(5/4))
```

Rubi [A] time = 0.183075, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{7/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -Sqrt[x]/(4*c*(b + c*x^2)^2) + Sqrt[x]/(16*b*c*(b + c*x^2)) - (3*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(5/4)) + (3*ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(5/4)) - (
3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b
^(7/4)*c^(5/4)) + (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(64*Sqrt[2]*b^(7/4)*c^(5/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^3}\,dx=\int \frac{x^{3/2}}{\left(b+cx^2\right)^3}\,dx$$

$$=-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}+\frac{\int \frac{1}{\sqrt{x}\left(b+cx^2\right)^2}\,dx}{8c}$$

$$=-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}+\frac{\sqrt{x}}{16bc\left(b+cx^2\right)}+\frac{3\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32bc}$$

$$=-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}+\frac{\sqrt{x}}{16bc\left(b+cx^2\right)}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16bc}$$

$$=-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}+\frac{\sqrt{x}}{16bc\left(b+cx^2\right)}+\frac{3\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{3/2}c}+\frac{3\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{3/2}c}$$

$$=-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}+\frac{\sqrt{x}}{16bc\left(b+cx^2\right)}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{3/2}c^{3/2}}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,d\right.}{64b^{3/2}c^{3/2}}$$

$$=-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}+\frac{\sqrt{x}}{16bc\left(b+cx^2\right)}-\frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{64\sqrt{2}b^{7/4}c^{5/4}}$$

$$=-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}+\frac{\sqrt{x}}{16bc\left(b+cx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{5/4}}-\frac{3\log\left(\sqrt{b}-\sqrt{2}\right.}{64\sqrt{2}}$$

Mathematica [A] time = 0.116256, size = 223, normalized size = 0.92

$$\frac{\frac{8\sqrt[4]{c}\sqrt{x}}{b^2+bcx^2}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{7/4}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{7/4}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{7/4}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{7/4}}-\frac{32\sqrt[4]{c}\sqrt{x}}{\left(b+cx^2\right)^2}}{128c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-32*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 + (8*c^(1/4)*Sqrt[x])/(b^2 + b*c*x^2)
- (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(7/4) + (6*Sq
```

```
rt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(7/4) - (3*Sqrt[2]*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4) + (3*Sqr
t[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4))/(
128*c^(5/4))
```

Maple [A] time = 0.06, size = 169, normalized size = 0.7

$$2\frac{1}{\left(cx^{2}+b\right)^{2}}\left(1/32\,\frac{x^{5/2}}{b}-\frac{3\,\sqrt{x}}{32\,c}\right)+\frac{3\,\sqrt{2}}{128\,b^{2}c}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{3\,\sqrt{2}}{64\,b^{2}c}\sqrt[4]{\frac{b}{c}}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32/b*x^(5/2)-3/32*x^(1/2)/c)/(c*x^2+b)^2+3/128/b^2/c*(b/c)^(1/4)*2^(1/
2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^
(1/2)+(b/c)^(1/2)))+3/64/b^2/c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/
4)*x^(1/2)+1)+3/64/b^2/c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38107, size = 608, normalized size = 2.51

$$\frac{12\left(bc^{3}x^{4}+2\,b^{2}c^{2}x^{2}+b^{3}c\right)\left(-\frac{1}{b^{7}c^{5}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^{4}c^{2}\sqrt{-\frac{1}{b^{7}c^{5}}}+xb^{5}c^{4}}\left(-\frac{1}{b^{7}c^{5}}\right)^{\frac{3}{4}}-b^{5}c^{4}\sqrt{x}\left(-\frac{1}{b^{7}c^{5}}\right)^{\frac{3}{4}}\right)+3\left(bc^{3}x^{4}+2\,b^{2}c^{2}x^{2}\right.}{64\left(bc^{3}x^{4}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4)*arctan(sq
rt(b^4*c^2*sqrt(-1/(b^7*c^5)) + x)*b^5*c^4*(-1/(b^7*c^5))^(3/4) - b^5*c^4*s
qrt(x)*(-1/(b^7*c^5))^(3/4)) + 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b
^7*c^5))^(1/4)*log(b^2*c*(-1/(b^7*c^5))^(1/4) + sqrt(x)) - 3*(b*c^3*x^4 + 2
*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4)*log(-b^2*c*(-1/(b^7*c^5))^(1/4)
+ sqrt(x)) + 4*(c*x^2 - 3*b)*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13433, size = 285, normalized size = 1.18

$$\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^2}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^2}+\frac{3\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\vphantom{x}}\right.}{128\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^2*c^2) + 3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 3/128*sqrt(2)*
(b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) -
3/128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c
))/(b^2*c^2) + 1/16*(c*x^(5/2) - 3*b*sqrt(x))/((c*x^2 + b)^2*b*c)
```

3.344 $\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=239

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{9/4}c^{3/4}}$$

```
[Out] x^(3/2)/(4*b*(b + c*x^2)^2) + (5*x^(3/2))/(16*b^2*(b + c*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(3/4))
+ (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(9/4)*c^(3/4)) - (5*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(3/4))
```

Rubi [A] time = 0.184859, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{9/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] x^(3/2)/(4*b*(b + c*x^2)^2) + (5*x^(3/2))/(16*b^2*(b + c*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(3/4))
+ (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(9/4)*c^(3/4)) - (5*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(3/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{\sqrt{x}}{\left(b+cx^2\right)^3}\,dx$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5\int \frac{\sqrt{x}}{\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^2}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^2\sqrt{c}} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^2\sqrt{c}}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^2c} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,d\right.}{64b^2c}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} + \frac{5\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{3/4}} - \frac{5\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{64\sqrt{2}b^{9/4}c^{3/4}}$$

$$= \frac{x^{3/2}}{4b\left(b+cx^2\right)^2} + \frac{5x^{3/2}}{16b^2\left(b+cx^2\right)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{3/4}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{3/4}} + \frac{5\log\left(\sqrt{b}-\sqrt{\right.}}{64\sqrt{}}$$

Mathematica [C] time = 0.0054439, size = 29, normalized size = 0.12

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{cx^2}{b}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((c*x^2)/b)])/(3*b^3)
```

Maple [A] time = 0.054, size = 175, normalized size = 0.7

$$\frac{1}{4\,b\left(cx^2+b\right)^2}x^{\frac{3}{2}}+\frac{5}{16\,b^2\left(cx^2+b\right)}x^{\frac{3}{2}}+\frac{5\sqrt{2}}{128\,b^2c}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{5\sqrt{2}}{64\,b^2c}\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x^(3/2)/b/(c*x^2+b)^2+5/16*x^(3/2)/b^2/(c*x^2+b)+5/128/b^2/c/(b/c)^(1/4
)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(
1/2)*2^(1/2)+(b/c)^(1/2)))+5/64/b^2/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)+1)+5/64/b^2/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1
/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38819, size = 602, normalized size = 2.52

$$-\frac{20\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{4}\right)\left(-\frac{1}{b^{9}c^{3}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b^{5}c\sqrt{-\frac{1}{b^{9}c^{3}}}+x}b^{2}c\left(-\frac{1}{b^{9}c^{3}}\right)^{\frac{1}{4}}-b^{2}c\sqrt{x}\left(-\frac{1}{b^{9}c^{3}}\right)^{\frac{1}{4}}\right)-5\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}\right.}{64\left(b^{2}c^{2}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(20*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*arctan(sqr
t(-b^5*c*sqrt(-1/(b^9*c^3)) + x)*b^2*c*(-1/(b^9*c^3))^(1/4) - b^2*c*sqrt(x)
*(-1/(b^9*c^3))^(1/4)) - 5*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))
^(1/4)*log(b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt(x)) + 5*(b^2*c^2*x^4 + 2*b^3
*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*log(-b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt
(x)) - 4*(5*c*x^3 + 9*b*x)*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15694, size = 282, normalized size = 1.18

$$\frac{5\,cx^{\frac{7}{2}}+9\,bx^{\frac{3}{2}}}{16\left(cx^{2}+b\right)^{2}b^{2}}+\frac{5\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c^{3}}+\frac{5\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c^{3}}-\frac{5\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\log\left(\cdots\right.}{12\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*c*x^(7/2) + 9*b*x^(3/2))/((c*x^2 + b)^2*b^2) + 5/64*sqrt(2)*(b*c^3)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b
^3*c^3) + 5/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) - 5/128*sqrt(2)*(b*c^3)^(3/4)*log(sq
rt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3) + 5/128*sqrt(2)*(b*c^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

3.345 $\int \frac{x^{11/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=239

$$\frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)}-\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}}+$$

```
[Out] Sqrt[x]/(4*b*(b + c*x^2)^2) + (7*Sqrt[x])/(16*b^2*(b + c*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(1/
4)) - (21*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*S
qrt[2]*b^(11/4)*c^(1/4)) + (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rubi [A] time = 0.186145, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1584, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)}-\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] Sqrt[x]/(4*b*(b + c*x^2)^2) + (7*Sqrt[x])/(16*b^2*(b + c*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(1/
4)) - (21*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*S
qrt[2]*b^(11/4)*c^(1/4)) + (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{\sqrt{x}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\int \frac{1}{\sqrt{x}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{5/2}} + \frac{21\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{\right.}}{32b^{5/2}}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^{5/2}\sqrt{c}} + \frac{21\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\right.}{64b^{5/2}\sqrt{c}}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} - \frac{21\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{64\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}} - \frac{21\log\left(\sqrt{b}\right.}{64}$$

Mathematica [A] time = 0.0860954, size = 220, normalized size = 0.92

$$\frac{\frac{32b^{7/4}\sqrt{x}}{\left(b+cx^2\right)^2}+\frac{56b^{3/4}\sqrt{x}}{b+cx^2}-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}}+\frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right.}{\sqrt[4]{c}}}{128b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((32*b^(7/4)*Sqrt[x])/(b + c*x^2)^2 + (56*b^(3/4)*Sqrt[x])/(b + c*x^2) - (4
2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) + (42*Sqrt
[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) - (21*Sqrt[2]*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4) + (21*Sqr
t[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4))/(
128*b^(11/4))
```

Maple [A] time = 0.054, size = 166, normalized size = 0.7

$$\frac{1}{4\,b\left(cx^2+b\right)^2}\sqrt{x}+\frac{7}{16\,b^2\left(cx^2+b\right)}\sqrt{x}+\frac{21\,\sqrt{2}}{128\,b^3}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{21\,\sqrt{2}}{64\,b^3}\sqrt[4]{\frac{b}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x^(1/2)/b/(c*x^2+b)^2+7/16*x^(1/2)/b^2/(c*x^2+b)+21/128/b^3*(b/c)^(1/4)
*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2)))+21/64/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)+1)+21/64/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)
*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40925, size = 576, normalized size = 2.41

$$\frac{84\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{4}\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^{6}\sqrt{-\frac{1}{b^{11}c}}+x}\,b^{8}c\left(-\frac{1}{b^{11}c}\right)^{\frac{3}{4}}-b^{8}c\sqrt{x}\left(-\frac{1}{b^{11}c}\right)^{\frac{3}{4}}\right)+21\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{\cdots}\right.}{64\left(b^{2}c^{2}x^{4}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(84*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*arctan(sqrt(
b^6*sqrt(-1/(b^11*c)) + x)*b^8*c*(-1/(b^11*c))^(3/4) - b^8*c*sqrt(x)*(-1/(b
^11*c))^(3/4)) + 21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*l
og(b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) - 21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4
)*(-1/(b^11*c))^(1/4)*log(-b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) + 4*(7*c*x^2
+ 11*b)*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14383, size = 282, normalized size = 1.18

$$\frac{21\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c}+\frac{21\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c}+\frac{21\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\cdots\right.}{128\,b^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/(b^3*c) + 21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 21/128*sqrt(2)*(
b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) - 21/
128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b^3*c) + 1/16*(7*c*x^(5/2) + 11*b*sqrt(x))/((c*x^2 + b)^2*b^2)
```

3.346 $\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=251

$$\frac{9}{16b^2\sqrt{x}\,(b+cx^2)}-\frac{45\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}}+\frac{45\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}}+\frac{45\sqrt[4]{c}\tan^{-1}\left(1-\right.}{32\sqrt{2}b^{13/}}$$

```
[Out] -45/(16*b^3*Sqrt[x]) + 1/(4*b*Sqrt[x]*(b + c*x^2)^2) + 9/(16*b^2*Sqrt[x]*(b
 + c*x^2)) + (45*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32
*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(32*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4)) + (45*c^(1/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.216856, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{16b^2\sqrt{x}\,(b+cx^2)}-\frac{45\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}}+\frac{45\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}}+\frac{45\sqrt[4]{c}\tan^{-1}\left(1-\right.}{32\sqrt{2}b^{13/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -45/(16*b^3*Sqrt[x]) + 1/(4*b*Sqrt[x]*(b + c*x^2)^2) + 9/(16*b^2*Sqrt[x]*(b
 + c*x^2)) + (45*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32
*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(32*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4)) + (45*c^(1/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{3/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9\int \frac{1}{x^{3/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} + \frac{45\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{(45c)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{(45c)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} + \frac{\left(45\sqrt{c}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^3} - \frac{\left(45\sqrt{c}\right)}{}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{45\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^3} - \frac{45\,\text{Subs}}{}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} - \frac{45\sqrt[4]{c}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}} + \frac{45\sqrt[4]{c}\,\text{l}}{}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2} + \frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)} + \frac{45\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}} - \frac{45\sqrt[4]{c}\tan^{-1}\left(1+\right.}{32\sqrt{2}b^{13/4}}$$

Mathematica [C] time = 0.0058236, size = 27, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-\frac{cx^2}{b}\right)}{b^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 3, 3/4, -((c*x^2)/b)])/(b^3*Sqrt[x])
```

Maple [A] time = 0.063, size = 178, normalized size = 0.7

$$-2\frac{1}{b^3\sqrt{x}}-\frac{13c^2}{16b^3(cx^2+b)^2}x^{\frac{7}{2}}-\frac{17c}{16b^2(cx^2+b)^2}x^{\frac{3}{2}}-\frac{45\sqrt{2}}{128b^3}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -2/b^3/x^(1/2)-13/16*c^2/b^3/(c*x^2+b)^2*x^(7/2)-17/16*c/b^2/(c*x^2+b)^2*x^
(3/2)-45/128/b^3/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c
)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-45/64/b^3/(b/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45/64/b^3/(b/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34425, size = 672, normalized size = 2.68

$$180\left(b^3c^2x^5+2b^4cx^3+b^5x\right)\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125b^3c\sqrt{x}\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-8303765625b^7c\sqrt{-\frac{c}{b^{13}}}+8303765625c^2xb^3}\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}}{91125c}\right)-45\left(b^3c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(180*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-c/b^13)^(1/4)*arctan(-1/911
25*(91125*b^3*c*sqrt(x)*(-c/b^13)^(1/4) - sqrt(-8303765625*b^7*c*sqrt(-c/b^
13) + 8303765625*c^2*x)*b^3*(-c/b^13)^(1/4))/c) - 45*(b^3*c^2*x^5 + 2*b^4*c
*x^3 + b^5*x)*(-c/b^13)^(1/4)*log(91125*b^10*(-c/b^13)^(3/4) + 91125*c*sqrt
(x)) + 45*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-c/b^13)^(1/4)*log(-91125*b^
10*(-c/b^13)^(3/4) + 91125*c*sqrt(x)) - 4*(45*c^2*x^4 + 81*b*c*x^2 + 32*b^2
)*sqrt(x))/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.158, size = 297, normalized size = 1.18

$$-\frac{2}{b^{3}\sqrt{x}}-\frac{45\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{4}c^{2}}-\frac{45\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{4}c^{2}}+\frac{45\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\right.}{128\,b^{4}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -2/(b^3*sqrt(x)) - 45/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 45/64*sqrt(2)*(b*c^3)^(3/
4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^4*
c^2) + 45/128*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + s
qrt(b/c))/(b^4*c^2) - 45/128*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 1/16*(13*c^2*x^(7/2) + 17*b*c*x^(3/2)
)/((c*x^2 + b)^2*b^3)
```

3.347 $\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=251

$$\frac{77c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{64\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{64\sqrt{2}b^{15/4}}+\frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\text{t}}{}$$

```
[Out] -77/(48*b^3*x^(3/2)) + 1/(4*b*x^(3/2)*(b + c*x^2)^2) + 11/(16*b^2*x^(3/2)*(
b + c*x^2)) + (77*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*b^(15/4)) + (77*c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4))
```

Rubi [A] time = 0.213661, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{64\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{cx}\right)}{64\sqrt{2}b^{15/4}}+\frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -77/(48*b^3*x^(3/2)) + 1/(4*b*x^(3/2)*(b + c*x^2)^2) + 11/(16*b^2*x^(3/2)*(
b + c*x^2)) + (77*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*b^(15/4)) + (77*c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{5/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11\int \frac{1}{x^{5/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} + \frac{77\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{(77c)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{(77c)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^3}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{(77c)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{7/2}} - \frac{(77c)\,\mathrm{Su}}{}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} - \frac{\left(77\sqrt{c}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^{7/2}} - \cdots$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} + \frac{77c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}} - \frac{77c^{3}}{}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}\left(b+cx^2\right)^2} + \frac{11}{16b^2x^{3/2}\left(b+cx^2\right)} + \frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}} - \frac{77c^{3/4}\tan^{-1}\left(1\right.}{32\sqrt{2}b^{1}}$$

Mathematica [C] time = 0.006744, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-\frac{cx^2}{b}\right)}{3b^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 3, 1/4, -((c*x^2)/b)])/(3*b^3*x^(3/2))
```

Maple [A] time = 0.066, size = 181, normalized size = 0.7

$$-\frac{2}{3 b^3} x^{-\frac{3}{2}} - \frac{15 c^2}{16 b^3 \left(c x^2+b\right)^2} x^{\frac{5}{2}} - \frac{19 c}{16 b^2 \left(c x^2+b\right)^2} \sqrt{x} - \frac{77 c \sqrt{2}}{128 b^4} \sqrt[4]{\frac{b}{c}} \ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -2/3/b^3/x^(3/2)-15/16/b^3*c^2/(c*x^2+b)^2*x^(5/2)-19/16/b^2*c/(c*x^2+b)^2*
x^(1/2)-77/128/b^4*c*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+
(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-77/64/b^4*c*(b/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-77/64/b^4*c*(b/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3736, size = 645, normalized size = 2.57

$$-\frac{924 \left(b^3 c^2 x^6 + 2 b^4 c x^4 + b^5 x^2\right) \left(-\frac{c^3}{b^{15}}\right)^{\frac{1}{4}} \arctan\left(-\frac{b^{11} c \sqrt{x} \left(-\frac{c^3}{b^{15}}\right)^{\frac{3}{4}} - \sqrt{b^8 \sqrt{-\frac{c^3}{b^{15}}} + c^2 x} b^{11} \left(-\frac{c^3}{b^{15}}\right)^{\frac{3}{4}}}{c^3}\right) + 231 \left(b^3 c^2 x^6 + 2 b^4 c x^4 + b^5 x^2\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(924*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/b^15)^(1/4)*arctan(
-(b^11*c*sqrt(x)*(-c^3/b^15)^(3/4) - sqrt(b^8*sqrt(-c^3/b^15) + c^2*x)*b^11
*(-c^3/b^15)^(3/4))/c^3) + 231*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/
b^15)^(1/4)*log(77*b^4*(-c^3/b^15)^(1/4) + 77*c*sqrt(x)) - 231*(b^3*c^2*x^6
 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/b^15)^(1/4)*log(-77*b^4*(-c^3/b^15)^(1/4) +
 77*c*sqrt(x)) + 4*(77*c^2*x^4 + 121*b*c*x^2 + 32*b^2)*sqrt(x))/(b^3*c^2*x^
6 + 2*b^4*c*x^4 + b^5*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16165, size = 281, normalized size = 1.12

$$-\frac{77\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4}-\frac{77\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4}-\frac{77\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x\right.}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/b^4 - 77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 - 77/128*sqrt(2)*(b*c^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 77/128*sqrt(2)
*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/16
*(15*c^2*x^(5/2) + 19*b*c*sqrt(x))/((c*x^2 + b)^2*b^3) - 2/3/(b^3*x^(3/2))
```

3.348 $\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=264

$$\frac{117c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}-\frac{117c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}}+\frac{117c}{}$$

```
[Out] -117/(80*b^3*x^(5/2)) + (117*c)/(16*b^4*Sqrt[x]) + 1/(4*b*x^(5/2)*(b + c*x^
2)^2) + 13/(16*b^2*x^(5/2)*(b + c*x^2)) - (117*c^(5/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17
/4)) - (117*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(64*Sqrt[2]*b^(17/4))
```

Rubi [A] time = 0.233198, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}-\frac{117c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}}+\frac{117c}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -117/(80*b^3*x^(5/2)) + (117*c)/(16*b^4*Sqrt[x]) + 1/(4*b*x^(5/2)*(b + c*x^
2)^2) + 13/(16*b^2*x^(5/2)*(b + c*x^2)) - (117*c^(5/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17
/4)) - (117*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(64*Sqrt[2]*b^(17/4))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{7/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^{5/2}\left(b+cx^2\right)^2} + \frac{13\int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{5/2}\left(b+cx^2\right)^2} + \frac{13}{16b^2x^{5/2}\left(b+cx^2\right)} + \frac{117\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{117}{80b^3x^{5/2}} + \frac{1}{4bx^{5/2}\left(b+cx^2\right)^2} + \frac{13}{16b^2x^{5/2}\left(b+cx^2\right)} - \frac{(117c)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}\left(b+cx^2\right)^2} + \frac{13}{16b^2x^{5/2}\left(b+cx^2\right)} + \frac{\left(117c^2\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^4}$$

$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}\left(b+cx^2\right)^2} + \frac{13}{16b^2x^{5/2}\left(b+cx^2\right)} + \frac{\left(117c^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^4}$$

$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}\left(b+cx^2\right)^2} + \frac{13}{16b^2x^{5/2}\left(b+cx^2\right)} - \frac{\left(117c^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \right.}{32b^4}$$

$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}\left(b+cx^2\right)^2} + \frac{13}{16b^2x^{5/2}\left(b+cx^2\right)} + \frac{(117c)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x\right.}{64b^4}$$

$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}\left(b+cx^2\right)^2} + \frac{13}{16b^2x^{5/2}\left(b+cx^2\right)} + \frac{117c^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{64\sqrt{2}b^{17/4}}$$

$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}\left(b+cx^2\right)^2} + \frac{13}{16b^2x^{5/2}\left(b+cx^2\right)} - \frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}} + \frac{117}{}$$

<u>Mathematica</u> [C] time = 0.0073955, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-\frac{cx^2}{b}\right)}{5b^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 3, -1/4, -((c*x^2)/b)])/(5*b^3*x^(5/2))
```

Maple [A] time = 0.064, size = 192, normalized size = 0.7

$$-\frac{2}{5\,b^3}x^{-\frac{5}{2}}+6\,\frac{c}{b^4\sqrt{x}}+\frac{21\,c^3}{16\,b^4\left(cx^2+b\right)^2}x^{\frac{7}{2}}+\frac{25\,c^2}{16\,b^3\left(cx^2+b\right)^2}x^{\frac{3}{2}}+\frac{117\,c\sqrt{2}}{128\,b^4}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -2/5/b^3/x^(5/2)+6*c/b^4/x^(1/2)+21/16*c^3/b^4/(c*x^2+b)^2*x^(7/2)+25/16*c^
2/b^3/(c*x^2+b)^2*x^(3/2)+117/128*c/b^4/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/
4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))
)+117/64*c/b^4/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+11
7/64*c/b^4/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40614, size = 778, normalized size = 2.95

$$-\frac{2340\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613\,b^4c^4\sqrt{x}\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}-\sqrt{-2565164201769\,b^9c^5\sqrt{-\frac{c^5}{b^{17}}}+2565164201769\,c^8xb^4\left(-\frac{c^5}{b^{17}}\right)}}{1601613\,c^5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")

[Out] -1/320*(2340*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*arctan(-1/1601613*(1601613*b^4*c^4*sqrt(x)*(-c^5/b^17)^(1/4) - sqrt(-2565164201769*b^9*c^5*sqrt(-c^5/b^17) + 2565164201769*c^8*x)*b^4*(-c^5/b^17)^(1/4))/c^5) - 585*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*log(1601613*b^13*(-c^5/b^17)^(3/4) + 1601613*c^4*sqrt(x)) + 585*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*log(-1601613*b^13*(-c^5/b^17)^(3/4) + 1601613*c^4*sqrt(x)) - 4*(585*c^3*x^6 + 1053*b*c^2*x^4 + 416*b^2*c*x^2 - 32*b^3)*sqrt(x))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)/(c*x**4+b*x**2)**3,x)

[Out] Timed out

Giac [A] time = 1.18525, size = 313, normalized size = 1.19

$$\frac{117\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c}+\frac{117\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c}-\frac{117\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\right.}{128\,b^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")

[Out] 117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) + 117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) - 117/128*sqrt(2

```
)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) +
117/128*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
/c))/(b^5*c) + 1/16*(21*c^3*x^(7/2) + 25*b*c^2*x^(3/2))/((c*x^2 + b)^2*b^4)
 + 2/5*(15*c*x^2 - b)/(b^4*x^(5/2))
```

3.349 $\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=264

$$-\frac{165c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}+\frac{165c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}+\frac{165c}{}$$

```
[Out] -165/(112*b^3*x^(7/2)) + (55*c)/(16*b^4*x^(3/2)) + 1/(4*b*x^(7/2)*(b + c*x^
2)^2) + 15/(16*b^2*x^(7/2)*(b + c*x^2)) - (165*c^(7/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) + (165*c^(7/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) - (165*c^(7/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19
/4)) + (165*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(64*Sqrt[2]*b^(19/4))
```

Rubi [A] time = 0.231428, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{165c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}+\frac{165c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}+\frac{165c}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -165/(112*b^3*x^(7/2)) + (55*c)/(16*b^4*x^(3/2)) + 1/(4*b*x^(7/2)*(b + c*x^
2)^2) + 15/(16*b^2*x^(7/2)*(b + c*x^2)) - (165*c^(7/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) + (165*c^(7/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) - (165*c^(7/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19
/4)) + (165*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(64*Sqrt[2]*b^(19/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^{9/2}\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15\int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}\,dx}{8b} \\
&= \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{165\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{32b^2} \\
&= -\frac{165}{112b^3x^{7/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{(165c)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^3} \\
&= -\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^2\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^4} \\
&= -\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\right.}{16b^4} \\
&= -\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^2\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,\right.}{32b^{9/2}} \\
&= -\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{\left(165c^{3/2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x}\right.}{64b^{9/2}} \\
&= -\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{165c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{64\sqrt{2}b^{19/4}} \\
&= -\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}+
\end{aligned}
$$

Mathematica [C] time = 0.0063249, size = 29, normalized size = 0.11

$$
-\frac{2\,{}_2F_1\left(-\frac{7}{4},3;-\frac{3}{4};-\frac{cx^2}{b}\right)}{7b^3x^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 3, -3/4, -((c*x^2)/b)])/(7*b^3*x^(7/2))
```

Maple [A] time = 0.065, size = 198, normalized size = 0.8

$$-\frac{2}{7b^3}x^{-\frac{7}{2}}+2\frac{c}{b^4x^{3/2}}+\frac{23c^3}{16b^4\left(cx^2+b\right)^2}x^{\frac{5}{2}}+\frac{27c^2}{16b^3\left(cx^2+b\right)^2}\sqrt{x}+\frac{165c^2\sqrt{2}}{128b^5}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -2/7/b^3/x^(7/2)+2*c/b^4/x^(3/2)+23/16/b^4*c^3/(c*x^2+b)^2*x^(5/2)+27/16/b^
3*c^2/(c*x^2+b)^2*x^(1/2)+165/128/b^5*c^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(
1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2
)))+165/64/b^5*c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1
)+165/64/b^5*c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40631, size = 689, normalized size = 2.61

$$4620\left(b^4c^2x^8+2b^5cx^6+b^6x^4\right)\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{14}c^2\sqrt{x}\left(-\frac{c^7}{b^{19}}\right)^{\frac{3}{4}}-\sqrt{b^{10}\sqrt{-\frac{c^7}{b^{19}}}+c^4x}\,b^{14}\left(-\frac{c^7}{b^{19}}\right)^{\frac{3}{4}}}{c^7}\right)+1155\left(b^4c^2x^8+2b^5cx^6+b^6x^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/448*(4620*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-c^7/b^19)^(1/4)*arctan(
-(b^14*c^2*sqrt(x)*(-c^7/b^19)^(3/4) - sqrt(b^10*sqrt(-c^7/b^19) + c^4*x)*b
^14*(-c^7/b^19)^(3/4))/c^7) + 1155*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-
c^7/b^19)^(1/4)*log(165*b^5*(-c^7/b^19)^(1/4) + 165*c^2*sqrt(x)) - 1155*(b^
4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-c^7/b^19)^(1/4)*log(-165*b^5*(-c^7/b^1
9)^(1/4) + 165*c^2*sqrt(x)) + 4*(385*c^3*x^6 + 605*b*c^2*x^4 + 160*b^2*c*x^
2 - 32*b^3)*sqrt(x))/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19944, size = 302, normalized size = 1.14

$$\frac{165\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{5}}+\frac{165\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{5}}+\frac{165\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\log\left(\sqrt{2}\sqrt{x}\left(\right.\right.}{128\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*
sqrt(x))/(b/c)^(1/4))/b^5 + 165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 + 165/128*sqrt(2)*(b
*c^3)^(1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 165/12
```

```
8*sqrt(2)*(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/b^5 + 1/16*(23*c^3*x^(5/2) + 27*b*c^2*sqrt(x))/((c*x^2 + b)^2*b^4) + 2/7*(
7*c*x^2 - b)/(b^4*x^(7/2))
```

3.350 $\int \frac{\sqrt{x}}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=279

$$-\frac{221c^2}{16b^5\sqrt{x}}-\frac{221c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{32\sqrt{2}b^{21/4}}$$

```
[Out] -221/(144*b^3*x^(9/2)) + (221*c)/(80*b^4*x^(5/2)) - (221*c^2)/(16*b^5*Sqrt[
x]) + 1/(4*b*x^(9/2)*(b + c*x^2)^2) + 17/(16*b^2*x^(9/2)*(b + c*x^2)) + (22
1*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/
4)) - (221*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[
2]*b^(21/4)) - (221*c^(9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4)) + (221*c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rubi [A] time = 0.267987, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{221c^2}{16b^5\sqrt{x}}-\frac{221c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{32\sqrt{2}b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -221/(144*b^3*x^(9/2)) + (221*c)/(80*b^4*x^(5/2)) - (221*c^2)/(16*b^5*Sqrt[
x]) + 1/(4*b*x^(9/2)*(b + c*x^2)^2) + 17/(16*b^2*x^(9/2)*(b + c*x^2)) + (22
1*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/
4)) - (221*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[
2]*b^(21/4)) - (221*c^(9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4)) + (221*c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17\int \frac{1}{x^{11/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{221\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{(221c)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{\left(221c^2\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^4}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{\left(221c^3\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^5}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{\left(221c^3\right)\operatorname{Subst}\left(\int \frac{x^2}{b+c}\right.}{16b^5}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{\left(221c^{5/2}\right)\operatorname{Subst}\left(\int \right.}{32b^5}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{\left(221c^2\right)\operatorname{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}}\right.}{64b}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{221c^{9/4}\log\left(\sqrt{b}-\sqrt{2}\right.}{64\sqrt{2}b}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{221c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{32\sqrt{2}b^{21/4}}$$

Mathematica [C] time = 0.0080898, size = 29, normalized size = 0.1

$$-\frac{2\,{}_2F_1\left(-\frac{9}{4},3;-\frac{5}{4};-\frac{cx^2}{b}\right)}{9b^3x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-2*Hypergeometric2F1[-9/4, 3, -5/4, -((c*x^2)/b)])/(9*b^3*x^(9/2))
```

Maple [A] time = 0.064, size = 209, normalized size = 0.8

$$-\frac{2}{9\,b^3}x^{-\frac{9}{2}}-12\,\frac{c^2}{b^5\sqrt{x}}+\frac{6\,c}{5\,b^4}x^{-\frac{5}{2}}-\frac{29\,c^4}{16\,b^5\left(cx^2+b\right)^2}x^{\frac{7}{2}}-\frac{33\,c^3}{16\,b^4\left(cx^2+b\right)^2}x^{\frac{3}{2}}-\frac{221\,c^2\sqrt{2}}{128\,b^5}\ln\left(\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -2/9/b^3/x^(9/2)-12*c^2/b^5/x^(1/2)+6/5*c/b^4/x^(5/2)-29/16*c^4/b^5/(c*x^2+
b)^2*x^(7/2)-33/16*c^3/b^4/(c*x^2+b)^2*x^(3/2)-221/128*c^2/b^5/(b/c)^(1/4)*
2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/
2)*2^(1/2)+(b/c)^(1/2)))-221/64*c^2/b^5/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(b/c)^(1/4)*x^(1/2)+1)-221/64*c^2/b^5/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42449, size = 828, normalized size = 2.97

$$\frac{39780\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}\arctan\left(-\frac{10793861\,b^5c^7\sqrt{x}\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}-\sqrt{-116507435287321\,b^{11}c^9\sqrt{-\frac{c^9}{b^{21}}}+116507435287321\,c^{14}xb^5\left(-\right.}}{10793861\,c^9}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2880*(39780*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*arcta
n(-1/10793861*(10793861*b^5*c^7*sqrt(x)*(-c^9/b^21)^(1/4) - sqrt(-116507435
287321*b^11*c^9*sqrt(-c^9/b^21) + 116507435287321*c^14*x)*b^5*(-c^9/b^21)^(
1/4))/c^9) - 9945*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*l
og(10793861*b^16*(-c^9/b^21)^(3/4) + 10793861*c^7*sqrt(x)) + 9945*(b^5*c^2*
x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*log(-10793861*b^16*(-c^9/b^2
1)^(3/4) + 10793861*c^7*sqrt(x)) - 4*(9945*c^4*x^8 + 17901*b*c^3*x^6 + 7072
*b^2*c^2*x^4 - 544*b^3*c*x^2 + 160*b^4)*sqrt(x))/(b^5*c^2*x^9 + 2*b^6*c*x^7
 + b^7*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16819, size = 312, normalized size = 1.12

$$-\frac{221\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}-\frac{221\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}+\frac{221\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}\right.}{128\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/b^6 - 221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 + 221/128*sqrt(2)*(b*c^
3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 - 221/128*sqr
t(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 -
1/16*(29*c^4*x^(7/2) + 33*b*c^3*x^(3/2))/((c*x^2 + b)^2*b^5) - 2/45*(270*c^
2*x^4 - 27*b*c*x^2 + 5*b^2)/(b^5*x^(9/2))
```

3.351 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=279

$$-\frac{95c^2}{16b^5x^{3/2}}+\frac{285c^{11/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}-\frac{285c^{11/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}+\frac{285c^{11/4}\tan^{-1}\left(1-\right.}{32\sqrt{2}b^{23/4}}$$

```
[Out] -285/(176*b^3*x^(11/2)) + (285*c)/(112*b^4*x^(7/2)) - (95*c^2)/(16*b^5*x^(3
/2)) + 1/(4*b*x^(11/2)*(b + c*x^2)^2) + 19/(16*b^2*x^(11/2)*(b + c*x^2)) +
(285*c^(11/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(23/4)) - (285*c^(11/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*
Sqrt[2]*b^(23/4)) + (285*c^(11/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4)) - (285*c^(11/4)*Log[Sqrt[b] + Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rubi [A] time = 0.260175, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1584, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{95c^2}{16b^5x^{3/2}}+\frac{285c^{11/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}-\frac{285c^{11/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}+\frac{285c^{11/4}\tan^{-1}\left(1-\right.}{32\sqrt{2}b^{23/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```


```
[Out] -285/(176*b^3*x^(11/2)) + (285*c)/(112*b^4*x^(7/2)) - (95*c^2)/(16*b^5*x^(3
/2)) + 1/(4*b*x^(11/2)*(b + c*x^2)^2) + 19/(16*b^2*x^(11/2)*(b + c*x^2)) +
(285*c^(11/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(23/4)) - (285*c^(11/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*
Sqrt[2]*b^(23/4)) + (285*c^(11/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4)) - (285*c^(11/4)*Log[Sqrt[b] + Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```


Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{13/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19\int \frac{1}{x^{13/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} + \frac{285\int \frac{1}{x^{13/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} - \frac{(285c)\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} + \frac{\left(285c^2\right)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^4}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} - \frac{\left(285c^3\right)\int \frac{}{\sqrt{}}}{32b}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} - \frac{\left(285c^3\right)\text{Sub}}{}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} - \frac{\left(285c^3\right)\text{Sub}}{}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} - \frac{\left(285c^{5/2}\right)\text{Su}}{}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} + \frac{285c^{11/4}\log}{}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\left(b+cx^2\right)^2} + \frac{19}{16b^2x^{11/2}\left(b+cx^2\right)} + \frac{285c^{11/4}\tan}{32}$$

Mathematica [C] time = 0.0078669, size = 29, normalized size = 0.1

$$-\frac{2\,{}_2F_1\left(-\frac{11}{4},3;-\frac{7}{4};-\frac{cx^2}{b}\right)}{11b^3x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-11/4, 3, -7/4, -((c*x^2)/b)])/(11*b^3*x^(11/2))
```

Maple [A] time = 0.067, size = 209, normalized size = 0.8

$$-\frac{2}{11\,b^3}x^{-\frac{11}{2}}-4\,\frac{c^2}{b^5x^{3/2}}+\frac{6\,c}{7\,b^4}x^{-\frac{7}{2}}-\frac{31\,c^4}{16\,b^5\left(cx^2+b\right)^2}x^{\frac{5}{2}}-\frac{35\,c^3}{16\,b^4\left(cx^2+b\right)^2}\sqrt{x}-\frac{285\,c^3\sqrt{2}}{128\,b^6}\sqrt[4]{\frac{b}{c}}\ln\left(\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] -2/11/b^3/x^(11/2)-4*c^2/b^5/x^(3/2)+6/7*c/b^4/x^(7/2)-31/16/b^5*c^4/(c*x^2
+b)^2*x^(5/2)-35/16/b^4*c^3/(c*x^2+b)^2*x^(1/2)-285/128/b^6*c^3*(b/c)^(1/4)
*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2)))-285/64/b^6*c^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(b/c)^(1/4)*x^(1/2)+1)-285/64/b^6*c^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44937, size = 744, normalized size = 2.67

$$-\frac{87780\left(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6\right)\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{17}c^3\sqrt{x}\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{3}{4}}-\sqrt{b^{12}\sqrt{-\frac{c^{11}}{b^{23}}}+c^6x}\,b^{17}\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{3}{4}}}{c^{11}}\right)+21945\left(b^5c^2x^{10}+2\,b^6cx^8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4928*(87780*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-c^11/b^23)^(1/4)*ar
ctan(-(b^17*c^3*sqrt(x)*(-c^11/b^23)^(3/4) - sqrt(b^12*sqrt(-c^11/b^23) + c
^6*x)*b^17*(-c^11/b^23)^(3/4))/c^11) + 21945*(b^5*c^2*x^10 + 2*b^6*c*x^8 +
b^7*x^6)*(-c^11/b^23)^(1/4)*log(285*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x
)) - 21945*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-c^11/b^23)^(1/4)*log(-2
85*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x)) + 4*(7315*c^4*x^8 + 11495*b*c^
3*x^6 + 3040*b^2*c^2*x^4 - 608*b^3*c*x^2 + 224*b^4)*sqrt(x))/(b^5*c^2*x^10
+ 2*b^6*c*x^8 + b^7*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15865, size = 328, normalized size = 1.18

$$-\frac{285\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}-\frac{285\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}-\frac{285\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}c^2\log\left(\sqrt{2}\right.}{128\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] -285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) +
 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(-1/2
*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/128*sqrt(
2)*(b*c^3)^(1/4)*c^2*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 +
 285/128*sqrt(2)*(b*c^3)^(1/4)*c^2*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + s
qrt(b/c))/b^6 - 1/16*(31*c^4*x^(5/2) + 35*b*c^3*sqrt(x))/((c*x^2 + b)^2*b^5
) - 2/77*(154*c^2*x^4 - 33*b*c*x^2 + 7*b^2)/(b^5*x^(11/2))
```

3.352 $\int x^{7/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=323

$$\frac{14b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}+\frac{28b^3x^{3/2}\left(b+cx^2\right)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx}}{585c^2}$$

```
[Out] (28*b^3*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
+ c*x^4]) - (28*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(585*c^2) + (4*b*x^(5/2)*S
qrt[b*x^2 + c*x^4])/(117*c) + (2*x^(9/2)*Sqrt[b*x^2 + c*x^4])/13 - (28*b^(1
3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 +
 c*x^4]) + (14*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^
(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.383018, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{28b^3x^{3/2}\left(b+cx^2\right)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{14b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}-\frac{28b^{13}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (28*b^3*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
+ c*x^4]) - (28*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(585*c^2) + (4*b*x^(5/2)*S
qrt[b*x^2 + c*x^4])/(117*c) + (2*x^(9/2)*Sqrt[b*x^2 + c*x^4])/13 - (28*b^(1
3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 +
 c*x^4]) + (14*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^
(11/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx = \frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{1}{13}(2b)\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}-\frac{\left(14b^2\right)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{117c}$$

$$= -\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(14b^3\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{195c^2}$$

$$= -\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(14b^3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{195c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(28b^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{195c^2\sqrt{bx^2+cx}}$$

$$= -\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(28b^{7/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\right.}{195c^{5/2}\sqrt{bx^2+c}}$$

$$= \frac{28b^3x^{3/2}\left(b+cx^2\right)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}$$

Mathematica [C] time = 0.0787154, size = 102, normalized size = 0.32

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{\frac{cx^2}{b}+1}\left(-7b^2+2bcx^2+9c^2x^4\right)+7b^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{117c^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(Sqrt[1 + (c*x^2)/b]*(-7*b^2 + 2*b*c*x^2 +
 9*c^2*x^4) + 7*b^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(117*
c^2*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.183, size = 237, normalized size = 0.7

$$\frac{2}{\left(585\,cx^2+585\,b\right)c^3}\sqrt{cx^4+bx^2}\left(45\,x^8c^4+55\,x^6bc^3+42\,b^4\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+}{\sqrt{-}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 2/585*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^3*(45*x^8*c^4+55*x^6*b*c^3+42
*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-21*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(
1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*x^4*b
^2*c^2-14*x^2*b^3*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+bx^2}x^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{cx^4+bx^2}x^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{7}{2}}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)*sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2}x^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

3.353 $\int x^{5/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=176

$$\frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{4bx^{3/2}\sqrt{bx^2+c}}{77c}$$

```
[Out] (-20*b^2*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (4*b*x^(3/2)*Sqrt[b*x^2 +
 c*x^4])/(77*c) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/11 + (10*b^(11/4)*x*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.248887, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2021, 2024, 2032, 329, 220}

$$-\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[b*x^2 + c*x^4],x]
```



```
[Out] (-20*b^2*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (4*b*x^(3/2)*Sqrt[b*x^2 +
 c*x^4])/(77*c) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/11 + (10*b^(11/4)*x*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx = \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{1}{11}(2b)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}-\frac{\left(10b^2\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c}$$

$$= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{\left(10b^3\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231c^2}$$

$$= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{\left(10b^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{231c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{\left(20b^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\right.}{231c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F}{231c^{9/4}\sqrt{bx^2+c}}$$

Mathematica [C] time = 0.0586175, size = 102, normalized size = 0.58

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{\frac{cx^2}{b}+1}\left(-5b^2+2bcx^2+7c^2x^4\right)+5b^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{77c^2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(Sqrt[1 + (c*x^2)/b]*(-5*b^2 + 2*b*c*x^2 + 7*c^2*x
^4) + 5*b^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(77*c^2*Sqrt[
x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.189, size = 157, normalized size = 0.9

$$\frac{2}{\left(231\,cx^2+231\,b\right)c^3}\sqrt{cx^4+bx^2}\left(21\,x^7c^4+5\,b^3\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(21*x^7*c^4+5*b^3*(-b*c)^(1/2)*
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2),1/2*2^(1/2))+27*x^5*b*c^3-4*b^2*c^2*x^3-10*x*b^3*c)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2}x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{cx^4 + bx^2}x^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{5}{2}}\sqrt{x^2\left(b + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)*sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2}x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

3.354 $\int x^{3/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=293

$$-\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}-\frac{4b^2x^{3/2}\left(b+cx^2\right)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)}{15}$$

```
[Out] (-4*b^2*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c) + (2*x^(5/2)*Sqrt[b*x^2
 + c*x^4])/9 + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c
^(7/4)*Sqrt[b*x^2 + c*x^4]) - (2*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.306951, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{4b^2x^{3/2}\left(b+cx^2\right)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}}{15c^{7/4}\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-4*b^2*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c) + (2*x^(5/2)*Sqrt[b*x^2
 + c*x^4])/9 + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c
^(7/4)*Sqrt[b*x^2 + c*x^4]) - (2*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
```

```
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
```

```
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx = \frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}+\frac{1}{9}(2b)\int\frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}-\frac{\left(2b^2\right)\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c}$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}-\frac{\left(2b^2x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15c\sqrt{bx^2+cx^4}}$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}-\frac{\left(4b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15c\sqrt{bx^2+cx^4}}$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}-\frac{\left(4b^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15c^{3/2}\sqrt{bx^2+cx^4}}+\frac{\left(4b^{5/2}x\right.}{}$$

$$= -\frac{4b^2x^{3/2}\left(b+cx^2\right)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)}{15c}$$

Mathematica [C] time = 0.0386211, size = 86, normalized size = 0.29

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}-b\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{9c\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b] - b*Hyper
geometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(9*c*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.186, size = 226, normalized size = 0.8

$$-\frac{2}{\left(45\,c x^2+45\,b\right)c^2}\sqrt{c x^4+b x^2}\left(-5\,c^3x^6+6\,b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^2*(-5*c^3*x^6+6*b^3*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))-3*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((
-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-7*b*c^2*x^4-2*b^2*c*x^
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2}x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{cx^4+bx^2}x^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2}x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

3.355 $\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=146

$$-\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/
7 - (2*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*c^(5/4)*Sqrt
[b*x^2 + c*x^4])
```

Rubi [A] time = 0.184986, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2021, 2024, 2032, 329, 220}

$$-\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/
7 - (2*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*c^(5/4)*Sqrt
[b*x^2 + c*x^4])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx &= \frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}+\frac{1}{7}(2b)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}-\frac{\left(2b^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c} \\
&= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}-\frac{\left(2b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21c\sqrt{bx^2+cx^4}} \\
&= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}-\frac{\left(4b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{21c\sqrt{bx^2+cx^4}} \\
&= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}-\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.0339688, size = 86, normalized size = 0.59

$$
\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)\sqrt{\frac{cx^2}{b}+1}-b\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{7c\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b] - b*Hypergeometri
c2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(7*c*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.182, size = 145, normalized size = 1.

$$
-\frac{2}{\left(21\,cx^2+21\,b\right)c^2}\sqrt{cx^4+bx^2}\left(b^2\sqrt{-bc}\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\left(cx+\right.}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(b^2*(-b*c)^(1/2)*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))-3*c^3*x^5-5*b*c^2*x^3-2*b^2*c*x)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2}\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{x^2\left(b + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

3.356 $\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

Optimal. Leaf size=263

$$\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}-\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}+$$

```
[Out] (4*b*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x
^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (4*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2*b^(5/4)*x*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.238227, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2021, 2032, 329, 305, 220, 1196}

$$\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}-\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}+\frac{4b}{5\sqrt{c}\left(\sqrt{b}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/Sqrt[x],x]
```

```
[Out] (4*b*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x
^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (4*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2*b^(5/4)*x*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
```

```
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx &= \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}+\frac{1}{5}(2b)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}+\frac{\left(2bx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}} \\
&= \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}+\frac{\left(4bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{bx^2+cx^4}} \\
&= \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}+\frac{\left(4b^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{c}\sqrt{bx^2+cx^4}}-\frac{\left(4b^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\cdots}{\sqrt{b+\cdots}}\cdots\right.}{5\sqrt{c}\sqrt{bx^2+cx^4}} \\
&= \frac{4bx^{3/2}\left(b+cx^2\right)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}-\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\cdots\right.\right.}{5c^{3/4}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.0141753, size = 57, normalized size = 0.22

$$
\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{3\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2
)/b)])/(3*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.184, size = 213, normalized size = 0.8

$$
\frac{2}{\left(5\,cx^2+5\,b\right)c}\sqrt{cx^4+bx^2}\left(2\,b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-b^2\sqrt{\left(cx+\cdots\right.}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(1/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c*(2*b^2*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))-b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+c^2*x^4+b*c*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/sqrt(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

3.357 $\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.133109, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2021, 2032, 329, 220}

$$\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx &= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{1}{3}(2b)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{\left(2bx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}}\\
&= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{\left(4bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{bx^2+cx^4}}\\
&= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.013025, size = 55, normalized size = 0.47

$$
\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)}{\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)])/(
Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.186, size = 130, normalized size = 1.1

$$\frac{2}{\left(3\,cx^{2}+3\,b\right)c}\sqrt{cx^{4}+bx^{2}}\left(b\sqrt{-bc}\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(3/2),x)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(b*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x
*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))+c^2*x^3+b*c*x)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{4}+bx^{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^{4}+bx^{2}}}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

3.358 $\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{2\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}}+\frac{4\sqrt{c}x^{3/2}\left(b+cx^2\right)}{\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}-\frac{4\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{\right.}}{}$$

```
[Out] (4*Sqrt[c]*x^(3/2)*(b + c*x^2))/((Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4])
 - (2*Sqrt[b*x^2 + c*x^4])/x^(3/2) - (4*b^(1/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*
Sqrt[x])/b^(1/4)], 1/2])/Sqrt[b*x^2 + c*x^4] + (2*b^(1/4)*c^(1/4)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/Sqrt[b*x^2 + c*x^4]
```

Rubi [A] time = 0.237952, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2020, 2032, 329, 305, 220, 1196}

$$\frac{4\sqrt{c}x^{3/2}\left(b+cx^2\right)}{\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}+\frac{2\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(5/2),x]
```


```
[Out] (4*Sqrt[c]*x^(3/2)*(b + c*x^2))/((Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4])
 - (2*Sqrt[b*x^2 + c*x^4])/x^(3/2) - (4*b^(1/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*
Sqrt[x])/b^(1/4)], 1/2])/Sqrt[b*x^2 + c*x^4] + (2*b^(1/4)*c^(1/4)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/Sqrt[b*x^2 + c*x^4]
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
```

```
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + (2c)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + \frac{\left(2cx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + \frac{\left(4cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + \frac{\left(4\sqrt{b}\sqrt{c}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}} - \frac{\left(4\sqrt{b}\sqrt{c}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-}{\sqrt{b}}\right.}{\sqrt{bx^2+cx^4}}$$

$$= \frac{4\sqrt{c}x^{3/2}\left(b+cx^2\right)}{\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} - \frac{4\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}} +$$

Mathematica [C] time = 0.0138967, size = 55, normalized size = 0.22

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)}{x^{3/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(5/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^2)/b)])
/(x^(3/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.186, size = 202, normalized size = 0.8

$$2\frac{\sqrt{cx^4+bx^2}}{x^{3/2}\left(cx^2+b\right)}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)b-\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(5/2),x)
```

```
[Out] 2*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2),1/2*2^(1/2))*b-c*x^2-b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 \left(b + cx^2\right)}}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

3.359 $\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*x^(5/2)) + (2*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.132003, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2020, 2032, 329, 220}

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(7/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*x^(5/2)) + (2*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{1}{3}(2c)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{\left(2cx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}}\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{\left(4cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{bx^2+cx^4}}\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.0138369, size = 57, normalized size = 0.48

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{3x^{5/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(7/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^2)/b)])
/(3*x^(5/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.183, size = 125, normalized size = 1.1

$$\frac{2}{3\,cx^2+3\,b}\sqrt{cx^4+bx^2}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(7/2),x)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)/(c*x^2+b)*(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*
c)^(1/2)*x-c*x^2-b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(7/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

3.360 $\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

Optimal. Leaf size=293

$$\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}+$$

```
[Out] (4*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x
^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*x^(7/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(5*b
*x^(3/2)) - (4*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/
4)*Sqrt[b*x^2 + c*x^4]) + (2*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^
2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.299394, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}+\frac{4c^{3/2}}{5b\left(\sqrt{b}+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(9/2),x]
```



```
[Out] (4*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x
^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*x^(7/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(5*b
*x^(3/2)) - (4*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/
4)*Sqrt[b*x^2 + c*x^4]) + (2*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^
2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
```

```
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
```

```
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} + \frac{1}{5}(2c)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(2c^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(2c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(4c^2x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(4c^{3/2}x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{b}\sqrt{bx^2+cx^4}} - \frac{\left(4c^{3/2}x\sqrt{b+cx^2}\right.}{5}$$

$$= \frac{4c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\,\mathrm{t}\right.}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.013435, size = 57, normalized size = 0.19

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5x^{7/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(9/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^2)/b)]
)/(5*x^(7/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.192, size = 224, normalized size = 0.8

$$\frac{2}{\left(5\,cx^{2}+5\,b\right)b}\sqrt{cx^{4}+bx^{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^{2}bc-\sqrt{\left(cx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(9/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(7/2)/(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^
2*b*c-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-2*c^2*x^4-3*b*c*x^2-b^2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{4}+bx^{2}}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^{4}+bx^{2}}}{x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b + cx^2\right)}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(9/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(9/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

3.361 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*x^(9/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(21*b*x^(5
/2)) - (2*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*b^(5/4)*S
qrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.182391, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2020, 2025, 2032, 329, 220}

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(11/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*x^(9/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(21*b*x^(5
/2)) - (2*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*b^(5/4)*S
qrt[b*x^2 + c*x^4])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}} + \frac{1}{7}(2c)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}} - \frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{(2c^2)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}} - \frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{\left(2c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}} - \frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{\left(4c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{21b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}} - \frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0154649, size = 57, normalized size = 0.39

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7x^{9/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(11/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^2)/b)]
)/(7*x^(9/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.181, size = 142, normalized size = 1.

$$-\frac{2}{\left(21\,cx^2+21\,b\right)b}\sqrt{cx^4+bx^2}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{\cdot}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(11/2),x)
```

```
[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(9/2)/(c*x^2+b)*((((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-
b*c)^(1/2)*x^3*c+2*c^2*x^4+5*b*c*x^2+3*b^2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

3.362 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

Optimal. Leaf size=323

$$-\frac{2c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}+\frac{4c^{9/}\cdots}{\cdots}$$

```
[Out] (-4*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*x^(11/2)) - (4*c*Sqrt[b*x^2 + c*x^4])
/(45*b*x^(7/2)) + (4*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^(3/2)) + (4*c^(9/4)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4
]) - (2*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqr
t[b*x^2 + c*x^4])
```

Rubi [A] time = 0.365863, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{4c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}-\frac{2c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}+\frac{4c^{9/4}x\left(\sqrt{b}+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(13/2),x]
```



```
[Out] (-4*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*x^(11/2)) - (4*c*Sqrt[b*x^2 + c*x^4])
/(45*b*x^(7/2)) + (4*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^(3/2)) + (4*c^(9/4)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4
]) - (2*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqr
t[b*x^2 + c*x^4])
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} + \frac{1}{9}(2c)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} - \frac{\left(2c^2\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{\left(2c^3\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{\left(2c^3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{\left(4c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{\left(4c^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15b^{3/2}\sqrt{bx^2+cx^4}} +$$

$$= -\frac{4c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} + \frac{4c^{9/4}x\left(\sqrt{b}+\right.}{}$$

Mathematica [C] time = 0.0159796, size = 57, normalized size = 0.18

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{9}{4},-\frac{1}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9x^{11/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(13/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-9/4, -1/2, -5/4, -((c*x^2)/b)]
)/(9*x^(11/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.19, size = 239, normalized size = 0.7

$$-\frac{2}{\left(45\,cx^{2}+45\,b\right)b^{2}}\sqrt{cx^{4}+bx^{2}}\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)x^{4}bc^{2}-3\,\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(13/2),x)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(11/2)/(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*x^4*b*c^2-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-6*c^3*x^6-4*b*c^2*x^4+7*
b^2*c*x^2+5*b^3)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^{4}+bx^{2}}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^{4}+bx^{2}}}{x^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

3.363 $\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

Optimal. Leaf size=176

$$\frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*x^(13/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(77*b*x^
(9/2)) + (20*c^2*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) + (10*c^(11/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.23427, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2020, 2025, 2032, 329, 220}

$$\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}+\frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(15/2),x]
```


```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*x^(13/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(77*b*x^
(9/2)) + (20*c^2*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) + (10*c^(11/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} + \frac{1}{11}(2c)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{(10c^2)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} + \frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} + \frac{(10c^3)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} + \frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} + \frac{\left(10c^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{231b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} + \frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} + \frac{\left(20c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{231b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} + \frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} + \frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\ldots}{\ldots}\right.\right.}{231b^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0140598, size = 57, normalized size = 0.32

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{2};-\frac{7}{4};-\frac{cx^2}{b}\right)}{11x^{13/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((c*x^2)/b)
])/(11*x^(13/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.186, size = 156, normalized size = 0.9

$$\frac{2}{\left(231\,cx^2+231\,b\right)b^2}\sqrt{cx^4+bx^2}\left(5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^5c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(15/2),x)
```

```
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(13/2)/(c*x^2+b)*(5*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*(-b*c)^(1/2)*x^5*c^2+10*c^3*x^6+4*b*c^2*x^4-27*b^2*c*x^2-21*b^3)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{15}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{15}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

3.364 $\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=350

$$\frac{28b^{17/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}+\frac{56b^4x^{3/2}\left(b+cx^2\right)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{56b^3\sqrt{x}\sqrt{bx^2+c}}{3315c^2}$$

```
[Out] (56*b^4*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
 + c*x^4]) - (56*b^3*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(3315*c^2) + (8*b^2*x^(5/
2)*Sqrt[b*x^2 + c*x^4])/(663*c) + (12*b*x^(9/2)*Sqrt[b*x^2 + c*x^4])/221 +
(2*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/17 - (56*b^(17/4)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (28*b^(17/4)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]
)
```

Rubi [A] time = 0.433556, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{56b^4x^{3/2}\left(b+cx^2\right)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{28b^{17/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}-\frac{56b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (56*b^4*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
 + c*x^4]) - (56*b^3*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(3315*c^2) + (8*b^2*x^(5/
2)*Sqrt[b*x^2 + c*x^4])/(663*c) + (12*b*x^(9/2)*Sqrt[b*x^2 + c*x^4])/221 +
(2*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/17 - (56*b^(17/4)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (28*b^(17/4)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]
)
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{17}(6b)\int x^{7/2}\sqrt{bx^2+cx^4}\,dx$$

$$=\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{221}\left(12b^2\right)\int\frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$=\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(28b^3\right)\int\frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{663c}$$

$$=-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}+$$

$$=-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}+$$

$$=-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}+$$

$$=-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}+$$

$$=\frac{56b^4x^{3/2}\left(b+cx^2\right)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{b}$$

Mathematica [C] time = 0.0618634, size = 101, normalized size = 0.29

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(7b^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)-\left(7b-13cx^2\right)\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\right)}{221c^2\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((7*b - 13*c*x^2)*(b + c*x^2)^2*Sqrt[1 +
 (c*x^2)/b]) + 7*b^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(221
*c^2*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.198, size = 248, normalized size = 0.7

$$\frac{2}{3315\left(cx^{2}+b\right)^{2}c^{3}}\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(195\,x^{10}c^{5}+480\,x^{8}bc^{4}+305\,x^{6}b^{2}c^{3}+84\,b^{5}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{El}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 2/3315*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^3*(195*x^10*c^5+480*x^8*b*
c^4+305*x^6*b^2*c^3+84*b^5*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elliptic
E(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-42*b^5*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))-8*x^4*b^3*c^2-28*x^2*b^4*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^{5}+bx^{3}\right)\sqrt{cx^{4}+bx^{2}}\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^5 + b*x^3)*sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2), x)
```

3.365 $\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=203

$$\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+}$$

```
[Out] (-8*b^3*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (8*b^2*x^(3/2)*Sqrt[b*x^2
+ c*x^4])/(385*c) + (4*b*x^(7/2)*Sqrt[b*x^2 + c*x^4])/55 + (2*x^(3/2)*(b*x^
2 + c*x^4)^(3/2))/15 + (4*b^(15/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.289279, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2021, 2024, 2032, 329, 220}

$$-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (-8*b^3*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (8*b^2*x^(3/2)*Sqrt[b*x^2
+ c*x^4])/(385*c) + (4*b*x^(7/2)*Sqrt[b*x^2 + c*x^4])/55 + (2*x^(3/2)*(b*x^
2 + c*x^4)^(3/2))/15 + (4*b^(15/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{5}(2b)\int x^{5/2}\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{55}\left(4b^2\right)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(4b^3\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c}$$
$$= -\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{\left(4b^4\right)\int}{}$$
$$= -\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{\left(4b^4x\sqrt{}\right.}{}$$
$$= -\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{\left(8b^4x\sqrt{}\right.}{}$$
$$= -\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{4b^{15/4}x}{}$$

Mathematica [C] time = 0.0595289, size = 101, normalized size = 0.5

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(5b^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)-\left(5b-11cx^2\right)\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}\right)}{165c^2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((5*b - 11*c*x^2)*(b + c*x^2)^2*Sqrt[1 + (c*x^2)
/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(165*c^2*Sqr
t[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.189, size = 168, normalized size = 0.8

$$\frac{2}{1155\left(cx^2+b\right)^2c^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(77\,x^9c^5+196\,x^7bc^4+10\,b^4\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)*x^(1/2),x)
```

```
[Out] 2/1155*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(77*x^9*c^5+196*x^7*b*c^4+10
*b^4*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+131*x^5*b^2*c^3-8*x^3*b^3*c^2
-20*x*b^4*c)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} \sqrt{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)
```

3.366 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=320

$$-\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}}-\frac{8b^3x^{3/2}\left(b+cx^2\right)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)}{6}$$

```
[Out] (-8*b^3*x^(3/2)*(b + c*x^2))/(65*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) + (8*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(195*c) + (4*b*x^(5/2)*Sqrt[
b*x^2 + c*x^4])/39 + (2*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/13 + (8*b^(13/4)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*c^(7/4)*Sqrt[b*x^2 + c*x^4]) -
 (4*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)
^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*c^(7/4)*Sqrt[b
*x^2 + c*x^4])
```

Rubi [A] time = 0.369991, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{8b^3x^{3/2}\left(b+cx^2\right)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}}+\frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}}{65c^{7/4}\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/Sqrt[x],x]
```


```
[Out] (-8*b^3*x^(3/2)*(b + c*x^2))/(65*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) + (8*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(195*c) + (4*b*x^(5/2)*Sqrt[
b*x^2 + c*x^4])/39 + (2*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/13 + (8*b^(13/4)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*c^(7/4)*Sqrt[b*x^2 + c*x^4]) -
 (4*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)
^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*c^(7/4)*Sqrt[b
*x^2 + c*x^4])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{13}(6b)\int x^{3/2}\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{39}\left(4b^2\right)\int\frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(4b^3\right)\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{65c}$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(4b^3x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{65c\sqrt{bx^2+cx^4}}$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(8b^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{b+\cdots}}\right.}{65c\sqrt{bx^2+cx^4}}$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(8b^{7/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{\cdots}{\sqrt{\cdots}}\right.}{65c^{3/2}\sqrt{bx^2+cx\cdots}}$$

$$= -\frac{8b^3x^{3/2}\left(b+cx^2\right)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)\cdots$$

Mathematica [C] time = 0.0480372, size = 90, normalized size = 0.28

$$\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}-b^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{13c\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b^2*H
ypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(13*c*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.187, size = 237, normalized size = 0.7

$$-\frac{2}{195\left(cx^2+b\right)^2c^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-15\,x^8c^4-40\,x^6bc^3+12\,b^4\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+}{\sqrt{}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(1/2),x)
```

```
[Out] -2/195*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^2*(-15*x^8*c^4-40*x^6*b*c^
3+12*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-29*x
^4*b^2*c^2-4*x^2*b^3*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4+bx^2}\left(cx^3+bx\right)\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^3 + b*x)*sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/sqrt(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)
```

3.367 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}}+\frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}}$$

```
[Out] (8*b^2*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) + (12*b*x^(3/2)*Sqrt[b*x^2 + c*x
^4])/77 + (2*(b*x^2 + c*x^4)^(3/2))/(11*Sqrt[x]) - (4*b^(11/4)*x*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c
^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.235517, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2021, 2024, 2032, 329, 220}

$$-\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}}+\frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(3/2),x]
```


```
[Out] (8*b^2*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) + (12*b*x^(3/2)*Sqrt[b*x^2 + c*x
^4])/77 + (2*(b*x^2 + c*x^4)^(3/2))/(11*Sqrt[x]) - (4*b^(11/4)*x*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c
^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{3/2}}\,dx = \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} + \frac{1}{11}(6b)\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} + \frac{1}{77}\left(12b^2\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} - \frac{\left(4b^3\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77c}$$

$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} - \frac{\left(4b^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{77c\sqrt{bx^2+cx^4}}$$

$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} - \frac{\left(8b^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x,\right.}{77c\sqrt{bx^2+cx^4}}$$

$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}} - \frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\,\text{ta}\right.}{77c^{5/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0390194, size = 90, normalized size = 0.52

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)^2\sqrt{\frac{cx^2}{b}+1}-b^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{11c\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b^2*Hypergeom
etric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(11*c*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.183, size = 157, normalized size = 0.9

$$-\frac{2}{77\left(cx^2+b\right)^2c^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-7x^7c^4+2b^3\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(3/2),x)
```

```
[Out] -2/77*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-7*x^7*c^4+2*b^3*(-b*c)^(1/2
)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))-20*x^5*b*c^3-17*b^2*c^2*x^3-4*x*b^3*c)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2}\left(cx^2 + b\right)\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)*sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(3/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2), x)
```

3.368 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=290

$$\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}-\frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}+$$

```
[Out] (8*b^2*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2*(b*x^2 + c*x^4)^(3/2))/
(9*x^(3/2)) - (8*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^
(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.298659, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2021, 2032, 329, 305, 220, 1196}

$$\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}-\frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}+\frac{8l}{15\sqrt{c}\left(\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(5/2),x]
```

```
[Out] (8*b^2*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2*(b*x^2 + c*x^4)^(3/2))/
(9*x^(3/2)) - (8*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^
(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
```

```
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} + \frac{1}{3}(2b)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$

$$= \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} + \frac{1}{15}(4b^2)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} + \frac{\left(4b^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15\sqrt{bx^2+cx^4}}$$

$$= \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} + \frac{\left(8b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15\sqrt{bx^2+cx^4}}$$

$$= \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} + \frac{\left(8b^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15\sqrt{c}\sqrt{bx^2+cx^4}} - \frac{\left(8b^{5/2}x\right.}{}$$

$$= \frac{8b^2x^{3/2}(b+cx^2)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} - \frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{}{(}}}{15c^{3/4}}$$

Mathematica [C] time = 0.0168486, size = 58, normalized size = 0.2

$$\frac{2b\sqrt{x}\sqrt{x^2(b+cx^2)}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{3\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x
^2)/b)])/(3*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.188, size = 226, normalized size = 0.8

$$\frac{2}{45\,(cx^2+b)^2\,c}(cx^4+bx^2)^{\frac{3}{2}}\left(5\,c^3x^6+12\,b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(5/2),x)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c*(5*c^3*x^6+12*b^3*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))-6*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+16*b*c^2*x^4+11*b^2*c*x
^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(cx^2 + b\right)}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(5/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2), x)
```

3.369 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=143

$$\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{7\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(7*Sqrt[x]) + (2*(b*x^2 + c*x^4)^(3/2))/(7*x^(5/2
)) + (4*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(7*c^(1/4)*Sqrt
[b*x^2 + c*x^4])
```

Rubi [A] time = 0.184849, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2021, 2032, 329, 220}

$$\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(7/2),x]
```

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(7*Sqrt[x]) + (2*(b*x^2 + c*x^4)^(3/2))/(7*x^(5/2
)) + (4*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(7*c^(1/4)*Sqrt
[b*x^2 + c*x^4])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx &= \frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}+\frac{1}{7}(6b)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx\\
&= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}+\frac{1}{7}\left(4b^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}+\frac{\left(4b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{7\sqrt{bx^2+cx^4}}\\
&= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}+\frac{\left(8b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{7\sqrt{bx^2+cx^4}}\\
&= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}+\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.0161236, size = 56, normalized size = 0.39

$$
\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)}{\sqrt{x}\sqrt{\frac{cx^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(7/2),x]
```

```
[Out] (2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)])
/(Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.184, size = 145, normalized size = 1.

$$\frac{2}{7\left(cx^2+b\right)^2 c}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2b^2\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)+c^3x^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(7/2),x)
```

```
[Out] 2/7*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(2*b^2*(-b*c)^(1/2)*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))+c^3*x^5+4*b*c^2*x^3+3*b^2*c*x)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(7/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2), x)
```

3.370 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=286

$$\frac{12b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{5\sqrt{bx^2+cx^4}}$$

```
[Out] (24*b*Sqrt[c]*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) + (12*c*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (2*(b*x^2 + c*x^4)^(3/2))/x^
(7/2) - (24*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*
Sqrt[b*x^2 + c*x^4]) + (12*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1
/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.301089, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2020, 2021, 2032, 329, 305, 220, 1196}

$$\frac{12b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}+\frac{}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(9/2),x]
```


```
[Out] (24*b*Sqrt[c]*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) + (12*c*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (2*(b*x^2 + c*x^4)^(3/2))/x^
(7/2) - (24*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*
Sqrt[b*x^2 + c*x^4]) + (12*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1
/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
```

```
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
```

```
x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx = -\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}} + (6c)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$

$$= \frac{12}{5}c\sqrt{x}\sqrt{bx^2+cx^4} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}} + \frac{1}{5}(12bc)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{12}{5}c\sqrt{x}\sqrt{bx^2+cx^4} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}} + \frac{\left(12bcx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{12}{5}c\sqrt{x}\sqrt{bx^2+cx^4} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}} + \frac{\left(24bcx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{12}{5}c\sqrt{x}\sqrt{bx^2+cx^4} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}} + \frac{\left(24b^{3/2}\sqrt{c}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}} - \frac{\left(24b^3\right.}{}$$

$$= \frac{24b\sqrt{c}x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{12}{5}c\sqrt{x}\sqrt{bx^2+cx^4} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}} - \frac{24b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b}{(\sqrt{b}}}}{5\sqrt{bx^2}}$$

Mathematica [C] time = 0.0158831, size = 56, normalized size = 0.2

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)}{x^{3/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(9/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^2)/b)
])/(x^(3/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.185, size = 216, normalized size = 0.8

$$\frac{2}{5\left(cx^{2}+b\right)^{2}}\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(12\,b^{2}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-6\,b^{2}\sqrt{\frac{cx}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(9/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(12*b^2*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))-6*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+c^2*x^4-4*b*c*x^2-5*b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^{4}+bx^{2}}\left(cx^{2}+b\right)}{x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(9/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(9/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2), x)
```

3.371 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}} \, dx$

Optimal. Leaf size=143

$$\frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt{bx^2+cx^4}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}+\frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

```
[Out] (4*c*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) - (2*(b*x^2 + c*x^4)^(3/2))/(3*x^(9/2
)) + (4*b^(3/4)*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*Sqrt
[b*x^2 + c*x^4])
```

Rubi [A] time = 0.187946, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2020, 2021, 2032, 329, 220}

$$\frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{bx^2+cx^4}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}+\frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(11/2),x]
```

```
[Out] (4*c*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) - (2*(b*x^2 + c*x^4)^(3/2))/(3*x^(9/2
)) + (4*b^(3/4)*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*Sqrt
[b*x^2 + c*x^4])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = -\frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + (2c)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$$
$$= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + \frac{1}{3}(4bc)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + \frac{\left(4bcx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}}$$
$$= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + \frac{\left(8bcx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3\sqrt{bx^2+cx^4}}$$
$$= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + \frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.016731, size = 58, normalized size = 0.41

$$-\frac{2b\sqrt{x^2(b+cx^2)}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{cx^2}{b}\right)}{3x^{5/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(11/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^2)/b)
])/(3*x^(5/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.187, size = 130, normalized size = 0.9

$$\frac{2}{3\left(cx^2+b\right)^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}xb+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(11/2),x)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(3/2)/x^(9/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
(-b*c)^(1/2)*x*b+c^2*x^4-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(cx^2 + b\right)}{x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2), x)
```

3.372 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}} dx$

Optimal. Leaf size=287

$$\frac{12\sqrt[4]{b}c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}+\frac{24c^{3/2}x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{24\sqrt[4]{b}c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\ldots}{5\ldots}$$

```
[Out] (24*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^
4]) - (12*c*Sqrt[b*x^2 + c*x^4])/(5*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(5
*x^(11/2)) - (24*b^(1/4)*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(5*Sqrt[b*x^2 + c*x^4]) + (12*b^(1/4)*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.304592, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2020, 2032, 329, 305, 220, 1196}

$$\frac{24c^{3/2}x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{12\sqrt[4]{b}c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24\sqrt[4]{b}c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\ldots}}\ldots}{5\sqrt{bx^2+c\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(13/2),x]
```



```
[Out] (24*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^
4]) - (12*c*Sqrt[b*x^2 + c*x^4])/(5*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(5
*x^(11/2)) - (24*b^(1/4)*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(5*Sqrt[b*x^2 + c*x^4]) + (12*b^(1/4)*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
```

```
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = -\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} + \frac{1}{5}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} + \frac{1}{5}(12c^2)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} + \frac{\left(12c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} + \frac{\left(24c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}}$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} + \frac{\left(24\sqrt{b}c^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}} - \frac{(24\sqrt{bc}\ldots}{\ldots}$$

$$= \frac{24c^{3/2}x^{3/2}(b+cx^2)}{5(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} - \frac{24\sqrt[4]{b}c^{5/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}\ldots}}}{5\sqrt{bx^2+c\ldots}}$$

Mathematica [C] time = 0.0165014, size = 58, normalized size = 0.2

$$-\frac{2b\sqrt{x^2(b+cx^2)}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5x^{7/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(13/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^2)/b
)])/(5*x^(7/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.19, size = 221, normalized size = 0.8

$$\frac{2}{5(cx^2+b)^2}(cx^4+bx^2)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^2bc-6\sqrt{\frac{cx\ldots}{\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(13/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(11/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)
^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)*x^2*b*c-6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-7*c^2*x^4-8*b*c*x^2-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(cx^2 + b\right)}{x^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2), x)
```

3.373 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

Optimal. Leaf size=143

$$\frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(7*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(7*x^(13
/2)) + (4*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(7*b^(1/4)*Sq
rt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.18338, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2020, 2032, 329, 220}

$$\frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(15/2),x]
```


```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(7*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(7*x^(13
/2)) + (4*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(7*b^(1/4)*Sq
rt[b*x^2 + c*x^4])
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx &= -\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}+\frac{1}{7}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx\\
&= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}+\frac{1}{7}\left(4c^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}+\frac{\left(4c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{7\sqrt{bx^2+cx^4}}\\
&= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}+\frac{\left(8c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{7\sqrt{bx^2+cx^4}}\\
&= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}+\frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0166989, size = 58, normalized size = 0.41

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7x^{9/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(15/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((c*x^2)/b
)])/(7*x^(9/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.185, size = 140, normalized size = 1.

$$\frac{2}{7\left(cx^2+b\right)^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^3c-3c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(15/2),x)
```

```
[Out] 2/7*(c*x^4+b*x^2)^(3/2)/x^(13/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*(-b*c)^(1/2)*x^3*c-3*c^2*x^4-4*b*c*x^2-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{15}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2), x)
```

3.374 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

Optimal. Leaf size=320

$$\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}-\frac{8c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}+$$

```
[Out] (8*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(15*x^(7/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4]
)/(15*b*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(9*x^(15/2)) - (8*c^(9/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) +
(4*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x
^2 + c*x^4])
```

Rubi [A] time = 0.359815, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}-\frac{8c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}+\frac{8c^{5/}}{15b\left(\sqrt{b}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(17/2),x]
```

```
[Out] (8*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(15*x^(7/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4]
)/(15*b*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(9*x^(15/2)) - (8*c^(9/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) +
(4*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x
^2 + c*x^4])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{17/2}}\,dx = -\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}} + \frac{1}{3}(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}} + \frac{1}{15}\left(4c^2\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}} + \frac{\left(4c^3\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}} + \frac{\left(4c^3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15b\sqrt{bx^2+cx^4}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}} + \frac{\left(8c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{}\right.}{15b\sqrt{bx^2+cx^4}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}} + \frac{\left(8c^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \right.}{15\sqrt{b}\sqrt{bx^2+cx^4}}$$

$$= \frac{8c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}} - \frac{8c^{9/4}x\left(\sqrt{b}\right.}{}$$

Mathematica [C] time = 0.0192765, size = 58, normalized size = 0.18

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9x^{11/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(17/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((c*x^2)/b
)])/(9*x^(11/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.192, size = 239, normalized size = 0.8

$$\frac{2}{45\left(cx^2+b\right)^2 b}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^4bc^2-6\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(17/2),x)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(15/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*x^4*b*c^2-6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-12*c^3*x^6-23*b*c^2*x^
4-16*b^2*c*x^2-5*b^3)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(17/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2), x)
```

3.375 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{19/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{bx^2+cx^4}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

```
[Out] (-12*c*Sqrt[b*x^2 + c*x^4])/(77*x^(9/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(77*
b*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(11*x^(17/2)) - (4*c^(11/4)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.237128, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2020, 2025, 2032, 329, 220}

$$-\frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{bx^2+cx^4}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(19/2),x]
```


```
[Out] (-12*c*Sqrt[b*x^2 + c*x^4])/(77*x^(9/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(77*
b*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(11*x^(17/2)) - (4*c^(11/4)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{19/2}}\,dx = -\frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} + \frac{1}{11}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$$
$$= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} + \frac{1}{77}(12c^2)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} - \frac{(4c^3)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77b}$$
$$= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} - \frac{\left(4c^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{77b\sqrt{bx^2+cx^4}}$$
$$= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} - \frac{\left(8c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right.}{77b\sqrt{bx^2+cx^4}}$$
$$= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} - \frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\right.}{77b^{5/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0178096, size = 58, normalized size = 0.34

$$-\frac{2b\sqrt{x^2(b+cx^2)}\,{}_2F_1\left(-\frac{11}{4},-\frac{3}{2};-\frac{7}{4};-\frac{cx^2}{b}\right)}{11x^{13/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(19/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((c*x^2)/
b)])/(11*x^(13/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.19, size = 156, normalized size = 0.9

$$-\frac{2}{77\left(cx^2+b\right)^2 b}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^5c^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(19/2),x)
```

```
[Out] -2/77*(c*x^4+b*x^2)^(3/2)/x^(17/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*(-b*c)^(1/2)*x^5*c^2+4*c^3*x^6+17*b*c^2*x^4+20*b^2*c*x^2+7*b^3)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(cx^2 + b\right)}{x^{\frac{15}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(15/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(19/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2), x)
```

3.376 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}\,dx$

Optimal. Leaf size=350

$$-\frac{4c^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}-\frac{8c^{7/2}x^{3/2}\left(b+cx^2\right)}{65b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}+\ldots$$

```
[Out] (-8*c^(7/2)*x^(3/2)*(b + c*x^2))/(65*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(39*x^(11/2)) - (8*c^2*Sqrt[b*x^2 + c*
x^4])/(195*b*x^(7/2)) + (8*c^3*Sqrt[b*x^2 + c*x^4])/(65*b^2*x^(3/2)) - (2*(
b*x^2 + c*x^4)^(3/2))/(13*x^(19/2)) + (8*c^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (4*c^(13/4)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.426244, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{8c^{7/2}x^{3/2}\left(b+cx^2\right)}{65b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{4c^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}+\frac{8c^{13/4}x\left(\sqrt{b}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(21/2),x]
```


```
[Out] (-8*c^(7/2)*x^(3/2)*(b + c*x^2))/(65*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(39*x^(11/2)) - (8*c^2*Sqrt[b*x^2 + c*
x^4])/(195*b*x^(7/2)) + (8*c^3*Sqrt[b*x^2 + c*x^4])/(65*b^2*x^(3/2)) - (2*(
b*x^2 + c*x^4)^(3/2))/(13*x^(19/2)) + (8*c^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (4*c^(13/4)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{21/2}}\,dx = -\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}+\frac{1}{13}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}+\frac{1}{39}\left(4c^2\right)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}-\frac{\left(4c^3\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{65b}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}-\frac{\left(4c^4\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{65b^2}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}-\frac{\left(4c^4x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+c}}}{65b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}-\frac{\left(8c^4x\sqrt{b+cx^2}\right)\text{Subst}}{65b^2\sqrt{bx^2}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}-\frac{\left(8c^{7/2}x\sqrt{b+cx^2}\right)\text{Subs}}{65b^{3/2}\sqrt{b}}$$

$$= -\frac{8c^{7/2}x^{3/2}\left(b+cx^2\right)}{65b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2\left(bx^2+c\right.}{13x^{19}}$$

Mathematica [C] time = 0.0187429, size = 58, normalized size = 0.17

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{13}{4},-\frac{3}{2};-\frac{9}{4};-\frac{cx^2}{b}\right)}{13x^{15/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(21/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-13/4, -3/2, -9/4, -((c*x^2)/
b)])/(13*x^(15/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.204, size = 250, normalized size = 0.7

$$-\frac{2}{195\left(cx^{2}+b\right)^{2}b^{2}}\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^{6}bc^{3}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(21/2),x)
```

```
[Out] -2/195*(c*x^4+b*x^2)^(3/2)/x^(19/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*x^6*b*c^3-6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^6*b*c^3-12*x^8*c^4-8*x^6*b*c
^3+29*x^4*b^2*c^2+40*x^2*b^3*c+15*b^4)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{x^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}(cx^2+b)}{x^{\frac{17}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(17/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(21/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2), x)
```

3.377 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx$

Optimal. Leaf size=203

$$\frac{4c^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{2\left(\right.}{}$$

```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(55*x^(13/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(385
*b*x^(9/2)) + (8*c^3*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) - (2*(b*x^2 + c
*x^4)^(3/2))/(15*x^(21/2)) + (4*c^(15/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.295741, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2020, 2025, 2032, 329, 220}

$$\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}+\frac{4c^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{2\left(bx^2+cx^4\right.}{15x^{21/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(23/2),x]
```


```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(55*x^(13/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(385
*b*x^(9/2)) + (8*c^3*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) - (2*(b*x^2 + c
*x^4)^(3/2))/(15*x^(21/2)) + (4*c^(15/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{23/2}}\,dx = -\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}+\frac{1}{5}(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$$
$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}+\frac{1}{55}\left(4c^2\right)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}-\frac{\left(4c^3\right)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b}$$
$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}+\frac{\left(4c^4\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231b^2}$$
$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}+\frac{\left(4c^4x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+}}}{231b^2\sqrt{bx^2+cx^4}}$$
$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}+\frac{\left(8c^4x\sqrt{b+cx^2}\right)\text{Subst}\left(\right.}{231b^2\sqrt{bx^2}}$$
$$= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}+\frac{4c^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{}{(\sqrt{}}}}{231b^{9/4}}$$

Mathematica [C] time = 0.0185506, size = 58, normalized size = 0.29

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{15}{4},-\frac{3}{2};-\frac{11}{4};-\frac{cx^2}{b}\right)}{15x^{17/2}\sqrt{\frac{cx^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(23/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((c*x^2)
/b)])/(15*x^(17/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] time = 0.206, size = 167, normalized size = 0.8

$$\frac{2}{1155\left(cx^2+b\right)^2b^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(10\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^7c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(23/2),x)
```

```
[Out] 2/1155*(c*x^4+b*x^2)^(3/2)/x^(21/2)/(c*x^2+b)^2*(10*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*(-b*c)^(1/2)*x^7*c^3+20*x^8*c^4+8*x^6*b*c^3-131*x^4*b^2*c^2-196*x^2*b^
3*c-77*b^4)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{23}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{19}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(19/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(23/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{23}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2), x)
```

3.378 $\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=179

$$-\frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{13/4}\sqrt{bx^2+cx^4}}+\frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}}-\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2x^{7/2}\sqrt{bx^2}}{11c}$$

```
[Out] (30*b^2*Sqrt[b*x^2 + c*x^4])/(77*c^3*Sqrt[x]) - (18*b*x^(3/2)*Sqrt[b*x^2 +
c*x^4])/(77*c^2) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(11*c) - (15*b^(11/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.244054, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2024, 2032, 329, 220}

$$\frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}}-\frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{13/4}\sqrt{bx^2+cx^4}}-\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (30*b^2*Sqrt[b*x^2 + c*x^4])/(77*c^3*Sqrt[x]) - (18*b*x^(3/2)*Sqrt[b*x^2 +
c*x^4])/(77*c^2) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(11*c) - (15*b^(11/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{(9b)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{11c}$$

$$= -\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} + \frac{\left(45b^2\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c^2}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(15b^3\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77c^3}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(15b^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{77c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(30b^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x\right.}{77c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\,\text{ta}\right.}{77c^{13/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0352537, size = 97, normalized size = 0.54

$$\frac{2x^{3/2}\left(-15b^3\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+6b^2cx^2+15b^3-2bc^2x^4+7c^3x^6\right)}{77c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(15*b^3 + 6*b^2*c*x^2 - 2*b*c^2*x^4 + 7*c^3*x^6 - 15*b^3*Sqrt[1
+ (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(77*c^3*Sqrt[
x^2*(b + c*x^2)])
```

Maple [A] time = 0.19, size = 148, normalized size = 0.8

$$-\frac{1}{77\,c^4}\sqrt{x}\left(-14\,x^7c^4+15\,b^3\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)+4\,x^5bc^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/77/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-14*x^7*c^4+15*b^3*(-b*c)^(1/2)*((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))+4*x^5*b*c^3-12*b^2*c^2*x^3-30*x*b^3*c)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}{cx^2+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(9/2)/(c*x^2 + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

3.379 $\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}+\frac{14b^2x^{3/2}\left(b+cx^2\right)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)}{15c^{1}}$$

```
[Out] (14*b^2*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (14*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^2) + (2*x^(5/2)*Sqrt[b*
x^2 + c*x^4])/(9*c) - (14*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (7*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.297892, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2024, 2032, 329, 305, 220, 1196}

$$\frac{14b^2x^{3/2}\left(b+cx^2\right)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}-\frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)}}}{15c^{11/4}\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (14*b^2*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (14*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^2) + (2*x^(5/2)*Sqrt[b*
x^2 + c*x^4])/(9*c) - (14*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (7*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
```

```
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{(7b)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{9c}$$

$$= -\frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(7b^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c^2}$$

$$= -\frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(7b^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(14b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(14b^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^{5/2}\sqrt{bx^2+cx^4}} - \frac{\left(14b^{5/2}\right.}{\ldots}$$

$$= \frac{14b^2x^{3/2}\left(b+cx^2\right)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\ldots}}{15c^{11/4}\ldots}$$

Mathematica [C] time = 0.0343946, size = 86, normalized size = 0.29

$$\frac{2x^{5/2}\left(7b^2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right) - 7b^2 - 2bcx^2 + 5c^2x^4\right)}{45c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(-7*b^2 - 2*b*c*x^2 + 5*c^2*x^4 + 7*b^2*Sqrt[1 + (c*x^2)/b]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(45*c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.19, size = 217, normalized size = 0.7

$$\frac{1}{45\,c^3}\sqrt{x}\left(10\,c^3x^6 + 42\,b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right) - 21\,b^3\sqrt{\frac{cx+\sqrt{-\ldots}}{\sqrt{-bc}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/45/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^3*(10*c^3*x^6+42*b^3*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c
/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))-21*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*b*c^2*x^4-14*b^2*c*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}x^{\frac{7}{2}}}{cx^2 + b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(7/2)/(c*x^2 + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

3.380 $\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=149

$$\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

```
[Out] (-10*b*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*x^2 + c*x^
4])/(7*c) + (5*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*c^(9
/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.183499, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2024, 2032, 329, 220}

$$\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (-10*b*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*x^2 + c*x^
4])/(7*c) + (5*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*c^(9
/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx &= \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{(5b)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{7c} \\
&= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(5b^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c^2} \\
&= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(5b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21c^2\sqrt{bx^2+cx^4}} \\
&= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(10b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{21c^2\sqrt{bx^2+cx^4}} \\
&= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.0353362, size = 86, normalized size = 0.58

$$\frac{2x^{3/2}\left(5b^2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-5b^2-2bcx^2+3c^2x^4\right)}{21c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(-5*b^2 - 2*b*c*x^2 + 3*c^2*x^4 + 5*b^2*Sqrt[1 + (c*x^2)/b]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(21*c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.183, size = 137, normalized size = 0.9

$$\frac{1}{21\,c^3}\sqrt{x}\left(5\,b^2\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)+6\,c^3x^5-4\,bc^2x^3-10\,b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/21/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(5*b^2*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-
b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))+6*c^3*x^5-4*b*c^2*x^3-10*b^2*c*x)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}}{cx^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(5/2)/(c*x^2 + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

3.381 $\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=266

$$-\frac{3b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}-$$

```
[Out] (-6*b*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (6*b^(5/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/
4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4]) - (3*b^(5/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.236616, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2024, 2032, 329, 305, 220, 1196}

$$-\frac{3b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}-\frac{6}{5c^{3/2}\left(\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-6*b*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (6*b^(5/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/
4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4]) - (3*b^(5/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
```

```
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{(3b)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(3bx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(6bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(6b^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5c^{3/2}\sqrt{bx^2+cx^4}} + \frac{\left(6b^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^{}}{\sqrt{b}}}{\sqrt{b+cx}}\right.}{5c^{3/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{6bx^{3/2}\left(b+cx^2\right)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} + \frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0248894, size = 70, normalized size = 0.26

$$\frac{2x^{5/2}\left(-b\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)+b+cx^2\right)}{5c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(b + c*x^2 - b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7
/4, -((c*x^2)/b)]))/(5*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.211, size = 206, normalized size = 0.8

$$-\frac{1}{5c^2}\sqrt{x}\left(6b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-3b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/5/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^2*(6*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2
))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-3*b
^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*c^2*x^4-2*b*c*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\, x^{\frac{3}{2}}}{cx^2 + b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(3/2)/(c*x^2 + b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

3.382 $\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.131017, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2024, 2032, 329, 220}

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[b*x^2 + c*x^4],x]
```


```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx &= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{b\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3c} \\
&= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{\left(bx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3c\sqrt{bx^2+cx^4}} \\
&= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{\left(2bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3c\sqrt{bx^2+cx^4}} \\
&= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.0266189, size = 70, normalized size = 0.58

$$
\frac{2x^{3/2}\left(-b\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+b+cx^2\right)}{3c\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(b + c*x^2 - b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5
/4, -((c*x^2)/b)]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.177, size = 123, normalized size = 1.

$$-\frac{1}{3c^2}\sqrt{x}\left(b\sqrt{-bc}\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(b*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)
^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)-2*c^2*x^3-2*b*c*x)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{cx^2+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^2 + b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

3.383 $\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=231

$$\frac{\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}+\ldots$$

```
[Out] (2*x^(3/2)*(b + c*x^2))/(Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4])
 - (2*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(3/4)*Sqrt[b*x
^2 + c*x^4]) + (b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(3/4
)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.176229, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2032, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}+\frac{2x^{3/\ldots}}{\sqrt{c}\left(\sqrt{b}+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(b + c*x^2))/(Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4])
 - (2*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(3/4)*Sqrt[b*x
^2 + c*x^4]) + (b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(3/4
)*Sqrt[b*x^2 + c*x^4])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
```

```
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{\left(x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{\sqrt{bx^2+cx^4}}$$
$$= \frac{\left(2x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}}$$
$$= \frac{\left(2\sqrt{b}x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{c}\sqrt{bx^2+cx^4}} - \frac{\left(2\sqrt{b}x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{c}\sqrt{bx^2+cx^4}}$$
$$= \frac{2x^{3/2}\left(b+cx^2\right)}{\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}} + \frac{\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{}\right.}{}$$

Mathematica [C] time = 0.0121433, size = 57, normalized size = 0.25

$$\frac{2x^{5/2}\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b
)])/(3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.174, size = 131, normalized size = 0.6

$$\frac{b\sqrt{2}}{c}\sqrt{x}\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\left(2\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-\mathrm{EllipticF}\left(\sqrt{\left(\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*b/c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*(
2*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-EllipticF(
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^3 + bx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^3 + b*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

3.384 $\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}} dx$

Optimal. Leaf size=90

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

```
[Out] (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(1/4)*c^(1/4)*Sqrt[b*x^2 +
c*x^4])
```

Rubi [A] time = 0.0890293, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 329, 220}

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(1/4)*c^(1/4)*Sqrt[b*x^2 +
c*x^4])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx &= \frac{\left(x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{\sqrt{bx^2+cx^4}}\\ &= \frac{\left(2x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{bx^2+cx^4}}\\ &= \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.0132592, size = 55, normalized size = 0.61

$$\frac{2x^{3/2}\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b
)])/Sqrt[x^2*(b + c*x^2)]
```

Maple [A] time = 0.183, size = 106, normalized size = 1.2

$$\frac{\sqrt{2}}{c}\sqrt{x}\sqrt{-bc}\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{cx^4+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^4 + bx^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

3.385 $\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=259

$$\frac{\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}+\frac{}{b(}$$

```
[Out] (2*Sqrt[c]*x^(3/2)*(b + c*x^2))/(b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4
]) - (2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (c^(1/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.231556, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{c}x^{3/2}}{b\left(\sqrt{b}+\sqrt{c}x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[c]*x^(3/2)*(b + c*x^2))/(b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4
]) - (2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (c^(1/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{c\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(cx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2\sqrt{c}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{bx^2+cx^4}} - \frac{\left(2\sqrt{c}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x}{\sqrt{b}}}{\sqrt{b+cx}\ldots}\right.}{\sqrt{b}\sqrt{bx^2+cx^4}}$$

$$= \frac{2\sqrt{c}x^{3/2}\left(b+cx^2\right)}{b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} - \frac{2\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0155287, size = 55, normalized size = 0.21

$$-\frac{2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{cx^2}{b}\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((c*x^2)
/b)])/Sqrt[x^2*(b + c*x^2)]
```

Maple [A] time = 0.18, size = 195, normalized size = 0.8

$$\frac{1}{b}\sqrt{x}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)b-\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*El
lipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*b-2*c*x^2-2*b)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^5 + bx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^5 + b*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

3.386 $\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.133416, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2025, 2032, 329, 220}

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```



```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{c\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3b} \\
&= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{\left(cx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3b\sqrt{bx^2+cx^4}} \\
&= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{\left(2cx\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3b\sqrt{bx^2+cx^4}} \\
&= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.0153623, size = 57, normalized size = 0.47

$$
-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{3\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((c*x^2)/b)])/(3
*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.184, size = 119, normalized size = 1.

$$-\frac{1}{3b}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}x+2\,cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3/(c*x^4+b*x^2)^(1/2)/x^(1/2)*(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*E
llipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*
x+2*c*x^2+2*b)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{cx^6+bx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^6 + b*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

3.387 $\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$-\frac{3c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}-\frac{6c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{}{(}}}{5b^{7/4}}$$

```
[Out] (-6*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) + (6*c*Sqrt[b*x^2 + c*x^4])
/(5*b^2*x^(3/2)) + (6*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(
5*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (3*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.293697, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$-\frac{6c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{3c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}+\frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E}{5b^{7/4}\sqrt{bx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-6*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) + (6*c*Sqrt[b*x^2 + c*x^4])
/(5*b^2*x^(3/2)) + (6*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(
5*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (3*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{(3c)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{5b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(3c^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(3c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(6c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(6c^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b^{3/2}\sqrt{bx^2+cx^4}} + \frac{\left(6c^{3/2}x\sqrt{b+\right.}{}$$

$$= -\frac{6c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)}}}{5b^{7/4}\sqrt{bx^2}}$$

Mathematica [C] time = 0.0142158, size = 57, normalized size = 0.19

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5x^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((c*x^2)/b)])/(
5*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.191, size = 215, normalized size = 0.7

$$-\frac{1}{5\,b^2}\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^2bc - 3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+}{\sqrt{-}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/5/(c*x^4+b*x^2)^(1/2)/x^(3/2)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-3*(
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-6*c^2*x^4-4*b*c*x^2+2*b^2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{x}}{cx^7 + bx^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^7 + b*x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

3.388 $\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=149

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) + (10*c*Sqrt[b*x^2 + c*x^4])/(21*b^2
*x^(5/2)) + (5*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*b^(9
/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.181635, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2025, 2032, 329, 220}

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) + (10*c*Sqrt[b*x^2 + c*x^4])/(21*b^2
*x^(5/2)) + (5*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*b^(9
/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}-\frac{(5c)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{7b}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}+\frac{\left(5c^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b^2}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}+\frac{\left(5c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21b^2\sqrt{bx^2+cx^4}}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}+\frac{\left(10c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{21b^2\sqrt{bx^2+cx^4}}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}+\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0141234, size = 57, normalized size = 0.38

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((c*x^2)/b)])/(
7*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.184, size = 134, normalized size = 0.9

$$\frac{1}{21\,b^2}\left(5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^3c+10\,c^2x^4+4\,bcx^2-6\,b^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/21/(c*x^4+b*x^2)^(1/2)/x^(5/2)*(5*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2
)*x^3*c+10*c^2*x^4+4*b*c*x^2-6*b^2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{cx^8+bx^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^8 + b*x^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

3.389 $\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=326

$$\frac{7c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}+\frac{14c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}}-\frac{14c^{9/4}}{}$$

```
[Out] (14*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) + (14*c*Sqrt[b*x^2 + c*x^
4])/(45*b^2*x^(7/2)) - (14*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^(3/2)) - (14*
c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2
 + c*x^4]) + (7*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(
11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.358624, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$\frac{14c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}}+\frac{7c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}-\frac{14c^{9/4}x\left(\sqrt{b}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```


```
[Out] (14*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) + (14*c*Sqrt[b*x^2 + c*x^
4])/(45*b^2*x^(7/2)) - (14*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^(3/2)) - (14*
c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2
 + c*x^4]) + (7*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(
11/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{(7c)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{9b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{\left(7c^2\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(7c^3\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b^3}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(7c^3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(14c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{\right.}{15b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(14c^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \right.}{15b^{5/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{14c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{14c^{9/4}x\left(\sqrt{b}\right.}{}$$

Mathematica [C] time = 0.0132543, size = 57, normalized size = 0.17

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9x^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-9/4, 1/2, -5/4, -((c*x^2)/b)])/(
9*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.188, size = 230, normalized size = 0.7

$$\frac{1}{45\,b^3}\left(42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^4bc^2-21\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/45/(c*x^4+b*x^2)^(1/2)/x^(7/2)*(42*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2
)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-
21*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-42*c^3*x^6-28*b*c^2*x^4+4*b^2*c*x^
2-10*b^3)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{cx^9+bx^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^9 + b*x^7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{9}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(9/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}\,x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

3.390 $\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=179

$$-\frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{13/4}\sqrt{bx^2+cx^4}}-\frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}}+\frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) + (18*c*Sqrt[b*x^2 + c*x^4])/(77*b
^2*x^(9/2)) - (30*c^2*Sqrt[b*x^2 + c*x^4])/(77*b^3*x^(5/2)) - (15*c^(11/4)*
x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF
[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*b^(13/4)*Sqrt[b*x^2 + c*x^4
])
```

Rubi [A] time = 0.233116, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2025, 2032, 329, 220}

$$-\frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}}-\frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{13/4}\sqrt{bx^2+cx^4}}+\frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```


```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) + (18*c*Sqrt[b*x^2 + c*x^4])/(77*b
^2*x^(9/2)) - (30*c^2*Sqrt[b*x^2 + c*x^4])/(77*b^3*x^(5/2)) - (15*c^(11/4)*
x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF
[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*b^(13/4)*Sqrt[b*x^2 + c*x^4
])
```


Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
```

```
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{(9c)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx}{11b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{\left(45c^2\right)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{\left(15c^3\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77b^3}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{\left(15c^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{77b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{\left(30c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \ldots\right.}{77b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan\ldots\right.}{77b^{13/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0134762, size = 57, normalized size = 0.32

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{11}{4},\frac{1}{2};-\frac{7}{4};-\frac{cx^2}{b}\right)}{11x^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-11/4, 1/2, -7/4, -((c*x^2)/b)])/
(11*x^(9/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.187, size = 147, normalized size = 0.8

$$-\frac{1}{77\,b^3}\left(15\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^5c^2+30\,c^3x^6+12\,bc^2x^4 \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/77/(c*x^4+b*x^2)^(1/2)/x^(9/2)*(15*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1
/2)*x^5*c^2+30*c^3*x^6+12*b*c^2*x^4-4*b^2*c*x^2+14*b^3)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{cx^{10}+bx^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c*x^10 + b*x^8), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

3.391 $\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14c^{13/4}\sqrt{bx^2+cx^4}}+\frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2}-\frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^(11/2)/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(7*c^3*Sqr
t[x]) + (9*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c^2) + (15*b^(7/4)*x*(Sqrt[b] +
Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^
(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.242447, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2022, 2024, 2032, 329, 220}

$$\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14c^{13/4}\sqrt{bx^2+cx^4}}+\frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2}-\frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] -(x^(11/2)/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(7*c^3*Sqr
t[x]) + (9*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c^2) + (15*b^(7/4)*x*(Sqrt[b] +
Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^
(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} + \frac{9\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} - \frac{(45b)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{14c^2}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{\left(15b^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{14c^3}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{\left(15b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{14c^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{\left(15b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{\right.}{7c^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\right.}{14c^{13/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0292939, size = 86, normalized size = 0.49

$$\frac{x^{3/2}\left(15b^2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-15b^2-6bcx^2+2c^2x^4\right)}{7c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-15*b^2 - 6*b*c*x^2 + 2*c^2*x^4 + 15*b^2*Sqrt[1 + (c*x^2)/b]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(7*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.191, size = 144, normalized size = 0.8

$$\frac{cx^2+b}{14\,c^4}x^{\frac{5}{2}}\left(15\,b^2\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)+4\,c^3x^5-12\,bc^2x^3-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/14/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(15*b^2*(-b*c)^(1/2)*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))+4*c^3*x^5-12*b*c^2*x^3-30*b^2*c*x)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{17}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\,x^{\frac{9}{2}}}{c^2x^4 + 2\,bcx^2 + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(9/2)/(c^2*x^4 + 2*b*c*x^2 + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{17}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.392 $\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}+\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^(9/2)/(c*Sqrt[b*x^2 + c*x^4])) - (21*b*x^(3/2)*(b + c*x^2))/(5*c^(5/2)*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (7*Sqrt[x]*Sqrt[b*x^2 + c*x^4]
)/(5*c^2) + (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(1
1/4)*Sqrt[b*x^2 + c*x^4]) - (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.29648, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2022, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}+\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}}-\frac{}{5c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] -(x^(9/2)/(c*Sqrt[b*x^2 + c*x^4])) - (21*b*x^(3/2)*(b + c*x^2))/(5*c^(5/2)*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (7*Sqrt[x]*Sqrt[b*x^2 + c*x^4]
)/(5*c^2) + (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(1
1/4)*Sqrt[b*x^2 + c*x^4]) - (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}+\frac{7\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}+\frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2}-\frac{(21b)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{10c^2}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}+\frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2}-\frac{\left(21bx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{10c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}+\frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2}-\frac{\left(21bx\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}+\frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2}-\frac{\left(21b^{3/2}x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c^{5/2}\sqrt{bx^2+cx^4}}+\frac{\left(21b^{3/2}x\sqrt{\right.}{}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}-\frac{21bx^{3/2}\left(b+cx^2\right)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2}+\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+c}{(\sqrt{b}+}}}{5c^{11/4}\sqrt{bx}}$$

Mathematica [C] time = 0.0264773, size = 72, normalized size = 0.25

$$\frac{2x^{5/2}\left(7b\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)-7b+cx^2\right)}{5c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*(-7*b + c*x^2 + 7*b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3
/2, 7/4, -((c*x^2)/b)]))/(5*c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.187, size = 213, normalized size = 0.7

$$-\frac{cx^2+b}{10\,c^3}x^{\frac{5}{2}}\left(42\,b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)-21\,b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(42*b^2*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))-21*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*c^2*x^4-14*b*c*x^2)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{15}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\,x^{\frac{7}{2}}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(7/2)/(c^2*x^4 + 2*b*c*x^2 + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{15}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.393 $\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}+\frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^(7/2)/(c*Sqrt[b*x^2 + c*x^4])) + (5*Sqrt[b*x^2 + c*x^4])/(3*c^2*Sqrt[x]
) - (5*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(6*c^(9/4)*Sqrt[
b*x^2 + c*x^4])
```

Rubi [A] time = 0.185086, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2022, 2024, 2032, 329, 220}

$$-\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}+\frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^(3/2),x]
```



```
[Out] -(x^(7/2)/(c*Sqrt[b*x^2 + c*x^4])) + (5*Sqrt[b*x^2 + c*x^4])/(3*c^2*Sqrt[x]
) - (5*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(6*c^(9/4)*Sqrt[
b*x^2 + c*x^4])
```


Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{(5b)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6c^2}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{\left(5bx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{6c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{\left(5bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.026614, size = 73, normalized size = 0.5

$$\frac{x^{3/2}\left(-5b\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+5b+2cx^2\right)}{3c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(5*b + 2*c*x^2 - 5*b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/
2, 5/4, -((c*x^2)/b)]))/(3*c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.186, size = 131, normalized size = 0.9

$$-\frac{cx^2+b}{6\,c^3}x^{\frac{5}{2}}\left(5\,b\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-4\,c^2x^3-10\,bcx\right)\left(cx^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(5*b*(-b*c)^(1/2)*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
,1/2*2^(1/2))-4*c^2*x^3-10*b*c*x)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{cx^4 + bx^2}\,x^{\frac{5}{2}}}{c^2x^4 + 2\,bcx^2 + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(5/2)/(c^2*x^4 + 2*b*c*x^2 + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.394 $\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=259

$$\frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{7/4}\sqrt{bx^2+cx^4}}+\frac{3x^{3/2}\left(b+cx^2\right)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b-}{(\sqrt{b}-}}}{c^{7/4}\sqrt{b}}$$

```
[Out] -(x^(5/2)/(c*Sqrt[b*x^2 + c*x^4])) + (3*x^(3/2)*(b + c*x^2))/(c^(3/2)*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(c^(7/4)*Sqrt[b*x^2 + c*x^4]) + (3*b^(1/4)*x*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c
^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.23408, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2022, 2032, 329, 305, 220, 1196}

$$\frac{3x^{3/2}\left(b+cx^2\right)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2c^{7/4}\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(\right.}{c^{7/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(5/2)/(c*Sqrt[b*x^2 + c*x^4])) + (3*x^(3/2)*(b + c*x^2))/(c^(3/2)*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(c^(7/4)*Sqrt[b*x^2 + c*x^4]) + (3*b^(1/4)*x*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c
^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
```

```
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}} + \frac{3\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}} + \frac{\left(3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2c\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}} + \frac{\left(3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{c\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}} + \frac{\left(3\sqrt{b}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{c^{3/2}\sqrt{bx^2+cx^4}} - \frac{\left(3\sqrt{b}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}}{\sqrt{}}}{\sqrt{b+c}}\right.}{c^{3/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}} + \frac{3x^{3/2}\left(b+cx^2\right)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{c^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0223051, size = 61, normalized size = 0.24

$$-\frac{2x^{5/2}\left(\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)-1\right)}{c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x^(5/2)*(-1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c
*x^2)/b)]))/(c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.216, size = 200, normalized size = 0.8

$$\frac{cx^2+b}{2\,c^2}x^{\frac{5}{2}}\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)b-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-
3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*b-2*c*x^2)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\,x^{\frac{3}{2}}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(3/2)/(c^2*x^4 + 2*b*c*x^2 + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.395 $\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=119

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^(3/2)/(c*Sqrt[b*x^2 + c*x^4])) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(2*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.138264, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2022, 2032, 329, 220}

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(3/2)/(c*Sqrt[b*x^2 + c*x^4])) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(2*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{2c}\\
&= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{2c\sqrt{bx^2+cx^4}}\\
&= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{c\sqrt{bx^2+cx^4}}\\
&= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0207832, size = 60, normalized size = 0.5

$$\frac{x^{3/2}\left(\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)-1\right)}{c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^
2)/b)]))/(c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.206, size = 120, normalized size = 1.

$$\frac{cx^2+b}{2c^2}x^{\frac{5}{2}}\left(\sqrt{-bc}\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\mathrm{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c
/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))-2*c*x)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{c^2x^4+2\,bcx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^4 + 2*b*c*x^2 + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.396 $\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x^{5/2}}{b\sqrt{bx^2+\dots}}$$

```
[Out] x^(5/2)/(b*Sqrt[b*x^2 + c*x^4]) - (x^(3/2)*(b + c*x^2))/(b*Sqrt[c]*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4]) - (x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(2*b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.236835, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2023, 2032, 329, 305, 220, 1196}

$$-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\dots$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] x^(5/2)/(b*Sqrt[b*x^2 + c*x^4]) - (x^(3/2)*(b + c*x^2))/(b*Sqrt[c]*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4]) - (x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(2*b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
```

```
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\left(x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2b\sqrt{bx^2+cx^4}}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{c}\sqrt{bx^2+cx^4}} + \frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right.}{\sqrt{b}\sqrt{c}\sqrt{bx^2+cx^4}}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{x^{3/2}\left(b+cx^2\right)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}} -$$

Mathematica [C] time = 0.0152684, size = 60, normalized size = 0.23

$$\frac{2x^{5/2}\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)}{3b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^2)/b
)])/(3*b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.187, size = 203, normalized size = 0.8

$$-\frac{cx^2+b}{2\,bc}x^{\frac{5}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)b-\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1
/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b
-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2),1/2*2^(1/2))*b-2*c*x^2)/c/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{x}}{c^2x^5 + 2\,bcx^3 + b^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^5 + 2*b*c*x^3 + b^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(7/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.397 $\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

```
[Out] x^(3/2)/(b*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(2*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.136454, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2023, 2032, 329, 220}

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^(3/2)/(b*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(2*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\
&= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{2b\sqrt{bx^2+cx^4}}\\
&= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}\\
&= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0195093, size = 60, normalized size = 0.51

$$\frac{x^{3/2}\left(\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)+1\right)}{b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2
)/b)]))/(b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.207, size = 123, normalized size = 1.

$$\frac{cx^2+b}{2\,bc}x^{\frac{5}{2}}\left(\sqrt{-bc}\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c
/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))+2*c*x)/c/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{5}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{c^2x^6+2\,bcx^4+b^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^6 + 2*b*c*x^4 + b^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(5/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.398 $\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{7/4}\sqrt{bx^2+cx^4}}+\frac{3\sqrt{c}x^{3/2}\left(b+cx^2\right)}{b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}-\frac{3\sqrt[4]{c}x\left(\sqrt{b}\right.}{}$$

```
[Out] Sqrt[x]/(b*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*x^(3/2)*(b + c*x^2))/(b^2*(Sqr
t[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(b^2*x^(3/
2)) - (3*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c
]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(7/4)*Sqrt[
b*x^2 + c*x^4]) + (3*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2
*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.298486, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt{c}x^{3/2}\left(b+cx^2\right)}{b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}+\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] Sqrt[x]/(b*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*x^(3/2)*(b + c*x^2))/(b^2*(Sqr
t[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(b^2*x^(3/
2)) - (3*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c
]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(7/4)*Sqrt[
b*x^2 + c*x^4]) + (3*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2
*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}+\frac{3\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}+\frac{(3c)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}+\frac{\left(3cx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}+\frac{\left(3cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}+\frac{\left(3\sqrt{c}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{b^{3/2}\sqrt{bx^2+cx^4}}-\frac{\left(3\sqrt{c}x\sqrt{b+cx^2}\right)}{b^{3/}}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}+\frac{3\sqrt{c}x^{3/2}\left(b+cx^2\right)}{b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}-\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\right.}{b^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0180669, size = 58, normalized size = 0.2

$$-\frac{2\sqrt{x}\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{cx^2}{b}\right)}{b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((c*x^2)
/b)])/(b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.201, size = 203, normalized size = 0.7

$$\frac{cx^2+b}{2\,b^2}x^{\frac{5}{2}}\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)b-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+}{\sqrt{-}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-
3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*b-6*c*x^2-4*b)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{c^2x^7+2\,bcx^5+b^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^7 + 2*b*c*x^5 + b^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(3/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.399 $\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^(5/2))
 - (5*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(6*b^(9/4)*Sqrt[b
*x^2 + c*x^4])
```

Rubi [A] time = 0.184625, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2023, 2025, 2032, 329, 220}

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] 1/(b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^(5/2))
 - (5*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(6*b^(9/4)*Sqrt[b
*x^2 + c*x^4])
```


Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{5\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{(5c)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6b^2}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{\left(5cx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{6b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{\left(5cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0194315, size = 60, normalized size = 0.41

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{3b\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((c*x^2)/b)])/(3
*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.217, size = 127, normalized size = 0.9

$$-\frac{cx^2+b}{6\,b^2}x^{\frac{3}{2}}\left(5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x+10\,cx^2+4\,b\right)\left(cx^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(3/2)*(c*x^2+b)*(5*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1
/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(
-b*c)^(1/2)*x+10*c*x^2+4*b)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\sqrt{x}}{c^2x^8 + 2\,bcx^6 + b^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

3.400 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=320

$$-\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}-\frac{21c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\cdots}{5b^{1\cdots}}$$

```
[Out] 1/(b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) - (21*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b^3
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*Sqrt[b*x^2 + c*x^4])/(5*b^
2*x^(7/2)) + (21*c*Sqrt[b*x^2 + c*x^4])/(5*b^3*x^(3/2)) + (21*c^(5/4)*x*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (
21*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sqrt[b*
x^2 + c*x^4])
```

Rubi [A] time = 0.364085, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2023, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{21c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}+\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)\cdots}}\cdots}{5b^{11/4}\sqrt{bx^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] 1/(b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) - (21*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b^3
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*Sqrt[b*x^2 + c*x^4])/(5*b^
2*x^(7/2)) + (21*c*Sqrt[b*x^2 + c*x^4])/(5*b^3*x^(3/2)) + (21*c^(5/4)*x*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (
21*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sqrt[b*
x^2 + c*x^4])
```


Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}+\frac{7\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}-\frac{(21c)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{10b^2}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{\left(21c^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{10b^3}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{\left(21c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{10b^3\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{\left(21c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,x\right.}{5b^3\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{\left(21c^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,\right.}{5b^{5/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{21c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}+\frac{21c^{5/4}x\left(\right.}{}$$

Mathematica [C] time = 0.0170526, size = 60, normalized size = 0.19

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5bx^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((c*x^2)/b)])/(
5*b*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.197, size = 222, normalized size = 0.7

$$-\frac{cx^2+b}{10\,b^3}\sqrt{x}\left(42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)x^2bc-21\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(1/2)*(c*x^2+b)*(42*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*x^2*b*c-21*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^2*b*c-42*c^2*x^4-28*b*c*x^2+4*b^2
)/b^3
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{c^2x^9+2\,bcx^7+b^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^9 + 2*b*c*x^7 + b^2*x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

3.401 $\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14b^{13/4}\sqrt{bx^2+cx^4}}+\frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}}-\frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) - (9*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^(9/2))
 + (15*c*Sqrt[b*x^2 + c*x^4])/(7*b^3*x^(5/2)) + (15*c^(7/4)*x*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1
/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.233287, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2023, 2025, 2032, 329, 220}

$$\frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{bx^2+cx^4}}+\frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}}-\frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] 1/(b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) - (9*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^(9/2))
 + (15*c*Sqrt[b*x^2 + c*x^4])/(7*b^3*x^(5/2)) + (15*c^(7/4)*x*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1
/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{9\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} - \frac{(45c)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{14b^2}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{\left(15c^2\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{14b^3}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{\left(15c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{14b^3\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{\left(15c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x,\right.}{7b^3\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan\right.}{14b^{13/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] time = 0.0171167, size = 60, normalized size = 0.35

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7bx^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-7/4, 3/2, -3/4, -((c*x^2)/b)])/(
7*b*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.217, size = 141, normalized size = 0.8

$$\frac{cx^2+b}{14\,b^3}\left(15\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}x^3c+30\,c^2x^4+12\,bcx^2-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/14/(c*x^4+b*x^2)^(3/2)/x^(1/2)*(c*x^2+b)*(15*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
(-b*c)^(1/2)*x^3*c+30*c^2*x^4+12*b*c*x^2-4*b^2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{c^2x^{10}+2\,bcx^8+b^2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^10 + 2*b*c*x^8 + b^2*x^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

3.402 $\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=350

$$\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}+\frac{77c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}}-$$

```
[Out] 1/(b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (77*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^
4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (11*Sqrt[b*x^2 + c*x^4])/(9*
b^2*x^(11/2)) + (77*c*Sqrt[b*x^2 + c*x^4])/(45*b^3*x^(7/2)) - (77*c^2*Sqrt[
b*x^2 + c*x^4])/(15*b^4*x^(3/2)) - (77*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) + (77*c^(9/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.426855, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{77c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}}+\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] 1/(b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (77*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^
4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (11*Sqrt[b*x^2 + c*x^4])/(9*
b^2*x^(11/2)) + (77*c*Sqrt[b*x^2 + c*x^4])/(45*b^3*x^(7/2)) - (77*c^2*Sqrt[
b*x^2 + c*x^4])/(15*b^4*x^(3/2)) - (77*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) + (77*c^(9/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2 + c*x^4])
```


Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{11\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} - \frac{(77c)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{18b^2}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{\left(77c^2\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{30b^3}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{\left(77c^3\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,d}{30b^4}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{\left(77c^3x\sqrt{b+cx^2}\right)}{30b^4\sqrt{bx^2+}}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{\left(77c^3x\sqrt{b+cx^2}\right)}{15b}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{\left(77c^{5/2}x\sqrt{b+cx^2}\right)}{15b}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{77c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2}{15}$$

Mathematica [C] time = 0.0178508, size = 60, normalized size = 0.17

$$-\frac{2\sqrt{\frac{cx^2}{b}+1}\,{}_2F_1\left(-\frac{9}{4},\frac{3}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9bx^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((c*x^2)/b)])/(
9*b*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.202, size = 237, normalized size = 0.7

$$\frac{cx^2+b}{90\,b^4}\left(462\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)x^4bc^2-231\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/90/(c*x^4+b*x^2)^(3/2)/x^(3/2)*(c*x^2+b)*(462*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*x^4*b*c^2-231*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^4*b*c^2-462*c^3*x^6-308*b*c^2*
x^4+44*b^2*c*x^2-20*b^3)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}\sqrt{x}}{c^2x^{11}+2\,bcx^9+b^2x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x)/(c^2*x^11 + 2*b*c*x^9 + b^2*x^7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}} \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**(5/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

3.403 $\int (cx)^m \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=73

$$\frac{3b^2cx^9(cx)^m}{m+9}+\frac{b^3x^7(cx)^m}{m+7}+\frac{3bc^2x^{11}(cx)^m}{m+11}+\frac{c^3x^{13}(cx)^m}{m+13}$$

```
[Out] (b^3*x^7*(c*x)^m)/(7 + m) + (3*b^2*c*x^9*(c*x)^m)/(9 + m) + (3*b*c^2*x^11*(
c*x)^m)/(11 + m) + (c^3*x^13*(c*x)^m)/(13 + m)
```

Rubi [A] time = 0.0498427, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 270}

$$\frac{3b^2cx^9(cx)^m}{m+9}+\frac{b^3x^7(cx)^m}{m+7}+\frac{3bc^2x^{11}(cx)^m}{m+11}+\frac{c^3x^{13}(cx)^m}{m+13}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (b^3*x^7*(c*x)^m)/(7 + m) + (3*b^2*c*x^9*(c*x)^m)/(9 + m) + (3*b*c^2*x^11*(
c*x)^m)/(11 + m) + (c^3*x^13*(c*x)^m)/(13 + m)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (cx)^m \left(bx^2+cx^4\right)^3 dx = \left(x^{-m}(cx)^m\right) \text{Subst}\left(\int x^m \left(bx^2+cx^4\right)^3 dx, x, x\right)$$
$$= \left(x^{-m}(cx)^m\right) \text{Subst}\left(\int x^{6+m} \left(b+cx^2\right)^3 dx, x, x\right)$$
$$= \left(x^{-m}(cx)^m\right) \text{Subst}\left(\int \left(b^3x^{6+m}+3b^2cx^{8+m}+3bc^2x^{10+m}+c^3x^{12+m}\right) dx, x, x\right)$$
$$= \frac{b^3x^7(cx)^m}{7+m}+\frac{3b^2cx^9(cx)^m}{9+m}+\frac{3bc^2x^{11}(cx)^m}{11+m}+\frac{c^3x^{13}(cx)^m}{13+m}$$

Mathematica [A] time = 0.0400591, size = 59, normalized size = 0.81

$$x^7(cx)^m\left(\frac{3b^2cx^2}{m+9}+\frac{b^3}{m+7}+\frac{3bc^2x^4}{m+11}+\frac{c^3x^6}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^7*(c*x)^m*(b^3/(7 + m) + (3*b^2*c*x^2)/(9 + m) + (3*b*c^2*x^4)/(11 + m) +
 (c^3*x^6)/(13 + m))
```

Maple [B] time = 0.048, size = 181, normalized size = 2.5

$$\frac{(cx)^m \left(c^3m^3x^6+27\,c^3m^2x^6+3\,bc^2m^3x^4+239\,c^3mx^6+87\,bc^2m^2x^4+693\,c^3x^6+3\,b^2cm^3x^2+813\,bc^2mx^4+93\,b^2cm^2x^2\right.}{(13+m)\,(11+m)\,(9+m)\,(7+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2)^3,x)
```

```
[Out] (c*x)^m*(c^3*m^3*x^6+27*c^3*m^2*x^6+3*b*c^2*m^3*x^4+239*c^3*m*x^6+87*b*c^2*
m^2*x^4+693*c^3*x^6+3*b^2*c*m^3*x^2+813*b*c^2*m*x^4+93*b^2*c*m^2*x^2+2457*b
*c^2*x^4+b^3*m^3+933*b^2*c*m*x^2+33*b^3*m^2+3003*b^2*c*x^2+359*b^3*m+1287*b
^3)*x^7/(13+m)/(11+m)/(9+m)/(7+m)
```

Maxima [A] time = 1.01244, size = 103, normalized size = 1.41

$$\frac{c^{m+3}x^{13}x^m}{m+13} + \frac{3\,bc^{m+2}x^{11}x^m}{m+11} + \frac{3\,b^2c^{m+1}x^9x^m}{m+9} + \frac{b^3c^mx^7x^m}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] c^(m + 3)*x^13*x^m/(m + 13) + 3*b*c^(m + 2)*x^11*x^m/(m + 11) + 3*b^2*c^(m
+ 1)*x^9*x^m/(m + 9) + b^3*c^m*x^7*x^m/(m + 7)
```

Fricas [B] time = 1.58996, size = 375, normalized size = 5.14

$$\frac{\left(\left(c^3m^3 + 27\,c^3m^2 + 239\,c^3m + 693\,c^3\right)x^{13} + 3\left(bc^2m^3 + 29\,bc^2m^2 + 271\,bc^2m + 819\,bc^2\right)x^{11} + 3\left(b^2cm^3 + 31\,b^2cm^2 + 311\right.\right.}{m^4 + 40\,m^3 + 590\,m^2 + 3800\,m + 9009}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^3 + 27*c^3*m^2 + 239*c^3*m + 693*c^3)*x^13 + 3*(b*c^2*m^3 + 29*b*c^
2*m^2 + 271*b*c^2*m + 819*b*c^2)*x^11 + 3*(b^2*c*m^3 + 31*b^2*c*m^2 + 311*b
^2*c*m + 1001*b^2*c)*x^9 + (b^3*m^3 + 33*b^3*m^2 + 359*b^3*m + 1287*b^3)*x^
7)*(c*x)^m/(m^4 + 40*m^3 + 590*m^2 + 3800*m + 9009)
```

Sympy [A] time = 5.30557, size = 758, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2)**3,x)
```

```
[Out] Piecewise(((-b**3/(6*x**6) - 3*b**2*c/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*l
og(x))/c**13, Eq(m, -13)), ((-b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**2*
log(x) + c**3*x**2/2)/c**11, Eq(m, -11)), ((-b**3/(2*x**2) + 3*b**2*c*log(x
) + 3*b*c**2*x**2/2 + c**3*x**4/4)/c**9, Eq(m, -9)), ((b**3*log(x) + 3*b**2
*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6)/c**7, Eq(m, -7)), (b**3*c**m*m**
```

```
3*x**7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 33*b**3*c**m*m**2
*x**7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 359*b**3*c**m*m*x*
*7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 1287*b**3*c**m*x**7*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b**2*c*c**m*m**3*x**9*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 93*b**2*c*c**m*m**2*x**9*
x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 933*b**2*c*c**m*m*x**9*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3003*b**2*c*c**m*x**9*x**
m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b*c**2*c**m*m**3*x**11*x*
*m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 87*b*c**2*c**m*m**2*x**11*
x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 813*b*c**2*c**m*m*x**11*
x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 2457*b*c**2*c**m*x**11*x
**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + c**3*c**m*m**3*x**13*x**m
/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 27*c**3*c**m*m**2*x**13*x**m
/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 239*c**3*c**m*m*x**13*x**m/(
m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 693*c**3*c**m*x**13*x**m/(m**4
 + 40*m**3 + 590*m**2 + 3800*m + 9009), True))
```

Giac [B] time = 1.1681, size = 356, normalized size = 4.88

$$\frac{(cx)^m c^3m^3x^{13} + 27\,(cx)^m c^3m^2x^{13} + 3\,(cx)^m bc^2m^3x^{11} + 239\,(cx)^m c^3mx^{13} + 87\,(cx)^m bc^2m^2x^{11} + 693\,(cx)^m c^3x^{13} + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] ((c*x)^m*c^3*m^3*x^13 + 27*(c*x)^m*c^3*m^2*x^13 + 3*(c*x)^m*b*c^2*m^3*x^11
+ 239*(c*x)^m*c^3*m*x^13 + 87*(c*x)^m*b*c^2*m^2*x^11 + 693*(c*x)^m*c^3*x^13
 + 3*(c*x)^m*b^2*c*m^3*x^9 + 813*(c*x)^m*b*c^2*m*x^11 + 93*(c*x)^m*b^2*c*m^
2*x^9 + 2457*(c*x)^m*b*c^2*x^11 + (c*x)^m*b^3*m^3*x^7 + 933*(c*x)^m*b^2*c*m
*x^9 + 33*(c*x)^m*b^3*m^2*x^7 + 3003*(c*x)^m*b^2*c*x^9 + 359*(c*x)^m*b^3*m*
x^7 + 1287*(c*x)^m*b^3*x^7)/(m^4 + 40*m^3 + 590*m^2 + 3800*m + 9009)
```

3.404 $\int (cx)^m \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=52

$$\frac{b^2x^5(cx)^m}{m+5}+\frac{2bcx^7(cx)^m}{m+7}+\frac{c^2x^9(cx)^m}{m+9}$$

```
[Out] (b^2*x^5*(c*x)^m)/(5 + m) + (2*b*c*x^7*(c*x)^m)/(7 + m) + (c^2*x^9*(c*x)^m)
/(9 + m)
```

Rubi [A] time = 0.0394327, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 270}

$$\frac{b^2x^5(cx)^m}{m+5}+\frac{2bcx^7(cx)^m}{m+7}+\frac{c^2x^9(cx)^m}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5*(c*x)^m)/(5 + m) + (2*b*c*x^7*(c*x)^m)/(7 + m) + (c^2*x^9*(c*x)^m)
/(9 + m)
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (cx)^m \left(bx^2+cx^4\right)^2 dx &= \left(x^{-m}(cx)^m\right) \operatorname{Subst}\left(\int x^m \left(bx^2+cx^4\right)^2 dx, x, x\right)\\
&= \left(x^{-m}(cx)^m\right) \operatorname{Subst}\left(\int x^{4+m} \left(b+cx^2\right)^2 dx, x, x\right)\\
&= \left(x^{-m}(cx)^m\right) \operatorname{Subst}\left(\int \left(b^2x^{4+m}+2bcx^{6+m}+c^2x^{8+m}\right) dx, x, x\right)\\
&= \frac{b^2x^5(cx)^m}{5+m}+\frac{2bcx^7(cx)^m}{7+m}+\frac{c^2x^9(cx)^m}{9+m}
\end{aligned}
$$

Mathematica [A] time = 0.034236, size = 43, normalized size = 0.83

$$
x^5(cx)^m \left(\frac{b^2}{m+5}+\frac{2bcx^2}{m+7}+\frac{c^2x^4}{m+9}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^5*(c*x)^m*(b^2/(5 + m) + (2*b*c*x^2)/(7 + m) + (c^2*x^4)/(9 + m))
```

Maple [A] time = 0.049, size = 96, normalized size = 1.9

$$
\frac{(cx)^m \left(c^2m^2x^4+12\,c^2mx^4+2\,bcm^2x^2+35\,c^2x^4+28\,bcmx^2+b^2m^2+90\,bcx^2+16\,b^2m+63\,b^2\right) x^5}{(9+m)(7+m)(5+m)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2)^2,x)
```

```
[Out] (c*x)^m*(c^2*m^2*x^4+12*c^2*m*x^4+2*b*c*m^2*x^2+35*c^2*x^4+28*b*c*m*x^2+b^2
*m^2+90*b*c*x^2+16*b^2*m+63*b^2)*x^5/(9+m)/(7+m)/(5+m)
```

Maxima [A] time = 1.00875, size = 74, normalized size = 1.42

$$
\frac{c^{m+2}x^9x^m}{m+9}+\frac{2\,bc^{m+1}x^7x^m}{m+7}+\frac{b^2c^mx^5x^m}{m+5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] c^(m + 2)*x^9*x^m/(m + 9) + 2*b*c^(m + 1)*x^7*x^m/(m + 7) + b^2*c^m*x^5*x^m
/(m + 5)
```

Fricas [A] time = 1.5866, size = 200, normalized size = 3.85

$$\frac{\left(\left(c^2m^2+12\,c^2m+35\,c^2\right)x^9+2\left(bcm^2+14\,bcm+45\,bc\right)x^7+\left(b^2m^2+16\,b^2m+63\,b^2\right)x^5\right)(cx)^m}{m^3+21\,m^2+143\,m+315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^2 + 12*c^2*m + 35*c^2)*x^9 + 2*(b*c*m^2 + 14*b*c*m + 45*b*c)*x^7 +
(b^2*m^2 + 16*b^2*m + 63*b^2)*x^5)*(c*x)^m/(m^3 + 21*m^2 + 143*m + 315)
```

Sympy [A] time = 2.1886, size = 352, normalized size = 6.77

$$\begin{cases} \frac{-\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)}{c^9} \\ \frac{-\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}}{c^7} \\ \frac{b^2\log(x)+bcx^2+\frac{c^2x^4}{4}}{c^5} \\ \frac{b^2c^mm^2x^5x^m}{m^3+21m^2+143m+315}+\frac{16b^2c^mmx^5x^m}{m^3+21m^2+143m+315}+\frac{63b^2c^mx^5x^m}{m^3+21m^2+143m+315}+\frac{2bcc^mm^2x^7x^m}{m^3+21m^2+143m+315}+\frac{28bcc^mmx^7x^m}{m^3+21m^2+143m+315}+\frac{90bcc^mx^7x^m}{m^3+21m^2+143m+315}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2)**2,x)
```

```
[Out] Piecewise(((-b**2/(4*x**4) - b*c/x**2 + c**2*log(x))/c**9, Eq(m, -9)), ((-b
**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/c**7, Eq(m, -7)), ((b**2*log(x)
+ b*c*x**2 + c**2*x**4/4)/c**5, Eq(m, -5)), (b**2*c**m*m**2*x**5*x**m/(m**3
 + 21*m**2 + 143*m + 315) + 16*b**2*c**m*m*x**5*x**m/(m**3 + 21*m**2 + 143*
m + 315) + 63*b**2*c**m*x**5*x**m/(m**3 + 21*m**2 + 143*m + 315) + 2*b*c*c*
*m*m**2*x**7*x**m/(m**3 + 21*m**2 + 143*m + 315) + 28*b*c*c**m*m*x**7*x**m/
```

```
(m**3 + 21*m**2 + 143*m + 315) + 90*b*c*c**m*x**7*x**m/(m**3 + 21*m**2 + 14
3*m + 315) + c**2*c**m*m**2*x**9*x**m/(m**3 + 21*m**2 + 143*m + 315) + 12*c
**2*c**m*m*x**9*x**m/(m**3 + 21*m**2 + 143*m + 315) + 35*c**2*c**m*x**9*x**
m/(m**3 + 21*m**2 + 143*m + 315), True))
```

Giac [B] time = 1.15981, size = 190, normalized size = 3.65

$$\frac{(cx)^m c^2m^2x^9 + 12\,(cx)^m c^2mx^9 + 2\,(cx)^m bcm^2x^7 + 35\,(cx)^m c^2x^9 + 28\,(cx)^m bcmx^7 + (cx)^m b^2m^2x^5 + 90\,(cx)^m bcx^7 +}{m^3 + 21\,m^2 + 143\,m + 315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] ((c*x)^m*c^2*m^2*x^9 + 12*(c*x)^m*c^2*m*x^9 + 2*(c*x)^m*b*c*m^2*x^7 + 35*(c
*x)^m*c^2*x^9 + 28*(c*x)^m*b*c*m*x^7 + (c*x)^m*b^2*m^2*x^5 + 90*(c*x)^m*b*c
*x^7 + 16*(c*x)^m*b^2*m*x^5 + 63*(c*x)^m*b^2*x^5)/(m^3 + 21*m^2 + 143*m + 3
15)
```

3.405 $\int (cx)^m \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=34

$$\frac{b(cx)^{m+3}}{c^3(m+3)}+\frac{(cx)^{m+5}}{c^4(m+5)}$$

```
[Out] (b*(c*x)^(3 + m))/(c^3*(3 + m)) + (c*x)^(5 + m)/(c^4*(5 + m))
```

Rubi [A] time = 0.014332, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{b(cx)^{m+3}}{c^3(m+3)}+\frac{(cx)^{m+5}}{c^4(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4),x]
```

```
[Out] (b*(c*x)^(3 + m))/(c^3*(3 + m)) + (c*x)^(5 + m)/(c^4*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(bx^2+cx^4\right) dx &= \int \left(\frac{b(cx)^{2+m}}{c^2}+\frac{(cx)^{4+m}}{c^3}\right) dx\\ &= \frac{b(cx)^{3+m}}{c^3(3+m)}+\frac{(cx)^{5+m}}{c^4(5+m)}\end{aligned}$$

Mathematica [A] time = 0.016443, size = 27, normalized size = 0.79

$$x^3(cx)^m\left(\frac{b}{m+3}+\frac{cx^2}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4),x]
```

```
[Out] x^3*(c*x)^m*(b/(3 + m) + (c*x^2)/(5 + m))
```

Maple [A] time = 0.044, size = 39, normalized size = 1.2

$$\frac{(cx)^m\left(cmx^2+3\,cx^2+bm+5\,b\right)x^3}{(5+m)\,(3+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2),x)
```

```
[Out] (c*x)^m*(c*m*x^2+3*c*x^2+b*m+5*b)*x^3/(5+m)/(3+m)
```

Maxima [A] time = 1.0151, size = 46, normalized size = 1.35

$$\frac{c^{m+1}x^5x^m}{m+5}+\frac{bc^mx^3x^m}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] c^(m + 1)*x^5*x^m/(m + 5) + b*c^m*x^3*x^m/(m + 3)
```

Fricas [A] time = 1.57861, size = 84, normalized size = 2.47

$$\frac{\left((cm+3\,c)x^5+(bm+5\,b)x^3\right)(cx)^m}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] ((c*m + 3*c)*x^5 + (b*m + 5*b)*x^3)*(c*x)^m/(m^2 + 8*m + 15)
```

Sympy [A] time = 0.711242, size = 119, normalized size = 3.5

$$\begin{cases} \frac{-\frac{b}{2x^2}+c\log(x)}{c^5} & \text{for } m=-5 \\ \frac{b\log(x)+\frac{cx^2}{2}}{c^3} & \text{for } m=-3 \\ \frac{bc^m mx^3x^m}{m^2+8m+15} + \frac{5bc^m x^3x^m}{m^2+8m+15} + \frac{cc^m mx^5x^m}{m^2+8m+15} + \frac{3cc^m x^5x^m}{m^2+8m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((((-b/(2*x**2) + c*log(x))/c**5, Eq(m, -5)), ((b*log(x) + c*x**2/2
)/c**3, Eq(m, -3)), (b*c**m*m*x**3*x**m/(m**2 + 8*m + 15) + 5*b*c**m*x**3*x
**m/(m**2 + 8*m + 15) + c*c**m*m*x**5*x**m/(m**2 + 8*m + 15) + 3*c*c**m*x**
5*x**m/(m**2 + 8*m + 15), True))
```

Giac [A] time = 1.21245, size = 76, normalized size = 2.24

$$\frac{(cx)^m cmx^5 + 3\,(cx)^m cx^5 + (cx)^m bmx^3 + 5\,(cx)^m bx^3}{m^2 + 8\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] ((c*x)^m*c*m*x^5 + 3*(c*x)^m*c*x^5 + (c*x)^m*b*m*x^3 + 5*(c*x)^m*b*x^3)/(m^
2 + 8*m + 15)
```

3.406 $\int \frac{(cx)^m}{bx^2+cx^4} \, dx$

Optimal. Leaf size=45

$$-\frac{(cx)^m \, {}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{b(1-m)x}$$

```
[Out] -((((c*x)^m*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*(1
 - m)*x))
```

Rubi [A] time = 0.0322719, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 364}

$$-\frac{(cx)^m \, {}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{b(1-m)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^2 + c*x^4),x]
```

```
[Out] -((((c*x)^m*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*(1
 - m)*x))
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^m}{bx^2+cx^4}\,dx &= \left(x^{-m}(cx)^m\right)\text{Subst}\left(\int \frac{x^m}{bx^2+cx^4}\,dx,x,x\right)\\
&= \left(x^{-m}(cx)^m\right)\text{Subst}\left(\int \frac{x^{-2+m}}{b+cx^2}\,dx,x,x\right)\\
&= -\frac{(cx)^m\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{cx^2}{b}\right)}{b(1-m)x}
\end{aligned}$$

Mathematica [A] time = 0.0111761, size = 42, normalized size = 0.93

$$\frac{(cx)^m\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{b(m-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^2 + c*x^4),x]
```

```
[Out] ((c*x)^m*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*(-1
+ m)*x)
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{cx^4+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(c*x^4+b*x^2),x)
```

```
[Out] int((c*x)^m/(c*x^4+b*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{cx^4 + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^m}{cx^4 + bx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(c*x^4 + b*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{x^2\left(b + cx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(c*x**4+b*x**2),x)
```

```
[Out] Integral((c*x)**m/(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{cx^4 + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2), x)
```

3.407 $\int \frac{(cx)^m}{\left(bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=45

$$-\frac{(cx)^m\,{}_2F_1\left(2,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{b^2(3-m)x^3}$$

```
[Out] -(((c*x)^m*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)])/(b^2
*(3 - m)*x^3))
```

Rubi [A] time = 0.0307726, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 364}

$$-\frac{(cx)^m\,{}_2F_1\left(2,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{b^2(3-m)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^2 + c*x^4)^2,x]
```


```
[Out] -(((c*x)^m*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)])/(b^2
*(3 - m)*x^3))
```


Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m}{\left(bx^2+cx^4\right)^2}\,dx &= \left(x^{-m}(cx)^m\right)\operatorname{Subst}\left(\int \frac{x^m}{\left(bx^2+cx^4\right)^2}\,dx,x,x\right)\\ &= \left(x^{-m}(cx)^m\right)\operatorname{Subst}\left(\int \frac{x^{-4+m}}{\left(b+cx^2\right)^2}\,dx,x,x\right)\\ &= -\frac{(cx)^m\,{}_2F_1\left(2,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\frac{cx^2}{b}\right)}{b^2(3-m)x^3}\end{aligned}$$

Mathematica [A] time = 0.0115341, size = 44, normalized size = 0.98

$$\frac{(cx)^m\,{}_2F_1\left(2,\frac{m-3}{2};\frac{m-3}{2}+1;-\frac{cx^2}{b}\right)}{b^2(m-3)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*x)^m*Hypergeometric2F1[2, (-3 + m)/2, 1 + (-3 + m)/2, -((c*x^2)/b)])/(b
^2*(-3 + m)*x^3)
```

Maple [F] time = 0.334, size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\left(cx^4+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(c*x^4+b*x^2)^2,x)
```

```
[Out] int((c*x)^m/(c*x^4+b*x^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(cx^4 + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^m}{c^2x^8 + 2\,bcx^6 + b^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{x^4 (b + cx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(c*x**4+b*x**2)**2,x)
```

```
[Out] Integral((c*x)**m/(x**4*(b + c*x**2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\left(cx^4 + bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^2, x)
```

3.408 $\int \frac{(cx)^m}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=45

$$-\frac{(cx)^m\,{}_2F_1\left(3,\frac{m-5}{2};\frac{m-3}{2};-\frac{cx^2}{b}\right)}{b^3(5-m)x^5}$$

```
[Out] -(((c*x)^m*Hypergeometric2F1[3, (-5 + m)/2, (-3 + m)/2, -((c*x^2)/b)])/(b^3
*(5 - m)*x^5))
```

Rubi [A] time = 0.0314853, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1142, 1584, 364}

$$-\frac{(cx)^m\,{}_2F_1\left(3,\frac{m-5}{2};\frac{m-3}{2};-\frac{cx^2}{b}\right)}{b^3(5-m)x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(((c*x)^m*Hypergeometric2F1[3, (-5 + m)/2, (-3 + m)/2, -((c*x^2)/b)])/(b^3
*(5 - m)*x^5))
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m}{(bx^2+cx^4)^3}\,dx &= (x^{-m}(cx)^m)\,\text{Subst}\left(\int \frac{x^m}{(bx^2+cx^4)^3}\,dx, x, x\right)\\ &= (x^{-m}(cx)^m)\,\text{Subst}\left(\int \frac{x^{-6+m}}{(b+cx^2)^3}\,dx, x, x\right)\\ &= -\frac{(cx)^m\,{}_2F_1\left(3,\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);-\frac{cx^2}{b}\right)}{b^3(5-m)x^5}\end{aligned}$$

Mathematica [A] time = 0.0127925, size = 44, normalized size = 0.98

$$\frac{(cx)^m\,{}_2F_1\left(3,\frac{m-5}{2};\frac{m-5}{2}+1;-\frac{cx^2}{b}\right)}{b^3(m-5)x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((c*x)^m*Hypergeometric2F1[3, (-5 + m)/2, 1 + (-5 + m)/2, -((c*x^2)/b)])/(b
^3*(-5 + m)*x^5)
```

Maple [F] time = 0.345, size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(cx^4+bx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(c*x^4+b*x^2)^3,x)
```

```
[Out] int((c*x)^m/(c*x^4+b*x^2)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(cx^4 + bx^2)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^m}{c^3x^{12} + 3\,bc^2x^{10} + 3\,b^2cx^8 + b^3x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(c^3*x^12 + 3*b*c^2*x^10 + 3*b^2*c*x^8 + b^3*x^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{x^6\,(b + cx^2)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(c*x**4+b*x**2)**3,x)
```

```
[Out] Integral((c*x)**m/(x**6*(b + c*x**2)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\left(cx^4 + bx^2\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^3, x)
```

3.409 $\int x^3 \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rubi [A] time = 0.0095719, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a^2+2abx^2+b^2x^4\right) dx &= \int \left(a^2x^3+2abx^5+b^2x^7\right) dx\\ &= \frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.0012921, size = 30, normalized size = 1.

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Maple [A] time = 0.041, size = 25, normalized size = 0.8

$$\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Maxima [A] time = 0.991554, size = 32, normalized size = 1.07

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Fricas [A] time = 1.29055, size = 55, normalized size = 1.83

$$\frac{1}{8}x^8b^2+\frac{1}{3}x^6ba+\frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2
```

Sympy [A] time = 0.06008, size = 24, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

Giac [A] time = 1.12783, size = 32, normalized size = 1.07

$$\frac{1}{8}\,b^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

3.410 $\int x^2 \left(a^2 + 2abx^2 + b^2x^4\right) dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rubi [A] time = 0.0090823, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(a^2 + 2abx^2 + b^2x^4\right) dx &= \int \left(a^2x^2 + 2abx^4 + b^2x^6\right) dx \\ &= \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.000928, size = 30, normalized size = 1.

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Maple [A] time = 0.04, size = 25, normalized size = 0.8

$$\frac{a^2x^3}{3}+\frac{2\,abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Maxima [A] time = 0.985624, size = 32, normalized size = 1.07

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Fricas [A] time = 1.2728, size = 55, normalized size = 1.83

$$\frac{1}{7}x^7b^2+\frac{2}{5}x^5ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2
```

Sympy [A] time = 0.060454, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

Giac [A] time = 1.11288, size = 32, normalized size = 1.07

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

3.411 $\int x\left(a^2+2abx^2+b^2x^4\right)\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}$$

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + (b^2*x^6)/6
```

Rubi [A] time = 0.0094222, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {14}

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + (b^2*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)\,dx&=\int\left(a^2x+2abx^3+b^2x^5\right)\,dx\\&=\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.0018107, size = 16, normalized size = 0.53

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

Maple [A] time = 0.041, size = 25, normalized size = 0.8

$$\frac{b^2x^6}{6} + \frac{abx^4}{2} + \frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/6*b^2*x^6+1/2*a*b*x^4+1/2*a^2*x^2
```

Maxima [A] time = 0.992762, size = 32, normalized size = 1.07

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Fricas [A] time = 1.2158, size = 55, normalized size = 1.83

$$\frac{1}{6}x^6b^2 + \frac{1}{2}x^4ba + \frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^2 + 1/2*x^4*b*a + 1/2*x^2*a^2
```

Sympy [A] time = 0.061395, size = 24, normalized size = 0.8

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

Giac [A] time = 1.18048, size = 32, normalized size = 1.07

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

3.412 $\int \left(a^2+2abx^2+b^2x^4\right)\,dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] time = 0.0049772, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)\,dx=a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Mathematica [A] time = 0.0000442, size = 25, normalized size = 1.

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A] time = 0.039, size = 22, normalized size = 0.9

$$a^2x+\frac{2\,abx^3}{3}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^2*x^4+2*a*b*x^2+a^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Maxima [A] time = 0.990951, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A] time = 1.28327, size = 47, normalized size = 1.88

$$\frac{1}{5}x^5b^2+\frac{2}{3}x^3ba+xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^2 + 2/3*x^3*b*a + x*a^2
```

Sympy [A] time = 0.061637, size = 22, normalized size = 0.88

$$a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**2*x**4+2*a*b*x**2+a**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Giac [A] time = 1.13614, size = 28, normalized size = 1.12

$$\frac{1}{5}\, b^2 x^5 + \frac{2}{3}\, abx^3 + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.413 $\int \frac{a^2+2abx^2+b^2x^4}{x} dx$

Optimal. Leaf size=23

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rubi [A] time = 0.0072947, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2 + 2abx^2 + b^2x^4}{x} dx &= \int \left(\frac{a^2}{x} + 2abx + b^2x^3\right) dx\\ &= abx^2 + \frac{b^2x^4}{4} + a^2 \log(x)\end{aligned}$$

Mathematica [A] time = 0.0009761, size = 23, normalized size = 1.

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Maple [A] time = 0.043, size = 22, normalized size = 1.

$$abx^2 + \frac{b^2x^4}{4} + a^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x,x)
```

```
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Maxima [A] time = 0.981399, size = 32, normalized size = 1.39

$$\frac{1}{4}b^2x^4 + abx^2 + \frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Fricas [A] time = 1.44811, size = 49, normalized size = 2.13

$$\frac{1}{4}b^2x^4 + abx^2 + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A] time = 0.247208, size = 20, normalized size = 0.87

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

Giac [A] time = 1.15573, size = 32, normalized size = 1.39

$$\frac{1}{4}\,b^2x^4 + abx^2 + \frac{1}{2}\,a^2 \log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

3.414 $\int \frac{a^2+2abx^2+b^2x^4}{x^2} dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rubi [A] time = 0.008317, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^2}\,dx &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0008292, size = 24, normalized size = 1.

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Maple [A] time = 0.044, size = 23, normalized size = 1.

$$-\frac{a^2}{x}+2\,abx+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^2,x)
```

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Maxima [A] time = 0.985642, size = 30, normalized size = 1.25

$$\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Fricas [A] time = 1.41526, size = 50, normalized size = 2.08

$$\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A] time = 0.24932, size = 19, normalized size = 0.79

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

Giac [A] time = 1.15915, size = 30, normalized size = 1.25

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

3.415 $\int \frac{a^2+2abx^2+b^2x^4}{x^3}\,dx$

Optimal. Leaf size=27

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rubi [A] time = 0.0091011, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
```

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^3}\,dx &= \int \left(\frac{a^2}{x^3}+\frac{2ab}{x}+b^2x\right)dx\\ &= -\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.0012949, size = 27, normalized size = 1.

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
```

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Maple [A] time = 0.056, size = 24, normalized size = 0.9

$$-\frac{a^2}{2\,x^2}+\frac{b^2x^2}{2}+2\,ab\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^3,x)
```

```
[Out] -1/2/x^2*a^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Maxima [A] time = 0.984704, size = 32, normalized size = 1.19

$$\frac{1}{2}\,b^2x^2+ab\log\left(x^2\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

Fricas [A] time = 1.45842, size = 59, normalized size = 2.19

$$\frac{b^2x^4+4\,abx^2\log\left(x\right)-a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

Sympy [A] time = 0.274006, size = 24, normalized size = 0.89

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

Giac [A] time = 1.13649, size = 43, normalized size = 1.59

$$\frac{1}{2}b^2x^2 + ab\log\left(x^2\right) - \frac{2\,abx^2 + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

3.416 $\int \frac{a^2+2abx^2+b^2x^4}{x^4} dx$

Optimal. Leaf size=23

$$-\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Rubi [A] time = 0.0089646, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^4} dx &= \int \left(b^2 + \frac{a^2}{x^4} + \frac{2ab}{x^2}\right) dx\\ &= -\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x\end{aligned}$$

Mathematica [A] time = 0.0009226, size = 23, normalized size = 1.

$$-\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Maple [A] time = 0.048, size = 22, normalized size = 1.

$$-\frac{a^2}{3\,x^3}-2\,\frac{ab}{x}+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^4,x)
```

```
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

Maxima [A] time = 0.986414, size = 30, normalized size = 1.3

$$b^2x-\frac{6\,abx^2+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="maxima")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Fricas [A] time = 1.37206, size = 53, normalized size = 2.3

$$\frac{3\,b^2x^4-6\,abx^2-a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

Sympy [A] time = 0.285569, size = 20, normalized size = 0.87

$$b^2x - \frac{a^2 + 6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**4,x)
```

```
[Out] b**2*x - (a**2 + 6*a*b*x**2)/(3*x**3)
```

Giac [A] time = 1.15332, size = 30, normalized size = 1.3

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="giac")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

3.417 $\int \frac{a^2+2abx^2+b^2x^4}{x^5} dx$

Optimal. Leaf size=24

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Rubi [A] time = 0.009459, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]

[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^5} dx &= \int \left(\frac{a^2}{x^5} + \frac{2ab}{x^3} + \frac{b^2}{x}\right) dx\\ &= -\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)\end{aligned}$$

Mathematica [A] time = 0.0010922, size = 24, normalized size = 1.

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Maple [A] time = 0.047, size = 23, normalized size = 1.

$$-\frac{a^2}{4\,x^4}-\frac{ab}{x^2}+b^2\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^5,x)
```

```
[Out] -1/4*a^2/x^4-1/x^2*a*b+b^2*ln(x)
```

Maxima [A] time = 0.970607, size = 35, normalized size = 1.46

$$\frac{1}{2}\,b^2\log\left(x^2\right)-\frac{4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

Fricas [A] time = 1.44448, size = 62, normalized size = 2.58

$$\frac{4\,b^2x^4\log\left(x\right)-4\,abx^2-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

Sympy [A] time = 0.319698, size = 22, normalized size = 0.92

$$b^2 \log(x) - \frac{a^2 + 4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**5,x)
```

```
[Out] b**2*log(x) - (a**2 + 4*a*b*x**2)/(4*x**4)
```

Giac [A] time = 1.17739, size = 46, normalized size = 1.92

$$\frac{1}{2} b^2 \log\left(x^2\right) - \frac{3 b^2 x^4 + 4 abx^2 + a^2}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

3.418 $\int \frac{a^2+2abx^2+b^2x^4}{x^6} dx$

Optimal. Leaf size=28

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Rubi [A] time = 0.0096647, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^6} dx &= \int \left(\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2}{x^2}\right) dx\\ &= -\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}\end{aligned}$$

Mathematica [A] time = 0.0008508, size = 28, normalized size = 1.

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Maple [A] time = 0.047, size = 25, normalized size = 0.9

$$-\frac{a^2}{5\,x^5}-\frac{2\,ab}{3\,x^3}-\frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^6,x)
```

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Maxima [A] time = 1.35489, size = 35, normalized size = 1.25

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Fricas [A] time = 1.38187, size = 61, normalized size = 2.18

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Sympy [A] time = 0.323813, size = 27, normalized size = 0.96

$$-\frac{3a^2 + 10abx^2 + 15b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**6,x)
```

```
[Out] -(3*a**2 + 10*a*b*x**2 + 15*b**2*x**4)/(15*x**5)
```

Giac [A] time = 1.12716, size = 35, normalized size = 1.25

$$-\frac{15\,b^2x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

3.419 $\int \frac{a^2+2abx^2+b^2x^4}{x^7}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Rubi [A] time = 0.0089832, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^7}\,dx &= \int \left(\frac{a^2}{x^7}+\frac{2ab}{x^5}+\frac{b^2}{x^3}\right)dx\\ &= -\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0010388, size = 30, normalized size = 1.

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Maple [A] time = 0.048, size = 25, normalized size = 0.8

$$-\frac{a^2}{6\,x^6}-\frac{ab}{2\,x^4}-\frac{b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^7,x)
```

```
[Out] -1/6*a^2/x^6-1/2*a*b/x^4-1/2*b^2/x^2
```

Maxima [A] time = 0.994061, size = 32, normalized size = 1.07

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Fricas [A] time = 1.3988, size = 54, normalized size = 1.8

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Sympy [A] time = 0.338559, size = 26, normalized size = 0.87

$$-\frac{a^2 + 3abx^2 + 3b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**7,x)
```

```
[Out] -(a**2 + 3*a*b*x**2 + 3*b**2*x**4)/(6*x**6)
```

Giac [A] time = 1.14528, size = 32, normalized size = 1.07

$$-\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

3.420 $\int \frac{a^2+2abx^2+b^2x^4}{x^8} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rubi [A] time = 0.0094775, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^8}\,dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2}{x^4}\right) dx\\ &= -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0008715, size = 30, normalized size = 1.

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Maple [A] time = 0.047, size = 25, normalized size = 0.8

$$-\frac{a^2}{7\,x^7}-\frac{2\,ab}{5\,x^5}-\frac{b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^8,x)
```

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Maxima [A] time = 1.00139, size = 35, normalized size = 1.17

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Fricas [A] time = 1.41524, size = 63, normalized size = 2.1

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Sympy [A] time = 0.348299, size = 27, normalized size = 0.9

$$-\frac{15a^2 + 42abx^2 + 35b^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**8,x)
```

```
[Out] -(15*a**2 + 42*a*b*x**2 + 35*b**2*x**4)/(105*x**7)
```

Giac [A] time = 1.13422, size = 35, normalized size = 1.17

$$-\frac{35\,b^2x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

3.421 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^2 dx$

Optimal. Leaf size=56

$$\frac{6}{11}a^2b^2x^{11}+\frac{4}{9}a^3bx^9+\frac{a^4x^7}{7}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 + (
b^4*x^15)/15
```

Rubi [A] time = 0.0283449, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{6}{11}a^2b^2x^{11}+\frac{4}{9}a^3bx^9+\frac{a^4x^7}{7}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 + (
b^4*x^15)/15
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^2 dx = \frac{\int x^6\left(ab+b^2x^2\right)^4 dx}{b^4}$$
$$= \frac{\int \left(a^4b^4x^6+4a^3b^5x^8+6a^2b^6x^{10}+4ab^7x^{12}+b^8x^{14}\right) dx}{b^4}$$
$$= \frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Mathematica [A] time = 0.0025841, size = 56, normalized size = 1.

$$\frac{6}{11}a^2b^2x^{11}+\frac{4}{9}a^3bx^9+\frac{a^4x^7}{7}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 + (
b^4*x^15)/15
```

Maple [A] time = 0.04, size = 47, normalized size = 0.8

$$\frac{a^4x^7}{7}+\frac{4\,a^3bx^9}{9}+\frac{6\,a^2b^2x^{11}}{11}+\frac{4\,ab^3x^{13}}{13}+\frac{b^4x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/7*a^4*x^7+4/9*a^3*b*x^9+6/11*a^2*b^2*x^11+4/13*a*b^3*x^13+1/15*b^4*x^15
```

Maxima [A] time = 0.989778, size = 62, normalized size = 1.11

$$\frac{1}{15}\,b^4x^{15}+\frac{4}{13}\,ab^3x^{13}+\frac{6}{11}\,a^2b^2x^{11}+\frac{4}{9}\,a^3bx^9+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*a
^4*x^7
```

Fricas [A] time = 1.3078, size = 112, normalized size = 2.

$$\frac{1}{15}x^{15}b^4 + \frac{4}{13}x^{13}b^3a + \frac{6}{11}x^{11}b^2a^2 + \frac{4}{9}x^9ba^3 + \frac{1}{7}x^7a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^4 + 4/13*x^13*b^3*a + 6/11*x^11*b^2*a^2 + 4/9*x^9*b*a^3 + 1/7*x
^7*a^4
```

Sympy [A] time = 0.073511, size = 53, normalized size = 0.95

$$\frac{a^4x^7}{7} + \frac{4a^3bx^9}{9} + \frac{6a^2b^2x^{11}}{11} + \frac{4ab^3x^{13}}{13} + \frac{b^4x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**7/7 + 4*a**3*b*x**9/9 + 6*a**2*b**2*x**11/11 + 4*a*b**3*x**13/13 +
b**4*x**15/15
```

Giac [A] time = 1.12032, size = 62, normalized size = 1.11

$$\frac{1}{15}\,b^4x^{15} + \frac{4}{13}\,ab^3x^{13} + \frac{6}{11}\,a^2b^2x^{11} + \frac{4}{9}\,a^3bx^9 + \frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*a
^4*x^7
```

3.422 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^2\right)^5}{10b^3}+\frac{\left(a+bx^2\right)^7}{14b^3}-\frac{a\left(a+bx^2\right)^6}{6b^3}$$

```
[Out] (a^2*(a + b*x^2)^5)/(10*b^3) - (a*(a + b*x^2)^6)/(6*b^3) + (a + b*x^2)^7/(1
4*b^3)
```

Rubi [A] time = 0.070219, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^2\left(a+bx^2\right)^5}{10b^3}+\frac{\left(a+bx^2\right)^7}{14b^3}-\frac{a\left(a+bx^2\right)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```


```
[Out] (a^2*(a + b*x^2)^5)/(10*b^3) - (a*(a + b*x^2)^6)/(6*b^3) + (a + b*x^2)^7/(1
4*b^3)
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int x^5\left(ab+b^2x^2\right)^4\,dx}{b^4} \\
&= \frac{\text{Subst}\left(\int x^2\left(ab+b^2x\right)^4\,dx,x,x^2\right)}{2b^4} \\
&= \frac{\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^4}{b^2}-\frac{2a(ab+b^2x)^5}{b^3}+\frac{(ab+b^2x)^6}{b^4}\right)dx,x,x^2\right)}{2b^4} \\
&= \frac{a^2\left(a+bx^2\right)^5}{10b^3}-\frac{a\left(a+bx^2\right)^6}{6b^3}+\frac{\left(a+bx^2\right)^7}{14b^3}
\end{aligned}$$

Mathematica [A] time = 0.002353, size = 56, normalized size = 1.06

$$\frac{3}{5}a^2b^2x^{10}+\frac{1}{2}a^3bx^8+\frac{a^4x^6}{6}+\frac{1}{3}ab^3x^{12}+\frac{b^4x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^6)/6 + (a^3*b*x^8)/2 + (3*a^2*b^2*x^10)/5 + (a*b^3*x^12)/3 + (b^4*x^
14)/14
```

Maple [A] time = 0.042, size = 47, normalized size = 0.9

$$\frac{b^4x^{14}}{14}+\frac{ab^3x^{12}}{3}+\frac{3\,a^2b^2x^{10}}{5}+\frac{a^3bx^8}{2}+\frac{a^4x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/14*b^4*x^14+1/3*a*b^3*x^12+3/5*a^2*b^2*x^10+1/2*a^3*b*x^8+1/6*a^4*x^6
```

Maxima [A] time = 0.986442, size = 62, normalized size = 1.17

$$\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^4
*x^6
```

Fricas [A] time = 1.24881, size = 109, normalized size = 2.06

$$\frac{1}{14}x^{14}b^4+\frac{1}{3}x^{12}b^3a+\frac{3}{5}x^{10}b^2a^2+\frac{1}{2}x^8ba^3+\frac{1}{6}x^6a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^4 + 1/3*x^12*b^3*a + 3/5*x^10*b^2*a^2 + 1/2*x^8*b*a^3 + 1/6*x^6
*a^4
```

Sympy [A] time = 0.073926, size = 49, normalized size = 0.92

$$\frac{a^4x^6}{6}+\frac{a^3bx^8}{2}+\frac{3a^2b^2x^{10}}{5}+\frac{ab^3x^{12}}{3}+\frac{b^4x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**6/6 + a**3*b*x**8/2 + 3*a**2*b**2*x**10/5 + a*b**3*x**12/3 + b**4*x
**14/14
```

Giac [A] time = 1.15813, size = 62, normalized size = 1.17

$$\frac{1}{14}\,b^4x^{14} + \frac{1}{3}\,ab^3x^{12} + \frac{3}{5}\,a^2b^2x^{10} + \frac{1}{2}\,a^3bx^8 + \frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^4
*x^6
```

3.423 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{2}{3}a^2b^2x^9+\frac{4}{7}a^3bx^7+\frac{a^4x^5}{5}+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b^
4*x^13)/13
```

Rubi [A] time = 0.0272856, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{2}{3}a^2b^2x^9+\frac{4}{7}a^3bx^7+\frac{a^4x^5}{5}+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b^
4*x^13)/13
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\int x^4\left(ab+b^2x^2\right)^4\,dx}{b^4}$$
$$=\frac{\int\left(a^4b^4x^4+4a^3b^5x^6+6a^2b^6x^8+4ab^7x^{10}+b^8x^{12}\right)dx}{b^4}$$
$$=\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Mathematica [A] time = 0.00262, size = 56, normalized size = 1.

$$\frac{2}{3}a^2b^2x^9+\frac{4}{7}a^3bx^7+\frac{a^4x^5}{5}+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b^
4*x^13)/13
```

Maple [A] time = 0.043, size = 47, normalized size = 0.8

$$\frac{a^4x^5}{5}+\frac{4\,a^3bx^7}{7}+\frac{2\,a^2b^2x^9}{3}+\frac{4\,ab^3x^{11}}{11}+\frac{b^4x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/5*a^4*x^5+4/7*a^3*b*x^7+2/3*a^2*b^2*x^9+4/11*a*b^3*x^11+1/13*b^4*x^13
```

Maxima [A] time = 0.992755, size = 62, normalized size = 1.11

$$\frac{1}{13}\,b^4x^{13}+\frac{4}{11}\,ab^3x^{11}+\frac{2}{3}\,a^2b^2x^9+\frac{4}{7}\,a^3bx^7+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^4
*x^5
```

Fricas [A] time = 1.50247, size = 109, normalized size = 1.95

$$\frac{1}{13}x^{13}b^4 + \frac{4}{11}x^{11}b^3a + \frac{2}{3}x^9b^2a^2 + \frac{4}{7}x^7ba^3 + \frac{1}{5}x^5a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^4 + 4/11*x^11*b^3*a + 2/3*x^9*b^2*a^2 + 4/7*x^7*b*a^3 + 1/5*x^5
*a^4
```

Sympy [A] time = 0.071213, size = 53, normalized size = 0.95

$$\frac{a^4x^5}{5} + \frac{4a^3bx^7}{7} + \frac{2a^2b^2x^9}{3} + \frac{4ab^3x^{11}}{11} + \frac{b^4x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**5/5 + 4*a**3*b*x**7/7 + 2*a**2*b**2*x**9/3 + 4*a*b**3*x**11/11 + b*
*4*x**13/13
```

Giac [A] time = 1.12688, size = 62, normalized size = 1.11

$$\frac{1}{13}b^4x^{13} + \frac{4}{11}ab^3x^{11} + \frac{2}{3}a^2b^2x^9 + \frac{4}{7}a^3bx^7 + \frac{1}{5}a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^4
*x^5
```

3.424 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^6}{12b^2}-\frac{a\left(a+bx^2\right)^5}{10b^2}$$

```
[Out] -(a*(a + b*x^2)^5)/(10*b^2) + (a + b*x^2)^6/(12*b^2)
```

Rubi [A] time = 0.0395182, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{\left(a+bx^2\right)^6}{12b^2}-\frac{a\left(a+bx^2\right)^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(a*(a + b*x^2)^5)/(10*b^2) + (a + b*x^2)^6/(12*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 dx = \frac{\int x^3 \left(ab+b^2x^2\right)^4 dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int x \left(ab+b^2x\right)^4 dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a\left(ab+b^2x\right)^4}{b}+\frac{\left(ab+b^2x\right)^5}{b^2}\right) dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a\left(a+bx^2\right)^5}{10b^2}+\frac{\left(a+bx^2\right)^6}{12b^2}$$

Mathematica [A] time = 0.0021975, size = 56, normalized size = 1.65

$$\frac{3}{4}a^2b^2x^8+\frac{2}{3}a^3bx^6+\frac{a^4x^4}{4}+\frac{2}{5}ab^3x^{10}+\frac{b^4x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^4)/4 + (2*a^3*b*x^6)/3 + (3*a^2*b^2*x^8)/4 + (2*a*b^3*x^10)/5 + (b^4
*x^12)/12
```

Maple [A] time = 0.041, size = 47, normalized size = 1.4

$$\frac{b^4x^{12}}{12}+\frac{2\,ab^3x^{10}}{5}+\frac{3\,a^2b^2x^8}{4}+\frac{2\,a^3bx^6}{3}+\frac{a^4x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/12*b^4*x^12+2/5*a*b^3*x^10+3/4*a^2*b^2*x^8+2/3*a^3*b*x^6+1/4*a^4*x^4
```

Maxima [A] time = 0.994285, size = 62, normalized size = 1.82

$$\frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4*
x^4
```

Fricas [A] time = 1.46452, size = 108, normalized size = 3.18

$$\frac{1}{12}x^{12}b^4+\frac{2}{5}x^{10}b^3a+\frac{3}{4}x^8b^2a^2+\frac{2}{3}x^6ba^3+\frac{1}{4}x^4a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^4 + 2/5*x^10*b^3*a + 3/4*x^8*b^2*a^2 + 2/3*x^6*b*a^3 + 1/4*x^4*
a^4
```

Sympy [A] time = 0.074655, size = 53, normalized size = 1.56

$$\frac{a^4x^4}{4}+\frac{2a^3bx^6}{3}+\frac{3a^2b^2x^8}{4}+\frac{2ab^3x^{10}}{5}+\frac{b^4x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**4/4 + 2*a**3*b*x**6/3 + 3*a**2*b**2*x**8/4 + 2*a*b**3*x**10/5 + b**
4*x**12/12
```

Giac [A] time = 1.13718, size = 62, normalized size = 1.82

$$\frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4*
x^4
```

3.425 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{6}{7}a^2b^2x^7+\frac{4}{5}a^3bx^5+\frac{a^4x^3}{3}+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4*
x^11)/11
```

Rubi [A] time = 0.0288531, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{6}{7}a^2b^2x^7+\frac{4}{5}a^3bx^5+\frac{a^4x^3}{3}+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4*
x^11)/11
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^2 dx = \frac{\int x^2 \left(ab+b^2x^2\right)^4 dx}{b^4}$$
$$= \frac{\int \left(a^4b^4x^2+4a^3b^5x^4+6a^2b^6x^6+4ab^7x^8+b^8x^{10}\right) dx}{b^4}$$
$$= \frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Mathematica [A] time = 0.0021344, size = 56, normalized size = 1.

$$\frac{6}{7}a^2b^2x^7+\frac{4}{5}a^3bx^5+\frac{a^4x^3}{3}+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4*
x^11)/11
```

Maple [A] time = 0.04, size = 47, normalized size = 0.8

$$\frac{a^4x^3}{3}+\frac{4\,a^3bx^5}{5}+\frac{6\,a^2b^2x^7}{7}+\frac{4\,ab^3x^9}{9}+\frac{b^4x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/3*a^4*x^3+4/5*a^3*b*x^5+6/7*a^2*b^2*x^7+4/9*a*b^3*x^9+1/11*b^4*x^11
```

Maxima [A] time = 0.989445, size = 62, normalized size = 1.11

$$\frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*x
^3
```

Fricas [A] time = 1.51636, size = 107, normalized size = 1.91

$$\frac{1}{11}x^{11}b^4 + \frac{4}{9}x^9b^3a + \frac{6}{7}x^7b^2a^2 + \frac{4}{5}x^5ba^3 + \frac{1}{3}x^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^4 + 4/9*x^9*b^3*a + 6/7*x^7*b^2*a^2 + 4/5*x^5*b*a^3 + 1/3*x^3*a
^4
```

Sympy [A] time = 0.070825, size = 53, normalized size = 0.95

$$\frac{a^4x^3}{3} + \frac{4a^3bx^5}{5} + \frac{6a^2b^2x^7}{7} + \frac{4ab^3x^9}{9} + \frac{b^4x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**3/3 + 4*a**3*b*x**5/5 + 6*a**2*b**2*x**7/7 + 4*a*b**3*x**9/9 + b**4
*x**11/11
```

Giac [A] time = 1.12835, size = 62, normalized size = 1.11

$$\frac{1}{11}\,b^4x^{11} + \frac{4}{9}\,ab^3x^9 + \frac{6}{7}\,a^2b^2x^7 + \frac{4}{5}\,a^3bx^5 + \frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*x
^3
```

3.426 $\int x \left(a^2 + 2abx^2 + b^2x^4\right)^2 \, dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^2\right)^5}{10b}$$

```
[Out] (a + b*x^2)^5/(10*b)
```

Rubi [A] time = 0.0048408, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$\frac{\left(a + bx^2\right)^5}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a + b*x^2)^5/(10*b)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\int x\left(ab+b^2x^2\right)^4\,dx}{b^4}$$
$$=\frac{\left(a+bx^2\right)^5}{10b}$$

Mathematica [A] time = 0.0022879, size = 16, normalized size = 1.

$$\frac{\left(a+bx^2\right)^5}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a + b*x^2)^5/(10*b)
```

Maple [B] time = 0.039, size = 45, normalized size = 2.8

$$\frac{b^4x^{10}}{10}+\frac{ab^3x^8}{2}+a^2b^2x^6+a^3bx^4+\frac{a^4x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/10*b^4*x^10+1/2*a*b^3*x^8+a^2*b^2*x^6+a^3*b*x^4+1/2*a^4*x^2
```

Maxima [B] time = 0.982767, size = 59, normalized size = 3.69

$$\frac{1}{10}\,b^4x^{10}+\frac{1}{2}\,ab^3x^8+a^2b^2x^6+a^3bx^4+\frac{1}{2}\,a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

Fricas [B] time = 1.466, size = 96, normalized size = 6.

$$\frac{1}{10}x^{10}b^4 + \frac{1}{2}x^8b^3a + x^6b^2a^2 + x^4ba^3 + \frac{1}{2}x^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^4 + 1/2*x^8*b^3*a + x^6*b^2*a^2 + x^4*b*a^3 + 1/2*x^2*a^4
```

Sympy [B] time = 0.072371, size = 44, normalized size = 2.75

$$\frac{a^4x^2}{2} + a^3bx^4 + a^2b^2x^6 + \frac{ab^3x^8}{2} + \frac{b^4x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**2/2 + a**3*b*x**4 + a**2*b**2*x**6 + a*b**3*x**8/2 + b**4*x**10/10
```

Giac [B] time = 1.14335, size = 59, normalized size = 3.69

$$\frac{1}{10}\,b^4x^{10} + \frac{1}{2}\,ab^3x^8 + a^2b^2x^6 + a^3bx^4 + \frac{1}{2}\,a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

3.427 $\int \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=51

$$\frac{6}{5}a^2b^2x^5+\frac{4}{3}a^3bx^3+a^4x+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

```
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/9
```

Rubi [A] time = 0.0238017, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {28, 194}

$$\frac{6}{5}a^2b^2x^5+\frac{4}{3}a^3bx^3+a^4x+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/9
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^2 dx = \frac{\int \left(ab+b^2x^2\right)^4 dx}{b^4}$$
$$= \frac{\int \left(a^4b^4+4a^3b^5x^2+6a^2b^6x^4+4ab^7x^6+b^8x^8\right) dx}{b^4}$$
$$= a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Mathematica [A] time = 0.0012044, size = 51, normalized size = 1.

$$\frac{6}{5}a^2b^2x^5+\frac{4}{3}a^3bx^3+a^4x+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/9
```

Maple [A] time = 0.04, size = 44, normalized size = 0.9

$$a^4x+\frac{4\,a^3bx^3}{3}+\frac{6\,a^2b^2x^5}{5}+\frac{4\,ab^3x^7}{7}+\frac{b^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

Maxima [A] time = 0.976963, size = 74, normalized size = 1.45

$$\frac{1}{9}\,b^4x^9+\frac{4}{7}\,ab^3x^7+\frac{4}{5}\,a^2b^2x^5+a^4x+\frac{2}{15}\left(3\,b^2x^5+10\,abx^3\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 4/5*a^2*b^2*x^5 + a^4*x + 2/15*(3*b^2*x^5 + 1
0*a*b*x^3)*a^2
```

Fricas [A] time = 1.4885, size = 96, normalized size = 1.88

$$\frac{1}{9}x^9b^4 + \frac{4}{7}x^7b^3a + \frac{6}{5}x^5b^2a^2 + \frac{4}{3}x^3ba^3 + xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^4 + 4/7*x^7*b^3*a + 6/5*x^5*b^2*a^2 + 4/3*x^3*b*a^3 + x*a^4
```

Sympy [A] time = 0.068264, size = 49, normalized size = 0.96

$$a^4x + \frac{4a^3bx^3}{3} + \frac{6a^2b^2x^5}{5} + \frac{4ab^3x^7}{7} + \frac{b^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x + 4*a**3*b*x**3/3 + 6*a**2*b**2*x**5/5 + 4*a*b**3*x**7/7 + b**4*x**9
/9
```

Giac [A] time = 1.16168, size = 58, normalized size = 1.14

$$\frac{1}{9}b^4x^9 + \frac{4}{7}ab^3x^7 + \frac{6}{5}a^2b^2x^5 + \frac{4}{3}a^3bx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a^4*x
```

3.428 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x} dx$

Optimal. Leaf size=50

$$\frac{3}{2}a^2b^2x^4 + 2a^3bx^2 + a^4\log(x) + \frac{2}{3}ab^3x^6 + \frac{b^4x^8}{8}$$

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 + a^4*Log[x
]
```

Rubi [A] time = 0.0337603, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3}{2}a^2b^2x^4 + 2a^3bx^2 + a^4\log(x) + \frac{2}{3}ab^3x^6 + \frac{b^4x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 + a^4*Log[x
]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(4a^3b^5+\frac{a^4b^4}{x}+6a^2b^6x+4ab^7x^2+b^8x^3\right)dx, x, x^2\right)}{2b^4}$$
$$= 2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}+a^4\log(x)$$

Mathematica [A] time = 0.0041856, size = 50, normalized size = 1.

$$\frac{3}{2}a^2b^2x^4+2a^3bx^2+a^4\log(x)+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 + a^4*Log[x
]
```

Maple [A] time = 0.041, size = 45, normalized size = 0.9

$$2\,a^3bx^2+\frac{3\,a^2b^2x^4}{2}+\frac{2\,ab^3x^6}{3}+\frac{b^4x^8}{8}+a^4\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x,x)
```

```
[Out] 2*a^3*b*x^2+3/2*a^2*b^2*x^4+2/3*a*b^3*x^6+1/8*b^4*x^8+a^4*ln(x)
```

Maxima [A] time = 0.976765, size = 63, normalized size = 1.26

$$\frac{1}{8}\,b^4x^8 + \frac{2}{3}\,ab^3x^6 + \frac{3}{2}\,a^2b^2x^4 + 2\,a^3bx^2 + \frac{1}{2}\,a^4\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2*a^4*log(x
^2)
```

Fricas [A] time = 1.67549, size = 100, normalized size = 2.

$$\frac{1}{8}\,b^4x^8 + \frac{2}{3}\,ab^3x^6 + \frac{3}{2}\,a^2b^2x^4 + 2\,a^3bx^2 + a^4\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4*log(x)
```

Sympy [A] time = 0.274536, size = 49, normalized size = 0.98

$$a^4\log\left(x\right) + 2a^3bx^2 + \frac{3a^2b^2x^4}{2} + \frac{2ab^3x^6}{3} + \frac{b^4x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x,x)
```

```
[Out] a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*x
**8/8
```

Giac [A] time = 1.11139, size = 63, normalized size = 1.26

$$\frac{1}{8}\,b^4x^8 + \frac{2}{3}\,ab^3x^6 + \frac{3}{2}\,a^2b^2x^4 + 2\,a^3bx^2 + \frac{1}{2}\,a^4\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2*a^4*log(x
^2)
```

3.429 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^2}\,dx$

Optimal. Leaf size=48

$$2a^2b^2x^3+4a^3bx-\frac{a^4}{x}+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

Rubi [A] time = 0.0255003, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$2a^2b^2x^3+4a^3bx-\frac{a^4}{x}+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^2}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^2}\,dx}{b^4}$$
$$= \frac{\int \left(4a^3b^5+\frac{a^4b^4}{x^2}+6a^2b^6x^2+4ab^7x^4+b^8x^6\right)dx}{b^4}$$
$$= -\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Mathematica [A] time = 0.0081709, size = 48, normalized size = 1.

$$2a^2b^2x^3+4a^3bx-\frac{a^4}{x}+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

Maple [A] time = 0.046, size = 45, normalized size = 0.9

$$-\frac{a^4}{x}+4\,xa^3b+2\,a^2b^2x^3+\frac{4\,ab^3x^5}{5}+\frac{b^4x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x)
```

```
[Out] -a^4/x+4*x*a^3*b+2*a^2*b^2*x^3+4/5*a*b^3*x^5+1/7*b^4*x^7
```

Maxima [A] time = 0.958235, size = 59, normalized size = 1.23

$$\frac{1}{7}\,b^4x^7+\frac{4}{5}\,ab^3x^5+2\,a^2b^2x^3+4\,a^3bx-\frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

Fricas [A] time = 1.64799, size = 104, normalized size = 2.17

$$\frac{5\,b^4x^8 + 28\,ab^3x^6 + 70\,a^2b^2x^4 + 140\,a^3bx^2 - 35\,a^4}{35\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^4*x^8 + 28*a*b^3*x^6 + 70*a^2*b^2*x^4 + 140*a^3*b*x^2 - 35*a^4)/x
```

Sympy [A] time = 0.273524, size = 44, normalized size = 0.92

$$-\frac{a^4}{x} + 4a^3bx + 2a^2b^2x^3 + \frac{4ab^3x^5}{5} + \frac{b^4x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**2,x)
```

```
[Out] -a**4/x + 4*a**3*b*x + 2*a**2*b**2*x**3 + 4*a*b**3*x**5/5 + b**4*x**7/7
```

Giac [A] time = 1.11292, size = 59, normalized size = 1.23

$$\frac{1}{7}\,b^4x^7 + \frac{4}{5}\,ab^3x^5 + 2\,a^2b^2x^3 + 4\,a^3bx - \frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

3.430 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3} dx$

Optimal. Leaf size=48

$$3a^2b^2x^2 + 4a^3b\log(x) - \frac{a^4}{2x^2} + ab^3x^4 + \frac{b^4x^6}{6}$$

```
[Out] -a^4/(2*x^2) + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*Log[x]
```

Rubi [A] time = 0.0372027, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$3a^2b^2x^2 + 4a^3b\log(x) - \frac{a^4}{2x^2} + ab^3x^4 + \frac{b^4x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
```

```
[Out] -a^4/(2*x^2) + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^3}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^3}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^2}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(6a^2b^6+\frac{a^4b^4}{x^2}+\frac{4a^3b^5}{x}+4ab^7x+b^8x^2\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{2x^2}+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}+4a^3b\log(x)$$

Mathematica [A] time = 0.0047694, size = 48, normalized size = 1.

$$3a^2b^2x^2+4a^3b\log(x)-\frac{a^4}{2x^2}+ab^3x^4+\frac{b^4x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
```

```
[Out] -a^4/(2*x^2) + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*Log[x]
```

Maple [A] time = 0.046, size = 45, normalized size = 0.9

$$-\frac{a^4}{2\,x^2}+3\,a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}+4\,a^3b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x)
```

```
[Out] -1/2*a^4/x^2+3*a^2*b^2*x^2+a*b^3*x^4+1/6*b^4*x^6+4*a^3*b*ln(x)
```

Maxima [A] time = 0.996465, size = 62, normalized size = 1.29

$$\frac{1}{6}\,b^4x^6+ab^3x^4+3\,a^2b^2x^2+2\,a^3b\log\left(x^2\right)-\frac{a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*a^4/x^2
```

Fricas [A] time = 1.75956, size = 108, normalized size = 2.25

$$\frac{b^4x^8 + 6\,ab^3x^6 + 18\,a^2b^2x^4 + 24\,a^3bx^2\log(x) - 3\,a^4}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 + 6*a*b^3*x^6 + 18*a^2*b^2*x^4 + 24*a^3*b*x^2*log(x) - 3*a^4)/
x^2
```

Sympy [A] time = 0.311597, size = 46, normalized size = 0.96

$$-\frac{a^4}{2x^2} + 4a^3b\log(x) + 3a^2b^2x^2 + ab^3x^4 + \frac{b^4x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**3,x)
```

```
[Out] -a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4 + b**4*x*
*6/6
```

Giac [A] time = 1.13619, size = 76, normalized size = 1.58

$$\frac{1}{6}\,b^4x^6 + ab^3x^4 + 3\,a^2b^2x^2 + 2\,a^3b\log\left(x^2\right) - \frac{4\,a^3bx^2 + a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*(4*a^3*b*x
^2 + a^4)/x^2
```

3.431 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^4} dx$

Optimal. Leaf size=50

$$6a^2b^2x - \frac{4a^3b}{x} - \frac{a^4}{3x^3} + \frac{4}{3}ab^3x^3 + \frac{b^4x^5}{5}$$

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rubi [A] time = 0.0261702, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$6a^2b^2x - \frac{4a^3b}{x} - \frac{a^4}{3x^3} + \frac{4}{3}ab^3x^3 + \frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
```

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^4}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^4}\,dx}{b^4}$$
$$= \frac{\int \left(6a^2b^6+\frac{a^4b^4}{x^4}+\frac{4a^3b^5}{x^2}+4ab^7x^2+b^8x^4\right)dx}{b^4}$$
$$= -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Mathematica [A] time = 0.0063078, size = 50, normalized size = 1.

$$6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
```

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Maple [A] time = 0.046, size = 45, normalized size = 0.9

$$-\frac{a^4}{3x^3}-4\frac{a^3b}{x}+6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x)
```

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Maxima [A] time = 1.01285, size = 61, normalized size = 1.22

$$\frac{1}{5}b^4x^5+\frac{4}{3}ab^3x^3+6a^2b^2x-\frac{12a^3bx^2+a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

Fricas [A] time = 1.70234, size = 104, normalized size = 2.08

$$\frac{3\,b^4x^8+20\,ab^3x^6+90\,a^2b^2x^4-60\,a^3bx^2-5\,a^4}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^3
```

Sympy [A] time = 0.315752, size = 48, normalized size = 0.96

$$6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}-\frac{a^4+12a^3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**4,x)
```

```
[Out] 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 - (a**4 + 12*a**3*b*x**2)/(3*
x**3)
```

Giac [A] time = 1.15611, size = 61, normalized size = 1.22

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

3.432 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5} dx$

Optimal. Leaf size=49

$$6a^2b^2\log(x) - \frac{2a^3b}{x^2} - \frac{a^4}{4x^4} + 2ab^3x^2 + \frac{b^4x^4}{4}$$

```
[Out] -a^4/(4*x^4) - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*b^2*Log[x]
```

Rubi [A] time = 0.0356296, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$6a^2b^2\log(x) - \frac{2a^3b}{x^2} - \frac{a^4}{4x^4} + 2ab^3x^2 + \frac{b^4x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
```

```
[Out] -a^4/(4*x^4) - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*b^2*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^5}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^5}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^3}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(4ab^7+\frac{a^4b^4}{x^3}+\frac{4a^3b^5}{x^2}+\frac{6a^2b^6}{x}+b^8x\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+2ab^3x^2+\frac{b^4x^4}{4}+6a^2b^2\log(x)$$

Mathematica [A] time = 0.0045238, size = 49, normalized size = 1.

$$6a^2b^2\log(x)-\frac{2a^3b}{x^2}-\frac{a^4}{4x^4}+2ab^3x^2+\frac{b^4x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
```

```
[Out] -a^4/(4*x^4) - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*b^2*Log[x]
```

Maple [A] time = 0.046, size = 46, normalized size = 0.9

$$-\frac{a^4}{4\,x^4}-2\,\frac{a^3b}{x^2}+2\,ab^3x^2+\frac{b^4x^4}{4}+6\,a^2b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x)
```

```
[Out] -1/4*a^4/x^4-2*a^3*b/x^2+2*a*b^3*x^2+1/4*b^4*x^4+6*a^2*b^2*ln(x)
```

Maxima [A] time = 0.996828, size = 65, normalized size = 1.33

$$\frac{1}{4}\,b^4x^4+2\,ab^3x^2+3\,a^2b^2\log\left(x^2\right)-\frac{8\,a^3bx^2+a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(8*a^3*b*x^2 + a^4)/x^
4
```

Fricas [A] time = 1.70179, size = 104, normalized size = 2.12

$$\frac{b^4x^8 + 8\,ab^3x^6 + 24\,a^2b^2x^4\log(x) - 8\,a^3bx^2 - a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^8 + 8*a*b^3*x^6 + 24*a^2*b^2*x^4*log(x) - 8*a^3*b*x^2 - a^4)/x^4
```

Sympy [A] time = 0.360037, size = 48, normalized size = 0.98

$$6a^2b^2\log(x) + 2ab^3x^2 + \frac{b^4x^4}{4} - \frac{a^4 + 8a^3bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**5,x)
```

```
[Out] 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4 - (a**4 + 8*a**3*b*x**2)/(
4*x**4)
```

Giac [A] time = 1.18422, size = 80, normalized size = 1.63

$$\frac{1}{4}\,b^4x^4 + 2\,ab^3x^2 + 3\,a^2b^2\log\left(x^2\right) - \frac{18\,a^2b^2x^4 + 8\,a^3bx^2 + a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(18*a^2*b^2*x^4 + 8*a^
3*b*x^2 + a^4)/x^4
```

3.433 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6} dx$

Optimal. Leaf size=50

$$-\frac{6a^2b^2}{x} - \frac{4a^3b}{3x^3} - \frac{a^4}{5x^5} + 4ab^3x + \frac{b^4x^3}{3}$$

```
[Out] -a^4/(5*x^5) - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

Rubi [A] time = 0.0263534, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{x} - \frac{4a^3b}{3x^3} - \frac{a^4}{5x^5} + 4ab^3x + \frac{b^4x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
```

```
[Out] -a^4/(5*x^5) - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^6}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^6}\,dx}{b^4}$$
$$= \frac{\int \left(4ab^7+\frac{a^4b^4}{x^6}+\frac{4a^3b^5}{x^4}+\frac{6a^2b^6}{x^2}+b^8x^2\right)dx}{b^4}$$
$$= -\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

Mathematica [A] time = 0.00808, size = 50, normalized size = 1.

$$-\frac{6a^2b^2}{x}-\frac{4a^3b}{3x^3}-\frac{a^4}{5x^5}+4ab^3x+\frac{b^4x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
```

```
[Out] -a^4/(5*x^5) - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

Maple [A] time = 0.046, size = 45, normalized size = 0.9

$$-\frac{a^4}{5\,x^5}-\frac{4\,a^3b}{3\,x^3}-6\,\frac{b^2a^2}{x}+4\,ab^3x+\frac{b^4x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x)
```

```
[Out] -1/5*a^4/x^5-4/3*a^3*b/x^3-6*a^2*b^2/x+4*a*b^3*x+1/3*b^4*x^3
```

Maxima [A] time = 0.993944, size = 63, normalized size = 1.26

$$\frac{1}{3}\,b^4x^3+4\,ab^3x-\frac{90\,a^2b^2x^4+20\,a^3bx^2+3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3*a^4)/x^5
```

Fricas [A] time = 1.67278, size = 104, normalized size = 2.08

$$\frac{5 b^4 x^8 + 60 a b^3 x^6 - 90 a^2 b^2 x^4 - 20 a^3 b x^2 - 3 a^4}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(5*b^4*x^8 + 60*a*b^3*x^6 - 90*a^2*b^2*x^4 - 20*a^3*b*x^2 - 3*a^4)/x^5
```

Sympy [A] time = 0.367966, size = 48, normalized size = 0.96

$$4ab^3x + \frac{b^4x^3}{3} - \frac{3a^4 + 20a^3bx^2 + 90a^2b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**6,x)
```

```
[Out] 4*a*b**3*x + b**4*x**3/3 - (3*a**4 + 20*a**3*b*x**2 + 90*a**2*b**2*x**4)/(1
5*x**5)
```

Giac [A] time = 1.11867, size = 63, normalized size = 1.26

$$\frac{1}{3} b^4 x^3 + 4 a b^3 x - \frac{90 a^2 b^2 x^4 + 20 a^3 b x^2 + 3 a^4}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3*a^4)/x^5
```

3.434 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^7}\,dx$

Optimal. Leaf size=49

$$-\frac{3a^2b^2}{x^2}-\frac{a^3b}{x^4}-\frac{a^4}{6x^6}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

```
[Out] -a^4/(6*x^6) - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*b^3*Log[x]
```

Rubi [A] time = 0.0326376, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{3a^2b^2}{x^2}-\frac{a^3b}{x^4}-\frac{a^4}{6x^6}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
```

```
[Out] -a^4/(6*x^6) - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*b^3*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^7}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^7}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^4}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(b^8+\frac{a^4b^4}{x^4}+\frac{4a^3b^5}{x^3}+\frac{6a^2b^6}{x^2}+\frac{4ab^7}{x}\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+\frac{b^4x^2}{2}+4ab^3\log(x)$$

Mathematica [A] time = 0.0045966, size = 49, normalized size = 1.

$$-\frac{3a^2b^2}{x^2}-\frac{a^3b}{x^4}-\frac{a^4}{6x^6}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
```

```
[Out] -a^4/(6*x^6) - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*b^3*Log[x]
```

Maple [A] time = 0.046, size = 46, normalized size = 0.9

$$-\frac{a^4}{6\,x^6}-\frac{a^3b}{x^4}-3\,\frac{b^2a^2}{x^2}+\frac{b^4x^2}{2}+4\,ab^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x)
```

```
[Out] -1/6*a^4/x^6-a^3*b/x^4-3*a^2*b^2/x^2+1/2*b^4*x^2+4*a*b^3*ln(x)
```

Maxima [A] time = 0.9814, size = 65, normalized size = 1.33

$$\frac{1}{2}\,b^4x^2+2\,ab^3\log\left(x^2\right)-\frac{18\,a^2b^2x^4+6\,a^3bx^2+a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(18*a^2*b^2*x^4 + 6*a^3*b*x^2 + a^4)/x
^6
```

Fricas [A] time = 1.6557, size = 108, normalized size = 2.2

$$\frac{3\,b^4x^8 + 24\,ab^3x^6\log\left(x\right) - 18\,a^2b^2x^4 - 6\,a^3bx^2 - a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*x^8 + 24*a*b^3*x^6*log(x) - 18*a^2*b^2*x^4 - 6*a^3*b*x^2 - a^4)/
x^6
```

Sympy [A] time = 0.419184, size = 48, normalized size = 0.98

$$4ab^3\log\left(x\right) + \frac{b^4x^2}{2} - \frac{a^4 + 6a^3bx^2 + 18a^2b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**7,x)
```

```
[Out] 4*a*b**3*log(x) + b**4*x**2/2 - (a**4 + 6*a**3*b*x**2 + 18*a**2*b**2*x**4)/
(6*x**6)
```

Giac [A] time = 1.10848, size = 77, normalized size = 1.57

$$\frac{1}{2}\,b^4x^2 + 2\,ab^3\log\left(x^2\right) - \frac{22\,ab^3x^6 + 18\,a^2b^2x^4 + 6\,a^3bx^2 + a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(22*a*b^3*x^6 + 18*a^2*b^2*x^4 + 6*a^3
*b*x^2 + a^4)/x^6
```

3.435 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^8}\,dx$

Optimal. Leaf size=47

$$-\frac{2a^2b^2}{x^3}-\frac{4a^3b}{5x^5}-\frac{a^4}{7x^7}-\frac{4ab^3}{x}+b^4x$$

```
[Out] -a^4/(7*x^7) - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x
```

Rubi [A] time = 0.02538, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{2a^2b^2}{x^3}-\frac{4a^3b}{5x^5}-\frac{a^4}{7x^7}-\frac{4ab^3}{x}+b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
```

```
[Out] -a^4/(7*x^7) - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^8}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^8}\,dx}{b^4}$$
$$= \frac{\int \left(b^8+\frac{a^4b^4}{x^8}+\frac{4a^3b^5}{x^6}+\frac{6a^2b^6}{x^4}+\frac{4ab^7}{x^2}\right)dx}{b^4}$$
$$= -\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

Mathematica [A] time = 0.0052106, size = 47, normalized size = 1.

$$-\frac{2a^2b^2}{x^3}-\frac{4a^3b}{5x^5}-\frac{a^4}{7x^7}-\frac{4ab^3}{x}+b^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
```

```
[Out] -a^4/(7*x^7) - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x
```

Maple [A] time = 0.049, size = 44, normalized size = 0.9

$$-\frac{a^4}{7\,x^7}-\frac{4\,a^3b}{5\,x^5}-2\,\frac{b^2a^2}{x^3}-4\,\frac{ab^3}{x}+b^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x)
```

```
[Out] -1/7*a^4/x^7-4/5*a^3*b/x^5-2*a^2*b^2/x^3-4*a*b^3/x+b^4*x
```

Maxima [A] time = 0.994078, size = 62, normalized size = 1.32

$$b^4x-\frac{140\,ab^3x^6+70\,a^2b^2x^4+28\,a^3bx^2+5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a^4)/x^7
```

Fricas [A] time = 1.59812, size = 107, normalized size = 2.28

$$\frac{35\,b^4x^8 - 140\,ab^3x^6 - 70\,a^2b^2x^4 - 28\,a^3bx^2 - 5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 - 140*a*b^3*x^6 - 70*a^2*b^2*x^4 - 28*a^3*b*x^2 - 5*a^4)/x
^7
```

Sympy [A] time = 0.42316, size = 46, normalized size = 0.98

$$b^4x - \frac{5a^4 + 28a^3bx^2 + 70a^2b^2x^4 + 140ab^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**8,x)
```

```
[Out] b**4*x - (5*a**4 + 28*a**3*b*x**2 + 70*a**2*b**2*x**4 + 140*a*b**3*x**6)/(3
5*x**7)
```

Giac [A] time = 1.10015, size = 62, normalized size = 1.32

$$b^4x - \frac{140\,ab^3x^6 + 70\,a^2b^2x^4 + 28\,a^3bx^2 + 5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="giac")
```

```
[Out] b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a^4)/x^7
```

3.436 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9}\,dx$

Optimal. Leaf size=50

$$-\frac{3a^2b^2}{2x^4}-\frac{2a^3b}{3x^6}-\frac{a^4}{8x^8}-\frac{2ab^3}{x^2}+b^4\log(x)$$

```
[Out] -a^4/(8*x^8) - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3)/x^2 + b^
4*Log[x]
```

Rubi [A] time = 0.0326616, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{3a^2b^2}{2x^4}-\frac{2a^3b}{3x^6}-\frac{a^4}{8x^8}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

```
[Out] -a^4/(8*x^8) - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3)/x^2 + b^
4*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^9}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^9}\,dx}{b^4}\\ &= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^5}\,dx,x,x^2\right)}{2b^4}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^4b^4}{x^5}+\frac{4a^3b^5}{x^4}+\frac{6a^2b^6}{x^3}+\frac{4ab^7}{x^2}+\frac{b^8}{x}\right)dx,x,x^2\right)}{2b^4}\\ &= -\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.0047331, size = 50, normalized size = 1.

$$-\frac{3a^2b^2}{2x^4}-\frac{2a^3b}{3x^6}-\frac{a^4}{8x^8}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

```
[Out] -a^4/(8*x^8) - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3)/x^2 + b^
4*Log[x]
```

Maple [A] time = 0.046, size = 45, normalized size = 0.9

$$-\frac{a^4}{8\,x^8}-\frac{2\,a^3b}{3\,x^6}-\frac{3\,b^2a^2}{2\,x^4}-2\,\frac{ab^3}{x^2}+b^4\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x)
```

```
[Out] -1/8*a^4/x^8-2/3*a^3*b/x^6-3/2*a^2*b^2/x^4-2*a*b^3/x^2+b^4*ln(x)
```

Maxima [A] time = 0.992852, size = 68, normalized size = 1.36

$$\frac{1}{2}\,b^4\log\left(x^2\right) - \frac{48\,ab^3x^6 + 36\,a^2b^2x^4 + 16\,a^3bx^2 + 3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(x^2) - 1/24*(48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a^3*b*x^2 + 3*a
^4)/x^8
```

Fricas [A] time = 1.70528, size = 115, normalized size = 2.3

$$\frac{24\,b^4x^8\log\left(x\right) - 48\,ab^3x^6 - 36\,a^2b^2x^4 - 16\,a^3bx^2 - 3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^4*x^8*log(x) - 48*a*b^3*x^6 - 36*a^2*b^2*x^4 - 16*a^3*b*x^2 - 3*
a^4)/x^8
```

Sympy [A] time = 0.489087, size = 48, normalized size = 0.96

$$b^4\log\left(x\right) - \frac{3a^4 + 16a^3bx^2 + 36a^2b^2x^4 + 48ab^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**9,x)
```

```
[Out] b**4*log(x) - (3*a**4 + 16*a**3*b*x**2 + 36*a**2*b**2*x**4 + 48*a*b**3*x**6
)/(24*x**8)
```

Giac [A] time = 1.11652, size = 78, normalized size = 1.56

$$\frac{1}{2}\,b^4\log\left(x^2\right)-\frac{25\,b^4x^8+48\,ab^3x^6+36\,a^2b^2x^4+16\,a^3bx^2+3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(x^2) - 1/24*(25*b^4*x^8 + 48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a^
3*b*x^2 + 3*a^4)/x^8
```

3.437 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=54

$$-\frac{6a^2b^2}{5x^5}-\frac{4a^3b}{7x^7}-\frac{a^4}{9x^9}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

```
[Out] -a^4/(9*x^9) - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
- b^4/x
```

Rubi [A] time = 0.0261705, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{5x^5}-\frac{4a^3b}{7x^7}-\frac{a^4}{9x^9}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] -a^4/(9*x^9) - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
- b^4/x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{10}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{10}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{x^{10}}+\frac{4a^3b^5}{x^8}+\frac{6a^2b^6}{x^6}+\frac{4ab^7}{x^4}+\frac{b^8}{x^2}\right)dx}{b^4}$$
$$= -\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Mathematica [A] time = 0.0084107, size = 54, normalized size = 1.

$$-\frac{6a^2b^2}{5x^5}-\frac{4a^3b}{7x^7}-\frac{a^4}{9x^9}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] -a^4/(9*x^9) - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
- b^4/x
```

Maple [A] time = 0.047, size = 47, normalized size = 0.9

$$-\frac{a^4}{9\,x^9}-\frac{4\,a^3b}{7\,x^7}-\frac{6\,b^2a^2}{5\,x^5}-\frac{4\,ab^3}{3\,x^3}-\frac{b^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x)
```

```
[Out] -1/9*a^4/x^9-4/7*a^3*b/x^7-6/5*a^2*b^2/x^5-4/3*a*b^3/x^3-b^4/x
```

Maxima [A] time = 0.984635, size = 65, normalized size = 1.2

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

Fricas [A] time = 1.68739, size = 115, normalized size = 2.13

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

Sympy [A] time = 0.489036, size = 51, normalized size = 0.94

$$-\frac{35a^4+180a^3bx^2+378a^2b^2x^4+420ab^3x^6+315b^4x^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**10,x)
```

```
[Out] -(35*a**4 + 180*a**3*b*x**2 + 378*a**2*b**2*x**4 + 420*a*b**3*x**6 + 315*b*
*4*x**8)/(315*x**9)
```

Giac [A] time = 1.1258, size = 65, normalized size = 1.2

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

3.438 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{11}}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(a+bx^2\right)^5}{10ax^{10}}$$

```
[Out] -(a + b*x^2)^5/(10*a*x^10)
```

Rubi [A] time = 0.0061135, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$-\frac{\left(a+bx^2\right)^5}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
```

```
[Out] -(a + b*x^2)^5/(10*a*x^10)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{11}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{11}}\,dx}{b^4}$$
$$= -\frac{\left(a+bx^2\right)^5}{10ax^{10}}$$

Mathematica [B] time = 0.0041784, size = 52, normalized size = 2.74

$$-\frac{a^2b^2}{x^6}-\frac{a^3b}{2x^8}-\frac{a^4}{10x^{10}}-\frac{ab^3}{x^4}-\frac{b^4}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
```

```
[Out] -a^4/(10*x^10) - (a^3*b)/(2*x^8) - (a^2*b^2)/x^6 - (a*b^3)/x^4 - b^4/(2*x^2
)
```

Maple [B] time = 0.048, size = 47, normalized size = 2.5

$$-\frac{a^4}{10\,x^{10}}-\frac{ab^3}{x^4}-\frac{b^4}{2\,x^2}-\frac{a^3b}{2\,x^8}-\frac{b^2a^2}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x)
```

```
[Out] -1/10*a^4/x^10-a*b^3/x^4-1/2*b^4/x^2-1/2*a^3*b/x^8-b^2*a^2/x^6
```

Maxima [B] time = 0.993977, size = 62, normalized size = 3.26

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

Fricas [B] time = 1.6613, size = 103, normalized size = 5.42

$$-\frac{5 b^4 x^8 + 10 a b^3 x^6 + 10 a^2 b^2 x^4 + 5 a^3 b x^2 + a^4}{10 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

Sympy [B] time = 0.506714, size = 49, normalized size = 2.58

$$-\frac{a^4 + 5a^3bx^2 + 10a^2b^2x^4 + 10ab^3x^6 + 5b^4x^8}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**11,x)
```

```
[Out] -(a**4 + 5*a**3*b*x**2 + 10*a**2*b**2*x**4 + 10*a*b**3*x**6 + 5*b**4*x**8)/
(10*x**10)
```

Giac [B] time = 1.20888, size = 62, normalized size = 3.26

$$-\frac{5 b^4 x^8 + 10 a b^3 x^6 + 10 a^2 b^2 x^4 + 5 a^3 b x^2 + a^4}{10 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

3.439 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=56

$$-\frac{6a^2b^2}{7x^7}-\frac{4a^3b}{9x^9}-\frac{a^4}{11x^{11}}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

```
[Out] -a^4/(11*x^11) - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^5
) - b^4/(3*x^3)
```

Rubi [A] time = 0.0263073, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{7x^7}-\frac{4a^3b}{9x^9}-\frac{a^4}{11x^{11}}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] -a^4/(11*x^11) - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^5
) - b^4/(3*x^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{12}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{12}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{x^{12}}+\frac{4a^3b^5}{x^{10}}+\frac{6a^2b^6}{x^8}+\frac{4ab^7}{x^6}+\frac{b^8}{x^4}\right)dx}{b^4}$$
$$= -\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Mathematica [A] time = 0.0066583, size = 56, normalized size = 1.

$$-\frac{6a^2b^2}{7x^7}-\frac{4a^3b}{9x^9}-\frac{a^4}{11x^{11}}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] -a^4/(11*x^11) - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^5
) - b^4/(3*x^3)
```

Maple [A] time = 0.049, size = 47, normalized size = 0.8

$$-\frac{a^4}{11\,x^{11}}-\frac{4\,a^3b}{9\,x^9}-\frac{6\,b^2a^2}{7\,x^7}-\frac{4\,ab^3}{5\,x^5}-\frac{b^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x)
```

```
[Out] -1/11*a^4/x^11-4/9*a^3*b/x^9-6/7*a^2*b^2/x^7-4/5*a*b^3/x^5-1/3*b^4/x^3
```

Maxima [A] time = 0.994372, size = 65, normalized size = 1.16

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

Fricas [A] time = 1.71152, size = 124, normalized size = 2.21

$$-\frac{1155\,b^4x^8 + 2772\,ab^3x^6 + 2970\,a^2b^2x^4 + 1540\,a^3bx^2 + 315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

Sympy [A] time = 0.528373, size = 51, normalized size = 0.91

$$-\frac{315a^4 + 1540a^3bx^2 + 2970a^2b^2x^4 + 2772ab^3x^6 + 1155b^4x^8}{3465x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**12,x)
```

```
[Out] -(315*a**4 + 1540*a**3*b*x**2 + 2970*a**2*b**2*x**4 + 2772*a*b**3*x**6 + 11
55*b**4*x**8)/(3465*x**11)
```

Giac [A] time = 1.15832, size = 65, normalized size = 1.16

$$-\frac{1155\,b^4x^8 + 2772\,ab^3x^6 + 2970\,a^2b^2x^4 + 1540\,a^3bx^2 + 315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

3.440 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{13}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^2\right)^5}{60a^2x^{10}}-\frac{\left(a+bx^2\right)^5}{12ax^{12}}$$

```
[Out] -(a + b*x^2)^5/(12*a*x^12) + (b*(a + b*x^2)^5)/(60*a^2*x^10)
```

Rubi [A] time = 0.0245002, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{b\left(a+bx^2\right)^5}{60a^2x^{10}}-\frac{\left(a+bx^2\right)^5}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

```
[Out] -(a + b*x^2)^5/(12*a*x^12) + (b*(a + b*x^2)^5)/(60*a^2*x^10)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
```

```
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{13}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{13}}\,dx}{b^4}\\
&= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^7}\,dx,x,x^2\right)}{2b^4}\\
&= -\frac{\left(a+bx^2\right)^5}{12ax^{12}}-\frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^6}\,dx,x,x^2\right)}{12ab^3}\\
&= -\frac{\left(a+bx^2\right)^5}{12ax^{12}}+\frac{b\left(a+bx^2\right)^5}{60a^2x^{10}}
\end{aligned}$$

Mathematica [A] time = 0.0041385, size = 56, normalized size = 1.4

$$-\frac{3a^2b^2}{4x^8}-\frac{2a^3b}{5x^{10}}-\frac{a^4}{12x^{12}}-\frac{2ab^3}{3x^6}-\frac{b^4}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

```
[Out] -a^4/(12*x^12) - (2*a^3*b)/(5*x^10) - (3*a^2*b^2)/(4*x^8) - (2*a*b^3)/(3*x^
6) - b^4/(4*x^4)
```

Maple [A] time = 0.048, size = 47, normalized size = 1.2

$$-\frac{2 a^3 b}{5 x^{10}}-\frac{a^4}{12 x^{12}}-\frac{b^4}{4 x^4}-\frac{3 b^2 a^2}{4 x^8}-\frac{2 a b^3}{3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x)
```

```
[Out] -2/5*a^3*b/x^10-1/12*a^4/x^12-1/4*b^4/x^4-3/4*b^2*a^2/x^8-2/3*a*b^3/x^6
```

Maxima [A] time = 0.980356, size = 65, normalized size = 1.62

$$-\frac{15 b^4 x^8+40 a b^3 x^6+45 a^2 b^2 x^4+24 a^3 b x^2+5 a^4}{60 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12
```

Fricas [A] time = 1.69737, size = 108, normalized size = 2.7

$$-\frac{15 b^4 x^8+40 a b^3 x^6+45 a^2 b^2 x^4+24 a^3 b x^2+5 a^4}{60 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12
```

Sympy [A] time = 0.546981, size = 51, normalized size = 1.27

$$-\frac{5a^4+24a^3bx^2+45a^2b^2x^4+40ab^3x^6+15b^4x^8}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**13,x)
```

```
[Out] -(5*a**4 + 24*a**3*b*x**2 + 45*a**2*b**2*x**4 + 40*a*b**3*x**6 + 15*b**4*x*
*8)/(60*x**12)
```

Giac [A] time = 1.14102, size = 65, normalized size = 1.62

$$-\frac{15 b^4 x^8 + 40 a b^3 x^6 + 45 a^2 b^2 x^4 + 24 a^3 b x^2 + 5 a^4}{60 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12
```

3.441 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{14}} dx$

Optimal. Leaf size=56

$$-\frac{2a^2b^2}{3x^9}-\frac{4a^3b}{11x^{11}}-\frac{a^4}{13x^{13}}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

```
[Out] -a^4/(13*x^13) - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*x
^7) - b^4/(5*x^5)
```

Rubi [A] time = 0.0274963, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{2a^2b^2}{3x^9}-\frac{4a^3b}{11x^{11}}-\frac{a^4}{13x^{13}}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
```

```
[Out] -a^4/(13*x^13) - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*x
^7) - b^4/(5*x^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{14}}\,dx=\frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{14}}\,dx}{b^4}$$
$$=\frac{\int \left(\frac{a^4b^4}{x^{14}}+\frac{4a^3b^5}{x^{12}}+\frac{6a^2b^6}{x^{10}}+\frac{4ab^7}{x^8}+\frac{b^8}{x^6}\right)dx}{b^4}$$
$$=-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Mathematica [A] time = 0.0081476, size = 56, normalized size = 1.

$$-\frac{2a^2b^2}{3x^9}-\frac{4a^3b}{11x^{11}}-\frac{a^4}{13x^{13}}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
```

```
[Out] -a^4/(13*x^13) - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*x
^7) - b^4/(5*x^5)
```

Maple [A] time = 0.046, size = 47, normalized size = 0.8

$$-\frac{a^4}{13\,x^{13}}-\frac{4\,a^3b}{11\,x^{11}}-\frac{2\,b^2a^2}{3\,x^9}-\frac{4\,ab^3}{7\,x^7}-\frac{b^4}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x)
```

```
[Out] -1/13*a^4/x^13-4/11*a^3*b/x^11-2/3*a^2*b^2/x^9-4/7*a*b^3/x^7-1/5*b^4/x^5
```

Maxima [A] time = 0.97777, size = 65, normalized size = 1.16

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^
2 + 1155*a^4)/x^13
```

Fricas [A] time = 1.61811, size = 128, normalized size = 2.29

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^
2 + 1155*a^4)/x^13
```

Sympy [A] time = 0.558685, size = 51, normalized size = 0.91

$$-\frac{1155a^4+5460a^3bx^2+10010a^2b^2x^4+8580ab^3x^6+3003b^4x^8}{15015x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**14,x)
```

```
[Out] -(1155*a**4 + 5460*a**3*b*x**2 + 10010*a**2*b**2*x**4 + 8580*a*b**3*x**6 +
3003*b**4*x**8)/(15015*x**13)
```

Giac [A] time = 1.20116, size = 65, normalized size = 1.16

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^
2 + 1155*a^4)/x^13
```

3.442 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}} dx$

Optimal. Leaf size=56

$$-\frac{3a^2b^2}{5x^{10}}-\frac{a^3b}{3x^{12}}-\frac{a^4}{14x^{14}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

```
[Out] -a^4/(14*x^14) - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3)/(2*x^8)
- b^4/(6*x^6)
```

Rubi [A] time = 0.0364443, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{3a^2b^2}{5x^{10}}-\frac{a^3b}{3x^{12}}-\frac{a^4}{14x^{14}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] -a^4/(14*x^14) - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3)/(2*x^8)
- b^4/(6*x^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{15}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{15}}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^4}{x^8}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4b^4}{x^8}+\frac{4a^3b^5}{x^7}+\frac{6a^2b^6}{x^6}+\frac{4ab^7}{x^5}+\frac{b^8}{x^4}\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Mathematica [A] time = 0.0040352, size = 56, normalized size = 1.

$$-\frac{3a^2b^2}{5x^{10}}-\frac{a^3b}{3x^{12}}-\frac{a^4}{14x^{14}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] -a^4/(14*x^14) - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3)/(2*x^8)
- b^4/(6*x^6)
```

Maple [A] time = 0.047, size = 47, normalized size = 0.8

$$-\frac{a^4}{14\,x^{14}}-\frac{a^3b}{3\,x^{12}}-\frac{3\,b^2a^2}{5\,x^{10}}-\frac{ab^3}{2\,x^8}-\frac{b^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x)
```

```
[Out] -1/14*a^4/x^14-1/3*a^3*b/x^12-3/5*a^2*b^2/x^10-1/2*a*b^3/x^8-1/6*b^4/x^6
```

Maxima [A] time = 0.992685, size = 65, normalized size = 1.16

$$-\frac{35\,b^4x^8 + 105\,ab^3x^6 + 126\,a^2b^2x^4 + 70\,a^3bx^2 + 15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="maxima")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

Fricas [A] time = 1.67089, size = 113, normalized size = 2.02

$$-\frac{35\,b^4x^8 + 105\,ab^3x^6 + 126\,a^2b^2x^4 + 70\,a^3bx^2 + 15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="fricas")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

Sympy [A] time = 0.591961, size = 51, normalized size = 0.91

$$-\frac{15a^4 + 70a^3bx^2 + 126a^2b^2x^4 + 105ab^3x^6 + 35b^4x^8}{210x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**15,x)
```

```
[Out] -(15*a**4 + 70*a**3*b*x**2 + 126*a**2*b**2*x**4 + 105*a*b**3*x**6 + 35*b**4
*x**8)/(210*x**14)
```

Giac [A] time = 1.22416, size = 65, normalized size = 1.16

$$-\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="giac")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

3.443 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{16}} \, dx$

Optimal. Leaf size=56

$$-\frac{6a^2b^2}{11x^{11}}-\frac{4a^3b}{13x^{13}}-\frac{a^4}{15x^{15}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

```
[Out] -a^4/(15*x^15) - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4*a*b^3)/(9
*x^9) - b^4/(7*x^7)
```

Rubi [A] time = 0.0252742, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{6a^2b^2}{11x^{11}}-\frac{4a^3b}{13x^{13}}-\frac{a^4}{15x^{15}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] -a^4/(15*x^15) - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4*a*b^3)/(9
*x^9) - b^4/(7*x^7)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{16}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{16}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{x^{16}}+\frac{4a^3b^5}{x^{14}}+\frac{6a^2b^6}{x^{12}}+\frac{4ab^7}{x^{10}}+\frac{b^8}{x^8}\right)dx}{b^4}$$
$$= -\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Mathematica [A] time = 0.0068133, size = 56, normalized size = 1.

$$-\frac{6a^2b^2}{11x^{11}}-\frac{4a^3b}{13x^{13}}-\frac{a^4}{15x^{15}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] -a^4/(15*x^15) - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4*a*b^3)/(9
*x^9) - b^4/(7*x^7)
```

Maple [A] time = 0.047, size = 47, normalized size = 0.8

$$-\frac{a^4}{15\,x^{15}}-\frac{4\,a^3b}{13\,x^{13}}-\frac{6\,b^2a^2}{11\,x^{11}}-\frac{4\,ab^3}{9\,x^9}-\frac{b^4}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x)
```

```
[Out] -1/15*a^4/x^15-4/13*a^3*b/x^13-6/11*a^2*b^2/x^11-4/9*a*b^3/x^9-1/7*b^4/x^7
```

Maxima [A] time = 0.985168, size = 65, normalized size = 1.16

$$-\frac{6435\,b^4x^8+20020\,ab^3x^6+24570\,a^2b^2x^4+13860\,a^3bx^2+3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

Fricas [A] time = 1.87557, size = 131, normalized size = 2.34

$$-\frac{6435\,b^4x^8 + 20020\,ab^3x^6 + 24570\,a^2b^2x^4 + 13860\,a^3bx^2 + 3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

Sympy [A] time = 0.575622, size = 51, normalized size = 0.91

$$-\frac{3003a^4 + 13860a^3bx^2 + 24570a^2b^2x^4 + 20020ab^3x^6 + 6435b^4x^8}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**16,x)
```

```
[Out] -(3003*a**4 + 13860*a**3*b*x**2 + 24570*a**2*b**2*x**4 + 20020*a*b**3*x**6
+ 6435*b**4*x**8)/(45045*x**15)
```

Giac [A] time = 1.29041, size = 65, normalized size = 1.16

$$-\frac{6435\,b^4x^8 + 20020\,ab^3x^6 + 24570\,a^2b^2x^4 + 13860\,a^3bx^2 + 3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

3.444 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=82

$$\frac{15}{17}a^2b^4x^{17}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{13}a^4b^2x^{13}+\frac{6}{11}a^5bx^{11}+\frac{a^6x^9}{9}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3
 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

Rubi [A] time = 0.0456947, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{17}a^2b^4x^{17}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{13}a^4b^2x^{13}+\frac{6}{11}a^5bx^{11}+\frac{a^6x^9}{9}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3
 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 dx = \frac{\int x^8 \left(ab+b^2x^2\right)^6 dx}{b^6}$$
$$= \frac{\int \left(a^6b^6x^8+6a^5b^7x^{10}+15a^4b^8x^{12}+20a^3b^9x^{14}+15a^2b^{10}x^{16}+6ab^{11}x^{18}+b^{12}x^{20}\right) dx}{b^6}$$
$$= \frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Mathematica [A] time = 0.0031235, size = 82, normalized size = 1.

$$\frac{15}{17}a^2b^4x^{17}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{13}a^4b^2x^{13}+\frac{6}{11}a^5bx^{11}+\frac{a^6x^9}{9}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3
 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

Maple [A] time = 0.042, size = 69, normalized size = 0.8

$$\frac{a^6x^9}{9}+\frac{6\,a^5bx^{11}}{11}+\frac{15\,a^4b^2x^{13}}{13}+\frac{4\,a^3b^3x^{15}}{3}+\frac{15\,a^2b^4x^{17}}{17}+\frac{6\,ab^5x^{19}}{19}+\frac{b^6x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/9*a^6*x^9+6/11*a^5*b*x^11+15/13*a^4*b^2*x^13+4/3*a^3*b^3*x^15+15/17*a^2*b
^4*x^17+6/19*a*b^5*x^19+1/21*b^6*x^21
```

Maxima [A] time = 0.980238, size = 92, normalized size = 1.12

$$\frac{1}{21}\,b^6x^{21}+\frac{6}{19}\,ab^5x^{19}+\frac{15}{17}\,a^2b^4x^{17}+\frac{4}{3}\,a^3b^3x^{15}+\frac{15}{13}\,a^4b^2x^{13}+\frac{6}{11}\,a^5bx^{11}+\frac{1}{9}\,a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 + 1
5/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

Fricas [A] time = 1.72178, size = 170, normalized size = 2.07

$$\frac{1}{21}x^{21}b^6 + \frac{6}{19}x^{19}b^5a + \frac{15}{17}x^{17}b^4a^2 + \frac{4}{3}x^{15}b^3a^3 + \frac{15}{13}x^{13}b^2a^4 + \frac{6}{11}x^{11}ba^5 + \frac{1}{9}x^9a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^6 + 6/19*x^19*b^5*a + 15/17*x^17*b^4*a^2 + 4/3*x^15*b^3*a^3 + 1
5/13*x^13*b^2*a^4 + 6/11*x^11*b*a^5 + 1/9*x^9*a^6
```

Sympy [A] time = 0.082118, size = 80, normalized size = 0.98

$$\frac{a^6x^9}{9} + \frac{6a^5bx^{11}}{11} + \frac{15a^4b^2x^{13}}{13} + \frac{4a^3b^3x^{15}}{3} + \frac{15a^2b^4x^{17}}{17} + \frac{6ab^5x^{19}}{19} + \frac{b^6x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**9/9 + 6*a**5*b*x**11/11 + 15*a**4*b**2*x**13/13 + 4*a**3*b**3*x**15
/3 + 15*a**2*b**4*x**17/17 + 6*a*b**5*x**19/19 + b**6*x**21/21
```

Giac [A] time = 1.17663, size = 92, normalized size = 1.12

$$\frac{1}{21}\,b^6x^{21} + \frac{6}{19}\,ab^5x^{19} + \frac{15}{17}\,a^2b^4x^{17} + \frac{4}{3}\,a^3b^3x^{15} + \frac{15}{13}\,a^4b^2x^{13} + \frac{6}{11}\,a^5bx^{11} + \frac{1}{9}\,a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 + 1
5/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

3.445 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3 dx$

Optimal. Leaf size=72

$$\frac{3a^2\left(a+bx^2\right)^8}{16b^4}-\frac{a^3\left(a+bx^2\right)^7}{14b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}-\frac{a\left(a+bx^2\right)^9}{6b^4}$$

```
[Out] -(a^3*(a + b*x^2)^7)/(14*b^4) + (3*a^2*(a + b*x^2)^8)/(16*b^4) - (a*(a + b*
x^2)^9)/(6*b^4) + (a + b*x^2)^10/(20*b^4)
```

Rubi [A] time = 0.116991, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3a^2\left(a+bx^2\right)^8}{16b^4}-\frac{a^3\left(a+bx^2\right)^7}{14b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}-\frac{a\left(a+bx^2\right)^9}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a^3*(a + b*x^2)^7)/(14*b^4) + (3*a^2*(a + b*x^2)^8)/(16*b^4) - (a*(a + b*
x^2)^9)/(6*b^4) + (a + b*x^2)^10/(20*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx &= \frac{\int x^7 \left(ab+b^2x^2\right)^6 \, dx}{b^6} \\
&= \frac{\text{Subst}\left(\int x^3 \left(ab+b^2x\right)^6 \, dx, x, x^2\right)}{2b^6} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a^3\left(ab+b^2x\right)^6}{b^3}+\frac{3a^2\left(ab+b^2x\right)^7}{b^4}-\frac{3a\left(ab+b^2x\right)^8}{b^5}+\frac{\left(ab+b^2x\right)^9}{b^6}\right) dx, x, x^2\right)}{2b^6} \\
&= -\frac{a^3\left(a+bx^2\right)^7}{14b^4}+\frac{3a^2\left(a+bx^2\right)^8}{16b^4}-\frac{a\left(a+bx^2\right)^9}{6b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}
\end{aligned}$$

Mathematica [A] time = 0.0024967, size = 82, normalized size = 1.14

$$\frac{15}{16}a^2b^4x^{16}+\frac{10}{7}a^3b^3x^{14}+\frac{5}{4}a^4b^2x^{12}+\frac{3}{5}a^5bx^{10}+\frac{a^6x^8}{8}+\frac{1}{3}ab^5x^{18}+\frac{b^6x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^8)/8 + (3*a^5*b*x^10)/5 + (5*a^4*b^2*x^12)/4 + (10*a^3*b^3*x^14)/7 +
 (15*a^2*b^4*x^16)/16 + (a*b^5*x^18)/3 + (b^6*x^20)/20
```

Maple [A] time = 0.043, size = 69, normalized size = 1.

$$\frac{b^6x^{20}}{20}+\frac{ab^5x^{18}}{3}+\frac{15\,a^2b^4x^{16}}{16}+\frac{10\,a^3b^3x^{14}}{7}+\frac{5\,a^4b^2x^{12}}{4}+\frac{3\,a^5bx^{10}}{5}+\frac{a^6x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/20*b^6*x^20+1/3*a*b^5*x^18+15/16*a^2*b^4*x^16+10/7*a^3*b^3*x^14+5/4*a^4*b
^2*x^12+3/5*a^5*b*x^10+1/8*a^6*x^8
```

Maxima [A] time = 0.984808, size = 92, normalized size = 1.28

$$\frac{1}{20}\,b^6x^{20}+\frac{1}{3}\,ab^5x^{18}+\frac{15}{16}\,a^2b^4x^{16}+\frac{10}{7}\,a^3b^3x^{14}+\frac{5}{4}\,a^4b^2x^{12}+\frac{3}{5}\,a^5bx^{10}+\frac{1}{8}\,a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 + 5
/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

Fricas [A] time = 1.71961, size = 166, normalized size = 2.31

$$\frac{1}{20}x^{20}b^6+\frac{1}{3}x^{18}b^5a+\frac{15}{16}x^{16}b^4a^2+\frac{10}{7}x^{14}b^3a^3+\frac{5}{4}x^{12}b^2a^4+\frac{3}{5}x^{10}ba^5+\frac{1}{8}x^8a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^6 + 1/3*x^18*b^5*a + 15/16*x^16*b^4*a^2 + 10/7*x^14*b^3*a^3 + 5
/4*x^12*b^2*a^4 + 3/5*x^10*b*a^5 + 1/8*x^8*a^6
```

Sympy [A] time = 0.080959, size = 78, normalized size = 1.08

$$\frac{a^6x^8}{8}+\frac{3a^5bx^{10}}{5}+\frac{5a^4b^2x^{12}}{4}+\frac{10a^3b^3x^{14}}{7}+\frac{15a^2b^4x^{16}}{16}+\frac{ab^5x^{18}}{3}+\frac{b^6x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**8/8 + 3*a**5*b*x**10/5 + 5*a**4*b**2*x**12/4 + 10*a**3*b**3*x**14/7
 + 15*a**2*b**4*x**16/16 + a*b**5*x**18/3 + b**6*x**20/20
```

Giac [A] time = 1.15233, size = 92, normalized size = 1.28

$$\frac{1}{20}\,b^{6}x^{20}+\frac{1}{3}\,ab^{5}x^{18}+\frac{15}{16}\,a^{2}b^{4}x^{16}+\frac{10}{7}\,a^{3}b^{3}x^{14}+\frac{5}{4}\,a^{4}b^{2}x^{12}+\frac{3}{5}\,a^{5}bx^{10}+\frac{1}{8}\,a^{6}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 + 5
/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

3.446 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=79

$$a^2b^4x^{15}+\frac{20}{13}a^3b^3x^{13}+\frac{15}{11}a^4b^2x^{11}+\frac{2}{3}a^5bx^9+\frac{a^6x^7}{7}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13
 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Rubi [A] time = 0.0380232, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$a^2b^4x^{15}+\frac{20}{13}a^3b^3x^{13}+\frac{15}{11}a^4b^2x^{11}+\frac{2}{3}a^5bx^9+\frac{a^6x^7}{7}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13
 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{\int x^6\left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$=\frac{\int\left(a^6b^6x^6+6a^5b^7x^8+15a^4b^8x^{10}+20a^3b^9x^{12}+15a^2b^{10}x^{14}+6ab^{11}x^{16}+b^{12}x^{18}\right)dx}{b^6}$$
$$=\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Mathematica [A] time = 0.0024519, size = 79, normalized size = 1.

$$a^2b^4x^{15}+\frac{20}{13}a^3b^3x^{13}+\frac{15}{11}a^4b^2x^{11}+\frac{2}{3}a^5bx^9+\frac{a^6x^7}{7}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13
 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Maple [A] time = 0.043, size = 68, normalized size = 0.9

$$\frac{a^6x^7}{7}+\frac{2\,a^5bx^9}{3}+\frac{15\,a^4b^2x^{11}}{11}+\frac{20\,a^3b^3x^{13}}{13}+a^2b^4x^{15}+\frac{6\,ab^5x^{17}}{17}+\frac{b^6x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/7*a^6*x^7+2/3*a^5*b*x^9+15/11*a^4*b^2*x^11+20/13*a^3*b^3*x^13+a^2*b^4*x^1
5+6/17*a*b^5*x^17+1/19*b^6*x^19
```

Maxima [A] time = 1.00599, size = 90, normalized size = 1.14

$$\frac{1}{19}\,b^6x^{19}+\frac{6}{17}\,ab^5x^{17}+a^2b^4x^{15}+\frac{20}{13}\,a^3b^3x^{13}+\frac{15}{11}\,a^4b^2x^{11}+\frac{2}{3}\,a^5bx^9+\frac{1}{7}\,a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/11
*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

Fricas [A] time = 1.45639, size = 162, normalized size = 2.05

$$\frac{1}{19}x^{19}b^6 + \frac{6}{17}x^{17}b^5a + x^{15}b^4a^2 + \frac{20}{13}x^{13}b^3a^3 + \frac{15}{11}x^{11}b^2a^4 + \frac{2}{3}x^9ba^5 + \frac{1}{7}x^7a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^6 + 6/17*x^17*b^5*a + x^15*b^4*a^2 + 20/13*x^13*b^3*a^3 + 15/11
*x^11*b^2*a^4 + 2/3*x^9*b*a^5 + 1/7*x^7*a^6
```

Sympy [A] time = 0.078663, size = 76, normalized size = 0.96

$$\frac{a^6x^7}{7} + \frac{2a^5bx^9}{3} + \frac{15a^4b^2x^{11}}{11} + \frac{20a^3b^3x^{13}}{13} + a^2b^4x^{15} + \frac{6ab^5x^{17}}{17} + \frac{b^6x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**7/7 + 2*a**5*b*x**9/3 + 15*a**4*b**2*x**11/11 + 20*a**3*b**3*x**13/
13 + a**2*b**4*x**15 + 6*a*b**5*x**17/17 + b**6*x**19/19
```

Giac [A] time = 1.14545, size = 90, normalized size = 1.14

$$\frac{1}{19}b^6x^{19} + \frac{6}{17}ab^5x^{17} + a^2b^4x^{15} + \frac{20}{13}a^3b^3x^{13} + \frac{15}{11}a^4b^2x^{11} + \frac{2}{3}a^5bx^9 + \frac{1}{7}a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/11
*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

3.447 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^2\right)^7}{14b^3}+\frac{\left(a+bx^2\right)^9}{18b^3}-\frac{a\left(a+bx^2\right)^8}{8b^3}$$

```
[Out] (a^2*(a + b*x^2)^7)/(14*b^3) - (a*(a + b*x^2)^8)/(8*b^3) + (a + b*x^2)^9/(1
8*b^3)
```

Rubi [A] time = 0.0853567, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^2\left(a+bx^2\right)^7}{14b^3}+\frac{\left(a+bx^2\right)^9}{18b^3}-\frac{a\left(a+bx^2\right)^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^2*(a + b*x^2)^7)/(14*b^3) - (a*(a + b*x^2)^8)/(8*b^3) + (a + b*x^2)^9/(1
8*b^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int x^5\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\text{Subst}\left(\int x^2\left(ab+b^2x\right)^6\,dx,x,x^2\right)}{2b^6}\\ &= \frac{\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^6}{b^2}-\frac{2a(ab+b^2x)^7}{b^3}+\frac{(ab+b^2x)^8}{b^4}\right)dx,x,x^2\right)}{2b^6}\\ &= \frac{a^2\left(a+bx^2\right)^7}{14b^3}-\frac{a\left(a+bx^2\right)^8}{8b^3}+\frac{\left(a+bx^2\right)^9}{18b^3}\end{aligned}$$

Mathematica [A] time = 0.0026915, size = 82, normalized size = 1.55

$$\frac{15}{14}a^2b^4x^{14}+\frac{5}{3}a^3b^3x^{12}+\frac{3}{2}a^4b^2x^{10}+\frac{3}{4}a^5bx^8+\frac{a^6x^6}{6}+\frac{3}{8}ab^5x^{16}+\frac{b^6x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^6)/6 + (3*a^5*b*x^8)/4 + (3*a^4*b^2*x^10)/2 + (5*a^3*b^3*x^12)/3 + (
15*a^2*b^4*x^14)/14 + (3*a*b^5*x^16)/8 + (b^6*x^18)/18
```

Maple [A] time = 0.042, size = 69, normalized size = 1.3

$$\frac{b^6x^{18}}{18}+\frac{3\,ab^5x^{16}}{8}+\frac{15\,a^2b^4x^{14}}{14}+\frac{5\,a^3b^3x^{12}}{3}+\frac{3\,a^4b^2x^{10}}{2}+\frac{3\,a^5bx^8}{4}+\frac{a^6x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/18*b^6*x^18+3/8*a*b^5*x^16+15/14*a^2*b^4*x^14+5/3*a^3*b^3*x^12+3/2*a^4*b^
2*x^10+3/4*a^5*b*x^8+1/6*a^6*x^6
```

Maxima [A] time = 0.980579, size = 92, normalized size = 1.74

$$\frac{1}{18}b^{6}x^{18} + \frac{3}{8}ab^{5}x^{16} + \frac{15}{14}a^{2}b^{4}x^{14} + \frac{5}{3}a^{3}b^{3}x^{12} + \frac{3}{2}a^{4}b^{2}x^{10} + \frac{3}{4}a^{5}bx^{8} + \frac{1}{6}a^{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3/
2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

Fricas [A] time = 1.51299, size = 163, normalized size = 3.08

$$\frac{1}{18}x^{18}b^{6} + \frac{3}{8}x^{16}b^{5}a + \frac{15}{14}x^{14}b^{4}a^{2} + \frac{5}{3}x^{12}b^{3}a^{3} + \frac{3}{2}x^{10}b^{2}a^{4} + \frac{3}{4}x^{8}ba^{5} + \frac{1}{6}x^{6}a^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^6 + 3/8*x^16*b^5*a + 15/14*x^14*b^4*a^2 + 5/3*x^12*b^3*a^3 + 3/
2*x^10*b^2*a^4 + 3/4*x^8*b*a^5 + 1/6*x^6*a^6
```

Sympy [A] time = 0.081546, size = 80, normalized size = 1.51

$$\frac{a^{6}x^{6}}{6} + \frac{3a^{5}bx^{8}}{4} + \frac{3a^{4}b^{2}x^{10}}{2} + \frac{5a^{3}b^{3}x^{12}}{3} + \frac{15a^{2}b^{4}x^{14}}{14} + \frac{3ab^{5}x^{16}}{8} + \frac{b^{6}x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**6/6 + 3*a**5*b*x**8/4 + 3*a**4*b**2*x**10/2 + 5*a**3*b**3*x**12/3 +
 15*a**2*b**4*x**14/14 + 3*a*b**5*x**16/8 + b**6*x**18/18
```

Giac [A] time = 1.1859, size = 92, normalized size = 1.74

$$\frac{1}{18}\,b^{6}x^{18}+\frac{3}{8}\,ab^{5}x^{16}+\frac{15}{14}\,a^{2}b^{4}x^{14}+\frac{5}{3}\,a^{3}b^{3}x^{12}+\frac{3}{2}\,a^{4}b^{2}x^{10}+\frac{3}{4}\,a^{5}bx^{8}+\frac{1}{6}\,a^{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3/
2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

3.448 $\int x^4 \left(a^2 + 2abx^2 + b^2x^4\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{15}{13}a^2b^4x^{13} + \frac{20}{11}a^3b^3x^{11} + \frac{5}{3}a^4b^2x^9 + \frac{6}{7}a^5bx^7 + \frac{a^6x^5}{5} + \frac{2}{5}ab^5x^{15} + \frac{b^6x^{17}}{17}$$

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
(15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Rubi [A] time = 0.0398535, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{13}a^2b^4x^{13} + \frac{20}{11}a^3b^3x^{11} + \frac{5}{3}a^4b^2x^9 + \frac{6}{7}a^5bx^7 + \frac{a^6x^5}{5} + \frac{2}{5}ab^5x^{15} + \frac{b^6x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
(15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^3 dx = \frac{\int x^4 \left(ab+b^2x^2\right)^6 dx}{b^6}$$
$$= \frac{\int \left(a^6b^6x^4+6a^5b^7x^6+15a^4b^8x^8+20a^3b^9x^{10}+15a^2b^{10}x^{12}+6ab^{11}x^{14}+b^{12}x^{16}\right) dx}{b^6}$$
$$= \frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

Mathematica [A] time = 0.002518, size = 82, normalized size = 1.

$$\frac{15}{13}a^2b^4x^{13}+\frac{20}{11}a^3b^3x^{11}+\frac{5}{3}a^4b^2x^9+\frac{6}{7}a^5bx^7+\frac{a^6x^5}{5}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
(15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Maple [A] time = 0.043, size = 69, normalized size = 0.8

$$\frac{a^6x^5}{5}+\frac{6\,a^5bx^7}{7}+\frac{5\,a^4b^2x^9}{3}+\frac{20\,a^3b^3x^{11}}{11}+\frac{15\,a^2b^4x^{13}}{13}+\frac{2\,ab^5x^{15}}{5}+\frac{b^6x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/5*a^6*x^5+6/7*a^5*b*x^7+5/3*a^4*b^2*x^9+20/11*a^3*b^3*x^11+15/13*a^2*b^4*
x^13+2/5*a*b^5*x^15+1/17*b^6*x^17
```

Maxima [A] time = 0.985375, size = 92, normalized size = 1.12

$$\frac{1}{17}\,b^6x^{17}+\frac{2}{5}\,ab^5x^{15}+\frac{15}{13}\,a^2b^4x^{13}+\frac{20}{11}\,a^3b^3x^{11}+\frac{5}{3}\,a^4b^2x^9+\frac{6}{7}\,a^5bx^7+\frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

Fricas [A] time = 1.49004, size = 165, normalized size = 2.01

$$\frac{1}{17}x^{17}b^6 + \frac{2}{5}x^{15}b^5a + \frac{15}{13}x^{13}b^4a^2 + \frac{20}{11}x^{11}b^3a^3 + \frac{5}{3}x^9b^2a^4 + \frac{6}{7}x^7ba^5 + \frac{1}{5}x^5a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^6 + 2/5*x^15*b^5*a + 15/13*x^13*b^4*a^2 + 20/11*x^11*b^3*a^3 +
5/3*x^9*b^2*a^4 + 6/7*x^7*b*a^5 + 1/5*x^5*a^6
```

Sympy [A] time = 0.07613, size = 80, normalized size = 0.98

$$\frac{a^6x^5}{5} + \frac{6a^5bx^7}{7} + \frac{5a^4b^2x^9}{3} + \frac{20a^3b^3x^{11}}{11} + \frac{15a^2b^4x^{13}}{13} + \frac{2ab^5x^{15}}{5} + \frac{b^6x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**5/5 + 6*a**5*b*x**7/7 + 5*a**4*b**2*x**9/3 + 20*a**3*b**3*x**11/11
+ 15*a**2*b**4*x**13/13 + 2*a*b**5*x**15/5 + b**6*x**17/17
```

Giac [A] time = 1.15811, size = 92, normalized size = 1.12

$$\frac{1}{17}\,b^6x^{17} + \frac{2}{5}\,ab^5x^{15} + \frac{15}{13}\,a^2b^4x^{13} + \frac{20}{11}\,a^3b^3x^{11} + \frac{5}{3}\,a^4b^2x^9 + \frac{6}{7}\,a^5bx^7 + \frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

3.449 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^8}{16b^2}-\frac{a\left(a+bx^2\right)^7}{14b^2}$$

```
[Out] -(a*(a + b*x^2)^7)/(14*b^2) + (a + b*x^2)^8/(16*b^2)
```

Rubi [A] time = 0.0456321, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{\left(a+bx^2\right)^8}{16b^2}-\frac{a\left(a+bx^2\right)^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a*(a + b*x^2)^7)/(14*b^2) + (a + b*x^2)^8/(16*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^2+b^2x^4\right)^3\,dx&=\frac{\int x^3\left(ab+b^2x^2\right)^6\,dx}{b^6}\\&=\frac{\operatorname{Subst}\left(\int x\left(ab+b^2x\right)^6\,dx,x,x^2\right)}{2b^6}\\&=\frac{\operatorname{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^6}{b}+\frac{\left(ab+b^2x\right)^7}{b^2}\right)dx,x,x^2\right)}{2b^6}\\&=-\frac{a\left(a+bx^2\right)^7}{14b^2}+\frac{\left(a+bx^2\right)^8}{16b^2}\end{aligned}$$

Mathematica [B] time = 0.0025139, size = 77, normalized size = 2.26

$$\frac{5}{4}a^2b^4x^{12}+2a^3b^3x^{10}+\frac{15}{8}a^4b^2x^8+a^5bx^6+\frac{a^6x^4}{4}+\frac{3}{7}ab^5x^{14}+\frac{b^6x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^4)/4 + a^5*b*x^6 + (15*a^4*b^2*x^8)/8 + 2*a^3*b^3*x^10 + (5*a^2*b^4*
x^12)/4 + (3*a*b^5*x^14)/7 + (b^6*x^16)/16
```

Maple [B] time = 0.042, size = 68, normalized size = 2.

$$\frac{b^6x^{16}}{16}+\frac{3\,ab^5x^{14}}{7}+\frac{5\,a^2b^4x^{12}}{4}+2\,a^3b^3x^{10}+\frac{15\,a^4b^2x^8}{8}+a^5bx^6+\frac{x^4a^6}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/16*b^6*x^16+3/7*a*b^5*x^14+5/4*a^2*b^4*x^12+2*a^3*b^3*x^10+15/8*a^4*b^2*x
^8+a^5*b*x^6+1/4*x^4*a^6
```

Maxima [B] time = 0.983868, size = 90, normalized size = 2.65

$$\frac{1}{16}\,b^6x^{16} + \frac{3}{7}\,ab^5x^{14} + \frac{5}{4}\,a^2b^4x^{12} + 2\,a^3b^3x^{10} + \frac{15}{8}\,a^4b^2x^8 + a^5bx^6 + \frac{1}{4}\,a^6x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*a
^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

Fricas [B] time = 1.50752, size = 153, normalized size = 4.5

$$\frac{1}{16}x^{16}b^6 + \frac{3}{7}x^{14}b^5a + \frac{5}{4}x^{12}b^4a^2 + 2x^{10}b^3a^3 + \frac{15}{8}x^8b^2a^4 + x^6ba^5 + \frac{1}{4}x^4a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^6 + 3/7*x^14*b^5*a + 5/4*x^12*b^4*a^2 + 2*x^10*b^3*a^3 + 15/8*x
^8*b^2*a^4 + x^6*b*a^5 + 1/4*x^4*a^6
```

Sympy [B] time = 0.077396, size = 75, normalized size = 2.21

$$\frac{a^6x^4}{4} + a^5bx^6 + \frac{15a^4b^2x^8}{8} + 2a^3b^3x^{10} + \frac{5a^2b^4x^{12}}{4} + \frac{3ab^5x^{14}}{7} + \frac{b^6x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**4/4 + a**5*b*x**6 + 15*a**4*b**2*x**8/8 + 2*a**3*b**3*x**10 + 5*a**
2*b**4*x**12/4 + 3*a*b**5*x**14/7 + b**6*x**16/16
```

Giac [B] time = 1.15961, size = 90, normalized size = 2.65

$$\frac{1}{16}\,b^6x^{16} + \frac{3}{7}\,ab^5x^{14} + \frac{5}{4}\,a^2b^4x^{12} + 2\,a^3b^3x^{10} + \frac{15}{8}\,a^4b^2x^8 + a^5bx^6 + \frac{1}{4}\,a^6x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*a
^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

3.450 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{15}{11}a^2b^4x^{11}+\frac{20}{9}a^3b^3x^9+\frac{15}{7}a^4b^2x^7+\frac{6}{5}a^5bx^5+\frac{a^6x^3}{3}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (
15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Rubi [A] time = 0.0384399, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{11}a^2b^4x^{11}+\frac{20}{9}a^3b^3x^9+\frac{15}{7}a^4b^2x^7+\frac{6}{5}a^5bx^5+\frac{a^6x^3}{3}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (
15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{\int x^2\left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$=\frac{\int\left(a^6b^6x^2+6a^5b^7x^4+15a^4b^8x^6+20a^3b^9x^8+15a^2b^{10}x^{10}+6ab^{11}x^{12}+b^{12}x^{14}\right)dx}{b^6}$$
$$=\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Mathematica [A] time = 0.0026024, size = 82, normalized size = 1.

$$\frac{15}{11}a^2b^4x^{11}+\frac{20}{9}a^3b^3x^9+\frac{15}{7}a^4b^2x^7+\frac{6}{5}a^5bx^5+\frac{a^6x^3}{3}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (
15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Maple [A] time = 0.041, size = 69, normalized size = 0.8

$$\frac{a^6x^3}{3}+\frac{6\,a^5bx^5}{5}+\frac{15\,a^4b^2x^7}{7}+\frac{20\,a^3b^3x^9}{9}+\frac{15\,a^2b^4x^{11}}{11}+\frac{6\,ab^5x^{13}}{13}+\frac{b^6x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/3*a^6*x^3+6/5*a^5*b*x^5+15/7*a^4*b^2*x^7+20/9*a^3*b^3*x^9+15/11*a^2*b^4*x
^11+6/13*a*b^5*x^13+1/15*b^6*x^15
```

Maxima [A] time = 0.983176, size = 92, normalized size = 1.12

$$\frac{1}{15}\,b^6x^{15}+\frac{6}{13}\,ab^5x^{13}+\frac{15}{11}\,a^2b^4x^{11}+\frac{20}{9}\,a^3b^3x^9+\frac{15}{7}\,a^4b^2x^7+\frac{6}{5}\,a^5bx^5+\frac{1}{3}\,a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 + 1
5/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

Fricas [A] time = 1.43171, size = 165, normalized size = 2.01

$$\frac{1}{15}x^{15}b^6 + \frac{6}{13}x^{13}b^5a + \frac{15}{11}x^{11}b^4a^2 + \frac{20}{9}x^9b^3a^3 + \frac{15}{7}x^7b^2a^4 + \frac{6}{5}x^5ba^5 + \frac{1}{3}x^3a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^6 + 6/13*x^13*b^5*a + 15/11*x^11*b^4*a^2 + 20/9*x^9*b^3*a^3 + 1
5/7*x^7*b^2*a^4 + 6/5*x^5*b*a^5 + 1/3*x^3*a^6
```

Sympy [A] time = 0.078218, size = 80, normalized size = 0.98

$$\frac{a^6x^3}{3} + \frac{6a^5bx^5}{5} + \frac{15a^4b^2x^7}{7} + \frac{20a^3b^3x^9}{9} + \frac{15a^2b^4x^{11}}{11} + \frac{6ab^5x^{13}}{13} + \frac{b^6x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**3/3 + 6*a**5*b*x**5/5 + 15*a**4*b**2*x**7/7 + 20*a**3*b**3*x**9/9 +
 15*a**2*b**4*x**11/11 + 6*a*b**5*x**13/13 + b**6*x**15/15
```

Giac [A] time = 1.17251, size = 92, normalized size = 1.12

$$\frac{1}{15}\,b^6x^{15} + \frac{6}{13}\,ab^5x^{13} + \frac{15}{11}\,a^2b^4x^{11} + \frac{20}{9}\,a^3b^3x^9 + \frac{15}{7}\,a^4b^2x^7 + \frac{6}{5}\,a^5bx^5 + \frac{1}{3}\,a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 + 1
5/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

3.451 $\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^7}{14b}$$

```
[Out] (a + b*x^2)^7/(14*b)
```

Rubi [A] time = 0.0048827, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$\frac{\left(a+bx^2\right)^7}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a + b*x^2)^7/(14*b)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{\int x\left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$=\frac{\left(a+bx^2\right)^7}{14b}$$

Mathematica [A] time = 0.0019941, size = 16, normalized size = 1.

$$\frac{\left(a+bx^2\right)^7}{14b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a + b*x^2)^7/(14*b)
```

Maple [B] time = 0.04, size = 69, normalized size = 4.3

$$\frac{b^6x^{14}}{14}+\frac{ab^5x^{12}}{2}+\frac{3\,a^2b^4x^{10}}{2}+\frac{5\,a^3b^3x^8}{2}+\frac{5\,a^4b^2x^6}{2}+\frac{3\,a^5bx^4}{2}+\frac{x^2a^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/14*b^6*x^14+1/2*a*b^5*x^12+3/2*a^2*b^4*x^10+5/2*a^3*b^3*x^8+5/2*a^4*b^2*x
^6+3/2*a^5*b*x^4+1/2*x^2*a^6
```

Maxima [B] time = 0.993829, size = 92, normalized size = 5.75

$$\frac{1}{14}\,b^6x^{14}+\frac{1}{2}\,ab^5x^{12}+\frac{3}{2}\,a^2b^4x^{10}+\frac{5}{2}\,a^3b^3x^8+\frac{5}{2}\,a^4b^2x^6+\frac{3}{2}\,a^5bx^4+\frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*a
^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

Fricas [B] time = 1.46501, size = 158, normalized size = 9.88

$$\frac{1}{14}x^{14}b^6 + \frac{1}{2}x^{12}b^5a + \frac{3}{2}x^{10}b^4a^2 + \frac{5}{2}x^8b^3a^3 + \frac{5}{2}x^6b^2a^4 + \frac{3}{2}x^4ba^5 + \frac{1}{2}x^2a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^6 + 1/2*x^12*b^5*a + 3/2*x^10*b^4*a^2 + 5/2*x^8*b^3*a^3 + 5/2*x
^6*b^2*a^4 + 3/2*x^4*b*a^5 + 1/2*x^2*a^6
```

Sympy [B] time = 0.076982, size = 78, normalized size = 4.88

$$\frac{a^6x^2}{2} + \frac{3a^5bx^4}{2} + \frac{5a^4b^2x^6}{2} + \frac{5a^3b^3x^8}{2} + \frac{3a^2b^4x^{10}}{2} + \frac{ab^5x^{12}}{2} + \frac{b^6x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**2/2 + 3*a**5*b*x**4/2 + 5*a**4*b**2*x**6/2 + 5*a**3*b**3*x**8/2 + 3
*a**2*b**4*x**10/2 + a*b**5*x**12/2 + b**6*x**14/14
```

Giac [B] time = 1.17679, size = 92, normalized size = 5.75

$$\frac{1}{14}b^6x^{14} + \frac{1}{2}ab^5x^{12} + \frac{3}{2}a^2b^4x^{10} + \frac{5}{2}a^3b^3x^8 + \frac{5}{2}a^4b^2x^6 + \frac{3}{2}a^5bx^4 + \frac{1}{2}a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*a
^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

3.452 $\int \left(a^2+2abx^2+b^2x^4\right)^3 dx$

Optimal. Leaf size=73

$$\frac{5}{3}a^2b^4x^9+\frac{20}{7}a^3b^3x^7+3a^4b^2x^5+2a^5bx^3+a^6x+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/
3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

Rubi [A] time = 0.0343958, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {28, 194}

$$\frac{5}{3}a^2b^4x^9+\frac{20}{7}a^3b^3x^7+3a^4b^2x^5+2a^5bx^3+a^6x+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/
3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^3\,dx = \frac{\int \left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$= \frac{\int \left(a^6b^6+6a^5b^7x^2+15a^4b^8x^4+20a^3b^9x^6+15a^2b^{10}x^8+6ab^{11}x^{10}+b^{12}x^{12}\right)dx}{b^6}$$
$$= a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Mathematica [A] time = 0.0013389, size = 73, normalized size = 1.

$$\frac{5}{3}a^2b^4x^9+\frac{20}{7}a^3b^3x^7+3a^4b^2x^5+2a^5bx^3+a^6x+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/
3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

Maple [A] time = 0.04, size = 66, normalized size = 0.9

$$a^6x+2\,a^5bx^3+3\,a^4b^2x^5+\frac{20\,a^3b^3x^7}{7}+\frac{5\,a^2b^4x^9}{3}+\frac{6\,ab^5x^{11}}{11}+\frac{b^6x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] a^6*x+2*a^5*b*x^3+3*a^4*b^2*x^5+20/7*a^3*b^3*x^7+5/3*a^2*b^4*x^9+6/11*a*b^5
*x^11+1/13*b^6*x^13
```

Maxima [A] time = 0.97231, size = 135, normalized size = 1.85

$$\frac{1}{13}\,b^6x^{13}+\frac{6}{11}\,ab^5x^{11}+\frac{4}{3}\,a^2b^4x^9+\frac{8}{7}\,a^3b^3x^7+a^6x+\frac{1}{5}\left(3\,b^2x^5+10\,abx^3\right)a^4+\frac{1}{105}\left(35\,b^4x^9+180\,ab^3x^7+252\,a^2b^2x^5\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 4/3*a^2*b^4*x^9 + 8/7*a^3*b^3*x^7 + a^6*x
 + 1/5*(3*b^2*x^5 + 10*a*b*x^3)*a^4 + 1/105*(35*b^4*x^9 + 180*a*b^3*x^7 + 2
52*a^2*b^2*x^5)*a^2
```

Fricas [A] time = 1.47239, size = 146, normalized size = 2.

$$\frac{1}{13}x^{13}b^6 + \frac{6}{11}x^{11}b^5a + \frac{5}{3}x^9b^4a^2 + \frac{20}{7}x^7b^3a^3 + 3x^5b^2a^4 + 2x^3ba^5 + xa^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^6 + 6/11*x^11*b^5*a + 5/3*x^9*b^4*a^2 + 20/7*x^7*b^3*a^3 + 3*x^
5*b^2*a^4 + 2*x^3*b*a^5 + x*a^6
```

Sympy [A] time = 0.075363, size = 73, normalized size = 1.

$$a^6x + 2a^5bx^3 + 3a^4b^2x^5 + \frac{20a^3b^3x^7}{7} + \frac{5a^2b^4x^9}{3} + \frac{6ab^5x^{11}}{11} + \frac{b^6x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x + 2*a**5*b*x**3 + 3*a**4*b**2*x**5 + 20*a**3*b**3*x**7/7 + 5*a**2*b*
*4*x**9/3 + 6*a*b**5*x**11/11 + b**6*x**13/13
```

Giac [A] time = 1.15488, size = 88, normalized size = 1.21

$$\frac{1}{13}\,b^6x^{13} + \frac{6}{11}\,ab^5x^{11} + \frac{5}{3}\,a^2b^4x^9 + \frac{20}{7}\,a^3b^3x^7 + 3\,a^4b^2x^5 + 2\,a^5bx^3 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 5/3*a^2*b^4*x^9 + 20/7*a^3*b^3*x^7 + 3*a^
4*b^2*x^5 + 2*a^5*b*x^3 + a^6*x
```

3.453 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x}\,dx$

Optimal. Leaf size=76

$$\frac{15}{8}a^2b^4x^8+\frac{10}{3}a^3b^3x^6+\frac{15}{4}a^4b^2x^4+3a^5bx^2+a^6\log(x)+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
+ (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Rubi [A] time = 0.0545137, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{15}{8}a^2b^4x^8+\frac{10}{3}a^3b^3x^6+\frac{15}{4}a^4b^2x^4+3a^5bx^2+a^6\log(x)+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
+ (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(6a^5b^7+\frac{a^6b^6}{x}+15a^4b^8x+20a^3b^9x^2+15a^2b^{10}x^3+6ab^{11}x^4+b^{12}x^5\right)dx, x, x^2\right)}{2b^6}$$
$$= 3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}+a^6\log(x)$$

Mathematica [A] time = 0.0044944, size = 76, normalized size = 1.

$$\frac{15}{8}a^2b^4x^8+\frac{10}{3}a^3b^3x^6+\frac{15}{4}a^4b^2x^4+3a^5bx^2+a^6\log(x)+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
+ (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Maple [A] time = 0.043, size = 67, normalized size = 0.9

$$3\,a^5bx^2+\frac{15\,a^4b^2x^4}{4}+\frac{10\,a^3b^3x^6}{3}+\frac{15\,a^2b^4x^8}{8}+\frac{3\,ab^5x^{10}}{5}+\frac{b^6x^{12}}{12}+a^6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x,x)
```

```
[Out] 3*a^5*b*x^2+15/4*a^4*b^2*x^4+10/3*a^3*b^3*x^6+15/8*a^2*b^4*x^8+3/5*a*b^5*x^
10+1/12*b^6*x^12+a^6*ln(x)
```

Maxima [A] time = 1.01517, size = 93, normalized size = 1.22

$$\frac{1}{12}\,b^{6}x^{12}+\frac{3}{5}\,ab^{5}x^{10}+\frac{15}{8}\,a^{2}b^{4}x^{8}+\frac{10}{3}\,a^{3}b^{3}x^{6}+\frac{15}{4}\,a^{4}b^{2}x^{4}+3\,a^{5}bx^{2}+\frac{1}{2}\,a^{6}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="maxima")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

Fricas [A] time = 1.75273, size = 157, normalized size = 2.07

$$\frac{1}{12}\,b^{6}x^{12}+\frac{3}{5}\,ab^{5}x^{10}+\frac{15}{8}\,a^{2}b^{4}x^{8}+\frac{10}{3}\,a^{3}b^{3}x^{6}+\frac{15}{4}\,a^{4}b^{2}x^{4}+3\,a^{5}bx^{2}+a^{6}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6*log(x)
```

Sympy [A] time = 0.30263, size = 76, normalized size = 1.

$$a^{6}\log\left(x\right)+3a^{5}bx^{2}+\frac{15a^{4}b^{2}x^{4}}{4}+\frac{10a^{3}b^{3}x^{6}}{3}+\frac{15a^{2}b^{4}x^{8}}{8}+\frac{3ab^{5}x^{10}}{5}+\frac{b^{6}x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x,x)
```

```
[Out] a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**4/4 + 10*a**3*b**3*x**6/3 + 1
5*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x**12/12
```

Giac [A] time = 1.12874, size = 93, normalized size = 1.22

$$\frac{1}{12}\,b^{6}x^{12} + \frac{3}{5}\,ab^{5}x^{10} + \frac{15}{8}\,a^{2}b^{4}x^{8} + \frac{10}{3}\,a^{3}b^{3}x^{6} + \frac{15}{4}\,a^{4}b^{2}x^{4} + 3\,a^{5}bx^{2} + \frac{1}{2}\,a^{6}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

3.454 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2} dx$

Optimal. Leaf size=72

$$\frac{15}{7}a^2b^4x^7 + 4a^3b^3x^5 + 5a^4b^2x^3 + 6a^5bx - \frac{a^6}{x} + \frac{2}{3}ab^5x^9 + \frac{b^6x^{11}}{11}$$

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7 +
 (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Rubi [A] time = 0.0392213, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$\frac{15}{7}a^2b^4x^7 + 4a^3b^3x^5 + 5a^4b^2x^3 + 6a^5bx - \frac{a^6}{x} + \frac{2}{3}ab^5x^9 + \frac{b^6x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7 +
 (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^2}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^2}\,dx}{b^6}$$
$$= \frac{\int \left(6a^5b^7+\frac{a^6b^6}{x^2}+15a^4b^8x^2+20a^3b^9x^4+15a^2b^{10}x^6+6ab^{11}x^8+b^{12}x^{10}\right)dx}{b^6}$$
$$= -\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

Mathematica [A] time = 0.0084472, size = 72, normalized size = 1.

$$\frac{15}{7}a^2b^4x^7+4a^3b^3x^5+5a^4b^2x^3+6a^5bx-\frac{a^6}{x}+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7 +
 (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Maple [A] time = 0.045, size = 67, normalized size = 0.9

$$-\frac{a^6}{x}+6\,a^5bx+5\,a^4b^2x^3+4\,a^3b^3x^5+\frac{15\,a^2b^4x^7}{7}+\frac{2\,ab^5x^9}{3}+\frac{b^6x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x)
```

```
[Out] -a^6/x+6*a^5*b*x+5*a^4*b^2*x^3+4*a^3*b^3*x^5+15/7*a^2*b^4*x^7+2/3*a*b^5*x^9
+1/11*b^6*x^11
```

Maxima [A] time = 0.991793, size = 89, normalized size = 1.24

$$\frac{1}{11}\,b^6x^{11}+\frac{2}{3}\,ab^5x^9+\frac{15}{7}\,a^2b^4x^7+4\,a^3b^3x^5+5\,a^4b^2x^3+6\,a^5bx-\frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5 + 5*a^4*b^
2*x^3 + 6*a^5*b*x - a^6/x
```

Fricas [A] time = 1.71723, size = 165, normalized size = 2.29

$$\frac{21\,b^6x^{12} + 154\,ab^5x^{10} + 495\,a^2b^4x^8 + 924\,a^3b^3x^6 + 1155\,a^4b^2x^4 + 1386\,a^5bx^2 - 231\,a^6}{231\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/231*(21*b^6*x^12 + 154*a*b^5*x^10 + 495*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 1
155*a^4*b^2*x^4 + 1386*a^5*b*x^2 - 231*a^6)/x
```

Sympy [A] time = 0.304902, size = 70, normalized size = 0.97

$$-\frac{a^6}{x} + 6a^5bx + 5a^4b^2x^3 + 4a^3b^3x^5 + \frac{15a^2b^4x^7}{7} + \frac{2ab^5x^9}{3} + \frac{b^6x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**2,x)
```

```
[Out] -a**6/x + 6*a**5*b*x + 5*a**4*b**2*x**3 + 4*a**3*b**3*x**5 + 15*a**2*b**4*x
**7/7 + 2*a*b**5*x**9/3 + b**6*x**11/11
```

Giac [A] time = 1.14083, size = 89, normalized size = 1.24

$$\frac{1}{11}\,b^6x^{11} + \frac{2}{3}\,ab^5x^9 + \frac{15}{7}\,a^2b^4x^7 + 4\,a^3b^3x^5 + 5\,a^4b^2x^3 + 6\,a^5bx - \frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5 + 5*a^4*b^
2*x^3 + 6*a^5*b*x - a^6/x
```

3.455 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3} dx$

Optimal. Leaf size=77

$$\frac{5}{2}a^2b^4x^6+5a^3b^3x^4+\frac{15}{2}a^4b^2x^2+6a^5b\log(x)-\frac{a^6}{2x^2}+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

```
[Out] -a^6/(2*x^2) + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2 + (3*
a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Rubi [A] time = 0.0562668, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{5}{2}a^2b^4x^6+5a^3b^3x^4+\frac{15}{2}a^4b^2x^2+6a^5b\log(x)-\frac{a^6}{2x^2}+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] -a^6/(2*x^2) + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2 + (3*
a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^3}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^3}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^2}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(15a^4b^8+\frac{a^6b^6}{x^2}+\frac{6a^5b^7}{x}+20a^3b^9x+15a^2b^{10}x^2+6ab^{11}x^3+b^{12}x^4\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{2x^2}+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}+6a^5b\log(x)$$

Mathematica [A] time = 0.0075471, size = 77, normalized size = 1.

$$\frac{5}{2}a^2b^4x^6+5a^3b^3x^4+\frac{15}{2}a^4b^2x^2+6a^5b\log(x)-\frac{a^6}{2x^2}+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] -a^6/(2*x^2) + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2 + (3*
a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Maple [A] time = 0.047, size = 68, normalized size = 0.9

$$-\frac{a^6}{2\,x^2}+\frac{15\,a^4b^2x^2}{2}+5\,a^3b^3x^4+\frac{5\,a^2b^4x^6}{2}+\frac{3\,ab^5x^8}{4}+\frac{b^6x^{10}}{10}+6\,a^5b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x)
```

```
[Out] -1/2*a^6/x^2+15/2*a^4*b^2*x^2+5*a^3*b^3*x^4+5/2*a^2*b^4*x^6+3/4*a*b^5*x^8+1
/10*b^6*x^10+6*a^5*b*ln(x)
```

Maxima [A] time = 0.998797, size = 93, normalized size = 1.21

$$\frac{1}{10}\,b^6x^{10} + \frac{3}{4}\,ab^5x^8 + \frac{5}{2}\,a^2b^4x^6 + 5\,a^3b^3x^4 + \frac{15}{2}\,a^4b^2x^2 + 3\,a^5b\log\left(x^2\right) - \frac{a^6}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 + 15/2*a^4*
b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*a^6/x^2
```

Fricas [A] time = 1.60346, size = 167, normalized size = 2.17

$$\frac{2\,b^6x^{12} + 15\,ab^5x^{10} + 50\,a^2b^4x^8 + 100\,a^3b^3x^6 + 150\,a^4b^2x^4 + 120\,a^5bx^2\log\left(x\right) - 10\,a^6}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^6*x^12 + 15*a*b^5*x^10 + 50*a^2*b^4*x^8 + 100*a^3*b^3*x^6 + 150*a
^4*b^2*x^4 + 120*a^5*b*x^2*log(x) - 10*a^6)/x^2
```

Sympy [A] time = 0.342027, size = 76, normalized size = 0.99

$$-\frac{a^6}{2x^2} + 6a^5b\log\left(x\right) + \frac{15a^4b^2x^2}{2} + 5a^3b^3x^4 + \frac{5a^2b^4x^6}{2} + \frac{3ab^5x^8}{4} + \frac{b^6x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**3,x)
```

```
[Out] -a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**2*x**2/2 + 5*a**3*b**3*x**4 +
 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x**10/10
```

Giac [A] time = 1.1932, size = 107, normalized size = 1.39

$$\frac{1}{10}\,b^{6}x^{10} + \frac{3}{4}\,ab^{5}x^{8} + \frac{5}{2}\,a^{2}b^{4}x^{6} + 5\,a^{3}b^{3}x^{4} + \frac{15}{2}\,a^{4}b^{2}x^{2} + 3\,a^{5}b\log\left(x^{2}\right) - \frac{6\,a^{5}bx^{2} + a^{6}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 + 15/2*a^4*
b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*(6*a^5*b*x^2 + a^6)/x^2
```

3.456 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^4}\,dx$

Optimal. Leaf size=74

$$3a^2b^4x^5+\frac{20}{3}a^3b^3x^3+15a^4b^2x-\frac{6a^5b}{x}-\frac{a^6}{3x^3}+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

```
[Out] -a^6/(3*x^3) - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4*
x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Rubi [A] time = 0.0364336, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$3a^2b^4x^5+\frac{20}{3}a^3b^3x^3+15a^4b^2x-\frac{6a^5b}{x}-\frac{a^6}{3x^3}+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] -a^6/(3*x^3) - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4*
x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^4} dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^4} dx}{b^6}$$
$$= \frac{\int \left(15a^4b^8 + \frac{a^6b^6}{x^4} + \frac{6a^5b^7}{x^2} + 20a^3b^9x^2 + 15a^2b^{10}x^4 + 6ab^{11}x^6 + b^{12}x^8\right) dx}{b^6}$$
$$= -\frac{a^6}{3x^3} - \frac{6a^5b}{x} + 15a^4b^2x + \frac{20}{3}a^3b^3x^3 + 3a^2b^4x^5 + \frac{6}{7}ab^5x^7 + \frac{b^6x^9}{9}$$

Mathematica [A] time = 0.0089066, size = 74, normalized size = 1.

$$3a^2b^4x^5 + \frac{20}{3}a^3b^3x^3 + 15a^4b^2x - \frac{6a^5b}{x} - \frac{a^6}{3x^3} + \frac{6}{7}ab^5x^7 + \frac{b^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] -a^6/(3*x^3) - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4*
x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Maple [A] time = 0.046, size = 67, normalized size = 0.9

$$-\frac{a^6}{3x^3} - 6\frac{a^5b}{x} + 15a^4b^2x + \frac{20a^3b^3x^3}{3} + 3a^2b^4x^5 + \frac{6ab^5x^7}{7} + \frac{b^6x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x)
```

```
[Out] -1/3*a^6/x^3-6*a^5*b/x+15*a^4*b^2*x+20/3*a^3*b^3*x^3+3*a^2*b^4*x^5+6/7*a*b^
5*x^7+1/9*b^6*x^9
```

Maxima [A] time = 1.01736, size = 90, normalized size = 1.22

$$\frac{1}{9}b^6x^9 + \frac{6}{7}ab^5x^7 + 3a^2b^4x^5 + \frac{20}{3}a^3b^3x^3 + 15a^4b^2x - \frac{18a^5bx^2 + a^6}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 + 15*a^4*b^2
*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

Fricas [A] time = 1.62616, size = 159, normalized size = 2.15

$$\frac{7 b^6 x^{12} + 54 a b^5 x^{10} + 189 a^2 b^4 x^8 + 420 a^3 b^3 x^6 + 945 a^4 b^2 x^4 - 378 a^5 b x^2 - 21 a^6}{63 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^6*x^12 + 54*a*b^5*x^10 + 189*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 945*
a^4*b^2*x^4 - 378*a^5*b*x^2 - 21*a^6)/x^3
```

Sympy [A] time = 0.34225, size = 73, normalized size = 0.99

$$15a^4b^2x + \frac{20a^3b^3x^3}{3} + 3a^2b^4x^5 + \frac{6ab^5x^7}{7} + \frac{b^6x^9}{9} - \frac{a^6 + 18a^5bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**4,x)
```

```
[Out] 15*a**4*b**2*x + 20*a**3*b**3*x**3/3 + 3*a**2*b**4*x**5 + 6*a*b**5*x**7/7 +
 b**6*x**9/9 - (a**6 + 18*a**5*b*x**2)/(3*x**3)
```

Giac [A] time = 1.12572, size = 90, normalized size = 1.22

$$\frac{1}{9} b^6 x^9 + \frac{6}{7} a b^5 x^7 + 3 a^2 b^4 x^5 + \frac{20}{3} a^3 b^3 x^3 + 15 a^4 b^2 x - \frac{18 a^5 b x^2 + a^6}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 + 15*a^4*b^2
*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

3.457 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^5}\,dx$

Optimal. Leaf size=72

$$\frac{15}{4}a^2b^4x^4+10a^3b^3x^2+15a^4b^2\log(x)-\frac{3a^5b}{x^2}-\frac{a^6}{4x^4}+ab^5x^6+\frac{b^6x^8}{8}$$

```
[Out] -a^6/(4*x^4) - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/4 + a*b^5*
x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Rubi [A] time = 0.052438, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{15}{4}a^2b^4x^4+10a^3b^3x^2+15a^4b^2\log(x)-\frac{3a^5b}{x^2}-\frac{a^6}{4x^4}+ab^5x^6+\frac{b^6x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] -a^6/(4*x^4) - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/4 + a*b^5*
x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^5}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^5}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^3}\,dx,x,x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(20a^3b^9+\frac{a^6b^6}{x^3}+\frac{6a^5b^7}{x^2}+\frac{15a^4b^8}{x}+15a^2b^{10}x+6ab^{11}x^2+b^{12}x^3\right)dx,x,x^2\right)}{2b^6}$$
$$= -\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}+15a^4b^2\log(x)$$

Mathematica [A] time = 0.0048738, size = 72, normalized size = 1.

$$\frac{15}{4}a^2b^4x^4+10a^3b^3x^2+15a^4b^2\log(x)-\frac{3a^5b}{x^2}-\frac{a^6}{4x^4}+ab^5x^6+\frac{b^6x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] -a^6/(4*x^4) - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/4 + a*b^5*
x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Maple [A] time = 0.049, size = 67, normalized size = 0.9

$$-\frac{a^6}{4\,x^4}-3\,\frac{a^5b}{x^2}+10\,x^2a^3b^3+\frac{15\,a^2b^4x^4}{4}+ab^5x^6+\frac{b^6x^8}{8}+15\,a^4b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x)
```

```
[Out] -1/4*a^6/x^4-3*a^5*b/x^2+10*x^2*a^3*b^3+15/4*a^2*b^4*x^4+a*b^5*x^6+1/8*b^6*
x^8+15*a^4*b^2*ln(x)
```

Maxima [A] time = 1.01768, size = 93, normalized size = 1.29

$$\frac{1}{8}\,b^6x^8 + ab^5x^6 + \frac{15}{4}\,a^2b^4x^4 + 10\,a^3b^3x^2 + \frac{15}{2}\,a^4b^2\log\left(x^2\right) - \frac{12\,a^5bx^2 + a^6}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/2*a^4*b^2*
log(x^2) - 1/4*(12*a^5*b*x^2 + a^6)/x^4
```

Fricas [A] time = 1.71567, size = 158, normalized size = 2.19

$$\frac{b^6x^{12} + 8\,ab^5x^{10} + 30\,a^2b^4x^8 + 80\,a^3b^3x^6 + 120\,a^4b^2x^4\log\left(x\right) - 24\,a^5bx^2 - 2\,a^6}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(b^6*x^12 + 8*a*b^5*x^10 + 30*a^2*b^4*x^8 + 80*a^3*b^3*x^6 + 120*a^4*b^
2*x^4*log(x) - 24*a^5*b*x^2 - 2*a^6)/x^4
```

Sympy [A] time = 0.395663, size = 71, normalized size = 0.99

$$15a^4b^2\log\left(x\right) + 10a^3b^3x^2 + \frac{15a^2b^4x^4}{4} + ab^5x^6 + \frac{b^6x^8}{8} - \frac{a^6 + 12a^5bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**5,x)
```

```
[Out] 15*a**4*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x**6
 + b**6*x**8/8 - (a**6 + 12*a**5*b*x**2)/(4*x**4)
```

Giac [A] time = 1.14632, size = 108, normalized size = 1.5

$$\frac{1}{8}b^6x^8 + ab^5x^6 + \frac{15}{4}a^2b^4x^4 + 10a^3b^3x^2 + \frac{15}{2}a^4b^2\log\left(x^2\right) - \frac{45a^4b^2x^4 + 12a^5bx^2 + a^6}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/2*a^4*b^2*
log(x^2) - 1/4*(45*a^4*b^2*x^4 + 12*a^5*b*x^2 + a^6)/x^4
```

3.458 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^6}\,dx$

Optimal. Leaf size=72

$$5a^2b^4x^3+20a^3b^3x-\frac{15a^4b^2}{x}-\frac{2a^5b}{x^3}-\frac{a^6}{5x^5}+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

```
[Out] -a^6/(5*x^5) - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x^
3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Rubi [A] time = 0.0408074, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$5a^2b^4x^3+20a^3b^3x-\frac{15a^4b^2}{x}-\frac{2a^5b}{x^3}-\frac{a^6}{5x^5}+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```


```
[Out] -a^6/(5*x^5) - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x^
3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^6}\,dx}{b^6}$$
$$= \frac{\int \left(20a^3b^9+\frac{a^6b^6}{x^6}+\frac{6a^5b^7}{x^4}+\frac{15a^4b^8}{x^2}+15a^2b^{10}x^2+6ab^{11}x^4+b^{12}x^6\right)dx}{b^6}$$
$$= -\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Mathematica [A] time = 0.0063316, size = 72, normalized size = 1.

$$5a^2b^4x^3+20a^3b^3x-\frac{15a^4b^2}{x}-\frac{2a^5b}{x^3}-\frac{a^6}{5x^5}+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

```
[Out] -a^6/(5*x^5) - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x^
3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Maple [A] time = 0.046, size = 67, normalized size = 0.9

$$-\frac{a^6}{5\,x^5}-2\,\frac{a^5b}{x^3}-15\,\frac{a^4b^2}{x}+20\,xa^3b^3+5\,a^2b^4x^3+\frac{6\,ab^5x^5}{5}+\frac{b^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x)
```

```
[Out] -1/5*a^6/x^5-2*a^5*b/x^3-15*a^4*b^2/x+20*x*a^3*b^3+5*a^2*b^4*x^3+6/5*a*b^5*
x^5+1/7*b^6*x^7
```

Maxima [A] time = 0.987634, size = 90, normalized size = 1.25

$$\frac{1}{7}\,b^6x^7+\frac{6}{5}\,ab^5x^5+5\,a^2b^4x^3+20\,a^3b^3x-\frac{75\,a^4b^2x^4+10\,a^5bx^2+a^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*(75*a^4*b^
2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

Fricas [A] time = 1.59192, size = 157, normalized size = 2.18

$$\frac{5\,b^6x^{12} + 42\,ab^5x^{10} + 175\,a^2b^4x^8 + 700\,a^3b^3x^6 - 525\,a^4b^2x^4 - 70\,a^5bx^2 - 7\,a^6}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^6*x^12 + 42*a*b^5*x^10 + 175*a^2*b^4*x^8 + 700*a^3*b^3*x^6 - 525*
a^4*b^2*x^4 - 70*a^5*b*x^2 - 7*a^6)/x^5
```

Sympy [A] time = 0.409117, size = 71, normalized size = 0.99

$$20a^3b^3x + 5a^2b^4x^3 + \frac{6ab^5x^5}{5} + \frac{b^6x^7}{7} - \frac{a^6 + 10a^5bx^2 + 75a^4b^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**6,x)
```

```
[Out] 20*a**3*b**3*x + 5*a**2*b**4*x**3 + 6*a*b**5*x**5/5 + b**6*x**7/7 - (a**6 +
 10*a**5*b*x**2 + 75*a**4*b**2*x**4)/(5*x**5)
```

Giac [A] time = 1.17887, size = 90, normalized size = 1.25

$$\frac{1}{7}\,b^6x^7 + \frac{6}{5}\,ab^5x^5 + 5\,a^2b^4x^3 + 20\,a^3b^3x - \frac{75\,a^4b^2x^4 + 10\,a^5bx^2 + a^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*(75*a^4*b^
2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

3.459 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^7}\,dx$

Optimal. Leaf size=79

$$-\frac{15a^4b^2}{2x^2}+\frac{15}{2}a^2b^4x^2+20a^3b^3\log(x)-\frac{3a^5b}{2x^4}-\frac{a^6}{6x^6}+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

```
[Out] -a^6/(6*x^6) - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2*b^4*x^2)/
2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Rubi [A] time = 0.0505286, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{15a^4b^2}{2x^2}+\frac{15}{2}a^2b^4x^2+20a^3b^3\log(x)-\frac{3a^5b}{2x^4}-\frac{a^6}{6x^6}+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] -a^6/(6*x^6) - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2*b^4*x^2)/
2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^7}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^7}\,dx}{b^6}\\ &= \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^4}\,dx,x,x^2\right)}{2b^6}\\ &= \frac{\operatorname{Subst}\left(\int \left(15a^2b^{10}+\frac{a^6b^6}{x^4}+\frac{6a^5b^7}{x^3}+\frac{15a^4b^8}{x^2}+\frac{20a^3b^9}{x}+6ab^{11}x+b^{12}x^2\right)dx,x,x^2\right)}{2b^6}\\ &= -\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}+20a^3b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0049735, size = 79, normalized size = 1.

$$-\frac{15a^4b^2}{2x^2}+\frac{15}{2}a^2b^4x^2+20a^3b^3\log(x)-\frac{3a^5b}{2x^4}-\frac{a^6}{6x^6}+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] -a^6/(6*x^6) - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2*b^4*x^2)/
2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Maple [A] time = 0.05, size = 68, normalized size = 0.9

$$-\frac{a^6}{6\,x^6}-\frac{3\,a^5b}{2\,x^4}-\frac{15\,a^4b^2}{2\,x^2}+\frac{15\,a^2b^4x^2}{2}+\frac{3\,ab^5x^4}{2}+\frac{b^6x^6}{6}+20\,a^3b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x)
```

```
[Out] -1/6*a^6/x^6-3/2*a^5*b/x^4-15/2*a^4*b^2/x^2+15/2*a^2*b^4*x^2+3/2*a*b^5*x^4+
1/6*b^6*x^6+20*a^3*b^3*ln(x)
```

Maxima [A] time = 0.989, size = 95, normalized size = 1.2

$$\frac{1}{6}\,b^{6}x^{6}+\frac{3}{2}\,ab^{5}x^{4}+\frac{15}{2}\,a^{2}b^{4}x^{2}+10\,a^{3}b^{3}\log\left(x^{2}\right)-\frac{45\,a^{4}b^{2}x^{4}+9\,a^{5}bx^{2}+a^{6}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x^2) - 1/6*
(45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

Fricas [A] time = 1.79303, size = 154, normalized size = 1.95

$$\frac{b^{6}x^{12}+9\,ab^{5}x^{10}+45\,a^{2}b^{4}x^{8}+120\,a^{3}b^{3}x^{6}\log\left(x\right)-45\,a^{4}b^{2}x^{4}-9\,a^{5}bx^{2}-a^{6}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(b^6*x^12 + 9*a*b^5*x^10 + 45*a^2*b^4*x^8 + 120*a^3*b^3*x^6*log(x) - 45
*a^4*b^2*x^4 - 9*a^5*b*x^2 - a^6)/x^6
```

Sympy [A] time = 0.478979, size = 75, normalized size = 0.95

$$20a^{3}b^{3}\log\left(x\right)+\frac{15a^{2}b^{4}x^{2}}{2}+\frac{3ab^{5}x^{4}}{2}+\frac{b^{6}x^{6}}{6}-\frac{a^{6}+9a^{5}bx^{2}+45a^{4}b^{2}x^{4}}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**7,x)
```

```
[Out] 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2 + b**6*x**6/6 -
 (a**6 + 9*a**5*b*x**2 + 45*a**4*b**2*x**4)/(6*x**6)
```

Giac [A] time = 1.14009, size = 109, normalized size = 1.38

$$\frac{1}{6}\,b^{6}x^{6}+\frac{3}{2}\,ab^{5}x^{4}+\frac{15}{2}\,a^{2}b^{4}x^{2}+10\,a^{3}b^{3}\log\left(x^{2}\right)-\frac{110\,a^{3}b^{3}x^{6}+45\,a^{4}b^{2}x^{4}+9\,a^{5}bx^{2}+a^{6}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x^2) - 1/6*
(110*a^3*b^3*x^6 + 45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

3.460 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8} dx$

Optimal. Leaf size=72

$$-\frac{5a^4b^2}{x^3} - \frac{20a^3b^3}{x} + 15a^2b^4x - \frac{6a^5b}{5x^5} - \frac{a^6}{7x^7} + 2ab^5x^3 + \frac{b^6x^5}{5}$$

```
[Out] -a^6/(7*x^7) - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a^
2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Rubi [A] time = 0.0369146, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{5a^4b^2}{x^3} - \frac{20a^3b^3}{x} + 15a^2b^4x - \frac{6a^5b}{5x^5} - \frac{a^6}{7x^7} + 2ab^5x^3 + \frac{b^6x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] -a^6/(7*x^7) - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a^
2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^8}\,dx}{b^6}$$
$$= \frac{\int \left(15a^2b^{10}+\frac{a^6b^6}{x^8}+\frac{6a^5b^7}{x^6}+\frac{15a^4b^8}{x^4}+\frac{20a^3b^9}{x^2}+6ab^{11}x^2+b^{12}x^4\right)dx}{b^6}$$
$$= -\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

Mathematica [A] time = 0.0092379, size = 72, normalized size = 1.

$$-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x-\frac{6a^5b}{5x^5}-\frac{a^6}{7x^7}+2ab^5x^3+\frac{b^6x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] -a^6/(7*x^7) - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a^
2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Maple [A] time = 0.049, size = 67, normalized size = 0.9

$$-\frac{a^6}{7\,x^7}-\frac{6\,a^5b}{5\,x^5}-5\,\frac{a^4b^2}{x^3}-20\,\frac{a^3b^3}{x}+15\,a^2b^4x+2\,ab^5x^3+\frac{b^6x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x)
```

```
[Out] -1/7*a^6/x^7-6/5*a^5*b/x^5-5*a^4*b^2/x^3-20*a^3*b^3/x+15*a^2*b^4*x+2*a*b^5*
x^3+1/5*b^6*x^5
```

Maxima [A] time = 0.97182, size = 93, normalized size = 1.29

$$\frac{1}{5}\,b^6x^5+2\,ab^5x^3+15\,a^2b^4x-\frac{700\,a^3b^3x^6+175\,a^4b^2x^4+42\,a^5bx^2+5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6 + 175*a^4*
b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

Fricas [A] time = 1.7017, size = 157, normalized size = 2.18

$$\frac{7\,b^6x^{12} + 70\,ab^5x^{10} + 525\,a^2b^4x^8 - 700\,a^3b^3x^6 - 175\,a^4b^2x^4 - 42\,a^5bx^2 - 5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(7*b^6*x^12 + 70*a*b^5*x^10 + 525*a^2*b^4*x^8 - 700*a^3*b^3*x^6 - 175*
a^4*b^2*x^4 - 42*a^5*b*x^2 - 5*a^6)/x^7
```

Sympy [A] time = 0.470031, size = 71, normalized size = 0.99

$$15a^2b^4x + 2ab^5x^3 + \frac{b^6x^5}{5} - \frac{5a^6 + 42a^5bx^2 + 175a^4b^2x^4 + 700a^3b^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**8,x)
```

```
[Out] 15*a**2*b**4*x + 2*a*b**5*x**3 + b**6*x**5/5 - (5*a**6 + 42*a**5*b*x**2 + 1
75*a**4*b**2*x**4 + 700*a**3*b**3*x**6)/(35*x**7)
```

Giac [A] time = 1.1558, size = 93, normalized size = 1.29

$$\frac{1}{5}\,b^6x^5 + 2\,ab^5x^3 + 15\,a^2b^4x - \frac{700\,a^3b^3x^6 + 175\,a^4b^2x^4 + 42\,a^5bx^2 + 5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="giac")
```

```
[Out] 1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6 + 175*a^4*
b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

3.461 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9}\,dx$

Optimal. Leaf size=73

$$-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)-\frac{a^5b}{x^6}-\frac{a^6}{8x^8}+3ab^5x^2+\frac{b^6x^4}{4}$$

```
[Out] -a^6/(8*x^8) - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/x^2 + 3*a*
b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Rubi [A] time = 0.0520787, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)-\frac{a^5b}{x^6}-\frac{a^6}{8x^8}+3ab^5x^2+\frac{b^6x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] -a^6/(8*x^8) - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/x^2 + 3*a*
b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^9}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^9}\,dx}{b^6}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^5}\,dx, x, x^2\right)}{2b^6}$$

$$= \frac{\text{Subst}\left(\int \left(6ab^{11}+\frac{a^6b^6}{x^5}+\frac{6a^5b^7}{x^4}+\frac{15a^4b^8}{x^3}+\frac{20a^3b^9}{x^2}+\frac{15a^2b^{10}}{x}+b^{12}x\right)dx, x, x^2\right)}{2b^6}$$

$$= -\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+3ab^5x^2+\frac{b^6x^4}{4}+15a^2b^4\log(x)$$

Mathematica [A] time = 0.0080131, size = 73, normalized size = 1.

$$-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)-\frac{a^5b}{x^6}-\frac{a^6}{8x^8}+3ab^5x^2+\frac{b^6x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] -a^6/(8*x^8) - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/x^2 + 3*a*
b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Maple [A] time = 0.049, size = 68, normalized size = 0.9

$$-\frac{a^6}{8\,x^8}-\frac{a^5b}{x^6}-\frac{15\,a^4b^2}{4\,x^4}-10\,\frac{a^3b^3}{x^2}+3\,ab^5x^2+\frac{b^6x^4}{4}+15\,a^2b^4\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x)
```

```
[Out] -1/8*a^6/x^8-a^5*b/x^6-15/4*a^4*b^2/x^4-10*a^3*b^3/x^2+3*a*b^5*x^2+1/4*b^6*
x^4+15*a^2*b^4*ln(x)
```

Maxima [A] time = 0.983707, size = 95, normalized size = 1.3

$$\frac{1}{4}\,b^6x^4 + 3\,ab^5x^2 + \frac{15}{2}\,a^2b^4\log\left(x^2\right) - \frac{80\,a^3b^3x^6 + 30\,a^4b^2x^4 + 8\,a^5bx^2 + a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(80*a^3*b^3*x^6 + 3
0*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8
```

Fricas [A] time = 1.69756, size = 158, normalized size = 2.16

$$\frac{2\,b^6x^{12} + 24\,ab^5x^{10} + 120\,a^2b^4x^8\log(x) - 80\,a^3b^3x^6 - 30\,a^4b^2x^4 - 8\,a^5bx^2 - a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^6*x^12 + 24*a*b^5*x^10 + 120*a^2*b^4*x^8*log(x) - 80*a^3*b^3*x^6 -
 30*a^4*b^2*x^4 - 8*a^5*b*x^2 - a^6)/x^8
```

Sympy [A] time = 0.527583, size = 71, normalized size = 0.97

$$15a^2b^4\log(x) + 3ab^5x^2 + \frac{b^6x^4}{4} - \frac{a^6 + 8a^5bx^2 + 30a^4b^2x^4 + 80a^3b^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**9,x)
```

```
[Out] 15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6*x**4/4 - (a**6 + 8*a**5*b*x**2 +
 30*a**4*b**2*x**4 + 80*a**3*b**3*x**6)/(8*x**8)
```

Giac [A] time = 1.16452, size = 109, normalized size = 1.49

$$\frac{1}{4}\,b^6x^4+3\,ab^5x^2+\frac{15}{2}\,a^2b^4\log\left(x^2\right)-\frac{125\,a^2b^4x^8+80\,a^3b^3x^6+30\,a^4b^2x^4+8\,a^5bx^2+a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(125*a^2*b^4*x^8 +
80*a^3*b^3*x^6 + 30*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8
```

3.462 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{10}}\,dx$

Optimal. Leaf size=74

$$-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}-\frac{6a^5b}{7x^7}-\frac{a^6}{9x^9}+6ab^5x+\frac{b^6x^3}{3}$$

```
[Out] -a^6/(9*x^9) - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3) -
 (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Rubi [A] time = 0.0393374, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}-\frac{6a^5b}{7x^7}-\frac{a^6}{9x^9}+6ab^5x+\frac{b^6x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] -a^6/(9*x^9) - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3) -
 (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{10}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{10}}\,dx}{b^6}$$
$$= \frac{\int \left(6ab^{11}+\frac{a^6b^6}{x^{10}}+\frac{6a^5b^7}{x^8}+\frac{15a^4b^8}{x^6}+\frac{20a^3b^9}{x^4}+\frac{15a^2b^{10}}{x^2}+b^{12}x^2\right)dx}{b^6}$$
$$= -\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Mathematica [A] time = 0.0090245, size = 74, normalized size = 1.

$$-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}-\frac{6a^5b}{7x^7}-\frac{a^6}{9x^9}+6ab^5x+\frac{b^6x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] -a^6/(9*x^9) - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3) -
 (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Maple [A] time = 0.048, size = 67, normalized size = 0.9

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-3\frac{a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-15\frac{a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x)
```

```
[Out] -1/9*a^6/x^9-6/7*a^5*b/x^7-3*a^4*b^2/x^5-20/3*a^3*b^3/x^3-15*a^2*b^4/x+6*a*
b^5*x+1/3*b^6*x^3
```

Maxima [A] time = 0.972736, size = 93, normalized size = 1.26

$$\frac{1}{3}b^6x^3+6ab^5x-\frac{945a^2b^4x^8+420a^3b^3x^6+189a^4b^2x^4+54a^5bx^2+7a^6}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 189*a^4
*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

Fricas [A] time = 1.64801, size = 159, normalized size = 2.15

$$\frac{21\,b^6x^{12} + 378\,ab^5x^{10} - 945\,a^2b^4x^8 - 420\,a^3b^3x^6 - 189\,a^4b^2x^4 - 54\,a^5bx^2 - 7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(21*b^6*x^12 + 378*a*b^5*x^10 - 945*a^2*b^4*x^8 - 420*a^3*b^3*x^6 - 18
9*a^4*b^2*x^4 - 54*a^5*b*x^2 - 7*a^6)/x^9
```

Sympy [A] time = 0.550926, size = 71, normalized size = 0.96

$$6ab^5x + \frac{b^6x^3}{3} - \frac{7a^6 + 54a^5bx^2 + 189a^4b^2x^4 + 420a^3b^3x^6 + 945a^2b^4x^8}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**10,x)
```

```
[Out] 6*a*b**5*x + b**6*x**3/3 - (7*a**6 + 54*a**5*b*x**2 + 189*a**4*b**2*x**4 +
420*a**3*b**3*x**6 + 945*a**2*b**4*x**8)/(63*x**9)
```

Giac [A] time = 1.15072, size = 93, normalized size = 1.26

$$\frac{1}{3}\,b^6x^3 + 6\,ab^5x - \frac{945\,a^2b^4x^8 + 420\,a^3b^3x^6 + 189\,a^4b^2x^4 + 54\,a^5bx^2 + 7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 189*a^4
*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

3.463 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{11}}\,dx$

Optimal. Leaf size=77

$$-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}-\frac{3a^5b}{4x^8}-\frac{a^6}{10x^{10}}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

```
[Out] -a^6/(10*x^10) - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3*b^3)/x^4
- (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Rubi [A] time = 0.0511226, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}-\frac{3a^5b}{4x^8}-\frac{a^6}{10x^{10}}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] -a^6/(10*x^10) - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3*b^3)/x^4
- (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{11}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{11}}\,dx}{b^6}\\ &= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^6}\,dx,x,x^2\right)}{2b^6}\\ &= \frac{\text{Subst}\left(\int \left(b^{12}+\frac{a^6b^6}{x^6}+\frac{6a^5b^7}{x^5}+\frac{15a^4b^8}{x^4}+\frac{20a^3b^9}{x^3}+\frac{15a^2b^{10}}{x^2}+\frac{6ab^{11}}{x}\right)dx,x,x^2\right)}{2b^6}\\ &= -\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+\frac{b^6x^2}{2}+6ab^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.0050888, size = 77, normalized size = 1.

$$-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}-\frac{3a^5b}{4x^8}-\frac{a^6}{10x^{10}}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] -a^6/(10*x^10) - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3*b^3)/x^4
- (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Maple [A] time = 0.049, size = 68, normalized size = 0.9

$$-\frac{a^6}{10\,x^{10}}-\frac{3\,a^5b}{4\,x^8}-\frac{5\,a^4b^2}{2\,x^6}-5\,\frac{a^3b^3}{x^4}-\frac{15\,a^2b^4}{2\,x^2}+\frac{b^6x^2}{2}+6\,ab^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x)
```

```
[Out] -1/10*a^6/x^10-3/4*a^5*b/x^8-5/2*a^4*b^2/x^6-5*a^3*b^3/x^4-15/2*a^2*b^4/x^2
+1/2*b^6*x^2+6*a*b^5*ln(x)
```

Maxima [A] time = 1.00177, size = 97, normalized size = 1.26

$$\frac{1}{2}\,b^6x^2 + 3\,ab^5\log\left(x^2\right) - \frac{150\,a^2b^4x^8 + 100\,a^3b^3x^6 + 50\,a^4b^2x^4 + 15\,a^5bx^2 + 2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(150*a^2*b^4*x^8 + 100*a^3*b^3*x^6 +
50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

Fricas [A] time = 1.63156, size = 169, normalized size = 2.19

$$\frac{10\,b^6x^{12} + 120\,ab^5x^{10}\log\left(x\right) - 150\,a^2b^4x^8 - 100\,a^3b^3x^6 - 50\,a^4b^2x^4 - 15\,a^5bx^2 - 2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^6*x^12 + 120*a*b^5*x^10*log(x) - 150*a^2*b^4*x^8 - 100*a^3*b^3*x
^6 - 50*a^4*b^2*x^4 - 15*a^5*b*x^2 - 2*a^6)/x^10
```

Sympy [A] time = 0.621393, size = 73, normalized size = 0.95

$$6ab^5\log\left(x\right) + \frac{b^6x^2}{2} - \frac{2a^6 + 15a^5bx^2 + 50a^4b^2x^4 + 100a^3b^3x^6 + 150a^2b^4x^8}{20x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**11,x)
```

```
[Out] 6*a*b**5*log(x) + b**6*x**2/2 - (2*a**6 + 15*a**5*b*x**2 + 50*a**4*b**2*x**
4 + 100*a**3*b**3*x**6 + 150*a**2*b**4*x**8)/(20*x**10)
```

Giac [A] time = 1.11706, size = 109, normalized size = 1.42

$$\frac{1}{2}\,b^6x^2+3\,ab^5\log\left(x^2\right)-\frac{137\,ab^5x^{10}+150\,a^2b^4x^8+100\,a^3b^3x^6+50\,a^4b^2x^4+15\,a^5bx^2+2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(137*a*b^5*x^10 + 150*a^2*b^4*x^8 + 1
00*a^3*b^3*x^6 + 50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

3.464 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{12}}\,dx$

Optimal. Leaf size=71

$$-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{2a^5b}{3x^9}-\frac{a^6}{11x^{11}}-\frac{6ab^5}{x}+b^6x$$

```
[Out] -a^6/(11*x^11) - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^5
 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Rubi [A] time = 0.0370139, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{2a^5b}{3x^9}-\frac{a^6}{11x^{11}}-\frac{6ab^5}{x}+b^6x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] -a^6/(11*x^11) - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^5
 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{12}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{12}}\,dx}{b^6}$$
$$= \frac{\int \left(b^{12}+\frac{a^6b^6}{x^{12}}+\frac{6a^5b^7}{x^{10}}+\frac{15a^4b^8}{x^8}+\frac{20a^3b^9}{x^6}+\frac{15a^2b^{10}}{x^4}+\frac{6ab^{11}}{x^2}\right)dx}{b^6}$$
$$= -\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

Mathematica [A] time = 0.0055391, size = 71, normalized size = 1.

$$-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{2a^5b}{3x^9}-\frac{a^6}{11x^{11}}-\frac{6ab^5}{x}+b^6x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] -a^6/(11*x^11) - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^5
 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Maple [A] time = 0.049, size = 66, normalized size = 0.9

$$-\frac{a^6}{11\,x^{11}}-\frac{2\,a^5b}{3\,x^9}-\frac{15\,a^4b^2}{7\,x^7}-4\,\frac{a^3b^3}{x^5}-5\,\frac{a^2b^4}{x^3}-6\,\frac{ab^5}{x}+b^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x)
```

```
[Out] -1/11*a^6/x^11-2/3*a^5*b/x^9-15/7*a^4*b^2/x^7-4*a^3*b^3/x^5-5*a^2*b^4/x^3-6
*a*b^5/x+b^6*x
```

Maxima [A] time = 0.983274, size = 92, normalized size = 1.3

$$b^6x-\frac{1386\,ab^5x^{10}+1155\,a^2b^4x^8+924\,a^3b^3x^6+495\,a^4b^2x^4+154\,a^5bx^2+21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 495*a
^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

Fricas [A] time = 1.73128, size = 169, normalized size = 2.38

$$\frac{231\,b^6x^{12} - 1386\,ab^5x^{10} - 1155\,a^2b^4x^8 - 924\,a^3b^3x^6 - 495\,a^4b^2x^4 - 154\,a^5bx^2 - 21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/231*(231*b^6*x^12 - 1386*a*b^5*x^10 - 1155*a^2*b^4*x^8 - 924*a^3*b^3*x^6
- 495*a^4*b^2*x^4 - 154*a^5*b*x^2 - 21*a^6)/x^11
```

Sympy [A] time = 0.636993, size = 70, normalized size = 0.99

$$b^6x - \frac{21a^6 + 154a^5bx^2 + 495a^4b^2x^4 + 924a^3b^3x^6 + 1155a^2b^4x^8 + 1386ab^5x^{10}}{231x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**12,x)
```

```
[Out] b**6*x - (21*a**6 + 154*a**5*b*x**2 + 495*a**4*b**2*x**4 + 924*a**3*b**3*x*
*6 + 1155*a**2*b**4*x**8 + 1386*a*b**5*x**10)/(231*x**11)
```

Giac [A] time = 1.1375, size = 92, normalized size = 1.3

$$b^6x - \frac{1386\,ab^5x^{10} + 1155\,a^2b^4x^8 + 924\,a^3b^3x^6 + 495\,a^4b^2x^4 + 154\,a^5bx^2 + 21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="giac")
```

```
[Out] b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 495*a
^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

3.465 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{13}}\,dx$

Optimal. Leaf size=76

$$-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3a^5b}{5x^{10}}-\frac{a^6}{12x^{12}}-\frac{3ab^5}{x^2}+b^6\log(x)$$

```
[Out] -a^6/(12*x^12) - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*a^3*b^3)/(
3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log[x]
```

Rubi [A] time = 0.0486256, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3a^5b}{5x^{10}}-\frac{a^6}{12x^{12}}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

```
[Out] -a^6/(12*x^12) - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*a^3*b^3)/(
3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{13}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{13}}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^7}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^6b^6}{x^7}+\frac{6a^5b^7}{x^6}+\frac{15a^4b^8}{x^5}+\frac{20a^3b^9}{x^4}+\frac{15a^2b^{10}}{x^3}+\frac{6ab^{11}}{x^2}+\frac{b^{12}}{x}\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Mathematica [A] time = 0.004745, size = 76, normalized size = 1.

$$-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3a^5b}{5x^{10}}-\frac{a^6}{12x^{12}}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

```
[Out] -a^6/(12*x^12) - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*a^3*b^3)/(
3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log[x]
```

Maple [A] time = 0.049, size = 67, normalized size = 0.9

$$-\frac{a^6}{12\,x^{12}}-\frac{3\,a^5b}{5\,x^{10}}-\frac{15\,a^4b^2}{8\,x^8}-\frac{10\,a^3b^3}{3\,x^6}-\frac{15\,a^2b^4}{4\,x^4}-3\,\frac{ab^5}{x^2}+b^6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x)
```

```
[Out] -1/12*a^6/x^12-3/5*a^5*b/x^10-15/8*a^4*b^2/x^8-10/3*a^3*b^3/x^6-15/4*a^2*b^
4/x^4-3*a*b^5/x^2+b^6*ln(x)
```

Maxima [A] time = 0.982927, size = 97, normalized size = 1.28

$$\frac{1}{2}\,b^6\log\left(x^2\right)-\frac{360\,ab^5x^{10}+450\,a^2b^4x^8+400\,a^3b^3x^6+225\,a^4b^2x^4+72\,a^5bx^2+10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*b^6*log(x^2) - 1/120*(360*a*b^5*x^10 + 450*a^2*b^4*x^8 + 400*a^3*b^3*x^
6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

Fricas [A] time = 1.59287, size = 174, normalized size = 2.29

$$\frac{120\,b^6x^{12}\log\left(x\right)-360\,ab^5x^{10}-450\,a^2b^4x^8-400\,a^3b^3x^6-225\,a^4b^2x^4-72\,a^5bx^2-10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^6*x^12*log(x) - 360*a*b^5*x^10 - 450*a^2*b^4*x^8 - 400*a^3*b^3
*x^6 - 225*a^4*b^2*x^4 - 72*a^5*b*x^2 - 10*a^6)/x^12
```

Sympy [A] time = 0.718432, size = 71, normalized size = 0.93

$$b^6\log\left(x\right)-\frac{10a^6+72a^5bx^2+225a^4b^2x^4+400a^3b^3x^6+450a^2b^4x^8+360ab^5x^{10}}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**13,x)
```

```
[Out] b**6*log(x) - (10*a**6 + 72*a**5*b*x**2 + 225*a**4*b**2*x**4 + 400*a**3*b**
3*x**6 + 450*a**2*b**4*x**8 + 360*a*b**5*x**10)/(120*x**12)
```

Giac [A] time = 1.15707, size = 108, normalized size = 1.42

$$\frac{1}{2}\,b^{6}\log\left(x^{2}\right)-\frac{147\,b^{6}x^{12}+360\,ab^{5}x^{10}+450\,a^{2}b^{4}x^{8}+400\,a^{3}b^{3}x^{6}+225\,a^{4}b^{2}x^{4}+72\,a^{5}bx^{2}+10\,a^{6}}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*b^6*log(x^2) - 1/120*(147*b^6*x^12 + 360*a*b^5*x^10 + 450*a^2*b^4*x^8 +
 400*a^3*b^3*x^6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

3.466 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{14}}\,dx$

Optimal. Leaf size=76

$$-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{6a^5b}{11x^{11}}-\frac{a^6}{13x^{13}}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

```
[Out] -a^6/(13*x^13) - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/(
7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Rubi [A] time = 0.0401458, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{6a^5b}{11x^{11}}-\frac{a^6}{13x^{13}}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] -a^6/(13*x^13) - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/(
7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{14}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{14}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{x^{14}}+\frac{6a^5b^7}{x^{12}}+\frac{15a^4b^8}{x^{10}}+\frac{20a^3b^9}{x^8}+\frac{15a^2b^{10}}{x^6}+\frac{6ab^{11}}{x^4}+\frac{b^{12}}{x^2}\right)dx}{b^6}$$
$$= -\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Mathematica [A] time = 0.0089975, size = 76, normalized size = 1.

$$-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{6a^5b}{11x^{11}}-\frac{a^6}{13x^{13}}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] -a^6/(13*x^13) - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/(
7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Maple [A] time = 0.048, size = 69, normalized size = 0.9

$$-\frac{a^6}{13\,x^{13}}-\frac{6\,a^5b}{11\,x^{11}}-\frac{5\,a^4b^2}{3\,x^9}-\frac{20\,a^3b^3}{7\,x^7}-3\,\frac{a^2b^4}{x^5}-2\,\frac{ab^5}{x^3}-\frac{b^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x)
```

```
[Out] -1/13*a^6/x^13-6/11*a^5*b/x^11-5/3*a^4*b^2/x^9-20/7*a^3*b^3/x^7-3*a^2*b^4/x
^5-2*a*b^5/x^3-b^6/x
```

Maxima [A] time = 1.01139, size = 95, normalized size = 1.25

$$-\frac{3003\,b^6x^{12}+6006\,ab^5x^{10}+9009\,a^2b^4x^8+8580\,a^3b^3x^6+5005\,a^4b^2x^4+1638\,a^5bx^2+231\,a^6}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

Fricas [A] time = 1.61484, size = 178, normalized size = 2.34

$$-\frac{3003\,b^{6}x^{12} + 6006\,ab^{5}x^{10} + 9009\,a^{2}b^{4}x^{8} + 8580\,a^{3}b^{3}x^{6} + 5005\,a^{4}b^{2}x^{4} + 1638\,a^{5}bx^{2} + 231\,a^{6}}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

Sympy [A] time = 0.736059, size = 75, normalized size = 0.99

$$-\frac{231a^{6} + 1638a^{5}bx^{2} + 5005a^{4}b^{2}x^{4} + 8580a^{3}b^{3}x^{6} + 9009a^{2}b^{4}x^{8} + 6006ab^{5}x^{10} + 3003b^{6}x^{12}}{3003x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**14,x)
```

```
[Out] -(231*a**6 + 1638*a**5*b*x**2 + 5005*a**4*b**2*x**4 + 8580*a**3*b**3*x**6 +
 9009*a**2*b**4*x**8 + 6006*a*b**5*x**10 + 3003*b**6*x**12)/(3003*x**13)
```

Giac [A] time = 1.13566, size = 95, normalized size = 1.25

$$-\frac{3003\,b^{6}x^{12} + 6006\,ab^{5}x^{10} + 9009\,a^{2}b^{4}x^{8} + 8580\,a^{3}b^{3}x^{6} + 5005\,a^{4}b^{2}x^{4} + 1638\,a^{5}bx^{2} + 231\,a^{6}}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="giac")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

3.467 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{15}}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(a+bx^2\right)^7}{14ax^{14}}$$

```
[Out] -(a + b*x^2)^7/(14*a*x^14)
```

Rubi [A] time = 0.0066284, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$-\frac{\left(a+bx^2\right)^7}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

```
[Out] -(a + b*x^2)^7/(14*a*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{15}}\,dx}{b^6}$$
$$= -\frac{(a+bx^2)^7}{14ax^{14}}$$

Mathematica [B] time = 0.007406, size = 82, normalized size = 4.32

$$-\frac{3a^4b^2}{2x^{10}}-\frac{5a^3b^3}{2x^8}-\frac{5a^2b^4}{2x^6}-\frac{a^5b}{2x^{12}}-\frac{a^6}{14x^{14}}-\frac{3ab^5}{2x^4}-\frac{b^6}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

```
[Out] -a^6/(14*x^14) - (a^5*b)/(2*x^12) - (3*a^4*b^2)/(2*x^10) - (5*a^3*b^3)/(2*x
^8) - (5*a^2*b^4)/(2*x^6) - (3*a*b^5)/(2*x^4) - b^6/(2*x^2)
```

Maple [B] time = 0.048, size = 69, normalized size = 3.6

$$-\frac{a^5b}{2\,x^{12}}-\frac{3\,a^4b^2}{2\,x^{10}}-\frac{3\,ab^5}{2\,x^4}-\frac{a^6}{14\,x^{14}}-\frac{b^6}{2\,x^2}-\frac{5\,a^3b^3}{2\,x^8}-\frac{5\,a^2b^4}{2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x)
```

```
[Out] -1/2*a^5*b/x^12-3/2*a^4*b^2/x^10-3/2*a*b^5/x^4-1/14*a^6/x^14-1/2*b^6/x^2-5/
2*a^3*b^3/x^8-5/2*a^2*b^4/x^6
```

Maxima [B] time = 0.9972, size = 92, normalized size = 4.84

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

Fricas [B] time = 1.68576, size = 151, normalized size = 7.95

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

Sympy [B] time = 0.757519, size = 73, normalized size = 3.84

$$-\frac{a^6+7a^5bx^2+21a^4b^2x^4+35a^3b^3x^6+35a^2b^4x^8+21ab^5x^{10}+7b^6x^{12}}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**15,x)
```

```
[Out] -(a**6 + 7*a**5*b*x**2 + 21*a**4*b**2*x**4 + 35*a**3*b**3*x**6 + 35*a**2*b*
*4*x**8 + 21*a*b**5*x**10 + 7*b**6*x**12)/(14*x**14)
```

Giac [B] time = 1.17481, size = 92, normalized size = 4.84

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="giac")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

3.468 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{16}} \, dx$

Optimal. Leaf size=82

$$-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6a^5b}{13x^{13}}-\frac{a^6}{15x^{15}}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

```
[Out] -a^6/(15*x^15) - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^3
)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

Rubi [A] time = 0.0383924, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6a^5b}{13x^{13}}-\frac{a^6}{15x^{15}}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] -a^6/(15*x^15) - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^3
)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{16}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{16}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{x^{16}}+\frac{6a^5b^7}{x^{14}}+\frac{15a^4b^8}{x^{12}}+\frac{20a^3b^9}{x^{10}}+\frac{15a^2b^{10}}{x^8}+\frac{6ab^{11}}{x^6}+\frac{b^{12}}{x^4}\right)dx}{b^6}$$
$$= -\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Mathematica [A] time = 0.0095794, size = 82, normalized size = 1.

$$-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6a^5b}{13x^{13}}-\frac{a^6}{15x^{15}}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] -a^6/(15*x^15) - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^3
)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

Maple [A] time = 0.05, size = 69, normalized size = 0.8

$$-\frac{a^6}{15\,x^{15}}-\frac{6\,a^5b}{13\,x^{13}}-\frac{15\,a^4b^2}{11\,x^{11}}-\frac{20\,a^3b^3}{9\,x^9}-\frac{15\,a^2b^4}{7\,x^7}-\frac{6\,ab^5}{5\,x^5}-\frac{b^6}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x)
```

```
[Out] -1/15*a^6/x^15-6/13*a^5*b/x^13-15/11*a^4*b^2/x^11-20/9*a^3*b^3/x^9-15/7*a^2
*b^4/x^7-6/5*a*b^5/x^5-1/3*b^6/x^3
```

Maxima [A] time = 1.01108, size = 95, normalized size = 1.16

$$-\frac{15015\,b^6x^{12}+54054\,ab^5x^{10}+96525\,a^2b^4x^8+100100\,a^3b^3x^6+61425\,a^4b^2x^4+20790\,a^5bx^2+3003\,a^6}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

Fricas [A] time = 1.6635, size = 190, normalized size = 2.32

$$-\frac{15015\,b^{6}x^{12}+54054\,ab^{5}x^{10}+96525\,a^{2}b^{4}x^{8}+100100\,a^{3}b^{3}x^{6}+61425\,a^{4}b^{2}x^{4}+20790\,a^{5}bx^{2}+3003\,a^{6}}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

Sympy [A] time = 0.750857, size = 75, normalized size = 0.91

$$-\frac{3003a^{6}+20790a^{5}bx^{2}+61425a^{4}b^{2}x^{4}+100100a^{3}b^{3}x^{6}+96525a^{2}b^{4}x^{8}+54054ab^{5}x^{10}+15015b^{6}x^{12}}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**16,x)
```

```
[Out] -(3003*a**6 + 20790*a**5*b*x**2 + 61425*a**4*b**2*x**4 + 100100*a**3*b**3*x
**6 + 96525*a**2*b**4*x**8 + 54054*a*b**5*x**10 + 15015*b**6*x**12)/(45045*
x**15)
```

Giac [A] time = 1.18698, size = 95, normalized size = 1.16

$$-\frac{15015\,b^{6}x^{12}+54054\,ab^{5}x^{10}+96525\,a^{2}b^{4}x^{8}+100100\,a^{3}b^{3}x^{6}+61425\,a^{4}b^{2}x^{4}+20790\,a^{5}bx^{2}+3003\,a^{6}}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

3.469 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{17}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^2\right)^7}{112a^2x^{14}}-\frac{\left(a+bx^2\right)^7}{16ax^{16}}$$

```
[Out] -(a + b*x^2)^7/(16*a*x^16) + (b*(a + b*x^2)^7)/(112*a^2*x^14)
```

Rubi [A] time = 0.0251543, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{b\left(a+bx^2\right)^7}{112a^2x^{14}}-\frac{\left(a+bx^2\right)^7}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] -(a + b*x^2)^7/(16*a*x^16) + (b*(a + b*x^2)^7)/(112*a^2*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
```

```
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{17}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{17}}\,dx}{b^6}\\
&= \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^9}\,dx, x, x^2\right)}{2b^6}\\
&= -\frac{\left(a+bx^2\right)^7}{16ax^{16}} - \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^8}\,dx, x, x^2\right)}{16ab^5}\\
&= -\frac{\left(a+bx^2\right)^7}{16ax^{16}} + \frac{b\left(a+bx^2\right)^7}{112a^2x^{14}}
\end{aligned}
$$

Mathematica [A] time = 0.0046634, size = 78, normalized size = 1.95

$$
-\frac{5a^4b^2}{4x^{12}} - \frac{2a^3b^3}{x^{10}} - \frac{15a^2b^4}{8x^8} - \frac{3a^5b}{7x^{14}} - \frac{a^6}{16x^{16}} - \frac{ab^5}{x^6} - \frac{b^6}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] -a^6/(16*x^16) - (3*a^5*b)/(7*x^14) - (5*a^4*b^2)/(4*x^12) - (2*a^3*b^3)/x^
10 - (15*a^2*b^4)/(8*x^8) - (a*b^5)/x^6 - b^6/(4*x^4)
```

Maple [A] time = 0.053, size = 69, normalized size = 1.7

$$-\frac{a^{6}}{16 x^{16}}-\frac{b^{6}}{4 x^{4}}-\frac{15 a^{2} b^{4}}{8 x^{8}}-\frac{3 a^{5} b}{7 x^{14}}-\frac{a b^{5}}{x^{6}}-2 \frac{a^{3} b^{3}}{x^{10}}-\frac{5 a^{4} b^{2}}{4 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x)
```

```
[Out] -1/16*a^6/x^16-1/4*b^6/x^4-15/8*a^2*b^4/x^8-3/7*a^5*b/x^14-a*b^5/x^6-2*a^3*
b^3/x^10-5/4*a^4*b^2/x^12
```

Maxima [A] time = 0.975522, size = 95, normalized size = 2.38

$$-\frac{28 b^{6} x^{12}+112 a b^{5} x^{10}+210 a^{2} b^{4} x^{8}+224 a^{3} b^{3} x^{6}+140 a^{4} b^{2} x^{4}+48 a^{5} b x^{2}+7 a^{6}}{112 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

Fricas [A] time = 1.72615, size = 163, normalized size = 4.08

$$-\frac{28 b^{6} x^{12}+112 a b^{5} x^{10}+210 a^{2} b^{4} x^{8}+224 a^{3} b^{3} x^{6}+140 a^{4} b^{2} x^{4}+48 a^{5} b x^{2}+7 a^{6}}{112 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

Sympy [B] time = 0.81678, size = 75, normalized size = 1.88

$$-\frac{7a^6+48a^5bx^2+140a^4b^2x^4+224a^3b^3x^6+210a^2b^4x^8+112ab^5x^{10}+28b^6x^{12}}{112x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**17,x)
```

```
[Out] -(7*a**6 + 48*a**5*b*x**2 + 140*a**4*b**2*x**4 + 224*a**3*b**3*x**6 + 210*a
**2*b**4*x**8 + 112*a*b**5*x**10 + 28*b**6*x**12)/(112*x**16)
```

Giac [A] time = 1.12667, size = 95, normalized size = 2.38

$$-\frac{28\,b^6x^{12}+112\,ab^5x^{10}+210\,a^2b^4x^8+224\,a^3b^3x^6+140\,a^4b^2x^4+48\,a^5bx^2+7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

3.470 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{18}}\,dx$

Optimal. Leaf size=82

$$-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{2a^5b}{5x^{15}}-\frac{a^6}{17x^{17}}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

```
[Out] -a^6/(17*x^17) - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (20*a^3*b^3)
/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) - b^6/(5*x^5)
```

Rubi [A] time = 0.0377724, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{2a^5b}{5x^{15}}-\frac{a^6}{17x^{17}}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] -a^6/(17*x^17) - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (20*a^3*b^3)
/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) - b^6/(5*x^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{18}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{18}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{x^{18}}+\frac{6a^5b^7}{x^{16}}+\frac{15a^4b^8}{x^{14}}+\frac{20a^3b^9}{x^{12}}+\frac{15a^2b^{10}}{x^{10}}+\frac{6ab^{11}}{x^8}+\frac{b^{12}}{x^6}\right)dx}{b^6}$$
$$= -\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Mathematica [A] time = 0.0068018, size = 82, normalized size = 1.

$$-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{2a^5b}{5x^{15}}-\frac{a^6}{17x^{17}}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] -a^6/(17*x^17) - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (20*a^3*b^3)
/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) - b^6/(5*x^5)
```

Maple [A] time = 0.049, size = 69, normalized size = 0.8

$$-\frac{a^6}{17\,x^{17}}-\frac{2\,a^5b}{5\,x^{15}}-\frac{15\,a^4b^2}{13\,x^{13}}-\frac{20\,a^3b^3}{11\,x^{11}}-\frac{5\,a^2b^4}{3\,x^9}-\frac{6\,ab^5}{7\,x^7}-\frac{b^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x)
```

```
[Out] -1/17*a^6/x^17-2/5*a^5*b/x^15-15/13*a^4*b^2/x^13-20/11*a^3*b^3/x^11-5/3*a^2
*b^4/x^9-6/7*a*b^5/x^7-1/5*b^6/x^5
```

Maxima [A] time = 0.999607, size = 95, normalized size = 1.16

$$-\frac{51051\,b^6x^{12}+218790\,ab^5x^{10}+425425\,a^2b^4x^8+464100\,a^3b^3x^6+294525\,a^4b^2x^4+102102\,a^5bx^2+15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="maxima")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

Fricas [A] time = 1.58771, size = 198, normalized size = 2.41

$$-\frac{51051\,b^{6}x^{12}+218790\,ab^{5}x^{10}+425425\,a^{2}b^{4}x^{8}+464100\,a^{3}b^{3}x^{6}+294525\,a^{4}b^{2}x^{4}+102102\,a^{5}bx^{2}+15015\,a^{6}}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="fricas")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

Sympy [A] time = 0.793198, size = 75, normalized size = 0.91

$$-\frac{15015a^{6}+102102a^{5}bx^{2}+294525a^{4}b^{2}x^{4}+464100a^{3}b^{3}x^{6}+425425a^{2}b^{4}x^{8}+218790ab^{5}x^{10}+51051b^{6}x^{12}}{255255x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**18,x)
```

```
[Out] -(15015*a**6 + 102102*a**5*b*x**2 + 294525*a**4*b**2*x**4 + 464100*a**3*b**
3*x**6 + 425425*a**2*b**4*x**8 + 218790*a*b**5*x**10 + 51051*b**6*x**12)/(2
55255*x**17)
```

Giac [A] time = 1.11932, size = 95, normalized size = 1.16

$$-\frac{51051\,b^{6}x^{12}+218790\,ab^{5}x^{10}+425425\,a^{2}b^{4}x^{8}+464100\,a^{3}b^{3}x^{6}+294525\,a^{4}b^{2}x^{4}+102102\,a^{5}bx^{2}+15015\,a^{6}}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="giac")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

3.471 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx$

Optimal. Leaf size=62

$$-\frac{b^2(a+bx^2)^7}{504a^3x^{14}}+\frac{b(a+bx^2)^7}{72a^2x^{16}}-\frac{(a+bx^2)^7}{18ax^{18}}$$

```
[Out] -(a + b*x^2)^7/(18*a*x^18) + (b*(a + b*x^2)^7)/(72*a^2*x^16) - (b^2*(a + b*
x^2)^7)/(504*a^3*x^14)
```

Rubi [A] time = 0.0386362, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$-\frac{b^2(a+bx^2)^7}{504a^3x^{14}}+\frac{b(a+bx^2)^7}{72a^2x^{16}}-\frac{(a+bx^2)^7}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] -(a + b*x^2)^7/(18*a*x^18) + (b*(a + b*x^2)^7)/(72*a^2*x^16) - (b^2*(a + b*
x^2)^7)/(504*a^3*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
```

```
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{19}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{19}}\,dx}{b^6}\\
&= \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^{10}}\,dx, x, x^2\right)}{2b^6}\\
&= -\frac{\left(a+bx^2\right)^7}{18ax^{18}} - \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^9}\,dx, x, x^2\right)}{9ab^5}\\
&= -\frac{\left(a+bx^2\right)^7}{18ax^{18}} + \frac{b\left(a+bx^2\right)^7}{72a^2x^{16}} + \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^8}\,dx, x, x^2\right)}{72a^2b^4}\\
&= -\frac{\left(a+bx^2\right)^7}{18ax^{18}} + \frac{b\left(a+bx^2\right)^7}{72a^2x^{16}} - \frac{b^2\left(a+bx^2\right)^7}{504a^3x^{14}}
\end{aligned}
$$

Mathematica [A] time = 0.0043377, size = 82, normalized size = 1.32

$$
-\frac{15a^4b^2}{14x^{14}} - \frac{5a^3b^3}{3x^{12}} - \frac{3a^2b^4}{2x^{10}} - \frac{3a^5b}{8x^{16}} - \frac{a^6}{18x^{18}} - \frac{3ab^5}{4x^8} - \frac{b^6}{6x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] -a^6/(18*x^18) - (3*a^5*b)/(8*x^16) - (15*a^4*b^2)/(14*x^14) - (5*a^3*b^3)/
(3*x^12) - (3*a^2*b^4)/(2*x^10) - (3*a*b^5)/(4*x^8) - b^6/(6*x^6)
```

Maple [A] time = 0.049, size = 69, normalized size = 1.1

$$-\frac{a^{6}}{18\,x^{18}}-\frac{3\,a^{5}b}{8\,x^{16}}-\frac{15\,a^{4}b^{2}}{14\,x^{14}}-\frac{3\,a^{2}b^{4}}{2\,x^{10}}-\frac{3\,ab^{5}}{4\,x^{8}}-\frac{b^{6}}{6\,x^{6}}-\frac{5\,a^{3}b^{3}}{3\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x)
```

```
[Out] -1/18*a^6/x^18-3/8*a^5*b/x^16-15/14*a^4*b^2/x^14-3/2*a^2*b^4/x^10-3/4*a*b^5
/x^8-1/6*b^6/x^6-5/3*a^3*b^3/x^12
```

Maxima [A] time = 0.977026, size = 95, normalized size = 1.53

$$-\frac{84\,b^{6}x^{12}+378\,ab^{5}x^{10}+756\,a^{2}b^{4}x^{8}+840\,a^{3}b^{3}x^{6}+540\,a^{4}b^{2}x^{4}+189\,a^{5}bx^{2}+28\,a^{6}}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

Fricas [A] time = 1.65314, size = 166, normalized size = 2.68

$$-\frac{84\,b^{6}x^{12}+378\,ab^{5}x^{10}+756\,a^{2}b^{4}x^{8}+840\,a^{3}b^{3}x^{6}+540\,a^{4}b^{2}x^{4}+189\,a^{5}bx^{2}+28\,a^{6}}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

Sympy [A] time = 0.861803, size = 75, normalized size = 1.21

$$-\frac{28a^6+189a^5bx^2+540a^4b^2x^4+840a^3b^3x^6+756a^2b^4x^8+378ab^5x^{10}+84b^6x^{12}}{504x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**19,x)
```

```
[Out] -(28*a**6 + 189*a**5*b*x**2 + 540*a**4*b**2*x**4 + 840*a**3*b**3*x**6 + 756
*a**2*b**4*x**8 + 378*a*b**5*x**10 + 84*b**6*x**12)/(504*x**18)
```

Giac [A] time = 1.12034, size = 95, normalized size = 1.53

$$-\frac{84\,b^6x^{12}+378\,ab^5x^{10}+756\,a^2b^4x^8+840\,a^3b^3x^6+540\,a^4b^2x^4+189\,a^5bx^2+28\,a^6}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

3.472 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{20}} dx$

Optimal. Leaf size=80

$$-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{6a^5b}{17x^{17}}-\frac{a^6}{19x^{19}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

```
[Out] -a^6/(19*x^19) - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b^3)/(13*x^
13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/(7*x^7)
```

Rubi [A] time = 0.0375387, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{6a^5b}{17x^{17}}-\frac{a^6}{19x^{19}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] -a^6/(19*x^19) - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b^3)/(13*x^
13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/(7*x^7)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{20}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{20}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{x^{20}}+\frac{6a^5b^7}{x^{18}}+\frac{15a^4b^8}{x^{16}}+\frac{20a^3b^9}{x^{14}}+\frac{15a^2b^{10}}{x^{12}}+\frac{6ab^{11}}{x^{10}}+\frac{b^{12}}{x^8}\right)dx}{b^6}$$
$$= -\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Mathematica [A] time = 0.0095767, size = 80, normalized size = 1.

$$-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{6a^5b}{17x^{17}}-\frac{a^6}{19x^{19}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] -a^6/(19*x^19) - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b^3)/(13*x^
13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/(7*x^7)
```

Maple [A] time = 0.047, size = 69, normalized size = 0.9

$$-\frac{a^6}{19\,x^{19}}-\frac{6\,a^5b}{17\,x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20\,a^3b^3}{13\,x^{13}}-\frac{15\,a^2b^4}{11\,x^{11}}-\frac{2\,ab^5}{3\,x^9}-\frac{b^6}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x)
```

```
[Out] -1/19*a^6/x^19-6/17*a^5*b/x^17-a^4*b^2/x^15-20/13*a^3*b^3/x^13-15/11*a^2*b^
4/x^11-2/3*a*b^5/x^9-1/7*b^6/x^7
```

Maxima [A] time = 0.999935, size = 95, normalized size = 1.19

$$-\frac{138567\,b^6x^{12}+646646\,ab^5x^{10}+1322685\,a^2b^4x^8+1492260\,a^3b^3x^6+969969\,a^4b^2x^4+342342\,a^5bx^2+51051\,a^6}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="maxima")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

Fricas [A] time = 1.63399, size = 203, normalized size = 2.54

$$-\frac{138567\, b^{6} x^{12} + 646646\, a b^{5} x^{10} + 1322685\, a^{2} b^{4} x^{8} + 1492260\, a^{3} b^{3} x^{6} + 969969\, a^{4} b^{2} x^{4} + 342342\, a^{5} b x^{2} + 51051\, a^{6}}{969969\, x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="fricas")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

Sympy [A] time = 0.862294, size = 75, normalized size = 0.94

$$-\frac{51051a^{6} + 342342a^{5}bx^{2} + 969969a^{4}b^{2}x^{4} + 1492260a^{3}b^{3}x^{6} + 1322685a^{2}b^{4}x^{8} + 646646ab^{5}x^{10} + 138567b^{6}x^{12}}{969969x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**20,x)
```

```
[Out] -(51051*a**6 + 342342*a**5*b*x**2 + 969969*a**4*b**2*x**4 + 1492260*a**3*b*
*3*x**6 + 1322685*a**2*b**4*x**8 + 646646*a*b**5*x**10 + 138567*b**6*x**12)
/(969969*x**19)
```

Giac [A] time = 1.14533, size = 95, normalized size = 1.19

$$-\frac{138567\, b^{6} x^{12} + 646646\, a b^{5} x^{10} + 1322685\, a^{2} b^{4} x^{8} + 1492260\, a^{3} b^{3} x^{6} + 969969\, a^{4} b^{2} x^{4} + 342342\, a^{5} b x^{2} + 51051\, a^{6}}{969969\, x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="giac")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

3.473 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{21}}\,dx$

Optimal. Leaf size=84

$$\frac{b^3(a+bx^2)^7}{1680a^4x^{14}}-\frac{b^2(a+bx^2)^7}{240a^3x^{16}}+\frac{b(a+bx^2)^7}{60a^2x^{18}}-\frac{(a+bx^2)^7}{20ax^{20}}$$

```
[Out] -(a + b*x^2)^7/(20*a*x^20) + (b*(a + b*x^2)^7)/(60*a^2*x^18) - (b^2*(a + b*
x^2)^7)/(240*a^3*x^16) + (b^3*(a + b*x^2)^7)/(1680*a^4*x^14)
```

Rubi [A] time = 0.056049, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{b^3(a+bx^2)^7}{1680a^4x^{14}}-\frac{b^2(a+bx^2)^7}{240a^3x^{16}}+\frac{b(a+bx^2)^7}{60a^2x^{18}}-\frac{(a+bx^2)^7}{20ax^{20}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] -(a + b*x^2)^7/(20*a*x^20) + (b*(a + b*x^2)^7)/(60*a^2*x^18) - (b^2*(a + b*
x^2)^7)/(240*a^3*x^16) + (b^3*(a + b*x^2)^7)/(1680*a^4*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
```

```
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{21}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{21}}\,dx}{b^6}\\
&= \frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^{11}}\,dx,x,x^2\right)}{2b^6}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}-\frac{3\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^{10}}\,dx,x,x^2\right)}{20ab^5}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}+\frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^9}\,dx,x,x^2\right)}{30a^2b^4}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}-\frac{\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x^8}\,dx,x,x^2\right)}{240a^3b^3}\\
&= -\frac{\left(a+bx^2\right)^7}{20ax^{20}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}+\frac{b^3\left(a+bx^2\right)^7}{1680a^4x^{14}}
\end{aligned}$$

Mathematica [A] time = 0.0078792, size = 82, normalized size = 0.98

$$-\frac{15a^4b^2}{16x^{16}}-\frac{10a^3b^3}{7x^{14}}-\frac{5a^2b^4}{4x^{12}}-\frac{a^5b}{3x^{18}}-\frac{a^6}{20x^{20}}-\frac{3ab^5}{5x^{10}}-\frac{b^6}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] -a^6/(20*x^20) - (a^5*b)/(3*x^18) - (15*a^4*b^2)/(16*x^16) - (10*a^3*b^3)/(
7*x^14) - (5*a^2*b^4)/(4*x^12) - (3*a*b^5)/(5*x^10) - b^6/(8*x^8)
```

Maple [A] time = 0.048, size = 69, normalized size = 0.8

$$-\frac{a^5 b}{3\,x^{18}}-\frac{15\,a^4 b^2}{16\,x^{16}}-\frac{5\,a^2 b^4}{4\,x^{12}}-\frac{3\,a b^5}{5\,x^{10}}-\frac{a^6}{20\,x^{20}}-\frac{10\,a^3 b^3}{7\,x^{14}}-\frac{b^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x)
```

```
[Out] -1/3*a^5*b/x^18-15/16*a^4*b^2/x^16-5/4*a^2*b^4/x^12-3/5*a*b^5/x^10-1/20*a^6
/x^20-10/7*a^3*b^3/x^14-1/8*b^6/x^8
```

Maxima [A] time = 1.0085, size = 95, normalized size = 1.13

$$-\frac{210\,b^6 x^{12}+1008\,a b^5 x^{10}+2100\,a^2 b^4 x^8+2400\,a^3 b^3 x^6+1575\,a^4 b^2 x^4+560\,a^5 b x^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="maxima")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

Fricas [A] time = 1.69188, size = 174, normalized size = 2.07

$$-\frac{210\,b^6 x^{12}+1008\,a b^5 x^{10}+2100\,a^2 b^4 x^8+2400\,a^3 b^3 x^6+1575\,a^4 b^2 x^4+560\,a^5 b x^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="fricas")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

Sympy [A] time = 0.902203, size = 75, normalized size = 0.89

$$-\frac{84a^6+560a^5bx^2+1575a^4b^2x^4+2400a^3b^3x^6+2100a^2b^4x^8+1008ab^5x^{10}+210b^6x^{12}}{1680x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**21,x)
```

```
[Out] -(84*a**6 + 560*a**5*b*x**2 + 1575*a**4*b**2*x**4 + 2400*a**3*b**3*x**6 + 2
100*a**2*b**4*x**8 + 1008*a*b**5*x**10 + 210*b**6*x**12)/(1680*x**20)
```

Giac [A] time = 1.14007, size = 95, normalized size = 1.13

$$-\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="giac")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

3.474 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{22}}\,dx$

Optimal. Leaf size=82

$$-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6a^5b}{19x^{19}}-\frac{a^6}{21x^{21}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

```
[Out] -a^6/(21*x^21) - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (4*a^3*b^3)
/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11) - b^6/(9*x^9)
```

Rubi [A] time = 0.0397837, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 270}

$$-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6a^5b}{19x^{19}}-\frac{a^6}{21x^{21}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] -a^6/(21*x^21) - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (4*a^3*b^3)
/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11) - b^6/(9*x^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{22}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{22}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{x^{22}}+\frac{6a^5b^7}{x^{20}}+\frac{15a^4b^8}{x^{18}}+\frac{20a^3b^9}{x^{16}}+\frac{15a^2b^{10}}{x^{14}}+\frac{6ab^{11}}{x^{12}}+\frac{b^{12}}{x^{10}}\right)dx}{b^6}$$
$$= -\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Mathematica [A] time = 0.0097603, size = 82, normalized size = 1.

$$-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6a^5b}{19x^{19}}-\frac{a^6}{21x^{21}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] -a^6/(21*x^21) - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (4*a^3*b^3)
/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11) - b^6/(9*x^9)
```

Maple [A] time = 0.047, size = 69, normalized size = 0.8

$$-\frac{a^6}{21\,x^{21}}-\frac{6\,a^5b}{19\,x^{19}}-\frac{15\,a^4b^2}{17\,x^{17}}-\frac{4\,a^3b^3}{3\,x^{15}}-\frac{15\,a^2b^4}{13\,x^{13}}-\frac{6\,ab^5}{11\,x^{11}}-\frac{b^6}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x)
```

```
[Out] -1/21*a^6/x^21-6/19*a^5*b/x^19-15/17*a^4*b^2/x^17-4/3*a^3*b^3/x^15-15/13*a^
2*b^4/x^13-6/11*a*b^5/x^11-1/9*b^6/x^9
```

Maxima [A] time = 1.23608, size = 95, normalized size = 1.16

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="maxima")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

Fricas [A] time = 1.66159, size = 208, normalized size = 2.54

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="fricas")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

Sympy [A] time = 0.896024, size = 75, normalized size = 0.91

$$-\frac{138567a^6+918918a^5bx^2+2567565a^4b^2x^4+3879876a^3b^3x^6+3357585a^2b^4x^8+1587222ab^5x^{10}+323323b^6x^{12}}{2909907x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**22,x)
```

```
[Out] -(138567*a**6 + 918918*a**5*b*x**2 + 2567565*a**4*b**2*x**4 + 3879876*a**3*
b**3*x**6 + 3357585*a**2*b**4*x**8 + 1587222*a*b**5*x**10 + 323323*b**6*x**
12)/(2909907*x**21)
```

Giac [A] time = 1.12113, size = 95, normalized size = 1.16

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="giac")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

3.475 $\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=83

$$\frac{3a^2x^4}{4b^4}-\frac{2a^3x^2}{b^5}+\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^
5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rubi [A] time = 0.0821507, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3a^2x^4}{4b^4}-\frac{2a^3x^2}{b^5}+\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```


```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^
5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^{11}}{\left(ab+b^2x^2\right)^2}\,dx\\ &= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(-\frac{4a^3}{b^7}+\frac{3a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{b^4}-\frac{a^5}{b^7(a+bx)^2}+\frac{5a^4}{b^7(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}\end{aligned}$$

Mathematica [A] time = 0.0203641, size = 72, normalized size = 0.87

$$\frac{18a^2b^2x^4-48a^3bx^2+\frac{12a^5}{a+bx^2}+60a^4\log\left(a+bx^2\right)-8ab^3x^6+3b^4x^8}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a + b
*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

Maple [A] time = 0.048, size = 74, normalized size = 0.9

$$-2\,\frac{x^2a^3}{b^5}+\frac{3\,a^2x^4}{4\,b^4}-\frac{ax^6}{3\,b^3}+\frac{x^8}{8\,b^2}+\frac{a^5}{2\,b^6\left(bx^2+a\right)}+\frac{5\,a^4\ln\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2
+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Maxima [A] time = 1.18691, size = 104, normalized size = 1.25

$$\frac{a^5}{2\left(b^7x^2+ab^6\right)}+\frac{5\,a^4\log\left(bx^2+a\right)}{2\,b^6}+\frac{3\,b^3x^8-8\,ab^2x^6+18\,a^2bx^4-48\,a^3x^2}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 -
8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

Fricas [A] time = 1.65008, size = 198, normalized size = 2.39

$$\frac{3\,b^5x^{10}-5\,ab^4x^8+10\,a^2b^3x^6-30\,a^3b^2x^4-48\,a^4bx^2+12\,a^5+60\left(a^4bx^2+a^5\right)\log\left(bx^2+a\right)}{24\left(b^7x^2+ab^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*b
*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

Sympy [A] time = 0.440946, size = 80, normalized size = 0.96

$$\frac{a^5}{2ab^6+2b^7x^2}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x*
*2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)
```

Giac [A] time = 1.20629, size = 124, normalized size = 1.49

$$\frac{5\,a^4\log\left(\left|bx^2+a\right|\right)}{2\,b^6}-\frac{5\,a^4bx^2+4\,a^5}{2\left(bx^2+a\right)b^6}+\frac{3\,b^6x^8-8\,ab^5x^6+18\,a^2b^4x^4-48\,a^3b^3x^2}{24\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*a^4*log(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^2 + a)*b^
6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48*a^3*b^3*x^2)/b^8
```

3.476 $\int \frac{x^9}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=70

$$\frac{3a^2x^2}{2b^4} - \frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3\log(a+bx^2)}{b^5} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2
)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rubi [A] time = 0.0625934, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{3a^2x^2}{2b^4} - \frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3\log(a+bx^2)}{b^5} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2
)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^9}{\left(ab+b^2x^2\right)^2}\,dx\\ &= \frac{1}{2}b^2\,\mathrm{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^2\,\mathrm{Subst}\left(\int \left(\frac{3a^2}{b^6}-\frac{2ax}{b^5}+\frac{x^2}{b^4}+\frac{a^4}{b^6(a+bx)^2}-\frac{4a^3}{b^6(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0224498, size = 60, normalized size = 0.86

$$\frac{9a^2bx^2-\frac{3a^4}{a+bx^2}-12a^3\log\left(a+bx^2\right)-3ab^2x^4+b^3x^6}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a +
 b*x^2])/(6*b^5)
```

Maple [A] time = 0.048, size = 63, normalized size = 0.9

$$\frac{3\,a^2x^2}{2\,b^4}-\frac{ax^4}{2\,b^3}+\frac{x^6}{6\,b^2}-\frac{a^4}{2\,b^5\left(bx^2+a\right)}-2\,\frac{a^3\ln\left(bx^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*
x^2+a)/b^5
```

Maxima [A] time = 0.974688, size = 88, normalized size = 1.26

$$-\frac{a^4}{2\left(b^6x^2+ab^5\right)}-\frac{2\,a^3\log\left(bx^2+a\right)}{b^5}+\frac{b^2x^6-3\,abx^4+9\,a^2x^2}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*x^6 - 3*a*
b*x^4 + 9*a^2*x^2)/b^4
```

Fricas [A] time = 1.69532, size = 166, normalized size = 2.37

$$\frac{b^4x^8-2\,ab^3x^6+6\,a^2b^2x^4+9\,a^3bx^2-3\,a^4-12\left(a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{6\left(b^6x^2+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4 - 12*(a^3*
b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

Sympy [A] time = 0.431309, size = 66, normalized size = 0.94

$$-\frac{a^4}{2ab^5+2b^6x^2}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2/
(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

Giac [A] time = 1.13857, size = 108, normalized size = 1.54

$$-\frac{2\,a^3\log\left(\left|bx^2+a\right|\right)}{b^5}+\frac{b^4x^6-3\,ab^3x^4+9\,a^2b^2x^2}{6\,b^6}+\frac{4\,a^3bx^2+3\,a^4}{2\left(bx^2+a\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^2*b^2*x^2
)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

3.477 $\int \frac{x^7}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=57

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/(2*b^4)
```

Rubi [A] time = 0.0524434, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/(2*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^7}{\left(ab+b^2x^2\right)^2}\,dx\\ &= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(-\frac{2a}{b^5}+\frac{x}{b^4}-\frac{a^3}{b^5(a+bx)^2}+\frac{3a^2}{b^5(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.016188, size = 49, normalized size = 0.86

$$\frac{\frac{2a^3}{a+bx^2}+6a^2\log\left(a+bx^2\right)-4abx^2+b^2x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2])/(4*b^4)
```

Maple [A] time = 0.048, size = 52, normalized size = 0.9

$$-\frac{ax^2}{b^3}+\frac{x^4}{4\,b^2}+\frac{a^3}{2\,b^4\left(bx^2+a\right)}+\frac{3\,a^2\ln\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

Maxima [A] time = 0.961409, size = 73, normalized size = 1.28

$$\frac{a^3}{2\left(b^5x^2 + ab^4\right)} + \frac{3\,a^2\log\left(bx^2 + a\right)}{2\,b^4} + \frac{bx^4 - 4\,ax^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x^4 - 4*a*x
^2)/b^3
```

Fricas [A] time = 1.72323, size = 143, normalized size = 2.51

$$\frac{b^3x^6 - 3\,ab^2x^4 - 4\,a^2bx^2 + 2\,a^3 + 6\left(a^2bx^2 + a^3\right)\log\left(bx^2 + a\right)}{4\left(b^5x^2 + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 + a^3)*log(
b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A] time = 0.403627, size = 53, normalized size = 0.93

$$\frac{a^3}{2ab^4 + 2b^5x^2} + \frac{3a^2\log\left(a + bx^2\right)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*x**2/b*
*3 + x**4/(4*b**2)
```

Giac [A] time = 1.12868, size = 90, normalized size = 1.58

$$\frac{3\,a^2\log\left(\left|bx^2+a\right|\right)}{2\,b^4}+\frac{b^2x^4-4\,abx^2}{4\,b^4}-\frac{3\,a^2bx^2+2\,a^3}{2\,\left(bx^2+a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 3/2*a^2*log(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 - 1/2*(3*a^
2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

3.478 $\int \frac{x^5}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=44

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rubi [A] time = 0.037158, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^5}{\left(ab+b^2x^2\right)^2}\,dx\\ &= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{b^4}+\frac{a^2}{b^4(a+bx)^2}-\frac{2a}{b^4(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{x^2}{2b^2}-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.0159307, size = 38, normalized size = 0.86

$$\frac{-\frac{a^2}{a+bx^2}-2a\log\left(a+bx^2\right)+bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

Maple [A] time = 0.047, size = 41, normalized size = 0.9

$$\frac{x^2}{2\,b^2}-\frac{a^2}{2\,b^3\left(bx^2+a\right)}-\frac{a\ln\left(bx^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Maxima [A] time = 1.00991, size = 58, normalized size = 1.32

$$-\frac{a^2}{2\left(b^4x^2+ab^3\right)}+\frac{x^2}{2\,b^2}-\frac{a\log\left(bx^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

Fricas [A] time = 1.659, size = 113, normalized size = 2.57

$$\frac{b^2x^4+abx^2-a^2-2\left(abx^2+a^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/(b^4*x^2 +
 a*b^3)
```

Sympy [A] time = 0.379487, size = 39, normalized size = 0.89

$$-\frac{a^2}{2ab^3+2b^4x^2}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2*b**2)
```

Giac [A] time = 1.1368, size = 66, normalized size = 1.5

$$\frac{x^2}{2\,b^2} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{b^3} + \frac{2\,abx^2 + a^2}{2\,(bx^2 + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^2 - a*log(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 + a
)*b^3)
```

3.479 $\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=33

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rubi [A] time = 0.0284405, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^3}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(-\frac{a}{b^3(a+bx)^2}+\frac{1}{b^3(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Mathematica [A] time = 0.0085283, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^2}+\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.046, size = 30, normalized size = 0.9

$$\frac{a}{2\,b^2\left(bx^2+a\right)}+\frac{\ln\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Maxima [A] time = 1.01066, size = 43, normalized size = 1.3

$$\frac{a}{2\left(b^3x^2 + ab^2\right)} + \frac{\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

Fricas [A] time = 1.66032, size = 76, normalized size = 2.3

$$\frac{\left(bx^2 + a\right)\log\left(bx^2 + a\right) + a}{2\left(b^3x^2 + ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 0.339552, size = 29, normalized size = 0.88

$$\frac{a}{2ab^2 + 2b^3x^2} + \frac{\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

Giac [A] time = 1.1272, size = 41, normalized size = 1.24

$$\frac{\log\left(\left|bx^2 + a\right|\right)}{2\,b^2} + \frac{a}{2\left(bx^2 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^2 + 1/2*a/((b*x^2 + a)*b^2)
```

3.480 $\int \frac{x}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\left(a+bx^2\right)}$$

```
[Out] -1/(2*b*(a + b*x^2))
```

Rubi [A] time = 0.0053895, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/(2*b*(a + b*x^2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a^2+2abx^2+b^2x^4} \, dx &= b^2 \int \frac{x}{\left(ab+b^2x^2\right)^2} \, dx\\ &= -\frac{1}{2b\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0028788, size = 16, normalized size = 1.

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/(2*b*(a + b*x^2))
```

Maple [A] time = 0.046, size = 15, normalized size = 0.9

$$-\frac{1}{2\,b\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/b/(b*x^2+a)
```

Maxima [A] time = 1.04055, size = 20, normalized size = 1.25

$$-\frac{1}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

Fricas [A] time = 1.57589, size = 30, normalized size = 1.88

$$-\frac{1}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

Sympy [A] time = 0.293952, size = 15, normalized size = 0.94

$$-\frac{1}{2ab + 2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -1/(2*a*b + 2*b**2*x**2)
```

Giac [A] time = 1.15217, size = 19, normalized size = 1.19

$$-\frac{1}{2\left(bx^2 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

3.481 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rubi [A] time = 0.0382369, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x\left(ab+b^2x^2\right)^2}\,dx\\ &= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{a^2b^2x}-\frac{1}{ab(a+bx)^2}-\frac{1}{a^2b(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{1}{2a\left(a+bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0126737, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2}-\log\left(a+bx^2\right)+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.056, size = 35, normalized size = 0.9

$$\frac{1}{2\,a\left(bx^2+a\right)}+\frac{\ln(x)}{a^2}-\frac{\ln\left(bx^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Maxima [A] time = 0.989865, size = 50, normalized size = 1.32

$$\frac{1}{2\left(abx^2+a^2\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

Fricas [A] time = 1.688, size = 108, normalized size = 2.84

$$-\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)-2\left(bx^2+a\right)\log\left(x\right)-a}{2\left(a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

Sympy [A] time = 0.440688, size = 34, normalized size = 0.89

$$\frac{1}{2a^2+2abx^2}+\frac{\log\left(x\right)}{a^2}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

Giac [A] time = 1.14674, size = 63, normalized size = 1.66

$$\frac{\log\left(x^2\right)}{2\,a^2} - \frac{\log\left(\left|bx^2 + a\right|\right)}{2\,a^2} + \frac{bx^2 + 2\,a}{2\left(bx^2 + a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

3.482 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rubi [A] time = 0.0489452, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a^2+2abx^2+b^4x^4\right)}\,dx &= b^2\int \frac{1}{x^3\left(ab+b^2x^2\right)^2}\,dx\\ &= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{a^2b^2x^2}-\frac{2}{a^3bx}+\frac{1}{a^2(a+bx)^2}+\frac{2}{a^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0362084, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/(2*a^3)
```

Maple [A] time = 0.055, size = 46, normalized size = 0.9

$$-\frac{1}{2\,a^2x^2}-\frac{b}{2\,a^2\left(bx^2+a\right)}-2\,\frac{b\ln(x)}{a^3}+\frac{b\ln\left(bx^2+a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Maxima [A] time = 0.991222, size = 70, normalized size = 1.43

$$-\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)}+\frac{b\log\left(bx^2+a\right)}{a^3}-\frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - b*log(x^2
)/a^3
```

Fricas [A] time = 1.7226, size = 157, normalized size = 3.2

$$-\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log\left(x\right)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] time = 0.534295, size = 49, normalized size = 1.

$$-\frac{a+2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log\left(x\right)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -(a + 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

Giac [A] time = 1.16721, size = 69, normalized size = 1.41

$$-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

3.483 $\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=66

$$\frac{b^2}{2a^3(a+bx^2)}-\frac{3b^2\log(a+bx^2)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

```
[Out] -1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rubi [A] time = 0.0537842, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{b^2}{2a^3(a+bx^2)}-\frac{3b^2\log(a+bx^2)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^5\left(ab+b^2x^2\right)^2}\,dx \\
&= \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{x^3\left(ab+b^2x\right)^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}b^2\,\text{Subst}\left(\int \left(\frac{1}{a^2b^2x^3}-\frac{2}{a^3bx^2}+\frac{3}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}\right)dx,x,x^2\right) \\
&= -\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.0497941, size = 57, normalized size = 0.86

$$\frac{a\left(\frac{2b^2}{a+bx^2}-\frac{a}{x^4}+\frac{4b}{x^2}\right)-6b^2\log\left(a+bx^2\right)+12b^2\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] - 6*b^2*Log
[a + b*x^2])/(4*a^4)
```

Maple [A] time = 0.053, size = 61, normalized size = 0.9

$$-\frac{1}{4\,a^2x^4}+\frac{b}{x^2a^3}+\frac{b^2}{2\,a^3\left(bx^2+a\right)}+3\,\frac{b^2\ln(x)}{a^4}-\frac{3\,b^2\ln\left(bx^2+a\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/4/a^2/x^4+b/x^2/a^3+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*x
^2+a)/a^4
```

Maxima [A] time = 0.984186, size = 95, normalized size = 1.44

$$\frac{6\,b^2x^4+3\,abx^2-a^2}{4\left(a^3bx^6+a^4x^4\right)}-\frac{3\,b^2\log\left(bx^2+a\right)}{2\,a^4}+\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*log(b*x^2
 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

Fricas [A] time = 1.77421, size = 184, normalized size = 2.79

$$\frac{6\,ab^2x^4+3\,a^2bx^2-a^3-6\left(b^3x^6+ab^2x^4\right)\log\left(bx^2+a\right)+12\left(b^3x^6+ab^2x^4\right)\log\left(x\right)}{4\left(a^4bx^6+a^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*log(b*x^2 +
a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

Sympy [A] time = 0.673053, size = 68, normalized size = 1.03

$$\frac{-a^2+3abx^2+6b^2x^4}{4a^4x^4+4a^3bx^6}+\frac{3b^2\log\left(x\right)}{a^4}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*l
og(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

Giac [A] time = 1.12036, size = 116, normalized size = 1.76

$$\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}-\frac{3\,b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^4}+\frac{3\,b^3x^2+4\,ab^2}{2\left(bx^2+a\right)a^4}-\frac{9\,b^2x^4-4\,abx^2+a^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(x^2)/a^4 - 3/2*b^2*log(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 + 4
*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

3.484 $\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=92

$$\frac{3a^2x^3}{2b^4}-\frac{9a^3x}{2b^5}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14
*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```

Rubi [A] time = 0.0555928, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{3a^2x^3}{2b^4}-\frac{9a^3x}{2b^5}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14
*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^{10}}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9}{2}\int \frac{x^8}{ab+b^2x^2}\,dx\\
&= -\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9}{2}\int \left(-\frac{a^3}{b^5}+\frac{a^2x^2}{b^4}-\frac{ax^4}{b^3}+\frac{x^6}{b^2}+\frac{a^4}{b^4\left(ab+b^2x^2\right)}\right)dx\\
&= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{\left(9a^4\right)\int \frac{1}{ab+b^2x^2}\,dx}{2b^4}\\
&= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}
\end{aligned}$$

Mathematica [A] time = 0.053298, size = 82, normalized size = 0.89

$$\frac{x\left(70a^2bx^2-\frac{35a^4}{a+bx^2}-280a^3-28ab^2x^4+10b^3x^6\right)}{70b^5}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*x
^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Maple [A] time = 0.051, size = 78, normalized size = 0.9

$$\frac{x^7}{7b^2} - \frac{2ax^5}{5b^3} + \frac{a^2x^3}{b^4} - 4\frac{xa^3}{b^5} - \frac{a^4x}{2b^5(bx^2+a)} + \frac{9a^4}{2b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/7*x^7/b^2-2/5*a*x^5/b^3+a^2*x^3/b^4-4*a^3*x/b^5-1/2/b^5*a^4*x/(b*x^2+a)+9
/2/b^5*a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65898, size = 459, normalized size = 4.99

$$\left[\frac{20b^4x^9 - 36ab^3x^7 + 84a^2b^2x^5 - 420a^3bx^3 - 630a^4x + 315(a^3bx^2 + a^4)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{140(b^6x^2 + ab^5)}, \frac{10b^4x^9 - 18ab^3x^7 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a^
4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2*b^
2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(
```

```
b*x*sqrt(a/b)/a))/(b^6*x^2 + a*b^5)]
```

Sympy [A] time = 0.465266, size = 134, normalized size = 1.46

$$-\frac{a^4x}{2ab^5+2b^6x^2}-\frac{4a^3x}{b^5}+\frac{a^2x^3}{b^4}-\frac{2ax^5}{5b^3}-\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x**
5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**7/b**11)/a**3)/4 + 9
*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b**11)/a**3)/4 + x**7/(7*b**2)
```

Giac [A] time = 1.14126, size = 113, normalized size = 1.23

$$\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5}-\frac{a^4x}{2\left(bx^2+a\right)b^5}+\frac{5\,b^{12}x^7-14\,ab^{11}x^5+35\,a^2b^{10}x^3-140\,a^3b^9x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2 + a)*b^5)
 + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 - 140*a^3*b^9*x)/b^14
```

3.485 $\int \frac{x^8}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=79

$$\frac{7a^2x}{2b^4} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}} - \frac{7ax^3}{6b^3} - \frac{x^7}{2b\left(a+bx^2\right)} + \frac{7x^5}{10b^2}$$

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2*b*(a + b*
x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rubi [A] time = 0.0452181, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{7a^2x}{2b^4} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}} - \frac{7ax^3}{6b^3} - \frac{x^7}{2b\left(a+bx^2\right)} + \frac{7x^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2*b*(a + b*
x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^8}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7}{2}\int \frac{x^6}{ab+b^2x^2}\,dx\\
&= -\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7}{2}\int \left(\frac{a^2}{b^4}-\frac{ax^2}{b^3}+\frac{x^4}{b^2}-\frac{a^3}{b^3\left(ab+b^2x^2\right)}\right)dx\\
&= \frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b\left(a+bx^2\right)}-\frac{\left(7a^3\right)\int \frac{1}{ab+b^2x^2}\,dx}{2b^3}\\
&= \frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b\left(a+bx^2\right)}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.045999, size = 71, normalized size = 0.9

$$\frac{x\left(\frac{15a^3}{a+bx^2}+90a^2-20abx^2+6b^2x^4\right)}{30b^4}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*b^4) - (7*
a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A] time = 0.051, size = 68, normalized size = 0.9

$$\frac{x^5}{5\,b^2}-\frac{2\,a x^3}{3\,b^3}+3\,\frac{a^2 x}{b^4}+\frac{x a^3}{2\,b^4\left(b x^2+a\right)}-\frac{7\,a^3}{2\,b^4}\arctan\left(b x\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/5*x^5/b^2-2/3*a*x^3/b^3+3*a^2*x/b^4+1/2/b^4*a^3*x/(b*x^2+a)-7/2/b^4*a^3/(
a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76173, size = 409, normalized size = 5.18

$$\left[\frac{12\,b^3x^7-28\,ab^2x^5+140\,a^2bx^3+210\,a^3x+105\left(a^2bx^2+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{60\left(b^5x^2+ab^4\right)},\ \frac{6\,b^3x^7-14\,ab^2x^5+70\,a^2bx^3+10}{30}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 105*(a^2*b*x
^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*
x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*x - 1
05*(a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^2 + a*b^4)]
```

Sympy [A] time = 0.461368, size = 124, normalized size = 1.57

$$\frac{a^3x}{2ab^4+2b^5x^2}+\frac{3a^2x}{b^4}-\frac{2ax^3}{3b^3}+\frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}-\frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}+\frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**3) + 7*sqr
t(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7*sqrt(-a**5/b**9)*lo
g(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5*b**2)
```

Giac [A] time = 1.18509, size = 99, normalized size = 1.25

$$-\frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4}+\frac{a^3x}{2\left(bx^2+a\right)b^4}+\frac{3\,b^8x^5-10\,ab^7x^3+45\,a^2b^6x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

3.486 $\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=66

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rubi [A] time = 0.037731, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```


```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^6}{\left(ab+b^2x^2\right)^2}\,dx\\ &= -\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5}{2}\int \frac{x^4}{ab+b^2x^2}\,dx\\ &= -\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5}{2}\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2\left(ab+b^2x^2\right)}\right)dx\\ &= -\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{\left(5a^2\right)\int \frac{1}{ab+b^2x^2}\,dx}{2b^2}\\ &= -\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0410447, size = 60, normalized size = 0.91

$$\frac{x\left(-\frac{3a^2}{a+bx^2}-12a+2bx^2\right)}{6b^3}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Maple [A] time = 0.051, size = 57, normalized size = 0.9

$$\frac{x^3}{3\,b^2}-2\,\frac{ax}{b^3}-\frac{a^2x}{2\,b^3\left(bx^2+a\right)}+\frac{5\,a^2}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/3*x^3/b^2-2*a*x/b^3-1/2*a^2/b^3*x/(b*x^2+a)+5/2*a^2/b^3/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71196, size = 348, normalized size = 5.27

$$\left[\frac{4\,b^2x^5-20\,abx^3-30\,a^2x+15\left(abx^2+a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{12\left(b^4x^2+ab^3\right)},\frac{2\,b^2x^5-10\,abx^3-15\,a^2x+15\left(abx^2+a^2\right)\sqrt{\frac{a}{b}}\,\text{arcta}}{6\left(b^4x^2+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(-a/b)*lo
g((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2 + a*b^3), 1/6*(2*b^
2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a))/(b^4*x^2 + a*b^3)]
```

Sympy [A] time = 0.4402, size = 107, normalized size = 1.62

$$-\frac{a^{2}x}{2ab^{3}+2b^{4}x^{2}}-\frac{2ax}{b^{3}}-\frac{5\sqrt{-\frac{a^{3}}{b^{7}}}\log\left(x-\frac{b^{3}\sqrt{-\frac{a^{3}}{b^{7}}}}{a}\right)}{4}+\frac{5\sqrt{-\frac{a^{3}}{b^{7}}}\log\left(x+\frac{b^{3}\sqrt{-\frac{a^{3}}{b^{7}}}}{a}\right)}{4}+\frac{x^{3}}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7)*log(x -
b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x + b**3*sqrt(-a**3/b**
7)/a)/4 + x**3/(3*b**2)
```

Giac [A] time = 1.13221, size = 82, normalized size = 1.24

$$\frac{5\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b^{3}}-\frac{a^{2}x}{2\left(bx^{2}+a\right)b^{3}}+\frac{b^{4}x^{3}-6\,ab^{3}x}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3)
 + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```

3.487 $\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(5/2))
```

Rubi [A] time = 0.0273325, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 321, 205}

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^4}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3}{2}\int \frac{x^2}{ab+b^2x^2}\,dx\\
&= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{(3a)\int \frac{1}{ab+b^2x^2}\,dx}{2b}\\
&= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.031698, size = 51, normalized size = 0.93

$$\frac{ax}{2b^2\left(a+bx^2\right)}-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*b^(5/2))
```

Maple [A] time = 0.05, size = 43, normalized size = 0.8

$$\frac{x}{b^2}+\frac{ax}{2\,b^2\left(bx^2+a\right)}-\frac{3\,a}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] x/b^2+1/2/b^2*a*x/(b*x^2+a)-3/2/b^2*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69614, size = 285, normalized size = 5.18

$$\left[\frac{4\,bx^3+3\left(bx^2+a\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,ax}{4\left(b^3x^2+ab^2\right)},\frac{2\,bx^3-3\left(bx^2+a\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,ax}{2\left(b^3x^2+ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3 - 3*(b*x^2 + a)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*a*x)/(b^3*x^2 + a*b^2)]
```

Sympy [A] time = 0.40696, size = 83, normalized size = 1.51

$$\frac{ax}{2ab^2+2b^3x^2}+\frac{3\sqrt{-\frac{a}{b^5}}\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{a}{b^5}}\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/b**5) + x)
/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/b**2
```

Giac [A] time = 1.17699, size = 57, normalized size = 1.04

$$-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}+\frac{ax}{2\left(bx^2+a\right)b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 + a)*b^2) +
x/b^2
```

3.488 $\int \frac{x^2}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rubi [A] time = 0.0172016, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^2}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x}{2b\left(a+bx^2\right)}+\frac{1}{2}\int \frac{1}{ab+b^2x^2}\,dx\\
&= -\frac{x}{2b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0196203, size = 45, normalized size = 1.

$$
\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Maple [A] time = 0.048, size = 36, normalized size = 0.8

$$
-\frac{x}{2\,b\left(bx^2+a\right)}+\frac{1}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68457, size = 263, normalized size = 5.84

$$\left[-\frac{2\,abx + \left(bx^2 + a\right)\sqrt{-ab}\log\left(\frac{bx^2 - 2\,\sqrt{-ab}x - a}{bx^2 + a}\right)}{4\left(ab^3x^2 + a^2b^2\right)}, -\frac{abx - \left(bx^2 + a\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\left(ab^3x^2 + a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x + (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*x - (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [B] time = 0.350125, size = 78, normalized size = 1.73

$$-\frac{x}{2ab + 2b^2x^2} - \frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3)) + x
)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4
```

Giac [A] time = 1.18315, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b} - \frac{x}{2\left(bx^2 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

3.489 $\int \frac{1}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.017064, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {28, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{b\int \frac{1}{ab+b^2x^2}\,dx}{2a}\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0235303, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Maple [A] time = 0.047, size = 36, normalized size = 0.8

$$\frac{x}{2\,a\left(bx^2+a\right)}+\frac{1}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75532, size = 261, normalized size = 5.8

$$\left[\frac{2\,abx - \left(bx^2 + a\right)\sqrt{-ab}\log\left(\frac{bx^2 - 2\,\sqrt{-ab}x - a}{bx^2 + a}\right)}{4\,\left(a^2b^2x^2 + a^3b\right)}, \frac{abx + \left(bx^2 + a\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,\left(a^2b^2x^2 + a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] time = 0.359495, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

Giac [A] time = 1.14594, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a}+\frac{x}{2\left(bx^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

3.490 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rubi [A] time = 0.0274065, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```


```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{1}{2ax\left(a+bx^2\right)}+\frac{(3b)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a}\\
&= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{\left(3b^2\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^2}\\
&= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0362392, size = 54, normalized size = 0.95

$$-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

Maple [A] time = 0.053, size = 46, normalized size = 0.8

$$-\frac{1}{a^2x}-\frac{bx}{2a^2(bx^2+a)}-\frac{3b}{2a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/a^2/x-1/2*b/a^2*x/(b*x^2+a)-3/2*b/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76015, size = 288, normalized size = 5.05

$$\left[-\frac{6bx^2-3(bx^3+ax)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4a}{4(a^2bx^3+a^3x)},-\frac{3bx^2+3(bx^3+ax)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2a}{2(a^2bx^3+a^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A] time = 0.452937, size = 90, normalized size = 1.58

$$\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{2a+3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 - (2*a + 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

Giac [A] time = 1.16988, size = 63, normalized size = 1.11

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}-\frac{3\,bx^2+2\,a}{2\,\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

3.491 $\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=68

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] time = 0.0359153, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^4\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{1}{2ax^3\left(a+bx^2\right)}+\frac{(5b)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{2a}\\
&= -\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}-\frac{\left(5b^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a^2}\\
&= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{\left(5b^3\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^3}\\
&= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.037667, size = 67, normalized size = 0.99

$$
\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(3*a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Maple [A] time = 0.054, size = 59, normalized size = 0.9

$$-\frac{1}{3\,a^2x^3}+2\,\frac{b}{xa^3}+\frac{b^2x}{2\,a^3\left(bx^2+a\right)}+\frac{5\,b^2}{2\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/3/x^3/a^2+2*b/a^3/x+1/2*b^2/a^3*x/(b*x^2+a)+5/2*b^2/a^3/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8174, size = 359, normalized size = 5.28

$$\left[\frac{30\,b^2x^4+20\,abx^2+15\left(b^2x^5+abx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-4\,a^2}{12\left(a^3bx^5+a^4x^3\right)},\frac{15\,b^2x^4+10\,abx^2+15\left(b^2x^5+abx^3\right)\sqrt{\frac{b}{a}}\arctan\left(x\right.}{6\left(a^3bx^5+a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
```

```
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [A] time = 0.550768, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Giac [A] time = 1.13932, size = 80, normalized size = 1.18

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}+\frac{b^2x}{2\left(bx^2+a\right)a^3}+\frac{6\,bx^2-a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

3.492 $\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=81

$$-\frac{7b^2}{2a^4x}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b
*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rubi [A] time = 0.0460682, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{7b^2}{2a^4x}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b
*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^6\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{1}{2ax^5\left(a+bx^2\right)}+\frac{(7b)\int \frac{1}{x^6\left(ab+b^2x^2\right)}\,dx}{2a}\\
&= -\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{\left(7b^2\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{2a^2}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}+\frac{1}{2ax^5\left(a+bx^2\right)}+\frac{\left(7b^3\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a^3}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{\left(7b^4\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^4}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0440794, size = 80, normalized size = 0.99

$$-\frac{b^3x}{2a^4\left(a+bx^2\right)}-\frac{3b^2}{a^4x}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{2b}{3a^3x^3}-\frac{1}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(5*a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a +
b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Maple [A] time = 0.055, size = 70, normalized size = 0.9

$$-\frac{1}{5\,a^2x^5}+\frac{2\,b}{3\,a^3x^3}-3\,\frac{b^2}{a^4x}-\frac{b^3x}{2\,a^4\left(bx^2+a\right)}-\frac{7\,b^3}{2\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/5/a^2/x^5+2/3*b/a^3/x^3-3*b^2/a^4/x-1/2*b^3/a^4*x/(b*x^2+a)-7/2*b^3/a^4/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7181, size = 423, normalized size = 5.22

$$\left[-\frac{210\,b^3x^6+140\,ab^2x^4-28\,a^2bx^2+12\,a^3-105\left(b^3x^7+ab^2x^5\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{60\left(a^4bx^7+a^5x^5\right)},-\frac{105\,b^3x^6+70\,ab^2x^4-14\,a^2bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7
+ a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a
^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a
^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b*x^7 +
a^5*x^5)]
```

Sympy [A] time = 0.69883, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}-\frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}-\frac{6a^3-14a^2bx^2+70ab^2x^4+105b^3x^6}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a*
*9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - (6*a**3 - 14*a**2*b*x**2 + 70*a
*b**2*x**4 + 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

Giac [A] time = 1.1281, size = 95, normalized size = 1.17

$$-\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4}-\frac{b^3x}{2\left(bx^2+a\right)a^4}-\frac{45\,b^2x^4-10\,abx^2+3\,a^2}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^4
) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

3.493 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{a^5}{6b^6\left(a+bx^2\right)^3}-\frac{5a^4}{4b^6\left(a+bx^2\right)^2}+\frac{5a^3}{b^6\left(a+bx^2\right)}+\frac{5a^2\log\left(a+bx^2\right)}{b^6}-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}$$

```
[Out] (-2*a*x^2)/b^5 + x^4/(4*b^4) + a^5/(6*b^6*(a + b*x^2)^3) - (5*a^4)/(4*b^6*(
a + b*x^2)^2) + (5*a^3)/(b^6*(a + b*x^2)) + (5*a^2*Log[a + b*x^2])/b^6
```

Rubi [A] time = 0.0926364, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^5}{6b^6\left(a+bx^2\right)^3}-\frac{5a^4}{4b^6\left(a+bx^2\right)^2}+\frac{5a^3}{b^6\left(a+bx^2\right)}+\frac{5a^2\log\left(a+bx^2\right)}{b^6}-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-2*a*x^2)/b^5 + x^4/(4*b^4) + a^5/(6*b^6*(a + b*x^2)^3) - (5*a^4)/(4*b^6*(
a + b*x^2)^2) + (5*a^3)/(b^6*(a + b*x^2)) + (5*a^2*Log[a + b*x^2])/b^6
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^{11}}{\left(ab+b^2x^2\right)^4}\,dx\\ &= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^4}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(-\frac{4a}{b^9}+\frac{x}{b^8}-\frac{a^5}{b^9(a+bx)^4}+\frac{5a^4}{b^9(a+bx)^3}-\frac{10a^3}{b^9(a+bx)^2}+\frac{10a^2}{b^9(a+bx)}\right)dx,x\right.\\ &= -\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}+\frac{a^5}{6b^6\left(a+bx^2\right)^3}-\frac{5a^4}{4b^6\left(a+bx^2\right)^2}+\frac{5a^3}{b^6\left(a+bx^2\right)}+\frac{5a^2\log\left(a+bx^2\right)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.0305118, size = 78, normalized size = 0.86

$$\frac{\frac{2a^5}{\left(a+bx^2\right)^3}-\frac{15a^4}{\left(a+bx^2\right)^2}+\frac{60a^3}{a+bx^2}+60a^2\log\left(a+bx^2\right)-24abx^2+3b^2x^4}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-24*a*b*x^2 + 3*b^2*x^4 + (2*a^5)/(a + b*x^2)^3 - (15*a^4)/(a + b*x^2)^2 +
 (60*a^3)/(a + b*x^2) + 60*a^2*Log[a + b*x^2])/(12*b^6)
```

Maple [A] time = 0.052, size = 86, normalized size = 1.

$$-2\,\frac{ax^2}{b^5}+\frac{x^4}{4\,b^4}+\frac{a^5}{6\,b^6\left(bx^2+a\right)^3}-\frac{5\,a^4}{4\,b^6\left(bx^2+a\right)^2}+5\,\frac{a^3}{b^6\left(bx^2+a\right)}+5\,\frac{a^2\ln\left(bx^2+a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -2*a*x^2/b^5+1/4*x^4/b^4+1/6*a^5/b^6/(b*x^2+a)^3-5/4*a^4/b^6/(b*x^2+a)^2+5*
a^3/b^6/(b*x^2+a)+5*a^2*ln(b*x^2+a)/b^6
```

Maxima [A] time = 1.00571, size = 134, normalized size = 1.47

$$\frac{60\,a^3b^2x^4 + 105\,a^4bx^2 + 47\,a^5}{12\left(b^9x^6 + 3\,ab^8x^4 + 3\,a^2b^7x^2 + a^3b^6\right)} + \frac{5\,a^2\log\left(bx^2 + a\right)}{b^6} + \frac{bx^4 - 8\,ax^2}{4\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*a^3*b^2*x^4 + 105*a^4*b*x^2 + 47*a^5)/(b^9*x^6 + 3*a*b^8*x^4 + 3*a
^2*b^7*x^2 + a^3*b^6) + 5*a^2*log(b*x^2 + a)/b^6 + 1/4*(b*x^4 - 8*a*x^2)/b^
5
```

Fricas [A] time = 1.63722, size = 285, normalized size = 3.13

$$\frac{3\,b^5x^{10} - 15\,ab^4x^8 - 63\,a^2b^3x^6 - 9\,a^3b^2x^4 + 81\,a^4bx^2 + 47\,a^5 + 60\left(a^2b^3x^6 + 3\,a^3b^2x^4 + 3\,a^4bx^2 + a^5\right)\log\left(bx^2 + a\right)}{12\left(b^9x^6 + 3\,ab^8x^4 + 3\,a^2b^7x^2 + a^3b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 - 15*a*b^4*x^8 - 63*a^2*b^3*x^6 - 9*a^3*b^2*x^4 + 81*a^4*b
*x^2 + 47*a^5 + 60*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*log(b*
x^2 + a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)
```

Sympy [A] time = 0.784291, size = 100, normalized size = 1.1

$$\frac{5a^2\log\left(a + bx^2\right)}{b^6} - \frac{2ax^2}{b^5} + \frac{47a^5 + 105a^4bx^2 + 60a^3b^2x^4}{12a^3b^6 + 36a^2b^7x^2 + 36ab^8x^4 + 12b^9x^6} + \frac{x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 5*a**2*log(a + b*x**2)/b**6 - 2*a*x**2/b**5 + (47*a**5 + 105*a**4*b*x**2 +
60*a**3*b**2*x**4)/(12*a**3*b**6 + 36*a**2*b**7*x**2 + 36*a*b**8*x**4 + 12*
b**9*x**6) + x**4/(4*b**4)
```

Giac [A] time = 1.13509, size = 123, normalized size = 1.35

$$\frac{5\,a^2\log\left(\left|bx^2+a\right|\right)}{b^6}+\frac{b^4x^4-8\,ab^3x^2}{4\,b^8}-\frac{110\,a^2b^3x^6+270\,a^3b^2x^4+225\,a^4bx^2+63\,a^5}{12\left(bx^2+a\right)^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*log(abs(b*x^2 + a))/b^6 + 1/4*(b^4*x^4 - 8*a*b^3*x^2)/b^8 - 1/12*(110
*a^2*b^3*x^6 + 270*a^3*b^2*x^4 + 225*a^4*b*x^2 + 63*a^5)/((b*x^2 + a)^3*b^6
)
```

3.494 $\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=77

$$-\frac{a^4}{6b^5(a+bx^2)^3}+\frac{a^3}{b^5(a+bx^2)^2}-\frac{3a^2}{b^5(a+bx^2)}-\frac{2a\log(a+bx^2)}{b^5}+\frac{x^2}{2b^4}$$

```
[Out] x^2/(2*b^4) - a^4/(6*b^5*(a + b*x^2)^3) + a^3/(b^5*(a + b*x^2)^2) - (3*a^2)
/(b^5*(a + b*x^2)) - (2*a*Log[a + b*x^2])/b^5
```

Rubi [A] time = 0.0733187, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^4}{6b^5(a+bx^2)^3}+\frac{a^3}{b^5(a+bx^2)^2}-\frac{3a^2}{b^5(a+bx^2)}-\frac{2a\log(a+bx^2)}{b^5}+\frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x^2/(2*b^4) - a^4/(6*b^5*(a + b*x^2)^3) + a^3/(b^5*(a + b*x^2)^2) - (3*a^2)
/(b^5*(a + b*x^2)) - (2*a*Log[a + b*x^2])/b^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^9}{\left(ab+b^2x^2\right)^4}\,dx\\ &= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^4}\,dx, x, x^2\right)\\ &= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(\frac{1}{b^8}+\frac{a^4}{b^8(a+bx)^4}-\frac{4a^3}{b^8(a+bx)^3}+\frac{6a^2}{b^8(a+bx)^2}-\frac{4a}{b^8(a+bx)}\right)dx, x, x^2\right)\\ &= \frac{x^2}{2b^4}-\frac{a^4}{6b^5\left(a+bx^2\right)^3}+\frac{a^3}{b^5\left(a+bx^2\right)^2}-\frac{3a^2}{b^5\left(a+bx^2\right)}-\frac{2a\log\left(a+bx^2\right)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0481209, size = 59, normalized size = 0.77

$$-\frac{\frac{a^2\left(13a^2+30abx^2+18b^2x^4\right)}{\left(a+bx^2\right)^3}+12a\log\left(a+bx^2\right)-3bx^2}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(-3*b*x^2 + (a^2*(13*a^2 + 30*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 + 12*a*
Log[a + b*x^2])/(6*b^5)
```

Maple [A] time = 0.051, size = 74, normalized size = 1.

$$\frac{x^2}{2\,b^4}-\frac{a^4}{6\,b^5\left(bx^2+a\right)^3}+\frac{a^3}{b^5\left(bx^2+a\right)^2}-3\,\frac{a^2}{b^5\left(bx^2+a\right)}-2\,\frac{a\ln\left(bx^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/2*x^2/b^4-1/6*a^4/b^5/(b*x^2+a)^3+a^3/b^5/(b*x^2+a)^2-3*a^2/b^5/(b*x^2+a)
-2*a*ln(b*x^2+a)/b^5
```

Maxima [A] time = 0.997592, size = 119, normalized size = 1.55

$$-\frac{18\,a^2b^2x^4+30\,a^3bx^2+13\,a^4}{6\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}+\frac{x^2}{2\,b^4}-\frac{2\,a\log\left(bx^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(18*a^2*b^2*x^4 + 30*a^3*b*x^2 + 13*a^4)/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^
2*b^6*x^2 + a^3*b^5) + 1/2*x^2/b^4 - 2*a*log(b*x^2 + a)/b^5
```

Fricas [A] time = 1.72482, size = 255, normalized size = 3.31

$$\frac{3\,b^4x^8+9\,ab^3x^6-9\,a^2b^2x^4-27\,a^3bx^2-13\,a^4-12\left(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{6\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*x^8 + 9*a*b^3*x^6 - 9*a^2*b^2*x^4 - 27*a^3*b*x^2 - 13*a^4 - 12*(
a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^8*x^6 + 3
*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

Sympy [A] time = 0.741864, size = 88, normalized size = 1.14

$$-\frac{2a\log\left(a+bx^2\right)}{b^5}-\frac{13a^4+30a^3bx^2+18a^2b^2x^4}{6a^3b^5+18a^2b^6x^2+18ab^7x^4+6b^8x^6}+\frac{x^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -2*a*log(a + b*x**2)/b**5 - (13*a**4 + 30*a**3*b*x**2 + 18*a**2*b**2*x**4)/
(6*a**3*b**5 + 18*a**2*b**6*x**2 + 18*a*b**7*x**4 + 6*b**8*x**6) + x**2/(2*
b**4)
```

Giac [A] time = 1.11294, size = 99, normalized size = 1.29

$$\frac{x^2}{2\,b^4} - \frac{2\,a\log\left(\left|bx^2 + a\right|\right)}{b^5} + \frac{22\,ab^3x^6 + 48\,a^2b^2x^4 + 36\,a^3bx^2 + 9\,a^4}{6\left(bx^2 + a\right)^3 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^4 - 2*a*log(abs(b*x^2 + a))/b^5 + 1/6*(22*a*b^3*x^6 + 48*a^2*b^2*
x^4 + 36*a^3*b*x^2 + 9*a^4)/((b*x^2 + a)^3*b^5)
```

3.495 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

```
[Out] a^3/(6*b^4*(a + b*x^2)^3) - (3*a^2)/(4*b^4*(a + b*x^2)^2) + (3*a)/(2*b^4*(a
 + b*x^2)) + Log[a + b*x^2]/(2*b^4)
```

Rubi [A] time = 0.0636041, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^3/(6*b^4*(a + b*x^2)^3) - (3*a^2)/(4*b^4*(a + b*x^2)^2) + (3*a)/(2*b^4*(a
 + b*x^2)) + Log[a + b*x^2]/(2*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^7}{\left(ab+b^2x^2\right)^4}\,dx\\
&= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^4}\,dx,x,x^2\right)\\
&= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(-\frac{a^3}{b^7(a+bx)^4}+\frac{3a^2}{b^7(a+bx)^3}-\frac{3a}{b^7(a+bx)^2}+\frac{1}{b^7(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] time = 0.0184551, size = 50, normalized size = 0.7

$$
\frac{\frac{a\left(11a^2+27abx^2+18b^2x^4\right)}{\left(a+bx^2\right)^3}+6\log\left(a+bx^2\right)}{12b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((a*(11*a^2 + 27*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 + 6*Log[a + b*x^2])/(
12*b^4)
```

Maple [A] time = 0.049, size = 64, normalized size = 0.9

$$
\frac{a^3}{6\,b^4\left(bx^2+a\right)^3}-\frac{3\,a^2}{4\,b^4\left(bx^2+a\right)^2}+\frac{3\,a}{2\,b^4\left(bx^2+a\right)}+\frac{\ln\left(bx^2+a\right)}{2\,b^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*a^3/b^4/(b*x^2+a)^3-3/4*a^2/b^4/(b*x^2+a)^2+3/2*a/b^4/(b*x^2+a)+1/2*ln(
b*x^2+a)/b^4
```

Maxima [A] time = 1.00506, size = 104, normalized size = 1.46

$$\frac{18\,ab^2x^4 + 27\,a^2bx^2 + 11\,a^3}{12\left(b^7x^6 + 3\,ab^6x^4 + 3\,a^2b^5x^2 + a^3b^4\right)} + \frac{\log\left(bx^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3)/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*
b^5*x^2 + a^3*b^4) + 1/2*log(b*x^2 + a)/b^4
```

Fricas [A] time = 1.72319, size = 213, normalized size = 3.

$$\frac{18\,ab^2x^4 + 27\,a^2bx^2 + 11\,a^3 + 6\left(b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3\right)\log\left(bx^2 + a\right)}{12\left(b^7x^6 + 3\,ab^6x^4 + 3\,a^2b^5x^2 + a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3 + 6*(b^3*x^6 + 3*a*b^2*x^4 + 3*a
^2*b*x^2 + a^3)*log(b*x^2 + a))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^
3*b^4)
```

Sympy [A] time = 0.654259, size = 76, normalized size = 1.07

$$\frac{11a^3 + 27a^2bx^2 + 18ab^2x^4}{12a^3b^4 + 36a^2b^5x^2 + 36ab^6x^4 + 12b^7x^6} + \frac{\log\left(a + bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (11*a**3 + 27*a**2*b*x**2 + 18*a*b**2*x**4)/(12*a**3*b**4 + 36*a**2*b**5*x*
*2 + 36*a*b**6*x**4 + 12*b**7*x**6) + log(a + b*x**2)/(2*b**4)
```

Giac [A] time = 1.16371, size = 72, normalized size = 1.01

$$\frac{\log\left(|bx^2+a|\right)}{2\,b^4}-\frac{11\,b^2x^6+15\,abx^4+6\,a^2x^2}{12\,\left(bx^2+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^4 - 1/12*(11*b^2*x^6 + 15*a*b*x^4 + 6*a^2*x^2)/((
b*x^2 + a)^3*b^3)
```

3.496 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=19

$$\frac{x^6}{6a\left(a+bx^2\right)^3}$$

```
[Out] x^6/(6*a*(a + b*x^2)^3)
```

Rubi [A] time = 0.006774, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$\frac{x^6}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x^6/(6*a*(a + b*x^2)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^5}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{x^6}{6a\left(a+bx^2\right)^3}$$

Mathematica [A] time = 0.0132677, size = 35, normalized size = 1.84

$$-\frac{a^2+3abx^2+3b^2x^4}{6b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(a^2 + 3*a*b*x^2 + 3*b^2*x^4)/(6*b^3*(a + b*x^2)^3)
```

Maple [B] time = 0.049, size = 48, normalized size = 2.5

$$-\frac{a^2}{6\,b^3\left(bx^2+a\right)^3}+\frac{a}{2\,b^3\left(bx^2+a\right)^2}-\frac{1}{2\,b^3\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/6*a^2/b^3/(b*x^2+a)^3+1/2*a/b^3/(b*x^2+a)^2-1/2/b^3/(b*x^2+a)
```

Maxima [B] time = 0.979301, size = 78, normalized size = 4.11

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 +
 a^3*b^3)
```

Fricas [B] time = 1.58219, size = 116, normalized size = 6.11

$$-\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\left(b^6x^6 + 3\,ab^5x^4 + 3\,a^2b^4x^2 + a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 +
 a^3*b^3)
```

Sympy [B] time = 0.58234, size = 60, normalized size = 3.16

$$-\frac{a^2 + 3abx^2 + 3b^2x^4}{6a^3b^3 + 18a^2b^4x^2 + 18ab^5x^4 + 6b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -(a**2 + 3*a*b*x**2 + 3*b**2*x**4)/(6*a**3*b**3 + 18*a**2*b**4*x**2 + 18*a*
b**5*x**4 + 6*b**6*x**6)
```

Giac [A] time = 1.13551, size = 45, normalized size = 2.37

$$-\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\left(bx^2 + a\right)^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/((b*x^2 + a)^3*b^3)
```

3.497 $\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=34

$$\frac{a}{6b^2\left(a+bx^2\right)^3}-\frac{1}{4b^2\left(a+bx^2\right)^2}$$

```
[Out] a/(6*b^2*(a + b*x^2)^3) - 1/(4*b^2*(a + b*x^2)^2)
```

Rubi [A] time = 0.0292993, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a}{6b^2\left(a+bx^2\right)^3}-\frac{1}{4b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a/(6*b^2*(a + b*x^2)^3) - 1/(4*b^2*(a + b*x^2)^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^3}{\left(ab+b^2x^2\right)^4}\,dx \\
&= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^4}\,dx,x,x^2\right) \\
&= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(-\frac{a}{b^5(a+bx)^4}+\frac{1}{b^5(a+bx)^3}\right)dx,x,x^2\right) \\
&= \frac{a}{6b^2\left(a+bx^2\right)^3}-\frac{1}{4b^2\left(a+bx^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.0074374, size = 24, normalized size = 0.71

$$-\frac{a+3bx^2}{12b^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(a + 3*b*x^2)/(12*b^2*(a + b*x^2)^3)
```

Maple [A] time = 0.047, size = 31, normalized size = 0.9

$$\frac{a}{6\,b^2\left(bx^2+a\right)^3}-\frac{1}{4\,b^2\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*a/b^2/(b*x^2+a)^3-1/4/b^2/(b*x^2+a)^2
```

Maxima [A] time = 0.993504, size = 63, normalized size = 1.85

$$-\frac{3\,bx^2+a}{12\,\left(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*b^2)
```

Fricas [A] time = 1.64251, size = 96, normalized size = 2.82

$$-\frac{3\,bx^2+a}{12\,\left(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [A] time = 0.556278, size = 48, normalized size = 1.41

$$-\frac{a+3bx^2}{12a^3b^2+36a^2b^3x^2+36ab^4x^4+12b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -(a + 3*b*x**2)/(12*a**3*b**2 + 36*a**2*b**3*x**2 + 36*a*b**4*x**4 + 12*b**
5*x**6)
```

Giac [A] time = 1.21101, size = 30, normalized size = 0.88

$$-\frac{3\,bx^2+a}{12\left(bx^2+a\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*(3*b*x^2 + a)/((b*x^2 + a)^3*b^2)
```

3.498 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

```
[Out] -1/(6*b*(a + b*x^2)^3)
```

Rubi [A] time = 0.0049299, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/(6*b*(a + b*x^2)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a^2 + 2abx^2 + b^2x^4\right)^2}\,dx = b^4 \int \frac{x}{\left(ab + b^2x^2\right)^4}\,dx$$
$$= -\frac{1}{6b\left(a + bx^2\right)^3}$$

Mathematica [A] time = 0.002448, size = 16, normalized size = 1.

$$-\frac{1}{6b\left(a + bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/(6*b*(a + b*x^2)^3)
```

Maple [A] time = 0.046, size = 15, normalized size = 0.9

$$-\frac{1}{6\,b\left(bx^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/6/b/(b*x^2+a)^3
```

Maxima [B] time = 0.977073, size = 50, normalized size = 3.12

$$-\frac{1}{6\left(b^4x^6 + 3\,ab^3x^4 + 3\,a^2b^2x^2 + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

Fricas [B] time = 1.76343, size = 73, normalized size = 4.56

$$-\frac{1}{6\left(b^4x^6 + 3\,ab^3x^4 + 3\,a^2b^2x^2 + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

Sympy [B] time = 0.540938, size = 39, normalized size = 2.44

$$-\frac{1}{6a^3b + 18a^2b^2x^2 + 18ab^3x^4 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -1/(6*a**3*b + 18*a**2*b**2*x**2 + 18*a*b**3*x**4 + 6*b**4*x**6)
```

Giac [A] time = 1.11452, size = 19, normalized size = 1.19

$$-\frac{1}{6\left(bx^2 + a\right)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/6/((b*x^2 + a)^3*b)
```

3.499 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=70

$$\frac{1}{2a^3\left(a+bx^2\right)}+\frac{1}{4a^2\left(a+bx^2\right)^2}-\frac{\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)}{a^4}+\frac{1}{6a\left(a+bx^2\right)^3}$$

```
[Out] 1/(6*a*(a + b*x^2)^3) + 1/(4*a^2*(a + b*x^2)^2) + 1/(2*a^3*(a + b*x^2)) + L
og[x]/a^4 - Log[a + b*x^2]/(2*a^4)
```

Rubi [A] time = 0.0758043, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{1}{2a^3\left(a+bx^2\right)}+\frac{1}{4a^2\left(a+bx^2\right)^2}-\frac{\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)}{a^4}+\frac{1}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] 1/(6*a*(a + b*x^2)^3) + 1/(4*a^2*(a + b*x^2)^2) + 1/(2*a^3*(a + b*x^2)) + L
og[x]/a^4 - Log[a + b*x^2]/(2*a^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(\frac{1}{a^4b^4x}-\frac{1}{ab^3(a+bx)^4}-\frac{1}{a^2b^3(a+bx)^3}-\frac{1}{a^3b^3(a+bx)^2}-\frac{1}{a^4b^3(a+bx)}\right)\right.$$
$$= \frac{1}{6a\left(a+bx^2\right)^3}+\frac{1}{4a^2\left(a+bx^2\right)^2}+\frac{1}{2a^3\left(a+bx^2\right)}+\frac{\log(x)}{a^4}-\frac{\log\left(a+bx^2\right)}{2a^4}$$

Mathematica [A] time = 0.0355674, size = 54, normalized size = 0.77

$$\frac{\frac{a\left(11a^2+15abx^2+6b^2x^4\right)}{\left(a+bx^2\right)^3}-6\log\left(a+bx^2\right)+12\log(x)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x^2 + 6*b^2*x^4))/(a + b*x^2)^3 + 12*Log[x] - 6*Log[a
+ b*x^2])/(12*a^4)
```

Maple [A] time = 0.054, size = 63, normalized size = 0.9

$$\frac{1}{6\,a\left(bx^2+a\right)^3}+\frac{1}{4\,a^2\left(bx^2+a\right)^2}+\frac{1}{2\,a^3\left(bx^2+a\right)}+\frac{\ln(x)}{a^4}-\frac{\ln\left(bx^2+a\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6/a/(b*x^2+a)^3+1/4/a^2/(b*x^2+a)^2+1/2/a^3/(b*x^2+a)+ln(x)/a^4-1/2*ln(b*
x^2+a)/a^4
```

Maxima [A] time = 0.997209, size = 111, normalized size = 1.59

$$\frac{6\,b^2x^4+15\,abx^2+11\,a^2}{12\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^4}+\frac{\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^2*x^4 + 15*a*b*x^2 + 11*a^2)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5
*b*x^2 + a^6) - 1/2*log(b*x^2 + a)/a^4 + 1/2*log(x^2)/a^4
```

Fricas [B] time = 1.92034, size = 288, normalized size = 4.11

$$\frac{6\,ab^2x^4+15\,a^2bx^2+11\,a^3-6\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)+12\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\log(x)}{12\left(a^4b^3x^6+3\,a^5b^2x^4+3\,a^6bx^2+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*b^2*x^4 + 15*a^2*b*x^2 + 11*a^3 - 6*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^
2*b*x^2 + a^3)*log(b*x^2 + a) + 12*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3)*log(x))/(a^4*b^3*x^6 + 3*a^5*b^2*x^4 + 3*a^6*b*x^2 + a^7)
```

Sympy [A] time = 0.811277, size = 80, normalized size = 1.14

$$\frac{11a^2+15abx^2+6b^2x^4}{12a^6+36a^5bx^2+36a^4b^2x^4+12a^3b^3x^6}+\frac{\log(x)}{a^4}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (11*a**2 + 15*a*b*x**2 + 6*b**2*x**4)/(12*a**6 + 36*a**5*b*x**2 + 36*a**4*b
**2*x**4 + 12*a**3*b**3*x**6) + log(x)/a**4 - log(a/b + x**2)/(2*a**4)
```

Giac [A] time = 1.14283, size = 95, normalized size = 1.36

$$\frac{\log\left(x^2\right)}{2\,a^4}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^4}+\frac{11\,b^3x^6+39\,ab^2x^4+48\,a^2bx^2+22\,a^3}{12\,\left(bx^2+a\right)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^4 - 1/2*log(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6 + 39*a*b^
2*x^4 + 48*a^2*b*x^2 + 22*a^3)/((b*x^2 + a)^3*a^4)
```

3.500 $\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}+\frac{2b\log\left(a+bx^2\right)}{a^5}-\frac{4b\log(x)}{a^5}-\frac{1}{2a^4x^2}$$

```
[Out] -1/(2*a^4*x^2) - b/(6*a^2*(a + b*x^2)^3) - b/(2*a^3*(a + b*x^2)^2) - (3*b)/
(2*a^4*(a + b*x^2)) - (4*b*Log[x])/a^5 + (2*b*Log[a + b*x^2])/a^5
```

Rubi [A] time = 0.0856393, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}+\frac{2b\log\left(a+bx^2\right)}{a^5}-\frac{4b\log(x)}{a^5}-\frac{1}{2a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/(2*a^4*x^2) - b/(6*a^2*(a + b*x^2)^3) - b/(2*a^3*(a + b*x^2)^2) - (3*b)/
(2*a^4*(a + b*x^2)) - (4*b*Log[x])/a^5 + (2*b*Log[a + b*x^2])/a^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{1}{x^3\left(ab+b^2x^2\right)^4}\,dx\\ &= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^4}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(\frac{1}{a^4b^4x^2}-\frac{4}{a^5b^3x}+\frac{1}{a^2b^2(a+bx)^4}+\frac{2}{a^3b^2(a+bx)^3}+\frac{3}{a^4b^2(a+bx)^2}+\frac{}{a^5}\right.\right.\\ &= -\frac{1}{2a^4x^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{4b\log(x)}{a^5}+\frac{2b\log\left(a+bx\right.}{a^5}\end{aligned}$$

Mathematica [A] time = 0.0642186, size = 70, normalized size = 0.83

$$-\frac{\frac{a\left(22a^2bx^2+3a^3+30ab^2x^4+12b^3x^6\right)}{x^2\left(a+bx^2\right)^3}-12b\log\left(a+bx^2\right)+24b\log(x)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -((a*(3*a^3 + 22*a^2*b*x^2 + 30*a*b^2*x^4 + 12*b^3*x^6))/(x^2*(a + b*x^2)^3
) + 24*b*Log[x] - 12*b*Log[a + b*x^2])/(6*a^5)
```

Maple [A] time = 0.056, size = 77, normalized size = 0.9

$$-\frac{1}{2\,a^4x^2}-\frac{b}{6\,a^2\left(bx^2+a\right)^3}-\frac{b}{2\,a^3\left(bx^2+a\right)^2}-\frac{3\,b}{2\,a^4\left(bx^2+a\right)}-4\,\frac{b\ln(x)}{a^5}+2\,\frac{b\ln\left(bx^2+a\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/2/a^4/x^2-1/6*b/a^2/(b*x^2+a)^3-1/2*b/a^3/(b*x^2+a)^2-3/2*b/a^4/(b*x^2+a
)-4*b*ln(x)/a^5+2*b*ln(b*x^2+a)/a^5
```

Maxima [A] time = 1.01153, size = 134, normalized size = 1.6

$$-\frac{12\,b^{3}x^{6}+30\,ab^{2}x^{4}+22\,a^{2}bx^{2}+3\,a^{3}}{6\left(a^{4}b^{3}x^{8}+3\,a^{5}b^{2}x^{6}+3\,a^{6}bx^{4}+a^{7}x^{2}\right)}+\frac{2\,b\log\left(bx^{2}+a\right)}{a^{5}}-\frac{2\,b\log\left(x^{2}\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(12*b^3*x^6 + 30*a*b^2*x^4 + 22*a^2*b*x^2 + 3*a^3)/(a^4*b^3*x^8 + 3*a^
5*b^2*x^6 + 3*a^6*b*x^4 + a^7*x^2) + 2*b*log(b*x^2 + a)/a^5 - 2*b*log(x^2)/
a^5
```

Fricas [B] time = 1.78155, size = 339, normalized size = 4.04

$$-\frac{12\,ab^{3}x^{6}+30\,a^{2}b^{2}x^{4}+22\,a^{3}bx^{2}+3\,a^{4}-12\left(b^{4}x^{8}+3\,ab^{3}x^{6}+3\,a^{2}b^{2}x^{4}+a^{3}bx^{2}\right)\log\left(bx^{2}+a\right)+24\left(b^{4}x^{8}+3\,ab^{3}x^{6}+\right.}{6\left(a^{5}b^{3}x^{8}+3\,a^{6}b^{2}x^{6}+3\,a^{7}bx^{4}+a^{8}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*b^3*x^6 + 30*a^2*b^2*x^4 + 22*a^3*b*x^2 + 3*a^4 - 12*(b^4*x^8 +
3*a*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(b*x^2 + a) + 24*(b^4*x^8 + 3*a
*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(x))/(a^5*b^3*x^8 + 3*a^6*b^2*x^6
+ 3*a^7*b*x^4 + a^8*x^2)
```

Sympy [A] time = 1.27347, size = 100, normalized size = 1.19

$$-\frac{3a^{3}+22a^{2}bx^{2}+30ab^{2}x^{4}+12b^{3}x^{6}}{6a^{7}x^{2}+18a^{6}bx^{4}+18a^{5}b^{2}x^{6}+6a^{4}b^{3}x^{8}}-\frac{4b\log\left(x\right)}{a^{5}}+\frac{2b\log\left(\frac{a}{b}+x^{2}\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -(3*a**3 + 22*a**2*b*x**2 + 30*a*b**2*x**4 + 12*b**3*x**6)/(6*a**7*x**2 + 1
8*a**6*b*x**4 + 18*a**5*b**2*x**6 + 6*a**4*b**3*x**8) - 4*b*log(x)/a**5 + 2
*b*log(a/b + x**2)/a**5
```

Giac [A] time = 1.13287, size = 126, normalized size = 1.5

$$-\frac{2\,b\log\left(x^2\right)}{a^5}+\frac{2\,b\log\left(\left|bx^2+a\right|\right)}{a^5}+\frac{4\,bx^2-a}{2\,a^5x^2}-\frac{22\,b^4x^6+75\,ab^3x^4+87\,a^2b^2x^2+35\,a^3b}{6\left(bx^2+a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(x^2)/a^5 + 2*b*log(abs(b*x^2 + a))/a^5 + 1/2*(4*b*x^2 - a)/(a^5*x^
2) - 1/6*(22*b^4*x^6 + 75*a*b^3*x^4 + 87*a^2*b^2*x^2 + 35*a^3*b)/((b*x^2 +
a)^3*a^5)
```

3.501 $\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}-\frac{5b^2\log\left(a+bx^2\right)}{a^6}+\frac{10b^2\log(x)}{a^6}+\frac{2b}{a^5x^2}-\frac{1}{4a^4x^4}$$

```
[Out] -1/(4*a^4*x^4) + (2*b)/(a^5*x^2) + b^2/(6*a^3*(a + b*x^2)^3) + (3*b^2)/(4*a
^4*(a + b*x^2)^2) + (3*b^2)/(a^5*(a + b*x^2)) + (10*b^2*Log[x])/a^6 - (5*b^
2*Log[a + b*x^2])/a^6
```

Rubi [A] time = 0.0981371, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}-\frac{5b^2\log\left(a+bx^2\right)}{a^6}+\frac{10b^2\log(x)}{a^6}+\frac{2b}{a^5x^2}-\frac{1}{4a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/(4*a^4*x^4) + (2*b)/(a^5*x^2) + b^2/(6*a^3*(a + b*x^2)^3) + (3*b^2)/(4*a
^4*(a + b*x^2)^2) + (3*b^2)/(a^5*(a + b*x^2)) + (10*b^2*Log[x])/a^6 - (5*b^
2*Log[a + b*x^2])/a^6
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^5\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \frac{1}{x^3\left(ab+b^2x\right)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\,\text{Subst}\left(\int \left(\frac{1}{a^4b^4x^3}-\frac{4}{a^5b^3x^2}+\frac{10}{a^6b^2x}-\frac{1}{a^3b(a+bx)^4}-\frac{3}{a^4b(a+bx)^3}-\frac{6}{a^5b(a+bx)^2}\right.\right.$$
$$= -\frac{1}{4a^4x^4}+\frac{2b}{a^5x^2}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}+\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{10b^2\log(x)}{a^6}-\frac{5b^2\,\mathrm{l}}{}$$

Mathematica [A] time = 0.054207, size = 85, normalized size = 0.84

$$\frac{\frac{a\left(110a^2b^2x^4+15a^3bx^2-3a^4+150ab^3x^6+60b^4x^8\right)}{x^4\left(a+bx^2\right)^3}-60b^2\log\left(a+bx^2\right)+120b^2\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((a*(-3*a^4 + 15*a^3*b*x^2 + 110*a^2*b^2*x^4 + 150*a*b^3*x^6 + 60*b^4*x^8))
/(x^4*(a + b*x^2)^3) + 120*b^2*Log[x] - 60*b^2*Log[a + b*x^2])/(12*a^6)
```

Maple [A] time = 0.055, size = 96, normalized size = 1.

$$-\frac{1}{4\,a^4x^4}+2\,\frac{b}{a^5x^2}+\frac{b^2}{6\,a^3\left(bx^2+a\right)^3}+\frac{3\,b^2}{4\,a^4\left(bx^2+a\right)^2}+3\,\frac{b^2}{a^5\left(bx^2+a\right)}+10\,\frac{b^2\ln(x)}{a^6}-5\,\frac{b^2\ln\left(bx^2+a\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/4/a^4/x^4+2*b/a^5/x^2+1/6*b^2/a^3/(b*x^2+a)^3+3/4*b^2/a^4/(b*x^2+a)^2+3*
b^2/a^5/(b*x^2+a)+10*b^2*ln(x)/a^6-5*b^2*ln(b*x^2+a)/a^6
```

Maxima [A] time = 1.02538, size = 154, normalized size = 1.52

$$\frac{60\,b^4x^8+150\,ab^3x^6+110\,a^2b^2x^4+15\,a^3bx^2-3\,a^4}{12\left(a^5b^3x^{10}+3\,a^6b^2x^8+3\,a^7bx^6+a^8x^4\right)}-\frac{5\,b^2\log\left(bx^2+a\right)}{a^6}+\frac{5\,b^2\log\left(x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^8 + 150*a*b^3*x^6 + 110*a^2*b^2*x^4 + 15*a^3*b*x^2 - 3*a^4)/
(a^5*b^3*x^10 + 3*a^6*b^2*x^8 + 3*a^7*b*x^6 + a^8*x^4) - 5*b^2*log(b*x^2 +
a)/a^6 + 5*b^2*log(x^2)/a^6
```

Fricas [A] time = 1.8134, size = 375, normalized size = 3.71

$$\frac{60\,ab^4x^8+150\,a^2b^3x^6+110\,a^3b^2x^4+15\,a^4bx^2-3\,a^5-60\left(b^5x^{10}+3\,ab^4x^8+3\,a^2b^3x^6+a^3b^2x^4\right)\log\left(bx^2+a\right)+120\,(b^5}{12\left(a^6b^3x^{10}+3\,a^7b^2x^8+3\,a^8bx^6+a^9x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^8 + 150*a^2*b^3*x^6 + 110*a^3*b^2*x^4 + 15*a^4*b*x^2 - 3*a
^5 - 60*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b^2*x^4)*log(b*x^2 +
a) + 120*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b^2*x^4)*log(x))/(a^
6*b^3*x^10 + 3*a^7*b^2*x^8 + 3*a^8*b*x^6 + a^9*x^4)
```

Sympy [A] time = 2.25184, size = 116, normalized size = 1.15

$$\frac{-3a^4+15a^3bx^2+110a^2b^2x^4+150ab^3x^6+60b^4x^8}{12a^8x^4+36a^7bx^6+36a^6b^2x^8+12a^5b^3x^{10}}+\frac{10b^2\log\left(x\right)}{a^6}-\frac{5b^2\log\left(\frac{a}{b}+x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x**2 + 110*a**2*b**2*x**4 + 150*a*b**3*x**6 + 60*b**4*
x**8)/(12*a**8*x**4 + 36*a**7*b*x**6 + 36*a**6*b**2*x**8 + 12*a**5*b**3*x**
10) + 10*b**2*log(x)/a**6 - 5*b**2*log(a/b + x**2)/a**6
```

Giac [A] time = 1.15233, size = 146, normalized size = 1.45

$$\frac{5\,b^2\log\left(x^2\right)}{a^6}-\frac{5\,b^2\log\left(\left|bx^2+a\right|\right)}{a^6}+\frac{110\,b^5x^6+366\,ab^4x^4+411\,a^2b^3x^2+157\,a^3b^2}{12\left(bx^2+a\right)^3a^6}-\frac{30\,b^2x^4-8\,abx^2+a^2}{4\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5*b^2*log(x^2)/a^6 - 5*b^2*log(abs(b*x^2 + a))/a^6 + 1/12*(110*b^5*x^6 + 36
6*a*b^4*x^4 + 411*a^2*b^3*x^2 + 157*a^3*b^2)/((b*x^2 + a)^3*a^6) - 1/4*(30*
b^2*x^4 - 8*a*b*x^2 + a^2)/(a^6*x^4)
```

3.502 $\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=117

$$\frac{231a^2x}{16b^6}-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{13/2}}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\frac{77ax^3}{16b^5}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}+\frac{231x^5}{80b^4}$$

```
[Out] (231*a^2*x)/(16*b^6) - (77*a*x^3)/(16*b^5) + (231*x^5)/(80*b^4) - x^11/(6*b
*(a + b*x^2)^3) - (11*x^9)/(24*b^2*(a + b*x^2)^2) - (33*x^7)/(16*b^3*(a + b
*x^2)) - (231*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(13/2))
```

Rubi [A] time = 0.072478, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{231a^2x}{16b^6}-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{13/2}}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\frac{77ax^3}{16b^5}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}+\frac{231x^5}{80b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (231*a^2*x)/(16*b^6) - (77*a*x^3)/(16*b^5) + (231*x^5)/(80*b^4) - x^11/(6*b
*(a + b*x^2)^3) - (11*x^9)/(24*b^2*(a + b*x^2)^2) - (33*x^7)/(16*b^3*(a + b
*x^2)) - (231*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^{12}}{\left(ab+b^2x^2\right)^4}\,dx \\
&= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}+\frac{1}{6}\left(11b^2\right)\int \frac{x^{10}}{\left(ab+b^2x^2\right)^3}\,dx \\
&= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}+\frac{33}{8}\int \frac{x^8}{\left(ab+b^2x^2\right)^2}\,dx \\
&= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}+\frac{231\int \frac{x^6}{ab+b^2x^2}\,dx}{16b^2} \\
&= -\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}+\frac{231\int \left(\frac{a^2}{b^4}-\frac{ax^2}{b^3}+\frac{x^4}{b^2}-\frac{a^3}{b^3\left(ab+b^2x^2\right)}\right)d}{16b^2} \\
&= \frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}+\frac{231x^5}{80b^4}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\frac{\left(231a^3\right)\int}{16} \\
&= \frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}+\frac{231x^5}{80b^4}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\frac{231a^{5/2}\,\mathrm{ta}}{16b}
\end{aligned}$$

Mathematica [A] time = 0.0568608, size = 99, normalized size = 0.85

$$\frac{1584a^2b^3x^7+7623a^3b^2x^5+9240a^4bx^3+3465a^5x-176ab^4x^9+48b^5x^{11}}{240b^6\left(a+bx^2\right)^3}-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (3465*a^5*x + 9240*a^4*b*x^3 + 7623*a^3*b^2*x^5 + 1584*a^2*b^3*x^7 - 176*a*
b^4*x^9 + 48*b^5*x^11)/(240*b^6*(a + b*x^2)^3) - (231*a^(5/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(16*b^(13/2))
```

Maple [A] time = 0.053, size = 108, normalized size = 0.9

$$\frac{x^5}{5\,b^4}-\frac{4\,ax^3}{3\,b^5}+10\,\frac{a^2x}{b^6}+\frac{89\,a^3x^5}{16\,b^4\left(bx^2+a\right)^3}+\frac{59\,a^4x^3}{6\,b^5\left(bx^2+a\right)^3}+\frac{71\,a^5x}{16\,b^6\left(bx^2+a\right)^3}-\frac{231\,a^3}{16\,b^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/5*x^5/b^4-4/3*a*x^3/b^5+10*a^2*x/b^6+89/16/b^4*a^3/(b*x^2+a)^3*x^5+59/6/b
^5*a^4/(b*x^2+a)^3*x^3+71/16/b^6*a^5/(b*x^2+a)^3*x-231/16/b^6*a^3/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73812, size = 706, normalized size = 6.03

$$\left[\frac{96\,b^5x^{11}-352\,ab^4x^9+3168\,a^2b^3x^7+15246\,a^3b^2x^5+18480\,a^4bx^3+6930\,a^5x+3465\left(a^2b^3x^6+3\,a^3b^2x^4+3\,a^4bx^2+a\right.}{480\left(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/480*(96*b^5*x^11 - 352*a*b^4*x^9 + 3168*a^2*b^3*x^7 + 15246*a^3*b^2*x^5
+ 18480*a^4*b*x^3 + 6930*a^5*x + 3465*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*
b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b
^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6), 1/240*(48*b^5*x^11 - 176*a
*b^4*x^9 + 1584*a^2*b^3*x^7 + 7623*a^3*b^2*x^5 + 9240*a^4*b*x^3 + 3465*a^5*
x - 3465*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan
(b*x*sqrt(a/b)/a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)]
```

Sympy [A] time = 0.853864, size = 172, normalized size = 1.47

$$\frac{10a^2x}{b^6} - \frac{4ax^3}{3b^5} + \frac{231\sqrt{-\frac{a^5}{b^{13}}}\log\left(x - \frac{b^6\sqrt{-\frac{a^5}{b^{13}}}}{a^2}\right)}{32} - \frac{231\sqrt{-\frac{a^5}{b^{13}}}\log\left(x + \frac{b^6\sqrt{-\frac{a^5}{b^{13}}}}{a^2}\right)}{32} + \frac{213a^5x + 472a^4bx^3 + 267a^3b^2x^5}{48a^3b^6 + 144a^2b^7x^2 + 144ab^8x^4 + 48b^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 10*a**2*x/b**6 - 4*a*x**3/(3*b**5) + 231*sqrt(-a**5/b**13)*log(x - b**6*sqr
t(-a**5/b**13)/a**2)/32 - 231*sqrt(-a**5/b**13)*log(x + b**6*sqrt(-a**5/b**
13)/a**2)/32 + (213*a**5*x + 472*a**4*b*x**3 + 267*a**3*b**2*x**5)/(48*a**3
*b**6 + 144*a**2*b**7*x**2 + 144*a*b**8*x**4 + 48*b**9*x**6) + x**5/(5*b**4
)
```

Giac [A] time = 1.12034, size = 130, normalized size = 1.11

$$-\frac{231\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^6} + \frac{267\,a^3b^2x^5 + 472\,a^4bx^3 + 213\,a^5x}{48\left(bx^2 + a\right)^3b^6} + \frac{3\,b^{16}x^5 - 20\,ab^{15}x^3 + 150\,a^2b^{14}x}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -231/16*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/48*(267*a^3*b^2*x^5 +
 472*a^4*b*x^3 + 213*a^5*x)/((b*x^2 + a)^3*b^6) + 1/15*(3*b^16*x^5 - 20*a*b
^15*x^3 + 150*a^2*b^14*x)/b^20
```

3.503 $\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^2} \, dx$

Optimal. Leaf size=104

$$\frac{105a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{11/2}} - \frac{3x^7}{8b^2\left(a+bx^2\right)^2} - \frac{21x^5}{16b^3\left(a+bx^2\right)} - \frac{105ax}{16b^5} - \frac{x^9}{6b\left(a+bx^2\right)^3} + \frac{35x^3}{16b^4}$$

```
[Out] (-105*a*x)/(16*b^5) + (35*x^3)/(16*b^4) - x^9/(6*b*(a + b*x^2)^3) - (3*x^7)
/(8*b^2*(a + b*x^2)^2) - (21*x^5)/(16*b^3*(a + b*x^2)) + (105*a^(3/2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(11/2))
```

Rubi [A] time = 0.0594884, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{105a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{11/2}} - \frac{3x^7}{8b^2\left(a+bx^2\right)^2} - \frac{21x^5}{16b^3\left(a+bx^2\right)} - \frac{105ax}{16b^5} - \frac{x^9}{6b\left(a+bx^2\right)^3} + \frac{35x^3}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```



```
[Out] (-105*a*x)/(16*b^5) + (35*x^3)/(16*b^4) - x^9/(6*b*(a + b*x^2)^3) - (3*x^7)
/(8*b^2*(a + b*x^2)^2) - (21*x^5)/(16*b^3*(a + b*x^2)) + (105*a^(3/2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(11/2))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{x^{10}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}+\frac{1}{2}\left(3b^2\right)\int \frac{x^8}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}+\frac{21}{8}\int \frac{x^6}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{105\int \frac{x^4}{ab+b^2x^2}\,dx}{16b^2}$$

$$= -\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{105\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2\left(ab+b^2x^2\right)}\right)dx}{16b^2}$$

$$= -\frac{105ax}{16b^5}+\frac{35x^3}{16b^4}-\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{\left(105a^2\right)\int \frac{1}{ab+b^2x^2}\,d}{16b^4}$$

$$= -\frac{105ax}{16b^5}+\frac{35x^3}{16b^4}-\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{105a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{16b^{11/2}}$$

Mathematica [A] time = 0.04635, size = 89, normalized size = 0.86

$$\frac{\frac{\sqrt{b}x\left(-693a^2b^2x^4-840a^3bx^2-315a^4-144ab^3x^6+16b^4x^8\right)}{\left(a+bx^2\right)^3}+315a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((Sqrt[b]*x*(-315*a^4 - 840*a^3*b*x^2 - 693*a^2*b^2*x^4 - 144*a*b^3*x^6 + 1
6*b^4*x^8))/(a + b*x^2)^3 + 315*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(48*b^
(11/2))
```

Maple [A] time = 0.053, size = 97, normalized size = 0.9

$$\frac{x^3}{3\,b^4}-4\,\frac{ax}{b^5}-\frac{55\,a^2x^5}{16\,b^3\left(bx^2+a\right)^3}-\frac{35\,a^3x^3}{6\,b^4\left(bx^2+a\right)^3}-\frac{41\,a^4x}{16\,b^5\left(bx^2+a\right)^3}+\frac{105\,a^2}{16\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/3*x^3/b^4-4*a*x/b^5-55/16/b^3*a^2/(b*x^2+a)^3*x^5-35/6/b^4*a^3/(b*x^2+a)^
3*x^3-41/16/b^5*a^4/(b*x^2+a)^3*x+105/16/b^5*a^2/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75441, size = 633, normalized size = 6.09

$$\left[\frac{32\,b^4x^9-288\,ab^3x^7-1386\,a^2b^2x^5-1680\,a^3bx^3-630\,a^4x+315\left(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}}{bx^2+a}\right.}{96\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(32*b^4*x^9 - 288*a*b^3*x^7 - 1386*a^2*b^2*x^5 - 1680*a^3*b*x^3 - 630
*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*sqrt(-a/b)*log
((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^
2*b^6*x^2 + a^3*b^5), 1/48*(16*b^4*x^9 - 144*a*b^3*x^7 - 693*a^2*b^2*x^5 -
840*a^3*b*x^3 - 315*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 +
a^4)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*
x^2 + a^3*b^5)]
```

Sympy [A] time = 0.812077, size = 155, normalized size = 1.49

$$-\frac{4ax}{b^5}-\frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32}+\frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32}-\frac{123a^4x+280a^3bx^3+165a^2b^2x^5}{48a^3b^5+144a^2b^6x^2+144ab^7x^4+48b^8x^6}+\frac{x^3}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -4*a*x/b**5 - 105*sqrt(-a**3/b**11)*log(x - b**5*sqrt(-a**3/b**11)/a)/32 +
105*sqrt(-a**3/b**11)*log(x + b**5*sqrt(-a**3/b**11)/a)/32 - (123*a**4*x +
280*a**3*b*x**3 + 165*a**2*b**2*x**5)/(48*a**3*b**5 + 144*a**2*b**6*x**2 +
144*a*b**7*x**4 + 48*b**8*x**6) + x**3/(3*b**4)
```

Giac [A] time = 1.13797, size = 113, normalized size = 1.09

$$\frac{105\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^5}-\frac{165\,a^2b^2x^5+280\,a^3bx^3+123\,a^4x}{48\left(bx^2+a\right)^3b^5}+\frac{b^8x^3-12\,ab^7x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 105/16*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/48*(165*a^2*b^2*x^5 +
280*a^3*b*x^3 + 123*a^4*x)/((b*x^2 + a)^3*b^5) + 1/3*(b^8*x^3 - 12*a*b^7*x)
```

/b^12

3.504 $\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{9/2}}-\frac{x^7}{6b\left(a+bx^2\right)^3}+\frac{35x}{16b^4}$$

```
[Out] (35*x)/(16*b^4) - x^7/(6*b*(a + b*x^2)^3) - (7*x^5)/(24*b^2*(a + b*x^2)^2)
- (35*x^3)/(48*b^3*(a + b*x^2)) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(16*b^(9/2))
```

Rubi [A] time = 0.0466813, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 321, 205}

$$-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{9/2}}-\frac{x^7}{6b\left(a+bx^2\right)^3}+\frac{35x}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (35*x)/(16*b^4) - x^7/(6*b*(a + b*x^2)^3) - (7*x^5)/(24*b^2*(a + b*x^2)^2)
- (35*x^3)/(48*b^3*(a + b*x^2)) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(16*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^8}{\left(ab+b^2x^2\right)^4}\,dx\\
&= -\frac{x^7}{6b\left(a+bx^2\right)^3}+\frac{1}{6}\left(7b^2\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^3}\,dx\\
&= -\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}+\frac{35}{24}\int \frac{x^4}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}+\frac{35\int \frac{x^2}{ab+b^2x^2}\,dx}{16b^2}\\
&= \frac{35x}{16b^4}-\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{(35a)\int \frac{1}{ab+b^2x^2}\,dx}{16b^3}\\
&= \frac{35x}{16b^4}-\frac{x^7}{6b\left(a+bx^2\right)^3}-\frac{7x^5}{24b^2\left(a+bx^2\right)^2}-\frac{35x^3}{48b^3\left(a+bx^2\right)}-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0438906, size = 77, normalized size = 0.83

$$\frac{280a^2bx^3+105a^3x+231ab^2x^5+48b^3x^7}{48b^4\left(a+bx^2\right)^3}-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (105*a^3*x + 280*a^2*b*x^3 + 231*a*b^2*x^5 + 48*b^3*x^7)/(48*b^4*(a + b*x^2
)^3) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(9/2))
```

Maple [A] time = 0.054, size = 83, normalized size = 0.9

$$\frac{x}{b^4}+\frac{29\,ax^5}{16\,b^2\left(bx^2+a\right)^3}+\frac{17\,a^2x^3}{6\,b^3\left(bx^2+a\right)^3}+\frac{19\,xa^3}{16\,b^4\left(bx^2+a\right)^3}-\frac{35\,a}{16\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] x/b^4+29/16/b^2*a/(b*x^2+a)^3*x^5+17/6/b^3*a^2/(b*x^2+a)^3*x^3+19/16/b^4*a^
3/(b*x^2+a)^3*x-35/16/b^4*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73532, size = 571, normalized size = 6.14

$$\left[\frac{96\,b^3x^7+462\,ab^2x^5+560\,a^2bx^3+210\,a^3x+105\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{96\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)},\ \frac{48\,b^3x^7+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(96*b^3*x^7 + 462*a*b^2*x^5 + 560*a^2*b*x^3 + 210*a^3*x + 105*(b^3*x^
6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/
b) - a)/(b*x^2 + a)))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4), 1/
48*(48*b^3*x^7 + 231*a*b^2*x^5 + 280*a^2*b*x^3 + 105*a^3*x - 105*(b^3*x^6 +
 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x
^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [A] time = 0.744338, size = 131, normalized size = 1.41

$$\frac{35\sqrt{-\frac{a}{b^9}}\log\left(-b^4\sqrt{-\frac{a}{b^9}}+x\right)}{32}-\frac{35\sqrt{-\frac{a}{b^9}}\log\left(b^4\sqrt{-\frac{a}{b^9}}+x\right)}{32}+\frac{57a^3x+136a^2bx^3+87ab^2x^5}{48a^3b^4+144a^2b^5x^2+144ab^6x^4+48b^7x^6}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 35*sqrt(-a/b**9)*log(-b**4*sqrt(-a/b**9) + x)/32 - 35*sqrt(-a/b**9)*log(b**
4*sqrt(-a/b**9) + x)/32 + (57*a**3*x + 136*a**2*b*x**3 + 87*a*b**2*x**5)/(4
8*a**3*b**4 + 144*a**2*b**5*x**2 + 144*a*b**6*x**4 + 48*b**7*x**6) + x/b**4
```

Giac [A] time = 1.1114, size = 88, normalized size = 0.95

$$-\frac{35\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^4}+\frac{x}{b^4}+\frac{87\,ab^2x^5+136\,a^2bx^3+57\,a^3x}{48\,\left(bx^2+a\right)^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -35/16*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + x/b^4 + 1/48*(87*a*b^2*x^5
 + 136*a^2*b*x^3 + 57*a^3*x)/((b*x^2 + a)^3*b^4)
```

3.505 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{5x}{16b^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{a}b^{7/2}}-\frac{x^5}{6b\left(a+bx^2\right)^3}$$

```
[Out] -x^5/(6*b*(a + b*x^2)^3) - (5*x^3)/(24*b^2*(a + b*x^2)^2) - (5*x)/(16*b^3*(
a + b*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

Rubi [A] time = 0.039702, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 288, 205}

$$-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{5x}{16b^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{a}b^{7/2}}-\frac{x^5}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x^5/(6*b*(a + b*x^2)^3) - (5*x^3)/(24*b^2*(a + b*x^2)^2) - (5*x)/(16*b^3*(
a + b*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^6}{\left(ab+b^2x^2\right)^4}\,dx\\
&= -\frac{x^5}{6b\left(a+bx^2\right)^3}+\frac{1}{6}\left(5b^2\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^3}\,dx\\
&= -\frac{x^5}{6b\left(a+bx^2\right)^3}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}+\frac{5}{8}\int \frac{x^2}{\left(ab+b^2x^2\right)^2}\,dx\\
&= -\frac{x^5}{6b\left(a+bx^2\right)^3}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{5x}{16b^3\left(a+bx^2\right)}+\frac{5\int \frac{1}{ab+b^2x^2}\,dx}{16b^2}\\
&= -\frac{x^5}{6b\left(a+bx^2\right)^3}-\frac{5x^3}{24b^2\left(a+bx^2\right)^2}-\frac{5x}{16b^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{a}b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.037152, size = 66, normalized size = 0.8

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{a}b^{7/2}}-\frac{x\left(15a^2+40abx^2+33b^2x^4\right)}{48b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(x*(15*a^2 + 40*a*b*x^2 + 33*b^2*x^4))/(48*b^3*(a + b*x^2)^3) + (5*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

Maple [A] time = 0.049, size = 58, normalized size = 0.7

$$\frac{1}{\left(bx^2+a\right)^3}\left(-\frac{11\,x^5}{16\,b}-\frac{5\,ax^3}{6\,b^2}-\frac{5\,a^2x}{16\,b^3}\right)+\frac{5}{16\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (-11/16/b*x^5-5/6*a*x^3/b^2-5/16*a^2/b^3*x)/(b*x^2+a)^3+5/16/b^3/(a*b)^(1/2
)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66429, size = 544, normalized size = 6.55

$$\left[-\frac{66\,ab^3x^5+80\,a^2b^2x^3+30\,a^3bx+15\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{96\left(ab^7x^6+3\,a^2b^6x^4+3\,a^3b^5x^2+a^4b^4\right)},-\frac{33\,ab^3x^5+40\,a^2b^2x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(66*a*b^3*x^5 + 80*a^2*b^2*x^3 + 30*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2*
x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)))/(a*b^7*x^6 + 3*a^2*b^6*x^4 + 3*a^3*b^5*x^2 + a^4*b^4), -1/48*(33*a*
b^3*x^5 + 40*a^2*b^2*x^3 + 15*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b
*x^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^7*x^6 + 3*a^2*b^6*x^4 + 3
*a^3*b^5*x^2 + a^4*b^4)]
```

Sympy [A] time = 0.657521, size = 133, normalized size = 1.6

$$-\frac{5\sqrt{-\frac{1}{ab^7}}\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{ab^7}}\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{32}-\frac{15a^2x+40abx^3+33b^2x^5}{48a^3b^3+144a^2b^4x^2+144ab^5x^4+48b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -5*sqrt(-1/(a*b**7))*log(-a*b**3*sqrt(-1/(a*b**7)) + x)/32 + 5*sqrt(-1/(a*b
**7))*log(a*b**3*sqrt(-1/(a*b**7)) + x)/32 - (15*a**2*x + 40*a*b*x**3 + 33*
b**2*x**5)/(48*a**3*b**3 + 144*a**2*b**4*x**2 + 144*a*b**5*x**4 + 48*b**6*x
**6)
```

Giac [A] time = 1.13428, size = 76, normalized size = 0.92

$$\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}b^3}-\frac{33\,b^2x^5+40\,abx^3+15\,a^2x}{48\left(bx^2+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/48*(33*b^2*x^5 + 40*a*b*x^3
+ 15*a^2*x)/((b*x^2 + a)^3*b^3)
```

3.506 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=84

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{x}{16ab^2\left(a+bx^2\right)}-\frac{x}{8b^2\left(a+bx^2\right)^2}-\frac{x^3}{6b\left(a+bx^2\right)^3}$$

```
[Out] -x^3/(6*b*(a + b*x^2)^3) - x/(8*b^2*(a + b*x^2)^2) + x/(16*a*b^2*(a + b*x^2
)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.0411935, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{x}{16ab^2\left(a+bx^2\right)}-\frac{x}{8b^2\left(a+bx^2\right)^2}-\frac{x^3}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x^3/(6*b*(a + b*x^2)^3) - x/(8*b^2*(a + b*x^2)^2) + x/(16*a*b^2*(a + b*x^2
)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^4}{\left(ab+b^2x^2\right)^4}\,dx \\
&= -\frac{x^3}{6b\left(a+bx^2\right)^3}+\frac{1}{2}b^2\int \frac{x^2}{\left(ab+b^2x^2\right)^3}\,dx \\
&= -\frac{x^3}{6b\left(a+bx^2\right)^3}-\frac{x}{8b^2\left(a+bx^2\right)^2}+\frac{1}{8}\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx \\
&= -\frac{x^3}{6b\left(a+bx^2\right)^3}-\frac{x}{8b^2\left(a+bx^2\right)^2}+\frac{x}{16ab^2\left(a+bx^2\right)}+\frac{\int \frac{1}{ab+b^2x^2}\,dx}{16ab} \\
&= -\frac{x^3}{6b\left(a+bx^2\right)^3}-\frac{x}{8b^2\left(a+bx^2\right)^2}+\frac{x}{16ab^2\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0448912, size = 69, normalized size = 0.82

$$
\frac{-3a^2x-8abx^3+3b^2x^5}{48ab^2\left(a+bx^2\right)^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-3*a^2*x - 8*a*b*x^3 + 3*b^2*x^5)/(48*a*b^2*(a + b*x^2)^3) + ArcTan[(Sqrt[
b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Maple [A] time = 0.049, size = 58, normalized size = 0.7

$$\frac{1}{\left(bx^2+a\right)^3}\left(\frac{x^5}{16\,a}-\frac{x^3}{6\,b}-\frac{ax}{16\,b^2}\right)+\frac{1}{16\,b^2a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (1/16/a*x^5-1/6/b*x^3-1/16/b^2*a*x)/(b*x^2+a)^3+1/16/b^2/a/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80431, size = 537, normalized size = 6.39

$$\left[\frac{6\,ab^3x^5-16\,a^2b^2x^3-6\,a^3bx-3\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{96\left(a^2b^6x^6+3\,a^3b^5x^4+3\,a^4b^4x^2+a^5b^3\right)},\frac{3\,ab^3x^5-8\,a^2b^2x^3-3\,a^3bx\,\cdots}{48\left(a^2b\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(6*a*b^3*x^5 - 16*a^2*b^2*x^3 - 6*a^3*b*x - 3*(b^3*x^6 + 3*a*b^2*x^4
+ 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
```

```
)))/(a^2*b^6*x^6 + 3*a^3*b^5*x^4 + 3*a^4*b^4*x^2 + a^5*b^3), 1/48*(3*a*b^3*
x^5 - 8*a^2*b^2*x^3 - 3*a^3*b*x + 3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 +
a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^6*x^6 + 3*a^3*b^5*x^4 + 3*a^4*
b^4*x^2 + a^5*b^3)]
```

Sympy [B] time = 0.618193, size = 143, normalized size = 1.7

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32}+\frac{-3a^2x-8abx^3+3b^2x^5}{48a^4b^2+144a^3b^3x^2+144a^2b^4x^4+48ab^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32 + sqrt(-1
/(a**3*b**5))*log(a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32 + (-3*a**2*x - 8*a
*b*x**3 + 3*b**2*x**5)/(48*a**4*b**2 + 144*a**3*b**3*x**2 + 144*a**2*b**4*x
**4 + 48*a*b**5*x**6)
```

Giac [A] time = 1.15908, size = 84, normalized size = 1.

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}ab^2}+\frac{3\,b^2x^5-8\,abx^3-3\,a^2x}{48\left(bx^2+a\right)^3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/48*(3*b^2*x^5 - 8*a*b*x^3
- 3*a^2*x)/((b*x^2 + a)^3*a*b^2)
```

3.507 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{x}{24ab\left(a+bx^2\right)^2}-\frac{x}{6b\left(a+bx^2\right)^3}$$

```
[Out] -x/(6*b*(a + b*x^2)^3) + x/(24*a*b*(a + b*x^2)^2) + x/(16*a^2*b*(a + b*x^2)
) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.0426251, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{x}{24ab\left(a+bx^2\right)^2}-\frac{x}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x/(6*b*(a + b*x^2)^3) + x/(24*a*b*(a + b*x^2)^2) + x/(16*a^2*b*(a + b*x^2)
) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x^2}{\left(ab+b^2x^2\right)^4}\,dx\\
&= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{1}{6}b^2\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx\\
&= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{x}{24ab\left(a+bx^2\right)^2}+\frac{b\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{8a}\\
&= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{x}{24ab\left(a+bx^2\right)^2}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{\int \frac{1}{ab+b^2x^2}\,dx}{16a^2}\\
&= -\frac{x}{6b\left(a+bx^2\right)^3}+\frac{x}{24ab\left(a+bx^2\right)^2}+\frac{x}{16a^2b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.037126, size = 69, normalized size = 0.81

$$
\frac{-3a^2x+8abx^3+3b^2x^5}{48a^2b\left(a+bx^2\right)^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-3*a^2*x + 8*a*b*x^3 + 3*b^2*x^5)/(48*a^2*b*(a + b*x^2)^3) + ArcTan[(Sqrt[
b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Maple [A] time = 0.049, size = 58, normalized size = 0.7

$$\frac{1}{\left(bx^{2}+a\right)^{3}}\left(\frac{bx^{5}}{16\,a^{2}}+\frac{x^{3}}{6\,a}-\frac{x}{16\,b}\right)+\frac{1}{16\,ba^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (1/16*b/a^2*x^5+1/6/a*x^3-1/16*x/b)/(b*x^2+a)^3+1/16/b/a^2/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72973, size = 537, normalized size = 6.32

$$\left[\frac{6\,ab^{3}x^{5}+16\,a^{2}b^{2}x^{3}-6\,a^{3}bx-3\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\sqrt{-ab}\log\left(\frac{bx^{2}-2\,\sqrt{-ab}x-a}{bx^{2}+a}\right)}{96\left(a^{3}b^{5}x^{6}+3\,a^{4}b^{4}x^{4}+3\,a^{5}b^{3}x^{2}+a^{6}b^{2}\right)},\frac{3\,ab^{3}x^{5}+8\,a^{2}b^{2}x^{3}-3\,a^{3}bx}{48\left(a^{3}b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(6*a*b^3*x^5 + 16*a^2*b^2*x^3 - 6*a^3*b*x - 3*(b^3*x^6 + 3*a*b^2*x^4
+ 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
```

```
)))/(a^3*b^5*x^6 + 3*a^4*b^4*x^4 + 3*a^5*b^3*x^2 + a^6*b^2), 1/48*(3*a*b^3*
x^5 + 8*a^2*b^2*x^3 - 3*a^3*b*x + 3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 +
a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^5*x^6 + 3*a^4*b^4*x^4 + 3*a^5*
b^3*x^2 + a^6*b^2)]
```

Sympy [B] time = 0.611216, size = 139, normalized size = 1.64

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^5b^3}}\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32}+\frac{-3a^2x+8abx^3+3b^2x^5}{48a^5b+144a^4b^2x^2+144a^3b^3x^4+48a^2b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + sqrt(-1/(a
**5*b**3))*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + (-3*a**2*x + 8*a*b*x**
3 + 3*b**2*x**5)/(48*a**5*b + 144*a**4*b**2*x**2 + 144*a**3*b**3*x**4 + 48*
a**2*b**4*x**6)
```

Giac [A] time = 1.16199, size = 84, normalized size = 0.99

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}a^2b}+\frac{3\,b^2x^5+8\,abx^3-3\,a^2x}{48\left(bx^2+a\right)^3a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/48*(3*b^2*x^5 + 8*a*b*x^3
- 3*a^2*x)/((b*x^2 + a)^3*a^2*b)
```

3.508 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=79

$$\frac{5x}{16a^3(a+bx^2)}+\frac{5x}{24a^2(a+bx^2)^2}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{x}{6a(a+bx^2)^3}$$

```
[Out] x/(6*a*(a + b*x^2)^3) + (5*x)/(24*a^2*(a + b*x^2)^2) + (5*x)/(16*a^3*(a + b
*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Rubi [A] time = 0.036823, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {28, 199, 205}

$$\frac{5x}{16a^3(a+bx^2)}+\frac{5x}{24a^2(a+bx^2)^2}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{x}{6a(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

```
[Out] x/(6*a*(a + b*x^2)^3) + (5*x)/(24*a^2*(a + b*x^2)^2) + (5*x)/(16*a^3*(a + b
*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx\\
&= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{\left(5b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{6a}\\
&= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{\left(5b^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{8a^2}\\
&= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{(5b)\int \frac{1}{ab+b^2x^2}\,dx}{16a^3}\\
&= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.0344089, size = 66, normalized size = 0.84

$$\frac{33a^2x+40abx^3+15b^2x^5}{48a^3\left(a+bx^2\right)^3}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

```
[Out] (33*a^2*x + 40*a*b*x^3 + 15*b^2*x^5)/(48*a^3*(a + b*x^2)^3) + (5*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Maple [A] time = 0.045, size = 66, normalized size = 0.8

$$\frac{x}{6\,a\left(bx^2+a\right)^3}+\frac{5\,x}{24\,a^2\left(bx^2+a\right)^2}+\frac{5\,x}{16\,a^3\left(bx^2+a\right)}+\frac{5}{16\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*x/a/(b*x^2+a)^3+5/24*x/a^2/(b*x^2+a)^2+5/16*x/a^3/(b*x^2+a)+5/16/a^3/(a
*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78971, size = 541, normalized size = 6.85

$$\left[\frac{30\,ab^3x^5+80\,a^2b^2x^3+66\,a^3bx-15\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{96\left(a^4b^4x^6+3\,a^5b^3x^4+3\,a^6b^2x^2+a^7b\right)},\frac{15\,ab^3x^5+40\,a^2b^2x^3+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(30*a*b^3*x^5 + 80*a^2*b^2*x^3 + 66*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*x
^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
+ a)))/(a^4*b^4*x^6 + 3*a^5*b^3*x^4 + 3*a^6*b^2*x^2 + a^7*b), 1/48*(15*a*b^
3*x^5 + 40*a^2*b^2*x^3 + 33*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x
^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^4*x^6 + 3*a^5*b^3*x^4 + 3
*a^6*b^2*x^2 + a^7*b)]
```

Sympy [A] time = 0.62227, size = 129, normalized size = 1.63

$$-\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32}+\frac{33a^2x+40abx^3+15b^2x^5}{48a^6+144a^5bx^2+144a^4b^2x^4+48a^3b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -5*sqrt(-1/(a**7*b))*log(-a**4*sqrt(-1/(a**7*b)) + x)/32 + 5*sqrt(-1/(a**7*
b))*log(a**4*sqrt(-1/(a**7*b)) + x)/32 + (33*a**2*x + 40*a*b*x**3 + 15*b**2
*x**5)/(48*a**6 + 144*a**5*b*x**2 + 144*a**4*b**2*x**4 + 48*a**3*b**3*x**6)
```

Giac [A] time = 1.15096, size = 76, normalized size = 0.96

$$\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}a^3}+\frac{15\,b^2x^5+40\,abx^3+33\,a^2x}{48\left(bx^2+a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/48*(15*b^2*x^5 + 40*a*b*x^3
+ 33*a^2*x)/((b*x^2 + a)^3*a^3)
```

3.509 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{35}{48a^3x\left(a+bx^2\right)}+\frac{7}{24a^2x\left(a+bx^2\right)^2}-\frac{35\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{35}{16a^4x}+\frac{1}{6ax\left(a+bx^2\right)^3}$$

```
[Out] -35/(16*a^4*x) + 1/(6*a*x*(a + b*x^2)^3) + 7/(24*a^2*x*(a + b*x^2)^2) + 35/
(48*a^3*x*(a + b*x^2)) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(9/
2))
```

Rubi [A] time = 0.0560155, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{35}{48a^3x\left(a+bx^2\right)}+\frac{7}{24a^2x\left(a+bx^2\right)^2}-\frac{35\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{35}{16a^4x}+\frac{1}{6ax\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -35/(16*a^4*x) + 1/(6*a*x*(a + b*x^2)^3) + 7/(24*a^2*x*(a + b*x^2)^2) + 35/
(48*a^3*x*(a + b*x^2)) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(9/
2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx\\
&= \frac{1}{6ax\left(a+bx^2\right)^3}+\frac{\left(7b^3\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^3}\,dx}{6a}\\
&= \frac{1}{6ax\left(a+bx^2\right)^3}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{\left(35b^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx}{24a^2}\\
&= \frac{1}{6ax\left(a+bx^2\right)^3}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{35}{48a^3x\left(a+bx^2\right)}+\frac{(35b)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{16a^3}\\
&= -\frac{35}{16a^4x}+\frac{1}{6ax\left(a+bx^2\right)^3}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{35}{48a^3x\left(a+bx^2\right)}-\frac{\left(35b^2\right)\int \frac{1}{ab+b^2x^2}\,dx}{16a^4}\\
&= -\frac{35}{16a^4x}+\frac{1}{6ax\left(a+bx^2\right)^3}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{35}{48a^3x\left(a+bx^2\right)}-\frac{35\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.0427745, size = 79, normalized size = 0.83

$$-\frac{231a^2bx^2+48a^3+280ab^2x^4+105b^3x^6}{48a^4x\left(a+bx^2\right)^3}-\frac{35\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -(48*a^3 + 231*a^2*b*x^2 + 280*a*b^2*x^4 + 105*b^3*x^6)/(48*a^4*x*(a + b*x^
2)^3) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(9/2))
```

Maple [A] time = 0.055, size = 86, normalized size = 0.9

$$-\frac{1}{a^4x}-\frac{19\,b^3x^5}{16\,a^4\left(bx^2+a\right)^3}-\frac{17\,b^2x^3}{6\,a^3\left(bx^2+a\right)^3}-\frac{29\,bx}{16\,a^2\left(bx^2+a\right)^3}-\frac{35\,b}{16\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/a^4/x-19/16/a^4*b^3/(b*x^2+a)^3*x^5-17/6/a^3*b^2/(b*x^2+a)^3*x^3-29/16/a
^2*b/(b*x^2+a)^3*x-35/16/a^4*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86338, size = 574, normalized size = 6.04

$$\left[-\frac{210\,b^3x^6+560\,ab^2x^4+462\,a^2bx^2+96\,a^3-105\left(b^3x^7+3\,ab^2x^5+3\,a^2bx^3+a^3x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{96\left(a^4b^3x^7+3\,a^5b^2x^5+3\,a^6bx^3+a^7x\right)},-\frac{105\,b^3x^6}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(210*b^3*x^6 + 560*a*b^2*x^4 + 462*a^2*b*x^2 + 96*a^3 - 105*(b^3*x^7
 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b
/a) - a)/(b*x^2 + a)))/(a^4*b^3*x^7 + 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x),
 -1/48*(105*b^3*x^6 + 280*a*b^2*x^4 + 231*a^2*b*x^2 + 48*a^3 + 105*(b^3*x^7
 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b
^3*x^7 + 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x)]
```

Sympy [A] time = 0.976444, size = 138, normalized size = 1.45

$$\frac{35\sqrt{-\frac{b}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}}{b}+x\right)}{32}-\frac{35\sqrt{-\frac{b}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}}{b}+x\right)}{32}-\frac{48a^3+231a^2bx^2+280ab^2x^4+105b^3x^6}{48a^7x+144a^6bx^3+144a^5b^2x^5+48a^4b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 35*sqrt(-b/a**9)*log(-a**5*sqrt(-b/a**9)/b + x)/32 - 35*sqrt(-b/a**9)*log(a
**5*sqrt(-b/a**9)/b + x)/32 - (48*a**3 + 231*a**2*b*x**2 + 280*a*b**2*x**4
+ 105*b**3*x**6)/(48*a**7*x + 144*a**6*b*x**3 + 144*a**5*b**2*x**5 + 48*a**
4*b**3*x**7)
```

Giac [A] time = 1.11721, size = 92, normalized size = 0.97

$$-\frac{35\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^4}-\frac{1}{a^4x}-\frac{57\,b^3x^5+136\,ab^2x^3+87\,a^2bx}{48\,\left(bx^2+a\right)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -35/16*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/(a^4*x) - 1/48*(57*b^3*x
^5 + 136*a*b^2*x^3 + 87*a^2*b*x)/((b*x^2 + a)^3*a^4)
```

3.510 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=106

$$\frac{105b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{11/2}}+\frac{21}{16a^3x^3\left(a+bx^2\right)}+\frac{3}{8a^2x^3\left(a+bx^2\right)^2}+\frac{105b}{16a^5x}-\frac{35}{16a^4x^3}+\frac{1}{6ax^3\left(a+bx^2\right)^3}$$

```
[Out] -35/(16*a^4*x^3) + (105*b)/(16*a^5*x) + 1/(6*a*x^3*(a + b*x^2)^3) + 3/(8*a^
2*x^3*(a + b*x^2)^2) + 21/(16*a^3*x^3*(a + b*x^2)) + (105*b^(3/2)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(16*a^(11/2))
```

Rubi [A] time = 0.0661874, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{105b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{11/2}}+\frac{21}{16a^3x^3\left(a+bx^2\right)}+\frac{3}{8a^2x^3\left(a+bx^2\right)^2}+\frac{105b}{16a^5x}-\frac{35}{16a^4x^3}+\frac{1}{6ax^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -35/(16*a^4*x^3) + (105*b)/(16*a^5*x) + 1/(6*a*x^3*(a + b*x^2)^3) + 3/(8*a^
2*x^3*(a + b*x^2)^2) + 21/(16*a^3*x^3*(a + b*x^2)) + (105*b^(3/2)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(16*a^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^4\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{\left(3b^3\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^3}\,dx}{2a}$$

$$= \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{\left(21b^2\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^2}\,dx}{8a^2}$$

$$= \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} + \frac{(105b)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{16a^3}$$

$$= -\frac{35}{16a^4x^3} + \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} - \frac{\left(105b^2\right)\int \frac{1}{x^2\left(ab+b^2 \ldots\right.}}{16a^4}$$

$$= -\frac{35}{16a^4x^3} + \frac{105b}{16a^5x} + \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} + \frac{\left(105b^3\right) \ldots}{1 \ldots}$$

$$= -\frac{35}{16a^4x^3} + \frac{105b}{16a^5x} + \frac{1}{6ax^3\left(a+bx^2\right)^3} + \frac{3}{8a^2x^3\left(a+bx^2\right)^2} + \frac{21}{16a^3x^3\left(a+bx^2\right)} + \frac{105b^{3/2}\ldots}{16\ldots}$$

Mathematica [A] time = 0.0474373, size = 91, normalized size = 0.86

$$\frac{\frac{\sqrt{a}\left(693a^2b^2x^4+144a^3bx^2-16a^4+840ab^3x^6+315b^4x^8\right)}{x^3\left(a+bx^2\right)^3}+315b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((Sqrt[a]*(-16*a^4 + 144*a^3*b*x^2 + 693*a^2*b^2*x^4 + 840*a*b^3*x^6 + 315*
b^4*x^8))/(x^3*(a + b*x^2)^3) + 315*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(4
8*a^(11/2))
```

Maple [A] time = 0.056, size = 99, normalized size = 0.9

$$-\frac{1}{3\,a^4x^3}+4\,\frac{b}{a^5x}+\frac{41\,b^4x^5}{16\,a^5\left(bx^2+a\right)^3}+\frac{35\,b^3x^3}{6\,a^4\left(bx^2+a\right)^3}+\frac{55\,b^2x}{16\,a^3\left(bx^2+a\right)^3}+\frac{105\,b^2}{16\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/3/a^4/x^3+4*b/a^5/x+41/16/a^5*b^4/(b*x^2+a)^3*x^5+35/6/a^4*b^3/(b*x^2+a)
^3*x^3+55/16/a^3*b^2/(b*x^2+a)^3*x+105/16/a^5*b^2/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76567, size = 644, normalized size = 6.08

$$\left[\frac{630\,b^4x^8+1680\,ab^3x^6+1386\,a^2b^2x^4+288\,a^3bx^2-32\,a^4+315\left(b^4x^9+3\,ab^3x^7+3\,a^2b^2x^5+a^3bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{96\left(a^5b^3x^9+3\,a^6b^2x^7+3\,a^7bx^5+a^8x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(630*b^4*x^8 + 1680*a*b^3*x^6 + 1386*a^2*b^2*x^4 + 288*a^3*b*x^2 - 32
*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5 + a^3*b*x^3)*sqrt(-b/a)*l
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^3*x^9 + 3*a^6*b^2*x^
7 + 3*a^7*b*x^5 + a^8*x^3), 1/48*(315*b^4*x^8 + 840*a*b^3*x^6 + 693*a^2*b^2
*x^4 + 144*a^3*b*x^2 - 16*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5
+ a^3*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^5*b^3*x^9 + 3*a^6*b^2*x^7 +
3*a^7*b*x^5 + a^8*x^3)]
```

Sympy [A] time = 1.62734, size = 162, normalized size = 1.53

$$-\frac{105\sqrt{-\frac{b^3}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2}+x\right)}{32}+\frac{105\sqrt{-\frac{b^3}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2}+x\right)}{32}+\frac{-16a^4+144a^3bx^2+693a^2b^2x^4+840ab^3x^6+315b^4x^8}{48a^8x^3+144a^7bx^5+144a^6b^2x^7+48a^5b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -105*sqrt(-b**3/a**11)*log(-a**6*sqrt(-b**3/a**11)/b**2 + x)/32 + 105*sqrt(
-b**3/a**11)*log(a**6*sqrt(-b**3/a**11)/b**2 + x)/32 + (-16*a**4 + 144*a**3
*b*x**2 + 693*a**2*b**2*x**4 + 840*a*b**3*x**6 + 315*b**4*x**8)/(48*a**8*x*
*3 + 144*a**7*b*x**5 + 144*a**6*b**2*x**7 + 48*a**5*b**3*x**9)
```

Giac [A] time = 1.15613, size = 111, normalized size = 1.05

$$\frac{105\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^5}+\frac{315\,b^4x^8+840\,ab^3x^6+693\,a^2b^2x^4+144\,a^3bx^2-16\,a^4}{48\left(bx^3+ax\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 105/16*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/48*(315*b^4*x^8 + 840*
a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16*a^4)/((b*x^3 + a*x)^3*a^5)
```

3.511 $\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{231b^2}{16a^6x}-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{13/2}}+\frac{77b}{16a^5x^3}+\frac{33}{16a^3x^5\left(a+bx^2\right)}+\frac{11}{24a^2x^5\left(a+bx^2\right)^2}-\frac{231}{80a^4x^5}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

```
[Out] -231/(80*a^4*x^5) + (77*b)/(16*a^5*x^3) - (231*b^2)/(16*a^6*x) + 1/(6*a*x^5
*(a + b*x^2)^3) + 11/(24*a^2*x^5*(a + b*x^2)^2) + 33/(16*a^3*x^5*(a + b*x^2
)) - (231*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

Rubi [A] time = 0.0785271, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{231b^2}{16a^6x}-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{13/2}}+\frac{77b}{16a^5x^3}+\frac{33}{16a^3x^5\left(a+bx^2\right)}+\frac{11}{24a^2x^5\left(a+bx^2\right)^2}-\frac{231}{80a^4x^5}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -231/(80*a^4*x^5) + (77*b)/(16*a^5*x^3) - (231*b^2)/(16*a^6*x) + 1/(6*a*x^5
*(a + b*x^2)^3) + 11/(24*a^2*x^5*(a + b*x^2)^2) + 33/(16*a^3*x^5*(a + b*x^2
)) - (231*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^6\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{\left(11b^3\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^3}\,dx}{6a}$$

$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{\left(33b^2\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^2}\,dx}{8a^2}$$

$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)} + \frac{(231b)\int \frac{1}{x^6\left(ab+b^2x^2\right)}\,dx}{16a^3}$$

$$= -\frac{231}{80a^4x^5} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)} - \frac{\left(231b^2\right)\int \frac{1}{x^4\left(ab+\right.}}{16a^4}$$

$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)} + \frac{\left(231b\right.}{}$$

$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} - \frac{231b^2}{16a^6x} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)}$$

$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} - \frac{231b^2}{16a^6x} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.051214, size = 101, normalized size = 0.85

$$-\frac{7623a^2b^3x^6+1584a^3b^2x^4-176a^4bx^2+48a^5+9240ab^4x^8+3465b^5x^{10}}{240a^6x^5\left(a+bx^2\right)^3} - \frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -(48*a^5 - 176*a^4*b*x^2 + 1584*a^3*b^2*x^4 + 7623*a^2*b^3*x^6 + 9240*a*b^4
*x^8 + 3465*b^5*x^10)/(240*a^6*x^5*(a + b*x^2)^3) - (231*b^(5/2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

Maple [A] time = 0.056, size = 110, normalized size = 0.9

$$-\frac{1}{5a^4x^5}-10\frac{b^2}{a^6x}+\frac{4b}{3a^5x^3}-\frac{71b^5x^5}{16a^6\left(bx^2+a\right)^3}-\frac{59b^4x^3}{6a^5\left(bx^2+a\right)^3}-\frac{89b^3x}{16a^4\left(bx^2+a\right)^3}-\frac{231b^3}{16a^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/5/a^4/x^5-10*b^2/a^6/x+4/3*b/a^5/x^3-71/16/a^6*b^5/(b*x^2+a)^3*x^5-59/6/
a^5*b^4/(b*x^2+a)^3*x^3-89/16/a^4*b^3/(b*x^2+a)^3*x-231/16/a^6*b^3/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7744, size = 725, normalized size = 6.09

$$\left[-\frac{6930\,b^5x^{10}+18480\,ab^4x^8+15246\,a^2b^3x^6+3168\,a^3b^2x^4-352\,a^4bx^2+96\,a^5-3465\left(b^5x^{11}+3\,ab^4x^9+3\,a^2b^3x^7+a^3\right.}{480\left(a^6b^3x^{11}+3\,a^7b^2x^9+3\,a^8bx^7+a^9x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/480*(6930*b^5*x^10 + 18480*a*b^4*x^8 + 15246*a^2*b^3*x^6 + 3168*a^3*b^2
*x^4 - 352*a^4*b*x^2 + 96*a^5 - 3465*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*x^
```

```
7 + a^3*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
)/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a^9*x^5), -1/240*(3465*b^5*
x^10 + 9240*a*b^4*x^8 + 7623*a^2*b^3*x^6 + 1584*a^3*b^2*x^4 - 176*a^4*b*x^2
 + 48*a^5 + 3465*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*x^7 + a^3*b^2*x^5)*sqr
t(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a
^9*x^5)]
```

Sympy [A] time = 3.05541, size = 173, normalized size = 1.45

$$\frac{231\sqrt{-\frac{b^5}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3}+x\right)}{32}-\frac{231\sqrt{-\frac{b^5}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3}+x\right)}{32}-\frac{48a^5-176a^4bx^2+1584a^3b^2x^4+7623a^2b^3x^6+9240ab^4}{240a^9x^5+720a^8bx^7+720a^7b^2x^9+240a^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 231*sqrt(-b**5/a**13)*log(-a**7*sqrt(-b**5/a**13)/b**3 + x)/32 - 231*sqrt(-
b**5/a**13)*log(a**7*sqrt(-b**5/a**13)/b**3 + x)/32 - (48*a**5 - 176*a**4*b
*x**2 + 1584*a**3*b**2*x**4 + 7623*a**2*b**3*x**6 + 9240*a*b**4*x**8 + 3465
*b**5*x**10)/(240*a**9*x**5 + 720*a**8*b*x**7 + 720*a**7*b**2*x**9 + 240*a*
*6*b**3*x**11)
```

Giac [A] time = 1.15355, size = 126, normalized size = 1.06

$$-\frac{231\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^6}-\frac{213\,b^5x^5+472\,ab^4x^3+267\,a^2b^3x}{48\left(bx^2+a\right)^3a^6}-\frac{150\,b^2x^4-20\,abx^2+3\,a^2}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -231/16*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/48*(213*b^5*x^5 + 472
*a*b^4*x^3 + 267*a^2*b^3*x)/((b*x^2 + a)^3*a^6) - 1/15*(150*b^2*x^4 - 20*a*
b*x^2 + 3*a^2)/(a^6*x^5)
```

3.512 $\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=133

$$\frac{a^7}{10b^8(a+bx^2)^5} - \frac{7a^6}{8b^8(a+bx^2)^4} + \frac{7a^5}{2b^8(a+bx^2)^3} - \frac{35a^4}{4b^8(a+bx^2)^2} + \frac{35a^3}{2b^8(a+bx^2)} + \frac{21a^2\log(a+bx^2)}{2b^8} - \frac{3ax^2}{b^7} + \frac{x^4}{4b^6}$$

```
[Out] (-3*a*x^2)/b^7 + x^4/(4*b^6) + a^7/(10*b^8*(a + b*x^2)^5) - (7*a^6)/(8*b^8*
(a + b*x^2)^4) + (7*a^5)/(2*b^8*(a + b*x^2)^3) - (35*a^4)/(4*b^8*(a + b*x^2
)^2) + (35*a^3)/(2*b^8*(a + b*x^2)) + (21*a^2*Log[a + b*x^2])/(2*b^8)
```

Rubi [A] time = 0.142724, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^7}{10b^8(a+bx^2)^5} - \frac{7a^6}{8b^8(a+bx^2)^4} + \frac{7a^5}{2b^8(a+bx^2)^3} - \frac{35a^4}{4b^8(a+bx^2)^2} + \frac{35a^3}{2b^8(a+bx^2)} + \frac{21a^2\log(a+bx^2)}{2b^8} - \frac{3ax^2}{b^7} + \frac{x^4}{4b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-3*a*x^2)/b^7 + x^4/(4*b^6) + a^7/(10*b^8*(a + b*x^2)^5) - (7*a^6)/(8*b^8*
(a + b*x^2)^4) + (7*a^5)/(2*b^8*(a + b*x^2)^3) - (35*a^4)/(4*b^8*(a + b*x^2
)^2) + (35*a^3)/(2*b^8*(a + b*x^2)) + (21*a^2*Log[a + b*x^2])/(2*b^8)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{15}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{15}}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\mathrm{Subst}\left(\int \frac{x^7}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\,\mathrm{Subst}\left(\int \left(-\frac{6a}{b^{13}}+\frac{x}{b^{12}}-\frac{a^7}{b^{13}(a+bx)^6}+\frac{7a^6}{b^{13}(a+bx)^5}-\frac{21a^5}{b^{13}(a+bx)^4}+\frac{35a^4}{b^{13}(a+bx)^3}\right.\right.$$
$$= -\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}+\frac{a^7}{10b^8\left(a+bx^2\right)^5}-\frac{7a^6}{8b^8\left(a+bx^2\right)^4}+\frac{7a^5}{2b^8\left(a+bx^2\right)^3}-\frac{35a^4}{4b^8\left(a+bx^2\right)^2}+\frac{}{2b^8}$$

Mathematica [A] time = 0.0243096, size = 114, normalized size = 0.86

$$\frac{-500a^2b^5x^{10}-400a^3b^4x^8+1300a^4b^3x^6+2700a^5b^2x^4+1875a^6bx^2+420a^2\left(a+bx^2\right)^5\log\left(a+bx^2\right)+459a^7-70ab^6x^{12}}{40b^8\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (459*a^7 + 1875*a^6*b*x^2 + 2700*a^5*b^2*x^4 + 1300*a^4*b^3*x^6 - 400*a^3*b
^4*x^8 - 500*a^2*b^5*x^10 - 70*a*b^6*x^12 + 10*b^7*x^14 + 420*a^2*(a + b*x^
2)^5*Log[a + b*x^2])/(40*b^8*(a + b*x^2)^5)
```

Maple [A] time = 0.055, size = 120, normalized size = 0.9

$$-3\,\frac{ax^2}{b^7}+\frac{x^4}{4\,b^6}+\frac{a^7}{10\,b^8\left(bx^2+a\right)^5}-\frac{7\,a^6}{8\,b^8\left(bx^2+a\right)^4}+\frac{7\,a^5}{2\,b^8\left(bx^2+a\right)^3}-\frac{35\,a^4}{4\,b^8\left(bx^2+a\right)^2}+\frac{35\,a^3}{2\,b^8\left(bx^2+a\right)}+\frac{21\,a^2\ln\left(bx^2+\right.}{2\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -3*a*x^2/b^7+1/4*x^4/b^6+1/10*a^7/b^8/(b*x^2+a)^5-7/8*a^6/b^8/(b*x^2+a)^4+7
/2*a^5/b^8/(b*x^2+a)^3-35/4*a^4/b^8/(b*x^2+a)^2+35/2*a^3/b^8/(b*x^2+a)+21/2
*a^2*ln(b*x^2+a)/b^8
```

Maxima [A] time = 1.17093, size = 193, normalized size = 1.45

$$\frac{700\,a^3b^4x^8+2450\,a^4b^3x^6+3290\,a^5b^2x^4+1995\,a^6bx^2+459\,a^7}{40\left(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8\right)}+\frac{21\,a^2\log\left(bx^2+a\right)}{2\,b^8}+\frac{bx^4-12\,ax^2}{4\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(700*a^3*b^4*x^8 + 2450*a^4*b^3*x^6 + 3290*a^5*b^2*x^4 + 1995*a^6*b*x^
2 + 459*a^7)/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4
+ 5*a^4*b^9*x^2 + a^5*b^8) + 21/2*a^2*log(b*x^2 + a)/b^8 + 1/4*(b*x^4 - 12*
a*x^2)/b^7
```

Fricas [A] time = 1.63616, size = 450, normalized size = 3.38

$$\frac{10\,b^7x^{14}-70\,ab^6x^{12}-500\,a^2b^5x^{10}-400\,a^3b^4x^8+1300\,a^4b^3x^6+2700\,a^5b^2x^4+1875\,a^6bx^2+459\,a^7+420\left(a^2b^5x^{10}+5\right.}{40\left(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(10*b^7*x^14 - 70*a*b^6*x^12 - 500*a^2*b^5*x^10 - 400*a^3*b^4*x^8 + 13
00*a^4*b^3*x^6 + 2700*a^5*b^2*x^4 + 1875*a^6*b*x^2 + 459*a^7 + 420*(a^2*b^5
*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7
)*log(b*x^2 + a))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10
*x^4 + 5*a^4*b^9*x^2 + a^5*b^8)
```

Sympy [A] time = 1.46852, size = 150, normalized size = 1.13

$$\frac{21a^2\log\left(a+bx^2\right)}{2b^8}-\frac{3ax^2}{b^7}+\frac{459a^7+1995a^6bx^2+3290a^5b^2x^4+2450a^4b^3x^6+700a^3b^4x^8}{40a^5b^8+200a^4b^9x^2+400a^3b^{10}x^4+400a^2b^{11}x^6+200ab^{12}x^8+40b^{13}x^{10}}+\frac{x^4}{4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 21*a**2*log(a + b*x**2)/(2*b**8) - 3*a*x**2/b**7 + (459*a**7 + 1995*a**6*b*
x**2 + 3290*a**5*b**2*x**4 + 2450*a**4*b**3*x**6 + 700*a**3*b**4*x**8)/(40*
a**5*b**8 + 200*a**4*b**9*x**2 + 400*a**3*b**10*x**4 + 400*a**2*b**11*x**6
+ 200*a*b**12*x**8 + 40*b**13*x**10) + x**4/(4*b**6)
```

Giac [A] time = 1.15741, size = 153, normalized size = 1.15

$$\frac{21\,a^2\log\left(\left|bx^2+a\right|\right)}{2\,b^8}+\frac{b^6x^4-12\,ab^5x^2}{4\,b^{12}}-\frac{959\,a^2b^5x^{10}+4095\,a^3b^4x^8+7140\,a^4b^3x^6+6300\,a^5b^2x^4+2800\,a^6bx^2+500\,a^7}{40\,\left(bx^2+a\right)^5b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 21/2*a^2*log(abs(b*x^2 + a))/b^8 + 1/4*(b^6*x^4 - 12*a*b^5*x^2)/b^12 - 1/40
*(959*a^2*b^5*x^10 + 4095*a^3*b^4*x^8 + 7140*a^4*b^3*x^6 + 6300*a^5*b^2*x^4
 + 2800*a^6*b*x^2 + 500*a^7)/((b*x^2 + a)^5*b^8)
```

3.513 $\int \frac{x^{13}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=118

$$-\frac{a^6}{10b^7\left(a+bx^2\right)^5}+\frac{3a^5}{4b^7\left(a+bx^2\right)^4}-\frac{5a^4}{2b^7\left(a+bx^2\right)^3}+\frac{5a^3}{b^7\left(a+bx^2\right)^2}-\frac{15a^2}{2b^7\left(a+bx^2\right)}-\frac{3a\log\left(a+bx^2\right)}{b^7}+\frac{x^2}{2b^6}$$

```
[Out] x^2/(2*b^6) - a^6/(10*b^7*(a + b*x^2)^5) + (3*a^5)/(4*b^7*(a + b*x^2)^4) -
(5*a^4)/(2*b^7*(a + b*x^2)^3) + (5*a^3)/(b^7*(a + b*x^2)^2) - (15*a^2)/(2*b
^7*(a + b*x^2)) - (3*a*Log[a + b*x^2])/b^7
```

Rubi [A] time = 0.114487, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^6}{10b^7\left(a+bx^2\right)^5}+\frac{3a^5}{4b^7\left(a+bx^2\right)^4}-\frac{5a^4}{2b^7\left(a+bx^2\right)^3}+\frac{5a^3}{b^7\left(a+bx^2\right)^2}-\frac{15a^2}{2b^7\left(a+bx^2\right)}-\frac{3a\log\left(a+bx^2\right)}{b^7}+\frac{x^2}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^2/(2*b^6) - a^6/(10*b^7*(a + b*x^2)^5) + (3*a^5)/(4*b^7*(a + b*x^2)^4) -
(5*a^4)/(2*b^7*(a + b*x^2)^3) + (5*a^3)/(b^7*(a + b*x^2)^2) - (15*a^2)/(2*b
^7*(a + b*x^2)) - (3*a*Log[a + b*x^2])/b^7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{13}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^{13}}{\left(ab+b^2x^2\right)^6}\,dx\\ &= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^6}{\left(ab+b^2x\right)^6}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{1}{b^{12}}+\frac{a^6}{b^{12}(a+bx)^6}-\frac{6a^5}{b^{12}(a+bx)^5}+\frac{15a^4}{b^{12}(a+bx)^4}-\frac{20a^3}{b^{12}(a+bx)^3}+\frac{15}{b^{12}(a\cdots}\right.\right.\\ &= \frac{x^2}{2b^6}-\frac{a^6}{10b^7\left(a+bx^2\right)^5}+\frac{3a^5}{4b^7\left(a+bx^2\right)^4}-\frac{5a^4}{2b^7\left(a+bx^2\right)^3}+\frac{5a^3}{b^7\left(a+bx^2\right)^2}-\frac{15a^2}{2b^7\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0268393, size = 101, normalized size = 0.86

$$-\frac{50a^2b^4x^8+400a^3b^3x^6+600a^4b^2x^4+375a^5bx^2+87a^6-50ab^5x^{10}+60a\left(a+bx^2\right)^5\log\left(a+bx^2\right)-10b^6x^{12}}{20b^7\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(87*a^6 + 375*a^5*b*x^2 + 600*a^4*b^2*x^4 + 400*a^3*b^3*x^6 + 50*a^2*b^4*x
^8 - 50*a*b^5*x^10 - 10*b^6*x^12 + 60*a*(a + b*x^2)^5*Log[a + b*x^2])/(20*b
^7*(a + b*x^2)^5)
```

Maple [A] time = 0.056, size = 109, normalized size = 0.9

$$\frac{x^2}{2\,b^6}-\frac{a^6}{10\,b^7\left(bx^2+a\right)^5}+\frac{3\,a^5}{4\,b^7\left(bx^2+a\right)^4}-\frac{5\,a^4}{2\,b^7\left(bx^2+a\right)^3}+5\,\frac{a^3}{b^7\left(bx^2+a\right)^2}-\frac{15\,a^2}{2\,b^7\left(bx^2+a\right)}-3\,\frac{a\ln\left(bx^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/2*x^2/b^6-1/10*a^6/b^7/(b*x^2+a)^5+3/4*a^5/b^7/(b*x^2+a)^4-5/2*a^4/b^7/(b
*x^2+a)^3+5*a^3/b^7/(b*x^2+a)^2-15/2*a^2/b^7/(b*x^2+a)-3*a*ln(b*x^2+a)/b^7
```

Maxima [A] time = 1.23894, size = 178, normalized size = 1.51

$$-\frac{150\,a^{2}b^{4}x^{8} + 500\,a^{3}b^{3}x^{6} + 650\,a^{4}b^{2}x^{4} + 385\,a^{5}bx^{2} + 87\,a^{6}}{20\left(b^{12}x^{10} + 5\,ab^{11}x^{8} + 10\,a^{2}b^{10}x^{6} + 10\,a^{3}b^{9}x^{4} + 5\,a^{4}b^{8}x^{2} + a^{5}b^{7}\right)} + \frac{x^{2}}{2\,b^{6}} - \frac{3\,a\log\left(bx^{2} + a\right)}{b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(150*a^2*b^4*x^8 + 500*a^3*b^3*x^6 + 650*a^4*b^2*x^4 + 385*a^5*b*x^2
+ 87*a^6)/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*
a^4*b^8*x^2 + a^5*b^7) + 1/2*x^2/b^6 - 3*a*log(b*x^2 + a)/b^7
```

Fricas [A] time = 1.73678, size = 412, normalized size = 3.49

$$\frac{10\,b^{6}x^{12} + 50\,ab^{5}x^{10} - 50\,a^{2}b^{4}x^{8} - 400\,a^{3}b^{3}x^{6} - 600\,a^{4}b^{2}x^{4} - 375\,a^{5}bx^{2} - 87\,a^{6} - 60\left(ab^{5}x^{10} + 5\,a^{2}b^{4}x^{8} + 10\,a^{3}b^{3}x^{6} + 1\right.}{20\left(b^{12}x^{10} + 5\,ab^{11}x^{8} + 10\,a^{2}b^{10}x^{6} + 10\,a^{3}b^{9}x^{4} + 5\,a^{4}b^{8}x^{2} + a^{5}b^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^6*x^12 + 50*a*b^5*x^10 - 50*a^2*b^4*x^8 - 400*a^3*b^3*x^6 - 600*
a^4*b^2*x^4 - 375*a^5*b*x^2 - 87*a^6 - 60*(a*b^5*x^10 + 5*a^2*b^4*x^8 + 10*
a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 + a^6)*log(b*x^2 + a))/(b^12*x^1
0 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b
^7)
```

Sympy [A] time = 1.4333, size = 136, normalized size = 1.15

$$-\frac{3a\log\left(a + bx^{2}\right)}{b^{7}} - \frac{87a^{6} + 385a^{5}bx^{2} + 650a^{4}b^{2}x^{4} + 500a^{3}b^{3}x^{6} + 150a^{2}b^{4}x^{8}}{20a^{5}b^{7} + 100a^{4}b^{8}x^{2} + 200a^{3}b^{9}x^{4} + 200a^{2}b^{10}x^{6} + 100ab^{11}x^{8} + 20b^{12}x^{10}} + \frac{x^{2}}{2b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*a*log(a + b*x**2)/b**7 - (87*a**6 + 385*a**5*b*x**2 + 650*a**4*b**2*x**4
 + 500*a**3*b**3*x**6 + 150*a**2*b**4*x**8)/(20*a**5*b**7 + 100*a**4*b**8*x
**2 + 200*a**3*b**9*x**4 + 200*a**2*b**10*x**6 + 100*a*b**11*x**8 + 20*b**1
2*x**10) + x**2/(2*b**6)
```

Giac [A] time = 1.13895, size = 128, normalized size = 1.08

$$\frac{x^2}{2\,b^6} - \frac{3\,a\log\left(\left|bx^2 + a\right|\right)}{b^7} + \frac{137\,ab^5x^{10} + 535\,a^2b^4x^8 + 870\,a^3b^3x^6 + 720\,a^4b^2x^4 + 300\,a^5bx^2 + 50\,a^6}{20\left(bx^2 + a\right)^5 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^6 - 3*a*log(abs(b*x^2 + a))/b^7 + 1/20*(137*a*b^5*x^10 + 535*a^2*
b^4*x^8 + 870*a^3*b^3*x^6 + 720*a^4*b^2*x^4 + 300*a^5*b*x^2 + 50*a^6)/((b*x
^2 + a)^5*b^7)
```

3.514 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=109

$$\frac{a^5}{10b^6\left(a+bx^2\right)^5}-\frac{5a^4}{8b^6\left(a+bx^2\right)^4}+\frac{5a^3}{3b^6\left(a+bx^2\right)^3}-\frac{5a^2}{2b^6\left(a+bx^2\right)^2}+\frac{5a}{2b^6\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^6}$$

```
[Out] a^5/(10*b^6*(a + b*x^2)^5) - (5*a^4)/(8*b^6*(a + b*x^2)^4) + (5*a^3)/(3*b^6
*(a + b*x^2)^3) - (5*a^2)/(2*b^6*(a + b*x^2)^2) + (5*a)/(2*b^6*(a + b*x^2))
 + Log[a + b*x^2]/(2*b^6)
```

Rubi [A] time = 0.102303, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a^5}{10b^6\left(a+bx^2\right)^5}-\frac{5a^4}{8b^6\left(a+bx^2\right)^4}+\frac{5a^3}{3b^6\left(a+bx^2\right)^3}-\frac{5a^2}{2b^6\left(a+bx^2\right)^2}+\frac{5a}{2b^6\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^5/(10*b^6*(a + b*x^2)^5) - (5*a^4)/(8*b^6*(a + b*x^2)^4) + (5*a^3)/(3*b^6
*(a + b*x^2)^3) - (5*a^2)/(2*b^6*(a + b*x^2)^2) + (5*a)/(2*b^6*(a + b*x^2))
 + Log[a + b*x^2]/(2*b^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{11}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$

$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(-\frac{a^5}{b^{11}(a+bx)^6}+\frac{5a^4}{b^{11}(a+bx)^5}-\frac{10a^3}{b^{11}(a+bx)^4}+\frac{10a^2}{b^{11}(a+bx)^3}-\frac{5a}{b^{11}(a+bx)}\right.\right.$$

$$= \frac{a^5}{10b^6\left(a+bx^2\right)^5}-\frac{5a^4}{8b^6\left(a+bx^2\right)^4}+\frac{5a^3}{3b^6\left(a+bx^2\right)^3}-\frac{5a^2}{2b^6\left(a+bx^2\right)^2}+\frac{5a}{2b^6\left(a+bx^2\right)}+\frac{\log}{}$$

Mathematica [A] time = 0.0232354, size = 72, normalized size = 0.66

$$\frac{\frac{a\left(1100a^2b^2x^4+625a^3bx^2+137a^4+900ab^3x^6+300b^4x^8\right)}{\left(a+bx^2\right)^5}+60\log\left(a+bx^2\right)}{120b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((a*(137*a^4 + 625*a^3*b*x^2 + 1100*a^2*b^2*x^4 + 900*a*b^3*x^6 + 300*b^4*x
^8))/(a + b*x^2)^5 + 60*Log[a + b*x^2])/(120*b^6)
```

Maple [A] time = 0.056, size = 98, normalized size = 0.9

$$\frac{a^5}{10\,b^6\left(bx^2+a\right)^5}-\frac{5\,a^4}{8\,b^6\left(bx^2+a\right)^4}+\frac{5\,a^3}{3\,b^6\left(bx^2+a\right)^3}-\frac{5\,a^2}{2\,b^6\left(bx^2+a\right)^2}+\frac{5\,a}{2\,b^6\left(bx^2+a\right)}+\frac{\ln\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*a^5/b^6/(b*x^2+a)^5-5/8*a^4/b^6/(b*x^2+a)^4+5/3*a^3/b^6/(b*x^2+a)^3-5/
2*a^2/b^6/(b*x^2+a)^2+5/2*a/b^6/(b*x^2+a)+1/2*ln(b*x^2+a)/b^6
```

Maxima [A] time = 1.20553, size = 163, normalized size = 1.5

$$\frac{300\,ab^4x^8 + 900\,a^2b^3x^6 + 1100\,a^3b^2x^4 + 625\,a^4bx^2 + 137\,a^5}{120\left(b^{11}x^{10} + 5\,ab^{10}x^8 + 10\,a^2b^9x^6 + 10\,a^3b^8x^4 + 5\,a^4b^7x^2 + a^5b^6\right)} + \frac{\log\left(bx^2 + a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a^4*b*x^2 +
 137*a^5)/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a
^4*b^7*x^2 + a^5*b^6) + 1/2*log(b*x^2 + a)/b^6
```

Fricas [A] time = 1.69063, size = 367, normalized size = 3.37

$$\frac{300\,ab^4x^8 + 900\,a^2b^3x^6 + 1100\,a^3b^2x^4 + 625\,a^4bx^2 + 137\,a^5 + 60\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4bx^2 + \cdots\right.}{120\left(b^{11}x^{10} + 5\,ab^{10}x^8 + 10\,a^2b^9x^6 + 10\,a^3b^8x^4 + 5\,a^4b^7x^2 + a^5b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a^4*b*x^2 +
 137*a^5 + 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5
*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^
6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

Sympy [A] time = 1.26721, size = 124, normalized size = 1.14

$$\frac{137a^5 + 625a^4bx^2 + 1100a^3b^2x^4 + 900a^2b^3x^6 + 300ab^4x^8}{120a^5b^6 + 600a^4b^7x^2 + 1200a^3b^8x^4 + 1200a^2b^9x^6 + 600ab^{10}x^8 + 120b^{11}x^{10}} + \frac{\log\left(a + bx^2\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (137*a**5 + 625*a**4*b*x**2 + 1100*a**3*b**2*x**4 + 900*a**2*b**3*x**6 + 30
0*a*b**4*x**8)/(120*a**5*b**6 + 600*a**4*b**7*x**2 + 1200*a**3*b**8*x**4 +
1200*a**2*b**9*x**6 + 600*a*b**10*x**8 + 120*b**11*x**10) + log(a + b*x**2)
/(2*b**6)
```

Giac [A] time = 1.13556, size = 101, normalized size = 0.93

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^6}-\frac{137\,b^4x^{10}+385\,ab^3x^8+470\,a^2b^2x^6+270\,a^3bx^4+60\,a^4x^2}{120\,\left(bx^2+a\right)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^6 - 1/120*(137*b^4*x^10 + 385*a*b^3*x^8 + 470*a^2
*b^2*x^6 + 270*a^3*b*x^4 + 60*a^4*x^2)/((b*x^2 + a)^5*b^5)
```

3.515 $\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=19

$$\frac{x^{10}}{10a\left(a+bx^2\right)^5}$$

```
[Out] x^10/(10*a*(a + b*x^2)^5)
```

Rubi [A] time = 0.0067681, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 264}

$$\frac{x^{10}}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^10/(10*a*(a + b*x^2)^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6 \int \frac{x^9}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{x^{10}}{10a\left(a+bx^2\right)^5}$$

Mathematica [B] time = 0.0158142, size = 57, normalized size = 3.

$$-\frac{10a^2b^2x^4+5a^3bx^2+a^4+10ab^3x^6+5b^4x^8}{10b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a^4 + 5*a^3*b*x^2 + 10*a^2*b^2*x^4 + 10*a*b^3*x^6 + 5*b^4*x^8)/(10*b^5*(a
 + b*x^2)^5)
```

Maple [B] time = 0.049, size = 81, normalized size = 4.3

$$\frac{a^3}{2\,b^5\left(bx^2+a\right)^4}+\frac{a}{b^5\left(bx^2+a\right)^2}-\frac{a^2}{b^5\left(bx^2+a\right)^3}-\frac{a^4}{10\,b^5\left(bx^2+a\right)^5}-\frac{1}{2\,b^5\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/2*a^3/b^5/(b*x^2+a)^4+1/b^5*a/(b*x^2+a)^2-a^2/b^5/(b*x^2+a)^3-1/10*a^4/b^
5/(b*x^2+a)^5-1/2/b^5/(b*x^2+a)
```

Maxima [B] time = 1.25286, size = 138, normalized size = 7.26

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\left(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/(b^10
*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5
*b^5)
```

Fricas [B] time = 1.67945, size = 212, normalized size = 11.16

$$-\frac{5\,b^4x^8 + 10\,ab^3x^6 + 10\,a^2b^2x^4 + 5\,a^3bx^2 + a^4}{10\left(b^{10}x^{10} + 5\,ab^9x^8 + 10\,a^2b^8x^6 + 10\,a^3b^7x^4 + 5\,a^4b^6x^2 + a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/(b^10
*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5
*b^5)
```

Sympy [B] time = 1.18708, size = 107, normalized size = 5.63

$$-\frac{a^4 + 5a^3bx^2 + 10a^2b^2x^4 + 10ab^3x^6 + 5b^4x^8}{10a^5b^5 + 50a^4b^6x^2 + 100a^3b^7x^4 + 100a^2b^8x^6 + 50ab^9x^8 + 10b^{10}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -(a**4 + 5*a**3*b*x**2 + 10*a**2*b**2*x**4 + 10*a*b**3*x**6 + 5*b**4*x**8)/
(10*a**5*b**5 + 50*a**4*b**6*x**2 + 100*a**3*b**7*x**4 + 100*a**2*b**8*x**6
 + 50*a*b**9*x**8 + 10*b**10*x**10)
```

Giac [B] time = 1.16209, size = 74, normalized size = 3.89

$$-\frac{5\,b^4x^8 + 10\,ab^3x^6 + 10\,a^2b^2x^4 + 5\,a^3bx^2 + a^4}{10\left(bx^2 + a\right)^5 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/((b*x
^2 + a)^5*b^5)
```

3.516 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=39

$$\frac{x^8}{40a^2\left(a+bx^2\right)^4}+\frac{x^8}{10a\left(a+bx^2\right)^5}$$

```
[Out] x^8/(10*a*(a + b*x^2)^5) + x^8/(40*a^2*(a + b*x^2)^4)
```

Rubi [A] time = 0.0261123, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 266, 45, 37}

$$\frac{x^8}{40a^2\left(a+bx^2\right)^4}+\frac{x^8}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^8/(10*a*(a + b*x^2)^5) + x^8/(40*a^2*(a + b*x^2)^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
```

```
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^7}{\left(ab+b^2x^2\right)^6}\,dx\\
&= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^6}\,dx,x,x^2\right)\\
&= \frac{x^8}{10a\left(a+bx^2\right)^5}+\frac{b^5\,\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{10a}\\
&= \frac{x^8}{10a\left(a+bx^2\right)^5}+\frac{x^8}{40a^2\left(a+bx^2\right)^4}
\end{aligned}$$

Mathematica [A] time = 0.0137822, size = 46, normalized size = 1.18

$$-\frac{5a^2bx^2+a^3+10ab^2x^4+10b^3x^6}{40b^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a^3 + 5*a^2*b*x^2 + 10*a*b^2*x^4 + 10*b^3*x^6)/(40*b^4*(a + b*x^2)^5)
```

Maple [A] time = 0.048, size = 65, normalized size = 1.7

$$\frac{a}{2\,b^{4}\left(b x^{2}+a\right)^{3}}+\frac{a^{3}}{10\,b^{4}\left(b x^{2}+a\right)^{5}}-\frac{3\,a^{2}}{8\,b^{4}\left(b x^{2}+a\right)^{4}}-\frac{1}{4\,b^{4}\left(b x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/2*a/b^4/(b*x^2+a)^3+1/10*a^3/b^4/(b*x^2+a)^5-3/8*a^2/b^4/(b*x^2+a)^4-1/4/
b^4/(b*x^2+a)^2
```

Maxima [B] time = 1.34569, size = 123, normalized size = 3.15

$$-\frac{10\,b^{3}x^{6}+10\,ab^{2}x^{4}+5\,a^{2}bx^{2}+a^{3}}{40\left(b^{9}x^{10}+5\,ab^{8}x^{8}+10\,a^{2}b^{7}x^{6}+10\,a^{3}b^{6}x^{4}+5\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 + 5*a*b^8*x
^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 + a^5*b^4)
```

Fricas [B] time = 1.69553, size = 189, normalized size = 4.85

$$-\frac{10\,b^{3}x^{6}+10\,ab^{2}x^{4}+5\,a^{2}bx^{2}+a^{3}}{40\left(b^{9}x^{10}+5\,ab^{8}x^{8}+10\,a^{2}b^{7}x^{6}+10\,a^{3}b^{6}x^{4}+5\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 + 5*a*b^8*x
^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 + a^5*b^4)
```

Sympy [B] time = 1.10337, size = 95, normalized size = 2.44

$$-\frac{a^3 + 5a^2bx^2 + 10ab^2x^4 + 10b^3x^6}{40a^5b^4 + 200a^4b^5x^2 + 400a^3b^6x^4 + 400a^2b^7x^6 + 200ab^8x^8 + 40b^9x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -(a**3 + 5*a**2*b*x**2 + 10*a*b**2*x**4 + 10*b**3*x**6)/(40*a**5*b**4 + 200
*a**4*b**5*x**2 + 400*a**3*b**6*x**4 + 400*a**2*b**7*x**6 + 200*a*b**8*x**8
 + 40*b**9*x**10)
```

Giac [A] time = 1.13036, size = 59, normalized size = 1.51

$$-\frac{10\,b^3x^6 + 10\,ab^2x^4 + 5\,a^2bx^2 + a^3}{40\,\left(bx^2 + a\right)^5 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/((b*x^2 + a)^5*b^4)
```

3.517 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}$$

```
[Out] -a^2/(10*b^3*(a + b*x^2)^5) + a/(4*b^3*(a + b*x^2)^4) - 1/(6*b^3*(a + b*x^2
)^3)
```

Rubi [A] time = 0.0453038, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$-\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -a^2/(10*b^3*(a + b*x^2)^5) + a/(4*b^3*(a + b*x^2)^4) - 1/(6*b^3*(a + b*x^2
)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^5}{\left(ab+b^2x^2\right)^6}\,dx\\
&= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^6}\,dx, x, x^2\right)\\
&= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{a^2}{b^8(a+bx)^6}-\frac{2a}{b^8(a+bx)^5}+\frac{1}{b^8(a+bx)^4}\right)dx, x, x^2\right)\\
&= -\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}
\end{aligned}$$

Mathematica [A] time = 0.0128499, size = 35, normalized size = 0.66

$$-\frac{a^2+5abx^2+10b^2x^4}{60b^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a^2 + 5*a*b*x^2 + 10*b^2*x^4)/(60*b^3*(a + b*x^2)^5)
```

Maple [A] time = 0.049, size = 48, normalized size = 0.9

$$-\frac{a^2}{10\,b^3\left(bx^2+a\right)^5}+\frac{a}{4\,b^3\left(bx^2+a\right)^4}-\frac{1}{6\,b^3\left(bx^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/10*a^2/b^3/(b*x^2+a)^5+1/4*a/b^3/(b*x^2+a)^4-1/6/b^3/(b*x^2+a)^3
```

Maxima [A] time = 1.39645, size = 108, normalized size = 2.04

$$-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,\left(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10*a^2*b^6*x
^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

Fricas [A] time = 1.57576, size = 166, normalized size = 3.13

$$-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,\left(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10*a^2*b^6*x
^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

Sympy [A] time = 1.05316, size = 83, normalized size = 1.57

$$-\frac{a^2+5abx^2+10b^2x^4}{60a^5b^3+300a^4b^4x^2+600a^3b^5x^4+600a^2b^6x^6+300ab^7x^8+60b^8x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -(a**2 + 5*a*b*x**2 + 10*b**2*x**4)/(60*a**5*b**3 + 300*a**4*b**4*x**2 + 60
0*a**3*b**5*x**4 + 600*a**2*b**6*x**6 + 300*a*b**7*x**8 + 60*b**8*x**10)
```

Giac [A] time = 1.16526, size = 45, normalized size = 0.85

$$-\frac{10\,b^2x^4 + 5\,abx^2 + a^2}{60\,\left(bx^2 + a\right)^5 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/((b*x^2 + a)^5*b^3)
```

3.518 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=34

$$\frac{a}{10b^2(a+bx^2)^5} - \frac{1}{8b^2(a+bx^2)^4}$$

```
[Out] a/(10*b^2*(a + b*x^2)^5) - 1/(8*b^2*(a + b*x^2)^4)
```

Rubi [A] time = 0.0307099, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 43}

$$\frac{a}{10b^2(a+bx^2)^5} - \frac{1}{8b^2(a+bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a/(10*b^2*(a + b*x^2)^5) - 1/(8*b^2*(a + b*x^2)^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^3}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\mathrm{Subst}\left(\int \frac{x}{\left(ab+b^2x\right)^6}\,dx,x,x^2\right)$$
$$= \frac{1}{2}b^6\,\mathrm{Subst}\left(\int \left(-\frac{a}{b^7(a+bx)^6}+\frac{1}{b^7(a+bx)^5}\right)dx,x,x^2\right)$$
$$= \frac{a}{10b^2\left(a+bx^2\right)^5}-\frac{1}{8b^2\left(a+bx^2\right)^4}$$

Mathematica [A] time = 0.0077173, size = 24, normalized size = 0.71

$$-\frac{a+5bx^2}{40b^2\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a + 5*b*x^2)/(40*b^2*(a + b*x^2)^5)
```

Maple [A] time = 0.049, size = 31, normalized size = 0.9

$$\frac{a}{10\,b^2\left(bx^2+a\right)^5}-\frac{1}{8\,b^2\left(bx^2+a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*a/b^2/(b*x^2+a)^5-1/8/b^2/(b*x^2+a)^4
```

Maxima [B] time = 1.42217, size = 93, normalized size = 2.74

$$-\frac{5\,bx^2+a}{40\,\left(b^7x^{10}+5\,ab^6x^8+10\,a^2b^5x^6+10\,a^3b^4x^4+5\,a^4b^3x^2+a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10*a^3*b^4*x
^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

Fricas [B] time = 1.92577, size = 143, normalized size = 4.21

$$-\frac{5\,bx^2+a}{40\,\left(b^7x^{10}+5\,ab^6x^8+10\,a^2b^5x^6+10\,a^3b^4x^4+5\,a^4b^3x^2+a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10*a^3*b^4*x
^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

Sympy [B] time = 1.01158, size = 71, normalized size = 2.09

$$-\frac{a+5bx^2}{40a^5b^2+200a^4b^3x^2+400a^3b^4x^4+400a^2b^5x^6+200ab^6x^8+40b^7x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -(a + 5*b*x**2)/(40*a**5*b**2 + 200*a**4*b**3*x**2 + 400*a**3*b**4*x**4 + 4
00*a**2*b**5*x**6 + 200*a*b**6*x**8 + 40*b**7*x**10)
```

Giac [A] time = 1.14657, size = 30, normalized size = 0.88

$$-\frac{5\,bx^2+a}{40\,\left(bx^2+a\right)^5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/40*(5*b*x^2 + a)/((b*x^2 + a)^5*b^2)
```

3.519 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

```
[Out] -1/(10*b*(a + b*x^2)^5)
```

Rubi [A] time = 0.0053067, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 261}

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/(10*b*(a + b*x^2)^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6 \int \frac{x}{\left(ab+b^2x^2\right)^6}\,dx = -\frac{1}{10b\left(a+bx^2\right)^5}$$

Mathematica [A] time = 0.0028531, size = 16, normalized size = 1.

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/(10*b*(a + b*x^2)^5)
```

Maple [A] time = 0.047, size = 15, normalized size = 0.9

$$-\frac{1}{10\,b\left(bx^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/10/b/(b*x^2+a)^5
```

Maxima [B] time = 1.08085, size = 80, normalized size = 5.

$$-\frac{1}{10\left(b^6x^{10}+5\,ab^5x^8+10\,a^2b^4x^6+10\,a^3b^3x^4+5\,a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 + 5*a^4*b^2
*x^2 + a^5*b)
```

Fricas [B] time = 1.68132, size = 122, normalized size = 7.62

$$-\frac{1}{10\left(b^6x^{10}+5\,ab^5x^8+10\,a^2b^4x^6+10\,a^3b^3x^4+5\,a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 + 5*a^4*b^2
*x^2 + a^5*b)
```

Sympy [B] time = 0.961724, size = 63, normalized size = 3.94

$$-\frac{1}{10a^5b+50a^4b^2x^2+100a^3b^3x^4+100a^2b^4x^6+50ab^5x^8+10b^6x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -1/(10*a**5*b + 50*a**4*b**2*x**2 + 100*a**3*b**3*x**4 + 100*a**2*b**4*x**6
 + 50*a*b**5*x**8 + 10*b**6*x**10)
```

Giac [A] time = 1.15108, size = 19, normalized size = 1.19

$$-\frac{1}{10\left(bx^2+a\right)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/10/((b*x^2 + a)^5*b)
```

3.520 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=102

$$\frac{1}{2a^5\left(a+bx^2\right)}+\frac{1}{4a^4\left(a+bx^2\right)^2}+\frac{1}{6a^3\left(a+bx^2\right)^3}+\frac{1}{8a^2\left(a+bx^2\right)^4}-\frac{\log\left(a+bx^2\right)}{2a^6}+\frac{\log(x)}{a^6}+\frac{1}{10a\left(a+bx^2\right)^5}$$

```
[Out] 1/(10*a*(a + b*x^2)^5) + 1/(8*a^2*(a + b*x^2)^4) + 1/(6*a^3*(a + b*x^2)^3)
+ 1/(4*a^4*(a + b*x^2)^2) + 1/(2*a^5*(a + b*x^2)) + Log[x]/a^6 - Log[a + b*
x^2]/(2*a^6)
```

Rubi [A] time = 0.105126, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{1}{2a^5\left(a+bx^2\right)}+\frac{1}{4a^4\left(a+bx^2\right)^2}+\frac{1}{6a^3\left(a+bx^2\right)^3}+\frac{1}{8a^2\left(a+bx^2\right)^4}-\frac{\log\left(a+bx^2\right)}{2a^6}+\frac{\log(x)}{a^6}+\frac{1}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] 1/(10*a*(a + b*x^2)^5) + 1/(8*a^2*(a + b*x^2)^4) + 1/(6*a^3*(a + b*x^2)^3)
+ 1/(4*a^4*(a + b*x^2)^2) + 1/(2*a^5*(a + b*x^2)) + Log[x]/a^6 - Log[a + b*
x^2]/(2*a^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{1}{a^6b^6x} - \frac{1}{ab^5(a+bx)^6} - \frac{1}{a^2b^5(a+bx)^5} - \frac{1}{a^3b^5(a+bx)^4} - \frac{1}{a^4b^5(a+bx)^3}\right.\right.$$
$$= \frac{1}{10a\left(a+bx^2\right)^5} + \frac{1}{8a^2\left(a+bx^2\right)^4} + \frac{1}{6a^3\left(a+bx^2\right)^3} + \frac{1}{4a^4\left(a+bx^2\right)^2} + \frac{1}{2a^5\left(a+bx^2\right)} +$$

Mathematica [A] time = 0.0490998, size = 76, normalized size = 0.75

$$\frac{\frac{a\left(470a^2b^2x^4+385a^3bx^2+137a^4+270ab^3x^6+60b^4x^8\right)}{\left(a+bx^2\right)^5} - 60\log\left(a+bx^2\right) + 120\log(x)}{120a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((a*(137*a^4 + 385*a^3*b*x^2 + 470*a^2*b^2*x^4 + 270*a*b^3*x^6 + 60*b^4*x^8
))/(a + b*x^2)^5 + 120*Log[x] - 60*Log[a + b*x^2])/(120*a^6)
```

Maple [A] time = 0.056, size = 91, normalized size = 0.9

$$\frac{1}{10\,a\left(bx^2+a\right)^5} + \frac{1}{8\,a^2\left(bx^2+a\right)^4} + \frac{1}{6\,a^3\left(bx^2+a\right)^3} + \frac{1}{4\,a^4\left(bx^2+a\right)^2} + \frac{1}{2\,a^5\left(bx^2+a\right)} + \frac{\ln(x)}{a^6} - \frac{\ln\left(bx^2+a\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10/a/(b*x^2+a)^5+1/8/a^2/(b*x^2+a)^4+1/6/a^3/(b*x^2+a)^3+1/4/a^4/(b*x^2+a
)^2+1/2/a^5/(b*x^2+a)+ln(x)/a^6-1/2*ln(b*x^2+a)/a^6
```

Maxima [A] time = 1.3507, size = 170, normalized size = 1.67

$$\frac{60\,b^4x^8+270\,ab^3x^6+470\,a^2b^2x^4+385\,a^3bx^2+137\,a^4}{120\left(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10}\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^6}+\frac{\log\left(x^2\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(60*b^4*x^8 + 270*a*b^3*x^6 + 470*a^2*b^2*x^4 + 385*a^3*b*x^2 + 137*a
^4)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9
*b*x^2 + a^10) - 1/2*log(b*x^2 + a)/a^6 + 1/2*log(x^2)/a^6
```

Fricas [B] time = 1.50483, size = 489, normalized size = 4.79

$$\frac{60\,ab^4x^8+270\,a^2b^3x^6+470\,a^3b^2x^4+385\,a^4bx^2+137\,a^5-60\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\mathrm{l}}{120\left(a^6b^5x^{10}+5\,a^7b^4x^8+10\,a^8b^3x^6+10\,a^9b^2x^4+5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(60*a*b^4*x^8 + 270*a^2*b^3*x^6 + 470*a^3*b^2*x^4 + 385*a^4*b*x^2 + 1
37*a^5 - 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a
^4*b*x^2 + a^5)*log(b*x^2 + a) + 120*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x
^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*log(x))/(a^6*b^5*x^10 + 5*a^7*b^4*
x^8 + 10*a^8*b^3*x^6 + 10*a^9*b^2*x^4 + 5*a^10*b*x^2 + a^11)
```

Sympy [A] time = 2.73205, size = 128, normalized size = 1.25

$$\frac{137a^4+385a^3bx^2+470a^2b^2x^4+270ab^3x^6+60b^4x^8}{120a^{10}+600a^9bx^2+1200a^8b^2x^4+1200a^7b^3x^6+600a^6b^4x^8+120a^5b^5x^{10}}+\frac{\log\left(x\right)}{a^6}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (137*a**4 + 385*a**3*b*x**2 + 470*a**2*b**2*x**4 + 270*a*b**3*x**6 + 60*b**
4*x**8)/(120*a**10 + 600*a**9*b*x**2 + 1200*a**8*b**2*x**4 + 1200*a**7*b**3
*x**6 + 600*a**6*b**4*x**8 + 120*a**5*b**5*x**10) + log(x)/a**6 - log(a/b +
 x**2)/(2*a**6)
```

Giac [A] time = 1.14841, size = 124, normalized size = 1.22

$$\frac{\log\left(x^2\right)}{2\,a^6} - \frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^6} + \frac{137\,b^5x^{10}+745\,ab^4x^8+1640\,a^2b^3x^6+1840\,a^3b^2x^4+1070\,a^4bx^2+274\,a^5}{120\left(bx^2+a\right)^5a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^6 - 1/2*log(abs(b*x^2 + a))/a^6 + 1/120*(137*b^5*x^10 + 745*
a*b^4*x^8 + 1640*a^2*b^3*x^6 + 1840*a^3*b^2*x^4 + 1070*a^4*b*x^2 + 274*a^5)
/((b*x^2 + a)^5*a^6)
```

3.521 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=116

$$-\frac{5b}{2a^6(a+bx^2)}-\frac{b}{a^5(a+bx^2)^2}-\frac{b}{2a^4(a+bx^2)^3}-\frac{b}{4a^3(a+bx^2)^4}-\frac{b}{10a^2(a+bx^2)^5}+\frac{3b\log(a+bx^2)}{a^7}-\frac{6b\log(x)}{a^7}-\frac{1}{2a^6x^2}$$

```
[Out] -1/(2*a^6*x^2) - b/(10*a^2*(a + b*x^2)^5) - b/(4*a^3*(a + b*x^2)^4) - b/(2*
a^4*(a + b*x^2)^3) - b/(a^5*(a + b*x^2)^2) - (5*b)/(2*a^6*(a + b*x^2)) - (6
*b*Log[x])/a^7 + (3*b*Log[a + b*x^2])/a^7
```

Rubi [A] time = 0.125673, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$-\frac{5b}{2a^6(a+bx^2)}-\frac{b}{a^5(a+bx^2)^2}-\frac{b}{2a^4(a+bx^2)^3}-\frac{b}{4a^3(a+bx^2)^4}-\frac{b}{10a^2(a+bx^2)^5}+\frac{3b\log(a+bx^2)}{a^7}-\frac{6b\log(x)}{a^7}-\frac{1}{2a^6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/(2*a^6*x^2) - b/(10*a^2*(a + b*x^2)^5) - b/(4*a^3*(a + b*x^2)^4) - b/(2*
a^4*(a + b*x^2)^3) - b/(a^5*(a + b*x^2)^2) - (5*b)/(2*a^6*(a + b*x^2)) - (6
*b*Log[x])/a^7 + (3*b*Log[a + b*x^2])/a^7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^3\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^6}\,dx,x,x^2\right)$$

$$= \frac{1}{2}b^6\,\text{Subst}\left(\int\left(\frac{1}{a^6b^6x^2}-\frac{6}{a^7b^5x}+\frac{1}{a^2b^4(a+bx)^6}+\frac{2}{a^3b^4(a+bx)^5}+\frac{3}{a^4b^4(a+bx)^4}+\frac{}{a}\right.\right.$$

$$= -\frac{1}{2a^6x^2}-\frac{b}{10a^2\left(a+bx^2\right)^5}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{5}{2a^6\left(a\right.}$$

Mathematica [A] time = 0.0805515, size = 92, normalized size = 0.79

$$-\frac{\frac{a\left(470a^2b^3x^6+385a^3b^2x^4+137a^4bx^2+10a^5+270ab^4x^8+60b^5x^{10}\right)}{x^2\left(a+bx^2\right)^5}-60b\log\left(a+bx^2\right)+120b\log(x)}{20a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -((a*(10*a^5 + 137*a^4*b*x^2 + 385*a^3*b^2*x^4 + 470*a^2*b^3*x^6 + 270*a*b^
4*x^8 + 60*b^5*x^10))/(x^2*(a + b*x^2)^5) + 120*b*Log[x] - 60*b*Log[a + b*x
^2])/(20*a^7)
```

Maple [A] time = 0.057, size = 107, normalized size = 0.9

$$-\frac{1}{2\,x^2a^6}-\frac{b}{10\,a^2\left(bx^2+a\right)^5}-\frac{b}{4\,a^3\left(bx^2+a\right)^4}-\frac{b}{2\,a^4\left(bx^2+a\right)^3}-\frac{b}{a^5\left(bx^2+a\right)^2}-\frac{5\,b}{2\,a^6\left(bx^2+a\right)}-6\,\frac{b\ln(x)}{a^7}+3\,\frac{b\ln\left(bx\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/2/x^2/a^6-1/10*b/a^2/(b*x^2+a)^5-1/4*b/a^3/(b*x^2+a)^4-1/2*b/a^4/(b*x^2+
a)^3-b/a^5/(b*x^2+a)^2-5/2*b/a^6/(b*x^2+a)-6*b*ln(x)/a^7+3*b*ln(b*x^2+a)/a^
7
```

Maxima [A] time = 1.02722, size = 193, normalized size = 1.66

$$-\frac{60\,b^5x^{10}+270\,ab^4x^8+470\,a^2b^3x^6+385\,a^3b^2x^4+137\,a^4bx^2+10\,a^5}{20\left(a^6b^5x^{12}+5\,a^7b^4x^{10}+10\,a^8b^3x^8+10\,a^9b^2x^6+5\,a^{10}bx^4+a^{11}x^2\right)}+\frac{3\,b\log\left(bx^2+a\right)}{a^7}-\frac{3\,b\log\left(x^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(60*b^5*x^10 + 270*a*b^4*x^8 + 470*a^2*b^3*x^6 + 385*a^3*b^2*x^4 + 13
7*a^4*b*x^2 + 10*a^5)/(a^6*b^5*x^12 + 5*a^7*b^4*x^10 + 10*a^8*b^3*x^8 + 10*
a^9*b^2*x^6 + 5*a^10*b*x^4 + a^11*x^2) + 3*b*log(b*x^2 + a)/a^7 - 3*b*log(x
^2)/a^7
```

Fricas [B] time = 1.47088, size = 545, normalized size = 4.7

$$-\frac{60\,ab^5x^{10}+270\,a^2b^4x^8+470\,a^3b^3x^6+385\,a^4b^2x^4+137\,a^5bx^2+10\,a^6-60\left(b^6x^{12}+5\,ab^5x^{10}+10\,a^2b^4x^8+10\,a^3b^3x^6+\right.}{20\left(a^7b^5x^{12}+5\,a^8b^4x^{10}+10\,a^9b^3x^8+10\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/20*(60*a*b^5*x^10 + 270*a^2*b^4*x^8 + 470*a^3*b^3*x^6 + 385*a^4*b^2*x^4
+ 137*a^5*b*x^2 + 10*a^6 - 60*(b^6*x^12 + 5*a*b^5*x^10 + 10*a^2*b^4*x^8 + 1
0*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2)*log(b*x^2 + a) + 120*(b^6*x^12 +
 5*a*b^5*x^10 + 10*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2
)*log(x))/(a^7*b^5*x^12 + 5*a^8*b^4*x^10 + 10*a^9*b^3*x^8 + 10*a^10*b^2*x^6
 + 5*a^11*b*x^4 + a^12*x^2)
```

Sympy [A] time = 6.15536, size = 148, normalized size = 1.28

$$-\frac{10a^5+137a^4bx^2+385a^3b^2x^4+470a^2b^3x^6+270ab^4x^8+60b^5x^{10}}{20a^{11}x^2+100a^{10}bx^4+200a^9b^2x^6+200a^8b^3x^8+100a^7b^4x^{10}+20a^6b^5x^{12}}-\frac{6b\log(x)}{a^7}+\frac{3b\log\left(\frac{a}{b}+x^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -(10*a**5 + 137*a**4*b*x**2 + 385*a**3*b**2*x**4 + 470*a**2*b**3*x**6 + 270
*a*b**4*x**8 + 60*b**5*x**10)/(20*a**11*x**2 + 100*a**10*b*x**4 + 200*a**9*
b**2*x**6 + 200*a**8*b**3*x**8 + 100*a**7*b**4*x**10 + 20*a**6*b**5*x**12)
- 6*b*log(x)/a**7 + 3*b*log(a/b + x**2)/a**7
```

Giac [A] time = 1.16342, size = 155, normalized size = 1.34

$$-\frac{3\,b\log\left(x^2\right)}{a^7}+\frac{3\,b\log\left(\left|bx^2+a\right|\right)}{a^7}+\frac{6\,bx^2-a}{2\,a^7x^2}-\frac{137\,b^6x^{10}+735\,ab^5x^8+1590\,a^2b^4x^6+1740\,a^3b^3x^4+970\,a^4b^2x^2+22}{20\left(bx^2+a\right)^5a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -3*b*log(x^2)/a^7 + 3*b*log(abs(b*x^2 + a))/a^7 + 1/2*(6*b*x^2 - a)/(a^7*x^
2) - 1/20*(137*b^6*x^10 + 735*a*b^5*x^8 + 1590*a^2*b^4*x^6 + 1740*a^3*b^3*x
^4 + 970*a^4*b^2*x^2 + 224*a^5*b)/((b*x^2 + a)^5*a^7)
```

3.522 $\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=140

$$\frac{15b^2}{2a^7\left(a+bx^2\right)}+\frac{5b^2}{2a^6\left(a+bx^2\right)^2}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+\frac{b^2}{10a^3\left(a+bx^2\right)^5}-\frac{21b^2\log\left(a+bx^2\right)}{2a^8}+\frac{21b^2\log(x)}{a^8}$$

```
[Out] -1/(4*a^6*x^4) + (3*b)/(a^7*x^2) + b^2/(10*a^3*(a + b*x^2)^5) + (3*b^2)/(8*
a^4*(a + b*x^2)^4) + b^2/(a^5*(a + b*x^2)^3) + (5*b^2)/(2*a^6*(a + b*x^2)^2
) + (15*b^2)/(2*a^7*(a + b*x^2)) + (21*b^2*Log[x])/a^8 - (21*b^2*Log[a + b*
x^2])/(2*a^8)
```

Rubi [A] time = 0.144679, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 266, 44}

$$\frac{15b^2}{2a^7\left(a+bx^2\right)}+\frac{5b^2}{2a^6\left(a+bx^2\right)^2}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+\frac{b^2}{10a^3\left(a+bx^2\right)^5}-\frac{21b^2\log\left(a+bx^2\right)}{2a^8}+\frac{21b^2\log(x)}{a^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```



```
[Out] -1/(4*a^6*x^4) + (3*b)/(a^7*x^2) + b^2/(10*a^3*(a + b*x^2)^5) + (3*b^2)/(8*
a^4*(a + b*x^2)^4) + b^2/(a^5*(a + b*x^2)^3) + (5*b^2)/(2*a^6*(a + b*x^2)^2
) + (15*b^2)/(2*a^7*(a + b*x^2)) + (21*b^2*Log[x])/a^8 - (21*b^2*Log[a + b*
x^2])/(2*a^8)
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^5\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \frac{1}{x^3\left(ab+b^2x\right)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\,\text{Subst}\left(\int \left(\frac{1}{a^6b^6x^3}-\frac{6}{a^7b^5x^2}+\frac{21}{a^8b^4x}-\frac{1}{a^3b^3(a+bx)^6}-\frac{3}{a^4b^3(a+bx)^5}-\frac{6}{a^5b^3(a+\right.\right.$$
$$= -\frac{1}{4a^6x^4}+\frac{3b}{a^7x^2}+\frac{b^2}{10a^3\left(a+bx^2\right)^5}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{5b^2}{2a^6\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.0592197, size = 107, normalized size = 0.76

$$\frac{\frac{a\left(3290a^2b^4x^8+2695a^3b^3x^6+959a^4b^2x^4+70a^5bx^2-10a^6+1890ab^5x^{10}+420b^6x^{12}\right)}{x^4\left(a+bx^2\right)^5}-420b^2\log\left(a+bx^2\right)+840b^2\log(x)}{40a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((a*(-10*a^6 + 70*a^5*b*x^2 + 959*a^4*b^2*x^4 + 2695*a^3*b^3*x^6 + 3290*a^2
*b^4*x^8 + 1890*a*b^5*x^10 + 420*b^6*x^12))/(x^4*(a + b*x^2)^5) + 840*b^2*L
og[x] - 420*b^2*Log[a + b*x^2])/(40*a^8)
```

Maple [A] time = 0.058, size = 129, normalized size = 0.9

$$-\frac{1}{4\,x^4a^6}+3\,\frac{b}{a^7x^2}+\frac{b^2}{10\,a^3\left(bx^2+a\right)^5}+\frac{3\,b^2}{8\,a^4\left(bx^2+a\right)^4}+\frac{b^2}{a^5\left(bx^2+a\right)^3}+\frac{5\,b^2}{2\,a^6\left(bx^2+a\right)^2}+\frac{15\,b^2}{2\,a^7\left(bx^2+a\right)}+21\,\frac{b^2\ln(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/4/x^4/a^6+3*b/a^7/x^2+1/10*b^2/a^3/(b*x^2+a)^5+3/8*b^2/a^4/(b*x^2+a)^4+b
^2/a^5/(b*x^2+a)^3+5/2*b^2/a^6/(b*x^2+a)^2+15/2*b^2/a^7/(b*x^2+a)+21*b^2*ln
(x)/a^8-21/2*b^2*ln(b*x^2+a)/a^8
```

Maxima [A] time = 1.04972, size = 213, normalized size = 1.52

$$\frac{420\,b^6x^{12}+1890\,ab^5x^{10}+3290\,a^2b^4x^8+2695\,a^3b^3x^6+959\,a^4b^2x^4+70\,a^5bx^2-10\,a^6}{40\left(a^7b^5x^{14}+5\,a^8b^4x^{12}+10\,a^9b^3x^{10}+10\,a^{10}b^2x^8+5\,a^{11}bx^6+a^{12}x^4\right)}-\frac{21\,b^2\log\left(bx^2+a\right)}{2\,a^8}+\frac{21\,b^2\log(}{2\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(420*b^6*x^12 + 1890*a*b^5*x^10 + 3290*a^2*b^4*x^8 + 2695*a^3*b^3*x^6
+ 959*a^4*b^2*x^4 + 70*a^5*b*x^2 - 10*a^6)/(a^7*b^5*x^14 + 5*a^8*b^4*x^12 +
 10*a^9*b^3*x^10 + 10*a^10*b^2*x^8 + 5*a^11*b*x^6 + a^12*x^4) - 21/2*b^2*lo
g(b*x^2 + a)/a^8 + 21/2*b^2*log(x^2)/a^8
```

Fricas [B] time = 1.54522, size = 586, normalized size = 4.19

$$\frac{420\,ab^6x^{12}+1890\,a^2b^5x^{10}+3290\,a^3b^4x^8+2695\,a^4b^3x^6+959\,a^5b^2x^4+70\,a^6bx^2-10\,a^7-420\left(b^7x^{14}+5\,ab^6x^{12}+10\,a^2b\right.}{40\left(a^8b^5x^{14}+5\,a^9b^4x^{12}+10\,a^1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(420*a*b^6*x^12 + 1890*a^2*b^5*x^10 + 3290*a^3*b^4*x^8 + 2695*a^4*b^3*
x^6 + 959*a^5*b^2*x^4 + 70*a^6*b*x^2 - 10*a^7 - 420*(b^7*x^14 + 5*a*b^6*x^1
2 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 + 5*a^4*b^3*x^6 + a^5*b^2*x^4)*log(b*x
^2 + a) + 840*(b^7*x^14 + 5*a*b^6*x^12 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 +
 5*a^4*b^3*x^6 + a^5*b^2*x^4)*log(x))/(a^8*b^5*x^14 + 5*a^9*b^4*x^12 + 10*a
^10*b^3*x^10 + 10*a^11*b^2*x^8 + 5*a^12*b*x^6 + a^13*x^4)
```

Sympy [A] time = 13.4509, size = 165, normalized size = 1.18

$$\frac{-10a^6+70a^5bx^2+959a^4b^2x^4+2695a^3b^3x^6+3290a^2b^4x^8+1890ab^5x^{10}+420b^6x^{12}}{40a^{12}x^4+200a^{11}bx^6+400a^{10}b^2x^8+400a^9b^3x^{10}+200a^8b^4x^{12}+40a^7b^5x^{14}}+\frac{21b^2\log(x)}{a^8}-\frac{21b^2\log\left(\frac{a}{b}+x^2\right)}{2a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-10*a**6 + 70*a**5*b*x**2 + 959*a**4*b**2*x**4 + 2695*a**3*b**3*x**6 + 329
0*a**2*b**4*x**8 + 1890*a*b**5*x**10 + 420*b**6*x**12)/(40*a**12*x**4 + 200
*a**11*b*x**6 + 400*a**10*b**2*x**8 + 400*a**9*b**3*x**10 + 200*a**8*b**4*x
**12 + 40*a**7*b**5*x**14) + 21*b**2*log(x)/a**8 - 21*b**2*log(a/b + x**2)/
(2*a**8)
```

Giac [A] time = 1.15962, size = 176, normalized size = 1.26

$$\frac{21\,b^2\log\left(x^2\right)}{2\,a^8}-\frac{21\,b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^8}-\frac{63\,b^2x^4-12\,abx^2+a^2}{4\,a^8x^4}+\frac{959\,b^7x^{10}+5095\,ab^6x^8+10890\,a^2b^5x^6+11730\,a^3b^4x^4+\cdots}{40\left(bx^2+a\right)^5a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 21/2*b^2*log(x^2)/a^8 - 21/2*b^2*log(abs(b*x^2 + a))/a^8 - 1/4*(63*b^2*x^4
- 12*a*b*x^2 + a^2)/(a^8*x^4) + 1/40*(959*b^7*x^10 + 5095*a*b^6*x^8 + 10890
*a^2*b^5*x^6 + 11730*a^3*b^4*x^4 + 6390*a^4*b^3*x^2 + 1418*a^5*b^2)/((b*x^2
 + a)^5*a^8)
```

3.523 $\int \frac{x^{16}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=155

$$\frac{9009a^2x}{256b^8}-\frac{9009a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{17/2}}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{1287x^7}{256b^5\left(a+bx^2\right)}-\frac{3003ax^3}{256b^7}$$

```
[Out] (9009*a^2*x)/(256*b^8) - (3003*a*x^3)/(256*b^7) + (9009*x^5)/(1280*b^6) - x
^15/(10*b*(a + b*x^2)^5) - (3*x^13)/(16*b^2*(a + b*x^2)^4) - (13*x^11)/(32*
b^3*(a + b*x^2)^3) - (143*x^9)/(128*b^4*(a + b*x^2)^2) - (1287*x^7)/(256*b^
5*(a + b*x^2)) - (9009*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(17/2))
```

Rubi [A] time = 0.105612, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{9009a^2x}{256b^8}-\frac{9009a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{17/2}}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{1287x^7}{256b^5\left(a+bx^2\right)}-\frac{3003ax^3}{256b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```



```
[Out] (9009*a^2*x)/(256*b^8) - (3003*a*x^3)/(256*b^7) + (9009*x^5)/(1280*b^6) - x
^15/(10*b*(a + b*x^2)^5) - (3*x^13)/(16*b^2*(a + b*x^2)^4) - (13*x^11)/(32*
b^3*(a + b*x^2)^3) - (143*x^9)/(128*b^4*(a + b*x^2)^2) - (1287*x^7)/(256*b^
5*(a + b*x^2)) - (9009*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(17/2))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{16}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{16}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}+\frac{1}{2}\left(3b^4\right)\int \frac{x^{14}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}+\frac{1}{16}\left(39b^2\right)\int \frac{x^{12}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}+\frac{143}{32}\int \frac{x^{10}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}+\frac{1287\int \frac{x^8}{\left(ab+b^2x^2\right.}}{128b^2}$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{1287x^7}{256b^5\left(a+bx^2\right.}$$

$$= -\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{1287x^7}{256b^5\left(a+bx^2\right.}$$

$$= \frac{9009a^2x}{256b^8}-\frac{3003ax^3}{256b^7}+\frac{9009x^5}{1280b^6}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\ldots$$

$$= \frac{9009a^2x}{256b^8}-\frac{3003ax^3}{256b^7}+\frac{9009x^5}{1280b^6}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\ldots$$

Mathematica [A] time = 0.0654545, size = 122, normalized size = 0.79

$$\frac{\frac{\sqrt{b}x\left(16640a^2b^5x^{10}+137995a^3b^4x^8+338910a^4b^3x^6+384384a^5b^2x^4+210210a^6bx^2+45045a^7-1280ab^6x^{12}+256b^7x^{14}\right)}{\left(a+bx^2\right)^5}-45045a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1280b^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(45045*a^7 + 210210*a^6*b*x^2 + 384384*a^5*b^2*x^4 + 338910*a^4
*b^3*x^6 + 137995*a^3*b^4*x^8 + 16640*a^2*b^5*x^10 - 1280*a*b^6*x^12 + 256*
```

```
b^7*x^14))/(a + b*x^2)^5 - 45045*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(1280
*b^(17/2))
```

Maple [A] time = 0.056, size = 148, normalized size = 1.

$$\frac{x^5}{5\,b^6}-2\,\frac{ax^3}{b^7}+21\,\frac{a^2x}{b^8}+\frac{5327\,a^3x^9}{256\,b^4\left(bx^2+a\right)^5}+\frac{9443\,a^4x^7}{128\,b^5\left(bx^2+a\right)^5}+\frac{1001\,a^5x^5}{10\,b^6\left(bx^2+a\right)^5}+\frac{7837\,a^6x^3}{128\,b^7\left(bx^2+a\right)^5}+\frac{3633\,a^7x}{256\,b^8\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/5*x^5/b^6-2*a*x^3/b^7+21*a^2*x/b^8+5327/256/b^4*a^3/(b*x^2+a)^5*x^9+9443/
128/b^5*a^4/(b*x^2+a)^5*x^7+1001/10/b^6*a^5/(b*x^2+a)^5*x^5+7837/128/b^7*a^
6/(b*x^2+a)^5*x^3+3633/256/b^8*a^7/(b*x^2+a)^5*x-9009/256/b^8*a^3/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55159, size = 1049, normalized size = 6.77

$$\left[\frac{512\,b^7x^{15}-2560\,ab^6x^{13}+33280\,a^2b^5x^{11}+275990\,a^3b^4x^9+677820\,a^4b^3x^7+768768\,a^5b^2x^5+420420\,a^6bx^3+90090}{2560\left(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(512*b^7*x^15 - 2560*a*b^6*x^13 + 33280*a^2*b^5*x^11 + 275990*a^3*b
^4*x^9 + 677820*a^4*b^3*x^7 + 768768*a^5*b^2*x^5 + 420420*a^6*b*x^3 + 90090
*a^7*x + 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*
x^4 + 5*a^6*b*x^2 + a^7)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4 + 5
*a^4*b^9*x^2 + a^5*b^8), 1/1280*(256*b^7*x^15 - 1280*a*b^6*x^13 + 16640*a^2
*b^5*x^11 + 137995*a^3*b^4*x^9 + 338910*a^4*b^3*x^7 + 384384*a^5*b^2*x^5 +
210210*a^6*b*x^3 + 45045*a^7*x - 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^8 + 10*a
^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7)*sqrt(a/b)*arctan(b*x*sqrt(
a/b)/a))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4 + 5*
a^4*b^9*x^2 + a^5*b^8)]
```

Sympy [A] time = 1.53132, size = 218, normalized size = 1.41

$$\frac{21a^2x}{b^8} - \frac{2ax^3}{b^7} + \frac{9009\sqrt{-\frac{a^5}{b^{17}}}\log\left(x - \frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512} - \frac{9009\sqrt{-\frac{a^5}{b^{17}}}\log\left(x + \frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512} + \frac{18165a^7x + 78370a^6bx^3 + 1281}{1280a^5b^8 + 6400a^4b^9x^2 + 12800a^3b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**16/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 21*a**2*x/b**8 - 2*a*x**3/b**7 + 9009*sqrt(-a**5/b**17)*log(x - b**8*sqrt(-
a**5/b**17)/a**2)/512 - 9009*sqrt(-a**5/b**17)*log(x + b**8*sqrt(-a**5/b**1
7)/a**2)/512 + (18165*a**7*x + 78370*a**6*b*x**3 + 128128*a**5*b**2*x**5 +
94430*a**4*b**3*x**7 + 26635*a**3*b**4*x**9)/(1280*a**5*b**8 + 6400*a**4*b*
*9*x**2 + 12800*a**3*b**10*x**4 + 12800*a**2*b**11*x**6 + 6400*a*b**12*x**8
 + 1280*b**13*x**10) + x**5/(5*b**6)
```

Giac [A] time = 1.13854, size = 158, normalized size = 1.02

$$-\frac{9009\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^8} + \frac{26635\,a^3b^4x^9 + 94430\,a^4b^3x^7 + 128128\,a^5b^2x^5 + 78370\,a^6bx^3 + 18165\,a^7x}{1280\left(bx^2+a\right)^5b^8} + \frac{b^{24}x^5 - 10\,ab^{23}x^3}{5\,b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -9009/256*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^8) + 1/1280*(26635*a^3*b^4
*x^9 + 94430*a^4*b^3*x^7 + 128128*a^5*b^2*x^5 + 78370*a^6*b*x^3 + 18165*a^7
*x)/((b*x^2 + a)^5*b^8) + 1/5*(b^24*x^5 - 10*a*b^23*x^3 + 105*a^2*b^22*x)/b
^30
```

3.524 $\int \frac{x^{14}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{3003a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{15/2}}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}-\frac{3003x^5}{1280b^5\left(a+bx^2\right)}-\frac{3003ax}{256b^7}-\frac{x^{1\cdots}}{10b\left(a+\cdots\right.}$$

```
[Out] (-3003*a*x)/(256*b^7) + (1001*x^3)/(256*b^6) - x^13/(10*b*(a + b*x^2)^5) -
(13*x^11)/(80*b^2*(a + b*x^2)^4) - (143*x^9)/(480*b^3*(a + b*x^2)^3) - (429
*x^7)/(640*b^4*(a + b*x^2)^2) - (3003*x^5)/(1280*b^5*(a + b*x^2)) + (3003*a
^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(15/2))
```

Rubi [A] time = 0.0927995, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 302, 205}

$$\frac{3003a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{15/2}}-\frac{13x^{11}}{80b^2\left(a+bx^2\right)^4}-\frac{143x^9}{480b^3\left(a+bx^2\right)^3}-\frac{429x^7}{640b^4\left(a+bx^2\right)^2}-\frac{3003x^5}{1280b^5\left(a+bx^2\right)}-\frac{3003ax}{256b^7}-\frac{x^{1\cdots}}{10b\left(a+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-3003*a*x)/(256*b^7) + (1001*x^3)/(256*b^6) - x^13/(10*b*(a + b*x^2)^5) -
(13*x^11)/(80*b^2*(a + b*x^2)^4) - (143*x^9)/(480*b^3*(a + b*x^2)^3) - (429
*x^7)/(640*b^4*(a + b*x^2)^2) - (3003*x^5)/(1280*b^5*(a + b*x^2)) + (3003*a
^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(15/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{14}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5} + \frac{1}{10}\left(13b^4\right)\int \frac{x^{12}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5} - \frac{13x^{11}}{80b^2\left(a+bx^2\right)^4} + \frac{1}{80}\left(143b^2\right)\int \frac{x^{10}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5} - \frac{13x^{11}}{80b^2\left(a+bx^2\right)^4} - \frac{143x^9}{480b^3\left(a+bx^2\right)^3} + \frac{429}{160}\int \frac{x^8}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5} - \frac{13x^{11}}{80b^2\left(a+bx^2\right)^4} - \frac{143x^9}{480b^3\left(a+bx^2\right)^3} - \frac{429x^7}{640b^4\left(a+bx^2\right)^2} + \frac{3003\int \frac{x^6}{(ab+b^2}}{640b^2}$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5} - \frac{13x^{11}}{80b^2\left(a+bx^2\right)^4} - \frac{143x^9}{480b^3\left(a+bx^2\right)^3} - \frac{429x^7}{640b^4\left(a+bx^2\right)^2} - \frac{3003x^5}{1280b^5\left(a+b\right.}$$

$$= -\frac{x^{13}}{10b\left(a+bx^2\right)^5} - \frac{13x^{11}}{80b^2\left(a+bx^2\right)^4} - \frac{143x^9}{480b^3\left(a+bx^2\right)^3} - \frac{429x^7}{640b^4\left(a+bx^2\right)^2} - \frac{3003x^5}{1280b^5\left(a+b\right.}$$

$$= -\frac{3003ax}{256b^7} + \frac{1001x^3}{256b^6} - \frac{x^{13}}{10b\left(a+bx^2\right)^5} - \frac{13x^{11}}{80b^2\left(a+bx^2\right)^4} - \frac{143x^9}{480b^3\left(a+bx^2\right)^3} - \frac{429x^7}{640b^4\left(a+b\right.}$$

$$= -\frac{3003ax}{256b^7} + \frac{1001x^3}{256b^6} - \frac{x^{13}}{10b\left(a+bx^2\right)^5} - \frac{13x^{11}}{80b^2\left(a+bx^2\right)^4} - \frac{143x^9}{480b^3\left(a+bx^2\right)^3} - \frac{429x^7}{640b^4\left(a+b\right.}$$

Mathematica [A] time = 0.0585399, size = 111, normalized size = 0.78

$$\frac{\frac{\sqrt{b}x\left(-137995a^2b^4x^8-338910a^3b^3x^6-384384a^4b^2x^4-210210a^5bx^2-45045a^6-16640ab^5x^{10}+1280b^6x^{12}\right)}{\left(a+bx^2\right)^5}+45045a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3840b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(-45045*a^6 - 210210*a^5*b*x^2 - 384384*a^4*b^2*x^4 - 338910*a^
3*b^3*x^6 - 137995*a^2*b^4*x^8 - 16640*a*b^5*x^10 + 1280*b^6*x^12))/(a + b*
```

```
x^2)^5 + 45045*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3840*b^(15/2))
```

Maple [A] time = 0.055, size = 137, normalized size = 1.

$$\frac{x^3}{3\,b^6}-6\,\frac{ax}{b^7}-\frac{2373\,a^2x^9}{256\,b^3\left(bx^2+a\right)^5}-\frac{12131\,a^3x^7}{384\,b^4\left(bx^2+a\right)^5}-\frac{1253\,a^4x^5}{30\,b^5\left(bx^2+a\right)^5}-\frac{9629\,a^5x^3}{384\,b^6\left(bx^2+a\right)^5}-\frac{1467\,a^6x}{256\,b^7\left(bx^2+a\right)^5}+\frac{3003}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/3*x^3/b^6-6*a*x/b^7-2373/256/b^3*a^2/(b*x^2+a)^5*x^9-12131/384/b^4*a^3/(b
*x^2+a)^5*x^7-1253/30/b^5*a^4/(b*x^2+a)^5*x^5-9629/384/b^6*a^5/(b*x^2+a)^5*
x^3-1467/256/b^7*a^6/(b*x^2+a)^5*x+3003/256/b^7*a^2/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53815, size = 990, normalized size = 6.97

$$\left[\frac{2560\,b^6x^{13}-33280\,ab^5x^{11}-275990\,a^2b^4x^9-677820\,a^3b^3x^7-768768\,a^4b^2x^5-420420\,a^5bx^3-90090\,a^6x+45045\,(a}{7680\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(2560*b^6*x^13 - 33280*a*b^5*x^11 - 275990*a^2*b^4*x^9 - 677820*a^3
*b^3*x^7 - 768768*a^4*b^2*x^5 - 420420*a^5*b*x^3 - 90090*a^6*x + 45045*(a*b
^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 + a
^6)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^12*x^10
+ 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7
), 1/3840*(1280*b^6*x^13 - 16640*a*b^5*x^11 - 137995*a^2*b^4*x^9 - 338910*a
^3*b^3*x^7 - 384384*a^4*b^2*x^5 - 210210*a^5*b*x^3 - 45045*a^6*x + 45045*(a
*b^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 +
 a^6)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2
*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)]
```

Sympy [A] time = 1.46659, size = 202, normalized size = 1.42

$$-\frac{6ax}{b^7} - \frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x - \frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512} + \frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x + \frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512} - \frac{22005a^6x + 96290a^5bx^3 + 160384a^4b^2x \ldots}{3840a^5b^7 + 19200a^4b^8x^2 + 38400a^3b^9x^4 + 384 \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -6*a*x/b**7 - 3003*sqrt(-a**3/b**15)*log(x - b**7*sqrt(-a**3/b**15)/a)/512
+ 3003*sqrt(-a**3/b**15)*log(x + b**7*sqrt(-a**3/b**15)/a)/512 - (22005*a**
6*x + 96290*a**5*b*x**3 + 160384*a**4*b**2*x**5 + 121310*a**3*b**3*x**7 + 3
5595*a**2*b**4*x**9)/(3840*a**5*b**7 + 19200*a**4*b**8*x**2 + 38400*a**3*b*
*9*x**4 + 38400*a**2*b**10*x**6 + 19200*a*b**11*x**8 + 3840*b**12*x**10) +
x**3/(3*b**6)
```

Giac [A] time = 1.16593, size = 143, normalized size = 1.01

$$\frac{3003\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}b^7} - \frac{35595\,a^2b^4x^9 + 121310\,a^3b^3x^7 + 160384\,a^4b^2x^5 + 96290\,a^5bx^3 + 22005\,a^6x}{3840\left(bx^2+a\right)^5b^7} + \frac{b^{12}x^3 - 18\,ab^{11}x}{3\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

[Out] 3003/256*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) - 1/3840*(35595*a^2*b^4*x^9 + 121310*a^3*b^3*x^7 + 160384*a^4*b^2*x^5 + 96290*a^5*b*x^3 + 22005*a^6*x)/((b*x^2 + a)^5*b^7) + 1/3*(b^12*x^3 - 18*a*b^11*x)/b^18

3.525 $\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=131

$$-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{231x^3}{256b^5\left(a+bx^2\right)}-\frac{693\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{13/2}}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}+\frac{69}{25}$$

```
[Out] (693*x)/(256*b^6) - x^11/(10*b*(a + b*x^2)^5) - (11*x^9)/(80*b^2*(a + b*x^2
)^4) - (33*x^7)/(160*b^3*(a + b*x^2)^3) - (231*x^5)/(640*b^4*(a + b*x^2)^2)
 - (231*x^3)/(256*b^5*(a + b*x^2)) - (693*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(256*b^(13/2))
```

Rubi [A] time = 0.0777952, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 321, 205}

$$-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{231x^3}{256b^5\left(a+bx^2\right)}-\frac{693\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{13/2}}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}+\frac{69}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (693*x)/(256*b^6) - x^11/(10*b*(a + b*x^2)^5) - (11*x^9)/(80*b^2*(a + b*x^2
)^4) - (33*x^7)/(160*b^3*(a + b*x^2)^3) - (231*x^5)/(640*b^4*(a + b*x^2)^2)
 - (231*x^3)/(256*b^5*(a + b*x^2)) - (693*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(256*b^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^{12}}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(11b^4\right)\int \frac{x^{10}}{\left(ab+b^2x^2\right)^5}\,dx$$
$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(99b^2\right)\int \frac{x^8}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}+\frac{231}{160}\int \frac{x^6}{\left(ab+b^2x^2\right)^3}\,dx$$
$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}+\frac{231\int \frac{x^4}{\left(ab+b^2x^2\right.}}{128b^2}$$
$$= -\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{231x^3}{256b^5\left(a+b\right.}$$
$$= \frac{693x}{256b^6}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{2}{256b^5}$$
$$= \frac{693x}{256b^6}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}-\frac{11x^9}{80b^2\left(a+bx^2\right)^4}-\frac{33x^7}{160b^3\left(a+bx^2\right)^3}-\frac{231x^5}{640b^4\left(a+bx^2\right)^2}-\frac{2}{256b^5}$$

Mathematica [A] time = 0.0509986, size = 100, normalized size = 0.76

$$\frac{\frac{\sqrt{b}x\left(26070a^2b^3x^6+29568a^3b^2x^4+16170a^4bx^2+3465a^5+10615ab^4x^8+1280b^5x^{10}\right)}{\left(a+bx^2\right)^5}-3465\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1280b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(3465*a^5 + 16170*a^4*b*x^2 + 29568*a^3*b^2*x^4 + 26070*a^2*b^3
*x^6 + 10615*a*b^4*x^8 + 1280*b^5*x^10))/(a + b*x^2)^5 - 3465*Sqrt[a]*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(1280*b^(13/2))
```

Maple [A] time = 0.056, size = 123, normalized size = 0.9

$$\frac{x}{b^6}+\frac{843\,a x^9}{256\,b^2\left(b x^2+a\right)^5}+\frac{1327\,a^2x^7}{128\,b^3\left(b x^2+a\right)^5}+\frac{131\,a^3x^5}{10\,b^4\left(b x^2+a\right)^5}+\frac{977\,a^4x^3}{128\,b^5\left(b x^2+a\right)^5}+\frac{437\,a^5x}{256\,b^6\left(b x^2+a\right)^5}-\frac{693\,a}{256\,b^6}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] x/b^6+843/256/b^2*a/(b*x^2+a)^5*x^9+1327/128/b^3*a^2/(b*x^2+a)^5*x^7+131/10
/b^4*a^3/(b*x^2+a)^5*x^5+977/128/b^5*a^4/(b*x^2+a)^5*x^3+437/256/b^6*a^5/(b
*x^2+a)^5*x-693/256/b^6*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50938, size = 903, normalized size = 6.89

$$\left[\frac{2560\,b^5x^{11}+21230\,ab^4x^9+52140\,a^2b^3x^7+59136\,a^3b^2x^5+32340\,a^4bx^3+6930\,a^5x+3465\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3\right.}{2560\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(2560*b^5*x^11 + 21230*a*b^4*x^9 + 52140*a^2*b^3*x^7 + 59136*a^3*b^
2*x^5 + 32340*a^4*b*x^3 + 6930*a^5*x + 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^
2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 - 2*b
*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6
```

```
 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6), 1/1280*(1280*b^5*x^11 + 10615
*a*b^4*x^9 + 26070*a^2*b^3*x^7 + 29568*a^3*b^2*x^5 + 16170*a^4*b*x^3 + 3465
*a^5*x - 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5
*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^11*x^10 + 5*a*b^10*
x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)]
```

Sympy [A] time = 1.38736, size = 178, normalized size = 1.36

$$\frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(-b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{512}-\frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{512}+\frac{2185a^5x+9770a^4bx^3+16768a^3b^2x^5+13270\cdots}{1280a^5b^6+6400a^4b^7x^2+12800a^3b^8x^4+12800a^2b^9x\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 693*sqrt(-a/b**13)*log(-b**6*sqrt(-a/b**13) + x)/512 - 693*sqrt(-a/b**13)*l
og(b**6*sqrt(-a/b**13) + x)/512 + (2185*a**5*x + 9770*a**4*b*x**3 + 16768*a
**3*b**2*x**5 + 13270*a**2*b**3*x**7 + 4215*a*b**4*x**9)/(1280*a**5*b**6 +
6400*a**4*b**7*x**2 + 12800*a**3*b**8*x**4 + 12800*a**2*b**9*x**6 + 6400*a*
b**10*x**8 + 1280*b**11*x**10) + x/b**6
```

Giac [A] time = 1.14657, size = 117, normalized size = 0.89

$$-\frac{693\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^6}+\frac{x}{b^6}+\frac{4215\,ab^4x^9+13270\,a^2b^3x^7+16768\,a^3b^2x^5+9770\,a^4bx^3+2185\,a^5x}{1280\left(bx^2+a\right)^5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -693/256*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + x/b^6 + 1/1280*(4215*a*b
^4*x^9 + 13270*a^2*b^3*x^7 + 16768*a^3*b^2*x^5 + 9770*a^4*b*x^3 + 2185*a^5*
x)/((b*x^2 + a)^5*b^6)
```

3.526 $\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=121

$$-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{63x}{256b^5\left(a+bx^2\right)}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{a}b^{11/2}}-\frac{x^9}{10b\left(a+bx^2\right)^5}$$

```
[Out] -x^9/(10*b*(a + b*x^2)^5) - (9*x^7)/(80*b^2*(a + b*x^2)^4) - (21*x^5)/(160*
b^3*(a + b*x^2)^3) - (21*x^3)/(128*b^4*(a + b*x^2)^2) - (63*x)/(256*b^5*(a
+ b*x^2)) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*Sqrt[a]*b^(11/2))
```

Rubi [A] time = 0.0688564, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 288, 205}

$$-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{63x}{256b^5\left(a+bx^2\right)}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{a}b^{11/2}}-\frac{x^9}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^9/(10*b*(a + b*x^2)^5) - (9*x^7)/(80*b^2*(a + b*x^2)^4) - (21*x^5)/(160*
b^3*(a + b*x^2)^3) - (21*x^3)/(128*b^4*(a + b*x^2)^2) - (63*x)/(256*b^5*(a
+ b*x^2)) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*Sqrt[a]*b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^{10}}{\left(ab+b^2x^2\right)^6}\,dx\\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(9b^4\right)\int \frac{x^8}{\left(ab+b^2x^2\right)^5}\,dx\\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(63b^2\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^4}\,dx\\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}+\frac{21}{32}\int \frac{x^4}{\left(ab+b^2x^2\right)^3}\,dx\\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}+\frac{63\int \frac{x^2}{\left(ab+b^2x^2\right)}}{128b^2}\\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{63x}{256b^5\left(a+bx\right.}\\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-\frac{63x}{256b^5\left(a+bx\right.}
\end{aligned}$$

Mathematica [A] time = 0.0509354, size = 88, normalized size = 0.73

$$\frac{63\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{a}b^{11/2}}-\frac{x\left(2688a^2b^2x^4+1470a^3bx^2+315a^4+2370ab^3x^6+965b^4x^8\right)}{1280b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(x*(315*a^4 + 1470*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 2370*a*b^3*x^6 + 965*b^4
*x^8))/(1280*b^5*(a + b*x^2)^5) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*Sqr
t[a]*b^(11/2))
```

Maple [A] time = 0.056, size = 80, normalized size = 0.7

$$\frac{1}{\left(bx^2+a\right)^5}\left(-\frac{193\,x^9}{256\,b}-\frac{237\,ax^7}{128\,b^2}-\frac{21\,a^2x^5}{10\,b^3}-\frac{147\,a^3x^3}{128\,b^4}-\frac{63\,a^4x}{256\,b^5}\right)+\frac{63}{256\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (-193/256/b*x^9-237/128/b^2*a*x^7-21/10*a^2/b^3*x^5-147/128*a^3/b^4*x^3-63/
256*a^4/b^5*x)/(b*x^2+a)^5+63/256/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.513, size = 861, normalized size = 7.12

$$\left[-\frac{1930\,ab^5x^9+4740\,a^2b^4x^7+5376\,a^3b^3x^5+2940\,a^4b^2x^3+630\,a^5bx+315\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+\right.}{2560\left(ab^{11}x^{10}+5\,a^2b^{10}x^8+10\,a^3b^9x^6+10\,a^4b^8x^4+5\,a^5b^7x^2+a^6b^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(1930*a*b^5*x^9 + 4740*a^2*b^4*x^7 + 5376*a^3*b^3*x^5 + 2940*a^4*b
^2*x^3 + 630*a^5*b*x + 315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^
3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a*b^11*x^10 + 5*a^2*b^10*x^8 + 10*a^3*b^9*x^6 + 10*a^4*b^8*x
^4 + 5*a^5*b^7*x^2 + a^6*b^6), -1/1280*(965*a*b^5*x^9 + 2370*a^2*b^4*x^7 +
2688*a^3*b^3*x^5 + 1470*a^4*b^2*x^3 + 315*a^5*b*x - 315*(b^5*x^10 + 5*a*b^4
*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arcta
n(sqrt(a*b)*x/a))/(a*b^11*x^10 + 5*a^2*b^10*x^8 + 10*a^3*b^9*x^6 + 10*a^4*b
^8*x^4 + 5*a^5*b^7*x^2 + a^6*b^6)]
```

Sympy [A] time = 1.27807, size = 180, normalized size = 1.49

$$-\frac{63\sqrt{-\frac{1}{ab^{11}}}\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}+\frac{63\sqrt{-\frac{1}{ab^{11}}}\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}-\frac{315a^4x+1470a^3bx^3+2688a^2b^2x^5+}{1280a^5b^5+6400a^4b^6x^2+12800a^3b^7x^4+12800a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -63*sqrt(-1/(a*b**11))*log(-a*b**5*sqrt(-1/(a*b**11)) + x)/512 + 63*sqrt(-1
/(a*b**11))*log(a*b**5*sqrt(-1/(a*b**11)) + x)/512 - (315*a**4*x + 1470*a**
3*b*x**3 + 2688*a**2*b**2*x**5 + 2370*a*b**3*x**7 + 965*b**4*x**9)/(1280*a*
*5*b**5 + 6400*a**4*b**6*x**2 + 12800*a**3*b**7*x**4 + 12800*a**2*b**8*x**6
 + 6400*a*b**9*x**8 + 1280*b**10*x**10)
```

Giac [A] time = 1.1735, size = 105, normalized size = 0.87

$$\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}b^5}-\frac{965\,b^4x^9+2370\,ab^3x^7+2688\,a^2b^2x^5+1470\,a^3bx^3+315\,a^4x}{1280\left(bx^2+a\right)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/1280*(965*b^4*x^9 + 2370*a
*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3 + 315*a^4*x)/((b*x^2 + a)^5*b^
5)
```

3.527 $\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=122

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}+\frac{7x}{256ab^4\left(a+bx^2\right)}-\frac{7x}{128b^4\left(a+bx^2\right)^2}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

```
[Out] -x^7/(10*b*(a + b*x^2)^5) - (7*x^5)/(80*b^2*(a + b*x^2)^4) - (7*x^3)/(96*b^
3*(a + b*x^2)^3) - (7*x)/(128*b^4*(a + b*x^2)^2) + (7*x)/(256*a*b^4*(a + b*
x^2)) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(3/2)*b^(9/2))
```

Rubi [A] time = 0.0721806, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}+\frac{7x}{256ab^4\left(a+bx^2\right)}-\frac{7x}{128b^4\left(a+bx^2\right)^2}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^7/(10*b*(a + b*x^2)^5) - (7*x^5)/(80*b^2*(a + b*x^2)^4) - (7*x^3)/(96*b^
3*(a + b*x^2)^3) - (7*x)/(128*b^4*(a + b*x^2)^2) + (7*x)/(256*a*b^4*(a + b*
x^2)) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(3/2)*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^8}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(7b^4\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}+\frac{1}{16}\left(7b^2\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}+\frac{7}{32}\int \frac{x^2}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x}{128b^4\left(a+bx^2\right)^2}+\frac{7\int \frac{1}{\left(ab+b^2x^2\right)^2}\,d}{128b^2}$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x}{128b^4\left(a+bx^2\right)^2}+\frac{7x}{256ab^4\left(a+bx\right.}$$

$$= -\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x}{128b^4\left(a+bx^2\right)^2}+\frac{7x}{256ab^4\left(a+bx\right.}$$

Mathematica [A] time = 0.0578995, size = 91, normalized size = 0.75

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}-\frac{x\left(896a^2b^2x^4+490a^3bx^2+105a^4+790ab^3x^6-105b^4x^8\right)}{3840ab^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(x*(105*a^4 + 490*a^3*b*x^2 + 896*a^2*b^2*x^4 + 790*a*b^3*x^6 - 105*b^4*x^
8))/(3840*a*b^4*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(3/
2)*b^(9/2))
```

Maple [A] time = 0.051, size = 80, normalized size = 0.7

$$\frac{1}{\left(bx^2+a\right)^5}\left(\frac{7\,x^9}{256\,a}-\frac{79\,x^7}{384\,b}-\frac{7\,ax^5}{30\,b^2}-\frac{49\,a^2x^3}{384\,b^3}-\frac{7\,xa^3}{256\,b^4}\right)+\frac{7}{256\,ab^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (7/256/a*x^9-79/384/b*x^7-7/30/b^2*a*x^5-49/384*a^2/b^3*x^3-7/256*a^3*x/b^4
)/(b*x^2+a)^5+7/256/a/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51243, size = 855, normalized size = 7.01

$$\left[\frac{210\,ab^5x^9-1580\,a^2b^4x^7-1792\,a^3b^3x^5-980\,a^4b^2x^3-210\,a^5bx-105\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx\right.}{7680\left(a^2b^{10}x^{10}+5\,a^3b^9x^8+10\,a^4b^8x^6+10\,a^5b^7x^4+5\,a^6b^6x^2+a^7b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(210*a*b^5*x^9 - 1580*a^2*b^4*x^7 - 1792*a^3*b^3*x^5 - 980*a^4*b^2*
x^3 - 210*a^5*b*x - 105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b
^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^10*x^10 + 5*a^3*b^9*x^8 + 10*a^4*b^8*x^6 + 10*a^5*b^7*x^4
 + 5*a^6*b^6*x^2 + a^7*b^5), 1/3840*(105*a*b^5*x^9 - 790*a^2*b^4*x^7 - 896*
a^3*b^3*x^5 - 490*a^4*b^2*x^3 - 105*a^5*b*x + 105*(b^5*x^10 + 5*a*b^4*x^8 +
 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt
(a*b)*x/a))/(a^2*b^10*x^10 + 5*a^3*b^9*x^8 + 10*a^4*b^8*x^6 + 10*a^5*b^7*x^
4 + 5*a^6*b^6*x^2 + a^7*b^5)]
```

Sympy [A] time = 1.2115, size = 194, normalized size = 1.59

$$-\frac{7\sqrt{-\frac{1}{a^3b^9}}\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{512}+\frac{7\sqrt{-\frac{1}{a^3b^9}}\log\left(a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{512}+\frac{-105a^4x-490a^3bx^3-896a^2b^2x^5}{3840a^6b^4+19200a^5b^5x^2+38400a^4b^6x^4+38400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -7*sqrt(-1/(a**3*b**9))*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/512 + 7*sq
rt(-1/(a**3*b**9))*log(a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/512 + (-105*a**4
*x - 490*a**3*b*x**3 - 896*a**2*b**2*x**5 - 790*a*b**3*x**7 + 105*b**4*x**9
)/(3840*a**6*b**4 + 19200*a**5*b**5*x**2 + 38400*a**4*b**6*x**4 + 38400*a**
3*b**7*x**6 + 19200*a**2*b**8*x**8 + 3840*a*b**9*x**10)
```

Giac [A] time = 1.13508, size = 113, normalized size = 0.93

$$\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}ab^4}+\frac{105\,b^4x^9-790\,ab^3x^7-896\,a^2b^2x^5-490\,a^3bx^3-105\,a^4x}{3840\left(bx^2+a\right)^5ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/3840*(105*b^4*x^9 - 790*a
*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 105*a^4*x)/((b*x^2 + a)^5*a*b^
4)
```

3.528 $\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=123

$$\frac{3x}{256a^2b^3(a+bx^2)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}-\frac{x^3}{16b^2(a+bx^2)^4}+\frac{x}{128ab^3(a+bx^2)^2}-\frac{x}{32b^3(a+bx^2)^3}-\frac{x^5}{10b(a+bx^2)^5}$$

```
[Out] -x^5/(10*b*(a + b*x^2)^5) - x^3/(16*b^2*(a + b*x^2)^4) - x/(32*b^3*(a + b*x
^2)^3) + x/(128*a*b^3*(a + b*x^2)^2) + (3*x)/(256*a^2*b^3*(a + b*x^2)) + (3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b^(7/2))
```

Rubi [A] time = 0.0721446, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{3x}{256a^2b^3(a+bx^2)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}-\frac{x^3}{16b^2(a+bx^2)^4}+\frac{x}{128ab^3(a+bx^2)^2}-\frac{x}{32b^3(a+bx^2)^3}-\frac{x^5}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^5/(10*b*(a + b*x^2)^5) - x^3/(16*b^2*(a + b*x^2)^4) - x/(32*b^3*(a + b*x
^2)^3) + x/(128*a*b^3*(a + b*x^2)^2) + (3*x)/(256*a^2*b^3*(a + b*x^2)) + (3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^6}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5} + \frac{1}{2}b^4\int \frac{x^4}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5} - \frac{x^3}{16b^2\left(a+bx^2\right)^4} + \frac{1}{16}\left(3b^2\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5} - \frac{x^3}{16b^2\left(a+bx^2\right)^4} - \frac{x}{32b^3\left(a+bx^2\right)^3} + \frac{1}{32}\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5} - \frac{x^3}{16b^2\left(a+bx^2\right)^4} - \frac{x}{32b^3\left(a+bx^2\right)^3} + \frac{x}{128ab^3\left(a+bx^2\right)^2} + \frac{3\int \frac{1}{\left(ab+b^2x^2\right.}}{128ab}$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5} - \frac{x^3}{16b^2\left(a+bx^2\right)^4} - \frac{x}{32b^3\left(a+bx^2\right)^3} + \frac{x}{128ab^3\left(a+bx^2\right)^2} + \frac{3x}{256a^2b^3\left(a\right.}$$

$$= -\frac{x^5}{10b\left(a+bx^2\right)^5} - \frac{x^3}{16b^2\left(a+bx^2\right)^4} - \frac{x}{32b^3\left(a+bx^2\right)^3} + \frac{x}{128ab^3\left(a+bx^2\right)^2} + \frac{3x}{256a^2b^3\left(a\right.}$$

Mathematica [A] time = 0.0585359, size = 91, normalized size = 0.74

$$\frac{-128a^2b^2x^5-70a^3bx^3-15a^4x+70ab^3x^7+15b^4x^9}{1280a^2b^3\left(a+bx^2\right)^5}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-15*a^4*x - 70*a^3*b*x^3 - 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b^4*x^9)/(1
280*a^2*b^3*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b
^(7/2))
```

Maple [A] time = 0.052, size = 78, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^5}\left(\frac{3\,bx^9}{256\,a^2}+\frac{7\,x^7}{128\,a}-\frac{x^5}{10\,b}-\frac{7\,ax^3}{128\,b^2}-\frac{3\,a^2x}{256\,b^3}\right)+\frac{3}{256\,a^2b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (3/256*b/a^2*x^9+7/128/a*x^7-1/10/b*x^5-7/128*a*x^3/b^2-3/256*a^2/b^3*x)/(b
*x^2+a)^5+3/256/a^2/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49462, size = 838, normalized size = 6.81

$$\left[\frac{30\,ab^5x^9 + 140\,a^2b^4x^7 - 256\,a^3b^3x^5 - 140\,a^4b^2x^3 - 30\,a^5bx - 15\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4bx^2 + a\right.}{2560\left(a^3b^9x^{10} + 5\,a^4b^8x^8 + 10\,a^5b^7x^6 + 10\,a^6b^6x^4 + 5\,a^7b^5x^2 + a^8b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(30*a*b^5*x^9 + 140*a^2*b^4*x^7 - 256*a^3*b^3*x^5 - 140*a^4*b^2*x^3
 - 30*a^5*b*x - 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^
4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)))/(a^3*b^9*x^10 + 5*a^4*b^8*x^8 + 10*a^5*b^7*x^6 + 10*a^6*b^6*x^4 + 5*a
^7*b^5*x^2 + a^8*b^4), 1/1280*(15*a*b^5*x^9 + 70*a^2*b^4*x^7 - 128*a^3*b^3*
x^5 - 70*a^4*b^2*x^3 - 15*a^5*b*x + 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3
*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))
/(a^3*b^9*x^10 + 5*a^4*b^8*x^8 + 10*a^5*b^7*x^6 + 10*a^6*b^6*x^4 + 5*a^7*b^
5*x^2 + a^8*b^4)]
```

Sympy [A] time = 1.14973, size = 196, normalized size = 1.59

$$-\frac{3\sqrt{-\frac{1}{a^5b^7}}\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{512}+\frac{3\sqrt{-\frac{1}{a^5b^7}}\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{512}+\frac{-15a^4x - 70a^3bx^3 - 128a^2b^2x^5}{1280a^7b^3 + 6400a^6b^4x^2 + 12800a^5b^5x^4 + 12800}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*b**7))*log(-a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/512 + 3*sq
rt(-1/(a**5*b**7))*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/512 + (-15*a**4*
x - 70*a**3*b*x**3 - 128*a**2*b**2*x**5 + 70*a*b**3*x**7 + 15*b**4*x**9)/(1
280*a**7*b**3 + 6400*a**6*b**4*x**2 + 12800*a**5*b**5*x**4 + 12800*a**4*b**
6*x**6 + 6400*a**3*b**7*x**8 + 1280*a**2*b**8*x**10)
```

Giac [A] time = 1.15764, size = 113, normalized size = 0.92

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^2b^3}+\frac{15\,b^4x^9 + 70\,ab^3x^7 - 128\,a^2b^2x^5 - 70\,a^3bx^3 - 15\,a^4x}{1280\left(bx^2 + a\right)^5a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/1280*(15*b^4*x^9 + 70*a
*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/((b*x^2 + a)^5*a^2*b^
3)
```

3.529 $\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal. Leaf size=124

$$\frac{3x}{256a^3b^2(a+bx^2)}+\frac{x}{128a^2b^2(a+bx^2)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{x}{160ab^2(a+bx^2)^3}-\frac{3x}{80b^2(a+bx^2)^4}-\frac{x^3}{10b(a+bx^2)^5}$$

```
[Out] -x^3/(10*b*(a + b*x^2)^5) - (3*x)/(80*b^2*(a + b*x^2)^4) + x/(160*a*b^2*(a
+ b*x^2)^3) + x/(128*a^2*b^2*(a + b*x^2)^2) + (3*x)/(256*a^3*b^2*(a + b*x^2
)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(7/2)*b^(5/2))
```

Rubi [A] time = 0.0729037, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{3x}{256a^3b^2(a+bx^2)}+\frac{x}{128a^2b^2(a+bx^2)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{x}{160ab^2(a+bx^2)^3}-\frac{3x}{80b^2(a+bx^2)^4}-\frac{x^3}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^3/(10*b*(a + b*x^2)^5) - (3*x)/(80*b^2*(a + b*x^2)^4) + x/(160*a*b^2*(a
+ b*x^2)^3) + x/(128*a^2*b^2*(a + b*x^2)^2) + (3*x)/(256*a^3*b^2*(a + b*x^2
)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(7/2)*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{x^4}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(3b^4\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(3b^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{b\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{32a}$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\frac{3\int \frac{1}{\left(ab+b^2\right.}}{128a}$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\frac{3x}{256a^3b^2\left(\right.}$$

$$= -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\frac{3x}{256a^3b^2\left(\right.}$$

Mathematica [A] time = 0.0473166, size = 91, normalized size = 0.73

$$\frac{128a^2b^2x^5-70a^3bx^3-15a^4x+70ab^3x^7+15b^4x^9}{1280a^3b^2\left(a+bx^2\right)^5}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-15*a^4*x - 70*a^3*b*x^3 + 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b^4*x^9)/(1
280*a^3*b^2*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(7/2)*b
^(5/2))
```

Maple [A] time = 0.052, size = 78, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^5}\left(\frac{3\,b^2x^9}{256\,a^3}+\frac{7\,bx^7}{128\,a^2}+\frac{x^5}{10\,a}-\frac{7\,x^3}{128\,b}-\frac{3\,ax}{256\,b^2}\right)+\frac{3}{256\,b^2a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (3/256*b^2/a^3*x^9+7/128*b/a^2*x^7+1/10/a*x^5-7/128/b*x^3-3/256/b^2*a*x)/(b
*x^2+a)^5+3/256/b^2/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50087, size = 838, normalized size = 6.76

$$\left[\frac{30\,ab^5x^9+140\,a^2b^4x^7+256\,a^3b^3x^5-140\,a^4b^2x^3-30\,a^5bx-15\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\cdots\right.}{2560\left(a^4b^8x^{10}+5\,a^5b^7x^8+10\,a^6b^6x^6+10\,a^7b^5x^4+5\,a^8b^4x^2+a^9b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(30*a*b^5*x^9 + 140*a^2*b^4*x^7 + 256*a^3*b^3*x^5 - 140*a^4*b^2*x^3
 - 30*a^5*b*x - 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^
4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)))/(a^4*b^8*x^10 + 5*a^5*b^7*x^8 + 10*a^6*b^6*x^6 + 10*a^7*b^5*x^4 + 5*a
^8*b^4*x^2 + a^9*b^3), 1/1280*(15*a*b^5*x^9 + 70*a^2*b^4*x^7 + 128*a^3*b^3*
x^5 - 70*a^4*b^2*x^3 - 15*a^5*b*x + 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3
*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))
/(a^4*b^8*x^10 + 5*a^5*b^7*x^8 + 10*a^6*b^6*x^6 + 10*a^7*b^5*x^4 + 5*a^8*b^
4*x^2 + a^9*b^3)]
```

Sympy [A] time = 1.11086, size = 196, normalized size = 1.58

$$-\frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{512}+\frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{512}+\frac{-15a^4x-70a^3bx^3+128a^2b^2x^5+\cdots}{1280a^8b^2+6400a^7b^3x^2+12800a^6b^4x^4+12800a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*sqrt(-1/(a**7*b**5))*log(-a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/512 + 3*sq
rt(-1/(a**7*b**5))*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/512 + (-15*a**4*
x - 70*a**3*b*x**3 + 128*a**2*b**2*x**5 + 70*a*b**3*x**7 + 15*b**4*x**9)/(1
280*a**8*b**2 + 6400*a**7*b**3*x**2 + 12800*a**6*b**4*x**4 + 12800*a**5*b**
5*x**6 + 6400*a**4*b**6*x**8 + 1280*a**3*b**7*x**10)
```

Giac [A] time = 1.1785, size = 113, normalized size = 0.91

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^3b^2}+\frac{15\,b^4x^9+70\,ab^3x^7+128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\left(bx^2+a\right)^5a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2) + 1/1280*(15*b^4*x^9 + 70*a
*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/((b*x^2 + a)^5*a^3*b^
2)
```

3.530 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=125

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{7x}{256a^4b\left(a+bx^2\right)}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{x}{80ab\left(a+bx^2\right)^4}-\frac{x}{10b\left(a+bx^2\right)^5}$$

```
[Out] -x/(10*b*(a + b*x^2)^5) + x/(80*a*b*(a + b*x^2)^4) + (7*x)/(480*a^2*b*(a +
b*x^2)^3) + (7*x)/(384*a^3*b*(a + b*x^2)^2) + (7*x)/(256*a^4*b*(a + b*x^2))
 + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/2)*b^(3/2))
```

Rubi [A] time = 0.0746596, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 288, 199, 205}

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{7x}{256a^4b\left(a+bx^2\right)}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{x}{80ab\left(a+bx^2\right)^4}-\frac{x}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x/(10*b*(a + b*x^2)^5) + x/(80*a*b*(a + b*x^2)^4) + (7*x)/(480*a^2*b*(a +
b*x^2)^3) + (7*x)/(384*a^3*b*(a + b*x^2)^2) + (7*x)/(256*a^4*b*(a + b*x^2))
 + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/2)*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x^2}{\left(ab+b^2x^2\right)^6}\,dx \\
&= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{1}{10}b^4\int \frac{1}{\left(ab+b^2x^2\right)^5}\,dx \\
&= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{\left(7b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx}{80a} \\
&= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{\left(7b^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{96a^2} \\
&= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{(7b)\int \ldots}{12\ldots} \\
&= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7\ldots}{256a^4b\left(\ldots\right.} \\
&= -\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7\ldots}{256a^4b\left(\ldots\right.}
\end{aligned}
$$

Mathematica [A] time = 0.0513873, size = 91, normalized size = 0.73

$$\frac{896a^2b^2x^5+790a^3bx^3-105a^4x+490ab^3x^7+105b^4x^9}{3840a^4b\left(a+bx^2\right)^5}+\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-105*a^4*x + 790*a^3*b*x^3 + 896*a^2*b^2*x^5 + 490*a*b^3*x^7 + 105*b^4*x^9
)/(3840*a^4*b*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/2)
*b^(3/2))
```

Maple [A] time = 0.051, size = 80, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^5}\left(\frac{7\,b^3x^9}{256\,a^4}+\frac{49\,b^2x^7}{384\,a^3}+\frac{7\,bx^5}{30\,a^2}+\frac{79\,x^3}{384\,a}-\frac{7\,x}{256\,b}\right)+\frac{7}{256\,a^4b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (7/256*b^3/a^4*x^9+49/384*b^2/a^3*x^7+7/30*b/a^2*x^5+79/384/a*x^3-7/256*x/b
)/(b*x^2+a)^5+7/256/a^4/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47419, size = 855, normalized size = 6.84

$$\left[\frac{210\,ab^5x^9 + 980\,a^2b^4x^7 + 1792\,a^3b^3x^5 + 1580\,a^4b^2x^3 - 210\,a^5bx - 105\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4\right.}{7680\left(a^5b^7x^{10} + 5\,a^6b^6x^8 + 10\,a^7b^5x^6 + 10\,a^8b^4x^4 + 5\,a^9b^3x^2 + a^{10}b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(210*a*b^5*x^9 + 980*a^2*b^4*x^7 + 1792*a^3*b^3*x^5 + 1580*a^4*b^2*
x^3 - 210*a^5*b*x - 105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b
^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^5*b^7*x^10 + 5*a^6*b^6*x^8 + 10*a^7*b^5*x^6 + 10*a^8*b^4*x^4
+ 5*a^9*b^3*x^2 + a^10*b^2), 1/3840*(105*a*b^5*x^9 + 490*a^2*b^4*x^7 + 896*
a^3*b^3*x^5 + 790*a^4*b^2*x^3 - 105*a^5*b*x + 105*(b^5*x^10 + 5*a*b^4*x^8 +
 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt
(a*b)*x/a))/(a^5*b^7*x^10 + 5*a^6*b^6*x^8 + 10*a^7*b^5*x^6 + 10*a^8*b^4*x^4
 + 5*a^9*b^3*x^2 + a^10*b^2)]
```

Sympy [A] time = 1.10457, size = 190, normalized size = 1.52

$$-\frac{7\sqrt{-\frac{1}{a^9b^3}}\log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}} + x\right)}{512} + \frac{7\sqrt{-\frac{1}{a^9b^3}}\log\left(a^5b\sqrt{-\frac{1}{a^9b^3}} + x\right)}{512} + \frac{-105a^4x + 790a^3bx^3 + 896a^2b^2x^5}{3840a^9b + 19200a^8b^2x^2 + 38400a^7b^3x^4 + 38400a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -7*sqrt(-1/(a**9*b**3))*log(-a**5*b*sqrt(-1/(a**9*b**3)) + x)/512 + 7*sqrt(
-1/(a**9*b**3))*log(a**5*b*sqrt(-1/(a**9*b**3)) + x)/512 + (-105*a**4*x + 7
90*a**3*b*x**3 + 896*a**2*b**2*x**5 + 490*a*b**3*x**7 + 105*b**4*x**9)/(384
0*a**9*b + 19200*a**8*b**2*x**2 + 38400*a**7*b**3*x**4 + 38400*a**6*b**4*x*
*6 + 19200*a**5*b**5*x**8 + 3840*a**4*b**6*x**10)
```

Giac [A] time = 1.16046, size = 113, normalized size = 0.9

$$\frac{7\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^4b} + \frac{105\,b^4x^9 + 490\,ab^3x^7 + 896\,a^2b^2x^5 + 790\,a^3bx^3 - 105\,a^4x}{3840\left(bx^2 + a\right)^5a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b) + 1/3840*(105*b^4*x^9 + 490*a
*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 105*a^4*x)/((b*x^2 + a)^5*a^4*
b)
```

3.531 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=113

$$\frac{63x}{256a^5\left(a+bx^2\right)}+\frac{21x}{128a^4\left(a+bx^2\right)^2}+\frac{21x}{160a^3\left(a+bx^2\right)^3}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}+\frac{x}{10a\left(a+bx^2\right)^5}$$

```
[Out] x/(10*a*(a + b*x^2)^5) + (9*x)/(80*a^2*(a + b*x^2)^4) + (21*x)/(160*a^3*(a
+ b*x^2)^3) + (21*x)/(128*a^4*(a + b*x^2)^2) + (63*x)/(256*a^5*(a + b*x^2))
 + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(11/2)*Sqrt[b])
```

Rubi [A] time = 0.0663419, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {28, 199, 205}

$$\frac{63x}{256a^5\left(a+bx^2\right)}+\frac{21x}{128a^4\left(a+bx^2\right)^2}+\frac{21x}{160a^3\left(a+bx^2\right)^3}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}+\frac{x}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] x/(10*a*(a + b*x^2)^5) + (9*x)/(80*a^2*(a + b*x^2)^4) + (21*x)/(160*a^3*(a
+ b*x^2)^3) + (21*x)/(128*a^4*(a + b*x^2)^2) + (63*x)/(256*a^5*(a + b*x^2))
 + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(11/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{x}{10a\left(a+bx^2\right)^5}+\frac{\left(9b^5\right)\int \frac{1}{\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{\left(63b^4\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{21x}{160a^3\left(a+bx^2\right)^3}+\frac{\left(21b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{32a^3}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{21x}{160a^3\left(a+bx^2\right)^3}+\frac{21x}{128a^4\left(a+bx^2\right)^2}+\frac{\left(63b^2\right)\int \frac{1}{\left(ab+b\right.}}{128a^4}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{21x}{160a^3\left(a+bx^2\right)^3}+\frac{21x}{128a^4\left(a+bx^2\right)^2}+\frac{63x}{256a^5\left(a+bx\right.}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5}+\frac{9x}{80a^2\left(a+bx^2\right)^4}+\frac{21x}{160a^3\left(a+bx^2\right)^3}+\frac{21x}{128a^4\left(a+bx^2\right)^2}+\frac{63x}{256a^5\left(a+bx\right.}$$

Mathematica [A] time = 0.0442066, size = 89, normalized size = 0.79

$$\frac{\frac{\sqrt{a}x\left(2688a^2b^2x^4+2370a^3bx^2+965a^4+1470ab^3x^6+315b^4x^8\right)}{\left(a+bx^2\right)^5}+\frac{315\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}{1280a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] ((Sqrt[a]*x*(965*a^4 + 2370*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 1470*a*b^3*x^6 +
 315*b^4*x^8))/(a + b*x^2)^5 + (315*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(
1280*a^(11/2))
```

Maple [A] time = 0.046, size = 96, normalized size = 0.9

$$\frac{x}{10\,a\left(bx^2+a\right)^5}+\frac{9\,x}{80\,a^2\left(bx^2+a\right)^4}+\frac{21\,x}{160\,a^3\left(bx^2+a\right)^3}+\frac{21\,x}{128\,a^4\left(bx^2+a\right)^2}+\frac{63\,x}{256\,a^5\left(bx^2+a\right)}+\frac{63}{256\,a^5}\arctan\left(bx\frac{1}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*x/a/(b*x^2+a)^5+9/80*x/a^2/(b*x^2+a)^4+21/160*x/a^3/(b*x^2+a)^3+21/128
*x/a^4/(b*x^2+a)^2+63/256*x/a^5/(b*x^2+a)+63/256/a^5/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51128, size = 859, normalized size = 7.6

$$\left[\frac{630\,ab^5x^9+2940\,a^2b^4x^7+5376\,a^3b^3x^5+4740\,a^4b^2x^3+1930\,a^5bx-315\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\right.}{2560\left(a^6b^6x^{10}+5\,a^7b^5x^8+10\,a^8b^4x^6+10\,a^9b^3x^4+5\,a^{10}b^2x^2+a^{11}b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(630*a*b^5*x^9 + 2940*a^2*b^4*x^7 + 5376*a^3*b^3*x^5 + 4740*a^4*b^2
*x^3 + 1930*a^5*b*x - 315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3
*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a^6*b^6*x^10 + 5*a^7*b^5*x^8 + 10*a^8*b^4*x^6 + 10*a^9*b^3*x^
4 + 5*a^10*b^2*x^2 + a^11*b), 1/1280*(315*a*b^5*x^9 + 1470*a^2*b^4*x^7 + 26
88*a^3*b^3*x^5 + 2370*a^4*b^2*x^3 + 965*a^5*b*x + 315*(b^5*x^10 + 5*a*b^4*x
^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a))/(a^6*b^6*x^10 + 5*a^7*b^5*x^8 + 10*a^8*b^4*x^6 + 10*a^9*b^3
*x^4 + 5*a^10*b^2*x^2 + a^11*b)]
```

Sympy [A] time = 1.12022, size = 177, normalized size = 1.57

$$-\frac{63\sqrt{-\frac{1}{a^{11}b}}\log\left(-a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{512}+\frac{63\sqrt{-\frac{1}{a^{11}b}}\log\left(a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{512}+\frac{965a^4x+2370a^3bx^3+2688a^2b^2x^5+14}{1280a^{10}+6400a^9bx^2+12800a^8b^2x^4+12800a^7b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -63*sqrt(-1/(a**11*b))*log(-a**6*sqrt(-1/(a**11*b)) + x)/512 + 63*sqrt(-1/(
a**11*b))*log(a**6*sqrt(-1/(a**11*b)) + x)/512 + (965*a**4*x + 2370*a**3*b*
x**3 + 2688*a**2*b**2*x**5 + 1470*a*b**3*x**7 + 315*b**4*x**9)/(1280*a**10
+ 6400*a**9*b*x**2 + 12800*a**8*b**2*x**4 + 12800*a**7*b**3*x**6 + 6400*a**
6*b**4*x**8 + 1280*a**5*b**5*x**10)
```

Giac [A] time = 1.14911, size = 105, normalized size = 0.93

$$\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}a^5}+\frac{315\,b^4x^9+1470\,ab^3x^7+2688\,a^2b^2x^5+2370\,a^3bx^3+965\,a^4x}{1280\left(bx^2+a\right)^5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/1280*(315*b^4*x^9 + 1470*a
*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3 + 965*a^4*x)/((b*x^2 + a)^5*a^
```

5）

3.532 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=133

$$\frac{231}{256a^5x\left(a+bx^2\right)}+\frac{231}{640a^4x\left(a+bx^2\right)^2}+\frac{33}{160a^3x\left(a+bx^2\right)^3}+\frac{11}{80a^2x\left(a+bx^2\right)^4}-\frac{693\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{693}{256a^6x}+\frac{}{10ax}$$

```
[Out] -693/(256*a^6*x) + 1/(10*a*x*(a + b*x^2)^5) + 11/(80*a^2*x*(a + b*x^2)^4) +
 33/(160*a^3*x*(a + b*x^2)^3) + 231/(640*a^4*x*(a + b*x^2)^2) + 231/(256*a^
5*x*(a + b*x^2)) - (693*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))
```

Rubi [A] time = 0.0893844, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{231}{256a^5x\left(a+bx^2\right)}+\frac{231}{640a^4x\left(a+bx^2\right)^2}+\frac{33}{160a^3x\left(a+bx^2\right)^3}+\frac{11}{80a^2x\left(a+bx^2\right)^4}-\frac{693\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{693}{256a^6x}+\frac{}{10ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -693/(256*a^6*x) + 1/(10*a*x*(a + b*x^2)^5) + 11/(80*a^2*x*(a + b*x^2)^4) +
 33/(160*a^3*x*(a + b*x^2)^3) + 231/(640*a^4*x*(a + b*x^2)^2) + 231/(256*a^
5*x*(a + b*x^2)) - (693*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^2\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{\left(11b^5\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{\left(99b^4\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{\left(231b^3\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^3}\,dx}{160a^3}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{231}{640a^4x\left(a+bx^2\right)^2} + \frac{\left(231b^2\right.}{}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{231}{640a^4x\left(a+bx^2\right)^2} + \frac{}{256a^5}$$

$$= -\frac{693}{256a^6x} + \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{231}{640a^4x\left(a+bx^2\right.}$$

$$= -\frac{693}{256a^6x} + \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{231}{640a^4x\left(a+bx^2\right.}$$

Mathematica [A] time = 0.0549293, size = 101, normalized size = 0.76

$$-\frac{29568a^2b^3x^6+26070a^3b^2x^4+10615a^4bx^2+1280a^5+16170ab^4x^8+3465b^5x^{10}}{1280a^6x\left(a+bx^2\right)^5} - \frac{693\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -(1280*a^5 + 10615*a^4*b*x^2 + 26070*a^3*b^2*x^4 + 29568*a^2*b^3*x^6 + 1617
0*a*b^4*x^8 + 3465*b^5*x^10)/(1280*a^6*x*(a + b*x^2)^5) - (693*Sqrt[b]*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))
```

Maple [A] time = 0.057, size = 126, normalized size = 1.

$$-\frac{1}{a^{6}x}-\frac{437\,b^{5}x^{9}}{256\,a^{6}\left(bx^{2}+a\right)^{5}}-\frac{977\,b^{4}x^{7}}{128\,a^{5}\left(bx^{2}+a\right)^{5}}-\frac{131\,b^{3}x^{5}}{10\,a^{4}\left(bx^{2}+a\right)^{5}}-\frac{1327\,b^{2}x^{3}}{128\,a^{3}\left(bx^{2}+a\right)^{5}}-\frac{843\,bx}{256\,a^{2}\left(bx^{2}+a\right)^{5}}-\frac{693\,b}{256\,a^{6}}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/a^6/x-437/256/a^6*b^5/(b*x^2+a)^5*x^9-977/128/a^5*b^4/(b*x^2+a)^5*x^7-13
1/10/a^4*b^3/(b*x^2+a)^5*x^5-1327/128/a^3*b^2/(b*x^2+a)^5*x^3-843/256/a^2*b
/(b*x^2+a)^5*x-693/256/a^6*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53474, size = 906, normalized size = 6.81

$$\left[-\frac{6930\,b^{5}x^{10}+32340\,ab^{4}x^{8}+59136\,a^{2}b^{3}x^{6}+52140\,a^{3}b^{2}x^{4}+21230\,a^{4}bx^{2}+2560\,a^{5}-3465\left(b^{5}x^{11}+5\,ab^{4}x^{9}+10\,a^{2}b^{3}\right.}{2560\left(a^{6}b^{5}x^{11}+5\,a^{7}b^{4}x^{9}+10\,a^{8}b^{3}x^{7}+10\,a^{9}b^{2}x^{5}+5\,a^{10}bx^{3}+a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(6930*b^5*x^10 + 32340*a*b^4*x^8 + 59136*a^2*b^3*x^6 + 52140*a^3*b
^2*x^4 + 21230*a^4*b*x^2 + 2560*a^5 - 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a^2
```

```
*b^3*x^7 + 10*a^3*b^2*x^5 + 5*a^4*b*x^3 + a^5*x)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^8*b^
3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x), -1/1280*(3465*b^5*x^10 + 1
6170*a*b^4*x^8 + 29568*a^2*b^3*x^6 + 26070*a^3*b^2*x^4 + 10615*a^4*b*x^2 +
1280*a^5 + 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a^2*b^3*x^7 + 10*a^3*b^2*x^5 +
 5*a^4*b*x^3 + a^5*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^5*x^11 + 5*a^7*
b^4*x^9 + 10*a^8*b^3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x)]
```

Sympy [A] time = 4.07907, size = 185, normalized size = 1.39

$$\frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b}{a^{13}}}}{b}+x\right)}{512}-\frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b}{a^{13}}}}{b}+x\right)}{512}-\frac{1280a^5+10615a^4bx^2+26070a^3b^2x^4+29568a^2b^3x^6}{1280a^{11}x+6400a^{10}bx^3+12800a^9b^2x^5+12800a^8b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 693*sqrt(-b/a**13)*log(-a**7*sqrt(-b/a**13)/b + x)/512 - 693*sqrt(-b/a**13)
*log(a**7*sqrt(-b/a**13)/b + x)/512 - (1280*a**5 + 10615*a**4*b*x**2 + 2607
0*a**3*b**2*x**4 + 29568*a**2*b**3*x**6 + 16170*a*b**4*x**8 + 3465*b**5*x**
10)/(1280*a**11*x + 6400*a**10*b*x**3 + 12800*a**9*b**2*x**5 + 12800*a**8*b
**3*x**7 + 6400*a**7*b**4*x**9 + 1280*a**6*b**5*x**11)
```

Giac [A] time = 1.13629, size = 122, normalized size = 0.92

$$-\frac{693\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^6}-\frac{1}{a^6x}-\frac{2185\,b^5x^9+9770\,ab^4x^7+16768\,a^2b^3x^5+13270\,a^3b^2x^3+4215\,a^4bx}{1280\,\left(bx^2+a\right)^5a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -693/256*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/(a^6*x) - 1/1280*(2185
*b^5*x^9 + 9770*a*b^4*x^7 + 16768*a^2*b^3*x^5 + 13270*a^3*b^2*x^3 + 4215*a^
4*b*x)/((b*x^2 + a)^5*a^6)
```

3.533 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=144

$$\frac{3003b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{15/2}}+\frac{3003}{1280a^5x^3\left(a+bx^2\right)}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{13}{80a^2x^3\left(a+bx^2\right)^4}+\frac{3003b}{256a^7x}$$

```
[Out] -1001/(256*a^6*x^3) + (3003*b)/(256*a^7*x) + 1/(10*a*x^3*(a + b*x^2)^5) + 1
3/(80*a^2*x^3*(a + b*x^2)^4) + 143/(480*a^3*x^3*(a + b*x^2)^3) + 429/(640*a
^4*x^3*(a + b*x^2)^2) + 3003/(1280*a^5*x^3*(a + b*x^2)) + (3003*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(15/2))
```

Rubi [A] time = 0.103098, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$\frac{3003b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{15/2}}+\frac{3003}{1280a^5x^3\left(a+bx^2\right)}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{13}{80a^2x^3\left(a+bx^2\right)^4}+\frac{3003b}{256a^7x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1001/(256*a^6*x^3) + (3003*b)/(256*a^7*x) + 1/(10*a*x^3*(a + b*x^2)^5) + 1
3/(80*a^2*x^3*(a + b*x^2)^4) + 143/(480*a^3*x^3*(a + b*x^2)^3) + 429/(640*a
^4*x^3*(a + b*x^2)^2) + 3003/(1280*a^5*x^3*(a + b*x^2)) + (3003*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(15/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^4\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{\left(13b^5\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{\left(143b^4\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{\left(429b^3\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^3}\,d}{160a^3}$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{429}{640a^4x^3\left(a+bx^2\right)^2} + \cdots$$

$$= \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{429}{640a^4x^3\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{1001}{256a^6x^3} + \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \frac{42\cdots}{640a^4x^3\left(\cdots\right.}$$

$$= -\frac{1001}{256a^6x^3} + \frac{3003b}{256a^7x} + \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \cdots$$

$$= -\frac{1001}{256a^6x^3} + \frac{3003b}{256a^7x} + \frac{1}{10ax^3\left(a+bx^2\right)^5} + \frac{13}{80a^2x^3\left(a+bx^2\right)^4} + \frac{143}{480a^3x^3\left(a+bx^2\right)^3} + \cdots$$

Mathematica [A] time = 0.059476, size = 113, normalized size = 0.78

$$\frac{\frac{\sqrt{a}\left(384384a^2b^4x^8+338910a^3b^3x^6+137995a^4b^2x^4+16640a^5bx^2-1280a^6+210210ab^5x^{10}+45045b^6x^{12}\right)}{x^3\left(a+bx^2\right)^5}+45045b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3840a^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((Sqrt[a]*(-1280*a^6 + 16640*a^5*b*x^2 + 137995*a^4*b^2*x^4 + 338910*a^3*b^
3*x^6 + 384384*a^2*b^4*x^8 + 210210*a*b^5*x^10 + 45045*b^6*x^12))/(x^3*(a +
 b*x^2)^5) + 45045*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3840*a^(15/2))
```

Maple [A] time = 0.062, size = 139, normalized size = 1.

$$-\frac{1}{3\,a^6x^3}+6\,\frac{b}{a^7x}+\frac{1467\,b^6x^9}{256\,a^7\left(bx^2+a\right)^5}+\frac{9629\,b^5x^7}{384\,a^6\left(bx^2+a\right)^5}+\frac{1253\,b^4x^5}{30\,a^5\left(bx^2+a\right)^5}+\frac{12131\,b^3x^3}{384\,a^4\left(bx^2+a\right)^5}+\frac{2373\,b^2x}{256\,a^3\left(bx^2+a\right)^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/3/a^6/x^3+6*b/a^7/x+1467/256/a^7*b^6/(b*x^2+a)^5*x^9+9629/384/a^6*b^5/(b
*x^2+a)^5*x^7+1253/30/a^5*b^4/(b*x^2+a)^5*x^5+12131/384/a^4*b^3/(b*x^2+a)^5
*x^3+2373/256/a^3*b^2/(b*x^2+a)^5*x+3003/256/a^7*b^2/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51335, size = 1006, normalized size = 6.99

$$\left[\frac{90090\,b^6x^{12}+420420\,ab^5x^{10}+768768\,a^2b^4x^8+677820\,a^3b^3x^6+275990\,a^4b^2x^4+33280\,a^5bx^2-2560\,a^6+45045\left(b^6x\right.}{7680\left(a^7b^5x^{13}+5\,a^8b^4x^{11}+10\,a^9b^3x^9+10\,a^{10}b^2x^7+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(90090*b^6*x^12 + 420420*a*b^5*x^10 + 768768*a^2*b^4*x^8 + 677820*a
^3*b^3*x^6 + 275990*a^4*b^2*x^4 + 33280*a^5*b*x^2 - 2560*a^6 + 45045*(b^6*x
^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x^7 + 5*a^4*b^2*x^5 + a^5*
b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^5
*x^13 + 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 +
a^12*x^3), 1/3840*(45045*b^6*x^12 + 210210*a*b^5*x^10 + 384384*a^2*b^4*x^8
+ 338910*a^3*b^3*x^6 + 137995*a^4*b^2*x^4 + 16640*a^5*b*x^2 - 1280*a^6 + 45
045*(b^6*x^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x^7 + 5*a^4*b^2*
x^5 + a^5*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^7*b^5*x^13 + 5*a^8*b^4*x
^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^12*x^3)]
```

Sympy [A] time = 9.09644, size = 209, normalized size = 1.45

$$-\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(-\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512}+\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512}+\frac{-1280a^6+16640a^5bx^2+137995a^4b^2x^4+338910\cdots}{3840a^{12}x^3+19200a^{11}bx^5+38400a^{10}b^2x^7\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3003*sqrt(-b**3/a**15)*log(-a**8*sqrt(-b**3/a**15)/b**2 + x)/512 + 3003*sq
rt(-b**3/a**15)*log(a**8*sqrt(-b**3/a**15)/b**2 + x)/512 + (-1280*a**6 + 16
640*a**5*b*x**2 + 137995*a**4*b**2*x**4 + 338910*a**3*b**3*x**6 + 384384*a*
*2*b**4*x**8 + 210210*a*b**5*x**10 + 45045*b**6*x**12)/(3840*a**12*x**3 + 1
9200*a**11*b*x**5 + 38400*a**10*b**2*x**7 + 38400*a**9*b**3*x**9 + 19200*a*
*8*b**4*x**11 + 3840*a**7*b**5*x**13)
```

Giac [A] time = 1.14116, size = 140, normalized size = 0.97

$$\frac{3003\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^7}+\frac{18\,bx^2-a}{3\,a^7x^3}+\frac{22005\,b^6x^9+96290\,ab^5x^7+160384\,a^2b^4x^5+121310\,a^3b^3x^3+35595\,a^4b^2x}{3840\left(bx^2+a\right)^5a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3003/256*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7) + 1/3*(18*b*x^2 - a)/(a^
7*x^3) + 1/3840*(22005*b^6*x^9 + 96290*a*b^5*x^7 + 160384*a^2*b^4*x^5 + 121
310*a^3*b^3*x^3 + 35595*a^4*b^2*x)/((b*x^2 + a)^5*a^7)
```

3.534 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{9009b^2}{256a^8x}-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{17/2}}+\frac{3003b}{256a^7x^3}+\frac{1287}{256a^5x^5\left(a+bx^2\right)}+\frac{143}{128a^4x^5\left(a+bx^2\right)^2}+\frac{13}{32a^3x^5\left(a+bx^2\right)^3}+\frac{}{16a^2x^5}$$

```
[Out] -9009/(1280*a^6*x^5) + (3003*b)/(256*a^7*x^3) - (9009*b^2)/(256*a^8*x) + 1/
(10*a*x^5*(a + b*x^2)^5) + 3/(16*a^2*x^5*(a + b*x^2)^4) + 13/(32*a^3*x^5*(a
 + b*x^2)^3) + 143/(128*a^4*x^5*(a + b*x^2)^2) + 1287/(256*a^5*x^5*(a + b*x
^2)) - (9009*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(17/2))
```

Rubi [A] time = 0.122192, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 290, 325, 205}

$$-\frac{9009b^2}{256a^8x}-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{17/2}}+\frac{3003b}{256a^7x^3}+\frac{1287}{256a^5x^5\left(a+bx^2\right)}+\frac{143}{128a^4x^5\left(a+bx^2\right)^2}+\frac{13}{32a^3x^5\left(a+bx^2\right)^3}+\frac{}{16a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -9009/(1280*a^6*x^5) + (3003*b)/(256*a^7*x^3) - (9009*b^2)/(256*a^8*x) + 1/
(10*a*x^5*(a + b*x^2)^5) + 3/(16*a^2*x^5*(a + b*x^2)^4) + 13/(32*a^3*x^5*(a
 + b*x^2)^3) + 143/(128*a^4*x^5*(a + b*x^2)^2) + 1287/(256*a^5*x^5*(a + b*x
^2)) - (9009*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(17/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^6\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{\left(3b^5\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^5}\,dx}{2a}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{\left(39b^4\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^4}\,dx}{16a^2}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{\left(143b^3\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^3}\,dx}{32a^3}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{143}{128a^4x^5\left(a+bx^2\right)^2} + \frac{(1}{}$$

$$= \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{143}{128a^4x^5\left(a+bx^2\right)^2} + \frac{}{25}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} + \frac{14}{128a^4x^5\left(a\right.}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a+bx^2\right)^3} +$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} - \frac{9009b^2}{256a^8x} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a\right.}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} - \frac{9009b^2}{256a^8x} + \frac{1}{10ax^5\left(a+bx^2\right)^5} + \frac{3}{16a^2x^5\left(a+bx^2\right)^4} + \frac{13}{32a^3x^5\left(a\right.}$$

Mathematica [A] time = 0.0628365, size = 123, normalized size = 0.78

$$-\frac{384384a^2b^5x^{10} + 338910a^3b^4x^8 + 137995a^4b^3x^6 + 16640a^5b^2x^4 - 1280a^6bx^2 + 256a^7 + 210210ab^6x^{12} + 45045b^7x^{14}}{1280a^8x^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -(256*a^7 - 1280*a^6*b*x^2 + 16640*a^5*b^2*x^4 + 137995*a^4*b^3*x^6 + 33891
0*a^3*b^4*x^8 + 384384*a^2*b^5*x^10 + 210210*a*b^6*x^12 + 45045*b^7*x^14)/(
1280*a^8*x^5*(a + b*x^2)^5) - (9009*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
56*a^(17/2))
```

Maple [A] time = 0.061, size = 150, normalized size = 1.

$$-\frac{1}{5\,a^{6}x^{5}}-21\,\frac{b^{2}}{a^{8}x}+2\,\frac{b}{a^{7}x^{3}}-\frac{3633\,b^{7}x^{9}}{256\,a^{8}\left(bx^{2}+a\right)^{5}}-\frac{7837\,b^{6}x^{7}}{128\,a^{7}\left(bx^{2}+a\right)^{5}}-\frac{1001\,b^{5}x^{5}}{10\,a^{6}\left(bx^{2}+a\right)^{5}}-\frac{9443\,b^{4}x^{3}}{128\,a^{5}\left(bx^{2}+a\right)^{5}}-\frac{5327\,b^{3}}{256\,a^{4}\left(bx^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/5/a^6/x^5-21*b^2/a^8/x+2*b/a^7/x^3-3633/256/a^8*b^7/(b*x^2+a)^5*x^9-7837
/128/a^7*b^6/(b*x^2+a)^5*x^7-1001/10/a^6*b^5/(b*x^2+a)^5*x^5-9443/128/a^5*b
^4/(b*x^2+a)^5*x^3-5327/256/a^4*b^3/(b*x^2+a)^5*x-9009/256/a^8*b^3/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53888, size = 1073, normalized size = 6.83

$$\left[-\frac{90090\,b^{7}x^{14}+420420\,ab^{6}x^{12}+768768\,a^{2}b^{5}x^{10}+677820\,a^{3}b^{4}x^{8}+275990\,a^{4}b^{3}x^{6}+33280\,a^{5}b^{2}x^{4}-2560\,a^{6}bx^{2}+512}{2560\left(a^{8}b^{5}x^{15}+5\,a^{9}b^{4}x^{13}+10\,a^{10}b^{3}x^{11}+10\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(90090*b^7*x^14 + 420420*a*b^6*x^12 + 768768*a^2*b^5*x^10 + 677820
*a^3*b^4*x^8 + 275990*a^4*b^3*x^6 + 33280*a^5*b^2*x^4 - 2560*a^6*b*x^2 + 51
2*a^7 - 45045*(b^7*x^15 + 5*a*b^6*x^13 + 10*a^2*b^5*x^11 + 10*a^3*b^4*x^9 +
 5*a^4*b^3*x^7 + a^5*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)
/(b*x^2 + a)))/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*a^11*
b^2*x^9 + 5*a^12*b*x^7 + a^13*x^5), -1/1280*(45045*b^7*x^14 + 210210*a*b^6*
x^12 + 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995*a^4*b^3*x^6 + 1664
0*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7 + 45045*(b^7*x^15 + 5*a*b^6*x^13 +
 10*a^2*b^5*x^11 + 10*a^3*b^4*x^9 + 5*a^4*b^3*x^7 + a^5*b^2*x^5)*sqrt(b/a)*
arctan(x*sqrt(b/a)))/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10
*a^11*b^2*x^9 + 5*a^12*b*x^7 + a^13*x^5)]
```

Sympy [A] time = 19.3699, size = 221, normalized size = 1.41

$$\frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(-\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3}+x\right)}{512}-\frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3}+x\right)}{512}-\frac{256a^7-1280a^6bx^2+16640a^5b^2x^4+137995a^4b^3x^6}{1280a^{13}x^5+6400a^{12}bx^7+12800a^{11}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 9009*sqrt(-b**5/a**17)*log(-a**9*sqrt(-b**5/a**17)/b**3 + x)/512 - 9009*sqr
t(-b**5/a**17)*log(a**9*sqrt(-b**5/a**17)/b**3 + x)/512 - (256*a**7 - 1280*
a**6*b*x**2 + 16640*a**5*b**2*x**4 + 137995*a**4*b**3*x**6 + 338910*a**3*b*
*4*x**8 + 384384*a**2*b**5*x**10 + 210210*a*b**6*x**12 + 45045*b**7*x**14)/
(1280*a**13*x**5 + 6400*a**12*b*x**7 + 12800*a**11*b**2*x**9 + 12800*a**10*
b**3*x**11 + 6400*a**9*b**4*x**13 + 1280*a**8*b**5*x**15)
```

Giac [A] time = 1.11959, size = 155, normalized size = 0.99

$$-\frac{9009\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^8}-\frac{45045\,b^7x^{14}+210210\,ab^6x^{12}+384384\,a^2b^5x^{10}+338910\,a^3b^4x^8+137995\,a^4b^3x^6+16640\,a^5}{1280\left(bx^3+ax\right)^5a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -9009/256*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8) - 1/1280*(45045*b^7*x^1
4 + 210210*a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995*a
^4*b^3*x^6 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7)/((b*x^3 + a*x)^5
*a^8)
```

3.535 $\int \frac{1}{1+2x^2+x^4} dx$

Optimal. Leaf size=19

$$\frac{x}{2(x^2+1)} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi [A] time = 0.0031113, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {28, 199, 203}

$$\frac{x}{2(x^2+1)} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+2x^2+x^4}\,dx &= \int \frac{1}{\left(1+x^2\right)^2}\,dx\\ &= \frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.005355, size = 16, normalized size = 0.84

$$\frac{1}{2}\left(\frac{x}{x^2+1}+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + x^4)^(-1),x]

[Out] (x/(1 + x^2) + ArcTan[x])/2
```

Maple [A] time = 0.044, size = 16, normalized size = 0.8

$$\frac{x}{2\,x^2+2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*x^2+1),x)

[Out] 1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A] time = 1.48803, size = 20, normalized size = 1.05

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A] time = 1.48237, size = 55, normalized size = 2.89

$$\frac{\left(x^2+1\right)\arctan\left(x\right)+x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] time = 0.098102, size = 12, normalized size = 0.63

$$\frac{x}{2x^2+2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*x**2+1),x)
```

```
[Out] x/(2*x**2 + 2) + atan(x)/2
```

Giac [A] time = 1.12473, size = 20, normalized size = 1.05

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.536 $\int \frac{x}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=11

$$-\frac{1}{2(x^2+1)}$$

[Out] -1/(2*(1 + x^2))

Rubi [A] time = 0.0021301, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 261}

$$-\frac{1}{2(x^2+1)}$$

Antiderivative was successfully verified.

[In] Int[x/(1 + 2*x^2 + x^4),x]

[Out] -1/(2*(1 + x^2))

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{1+2x^2+x^4} \, dx &= \int \frac{x}{\left(1+x^2\right)^2} \, dx \\ &= -\frac{1}{2\left(1+x^2\right)} \end{aligned}$$

Mathematica [A] time = 0.0011637, size = 11, normalized size = 1.

$$-\frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/(2*(1 + x^2))
```

Maple [A] time = 0.043, size = 10, normalized size = 0.9

$$-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+1),x)
```

```
[Out] -1/2/(x^2+1)
```

Maxima [A] time = 0.991789, size = 12, normalized size = 1.09

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2/(x^2 + 1)
```

Fricas [A] time = 1.45597, size = 22, normalized size = 2.

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2/(x^2 + 1)
```

Sympy [A] time = 0.080482, size = 8, normalized size = 0.73

$$-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**2+1),x)
```

```
[Out] -1/(2*x**2 + 2)
```

Giac [A] time = 1.14512, size = 12, normalized size = 1.09

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2/(x^2 + 1)
```

3.537 $\int \frac{x^2}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi [A] time = 0.0048065, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 288, 203}

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 2*x^2 + x^4),x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = \int \frac{x^2}{\left(1+x^2\right)^2}\,dx$$
$$= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x)$$

Mathematica [A] time = 0.0072601, size = 19, normalized size = 1.

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 2*x^2 + x^4),x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Maple [A] time = 0.047, size = 16, normalized size = 0.8

$$-\frac{x}{2\,x^2+2} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+2*x^2+1),x)
```

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A] time = 1.49993, size = 20, normalized size = 1.05

$$-\frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A] time = 1.43236, size = 55, normalized size = 2.89

$$\frac{(x^2+1)\arctan(x)-x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] time = 0.096133, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+2*x**2+1),x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

Giac [A] time = 1.13518, size = 20, normalized size = 1.05

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.538 $\int \frac{x^3}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=22

$$\frac{1}{2(x^2+1)} + \frac{1}{2}\log(x^2+1)$$

```
[Out] 1/(2*(1 + x^2)) + Log[1 + x^2]/2
```

Rubi [A] time = 0.0108123, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{1}{2(x^2+1)} + \frac{1}{2}\log(x^2+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 2*x^2 + x^4),x]
```

```
[Out] 1/(2*(1 + x^2)) + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1+2x^2+x^4}\,dx &= \int \frac{x^3}{\left(1+x^2\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(1+x)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx,x,x^2\right)\\ &= \frac{1}{2\left(1+x^2\right)}+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0045505, size = 18, normalized size = 0.82

$$\frac{1}{2}\left(\frac{1}{x^2+1}+\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 2*x^2 + x^4),x]
```

```
[Out] ((1 + x^2)^(-1) + Log[1 + x^2])/2
```

Maple [A] time = 0.047, size = 19, normalized size = 0.9

$$\frac{1}{2\,x^2+2}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+2*x^2+1),x)
```

```
[Out] 1/2/(x^2+1)+1/2*ln(x^2+1)
```

Maxima [A] time = 0.965903, size = 24, normalized size = 1.09

$$\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 1.43392, size = 59, normalized size = 2.68

$$\frac{\left(x^2+1\right)\log\left(x^2+1\right)+1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*log(x^2 + 1) + 1)/(x^2 + 1)
```

Sympy [A] time = 0.083989, size = 15, normalized size = 0.68

$$\frac{\log\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+2*x**2+1),x)
```

```
[Out] log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

Giac [A] time = 1.14005, size = 24, normalized size = 1.09

$$\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

3.539 $\int \frac{x}{81-18x^2+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{2\left(9-x^2\right)}$$

```
[Out] 1/(2*(9 - x^2))
```

Rubi [A] time = 0.0021261, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 261}

$$\frac{1}{2\left(9-x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(81 - 18*x^2 + x^4),x]
```

```
[Out] 1/(2*(9 - x^2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{81-18x^2+x^4}\,dx = \int \frac{x}{\left(-9+x^2\right)^2}\,dx$$
$$= \frac{1}{2\left(9-x^2\right)}$$

Mathematica [A] time = 0.0018655, size = 11, normalized size = 0.85

$$-\frac{1}{2\left(x^2-9\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(81 - 18*x^2 + x^4),x]
```

```
[Out] -1/(2*(-9 + x^2))
```

Maple [A] time = 0.045, size = 10, normalized size = 0.8

$$-\frac{1}{2\,x^2-18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-18*x^2+81),x)
```

```
[Out] -1/2/(x^2-9)
```

Maxima [A] time = 0.985302, size = 12, normalized size = 0.92

$$-\frac{1}{2\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="maxima")
```

```
[Out] -1/2/(x^2 - 9)
```

Fricas [A] time = 1.68699, size = 22, normalized size = 1.69

$$-\frac{1}{2\left(x^{2}-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="fricas")
```

```
[Out] -1/2/(x^2 - 9)
```

Sympy [A] time = 0.08119, size = 8, normalized size = 0.62

$$-\frac{1}{2x^{2}-18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4-18*x**2+81),x)
```

```
[Out] -1/(2*x**2 - 18)
```

Giac [A] time = 1.12867, size = 12, normalized size = 0.92

$$-\frac{1}{2\left(x^{2}-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="giac")
```

```
[Out] -1/2/(x^2 - 9)
```

3.540 $\int \frac{x^3}{16-8x^2+x^4} \, dx$

Optimal. Leaf size=24

$$\frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)$$

```
[Out] 2/(4 - x^2) + Log[4 - x^2]/2
```

Rubi [A] time = 0.0147868, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 266, 43}

$$\frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(16 - 8*x^2 + x^4),x]
```

```
[Out] 2/(4 - x^2) + Log[4 - x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{16-8x^2+x^4}\,dx = \int \frac{x^3}{\left(-4+x^2\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(-4+x)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{4}{(-4+x)^2} + \frac{1}{-4+x}\right) dx, x, x^2\right)$$
$$= \frac{2}{4-x^2} + \frac{1}{2}\log\left(4-x^2\right)$$

Mathematica [A] time = 0.0055415, size = 20, normalized size = 0.83

$$\frac{1}{2}\log\left(x^2-4\right) - \frac{2}{x^2-4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(16 - 8*x^2 + x^4),x]
```

```
[Out] -2/(-4 + x^2) + Log[-4 + x^2]/2
```

Maple [A] time = 0.049, size = 19, normalized size = 0.8

$$-2\left(x^2-4\right)^{-1} + \frac{\ln\left(x^2-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4-8*x^2+16),x)
```

```
[Out] -2/(x^2-4)+1/2*ln(x^2-4)
```

Maxima [A] time = 0.982657, size = 24, normalized size = 1.

$$-\frac{2}{x^2-4} + \frac{1}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="maxima")
```

```
[Out] -2/(x^2 - 4) + 1/2*log(x^2 - 4)
```

Fricas [A] time = 1.66334, size = 59, normalized size = 2.46

$$\frac{(x^2-4)\log(x^2-4)-4}{2(x^2-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 4)*log(x^2 - 4) - 4)/(x^2 - 4)
```

Sympy [A] time = 0.089023, size = 14, normalized size = 0.58

$$\frac{\log(x^2-4)}{2}-\frac{2}{x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4-8*x**2+16),x)
```

```
[Out] log(x**2 - 4)/2 - 2/(x**2 - 4)
```

Giac [A] time = 1.15778, size = 26, normalized size = 1.08

$$-\frac{2}{x^2-4}+\frac{1}{2}\log(|x^2-4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="giac")
```

```
[Out] -2/(x^2 - 4) + 1/2*log(abs(x^2 - 4))
```

3.541 $\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}$$

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2))
```

Rubi [A] time = 0.0643209, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int x^2\left(ab+b^2x\right)\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int\left(abx^2+b^2x^3\right)\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.0142518, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(4ax^6+3bx^8\right)}{24\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(4*a*x^6 + 3*b*x^8))/(24*(a + b*x^2))
```

Maple [A] time = 0.172, size = 36, normalized size = 0.5

$$\frac{x^6\left(3\,bx^2+4\,a\right)}{24\,bx^2+24\,a}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/24*x^6*(3*b*x^2+4*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68822, size = 31, normalized size = 0.39

$$\frac{1}{8}\,bx^8 + \frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*b*x^8 + 1/6*a*x^6
```

Sympy [A] time = 0.093207, size = 12, normalized size = 0.15

$$\frac{ax^6}{6} + \frac{bx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**6/6 + b*x**8/8
```

Giac [A] time = 1.14404, size = 39, normalized size = 0.49

$$\frac{1}{8}\,bx^8\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{6}\,ax^6\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b*x^8*sgn(b*x^2 + a) + 1/6*a*x^6*sgn(b*x^2 + a)
```

3.542 $\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2}-\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}$$

```
[Out] -(a*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b^2) + (a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/2)/(6*b^2)
```

Rubi [A] time = 0.0516276, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 609}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2}-\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(a*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b^2) + (a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/2)/(6*b^2)
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
```

```
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)\\
&= \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2}-\frac{a\operatorname{Subst}\left(\int\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)}{2b}\\
&= -\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{6b^2}
\end{aligned}
$$

Mathematica [A] time = 0.0075663, size = 39, normalized size = 0.58

$$
\frac{\sqrt{\left(a+bx^2\right)^2}\left(3ax^4+2bx^6\right)}{12\left(a+bx^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]

[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x^4 + 2*b*x^6))/(12*(a + b*x^2))
```

Maple [A] time = 0.043, size = 36, normalized size = 0.5

$$
\frac{x^4\left(2\,bx^2+3\,a\right)}{12\,bx^2+12\,a}\sqrt{\left(bx^2+a\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)^2)^(1/2),x)

[Out] 1/12*x^4*(2*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58748, size = 31, normalized size = 0.46

$$\frac{1}{6}\,bx^{6} + \frac{1}{4}\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Sympy [A] time = 0.092378, size = 12, normalized size = 0.18

$$\frac{ax^{4}}{4} + \frac{bx^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

Giac [A] time = 1.12722, size = 31, normalized size = 0.46

$$\frac{1}{12}\left(2\,bx^{6} + 3\,ax^{4}\right)\operatorname{sgn}\left(bx^{2} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*b*x^6 + 3*a*x^4)*sgn(b*x^2 + a)
```

3.543 $\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

```
[Out] ((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b)
```

Rubi [A] time = 0.0255502, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 609}

$$\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b)
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)\\ &= \frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}\end{aligned}$$

Mathematica [A] time = 0.0071594, size = 38, normalized size = 1.06

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(2ax^2+bx^4\right)}{4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(2*a*x^2 + b*x^4))/(4*(a + b*x^2))
```

Maple [A] time = 0.043, size = 35, normalized size = 1.

$$\frac{x^2\left(bx^2+2\,a\right)}{4\,bx^2+4\,a}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4*x^2*(b*x^2+2*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45774, size = 31, normalized size = 0.86

$$\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A] time = 0.091266, size = 12, normalized size = 0.33

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

Giac [A] time = 1.14333, size = 30, normalized size = 0.83

$$\frac{1}{4}\left(bx^4 + 2\,ax^2\right)\mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(b*x^4 + 2*a*x^2)*sgn(b*x^2 + a)
```

3.544 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x} dx$

Optimal. Leaf size=75

$$\frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] (b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (a*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0219498, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

```
[Out] (b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (a*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x}+b^2x\right)dx}{ab+b^2x^2}$$
$$= \frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)} + \frac{a\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

Mathematica [A] time = 0.0121508, size = 37, normalized size = 0.49

$$\frac{\sqrt{(a+bx^2)^2}\,(2a\log(x)+bx^2)}{2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2 + 2*a*Log[x]))/(2*(a + b*x^2))
```

Maple [A] time = 0.212, size = 34, normalized size = 0.5

$$\frac{bx^2+2\,a\ln(x)}{2\,bx^2+2\,a}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x,x)
```

```
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(b*x^2+2*a*ln(x))/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48814, size = 30, normalized size = 0.4

$$\frac{1}{2}\,bx^2 + a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

Sympy [A] time = 0.106605, size = 10, normalized size = 0.13

$$a\log\left(x\right) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] a*log(x) + b*x**2/2
```

Giac [A] time = 1.12615, size = 41, normalized size = 0.55

$$\frac{1}{2}\,bx^2\mathrm{sgn}\left(bx^2 + a\right) + \frac{1}{2}\,a\log\left(x^2\right)\mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*sgn(b*x^2 + a) + 1/2*a*log(x^2)*sgn(b*x^2 + a)
```

3.545 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal. Leaf size=75

$$\frac{b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0218993, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^3}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^3}+\frac{b^2}{x}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)} + \frac{b\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

Mathematica [A] time = 0.0109884, size = 39, normalized size = 0.52

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a-2bx^2\log(x)\right)}{2x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a - 2*b*x^2*Log[x]))/(2*x^2*(a + b*x^2))
```

Maple [A] time = 0.173, size = 38, normalized size = 0.5

$$\frac{2\,b\ln(x)\,x^2-a}{2\,x^2\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^3,x)
```

```
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(2*b*ln(x)*x^2-a)/x^2/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50795, size = 41, normalized size = 0.55

$$\frac{2\,bx^2\log\left(x\right)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*log(x) - a)/x^2
```

Sympy [A] time = 0.274169, size = 10, normalized size = 0.13

$$-\frac{a}{2x^2}+b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x)
```

Giac [A] time = 1.15268, size = 61, normalized size = 0.81

$$\frac{1}{2}\,b\log\left(x^2\right)\operatorname{sgn}\left(bx^2+a\right)-\frac{bx^2\operatorname{sgn}\left(bx^2+a\right)+a\operatorname{sgn}\left(bx^2+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*log(x^2)*sgn(b*x^2 + a) - 1/2*(b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a
))/x^2
```

3.546 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5} dx$

Optimal. Leaf size=39

$$-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^4)
```

Rubi [A] time = 0.0382253, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
```

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^4)
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{ab+b^2x}{x^3}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}$$
$$= -\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

Mathematica [A] time = 0.008054, size = 37, normalized size = 0.95

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a+2bx^2\right)}{4x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a + 2*b*x^2))/(4*x^4*(a + b*x^2))
```

Maple [A] time = 0.042, size = 34, normalized size = 0.9

$$-\frac{2\,bx^2+a}{4\,x^4\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^5,x)
```

```
[Out] -1/4*(2*b*x^2+a)*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48193, size = 32, normalized size = 0.82

$$-\frac{2\,bx^2 + a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2 + a)/x^4
```

Sympy [A] time = 0.291191, size = 14, normalized size = 0.36

$$-\frac{a + 2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**5,x)
```

```
[Out] -(a + 2*b*x**2)/(4*x**4)
```

Giac [A] time = 1.15952, size = 41, normalized size = 1.05

$$-\frac{2\,bx^2\mathrm{sgn}\left(bx^2 + a\right) + a\mathrm{sgn}\left(bx^2 + a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a))/x^4
```

3.547 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}$$

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^6) + (a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/2)/(12*a^2*x^6)
```

Rubi [A] time = 0.0155318, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1110}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
```

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^6) + (a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/2)/(12*a^2*x^6)
```

Rule 1110

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(4*a*d*(p + 1)*(2*p +
1)), x] - Simp[((d*x)^(m + 1)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^p)/(4*a*d*(
2*p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx=-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}$$

Mathematica [A] time = 0.0085409, size = 39, normalized size = 0.54

$$-\frac{\sqrt{(a+bx^2)^2}\,(2a+3bx^2)}{12x^6\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(2*a + 3*b*x^2))/(12*x^6*(a + b*x^2))
```

Maple [A] time = 0.043, size = 36, normalized size = 0.5

$$-\frac{3\,bx^2+2\,a}{12\,x^6\,(bx^2+a)}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^7,x)
```

```
[Out] -1/12*(3*b*x^2+2*a)*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42886, size = 36, normalized size = 0.5

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

Sympy [A] time = 0.306986, size = 15, normalized size = 0.21

$$-\frac{2a+3bx^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**7,x)
```

```
[Out] -(2*a + 3*b*x**2)/(12*x**6)
```

Giac [A] time = 1.14877, size = 42, normalized size = 0.58

$$-\frac{3\,bx^2\mathrm{sgn}\left(bx^2+a\right)+2\,a\mathrm{sgn}\left(bx^2+a\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(3*b*x^2*sgn(b*x^2 + a) + 2*a*sgn(b*x^2 + a))/x^6
```

3.548 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9} dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2))
```

Rubi [A] time = 0.0594119, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{ab+b^2x}{x^5}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(\frac{ab}{x^5}+\frac{b^2}{x^4}\right)dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.0084244, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a+4bx^2\right)}{24x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(3*a + 4*b*x^2))/(24*x^8*(a + b*x^2))
```

Maple [A] time = 0.042, size = 36, normalized size = 0.5

$$-\frac{4\,bx^2+3\,a}{24\,x^8\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^9,x)
```

```
[Out] -1/24*(4*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43762, size = 36, normalized size = 0.46

$$-\frac{4\,bx^2+3\,a}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(4*b*x^2 + 3*a)/x^8
```

Sympy [A] time = 0.327124, size = 15, normalized size = 0.19

$$-\frac{3a+4bx^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**9,x)
```

```
[Out] -(3*a + 4*b*x**2)/(24*x**8)
```

Giac [A] time = 1.16272, size = 42, normalized size = 0.53

$$-\frac{4\,bx^2\mathrm{sgn}\left(bx^2+a\right)+3\,a\mathrm{sgn}\left(bx^2+a\right)}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/24*(4*b*x^2*sgn(b*x^2 + a) + 3*a*sgn(b*x^2 + a))/x^8
```

3.549 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rubi [A] time = 0.0581569, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{ab+b^2x}{x^6}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(\frac{ab}{x^6}+\frac{b^2}{x^5}\right)dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.0080135, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(4a+5bx^2\right)}{40x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(4*a + 5*b*x^2))/(40*x^10*(a + b*x^2))
```

Maple [A] time = 0.044, size = 36, normalized size = 0.5

$$-\frac{5\,bx^2+4\,a}{40\,x^{10}\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^11,x)
```

```
[Out] -1/40*(5*b*x^2+4*a)*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43148, size = 38, normalized size = 0.48

$$-\frac{5\,bx^2+4\,a}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/40*(5*b*x^2 + 4*a)/x^10
```

Sympy [A] time = 0.345309, size = 15, normalized size = 0.19

$$-\frac{4a+5bx^2}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**11,x)
```

```
[Out] -(4*a + 5*b*x**2)/(40*x**10)
```

Giac [A] time = 1.15097, size = 42, normalized size = 0.53

$$-\frac{5\,bx^2\mathrm{sgn}\left(bx^2+a\right)+4\,a\mathrm{sgn}\left(bx^2+a\right)}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/40*(5*b*x^2*sgn(b*x^2 + a) + 4*a*sgn(b*x^2 + a))/x^10
```

3.550 $\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rubi [A] time = 0.0226354, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```


```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^4\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(abx^4+b^2x^6\right)dx}{ab+b^2x^2}$$
$$= \frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)} + \frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0071378, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(7ax^5+5bx^7\right)}{35\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(7*a*x^5 + 5*b*x^7))/(35*(a + b*x^2))
```

Maple [A] time = 0.042, size = 36, normalized size = 0.5

$$\frac{x^5\left(5\,bx^2+7\,a\right)}{35\,bx^2+35\,a}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/35*x^5*(5*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 0.993339, size = 18, normalized size = 0.23

$$\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

Fricas [A] time = 1.48732, size = 31, normalized size = 0.39

$$\frac{1}{7}bx^7 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

Sympy [A] time = 0.091937, size = 12, normalized size = 0.15

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**7/7
```

Giac [A] time = 1.1356, size = 39, normalized size = 0.49

$$\frac{1}{7}bx^7\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{5}ax^5\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7*b*x^7*sgn(b*x^2 + a) + 1/5*a*x^5*sgn(b*x^2 + a)
```

3.551 $\int x^2 \sqrt{a^2 + 2abx^2 + b^2x^4} \, dx$

Optimal. Leaf size=79

$$\frac{bx^5\sqrt{a^2 + 2abx^2 + b^2x^4}}{5(a + bx^2)} + \frac{ax^3\sqrt{a^2 + 2abx^2 + b^2x^4}}{3(a + bx^2)}$$

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rubi [A] time = 0.023179, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx^5\sqrt{a^2 + 2abx^2 + b^2x^4}}{5(a + bx^2)} + \frac{ax^3\sqrt{a^2 + 2abx^2 + b^2x^4}}{3(a + bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2\left(ab+b^2x^2\right)\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(abx^2+b^2x^4\right)\,dx}{ab+b^2x^2}$$
$$= \frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0067569, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(5ax^3+3bx^5\right)}{15\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(5*a*x^3 + 3*b*x^5))/(15*(a + b*x^2))
```

Maple [A] time = 0.042, size = 36, normalized size = 0.5

$$\frac{x^3\left(3\,bx^2+5\,a\right)}{15\,bx^2+15\,a}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/15*x^3*(3*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 0.997414, size = 18, normalized size = 0.23

$$\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A] time = 1.48458, size = 31, normalized size = 0.39

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A] time = 0.091384, size = 12, normalized size = 0.15

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

Giac [A] time = 1.10858, size = 39, normalized size = 0.49

$$\frac{1}{5}\,bx^5\mathrm{sgn}\left(bx^2 + a\right) + \frac{1}{3}\,ax^3\mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*sgn(b*x^2 + a) + 1/3*a*x^3*sgn(b*x^2 + a)
```

3.552 $\int \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=74

$$\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*x^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rubi [A] time = 0.0136244, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1088}

$$\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```


```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*x^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(3*(a + b*x^2))
```


Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(2ab+2b^2x^2\right)\,dx}{2ab+2b^2x^2}\\ &= \frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0068423, size = 36, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(3ax+bx^3\right)}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x + b*x^3))/(3*(a + b*x^2))
```

Maple [A] time = 0.042, size = 33, normalized size = 0.5

$$\frac{x\left(bx^2+3\,a\right)}{3\,bx^2+3\,a}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*x*(b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 1.00162, size = 14, normalized size = 0.19

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + a*x
```

Fricas [A] time = 1.49377, size = 23, normalized size = 0.31

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + a*x
```

Sympy [A] time = 0.088153, size = 8, normalized size = 0.11

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**3/3
```

Giac [A] time = 1.1003, size = 27, normalized size = 0.36

$$\frac{1}{3}\left(bx^3 + 3\,ax\right)\mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x^3 + 3*a*x)*sgn(b*x^2 + a)
```

3.553 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2} dx$

Optimal. Leaf size=72

$$\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (b*x*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rubi [A] time = 0.0203706, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (b*x*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^2}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2+\frac{ab}{x^2}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)} + \frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Mathematica [A] time = 0.0074029, size = 35, normalized size = 0.49

$$\frac{\left(bx^2-a\right)\sqrt{\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
```

```
[Out] ((-a + b*x^2)*Sqrt[(a + b*x^2)^2])/(x*(a + b*x^2))
```

Maple [A] time = 0.042, size = 34, normalized size = 0.5

$$-\frac{-bx^2+a}{x\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^2,x)
```

```
[Out] -(-b*x^2+a)*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)
```

Maxima [A] time = 1.00983, size = 18, normalized size = 0.25

$$\frac{bx^2-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] (b*x^2 - a)/x
```

Fricas [A] time = 1.49991, size = 20, normalized size = 0.28

$$\frac{bx^2 - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (b*x^2 - a)/x
```

Sympy [A] time = 0.257612, size = 5, normalized size = 0.07

$$-\frac{a}{x} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] -a/x + b*x
```

Giac [A] time = 1.16587, size = 35, normalized size = 0.49

$$bx\text{sgn}\left(bx^2 + a\right) - \frac{a\text{sgn}\left(bx^2 + a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*x*sgn(b*x^2 + a) - a*sgn(b*x^2 + a)/x
```

3.554 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$

Optimal. Leaf size=77

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rubi [A] time = 0.0230154, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^4}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^4}+\frac{b^2}{x^2}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0069249, size = 37, normalized size = 0.48

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a+3bx^2\right)}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a + 3*b*x^2))/(3*x^3*(a + b*x^2))
```

Maple [A] time = 0.041, size = 34, normalized size = 0.4

$$-\frac{3\,bx^2+a}{3\,x^3\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^4,x)
```

```
[Out] -1/3*(3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)
```

Maxima [A] time = 1.01073, size = 18, normalized size = 0.23

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

Fricas [A] time = 1.42792, size = 32, normalized size = 0.42

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

Sympy [A] time = 0.286232, size = 14, normalized size = 0.18

$$-\frac{a+3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**4,x)
```

```
[Out] -(a + 3*b*x**2)/(3*x**3)
```

Giac [A] time = 1.13392, size = 41, normalized size = 0.53

$$-\frac{3\,bx^2\mathrm{sgn}\left(bx^2+a\right)+a\mathrm{sgn}\left(bx^2+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a))/x^3
```

3.555 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rubi [A] time = 0.0205168, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^6}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^6}+\frac{b^2}{x^4}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0072181, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a+5bx^2\right)}{15x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(3*a + 5*b*x^2))/(15*x^5*(a + b*x^2))
```

Maple [A] time = 0.042, size = 36, normalized size = 0.5

$$-\frac{5\,bx^2+3\,a}{15\,x^5\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^6,x)
```

```
[Out] -1/15*(5*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)
```

Maxima [A] time = 1.01464, size = 20, normalized size = 0.25

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Fricas [A] time = 1.47216, size = 36, normalized size = 0.46

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Sympy [A] time = 0.306549, size = 15, normalized size = 0.19

$$-\frac{3a+5bx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**6,x)
```

```
[Out] -(3*a + 5*b*x**2)/(15*x**5)
```

Giac [A] time = 1.16003, size = 42, normalized size = 0.53

$$-\frac{5\,bx^2\mathrm{sgn}\left(bx^2+a\right)+3\,a\mathrm{sgn}\left(bx^2+a\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(5*b*x^2*sgn(b*x^2 + a) + 3*a*sgn(b*x^2 + a))/x^5
```

3.556 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rubi [A] time = 0.0216657, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^8}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^8}+\frac{b^2}{x^6}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0075454, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(5a+7bx^2\right)}{35x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(5*a + 7*b*x^2))/(35*x^7*(a + b*x^2))
```

Maple [A] time = 0.042, size = 36, normalized size = 0.5

$$-\frac{7\,bx^2+5\,a}{35\,x^7\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^8,x)
```

```
[Out] -1/35*(7*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)
```

Maxima [A] time = 0.994376, size = 20, normalized size = 0.25

$$-\frac{7\,bx^2+5\,a}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/35*(7*b*x^2 + 5*a)/x^7
```

Fricas [A] time = 1.41808, size = 36, normalized size = 0.46

$$-\frac{7\,bx^2+5\,a}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b*x^2 + 5*a)/x^7
```

Sympy [A] time = 0.316303, size = 15, normalized size = 0.19

$$-\frac{5a+7bx^2}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**8,x)
```

```
[Out] -(5*a + 7*b*x**2)/(35*x**7)
```

Giac [A] time = 1.09939, size = 42, normalized size = 0.53

$$-\frac{7\,bx^2\mathrm{sgn}\left(bx^2+a\right)+5\,a\mathrm{sgn}\left(bx^2+a\right)}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(7*b*x^2*sgn(b*x^2 + a) + 5*a*sgn(b*x^2 + a))/x^7
```

3.557 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rubi [A] time = 0.0222312, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (b*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^{10}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^{10}}+\frac{b^2}{x^8}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0076428, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(7a+9bx^2\right)}{63x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(7*a + 9*b*x^2))/(63*x^9*(a + b*x^2))
```

Maple [A] time = 0.042, size = 36, normalized size = 0.5

$$-\frac{9\,bx^2+7\,a}{63\,x^9\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^10,x)
```

```
[Out] -1/63*(9*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

Maxima [A] time = 1.0145, size = 20, normalized size = 0.25

$$-\frac{9\,bx^2+7\,a}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/63*(9*b*x^2 + 7*a)/x^9
```

Fricas [A] time = 1.47957, size = 36, normalized size = 0.46

$$-\frac{9\,bx^2+7\,a}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/63*(9*b*x^2 + 7*a)/x^9
```

Sympy [A] time = 0.343235, size = 15, normalized size = 0.19

$$-\frac{7a+9bx^2}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**10,x)
```

```
[Out] -(7*a + 9*b*x**2)/(63*x**9)
```

Giac [A] time = 1.11281, size = 42, normalized size = 0.53

$$-\frac{9\,bx^2\mathrm{sgn}\left(bx^2+a\right)+7\,a\mathrm{sgn}\left(bx^2+a\right)}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/63*(9*b*x^2*sgn(b*x^2 + a) + 7*a*sgn(b*x^2 + a))/x^9
```

3.558 $\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}$$

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a^2*b*x^12*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a*b^2*x^14*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(16*(a + b*x^2))
```

Rubi [A] time = 0.113616, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a^2*b*x^12*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a*b^2*x^14*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(16*(a + b*x^2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^9\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^4\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,x^2\right)\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int x^4\left(ab+b^2x\right)^3\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int\left(a^3b^3x^4+3a^2b^4x^5+3ab^5x^6+b^6x^7\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\ &= \frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0189648, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{\left(a+bx^2\right)^2}\left(140a^2bx^2+56a^3+120ab^2x^4+35b^3x^6\right)}{560\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(56*a^3 + 140*a^2*b*x^2 + 120*a*b^2*x^4 + 35*b^3*
x^6))/(560*(a + b*x^2))
```

Maple [A] time = 0.169, size = 58, normalized size = 0.4

$$\frac{x^{10}\left(35\,b^3x^6+120\,b^2ax^4+140\,a^2bx^2+56\,a^3\right)}{560\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/560*x^10*(35*b^3*x^6+120*a*b^2*x^4+140*a^2*b*x^2+56*a^3)*((b*x^2+a)^2)^(3
/2)/(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51797, size = 89, normalized size = 0.53

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^9\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**9*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.10875, size = 90, normalized size = 0.54

$$\frac{1}{16}\,b^3x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{14}\,ab^2x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,a^2bx^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{10}\,a^3x^{10}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16*sgn(b*x^2 + a) + 3/14*a*b^2*x^14*sgn(b*x^2 + a) + 1/4*a^2*b*x
^12*sgn(b*x^2 + a) + 1/10*a^3*x^10*sgn(b*x^2 + a)
```

3.559 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a*b^2*x^12*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(14*(a + b*x^2))
```

Rubi [A] time = 0.113862, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a*b^2*x^12*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(14*(a + b*x^2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^3\left(a^2+2abx+b^2x^2\right)^{3/2}dx,x,x^2\right)\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int x^3\left(ab+b^2x\right)^3dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int\left(a^3b^3x^3+3a^2b^4x^4+3ab^5x^5+b^6x^6\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\ &= \frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0154582, size = 61, normalized size = 0.37

$$\frac{x^8\sqrt{\left(a+bx^2\right)^2}\left(84a^2bx^2+35a^3+70ab^2x^4+20b^3x^6\right)}{280\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(35*a^3 + 84*a^2*b*x^2 + 70*a*b^2*x^4 + 20*b^3*x^6
))/(280*(a + b*x^2))
```

Maple [A] time = 0.171, size = 58, normalized size = 0.4

$$\frac{x^8\left(20\,b^3x^6+70\,b^2ax^4+84\,a^2bx^2+35\,a^3\right)}{280\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/280*x^8*(20*b^3*x^6+70*a*b^2*x^4+84*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48179, size = 86, normalized size = 0.51

$$\frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**7*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.14016, size = 90, normalized size = 0.54

$$\frac{1}{14}\,b^3x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,ab^2x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{10}\,a^2bx^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{8}\,a^3x^8\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*sgn(b*x^2 + a) + 1/4*a*b^2*x^12*sgn(b*x^2 + a) + 3/10*a^2*b*x
^10*sgn(b*x^2 + a) + 1/8*a^3*x^8*sgn(b*x^2 + a)
```

3.560 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=106

$$\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^3}-\frac{a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{5b^3}+\frac{a^2(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^3}$$

```
[Out] (a^2*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b^3) - (a*(a^2 + 2*a
*b*x^2 + b^2*x^4)^(5/2))/(5*b^3) + ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)
^(5/2))/(12*b^3)
```

Rubi [A] time = 0.0834046, antiderivative size = 119, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1111, 645}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^4}{5b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^2*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^3) - (a*(a + b*x^2
)^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*b^3) + ((a + b*x^2)^5*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(12*b^3)
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rubi steps

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x^2 \left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^2\left(ab+b^2x\right)^3}{b^2}-\frac{2a\left(ab+b^2x\right)^4}{b^3}+\frac{\left(ab+b^2x\right)^5}{b^4}\right) dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8b^3}-\frac{a\left(a+bx^2\right)^4\sqrt{a^2+2abx^2+b^2x^4}}{5b^3}+\frac{\left(a+bx^2\right)^5\sqrt{a^2}}{1}$$

Mathematica [A] time = 0.0153002, size = 61, normalized size = 0.58

$$\frac{x^6\sqrt{\left(a+bx^2\right)^2}\left(45a^2bx^2+20a^3+36ab^2x^4+10b^3x^6\right)}{120\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3*x^6
))/(120*(a + b*x^2))
```

Maple [A] time = 0.175, size = 58, normalized size = 0.6

$$\frac{x^6\left(10\,b^3x^6+36\,ab^2x^4+45\,a^2bx^2+20\,a^3\right)}{120\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49195, size = 85, normalized size = 0.8

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.1235, size = 90, normalized size = 0.85

$$\frac{1}{12}\,b^3x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{10}\,ab^2x^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{8}\,a^2bx^8\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{6}\,a^3x^6\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12*sgn(b*x^2 + a) + 3/10*a*b^2*x^10*sgn(b*x^2 + a) + 3/8*a^2*b*x
^8*sgn(b*x^2 + a) + 1/6*a^3*x^6*sgn(b*x^2 + a)
```

3.561 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2}$$

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b^2) + (a^2 + 2*a*b*x
^2 + b^2*x^4)^(5/2)/(10*b^2)
```

Rubi [A] time = 0.0509264, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 609}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```


```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b^2) + (a^2 + 2*a*b*x
^2 + b^2*x^4)^(5/2)/(10*b^2)
```


Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,x^2\right)\\&=\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}-\frac{a\,\text{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,x^2\right)}{2b}\\&=-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}\end{aligned}$$

Mathematica [A] time = 0.0153101, size = 61, normalized size = 0.91

$$\frac{x^4\sqrt{\left(a+bx^2\right)^2}\left(20a^2bx^2+10a^3+15ab^2x^4+4b^3x^6\right)}{40\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*x^6)
)/(40*(a + b*x^2))
```

Maple [A] time = 0.173, size = 58, normalized size = 0.9

$$\frac{x^4\left(4\,b^3x^6+15\,ab^2x^4+20\,a^2bx^2+10\,a^3\right)}{40\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*((b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47925, size = 82, normalized size = 1.22

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{2}\,a^2bx^6+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**3*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.12088, size = 61, normalized size = 0.91

$$\frac{1}{40}\left(4\,b^3x^{10}+15\,ab^2x^8+20\,a^2bx^6+10\,a^3x^4\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*sgn(b*x^2 + a)
```

3.562 $\int x \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}$$

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b)
```

Rubi [A] time = 0.0259699, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 609}

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b)
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx, x, x^2\right)\\ &= \frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}\end{aligned}$$

Mathematica [A] time = 0.0118121, size = 27, normalized size = 0.75

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^(3/2))/(8*b)
```

Maple [A] time = 0.171, size = 57, normalized size = 1.6

$$\frac{x^2\left(b^3x^6+4\,ab^2x^4+6\,a^2bx^2+4\,a^3\right)}{8\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*x^2*(b^3*x^6+4*a*b^2*x^4+6*a^2*b*x^2+4*a^3)*((b*x^2+a)^2)^(3/2)/(b*x^2+
a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47199, size = 80, normalized size = 2.22

$$\frac{1}{8}\,b^3x^8+\frac{1}{2}\,ab^2x^6+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.09965, size = 59, normalized size = 1.64

$$\frac{1}{8}\left(b^3x^8+4\,ab^2x^6+6\,a^2bx^4+4\,a^3x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(b^3*x^8 + 4*a*b^2*x^6 + 6*a^2*b*x^4 + 4*a^3*x^2)*sgn(b*x^2 + a)
```

3.563 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=163

$$\frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] (3*a^2*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*x^
4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x^6*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*
Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0492274, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*x^
4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x^6*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*
Log[x])/(a + b*x^2)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x}\,dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)} + \frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)} + \frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)} +$$

Mathematica [A] time = 0.0197298, size = 60, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(bx^2\left(18a^2+9abx^2+2b^2x^4\right)+12a^3\log(x)\right)}{12\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12*a^3*Log[x
]))/(12*(a + b*x^2))
```

Maple [A] time = 0.22, size = 57, normalized size = 0.4

$$\frac{2\,b^{3}x^{6}+9\,ax^{4}b^{2}+18\,a^{2}bx^{2}+12\,a^{3}\ln\left(x\right)}{12\,\left(bx^{2}+a\right)^{3}}\left(\left(bx^{2}+a\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(3/2)*(2*b^3*x^6+9*a*x^4*b^2+18*a^2*b*x^2+12*a^3*ln(x))/
(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54085, size = 78, normalized size = 0.48

$$\frac{1}{6}\,b^{3}x^{6}+\frac{3}{4}\,ab^{2}x^{4}+\frac{3}{2}\,a^{2}bx^{2}+a^{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x, x)
```

Giac [A] time = 1.11943, size = 92, normalized size = 0.56

$$\frac{1}{6}\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^3\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sgn(b*x^2 + a) + 3/4*a*b^2*x^4*sgn(b*x^2 + a) + 3/2*a^2*b*x^2*s
gn(b*x^2 + a) + 1/2*a^3*log(x^2)*sgn(b*x^2 + a)
```

3.564 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=164

$$\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (3*a*b^2*x^2*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (b^3*x^4*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0470999, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```


```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (3*a*b^2*x^2*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (b^3*x^4*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^3}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^2}\,dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(3ab^5+\frac{a^3b^3}{x^2}+\frac{3a^2b^4}{x}+b^6x\right)dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a}{}$$

Mathematica [A] time = 0.020755, size = 62, normalized size = 0.38

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(12a^2bx^2\log(x)-2a^3+6ab^2x^4+b^3x^6\right)}{4x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-2*a^3 + 6*a*b^2*x^4 + b^3*x^6 + 12*a^2*b*x^2*Log[x])
)/(4*x^2*(a + b*x^2))
```

Maple [A] time = 0.223, size = 59, normalized size = 0.4

$$\frac{b^{3}x^{6}+6\,ax^{4}b^{2}+12\,ba^{2}\ln\left(x\right)x^{2}-2\,a^{3}}{4\left(bx^{2}+a\right)^{3}x^{2}}\left(\left(bx^{2}+a\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x)
```

```
[Out] 1/4*((b*x^2+a)^2)^(3/2)*(b^3*x^6+6*a*x^4*b^2+12*b*a^2*ln(x)*x^2-2*a^3)/(b*x
^2+a)^3/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4878, size = 85, normalized size = 0.52

$$\frac{b^{3}x^{6}+6\,ab^{2}x^{4}+12\,a^{2}bx^{2}\log\left(x\right)-2\,a^{3}}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**3, x)
```

Giac [A] time = 1.10966, size = 117, normalized size = 0.71

$$\frac{1}{4}b^3x^4\operatorname{sgn}\left(bx^2+a\right)+\frac{3}{2}ab^2x^2\operatorname{sgn}\left(bx^2+a\right)+\frac{3}{2}a^2b\log\left(x^2\right)\operatorname{sgn}\left(bx^2+a\right)-\frac{3\,a^2bx^2\operatorname{sgn}\left(bx^2+a\right)+a^3\operatorname{sgn}\left(bx^2+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sgn(b*x^2 + a) + 3/2*a*b^2*x^2*sgn(b*x^2 + a) + 3/2*a^2*b*log(x
^2)*sgn(b*x^2 + a) - 1/2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/
x^2
```

3.565 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=164

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*x^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0487639, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*x^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^5}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^3}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(b^6+\frac{a^3b^3}{x^3}+\frac{3a^2b^4}{x^2}+\frac{3ab^5}{x}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)} + \frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)} + 3ab^2$$

Mathematica [A] time = 0.0151436, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(6a^2bx^2+a^3-12ab^2x^4\log(x)-2b^3x^6\right)}{4x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a^3 + 6*a^2*b*x^2 - 2*b^3*x^6 - 12*a*b^2*x^4*Log[x])
)/(4*x^4*(a + b*x^2))
```

Maple [A] time = 0.23, size = 60, normalized size = 0.4

$$\frac{2 b^3 x^6 + 12 b^2 a \ln (x) x^4 - 6 a^2 b x^2 - a^3}{4 \left(b x^2 + a\right)^3 x^4} \left(\left(b x^2 + a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x)
```

```
[Out] 1/4*((b*x^2+a)^2)^(3/2)*(2*b^3*x^6+12*b^2*a*ln(x)*x^4-6*a^2*b*x^2-a^3)/(b*x
^2+a)^3/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42064, size = 85, normalized size = 0.52

$$\frac{2 b^3 x^6 + 12 a b^2 x^4 \log (x) - 6 a^2 b x^2 - a^3}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^2\right)^2\right)^{\frac{3}{2}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**5, x)
```

Giac [A] time = 1.11243, size = 117, normalized size = 0.71

$$\frac{1}{2}b^3x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}ab^2\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{9\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+6\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+a^3\mathrm{sgn}\left(bx^2+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^2 + a) + 3/2*a*b^2*log(x^2)*sgn(b*x^2 + a) - 1/4*(9*a*b
^2*x^4*sgn(b*x^2 + a) + 6*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^
4
```

3.566 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=163

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0461357, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```



```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
*Log[x])/(a + b*x^2)
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^7}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^7}\,dx}{b^2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^4}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^3}+\frac{3ab^5}{x^2}+\frac{b^6}{x}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}\\
&= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3\sqrt{a
\end{aligned}$$

Mathematica [A] time = 0.021217, size = 63, normalized size = 0.39

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a\left(2a^2+9abx^2+18b^2x^4\right)-12b^3x^6\log(x)\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a*(2*a^2 + 9*a*b*x^2 + 18*b^2*x^4) - 12*b^3*x^6*Log[
x]))/(12*x^6*(a + b*x^2))
```

Maple [A] time = 0.224, size = 60, normalized size = 0.4

$$\frac{12 b^3 \ln(x) x^6 - 18 a x^4 b^2 - 9 a^2 b x^2 - 2 a^3}{12 \left(b x^2 + a\right)^3 x^6} \left(\left(b x^2 + a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(3/2)*(12*b^3*ln(x)*x^6-18*a*x^4*b^2-9*a^2*b*x^2-2*a^3)/
(b*x^2+a)^3/x^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48674, size = 90, normalized size = 0.55

$$\frac{12 b^3 x^6 \log(x) - 18 a b^2 x^4 - 9 a^2 b x^2 - 2 a^3}{12 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^2\right)^2\right)^{\frac{3}{2}}}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**7, x)
```

Giac [A] time = 1.11445, size = 117, normalized size = 0.72

$$\frac{1}{2}\,b^3\log\left(x^2\right)\operatorname{sgn}\left(bx^2+a\right)-\frac{11\,b^3x^6\operatorname{sgn}\left(bx^2+a\right)+18\,ab^2x^4\operatorname{sgn}\left(bx^2+a\right)+9\,a^2bx^2\operatorname{sgn}\left(bx^2+a\right)+2\,a^3\operatorname{sgn}\left(bx^2+a\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(x^2)*sgn(b*x^2 + a) - 1/12*(11*b^3*x^6*sgn(b*x^2 + a) + 18*a*b^
2*x^4*sgn(b*x^2 + a) + 9*a^2*b*x^2*sgn(b*x^2 + a) + 2*a^3*sgn(b*x^2 + a))/x
^6
```

3.567 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

```
[Out] -((a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*a*x^8)
```

Rubi [A] time = 0.0394895, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$-\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

```
[Out] -((a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*a*x^8)
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^9}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^5}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^5}\,dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

Mathematica [A] time = 0.0148303, size = 59, normalized size = 1.44

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(4a^2bx^2+a^3+6ab^2x^4+4b^3x^6\right)}{8x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a^3 + 4*a^2*b*x^2 + 6*a*b^2*x^4 + 4*b^3*x^6))/(8*x^8
*(a + b*x^2))
```

Maple [A] time = 0.176, size = 56, normalized size = 1.4

$$-\frac{4\,b^3x^6+6\,ax^4b^2+4\,a^2bx^2+a^3}{8\,x^8\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x)
```

```
[Out] -1/8*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^8/(b*x^2
+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42766, size = 76, normalized size = 1.85

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**9, x)
```

Giac [B] time = 1.13841, size = 92, normalized size = 2.24

$$-\frac{4\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+6\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+4\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+a^3\mathrm{sgn}\left(bx^2+a\right)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*x^6*sgn(b*x^2 + a) + 6*a*b^2*x^4*sgn(b*x^2 + a) + 4*a^2*b*x^2*s
gn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^8
```

3.568 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}$$

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*a*x^10) + (a^2 + 2*a*b*
x^2 + b^2*x^4)^(5/2)/(40*a^2*x^10)
```

Rubi [A] time = 0.0167922, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1110}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*a*x^10) + (a^2 + 2*a*b*
x^2 + b^2*x^4)^(5/2)/(40*a^2*x^10)
```

Rule 1110

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(4*a*d*(p + 1)*(2*p +
1)), x] - Simp[((d*x)^(m + 1)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^p)/(4*a*d*(
2*p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{11}}\,dx=-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}$$

Mathematica [A] time = 0.0132023, size = 61, normalized size = 0.85

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(15a^2bx^2+4a^3+20ab^2x^4+10b^3x^6\right)}{40x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(4*a^3 + 15*a^2*b*x^2 + 20*a*b^2*x^4 + 10*b^3*x^6))/(
40*x^10*(a + b*x^2))
```

Maple [A] time = 0.164, size = 58, normalized size = 0.8

$$-\frac{10\,b^3x^6+20\,ax^4b^2+15\,a^2bx^2+4\,a^3}{40\,x^{10}\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x)
```

```
[Out] -1/40*(10*b^3*x^6+20*a*b^2*x^4+15*a^2*b*x^2+4*a^3)*((b*x^2+a)^2)^(3/2)/x^10
/(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45071, size = 85, normalized size = 1.18

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**11, x)
```

Giac [A] time = 1.13632, size = 93, normalized size = 1.29

$$-\frac{10\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+20\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+15\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+4\,a^3\mathrm{sgn}\left(bx^2+a\right)}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6*sgn(b*x^2 + a) + 20*a*b^2*x^4*sgn(b*x^2 + a) + 15*a^2*b*x
^2*sgn(b*x^2 + a) + 4*a^3*sgn(b*x^2 + a))/x^10
```

3.569 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(6*x^6*(a + b*x^2))
```

Rubi [A] time = 0.104971, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```


```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(6*x^6*(a + b*x^2))
```


Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{13}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^7}\,dx,x,x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^7}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^7}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^5}+\frac{b^6}{x^4}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)} - \frac{b^3\sqrt{a^2}}{6x}$$

Mathematica [A] time = 0.0136877, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(36a^2bx^2+10a^3+45ab^2x^4+20b^3x^6\right)}{120x^{12}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(10*a^3 + 36*a^2*b*x^2 + 45*a*b^2*x^4 + 20*b^3*x^6))/
(120*x^12*(a + b*x^2))
```

Maple [A] time = 0.166, size = 58, normalized size = 0.4

$$-\frac{20\,b^{3}x^{6}+45\,a x^{4}b^{2}+36\,a^{2}b x^{2}+10\,a^{3}}{120\,x^{12}\left(b x^{2}+a\right)^{3}}\left(\left(b x^{2}+a\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x)
```

```
[Out] -1/120*(20*b^3*x^6+45*a*b^2*x^4+36*a^2*b*x^2+10*a^3)*((b*x^2+a)^2)^(3/2)/x^
12/(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46861, size = 88, normalized size = 0.53

$$-\frac{20\,b^{3}x^{6}+45\,a b^{2}x^{4}+36\,a^{2}b x^{2}+10\,a^{3}}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+b x^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**13, x)
```

Giac [A] time = 1.11735, size = 93, normalized size = 0.56

$$-\frac{20\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+45\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+36\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+10\,a^3\mathrm{sgn}\left(bx^2+a\right)}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(20*b^3*x^6*sgn(b*x^2 + a) + 45*a*b^2*x^4*sgn(b*x^2 + a) + 36*a^2*b*
x^2*sgn(b*x^2 + a) + 10*a^3*sgn(b*x^2 + a))/x^12
```

3.570 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(8*x^8*(a + b*x^2))
```

Rubi [A] time = 0.105819, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(8*x^8*(a + b*x^2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{15}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^8}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^8}\,dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^7}+\frac{3ab^5}{x^6}+\frac{b^6}{x^5}\right)dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)} - \frac{b^3\sqrt{a^2+}}{8x^8}$$

Mathematica [A] time = 0.0172557, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(70a^2bx^2+20a^3+84ab^2x^4+35b^3x^6\right)}{280x^{14}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(20*a^3 + 70*a^2*b*x^2 + 84*a*b^2*x^4 + 35*b^3*x^6))/
(280*x^14*(a + b*x^2))
```

Maple [A] time = 0.176, size = 58, normalized size = 0.4

$$-\frac{35 b^3x^6 + 84 ax^4b^2 + 70 a^2bx^2 + 20 a^3}{280 x^{14} \left(bx^2 + a\right)^3} \left(\left(bx^2 + a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x)
```

```
[Out] -1/280*(35*b^3*x^6+84*a*b^2*x^4+70*a^2*b*x^2+20*a^3)*((b*x^2+a)^2)^(3/2)/x^
14/(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44522, size = 88, normalized size = 0.53

$$-\frac{35 b^3x^6 + 84 ab^2x^4 + 70 a^2bx^2 + 20 a^3}{280 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^{15}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**15, x)
```

Giac [A] time = 1.11401, size = 93, normalized size = 0.56

$$-\frac{35\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+84\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+70\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+20\,a^3\mathrm{sgn}\left(bx^2+a\right)}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/280*(35*b^3*x^6*sgn(b*x^2 + a) + 84*a*b^2*x^4*sgn(b*x^2 + a) + 70*a^2*b*
x^2*sgn(b*x^2 + a) + 20*a^3*sgn(b*x^2 + a))/x^14
```

3.571 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(10*x^10*(a + b*x^2))
```

Rubi [A] time = 0.104589, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(10*x^10*(a + b*x^2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{17}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{3/2}}{x^9}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^9}\,dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^9}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^7}+\frac{b^6}{x^6}\right)dx, x, x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)} - \frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)} - \frac{b^3\sqrt{a^2+}}{10x^1}$$

Mathematica [A] time = 0.014356, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(120a^2bx^2+35a^3+140ab^2x^4+56b^3x^6\right)}{560x^{16}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(35*a^3 + 120*a^2*b*x^2 + 140*a*b^2*x^4 + 56*b^3*x^6)
)/(560*x^16*(a + b*x^2))
```

Maple [A] time = 0.166, size = 58, normalized size = 0.4

$$-\frac{56\,b^{3}x^{6}+140\,a x^{4}b^{2}+120\,a^{2}b x^{2}+35\,a^{3}}{560\,x^{16}\left(b x^{2}+a\right)^{3}}\left(\left(b x^{2}+a\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x)
```

```
[Out] -1/560*(56*b^3*x^6+140*a*b^2*x^4+120*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)/
x^16/(b*x^2+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.477, size = 90, normalized size = 0.54

$$-\frac{56\,b^{3}x^{6}+140\,a b^{2}x^{4}+120\,a^{2}b x^{2}+35\,a^{3}}{560\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/560*(56*b^3*x^6 + 140*a*b^2*x^4 + 120*a^2*b*x^2 + 35*a^3)/x^16
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+b x^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**17, x)
```

Giac [A] time = 1.1141, size = 93, normalized size = 0.56

$$-\frac{56\, b^3x^6\mathrm{sgn}\left(bx^2+a\right)+140\, ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+120\, a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+35\, a^3\mathrm{sgn}\left(bx^2+a\right)}{560\, x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/560*(56*b^3*x^6*sgn(b*x^2 + a) + 140*a*b^2*x^4*sgn(b*x^2 + a) + 120*a^2*
b*x^2*sgn(b*x^2 + a) + 35*a^3*sgn(b*x^2 + a))/x^16
```

3.572 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (b^3*x^15*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(15*(a + b*x^2))
```

Rubi [A] time = 0.0420411, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```


```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (b^3*x^15*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(15*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^8 \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3x^8+3a^2b^4x^{10}+3ab^5x^{12}+b^6x^{14}\right) dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)} + \frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)} + \frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0150427, size = 61, normalized size = 0.37

$$\frac{x^9\sqrt{\left(a+bx^2\right)^2}\left(1755a^2bx^2+715a^3+1485ab^2x^4+429b^3x^6\right)}{6435\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]

[Out] (x^9*Sqrt[(a + b*x^2)^2]*(715*a^3 + 1755*a^2*b*x^2 + 1485*a*b^2*x^4 + 429*b
^3*x^6))/(6435*(a + b*x^2))
```

Maple [A] time = 0.164, size = 58, normalized size = 0.4

$$\frac{x^9\left(429\,b^3x^6+1485\,ax^4b^2+1755\,a^2bx^2+715\,a^3\right)}{6435\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)

[Out] 1/6435*x^9*(429*b^3*x^6+1485*a*b^2*x^4+1755*a^2*b*x^2+715*a^3)*((b*x^2+a)^2
)^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 1.00826, size = 47, normalized size = 0.28

$$\frac{1}{15}\,b^3x^{15} + \frac{3}{13}\,ab^2x^{13} + \frac{3}{11}\,a^2bx^{11} + \frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

Fricas [A] time = 1.43779, size = 88, normalized size = 0.53

$$\frac{1}{15}\,b^3x^{15} + \frac{3}{13}\,ab^2x^{13} + \frac{3}{11}\,a^2bx^{11} + \frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**8*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.12603, size = 90, normalized size = 0.54

$$\frac{1}{15}\,b^3x^{15}\mathrm{sgn}\left(bx^2 + a\right) + \frac{3}{13}\,ab^2x^{13}\mathrm{sgn}\left(bx^2 + a\right) + \frac{3}{11}\,a^2bx^{11}\mathrm{sgn}\left(bx^2 + a\right) + \frac{1}{9}\,a^3x^9\mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*b^3*x^15*sgn(b*x^2 + a) + 3/13*a*b^2*x^13*sgn(b*x^2 + a) + 3/11*a^2*b*
x^11*sgn(b*x^2 + a) + 1/9*a^3*x^9*sgn(b*x^2 + a)
```

3.573 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a^2*b*x^9*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a*b^2*x^11*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(13*(a + b*x^2))
```

Rubi [A] time = 0.0405747, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a^2*b*x^9*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a*b^2*x^11*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(13*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^6 \left(ab+b^2x^2\right)^3 dx}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3x^6+3a^2b^4x^8+3ab^5x^{10}+b^6x^{12}\right) dx}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)} + \frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} + \frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)} +$$

Mathematica [A] time = 0.0152198, size = 61, normalized size = 0.37

$$\frac{x^7\sqrt{\left(a+bx^2\right)^2} \left(1001a^2bx^2+429a^3+819ab^2x^4+231b^3x^6\right)}{3003\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^2)^2]*(429*a^3 + 1001*a^2*b*x^2 + 819*a*b^2*x^4 + 231*b^
3*x^6))/(3003*(a + b*x^2))
```

Maple [A] time = 0.167, size = 58, normalized size = 0.4

$$\frac{x^7\left(231\,b^3x^6+819\,ax^4b^2+1001\,a^2bx^2+429\,a^3\right)}{3003\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/3003*x^7*(231*b^3*x^6+819*a*b^2*x^4+1001*a^2*b*x^2+429*a^3)*((b*x^2+a)^2)
^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.997596, size = 47, normalized size = 0.28

$$\frac{1}{13}\,b^{3}x^{13}+\frac{3}{11}\,ab^{2}x^{11}+\frac{1}{3}\,a^{2}bx^{9}+\frac{1}{7}\,a^{3}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Fricas [A] time = 1.47817, size = 85, normalized size = 0.51

$$\frac{1}{13}\,b^{3}x^{13}+\frac{3}{11}\,ab^{2}x^{11}+\frac{1}{3}\,a^{2}bx^{9}+\frac{1}{7}\,a^{3}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{6}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**6*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.09411, size = 90, normalized size = 0.54

$$\frac{1}{13}\,b^{3}x^{13}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{3}{11}\,ab^{2}x^{11}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{1}{3}\,a^{2}bx^{9}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{1}{7}\,a^{3}x^{7}\mathrm{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*sgn(b*x^2 + a) + 3/11*a*b^2*x^11*sgn(b*x^2 + a) + 1/3*a^2*b*x
^9*sgn(b*x^2 + a) + 1/7*a^3*x^7*sgn(b*x^2 + a)
```

3.574 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a*b^2*x^9*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(11*(a + b*x^2))
```

Rubi [A] time = 0.0432692, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```



```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a*b^2*x^9*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(11*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^4\left(ab+b^2x^2\right)^3dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^3b^3x^4+3a^2b^4x^6+3ab^5x^8+b^6x^{10}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+$$

Mathematica [A] time = 0.0154241, size = 61, normalized size = 0.37

$$\frac{x^5\sqrt{\left(a+bx^2\right)^2}\left(495a^2bx^2+231a^3+385ab^2x^4+105b^3x^6\right)}{1155\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^2)^2]*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3
*x^6))/(1155*(a + b*x^2))
```

Maple [A] time = 0.164, size = 58, normalized size = 0.4

$$\frac{x^5\left(105\,b^3x^6+385\,ax^4b^2+495\,a^2bx^2+231\,a^3\right)}{1155\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*((b*x^2+a)^2)^
(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 1.00699, size = 47, normalized size = 0.28

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Fricas [A] time = 1.50835, size = 82, normalized size = 0.49

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.12588, size = 90, normalized size = 0.54

$$\frac{1}{11}\,b^3x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,ab^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{7}\,a^2bx^7\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{5}\,a^3x^5\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*sgn(b*x^2 + a) + 1/3*a*b^2*x^9*sgn(b*x^2 + a) + 3/7*a^2*b*x^7
*sgn(b*x^2 + a) + 1/5*a^3*x^5*sgn(b*x^2 + a)
```

3.575 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a*b^2*x^7*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(9*(a + b*x^2))
```

Rubi [A] time = 0.0407834, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a*b^2*x^7*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(9*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^2 \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3x^2+3a^2b^4x^4+3ab^5x^6+b^6x^8\right) dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} + \frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)} + \frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)} +$$

Mathematica [A] time = 0.0115577, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(189a^2bx^5+105a^3x^3+135ab^2x^7+35b^3x^9\right)}{315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*b^3*
x^9))/(315*(a + b*x^2))
```

Maple [A] time = 0.166, size = 58, normalized size = 0.4

$$\frac{x^3\left(35\,b^3x^6+135\,ax^4b^2+189\,a^2bx^2+105\,a^3\right)}{315\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*((b*x^2+a)^2)^(3
/2)/(b*x^2+a)^3
```

Maxima [A] time = 1.04189, size = 47, normalized size = 0.28

$$\frac{1}{9}b^3x^9 + \frac{3}{7}ab^2x^7 + \frac{3}{5}a^2bx^5 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Fricas [A] time = 1.46841, size = 80, normalized size = 0.48

$$\frac{1}{9}b^3x^9 + \frac{3}{7}ab^2x^7 + \frac{3}{5}a^2bx^5 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.13842, size = 90, normalized size = 0.54

$$\frac{1}{9}b^3x^9\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{7}ab^2x^7\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{5}a^2bx^5\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{3}a^3x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sgn(b*x^2 + a) + 3/7*a*b^2*x^7*sgn(b*x^2 + a) + 3/5*a^2*b*x^5*s
gn(b*x^2 + a) + 1/3*a^3*x^3*sgn(b*x^2 + a)
```

3.576 $\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=159

$$\frac{b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{7\left(a+bx^2\right)^3}+\frac{3ab^2x^5\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{5\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{a^3x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}$$

```
[Out] (a^3*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*x^3*(a^2 +
 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x^5*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^
(3/2))/(7*(a + b*x^2)^3)
```

Rubi [A] time = 0.0332733, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1088, 194}

$$\frac{b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{7\left(a+bx^2\right)^3}+\frac{3ab^2x^5\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{5\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{a^3x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*x^3*(a^2 +
 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x^5*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^
(3/2))/(7*(a + b*x^2)^3)
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}\int \left(2ab+2b^2x^2\right)^3\,dx}{\left(2ab+2b^2x^2\right)^3}$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}\int \left(8a^3b^3+24a^2b^4x^2+24ab^5x^4+8b^6x^6\right)dx}{\left(2ab+2b^2x^2\right)^3}$$
$$= \frac{a^3x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{3ab^2x^5\left(a^2+2abx^2+b^2x^4\right)}{5\left(a+bx^2\right)^3}$$

Mathematica [A] time = 0.0122895, size = 59, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(35a^2bx^3+35a^3x+21ab^2x^5+5b^3x^7\right)}{35\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^7))/
(35*(a + b*x^2))
```

Maple [A] time = 0.042, size = 56, normalized size = 0.4

$$\frac{x\left(5\,b^3x^6+21\,ax^4b^2+35\,a^2bx^2+35\,a^3\right)}{35\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)/(b*
x^2+a)^3
```

Maxima [A] time = 1.00336, size = 42, normalized size = 0.26

$$\frac{1}{7}b^3x^7 + \frac{3}{5}ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Fricas [A] time = 1.4736, size = 66, normalized size = 0.42

$$\frac{1}{7}b^3x^7 + \frac{3}{5}ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2), x)
```

Giac [A] time = 1.09579, size = 85, normalized size = 0.53

$$\frac{1}{7}b^3x^7\mathrm{sgn}\left(bx^2 + a\right) + \frac{3}{5}ab^2x^5\mathrm{sgn}\left(bx^2 + a\right) + a^2bx^3\mathrm{sgn}\left(bx^2 + a\right) + a^3x\mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*sgn(b*x^2 + a) + 3/5*a*b^2*x^5*sgn(b*x^2 + a) + a^2*b*x^3*sgn(b
*x^2 + a) + a^3*x*sgn(b*x^2 + a)
```

3.577 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=158

$$\frac{b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (3*a^2*b*x*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (a*b^2*x^3*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(a + b*x^2) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a +
b*x^2))
```

Rubi [A] time = 0.0397997, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (3*a^2*b*x*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (a*b^2*x^3*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(a + b*x^2) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a +
b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^2}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(3a^2b^4+\frac{a^3b^3}{x^2}+3ab^5x^2+b^6x^4\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^5}{}$$

Mathematica [A] time = 0.0151985, size = 60, normalized size = 0.38

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(15a^2bx^2-5a^3+5ab^2x^4+b^3x^6\right)}{5x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(5*x*
(a + b*x^2))
```

Maple [A] time = 0.166, size = 58, normalized size = 0.4

$$-\frac{-b^3x^6-5\,ax^4b^2-15\,a^2bx^2+5\,a^3}{5\,x\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x)
```

```
[Out] -1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^(3/2)/x/(b*x^2
+a)^3
```

Maxima [A] time = 1.00562, size = 49, normalized size = 0.31

$$\frac{b^3x^6 + 5\,ab^2x^4 + 15\,a^2bx^2 - 5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

Fricas [A] time = 1.42198, size = 73, normalized size = 0.46

$$\frac{b^3x^6 + 5\,ab^2x^4 + 15\,a^2bx^2 - 5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**2, x)
```

Giac [A] time = 1.11577, size = 86, normalized size = 0.54

$$\frac{1}{5}\,b^3x^5\mathrm{sgn}\left(bx^2 + a\right) + ab^2x^3\mathrm{sgn}\left(bx^2 + a\right) + 3\,a^2bx\mathrm{sgn}\left(bx^2 + a\right) - \frac{a^3\mathrm{sgn}\left(bx^2 + a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*sgn(b*x^2 + a) + a*b^2*x^3*sgn(b*x^2 + a) + 3*a^2*b*x*sgn(b*x^2
 + a) - a^3*sgn(b*x^2 + a)/x
```

3.578 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=161

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (3*a*b^2*x*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(a + b*x^2) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(
a + b*x^2))
```

Rubi [A] time = 0.0403986, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (3*a*b^2*x*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(a + b*x^2) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(
a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^4}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(3ab^5+\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^2}+b^6x^2\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)} + \frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b^3x^3}{}$$

Mathematica [A] time = 0.0131741, size = 59, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(9a^2bx^2+a^3-9ab^2x^4-b^3x^6\right)}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a^3 + 9*a^2*b*x^2 - 9*a*b^2*x^4 - b^3*x^6))/(3*x^3*(
a + b*x^2))
```

Maple [A] time = 0.166, size = 56, normalized size = 0.4

$$-\frac{-b^3x^6-9\,ax^4b^2+9\,a^2bx^2+a^3}{3\,x^3\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x)
```

```
[Out] -1/3*(-b^3*x^6-9*a*b^2*x^4+9*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^3/(b*x^2+
a)^3
```

Maxima [A] time = 1.01653, size = 49, normalized size = 0.3

$$\frac{b^3x^6 + 9\,ab^2x^4 - 9\,a^2bx^2 - a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

Fricas [A] time = 1.45455, size = 72, normalized size = 0.45

$$\frac{b^3x^6 + 9\,ab^2x^4 - 9\,a^2bx^2 - a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**4, x)
```

Giac [A] time = 1.13596, size = 90, normalized size = 0.56

$$\frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx^2 + a\right) + 3\,ab^2x\mathrm{sgn}\left(bx^2 + a\right) - \frac{9\,a^2bx^2\mathrm{sgn}\left(bx^2 + a\right) + a^3\mathrm{sgn}\left(bx^2 + a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sgn(b*x^2 + a) + 3*a*b^2*x*sgn(b*x^2 + a) - 1/3*(9*a^2*b*x^2*sg
n(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^3
```

3.579 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^6} dx$

Optimal. Leaf size=158

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(x*(a + b*x^2)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a +
 b*x^2)
```

Rubi [A] time = 0.0403273, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a^2*b*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(x*(a + b*x^2)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a +
 b*x^2)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^6}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^6+\frac{a^3b^3}{x^6}+\frac{3a^2b^4}{x^4}+\frac{3ab^5}{x^2}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2}}{}$$

Mathematica [A] time = 0.0145402, size = 59, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(5a^2bx^2+a^3+15ab^2x^4-5b^3x^6\right)}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a^3 + 5*a^2*b*x^2 + 15*a*b^2*x^4 - 5*b^3*x^6))/(5*x^
5*(a + b*x^2))
```

Maple [A] time = 0.165, size = 56, normalized size = 0.4

$$-\frac{-5\,b^3x^6+15\,ax^4b^2+5\,a^2bx^2+a^3}{5\,x^5\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x)
```

```
[Out] -1/5*(-5*b^3*x^6+15*a*b^2*x^4+5*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^5/(b*x
^2+a)^3
```

Maxima [A] time = 1.09508, size = 50, normalized size = 0.32

$$\frac{5 b^3 x^6 - 15 a b^2 x^4 - 5 a^2 b x^2 - a^3}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

Fricas [A] time = 1.47408, size = 76, normalized size = 0.48

$$\frac{5 b^3 x^6 - 15 a b^2 x^4 - 5 a^2 b x^2 - a^3}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^2\right)^2\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**6, x)
```

Giac [A] time = 1.11378, size = 89, normalized size = 0.56

$$b^3 x \operatorname{sgn}\left(b x^2 + a\right) - \frac{15 a b^2 x^4 \operatorname{sgn}\left(b x^2 + a\right) + 5 a^2 b x^2 \operatorname{sgn}\left(b x^2 + a\right) + a^3 \operatorname{sgn}\left(b x^2 + a\right)}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] b^3*x*sgn(b*x^2 + a) - 1/5*(15*a*b^2*x^4*sgn(b*x^2 + a) + 5*a^2*b*x^2*sgn(b
*x^2 + a) + a^3*sgn(b*x^2 + a))/x^5
```

3.580 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^8} dx$

Optimal. Leaf size=163

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x
*(a + b*x^2))
```

Rubi [A] time = 0.0432572, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x
*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

IGtQ[p, 0]

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^8}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^4}+\frac{b^6}{x^2}\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)} - \frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)} - \frac{b^3\sqrt{a^2+}}{x\,(}$$

Mathematica [A] time = 0.0125719, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(21a^2bx^2+5a^3+35ab^2x^4+35b^3x^6\right)}{35x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(5*a^3 + 21*a^2*b*x^2 + 35*a*b^2*x^4 + 35*b^3*x^6))/(
35*x^7*(a + b*x^2))
```

Maple [A] time = 0.166, size = 58, normalized size = 0.4

$$-\frac{35\,b^3x^6+35\,ax^4b^2+21\,a^2bx^2+5\,a^3}{35\,x^7\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x)
```

```
[Out] -1/35*(35*b^3*x^6+35*a*b^2*x^4+21*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^(3/2)/x^7/
(b*x^2+a)^3
```

Maxima [A] time = 1.20409, size = 50, normalized size = 0.31

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Fricas [A] time = 1.45199, size = 84, normalized size = 0.52

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**8, x)
```

Giac [A] time = 1.1118, size = 93, normalized size = 0.57

$$-\frac{35\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+35\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+21\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+5\,a^3\mathrm{sgn}\left(bx^2+a\right)}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(35*b^3*x^6*sgn(b*x^2 + a) + 35*a*b^2*x^4*sgn(b*x^2 + a) + 21*a^2*b*x
^2*sgn(b*x^2 + a) + 5*a^3*sgn(b*x^2 + a))/x^7
```

3.581 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(3*x^3*(a + b*x^2))
```

Rubi [A] time = 0.0394533, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (3*a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(3*x^3*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{10}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{10}}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^6}+\frac{b^6}{x^4}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)} - \frac{b^3\sqrt{a^2}}{3x}$$

Mathematica [A] time = 0.0127908, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(135a^2bx^2+35a^3+189ab^2x^4+105b^3x^6\right)}{315x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(35*a^3 + 135*a^2*b*x^2 + 189*a*b^2*x^4 + 105*b^3*x^6
))/(315*x^9*(a + b*x^2))
```

Maple [A] time = 0.162, size = 58, normalized size = 0.4

$$-\frac{105\,b^3x^6+189\,ax^4b^2+135\,a^2bx^2+35\,a^3}{315\,x^9\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x)
```

```
[Out] -1/315*(105*b^3*x^6+189*a*b^2*x^4+135*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)
/x^9/(b*x^2+a)^3
```

Maxima [A] time = 1.01958, size = 50, normalized size = 0.3

$$-\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Fricas [A] time = 1.48877, size = 90, normalized size = 0.54

$$-\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**10, x)
```

Giac [A] time = 1.11884, size = 93, normalized size = 0.56

$$-\frac{105\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+189\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+135\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+35\,a^3\mathrm{sgn}\left(bx^2+a\right)}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(105*b^3*x^6*sgn(b*x^2 + a) + 189*a*b^2*x^4*sgn(b*x^2 + a) + 135*a^2
*b*x^2*sgn(b*x^2 + a) + 35*a^3*sgn(b*x^2 + a))/x^9
```

3.582 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(5*x^5*(a + b*x^2))
```

Rubi [A] time = 0.0439584, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(5*x^5*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{12}}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{12}}+\frac{3a^2b^4}{x^{10}}+\frac{3ab^5}{x^8}+\frac{b^6}{x^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{b^3\sqrt{a^2+}}{5x^5}$$

Mathematica [A] time = 0.0166941, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(385a^2bx^2+105a^3+495ab^2x^4+231b^3x^6\right)}{1155x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(105*a^3 + 385*a^2*b*x^2 + 495*a*b^2*x^4 + 231*b^3*x^
6))/(1155*x^11*(a + b*x^2))
```

Maple [A] time = 0.16, size = 58, normalized size = 0.4

$$-\frac{231\,b^3x^6+495\,ax^4b^2+385\,a^2bx^2+105\,a^3}{1155\,x^{11}\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x)
```

```
[Out] -1/1155*(231*b^3*x^6+495*a*b^2*x^4+385*a^2*b*x^2+105*a^3)*((b*x^2+a)^2)^(3/
2)/x^11/(b*x^2+a)^3
```

Maxima [A] time = 1.00558, size = 50, normalized size = 0.3

$$-\frac{231\,b^{3}x^{6}+495\,ab^{2}x^{4}+385\,a^{2}bx^{2}+105\,a^{3}}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Fricas [A] time = 1.42585, size = 95, normalized size = 0.57

$$-\frac{231\,b^{3}x^{6}+495\,ab^{2}x^{4}+385\,a^{2}bx^{2}+105\,a^{3}}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**12, x)
```

Giac [A] time = 1.13327, size = 93, normalized size = 0.56

$$-\frac{231\,b^{3}x^{6}\mathrm{sgn}\left(bx^{2}+a\right)+495\,ab^{2}x^{4}\mathrm{sgn}\left(bx^{2}+a\right)+385\,a^{2}bx^{2}\mathrm{sgn}\left(bx^{2}+a\right)+105\,a^{3}\mathrm{sgn}\left(bx^{2}+a\right)}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/1155*(231*b^3*x^6*sgn(b*x^2 + a) + 495*a*b^2*x^4*sgn(b*x^2 + a) + 385*a^
2*b*x^2*sgn(b*x^2 + a) + 105*a^3*sgn(b*x^2 + a))/x^11
```

3.583 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(7*x^7*(a + b*x^2))
```

Rubi [A] time = 0.0403642, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(7*x^7*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{14}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{14}}+\frac{3a^2b^4}{x^{12}}+\frac{3ab^5}{x^{10}}+\frac{b^6}{x^8}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)} - \frac{b^3\sqrt{a^2+}}{7x^7}$$

Mathematica [A] time = 0.0134227, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(819a^2bx^2+231a^3+1001ab^2x^4+429b^3x^6\right)}{3003x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(231*a^3 + 819*a^2*b*x^2 + 1001*a*b^2*x^4 + 429*b^3*x
^6))/(3003*x^13*(a + b*x^2))
```

Maple [A] time = 0.164, size = 58, normalized size = 0.4

$$-\frac{429\,b^3x^6+1001\,ax^4b^2+819\,a^2bx^2+231\,a^3}{3003\,x^{13}\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x)
```

```
[Out] -1/3003*(429*b^3*x^6+1001*a*b^2*x^4+819*a^2*b*x^2+231*a^3)*((b*x^2+a)^2)^(3
/2)/x^13/(b*x^2+a)^3
```

Maxima [A] time = 1.04246, size = 50, normalized size = 0.3

$$-\frac{429\,b^3x^6 + 1001\,ab^2x^4 + 819\,a^2bx^2 + 231\,a^3}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/3003*(429*b^3*x^6 + 1001*a*b^2*x^4 + 819*a^2*b*x^2 + 231*a^3)/x^13
```

Fricas [A] time = 1.46536, size = 96, normalized size = 0.57

$$-\frac{429\,b^3x^6 + 1001\,ab^2x^4 + 819\,a^2bx^2 + 231\,a^3}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/3003*(429*b^3*x^6 + 1001*a*b^2*x^4 + 819*a^2*b*x^2 + 231*a^3)/x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^{14}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**14, x)
```

Giac [A] time = 1.09864, size = 93, normalized size = 0.56

$$-\frac{429\,b^3x^6\mathrm{sgn}\left(bx^2 + a\right) + 1001\,ab^2x^4\mathrm{sgn}\left(bx^2 + a\right) + 819\,a^2bx^2\mathrm{sgn}\left(bx^2 + a\right) + 231\,a^3\mathrm{sgn}\left(bx^2 + a\right)}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/3003*(429*b^3*x^6*sgn(b*x^2 + a) + 1001*a*b^2*x^4*sgn(b*x^2 + a) + 819*a
^2*b*x^2*sgn(b*x^2 + a) + 231*a^3*sgn(b*x^2 + a))/x^13
```

3.584 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{16}} dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(9*x^9*(a + b*x^2))
```

Rubi [A] time = 0.0411472, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```


```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(9*x^9*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{16}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{16}}+\frac{3a^2b^4}{x^{14}}+\frac{3ab^5}{x^{12}}+\frac{b^6}{x^{10}}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{b^3\sqrt{a^2}}{9x}$$

Mathematica [A] time = 0.0141802, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(1485a^2bx^2+429a^3+1755ab^2x^4+715b^3x^6\right)}{6435x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(429*a^3 + 1485*a^2*b*x^2 + 1755*a*b^2*x^4 + 715*b^3*
x^6))/(6435*x^15*(a + b*x^2))
```

Maple [A] time = 0.163, size = 58, normalized size = 0.4

$$-\frac{715\,b^3x^6+1755\,ax^4b^2+1485\,a^2bx^2+429\,a^3}{6435\,x^{15}\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x)
```

```
[Out] -1/6435*(715*b^3*x^6+1755*a*b^2*x^4+1485*a^2*b*x^2+429*a^3)*((b*x^2+a)^2)^(
3/2)/x^15/(b*x^2+a)^3
```

Maxima [A] time = 0.997737, size = 50, normalized size = 0.3

$$-\frac{715\,b^3x^6+1755\,ab^2x^4+1485\,a^2bx^2+429\,a^3}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/6435*(715*b^3*x^6 + 1755*a*b^2*x^4 + 1485*a^2*b*x^2 + 429*a^3)/x^15
```

Fricas [A] time = 1.49762, size = 97, normalized size = 0.58

$$-\frac{715\,b^3x^6+1755\,ab^2x^4+1485\,a^2bx^2+429\,a^3}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/6435*(715*b^3*x^6 + 1755*a*b^2*x^4 + 1485*a^2*b*x^2 + 429*a^3)/x^15
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**16, x)
```

Giac [A] time = 1.14869, size = 93, normalized size = 0.56

$$-\frac{715\,b^3x^6\operatorname{sgn}\left(bx^2+a\right)+1755\,ab^2x^4\operatorname{sgn}\left(bx^2+a\right)+1485\,a^2bx^2\operatorname{sgn}\left(bx^2+a\right)+429\,a^3\operatorname{sgn}\left(bx^2+a\right)}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/6435*(715*b^3*x^6*sgn(b*x^2 + a) + 1755*a*b^2*x^4*sgn(b*x^2 + a) + 1485*
a^2*b*x^2*sgn(b*x^2 + a) + 429*a^3*sgn(b*x^2 + a))/x^15
```

3.585 $\int x^{13} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{24}\sqrt{a^2+2abx^2+b^2x^4}}{24\left(a+bx^2\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*(a + b*x^2)) + (5*a^3*b^2*x^18*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(22*(a + b*x^2)) + (b^5*x^24*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*(a
 + b*x^2))
```

Rubi [A] time = 0.161685, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^5x^{24}\sqrt{a^2+2abx^2+b^2x^4}}{24\left(a+bx^2\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```



```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*(a + b*x^2)) + (5*a^3*b^2*x^18*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(22*(a + b*x^2)) + (b^5*x^24*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*(a
 + b*x^2))
```


Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int x^6\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int x^6\left(ab+b^2x\right)^5\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int\left(a^5b^5x^6+5a^4b^6x^7+10a^3b^7x^8+10a^2b^8x^9+5ab^9x^{10}+\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$=\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{5a^3b^2x^{18}\sqrt{a^2+2abx^2+b^2}}{9\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0246346, size = 83, normalized size = 0.33

$$\frac{x^{14}\sqrt{\left(a+bx^2\right)^2}\left(5544a^2b^3x^6+6160a^3b^2x^4+3465a^4bx^2+792a^5+2520ab^4x^8+462b^5x^{10}\right)}{11088\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^14*Sqrt[(a + b*x^2)^2]*(792*a^5 + 3465*a^4*b*x^2 + 6160*a^3*b^2*x^4 + 55
44*a^2*b^3*x^6 + 2520*a*b^4*x^8 + 462*b^5*x^10))/(11088*(a + b*x^2))
```

Maple [A] time = 0.164, size = 80, normalized size = 0.3

$$\frac{x^{14} \left(462 b^{5} x^{10}+2520 a b^{4} x^{8}+5544 a^{2} b^{3} x^{6}+6160 b^{2} a^{3} x^{4}+3465 a^{4} b x^{2}+792 a^{5}\right)}{11088 \left(b x^{2}+a\right)^{5}} \left(\left(b x^{2}+a\right)^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/11088*x^14*(462*b^5*x^10+2520*a*b^4*x^8+5544*a^2*b^3*x^6+6160*a^3*b^2*x^4
+3465*a^4*b*x^2+792*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42539, size = 142, normalized size = 0.56

$$\frac{1}{24} b^{5} x^{24}+\frac{5}{22} a b^{4} x^{22}+\frac{1}{2} a^{2} b^{3} x^{20}+\frac{5}{9} a^{3} b^{2} x^{18}+\frac{5}{16} a^{4} b x^{16}+\frac{1}{14} a^{5} x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{13} \left(\left(a+b x^{2}\right)^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**13*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.11661, size = 142, normalized size = 0.56

$$\frac{1}{24}\,b^5x^{24}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{22}\,ab^4x^{22}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^2b^3x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^3b^2x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{16}\,a^4bx^{16}\mathrm{sgn}\left(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24*sgn(b*x^2 + a) + 5/22*a*b^4*x^22*sgn(b*x^2 + a) + 1/2*a^2*b^3
*x^20*sgn(b*x^2 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^2 + a) + 5/16*a^4*b*x^16*sg
n(b*x^2 + a) + 1/14*a^5*x^14*sgn(b*x^2 + a)
```

3.586 $\int x^{11} \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

```
[Out] (a^5*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*(a + b*x^2)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (5*a^2*b^3*x^18*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(4*(a + b*x^2)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*(a
+ b*x^2))
```

Rubi [A] time = 0.15898, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{b^5x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*(a + b*x^2)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (5*a^2*b^3*x^18*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(4*(a + b*x^2)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*(a
+ b*x^2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int x^5\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^5\left(ab+b^2x\right)^5\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(a^5b^5x^5+5a^4b^6x^6+10a^3b^7x^7+10a^2b^8x^8+5ab^9x^9+b\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$= \frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12\left(a+bx^2\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^2+b^2}}{8\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0206497, size = 83, normalized size = 0.33

$$\frac{x^{12}\sqrt{\left(a+bx^2\right)^2}\left(3080a^2b^3x^6+3465a^3b^2x^4+1980a^4bx^2+462a^5+1386ab^4x^8+252b^5x^{10}\right)}{5544\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^12*Sqrt[(a + b*x^2)^2]*(462*a^5 + 1980*a^4*b*x^2 + 3465*a^3*b^2*x^4 + 30
80*a^2*b^3*x^6 + 1386*a*b^4*x^8 + 252*b^5*x^10))/(5544*(a + b*x^2))
```

Maple [A] time = 0.163, size = 80, normalized size = 0.3

$$\frac{x^{12}\left(252\,b^{5}x^{10}+1386\,a b^{4}x^{8}+3080\,a^{2}b^{3}x^{6}+3465\,b^{2}a^{3}x^{4}+1980\,a^{4}b x^{2}+462\,a^{5}\right)}{5544\left(b x^{2}+a\right)^{5}}\left(\left(b x^{2}+a\right)^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/5544*x^12*(252*b^5*x^10+1386*a*b^4*x^8+3080*a^2*b^3*x^6+3465*a^3*b^2*x^4+
1980*a^4*b*x^2+462*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46215, size = 140, normalized size = 0.55

$$\frac{1}{22}\,b^{5}x^{22}+\frac{1}{4}\,a b^{4}x^{20}+\frac{5}{9}\,a^{2}b^{3}x^{18}+\frac{5}{8}\,a^{3}b^{2}x^{16}+\frac{5}{14}\,a^{4}b x^{14}+\frac{1}{12}\,a^{5}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{11}\left(\left(a+b x^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**11*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.12745, size = 142, normalized size = 0.56

$$\frac{1}{22}\,b^5x^{22}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,ab^4x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^2b^3x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^3b^2x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{14}\,a^4bx^{14}\mathrm{sgn}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22*sgn(b*x^2 + a) + 1/4*a*b^4*x^20*sgn(b*x^2 + a) + 5/9*a^2*b^3*
x^18*sgn(b*x^2 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^2 + a) + 5/14*a^4*b*x^14*sgn
(b*x^2 + a) + 1/12*a^5*x^12*sgn(b*x^2 + a)
```

3.587 $\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=201

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^9}{20b^5}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{8b^5}-\frac{2a^3\sqrt{a^2+2}}{}$$

```
[Out] (a^4*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^5) - (2*a^3*(a +
b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^5) + (3*a^2*(a + b*x^2)^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^5) - (2*a*(a + b*x^2)^8*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(9*b^5) + ((a + b*x^2)^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(20*b^5)
```

Rubi [A] time = 0.131525, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1111, 645}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^9}{20b^5}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{8b^5}-\frac{2a^3\sqrt{a^2+2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (a^4*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^5) - (2*a^3*(a +
b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^5) + (3*a^2*(a + b*x^2)^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^5) - (2*a*(a + b*x^2)^8*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(9*b^5) + ((a + b*x^2)^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(20*b^5)
```


Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
```

```
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rubi steps

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{2}\operatorname{Subst}\left(\int x^4 \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(\frac{a^4\left(ab+b^2x\right)^5}{b^4} - \frac{4a^3\left(ab+b^2x\right)^6}{b^5} + \frac{6a^2\left(ab+b^2x\right)^7}{b^6} - \frac{4a\left(ab+b^2x\right)^8}{b^7} + \frac{(a\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{a^4\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^5} - \frac{2a^3\left(a+bx^2\right)^6\sqrt{a^2+2abx^2+b^2x^4}}{7b^5} + \frac{3a^2\left(a+bx^2\right.}{}$$

Mathematica [A] time = 0.0195426, size = 83, normalized size = 0.41

$$\frac{x^{10}\sqrt{\left(a+bx^2\right)^2}\left(1575a^2b^3x^6+1800a^3b^2x^4+1050a^4bx^2+252a^5+700ab^4x^8+126b^5x^{10}\right)}{2520\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(252*a^5 + 1050*a^4*b*x^2 + 1800*a^3*b^2*x^4 + 15
75*a^2*b^3*x^6 + 700*a*b^4*x^8 + 126*b^5*x^10))/(2520*(a + b*x^2))
```

Maple [A] time = 0.162, size = 80, normalized size = 0.4

$$\frac{x^{10}\left(126\,b^5x^{10}+700\,ab^4x^8+1575\,a^2b^3x^6+1800\,b^2a^3x^4+1050\,a^4bx^2+252\,a^5\right)}{2520\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/2520*x^10*(126*b^5*x^10+700*a*b^4*x^8+1575*a^2*b^3*x^6+1800*a^3*b^2*x^4+1
050*a^4*b*x^2+252*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51279, size = 142, normalized size = 0.71

$$\frac{1}{20}\,b^5x^{20} + \frac{5}{18}\,ab^4x^{18} + \frac{5}{8}\,a^2b^3x^{16} + \frac{5}{7}\,a^3b^2x^{14} + \frac{5}{12}\,a^4bx^{12} + \frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^9\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**9*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.11674, size = 142, normalized size = 0.71

$$\frac{1}{20}\,b^5x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{18}\,ab^4x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^2b^3x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,a^3b^2x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{12}\,a^4bx^{12}\mathrm{sgn}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20*sgn(b*x^2 + a) + 5/18*a*b^4*x^18*sgn(b*x^2 + a) + 5/8*a^2*b^3
*x^16*sgn(b*x^2 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^2 + a) + 5/12*a^4*b*x^12*sg
n(b*x^2 + a) + 1/10*a^5*x^10*sgn(b*x^2 + a)
```

3.588 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^8}{18b^4}-\frac{3a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^4}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{14b^4}-\frac{a^3\sqrt{a^2+2a}}{}$$

```
[Out] -(a^3*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^4) + (3*a^2*(a +
 b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*b^4) - (3*a*(a + b*x^2)^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*b^4) + ((a + b*x^2)^8*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(18*b^4)
```

Rubi [A] time = 0.118328, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^8}{18b^4}-\frac{3a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^4}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{14b^4}-\frac{a^3\sqrt{a^2+2a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] -(a^3*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^4) + (3*a^2*(a +
 b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*b^4) - (3*a*(a + b*x^2)^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*b^4) + ((a + b*x^2)^8*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(18*b^4)
```


Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
```

```
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{2}\,\text{Subst}\left(\int x^3 \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^3 \left(ab+b^2x\right)^5 dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(-\frac{a^3(ab+b^2x)^5}{b^3}+\frac{3a^2(ab+b^2x)^6}{b^4}-\frac{3a(ab+b^2x)^7}{b^5}+\frac{(ab+b^2x)^8}{b^6}\right) dx, x\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^4}+\frac{3a^2\left(a+bx^2\right)^6\sqrt{a^2+2abx^2+b^2x^4}}{14b^4}-\frac{3a\left(a+bx\right.}{}$$

Mathematica [A] time = 0.0215497, size = 83, normalized size = 0.52

$$\frac{x^8\sqrt{\left(a+bx^2\right)^2}\left(720a^2b^3x^6+840a^3b^2x^4+504a^4bx^2+126a^5+315ab^4x^8+56b^5x^{10}\right)}{1008\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(126*a^5 + 504*a^4*b*x^2 + 840*a^3*b^2*x^4 + 720*a
^2*b^3*x^6 + 315*a*b^4*x^8 + 56*b^5*x^10))/(1008*(a + b*x^2))
```

Maple [A] time = 0.162, size = 80, normalized size = 0.5

$$\frac{x^8\left(56\,b^5x^{10}+315\,ab^4x^8+720\,a^2b^3x^6+840\,b^2a^3x^4+504\,a^4bx^2+126\,a^5\right)}{1008\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/1008*x^8*(56*b^5*x^10+315*a*b^4*x^8+720*a^2*b^3*x^6+840*a^3*b^2*x^4+504*a
^4*b*x^2+126*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47461, size = 138, normalized size = 0.86

$$\frac{1}{18}\,b^5x^{18} + \frac{5}{16}\,ab^4x^{16} + \frac{5}{7}\,a^2b^3x^{14} + \frac{5}{6}\,a^3b^2x^{12} + \frac{1}{2}\,a^4bx^{10} + \frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**7*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.13432, size = 142, normalized size = 0.89

$$\frac{1}{18}b^5x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{16}ab^4x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}a^2b^3x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{6}a^3b^2x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}a^4bx^{10}\mathrm{sgn}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/18*b^5*x^18*sgn(b*x^2 + a) + 5/16*a*b^4*x^16*sgn(b*x^2 + a) + 5/7*a^2*b^3
*x^14*sgn(b*x^2 + a) + 5/6*a^3*b^2*x^12*sgn(b*x^2 + a) + 1/2*a^4*b*x^10*sgn
(b*x^2 + a) + 1/8*a^5*x^8*sgn(b*x^2 + a)
```

3.589 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{12b^3}$$

```
[Out] (a^2*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^3) - (a*(a + b*x^
2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^3) + ((a + b*x^2)^7*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(16*b^3)
```

Rubi [A] time = 0.0971745, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (a^2*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^3) - (a*(a + b*x^
2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^3) + ((a + b*x^2)^7*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(16*b^3)
```


Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2}dx,x,x^2\right)\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int x^2\left(ab+b^2x\right)^5dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^5}{b^2}-\frac{2a\left(ab+b^2x\right)^6}{b^3}+\frac{\left(ab+b^2x\right)^7}{b^4}\right)dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}\\ &= \frac{a^2\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^3}-\frac{a\left(a+bx^2\right)^6\sqrt{a^2+2abx^2+b^2x^4}}{7b^3}+\frac{\left(a+bx^2\right)^7\sqrt{a}}{1}\end{aligned}$$

Mathematica [A] time = 0.021695, size = 83, normalized size = 0.7

$$\frac{x^6\sqrt{\left(a+bx^2\right)^2}\left(280a^2b^3x^6+336a^3b^2x^4+210a^4bx^2+56a^5+120ab^4x^8+21b^5x^{10}\right)}{336\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(56*a^5 + 210*a^4*b*x^2 + 336*a^3*b^2*x^4 + 280*a^
2*b^3*x^6 + 120*a*b^4*x^8 + 21*b^5*x^10))/(336*(a + b*x^2))
```

Maple [A] time = 0.164, size = 80, normalized size = 0.7

$$\frac{x^6\left(21\,b^5x^{10}+120\,ab^4x^8+280\,a^2b^3x^6+336\,b^2a^3x^4+210\,a^4bx^2+56\,a^5\right)}{336\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/336*x^6*(21*b^5*x^10+120*a*b^4*x^8+280*a^2*b^3*x^6+336*a^3*b^2*x^4+210*a^
4*b*x^2+56*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44155, size = 131, normalized size = 1.1

$$\frac{1}{16}b^5x^{16} + \frac{5}{14}ab^4x^{14} + \frac{5}{6}a^2b^3x^{12} + a^3b^2x^{10} + \frac{5}{8}a^4bx^8 + \frac{1}{6}a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**5*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.13122, size = 140, normalized size = 1.18

$$\frac{1}{16}\,b^5x^{16}\text{sgn}\left(bx^2+a\right)+\frac{5}{14}\,ab^4x^{14}\text{sgn}\left(bx^2+a\right)+\frac{5}{6}\,a^2b^3x^{12}\text{sgn}\left(bx^2+a\right)+a^3b^2x^{10}\text{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^4bx^8\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16*sgn(b*x^2 + a) + 5/14*a*b^4*x^14*sgn(b*x^2 + a) + 5/6*a^2*b^3
*x^12*sgn(b*x^2 + a) + a^3*b^2*x^10*sgn(b*x^2 + a) + 5/8*a^4*b*x^8*sgn(b*x^
2 + a) + 1/6*a^5*x^6*sgn(b*x^2 + a)
```

3.590 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}$$

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b^2) + (a^2 + 2*a*b*
x^2 + b^2*x^4)^(7/2)/(14*b^2)
```

Rubi [A] time = 0.0516458, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 609}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b^2) + (a^2 + 2*a*b*
x^2 + b^2*x^4)^(7/2)/(14*b^2)
```


Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)\\&=\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a\operatorname{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)}{2b}\\&=-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}\end{aligned}$$

Mathematica [A] time = 0.021543, size = 83, normalized size = 1.24

$$\frac{x^4\sqrt{\left(a+bx^2\right)^2}\left(84a^2b^3x^6+105a^3b^2x^4+70a^4bx^2+21a^5+35ab^4x^8+6b^5x^{10}\right)}{84\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(21*a^5 + 70*a^4*b*x^2 + 105*a^3*b^2*x^4 + 84*a^2*
b^3*x^6 + 35*a*b^4*x^8 + 6*b^5*x^10))/(84*(a + b*x^2))
```

Maple [A] time = 0.165, size = 80, normalized size = 1.2

$$\frac{x^4\left(6\,b^5x^{10}+35\,ab^4x^8+84\,a^2b^3x^6+105\,b^2a^3x^4+70\,a^4bx^2+21\,a^5\right)}{84\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/84*x^4*(6*b^5*x^10+35*a*b^4*x^8+84*a^2*b^3*x^6+105*a^3*b^2*x^4+70*a^4*b*x
^2+21*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43888, size = 130, normalized size = 1.94

$$\frac{1}{14}\,b^{5}x^{14}+\frac{5}{12}\,ab^{4}x^{12}+a^{2}b^{3}x^{10}+\frac{5}{4}\,a^{3}b^{2}x^{8}+\frac{5}{6}\,a^{4}bx^{6}+\frac{1}{4}\,a^{5}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**3*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.17274, size = 90, normalized size = 1.34

$$\frac{1}{84}\left(6\,b^{5}x^{14}+35\,ab^{4}x^{12}+84\,a^{2}b^{3}x^{10}+105\,a^{3}b^{2}x^{8}+70\,a^{4}bx^{6}+21\,a^{5}x^{4}\right)\mathrm{sgn}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/84*(6*b^5*x^14 + 35*a*b^4*x^12 + 84*a^2*b^3*x^10 + 105*a^3*b^2*x^8 + 70*a
^4*b*x^6 + 21*a^5*x^4)*sgn(b*x^2 + a)
```

3.591 $\int x \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}$$

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b)
```

Rubi [A] time = 0.0258815, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 609}

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b)
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)\\ &= \frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}\end{aligned}$$

Mathematica [A] time = 0.0138417, size = 27, normalized size = 0.75

$$\frac{(a+bx^2)\left((a+bx^2)^2\right)^{5/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^(5/2))/(12*b)
```

Maple [B] time = 0.166, size = 79, normalized size = 2.2

$$\frac{x^2\left(b^5x^{10}+6\,ab^4x^8+15\,a^2b^3x^6+20\,a^3b^2x^4+15\,a^4bx^2+6\,a^5\right)}{12\,(bx^2+a)^5}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/12*x^2*(b^5*x^10+6*a*b^4*x^8+15*a^2*b^3*x^6+20*a^3*b^2*x^4+15*a^4*b*x^2+6
*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45982, size = 132, normalized size = 3.67

$$\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a^
4*b*x^4 + 1/2*a^5*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**2)**2)**(5/2), x)
```

Giac [B] time = 1.17384, size = 89, normalized size = 2.47

$$\frac{1}{12} \left(b^5 x^{12} + 6\, ab^4 x^{10} + 15\, a^2 b^3 x^8 + 20\, a^3 b^2 x^6 + 15\, a^4 b x^4 + 6\, a^5 x^2\right) \mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(b^5*x^12 + 6*a*b^4*x^10 + 15*a^2*b^3*x^8 + 20*a^3*b^2*x^6 + 15*a^4*b*
x^4 + 6*a^5*x^2)*sgn(b*x^2 + a)
```

3.592 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x} dx$

Optimal. Leaf size=251

$$\frac{b^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10(a+bx^2)}+\frac{5ab^4x^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+$$

```
[Out] (5*a^4*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a^3*b^2*
x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a^2*b^3*x^6*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(10*(a + b*x^2)) + (a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x
^2)
```

Rubi [A] time = 0.070981, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10(a+bx^2)}+\frac{5ab^4x^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```


```
[Out] (5*a^4*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a^3*b^2*
x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a^2*b^3*x^6*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(10*(a + b*x^2)) + (a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x
^2)
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(5a^4b^6+\frac{a^5b^5}{x}+10a^3b^7x+10a^2b^8x^2+5ab^9x^3+b^{10}x^4\right) d}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{5a^4bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2}}{3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0249113, size = 82, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(bx^2\left(200a^2b^2x^4+300a^3bx^2+300a^4+75ab^3x^6+12b^4x^8\right)+120a^5\log(x)\right)}{120\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2*(300*a^4 + 300*a^3*b*x^2 + 200*a^2*b^2*x^4 + 75
*a*b^3*x^6 + 12*b^4*x^8) + 120*a^5*Log[x]))/(120*(a + b*x^2))
```

Maple [A] time = 0.211, size = 79, normalized size = 0.3

$$\frac{12\,b^5x^{10}+75\,ab^4x^8+200\,a^2b^3x^6+300\,b^2a^3x^4+300\,a^4bx^2+120\,a^5\ln(x)}{120\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x)
```

```
[Out] 1/120*((b*x^2+a)^2)^(5/2)*(12*b^5*x^10+75*a*b^4*x^8+200*a^2*b^3*x^6+300*b^2
*a^3*x^4+300*a^4*b*x^2+120*a^5*ln(x))/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55215, size = 130, normalized size = 0.52

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x, x)
```

Giac [A] time = 1.12498, size = 143, normalized size = 0.57

$$\frac{1}{10}\,b^5x^{10}\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{3}\,a^2b^3x^6\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{2}\,a^3b^2x^4\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{2}\,a^4bx^2\mathrm{sgn}\left(bx^2 + a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sgn(b*x^2 + a) + 5/8*a*b^4*x^8*sgn(b*x^2 + a) + 5/3*a^2*b^3*x
^6*sgn(b*x^2 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^2 + a) + 5/2*a^4*b*x^2*sgn(b*x^
2 + a) + 1/2*a^5*log(x^2)*sgn(b*x^2 + a)
```

3.593 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^3*b^2*x^2
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a^2*b^3*x^4*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(6*(a + b*x^2)) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a
 + b*x^2)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0714932, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^5x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^3*b^2*x^2
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a^2*b^3*x^4*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(6*(a + b*x^2)) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a
 + b*x^2)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^3}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^2}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(10a^3b^7+\frac{a^5b^5}{x^2}+\frac{5a^4b^6}{x}+10a^2b^8x+5ab^9x^2+b^{10}x^3\right)dx,x\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0271085, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(60a^2b^3x^6+120a^3b^2x^4+120a^4bx^2\log(x)-12a^5+20ab^4x^8+3b^5x^{10}\right)}{24x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-12*a^5 + 120*a^3*b^2*x^4 + 60*a^2*b^3*x^6 + 20*a*b^4
*x^8 + 3*b^5*x^10 + 120*a^4*b*x^2*Log[x]))/(24*x^2*(a + b*x^2))
```

Maple [A] time = 0.214, size = 82, normalized size = 0.3

$$\frac{3\,b^{5}x^{10}+20\,ab^{4}x^{8}+60\,a^{2}b^{3}x^{6}+120\,b^{2}a^{3}x^{4}+120\,a^{4}b\ln\left(x\right)x^{2}-12\,a^{5}}{24\,\left(bx^{2}+a\right)^{5}x^{2}}\left(\left(bx^{2}+a\right)^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x)
```

```
[Out] 1/24*((b*x^2+a)^2)^(5/2)*(3*b^5*x^10+20*a*b^4*x^8+60*a^2*b^3*x^6+120*b^2*a^
3*x^4+120*a^4*b*ln(x)*x^2-12*a^5)/(b*x^2+a)^5/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51343, size = 142, normalized size = 0.57

$$\frac{3\,b^{5}x^{10}+20\,ab^{4}x^{8}+60\,a^{2}b^{3}x^{6}+120\,a^{3}b^{2}x^{4}+120\,a^{4}bx^{2}\log\left(x\right)-12\,a^{5}}{24\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a^
4*b*x^2*log(x) - 12*a^5)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**3, x)
```

Giac [A] time = 1.12865, size = 169, normalized size = 0.68

$$\frac{1}{8}b^5x^8\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{6}ab^4x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}a^2b^3x^4\mathrm{sgn}\left(bx^2+a\right)+5\,a^3b^2x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}a^4b\log\left(x^2\right)\mathrm{sgn}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sgn(b*x^2 + a) + 5/6*a*b^4*x^6*sgn(b*x^2 + a) + 5/2*a^2*b^3*x^4
*sgn(b*x^2 + a) + 5*a^3*b^2*x^2*sgn(b*x^2 + a) + 5/2*a^4*b*log(x^2)*sgn(b*x
^2 + a) - 1/2*(5*a^4*b*x^2*sgn(b*x^2 + a) + a^5*sgn(b*x^2 + a))/x^2
```

3.594 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^5} dx$

Optimal. Leaf size=250

$$\frac{b^5x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{5ab^4x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}-\frac{a^5\sqrt{\ldots}}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^2*b^3*x^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(4*(a + b*x^2)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a +
 b*x^2)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0700548, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$\frac{b^5x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{5ab^4x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}-\frac{a^5\sqrt{\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^2*b^3*x^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(4*(a + b*x^2)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a +
 b*x^2)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^5}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^3}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(10a^2b^8+\frac{a^5b^5}{x^3}+\frac{5a^4b^6}{x^2}+\frac{10a^3b^7}{x}+5ab^9x+b^{10}x^2\right)dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)} + \frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{5 \ldots}{\ldots}$$

Mathematica [A] time = 0.0264782, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(60a^2b^3x^6+120a^3b^2x^4\log(x)-30a^4bx^2-3a^5+15ab^4x^8+2b^5x^{10}\right)}{12x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 30*a^4*b*x^2 + 60*a^2*b^3*x^6 + 15*a*b^4*x^8
 + 2*b^5*x^10 + 120*a^3*b^2*x^4*Log[x]))/(12*x^4*(a + b*x^2))
```

Maple [A] time = 0.213, size = 82, normalized size = 0.3

$$\frac{2\,b^{5}x^{10}+15\,ab^{4}x^{8}+60\,a^{2}b^{3}x^{6}+120\,b^{2}a^{3}\ln\left(x\right)x^{4}-30\,a^{4}bx^{2}-3\,a^{5}}{12\,\left(bx^{2}+a\right)^{5}x^{4}}\left(\left(bx^{2}+a\right)^{2}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(5/2)*(2*b^5*x^10+15*a*b^4*x^8+60*a^2*b^3*x^6+120*b^2*a^
3*ln(x)*x^4-30*a^4*b*x^2-3*a^5)/(b*x^2+a)^5/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46848, size = 139, normalized size = 0.56

$$\frac{2\,b^{5}x^{10}+15\,ab^{4}x^{8}+60\,a^{2}b^{3}x^{6}+120\,a^{3}b^{2}x^{4}\log\left(x\right)-30\,a^{4}bx^{2}-3\,a^{5}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*log(x) -
 30*a^4*b*x^2 - 3*a^5)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**5, x)
```

Giac [A] time = 1.13494, size = 171, normalized size = 0.68

$$\frac{1}{6}b^5x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{4}ab^4x^4\mathrm{sgn}\left(bx^2+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^2+a\right)+5\,a^3b^2\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{30\,a^3b^2x^4\mathrm{sgn}\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sgn(b*x^2 + a) + 5/4*a*b^4*x^4*sgn(b*x^2 + a) + 5*a^2*b^3*x^2*s
gn(b*x^2 + a) + 5*a^3*b^2*log(x^2)*sgn(b*x^2 + a) - 1/4*(30*a^3*b^2*x^4*sgn
(b*x^2 + a) + 10*a^4*b*x^2*sgn(b*x^2 + a) + a^5*sgn(b*x^2 + a))/x^4
```

3.595 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=250

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{b^5x^4\sqrt{a\ldots}}{4\ldots}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(2*(a + b*x^2)) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a
 + b*x^2)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2
)
```

Rubi [A] time = 0.0724194, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{b^5x^4\sqrt{a\ldots}}{4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(2*(a + b*x^2)) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a
 + b*x^2)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2
)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^7}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^4}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(5ab^9+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^3}+\frac{10a^3b^7}{x^2}+\frac{10a^2b^8}{x}+b^{10}x\right)dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}+\frac{5ab}{}$$

Mathematica [A] time = 0.0243038, size = 85, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-60a^3b^2x^4+120a^2b^3x^6\log(x)-15a^4bx^2-2a^5+30ab^4x^8+3b^5x^{10}\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-2*a^5 - 15*a^4*b*x^2 - 60*a^3*b^2*x^4 + 30*a*b^4*x^8
 + 3*b^5*x^10 + 120*a^2*b^3*x^6*Log[x]))/(12*x^6*(a + b*x^2))
```

Maple [A] time = 0.212, size = 82, normalized size = 0.3

$$\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3\ln\left(x\right)x^6-60\,b^2a^3x^4-15\,a^4bx^2-2\,a^5}{12\,\left(bx^2+a\right)^5x^6}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(5/2)*(3*b^5*x^10+30*a*b^4*x^8+120*a^2*b^3*ln(x)*x^6-60*
b^2*a^3*x^4-15*a^4*b*x^2-2*a^5)/(b*x^2+a)^5/x^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5152, size = 139, normalized size = 0.56

$$\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3x^6\log\left(x\right)-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3*b^2*x^4 -
 15*a^4*b*x^2 - 2*a^5)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**7, x)
```

Giac [A] time = 1.12269, size = 173, normalized size = 0.69

$$\frac{1}{4}\,b^5x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,ab^4x^2\mathrm{sgn}\left(bx^2+a\right)+5\,a^2b^3\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{110\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+60\,a^3b^2x^4\mathrm{sgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sgn(b*x^2 + a) + 5/2*a*b^4*x^2*sgn(b*x^2 + a) + 5*a^2*b^3*log(x
^2)*sgn(b*x^2 + a) - 1/12*(110*a^2*b^3*x^6*sgn(b*x^2 + a) + 60*a^3*b^2*x^4*
sgn(b*x^2 + a) + 15*a^4*b*x^2*sgn(b*x^2 + a) + 2*a^5*sgn(b*x^2 + a))/x^6
```

3.596 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=250

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}+\frac{b^5x^2\sqrt{a^2}}{2(}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(x^2*(a + b*x^2)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*
(a + b*x^2)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0707543, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}+\frac{b^5x^2\sqrt{a^2}}{2(}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(x^2*(a + b*x^2)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*
(a + b*x^2)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^9}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^9}\,dx}{b^4\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^5}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(b^{10}+\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^3}+\frac{10a^2b^8}{x^2}+\frac{5ab^9}{x}\right)dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}\\
&= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}-\frac{5a^2b
\end{aligned}$$

Mathematica [A] time = 0.0196025, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(120a^2b^3x^6+60a^3b^2x^4+20a^4bx^2+3a^5-120ab^4x^8\log(x)-12b^5x^{10}\right)}{24x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(3*a^5 + 20*a^4*b*x^2 + 60*a^3*b^2*x^4 + 120*a^2*b^3*
x^6 - 12*b^5*x^10 - 120*a*b^4*x^8*Log[x]))/(24*x^8*(a + b*x^2))
```

Maple [A] time = 0.208, size = 82, normalized size = 0.3

$$\frac{12\,b^5x^{10}+120\,ab^4\ln(x)\,x^8-120\,a^2b^3x^6-60\,b^2a^3x^4-20\,a^4bx^2-3\,a^5}{24\,\left(bx^2+a\right)^5x^8}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x)
```

```
[Out] 1/24*((b*x^2+a)^2)^(5/2)*(12*b^5*x^10+120*a*b^4*ln(x)*x^8-120*a^2*b^3*x^6-6
0*b^2*a^3*x^4-20*a^4*b*x^2-3*a^5)/(b*x^2+a)^5/x^8
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46922, size = 142, normalized size = 0.57

$$\frac{12\,b^5x^{10}+120\,ab^4x^8\log(x)-120\,a^2b^3x^6-60\,a^3b^2x^4-20\,a^4bx^2-3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^4
 - 20*a^4*b*x^2 - 3*a^5)/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**9, x)
```

Giac [A] time = 1.14089, size = 170, normalized size = 0.68

$$\frac{1}{2}\,b^5x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,ab^4\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{125\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+120\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+60\,a^3b^2x^4\mathrm{sgn}}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sgn(b*x^2 + a) + 5/2*a*b^4*log(x^2)*sgn(b*x^2 + a) - 1/24*(125*
a*b^4*x^8*sgn(b*x^2 + a) + 120*a^2*b^3*x^6*sgn(b*x^2 + a) + 60*a^3*b^2*x^4*
sgn(b*x^2 + a) + 20*a^4*b*x^2*sgn(b*x^2 + a) + 3*a^5*sgn(b*x^2 + a))/x^8
```

3.597 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}} dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^4(a+bx^2)}-\frac{5ab^4\sqrt{a^2}}{2x^2}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(3*x^6*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(2*x^2*(a + b*x^2)) + (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*
x^2)
```

Rubi [A] time = 0.0682964, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^4(a+bx^2)}-\frac{5ab^4\sqrt{a^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(3*x^6*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(2*x^2*(a + b*x^2)) + (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*
x^2)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{11}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{11}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^6}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^3}+\frac{5ab^9}{x^2}+\frac{b^{10}}{x}\right)dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6\left(a+bx^2\right)} - \frac{5a^2b}{}$$

Mathematica [A] time = 0.0279921, size = 85, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a\left(200a^2b^2x^4+75a^3bx^2+12a^4+300ab^3x^6+300b^4x^8\right)-120b^5x^{10}\log(x)\right)}{120x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a*(12*a^4 + 75*a^3*b*x^2 + 200*a^2*b^2*x^4 + 300*a*b
^3*x^6 + 300*b^4*x^8) - 120*b^5*x^10*Log[x]))/(120*x^10*(a + b*x^2))
```

Maple [A] time = 0.212, size = 82, normalized size = 0.3

$$\frac{120\, b^5 \ln(x)\, x^{10} - 300\, a b^4 x^8 - 300\, a^2 b^3 x^6 - 200\, b^2 a^3 x^4 - 75\, a^4 b x^2 - 12\, a^5}{120\, \left(b x^2 + a\right)^5 x^{10}} \left(\left(b x^2 + a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x)
```

```
[Out] 1/120*((b*x^2+a)^2)^(5/2)*(120*b^5*ln(x)*x^10-300*a*b^4*x^8-300*a^2*b^3*x^6
-200*b^2*a^3*x^4-75*a^4*b*x^2-12*a^5)/(b*x^2+a)^5/x^10
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48737, size = 149, normalized size = 0.59

$$\frac{120\, b^5 x^{10} \log(x) - 300\, a b^4 x^8 - 300\, a^2 b^3 x^6 - 200\, a^3 b^2 x^4 - 75\, a^4 b x^2 - 12\, a^5}{120\, x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2*
x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**11, x)
```

Giac [A] time = 1.15287, size = 169, normalized size = 0.67

$$\frac{1}{2}\, b^5 \log\left(x^2\right) \operatorname{sgn}\left(bx^2 + a\right) - \frac{137\, b^5 x^{10} \operatorname{sgn}\left(bx^2 + a\right) + 300\, ab^4 x^8 \operatorname{sgn}\left(bx^2 + a\right) + 300\, a^2 b^3 x^6 \operatorname{sgn}\left(bx^2 + a\right) + 200\, a^3 b^2 x^4}{120\, x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(x^2)*sgn(b*x^2 + a) - 1/120*(137*b^5*x^10*sgn(b*x^2 + a) + 300*
a*b^4*x^8*sgn(b*x^2 + a) + 300*a^2*b^3*x^6*sgn(b*x^2 + a) + 200*a^3*b^2*x^4
*sgn(b*x^2 + a) + 75*a^4*b*x^2*sgn(b*x^2 + a) + 12*a^5*sgn(b*x^2 + a))/x^10
```

3.598 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*a*x^12)
```

Rubi [A] time = 0.0392379, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$-\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*a*x^12)
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^7}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^7}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$=-\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

Mathematica [A] time = 0.0179195, size = 81, normalized size = 1.98

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(20a^2b^3x^6+15a^3b^2x^4+6a^4bx^2+a^5+15ab^4x^8+6b^5x^{10}\right)}{12x^{12}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a^5 + 6*a^4*b*x^2 + 15*a^3*b^2*x^4 + 20*a^2*b^3*x^6
+ 15*a*b^4*x^8 + 6*b^5*x^10))/(12*x^12*(a + b*x^2))
```

Maple [B] time = 0.159, size = 78, normalized size = 1.9

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,b^2a^3x^4+6\,a^4bx^2+a^5}{12\,x^{12}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x)
```

```
[Out] -1/12*(6*b^5*x^10+15*a*b^4*x^8+20*a^2*b^3*x^6+15*a^3*b^2*x^4+6*a^4*b*x^2+a^
5)*((b*x^2+a)^2)^(5/2)/x^12/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.47602, size = 127, normalized size = 3.1

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**13, x)
```

Giac [B] time = 1.11985, size = 143, normalized size = 3.49

$$-\frac{6\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+15\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+20\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+15\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+6\,a^4bx^2\mathrm{sgn}\left(bx^2+\right.}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*x^10*sgn(b*x^2 + a) + 15*a*b^4*x^8*sgn(b*x^2 + a) + 20*a^2*b^3
*x^6*sgn(b*x^2 + a) + 15*a^3*b^2*x^4*sgn(b*x^2 + a) + 6*a^4*b*x^2*sgn(b*x^2
 + a) + a^5*sgn(b*x^2 + a))/x^12
```

3.599 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{15}}\,dx$

Optimal. Leaf size=72

$$\frac{(a^2+2abx^2+b^2x^4)^{7/2}}{84a^2x^{14}}-\frac{(a+bx^2)(a^2+2abx^2+b^2x^4)^{5/2}}{12ax^{14}}$$

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*a*x^14) + (a^2 + 2*a*b
*x^2 + b^2*x^4)^(7/2)/(84*a^2*x^14)
```

Rubi [A] time = 0.0168661, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1110}

$$\frac{(a^2+2abx^2+b^2x^4)^{7/2}}{84a^2x^{14}}-\frac{(a+bx^2)(a^2+2abx^2+b^2x^4)^{5/2}}{12ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*a*x^14) + (a^2 + 2*a*b
*x^2 + b^2*x^4)^(7/2)/(84*a^2*x^14)
```

Rule 1110

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(4*a*d*(p + 1)*(2*p +
1)), x] - Simp[((d*x)^(m + 1)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^p)/(4*a*d*(
2*p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{15}}\,dx=-\frac{(a+bx^2)(a^2+2abx^2+b^2x^4)^{5/2}}{12ax^{14}}+\frac{(a^2+2abx^2+b^2x^4)^{7/2}}{84a^2x^{14}}$$

Mathematica [A] time = 0.0197089, size = 83, normalized size = 1.15

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(105a^2b^3x^6+84a^3b^2x^4+35a^4bx^2+6a^5+70ab^4x^8+21b^5x^{10}\right)}{84x^{14}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(6*a^5 + 35*a^4*b*x^2 + 84*a^3*b^2*x^4 + 105*a^2*b^3*
x^6 + 70*a*b^4*x^8 + 21*b^5*x^10))/(84*x^14*(a + b*x^2))
```

Maple [A] time = 0.173, size = 80, normalized size = 1.1

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,b^2a^3x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x)
```

```
[Out] -1/84*(21*b^5*x^10+70*a*b^4*x^8+105*a^2*b^3*x^6+84*a^3*b^2*x^4+35*a^4*b*x^2
+6*a^5)*((b*x^2+a)^2)^(5/2)/x^14/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47869, size = 134, normalized size = 1.86

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**15, x)
```

Giac [A] time = 1.11714, size = 144, normalized size = 2.

$$-\frac{21\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+70\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+105\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+84\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+35\,a^4bx^2\mathrm{sgn}\left(bx^2\right.}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(21*b^5*x^10*sgn(b*x^2 + a) + 70*a*b^4*x^8*sgn(b*x^2 + a) + 105*a^2*b
^3*x^6*sgn(b*x^2 + a) + 84*a^3*b^2*x^4*sgn(b*x^2 + a) + 35*a^4*b*x^2*sgn(b*
x^2 + a) + 6*a^5*sgn(b*x^2 + a))/x^14
```

3.600 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{17}}\,dx$

Optimal. Leaf size=128

$$-\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{336a^3x^{12}}+\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{56a^2x^{14}}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{16ax^{16}}$$

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*a*x^16) + (b*(a + b*x^
2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(56*a^2*x^14) - (b^2*(a + b*x^2)^5*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(336*a^3*x^12)
```

Rubi [A] time = 0.0911089, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1111, 646, 45, 37}

$$-\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{336a^3x^{12}}+\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{56a^2x^{14}}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*a*x^16) + (b*(a + b*x^
2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(56*a^2*x^14) - (b^2*(a + b*x^2)^5*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(336*a^3*x^12)
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{17}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^9}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^9}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}\\
&= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} - \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^8}\,dx, x, x^2\right)}{8ab^3\left(ab+b^2x^2\right)}\\
&= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} + \frac{b\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{56a^2x^{14}} + \frac{\sqrt{a^2+2abx^2+b^2}\cdots}{56\cdots}\\
&= -\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} + \frac{b\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{56a^2x^{14}} - \frac{b^2\left(a+bx^2\right)^5\sqrt{a^2}\cdots}{336a^3\cdots}
\end{aligned}$$

Mathematica [A] time = 0.0177042, size = 83, normalized size = 0.65

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(336a^2b^3x^6+280a^3b^2x^4+120a^4bx^2+21a^5+210ab^4x^8+56b^5x^{10}\right)}{336x^{16}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(21*a^5 + 120*a^4*b*x^2 + 280*a^3*b^2*x^4 + 336*a^2*b
^3*x^6 + 210*a*b^4*x^8 + 56*b^5*x^10))/(336*x^16*(a + b*x^2))
```

Maple [A] time = 0.165, size = 80, normalized size = 0.6

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,b^2a^3x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x)
```

```
[Out] -1/336*(56*b^5*x^10+210*a*b^4*x^8+336*a^2*b^3*x^6+280*a^3*b^2*x^4+120*a^4*b
*x^2+21*a^5)*((b*x^2+a)^2)^(5/2)/x^16/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.488, size = 140, normalized size = 1.09

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**17, x)
```

Giac [A] time = 1.1318, size = 144, normalized size = 1.12

$$-\frac{56\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+210\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+336\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+280\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+120\,a^4bx^2\mathrm{sgn}}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(56*b^5*x^10*sgn(b*x^2 + a) + 210*a*b^4*x^8*sgn(b*x^2 + a) + 336*a^2
*b^3*x^6*sgn(b*x^2 + a) + 280*a^3*b^2*x^4*sgn(b*x^2 + a) + 120*a^4*b*x^2*sg
n(b*x^2 + a) + 21*a^5*sgn(b*x^2 + a))/x^16
```

3.601 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{19}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2}}{2x}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(6*x^12*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(2*x^10*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a +
 b*x^2))
```

Rubi [A] time = 0.15505, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(6*x^12*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(2*x^10*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a +
 b*x^2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{19}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{10}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{10}}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^{10}}+\frac{5a^4b^6}{x^9}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^7}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^5}\right)dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)} - \frac{5a^2b^3}{}$$

Mathematica [A] time = 0.0187703, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(840a^2b^3x^6+720a^3b^2x^4+315a^4bx^2+56a^5+504ab^4x^8+126b^5x^{10}\right)}{1008x^{18}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(56*a^5 + 315*a^4*b*x^2 + 720*a^3*b^2*x^4 + 840*a^2*b
^3*x^6 + 504*a*b^4*x^8 + 126*b^5*x^10))/(1008*x^18*(a + b*x^2))
```

Maple [A] time = 0.163, size = 80, normalized size = 0.3

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,b^2a^3x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x)
```

```
[Out] -1/1008*(126*b^5*x^10+504*a*b^4*x^8+840*a^2*b^3*x^6+720*a^3*b^2*x^4+315*a^4
*b*x^2+56*a^5)*((b*x^2+a)^2)^(5/2)/x^18/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64649, size = 143, normalized size = 0.56

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{19}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**19,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**19, x)
```

Giac [A] time = 1.12298, size = 144, normalized size = 0.56

$$-\frac{126\, b^5x^{10}\text{sgn}\left(bx^2 + a\right) + 504\, ab^4x^8\text{sgn}\left(bx^2 + a\right) + 840\, a^2b^3x^6\text{sgn}\left(bx^2 + a\right) + 720\, a^3b^2x^4\text{sgn}\left(bx^2 + a\right) + 315\, a^4bx^2\text{sgn}}{1008\, x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10*sgn(b*x^2 + a) + 504*a*b^4*x^8*sgn(b*x^2 + a) + 840*a
^2*b^3*x^6*sgn(b*x^2 + a) + 720*a^3*b^2*x^4*sgn(b*x^2 + a) + 315*a^4*b*x^2*
sgn(b*x^2 + a) + 56*a^5*sgn(b*x^2 + a))/x^18
```

3.602 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{21}} dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a}}{12}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(20*x^20*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(8*x^16*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(12*x^12*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10
*(a + b*x^2))
```

Rubi [A] time = 0.151046, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(20*x^20*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(8*x^16*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(12*x^12*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10
*(a + b*x^2))
```


Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{21}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{11}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{11}}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^{11}}+\frac{5a^4b^6}{x^{10}}+\frac{10a^3b^7}{x^9}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^7}+\frac{b^{10}}{x^6}\right)dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)} - \frac{5a^2b^3}{}$$

Mathematica [A] time = 0.0180073, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(1800a^2b^3x^6+1575a^3b^2x^4+700a^4bx^2+126a^5+1050ab^4x^8+252b^5x^{10}\right)}{2520x^{20}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(126*a^5 + 700*a^4*b*x^2 + 1575*a^3*b^2*x^4 + 1800*a^
2*b^3*x^6 + 1050*a*b^4*x^8 + 252*b^5*x^10))/(2520*x^20*(a + b*x^2))
```

Maple [A] time = 0.167, size = 80, normalized size = 0.3

$$-\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,b^2a^3x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x)
```

```
[Out] -1/2520*(252*b^5*x^10+1050*a*b^4*x^8+1800*a^2*b^3*x^6+1575*a^3*b^2*x^4+700*
a^4*b*x^2+126*a^5)*((b*x^2+a)^2)^(5/2)/x^20/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81811, size = 149, normalized size = 0.58

$$-\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{21}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**21,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**21, x)
```

Giac [A] time = 1.13888, size = 144, normalized size = 0.56

$$-\frac{252\, b^5 x^{10} \mathrm{sgn}\left(bx^2+a\right) + 1050\, ab^4 x^8 \mathrm{sgn}\left(bx^2+a\right) + 1800\, a^2 b^3 x^6 \mathrm{sgn}\left(bx^2+a\right) + 1575\, a^3 b^2 x^4 \mathrm{sgn}\left(bx^2+a\right) + 700\, a^4 b x^2}{2520\, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="giac")
```

```
[Out] -1/2520*(252*b^5*x^10*sgn(b*x^2 + a) + 1050*a*b^4*x^8*sgn(b*x^2 + a) + 1800
*a^2*b^3*x^6*sgn(b*x^2 + a) + 1575*a^3*b^2*x^4*sgn(b*x^2 + a) + 700*a^4*b*x
^2*sgn(b*x^2 + a) + 126*a^5*sgn(b*x^2 + a))/x^20
```

3.603 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{23}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2}}{14x}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^20*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(8*x^16*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(14*x^14*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a
 + b*x^2))
```

Rubi [A] time = 0.153309, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2}}{14x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^20*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(8*x^16*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(14*x^14*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a
 + b*x^2))
```


Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{23}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{12}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{12}}\,dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^{11}}+\frac{10a^3b^7}{x^{10}}+\frac{10a^2b^8}{x^9}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^7}\right) dx, x, x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)} - \frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}\left(a+bx^2\right)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)} - \frac{5a^2b^3\sqrt{}}{8}$$

Mathematica [A] time = 0.0175634, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3465a^2b^3x^6+3080a^3b^2x^4+1386a^4bx^2+252a^5+1980ab^4x^8+462b^5x^{10}\right)}{5544x^{22}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(252*a^5 + 1386*a^4*b*x^2 + 3080*a^3*b^2*x^4 + 3465*a
^2*b^3*x^6 + 1980*a*b^4*x^8 + 462*b^5*x^10))/(5544*x^22*(a + b*x^2))
```

Maple [A] time = 0.163, size = 80, normalized size = 0.3

$$-\frac{462\,b^5x^{10}+1980\,ab^4x^8+3465\,a^2b^3x^6+3080\,b^2a^3x^4+1386\,a^4bx^2+252\,a^5}{5544\,x^{22}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x)
```

```
[Out] -1/5544*(462*b^5*x^10+1980*a*b^4*x^8+3465*a^2*b^3*x^6+3080*a^3*b^2*x^4+1386
*a^4*b*x^2+252*a^5)*((b*x^2+a)^2)^(5/2)/x^22/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.25197, size = 150, normalized size = 0.59

$$-\frac{462\,b^5x^{10}+1980\,ab^4x^8+3465\,a^2b^3x^6+3080\,a^3b^2x^4+1386\,a^4bx^2+252\,a^5}{5544\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/5544*(462*b^5*x^10 + 1980*a*b^4*x^8 + 3465*a^2*b^3*x^6 + 3080*a^3*b^2*x^
4 + 1386*a^4*b*x^2 + 252*a^5)/x^22
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{23}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**23,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**23, x)
```

Giac [A] time = 1.12091, size = 144, normalized size = 0.56

$$-\frac{462\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+1980\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+3465\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+3080\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+1386\,a^4bx}{5544\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/5544*(462*b^5*x^10*sgn(b*x^2 + a) + 1980*a*b^4*x^8*sgn(b*x^2 + a) + 3465
*a^2*b^3*x^6*sgn(b*x^2 + a) + 3080*a^3*b^2*x^4*sgn(b*x^2 + a) + 1386*a^4*b*
x^2*sgn(b*x^2 + a) + 252*a^5*sgn(b*x^2 + a))/x^22
```

3.604 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{25}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2}}{16x}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*x^24*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(2*x^20*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(16*x^16*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(
a + b*x^2))
```

Rubi [A] time = 0.151287, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2}}{16x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*x^24*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(2*x^20*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(16*x^16*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(
a + b*x^2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{25}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a^2+2abx+b^2x^2\right)^{5/2}}{x^{13}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \frac{\left(ab+b^2x\right)^5}{x^{13}}\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\operatorname{Subst}\left(\int \left(\frac{a^5b^5}{x^{13}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^{11}}+\frac{10a^2b^8}{x^{10}}+\frac{5ab^9}{x^9}+\frac{b^{10}}{x^8}\right)dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{}}{9}\end{aligned}$$

Mathematica [A] time = 0.0209461, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(6160a^2b^3x^6+5544a^3b^2x^4+2520a^4bx^2+462a^5+3465ab^4x^8+792b^5x^{10}\right)}{11088x^{24}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(462*a^5 + 2520*a^4*b*x^2 + 5544*a^3*b^2*x^4 + 6160*a
^2*b^3*x^6 + 3465*a*b^4*x^8 + 792*b^5*x^10))/(11088*x^24*(a + b*x^2))
```

Maple [A] time = 0.163, size = 80, normalized size = 0.3

$$-\frac{792\,b^5x^{10}+3465\,ab^4x^8+6160\,a^2b^3x^6+5544\,b^2a^3x^4+2520\,a^4bx^2+462\,a^5}{11088\,x^{24}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x)
```

```
[Out] -1/11088*(792*b^5*x^10+3465*a*b^4*x^8+6160*a^2*b^3*x^6+5544*a^3*b^2*x^4+252
0*a^4*b*x^2+462*a^5)*((b*x^2+a)^2)^(5/2)/x^24/(b*x^2+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30539, size = 151, normalized size = 0.59

$$-\frac{792\,b^5x^{10}+3465\,ab^4x^8+6160\,a^2b^3x^6+5544\,a^3b^2x^4+2520\,a^4bx^2+462\,a^5}{11088\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="fricas")
```

```
[Out] -1/11088*(792*b^5*x^10 + 3465*a*b^4*x^8 + 6160*a^2*b^3*x^6 + 5544*a^3*b^2*x
^4 + 2520*a^4*b*x^2 + 462*a^5)/x^24
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**25,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**25, x)
```

Giac [A] time = 1.13436, size = 144, normalized size = 0.56

$$-\frac{792\,b^5x^{10}\mathrm{sgn}\left(bx^2 + a\right) + 3465\,ab^4x^8\mathrm{sgn}\left(bx^2 + a\right) + 6160\,a^2b^3x^6\mathrm{sgn}\left(bx^2 + a\right) + 5544\,a^3b^2x^4\mathrm{sgn}\left(bx^2 + a\right) + 2520\,a^4bx}{11088\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="giac")
```

```
[Out] -1/11088*(792*b^5*x^10*sgn(b*x^2 + a) + 3465*a*b^4*x^8*sgn(b*x^2 + a) + 616
0*a^2*b^3*x^6*sgn(b*x^2 + a) + 5544*a^3*b^2*x^4*sgn(b*x^2 + a) + 2520*a^4*b
*x^2*sgn(b*x^2 + a) + 462*a^5*sgn(b*x^2 + a))/x^24
```

3.605 $\int x^{12} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{23}\sqrt{a^2+2abx^2+b^2x^4}}{23\left(a+bx^2\right)}+\frac{5ab^4x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^2+b}}{17\left(a+bx^2\right)}$$

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a^4*b*x^15*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a^3*b^2*x^17*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(19*(a + b*x^2)) + (5*a*b^4*x^21*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(21*(a + b*x^2)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23
*(a + b*x^2))
```

Rubi [A] time = 0.0621528, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{23}\sqrt{a^2+2abx^2+b^2x^4}}{23\left(a+bx^2\right)}+\frac{5ab^4x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^2+b}}{17\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a^4*b*x^15*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a^3*b^2*x^17*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(19*(a + b*x^2)) + (5*a*b^4*x^21*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(21*(a + b*x^2)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23
*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^{12}\left(ab+b^2x^2\right)^5dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5x^{12}+5a^4b^6x^{14}+10a^3b^7x^{16}+10a^2b^8x^{18}+5ab^9x^{20}+b^{10}x\right.}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{a^4bx^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^2+b^2}}{17\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0232947, size = 83, normalized size = 0.33

$$\frac{x^{13}\sqrt{\left(a+bx^2\right)^2}\left(1067430a^2b^3x^6+1193010a^3b^2x^4+676039a^4bx^2+156009a^5+482885ab^4x^8+88179b^5x^{10}\right)}{2028117\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^2)^2]*(156009*a^5 + 676039*a^4*b*x^2 + 1193010*a^3*b^2*
x^4 + 1067430*a^2*b^3*x^6 + 482885*a*b^4*x^8 + 88179*b^5*x^10))/(2028117*(a
 + b*x^2))
```

Maple [A] time = 0.163, size = 80, normalized size = 0.3

$$\frac{x^{13}\left(88179\,b^5x^{10}+482885\,ab^4x^8+1067430\,a^2b^3x^6+1193010\,b^2a^3x^4+676039\,a^4bx^2+156009\,a^5\right)}{2028117\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/2028117*x^13*(88179*b^5*x^10+482885*a*b^4*x^8+1067430*a^2*b^3*x^6+1193010
*a^3*b^2*x^4+676039*a^4*b*x^2+156009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 1.00761, size = 77, normalized size = 0.3

$$\frac{1}{23}\,b^5x^{23}+\frac{5}{21}\,ab^4x^{21}+\frac{10}{19}\,a^2b^3x^{19}+\frac{10}{17}\,a^3b^2x^{17}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*b^2*x^17 +
 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

Fricas [A] time = 1.26959, size = 146, normalized size = 0.57

$$\frac{1}{23}\,b^5x^{23}+\frac{5}{21}\,ab^4x^{21}+\frac{10}{19}\,a^2b^3x^{19}+\frac{10}{17}\,a^3b^2x^{17}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*b^2*x^17 +
 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{12}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**12*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.12596, size = 142, normalized size = 0.56

$$\frac{1}{23}\,b^5x^{23}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{21}\,ab^4x^{21}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{19}\,a^2b^3x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{17}\,a^3b^2x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,a^4bx^{15}\mathrm{sgn}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/23*b^5*x^23*sgn(b*x^2 + a) + 5/21*a*b^4*x^21*sgn(b*x^2 + a) + 10/19*a^2*b
^3*x^19*sgn(b*x^2 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^2 + a) + 1/3*a^4*b*x^15
*sgn(b*x^2 + a) + 1/13*a^5*x^13*sgn(b*x^2 + a)
```

3.606 $\int x^{10} \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=255

$$\frac{b^5x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2abx^2+b^2}}{3\left(a+bx^2\right)}$$

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2*a^3*b^2*x^15*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(19*(a + b*x^2)) + (b^5*x^21*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2
1*(a + b*x^2))
```

Rubi [A] time = 0.0577635, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2abx^2+b^2}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2*a^3*b^2*x^15*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(19*(a + b*x^2)) + (b^5*x^21*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2
1*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^{10}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5x^{10}+5a^4b^6x^{12}+10a^3b^7x^{14}+10a^2b^8x^{16}+5ab^9x^{18}+b^{10}x\right.}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2abx^2+b}}{3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0208419, size = 83, normalized size = 0.33

$$\frac{x^{11}\sqrt{\left(a+bx^2\right)^2}\left(570570a^2b^3x^6+646646a^3b^2x^4+373065a^4bx^2+88179a^5+255255ab^4x^8+46189b^5x^{10}\right)}{969969\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^11*Sqrt[(a + b*x^2)^2]*(88179*a^5 + 373065*a^4*b*x^2 + 646646*a^3*b^2*x^
4 + 570570*a^2*b^3*x^6 + 255255*a*b^4*x^8 + 46189*b^5*x^10))/(969969*(a + b
*x^2))
```

Maple [A] time = 0.162, size = 80, normalized size = 0.3

$$\frac{x^{11}\left(46189\,b^5x^{10}+255255\,ab^4x^8+570570\,a^2b^3x^6+646646\,b^2a^3x^4+373065\,a^4bx^2+88179\,a^5\right)}{969969\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/969969*x^11*(46189*b^5*x^10+255255*a*b^4*x^8+570570*a^2*b^3*x^6+646646*a^
3*b^2*x^4+373065*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 0.997234, size = 77, normalized size = 0.3

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{19}\,ab^4x^{19}+\frac{10}{17}\,a^2b^3x^{17}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^2*x^15 + 5
/13*a^4*b*x^13 + 1/11*a^5*x^11
```

Fricas [A] time = 1.266, size = 144, normalized size = 0.56

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{19}\,ab^4x^{19}+\frac{10}{17}\,a^2b^3x^{17}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^2*x^15 + 5
/13*a^4*b*x^13 + 1/11*a^5*x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{10}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**10*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.13374, size = 142, normalized size = 0.56

$$\frac{1}{21}\,b^5x^{21}\text{sgn}\left(bx^2+a\right)+\frac{5}{19}\,ab^4x^{19}\text{sgn}\left(bx^2+a\right)+\frac{10}{17}\,a^2b^3x^{17}\text{sgn}\left(bx^2+a\right)+\frac{2}{3}\,a^3b^2x^{15}\text{sgn}\left(bx^2+a\right)+\frac{5}{13}\,a^4bx^{13}\text{sgn}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/21*b^5*x^21*sgn(b*x^2 + a) + 5/19*a*b^4*x^19*sgn(b*x^2 + a) + 10/17*a^2*b
^3*x^17*sgn(b*x^2 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^2 + a) + 5/13*a^4*b*x^13*
sgn(b*x^2 + a) + 1/11*a^5*x^11*sgn(b*x^2 + a)
```

3.607 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^2b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^2+b^2}}{13\left(a+bx^2\right)}$$

```
[Out] (a^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*a^3*b^2*x^13*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2*a^2*b^3*x^15*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(17*(a + b*x^2)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*
(a + b*x^2))
```

Rubi [A] time = 0.0608189, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^2b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^2+b^2}}{13\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*a^3*b^2*x^13*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2*a^2*b^3*x^15*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(17*(a + b*x^2)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*
(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^8\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^8\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5x^8+5a^4b^6x^{10}+10a^3b^7x^{12}+10a^2b^8x^{14}+5ab^9x^{16}+b^{10}x^{18}\right)}{b^4\left(ab+b^2x^2\right)}$$

$$=\frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^2+b^2}}{13\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0215853, size = 83, normalized size = 0.33

$$\frac{x^9\sqrt{\left(a+bx^2\right)^2}\left(277134a^2b^3x^6+319770a^3b^2x^4+188955a^4bx^2+46189a^5+122265ab^4x^8+21879b^5x^{10}\right)}{415701\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^2)^2]*(46189*a^5 + 188955*a^4*b*x^2 + 319770*a^3*b^2*x^4
 + 277134*a^2*b^3*x^6 + 122265*a*b^4*x^8 + 21879*b^5*x^10))/(415701*(a + b*
x^2))
```

Maple [A] time = 0.161, size = 80, normalized size = 0.3

$$\frac{x^9\left(21879\,b^5x^{10}+122265\,ab^4x^8+277134\,a^2b^3x^6+319770\,b^2a^3x^4+188955\,a^4bx^2+46189\,a^5\right)}{415701\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/415701*x^9*(21879*b^5*x^10+122265*a*b^4*x^8+277134*a^2*b^3*x^6+319770*a^3
*b^2*x^4+188955*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 1.00446, size = 77, normalized size = 0.3

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Fricas [A] time = 1.25025, size = 142, normalized size = 0.56

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**8*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.10924, size = 142, normalized size = 0.56

$$\frac{1}{19}\,b^5x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\,a^2b^3x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{sgn}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19*sgn(b*x^2 + a) + 5/17*a*b^4*x^17*sgn(b*x^2 + a) + 2/3*a^2*b^3
*x^15*sgn(b*x^2 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^2 + a) + 5/11*a^4*b*x^11*
sgn(b*x^2 + a) + 1/9*a^5*x^9*sgn(b*x^2 + a)
```

3.608 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{ab^4x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^2+b^2}}{11\left(a+bx^2\right)}$$

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (5*a^4*b*x^9*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a*b^4*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(3*(a + b*x^2)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a
+ b*x^2))
```

Rubi [A] time = 0.0584595, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{ab^4x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^2+b^2}}{11\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (5*a^4*b*x^9*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a*b^4*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(3*(a + b*x^2)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a
+ b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^6 \left(ab+b^2x^2\right)^5 dx}{b^4 \left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^5b^5x^6+5a^4b^6x^8+10a^3b^7x^{10}+10a^2b^8x^{12}+5ab^9x^{14}+b^{10}x^{16}\right)}{b^4 \left(ab+b^2x^2\right)}$$

$$= \frac{a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)} + \frac{5a^4bx^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)} + \frac{10a^3b^2x^{11}\sqrt{a^2+2abx^2+b^2x}}{11\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0203568, size = 83, normalized size = 0.33

$$\frac{x^7\sqrt{\left(a+bx^2\right)^2}\left(117810a^2b^3x^6+139230a^3b^2x^4+85085a^4bx^2+21879a^5+51051ab^4x^8+9009b^5x^{10}\right)}{153153\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^2)^2]*(21879*a^5 + 85085*a^4*b*x^2 + 139230*a^3*b^2*x^4
+ 117810*a^2*b^3*x^6 + 51051*a*b^4*x^8 + 9009*b^5*x^10))/(153153*(a + b*x^2
))
```

Maple [A] time = 0.164, size = 80, normalized size = 0.3

$$\frac{x^7\left(9009\,b^5x^{10}+51051\,ab^4x^8+117810\,a^2b^3x^6+139230\,b^2a^3x^4+85085\,a^4bx^2+21879\,a^5\right)}{153153\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/153153*x^7*(9009*b^5*x^10+51051*a*b^4*x^8+117810*a^2*b^3*x^6+139230*a^3*b
^2*x^4+85085*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 1.0171, size = 77, normalized size = 0.3

$$\frac{1}{17}\,b^{5}x^{17}+\frac{1}{3}\,ab^{4}x^{15}+\frac{10}{13}\,a^{2}b^{3}x^{13}+\frac{10}{11}\,a^{3}b^{2}x^{11}+\frac{5}{9}\,a^{4}bx^{9}+\frac{1}{7}\,a^{5}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Fricas [A] time = 1.34945, size = 140, normalized size = 0.55

$$\frac{1}{17}\,b^{5}x^{17}+\frac{1}{3}\,ab^{4}x^{15}+\frac{10}{13}\,a^{2}b^{3}x^{13}+\frac{10}{11}\,a^{3}b^{2}x^{11}+\frac{5}{9}\,a^{4}bx^{9}+\frac{1}{7}\,a^{5}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{6}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**6*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.11621, size = 142, normalized size = 0.56

$$\frac{1}{17}\,b^5x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,ab^4x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{13}\,a^2b^3x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{11}\,a^3b^2x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^4bx^9\mathrm{sgn}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sgn(b*x^2 + a) + 1/3*a*b^4*x^15*sgn(b*x^2 + a) + 10/13*a^2*b^
3*x^13*sgn(b*x^2 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^2 + a) + 5/9*a^4*b*x^9*s
gn(b*x^2 + a) + 1/7*a^5*x^7*sgn(b*x^2 + a)
```

3.609 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{5ab^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^3b^2x^9\sqrt{a^2+2abx^2+b^2}}{9\left(a+bx^2\right)}$$

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^3*b^2*x^9*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*a*b^4*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(13*(a + b*x^2)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*(a
 + b*x^2))
```

Rubi [A] time = 0.0594737, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{5ab^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^3b^2x^9\sqrt{a^2+2abx^2+b^2}}{9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^3*b^2*x^9*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*a*b^4*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(13*(a + b*x^2)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*(a
 + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^4\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5x^4+5a^4b^6x^6+10a^3b^7x^8+10a^2b^8x^{10}+5ab^9x^{12}+b^{10}x^{14}\right)}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{10a^3b^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0210503, size = 83, normalized size = 0.33

$$\frac{x^5\sqrt{\left(a+bx^2\right)^2}\left(40950a^2b^3x^6+50050a^3b^2x^4+32175a^4bx^2+9009a^5+17325ab^4x^8+3003b^5x^{10}\right)}{45045\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^2)^2]*(9009*a^5 + 32175*a^4*b*x^2 + 50050*a^3*b^2*x^4 +
40950*a^2*b^3*x^6 + 17325*a*b^4*x^8 + 3003*b^5*x^10))/(45045*(a + b*x^2))
```

Maple [A] time = 0.164, size = 80, normalized size = 0.3

$$\frac{x^5\left(3003\,b^5x^{10}+17325\,ab^4x^8+40950\,a^2b^3x^6+50050\,b^2a^3x^4+32175\,a^4bx^2+9009\,a^5\right)}{45045\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/45045*x^5*(3003*b^5*x^10+17325*a*b^4*x^8+40950*a^2*b^3*x^6+50050*a^3*b^2*
x^4+32175*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 1.01416, size = 77, normalized size = 0.3

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Fricas [A] time = 1.32816, size = 139, normalized size = 0.55

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.12896, size = 142, normalized size = 0.56

$$\frac{1}{15} b^5 x^{15} \operatorname{sgn}\left(bx^2 + a\right) + \frac{5}{13} ab^4 x^{13} \operatorname{sgn}\left(bx^2 + a\right) + \frac{10}{11} a^2 b^3 x^{11} \operatorname{sgn}\left(bx^2 + a\right) + \frac{10}{9} a^3 b^2 x^9 \operatorname{sgn}\left(bx^2 + a\right) + \frac{5}{7} a^4 bx^7 \operatorname{sgn}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15*sgn(b*x^2 + a) + 5/13*a*b^4*x^13*sgn(b*x^2 + a) + 10/11*a^2*b
^3*x^11*sgn(b*x^2 + a) + 10/9*a^3*b^2*x^9*sgn(b*x^2 + a) + 5/7*a^4*b*x^7*sg
n(b*x^2 + a) + 1/5*a^5*x^5*sgn(b*x^2 + a)
```

3.610 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=252

$$\frac{b^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{10a^3b^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

```
[Out] (a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x^7*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^2*b^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(9*(a + b*x^2)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
11*(a + b*x^2)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2
))
```

Rubi [A] time = 0.0598965, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{10a^3b^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```



```
[Out] (a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x^7*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^2*b^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(9*(a + b*x^2)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
11*(a + b*x^2)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2
))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2 \left(ab+b^2x^2\right)^5 dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5b^5x^2+5a^4b^6x^4+10a^3b^7x^6+10a^2b^8x^8+5ab^9x^{10}+b^{10}x^{12}\right) d}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} + \frac{a^4bx^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{10a^3b^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0218006, size = 83, normalized size = 0.33

$$\frac{x^3\sqrt{\left(a+bx^2\right)^2}\left(10010a^2b^3x^6+12870a^3b^2x^4+9009a^4bx^2+3003a^5+4095ab^4x^8+693b^5x^{10}\right)}{9009\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^2)^2]*(3003*a^5 + 9009*a^4*b*x^2 + 12870*a^3*b^2*x^4 + 1
0010*a^2*b^3*x^6 + 4095*a*b^4*x^8 + 693*b^5*x^10))/(9009*(a + b*x^2))
```

Maple [A] time = 0.163, size = 80, normalized size = 0.3

$$\frac{x^3\left(693\,b^5x^{10}+4095\,ab^4x^8+10010\,a^2b^3x^6+12870\,b^2a^3x^4+9009\,a^4bx^2+3003\,a^5\right)}{9009\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/9009*x^3*(693*b^5*x^10+4095*a*b^4*x^8+10010*a^2*b^3*x^6+12870*a^3*b^2*x^4
+9009*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 0.993534, size = 76, normalized size = 0.3

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

Fricas [A] time = 1.23806, size = 131, normalized size = 0.52

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.13055, size = 140, normalized size = 0.56

$$\frac{1}{13}\,b^5x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{11}\,ab^4x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{9}\,a^2b^3x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{7}\,a^3b^2x^7\mathrm{sgn}\left(bx^2+a\right)+a^4bx^5\mathrm{sgn}\left(bx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13*sgn(b*x^2 + a) + 5/11*a*b^4*x^11*sgn(b*x^2 + a) + 10/9*a^2*b^
3*x^9*sgn(b*x^2 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^2 + a) + a^4*b*x^5*sgn(b*x^
2 + a) + 1/3*a^5*x^3*sgn(b*x^2 + a)
```

3.611 $\int \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=248

$$\frac{b^5x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{11\left(a+bx^2\right)^5}+\frac{5ab^4x^9\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{9\left(a+bx^2\right)^5}+\frac{10a^2b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{7\left(a+bx^2\right)^5}+\frac{2a^3b^2x^5\left(a^2+2abx \cdots\right.}{\left(a+bx^2\right. \cdots}$$

```
[Out] (a^5*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (5*a^4*b*x^3*(a^2
 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(3*(a + b*x^2)^5) + (2*a^3*b^2*x^5*(a^2 + 2*
a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (10*a^2*b^3*x^7*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(5/2))/(7*(a + b*x^2)^5) + (5*a*b^4*x^9*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(5/2))/(9*(a + b*x^2)^5) + (b^5*x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
/(11*(a + b*x^2)^5)
```

Rubi [A] time = 0.051071, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1088, 194}

$$\frac{b^5x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{11\left(a+bx^2\right)^5}+\frac{5ab^4x^9\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{9\left(a+bx^2\right)^5}+\frac{10a^2b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{7\left(a+bx^2\right)^5}+\frac{2a^3b^2x^5\left(a^2+2abx \cdots\right.}{\left(a+bx^2\right. \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (5*a^4*b*x^3*(a^2
 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(3*(a + b*x^2)^5) + (2*a^3*b^2*x^5*(a^2 + 2*
a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (10*a^2*b^3*x^7*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(5/2))/(7*(a + b*x^2)^5) + (5*a*b^4*x^9*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(5/2))/(9*(a + b*x^2)^5) + (b^5*x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
/(11*(a + b*x^2)^5)
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2} \int \left(2ab+2b^2x^2\right)^5 dx}{\left(2ab+2b^2x^2\right)^5}$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2} \int \left(32a^5b^5+160a^4b^6x^2+320a^3b^7x^4+320a^2b^8x^6+160ab^9x^8+32\right.}{\left(2ab+2b^2x^2\right)^5}$$
$$= \frac{a^5x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5} + \frac{5a^4bx^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{3\left(a+bx^2\right)^5} + \frac{2a^3b^2x^5\left(a^2+2abx^2+b^2x\right.}{\left(a+bx^2\right)^5}$$

Mathematica [A] time = 0.016422, size = 81, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(990a^2b^3x^7+1386a^3b^2x^5+1155a^4bx^3+693a^5x+385ab^4x^9+63b^5x^{11}\right)}{693\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(693*a^5*x + 1155*a^4*b*x^3 + 1386*a^3*b^2*x^5 + 990*a
^2*b^3*x^7 + 385*a*b^4*x^9 + 63*b^5*x^11))/(693*(a + b*x^2))
```

Maple [A] time = 0.044, size = 78, normalized size = 0.3

$$\frac{x\left(63\,b^5x^{10}+385\,ab^4x^8+990\,a^2b^3x^6+1386\,b^2a^3x^4+1155\,a^4bx^2+693\,a^5\right)}{693\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/693*x*(63*b^5*x^10+385*a*b^4*x^8+990*a^2*b^3*x^6+1386*a^3*b^2*x^4+1155*a^
4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 0.998212, size = 73, normalized size = 0.29

$$\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

Fricas [A] time = 1.36876, size = 122, normalized size = 0.49

$$\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2), x)
```

Giac [A] time = 1.11997, size = 138, normalized size = 0.56

$$\frac{1}{11}\,b^5x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,ab^4x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{7}\,a^2b^3x^7\mathrm{sgn}\left(bx^2+a\right)+2\,a^3b^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}\,a^4bx^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sgn(b*x^2 + a) + 5/9*a*b^4*x^9*sgn(b*x^2 + a) + 10/7*a^2*b^3*
x^7*sgn(b*x^2 + a) + 2*a^3*b^2*x^5*sgn(b*x^2 + a) + 5/3*a^4*b*x^3*sgn(b*x^2
 + a) + a^5*x*sgn(b*x^2 + a)
```

3.612 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (5*a^4*b*x*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(3*(a + b*x^2)) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(a + b*x^2) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a +
b*x^2)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
```

Rubi [A] time = 0.0581297, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (5*a^4*b*x*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(3*(a + b*x^2)) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(a + b*x^2) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a +
b*x^2)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^2}\,dx}{b^4\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(5a^4b^6+\frac{a^5b^5}{x^2}+10a^3b^7x^2+10a^2b^8x^4+5ab^9x^6+b^{10}x^8\right)dx}{b^4\left(ab+b^2x^2\right)}\\ &= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{5a^4bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\end{aligned}$$

Mathematica [A] time = 0.0203504, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(126a^2b^3x^6+210a^3b^2x^4+315a^4bx^2-63a^5+45ab^4x^8+7b^5x^{10}\right)}{63x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-63*a^5 + 315*a^4*b*x^2 + 210*a^3*b^2*x^4 + 126*a^2*b
^3*x^6 + 45*a*b^4*x^8 + 7*b^5*x^10))/(63*x*(a + b*x^2))
```

Maple [A] time = 0.171, size = 80, normalized size = 0.3

$$-\frac{-7\,b^5x^{10}-45\,ab^4x^8-126\,a^2b^3x^6-210\,b^2a^3x^4-315\,a^4bx^2+63\,a^5}{63\,x\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x)
```

```
[Out] -1/63*(-7*b^5*x^10-45*a*b^4*x^8-126*a^2*b^3*x^6-210*a^3*b^2*x^4-315*a^4*b*x
^2+63*a^5)*((b*x^2+a)^2)^(5/2)/x/(b*x^2+a)^5
```

Maxima [A] time = 1.00605, size = 80, normalized size = 0.32

$$\frac{7\,b^5x^{10}+45\,ab^4x^8+126\,a^2b^3x^6+210\,a^3b^2x^4+315\,a^4bx^2-63\,a^5}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*a
^4*b*x^2 - 63*a^5)/x
```

Fricas [A] time = 1.18887, size = 131, normalized size = 0.53

$$\frac{7\,b^5x^{10}+45\,ab^4x^8+126\,a^2b^3x^6+210\,a^3b^2x^4+315\,a^4bx^2-63\,a^5}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*a
^4*b*x^2 - 63*a^5)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**2, x)
```

Giac [A] time = 1.11001, size = 139, normalized size = 0.56

$$\frac{1}{9}b^5x^9\operatorname{sgn}\left(bx^2+a\right)+\frac{5}{7}ab^4x^7\operatorname{sgn}\left(bx^2+a\right)+2a^2b^3x^5\operatorname{sgn}\left(bx^2+a\right)+\frac{10}{3}a^3b^2x^3\operatorname{sgn}\left(bx^2+a\right)+5a^4bx\operatorname{sgn}\left(bx^2+a\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sgn(b*x^2 + a) + 5/7*a*b^4*x^7*sgn(b*x^2 + a) + 2*a^2*b^3*x^5*s
gn(b*x^2 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^2 + a) + 5*a^4*b*x*sgn(b*x^2 + a)
- a^5*sgn(b*x^2 + a)/x
```

3.613 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=246

$$\frac{b^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^3*b^2*x*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^2*b^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(3*(a + b*x^2)) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*
x^2) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rubi [A] time = 0.06027, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$\frac{b^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^3*b^2*x*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^2*b^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(3*(a + b*x^2)) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*
x^2) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^4}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(10a^3b^7+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^2}+10a^2b^8x^2+5ab^9x^4+b^{10}x^6\right)dx}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)} + \frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{10a}{}$$

Mathematica [A] time = 0.0228803, size = 83, normalized size = 0.34

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(70a^2b^3x^6+210a^3b^2x^4-105a^4bx^2-7a^5+21ab^4x^8+3b^5x^{10}\right)}{21x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-7*a^5 - 105*a^4*b*x^2 + 210*a^3*b^2*x^4 + 70*a^2*b^3
*x^6 + 21*a*b^4*x^8 + 3*b^5*x^10))/(21*x^3*(a + b*x^2))
```

Maple [A] time = 0.163, size = 80, normalized size = 0.3

$$-\frac{-3\,b^5x^{10}-21\,ab^4x^8-70\,a^2b^3x^6-210\,b^2a^3x^4+105\,a^4bx^2+7\,a^5}{21\,x^3\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x)
```

```
[Out] -1/21*(-3*b^5*x^10-21*a*b^4*x^8-70*a^2*b^3*x^6-210*a^3*b^2*x^4+105*a^4*b*x^
2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^3/(b*x^2+a)^5
```

Maxima [A] time = 1.0072, size = 80, normalized size = 0.33

$$\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a^
4*b*x^2 - 7*a^5)/x^3
```

Fricas [A] time = 1.33215, size = 131, normalized size = 0.53

$$\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a^
4*b*x^2 - 7*a^5)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**4,x)
```

[Out] Integral(((a + b*x**2)**2)**(5/2)/x**4, x)

Giac [A] time = 1.15666, size = 140, normalized size = 0.57

$$\frac{1}{7}b^5x^7\mathrm{sgn}\left(bx^2+a\right)+ab^4x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{3}a^2b^3x^3\mathrm{sgn}\left(bx^2+a\right)+10\,a^3b^2x\mathrm{sgn}\left(bx^2+a\right)-\frac{15\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="giac")

[Out] 1/7*b^5*x^7*sgn(b*x^2 + a) + a*b^4*x^5*sgn(b*x^2 + a) + 10/3*a^2*b^3*x^3*sg
n(b*x^2 + a) + 10*a^3*b^2*x*sgn(b*x^2 + a) - 1/3*(15*a^4*b*x^2*sgn(b*x^2 +
a) + a^5*sgn(b*x^2 + a))/x^3

3.614 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^6} dx$

Optimal. Leaf size=249

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5ab}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a
 + b*x^2)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rubi [A] time = 0.0578224, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a
 + b*x^2)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^6}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(10a^2b^8+\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^2}+5ab^9x^2+b^{10}x^4\right)dx}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{10a^2}{}$$

Mathematica [A] time = 0.0193039, size = 83, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(150a^2b^3x^6-150a^3b^2x^4-25a^4bx^2-3a^5+25ab^4x^8+3b^5x^{10}\right)}{15x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 25*a^4*b*x^2 - 150*a^3*b^2*x^4 + 150*a^2*b^3
*x^6 + 25*a*b^4*x^8 + 3*b^5*x^10))/(15*x^5*(a + b*x^2))
```

Maple [A] time = 0.163, size = 80, normalized size = 0.3

$$-\frac{-3\,b^5x^{10}-25\,ab^4x^8-150\,a^2b^3x^6+150\,b^2a^3x^4+25\,a^4bx^2+3\,a^5}{15\,x^5\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x)
```

```
[Out] -1/15*(-3*b^5*x^10-25*a*b^4*x^8-150*a^2*b^3*x^6+150*a^3*b^2*x^4+25*a^4*b*x^
2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^5/(b*x^2+a)^5
```

Maxima [A] time = 1.01609, size = 80, normalized size = 0.32

$$\frac{3\,b^5x^{10} + 25\,ab^4x^8 + 150\,a^2b^3x^6 - 150\,a^3b^2x^4 - 25\,a^4bx^2 - 3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a^
4*b*x^2 - 3*a^5)/x^5
```

Fricas [A] time = 1.34473, size = 131, normalized size = 0.53

$$\frac{3\,b^5x^{10} + 25\,ab^4x^8 + 150\,a^2b^3x^6 - 150\,a^3b^2x^4 - 25\,a^4bx^2 - 3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a^
4*b*x^2 - 3*a^5)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**6, x)
```

Giac [A] time = 1.11859, size = 143, normalized size = 0.57

$$\frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}\,ab^4x^3\mathrm{sgn}\left(bx^2+a\right)+10\,a^2b^3x\mathrm{sgn}\left(bx^2+a\right)-\frac{150\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+25\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sgn(b*x^2 + a) + 5/3*a*b^4*x^3*sgn(b*x^2 + a) + 10*a^2*b^3*x*sg
n(b*x^2 + a) - 1/15*(150*a^3*b^2*x^4*sgn(b*x^2 + a) + 25*a^4*b*x^2*sgn(b*x^
2 + a) + 3*a^5*sgn(b*x^2 + a))/x^5
```

3.615 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=247

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{5ab^4x}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a^4*b*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(x*(a + b*x^2)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a +
b*x^2) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rubi [A] time = 0.0583021, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{5ab^4x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a^4*b*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(x*(a + b*x^2)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a +
b*x^2) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^8}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(5ab^9+\frac{a^5b^5}{x^8}+\frac{5a^4b^6}{x^6}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^2}+b^{10}x^2\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{10a^2b}{}$$

Mathematica [A] time = 0.0171713, size = 83, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(210a^2b^3x^6+70a^3b^2x^4+21a^4bx^2+3a^5-105ab^4x^8-7b^5x^{10}\right)}{21x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(3*a^5 + 21*a^4*b*x^2 + 70*a^3*b^2*x^4 + 210*a^2*b^3*
x^6 - 105*a*b^4*x^8 - 7*b^5*x^10))/(21*x^7*(a + b*x^2))
```

Maple [A] time = 0.168, size = 80, normalized size = 0.3

$$-\frac{-7\,b^5x^{10}-105\,ab^4x^8+210\,a^2b^3x^6+70\,b^2a^3x^4+21\,a^4bx^2+3\,a^5}{21\,x^7\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x)
```

```
[Out] -1/21*(-7*b^5*x^10-105*a*b^4*x^8+210*a^2*b^3*x^6+70*a^3*b^2*x^4+21*a^4*b*x^
2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^7/(b*x^2+a)^5
```

Maxima [A] time = 0.997564, size = 80, normalized size = 0.32

$$\frac{7\,b^5x^{10} + 105\,ab^4x^8 - 210\,a^2b^3x^6 - 70\,a^3b^2x^4 - 21\,a^4bx^2 - 3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a^
4*b*x^2 - 3*a^5)/x^7
```

Fricas [A] time = 1.39339, size = 131, normalized size = 0.53

$$\frac{7\,b^5x^{10} + 105\,ab^4x^8 - 210\,a^2b^3x^6 - 70\,a^3b^2x^4 - 21\,a^4bx^2 - 3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a^
4*b*x^2 - 3*a^5)/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**8, x)
```

Giac [A] time = 1.13044, size = 143, normalized size = 0.58

$$\frac{1}{3}b^5x^3\operatorname{sgn}\left(bx^2+a\right)+5\,ab^4x\operatorname{sgn}\left(bx^2+a\right)-\frac{210\,a^2b^3x^6\operatorname{sgn}\left(bx^2+a\right)+70\,a^3b^2x^4\operatorname{sgn}\left(bx^2+a\right)+21\,a^4bx^2\operatorname{sgn}\left(bx^2+a\right)}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sgn(b*x^2 + a) + 5*a*b^4*x*sgn(b*x^2 + a) - 1/21*(210*a^2*b^3*x
^6*sgn(b*x^2 + a) + 70*a^3*b^2*x^4*sgn(b*x^2 + a) + 21*a^4*b*x^2*sgn(b*x^2
+ a) + 3*a^5*sgn(b*x^2 + a))/x^7
```

3.616 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=246

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{5ab^4\sqrt{}}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x
*(a + b*x^2)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rubi [A] time = 0.0580791, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{5ab^4\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^3*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x
*(a + b*x^2)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{10}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^{10}+\frac{a^5b^5}{x^{10}}+\frac{5a^4b^6}{x^8}+\frac{10a^3b^7}{x^6}+\frac{10a^2b^8}{x^4}+\frac{5ab^9}{x^2}\right)dx}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)} - 10a^2b$$

Mathematica [A] time = 0.0181717, size = 83, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(210a^2b^3x^6+126a^3b^2x^4+45a^4bx^2+7a^5+315ab^4x^8-63b^5x^{10}\right)}{63x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(7*a^5 + 45*a^4*b*x^2 + 126*a^3*b^2*x^4 + 210*a^2*b^3
*x^6 + 315*a*b^4*x^8 - 63*b^5*x^10))/(63*x^9*(a + b*x^2))
```

Maple [A] time = 0.171, size = 80, normalized size = 0.3

$$-\frac{-63\,b^5x^{10}+315\,ab^4x^8+210\,a^2b^3x^6+126\,b^2a^3x^4+45\,a^4bx^2+7\,a^5}{63\,x^9\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x)
```

```
[Out] -1/63*(-63*b^5*x^10+315*a*b^4*x^8+210*a^2*b^3*x^6+126*a^3*b^2*x^4+45*a^4*b*
x^2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^9/(b*x^2+a)^5
```

Maxima [A] time = 1.00842, size = 80, normalized size = 0.33

$$\frac{63\,b^5x^{10} - 315\,ab^4x^8 - 210\,a^2b^3x^6 - 126\,a^3b^2x^4 - 45\,a^4bx^2 - 7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45*
a^4*b*x^2 - 7*a^5)/x^9
```

Fricas [A] time = 1.26493, size = 134, normalized size = 0.54

$$\frac{63\,b^5x^{10} - 315\,ab^4x^8 - 210\,a^2b^3x^6 - 126\,a^3b^2x^4 - 45\,a^4bx^2 - 7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45*
a^4*b*x^2 - 7*a^5)/x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**10, x)
```

Giac [A] time = 1.1324, size = 142, normalized size = 0.58

$$b^5 x \operatorname{sgn}\left(bx^2+a\right) - \frac{315\, ab^4x^8 \operatorname{sgn}\left(bx^2+a\right) + 210\, a^2b^3x^6 \operatorname{sgn}\left(bx^2+a\right) + 126\, a^3b^2x^4 \operatorname{sgn}\left(bx^2+a\right) + 45\, a^4bx^2 \operatorname{sgn}\left(bx^2+a\right.}{63\, x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] b^5*x*sgn(b*x^2 + a) - 1/63*(315*a*b^4*x^8*sgn(b*x^2 + a) + 210*a^2*b^3*x^6
*sgn(b*x^2 + a) + 126*a^3*b^2*x^4*sgn(b*x^2 + a) + 45*a^4*b*x^2*sgn(b*x^2 +
 a) + 7*a^5*sgn(b*x^2 + a))/x^9
```

3.617 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{12}} \, dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{5ab^4\sqrt{}}{3}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(x^5*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/
(3*x^3*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rubi [A] time = 0.0584969, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{5ab^4\sqrt{}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(x^5*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/
(3*x^3*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{12}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^{10}}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^6}+\frac{5ab^9}{x^4}+\frac{b^{10}}{x^2}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^2b}{}$$

Mathematica [A] time = 0.0170295, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(1386a^2b^3x^6+990a^3b^2x^4+385a^4bx^2+63a^5+1155ab^4x^8+693b^5x^{10}\right)}{693x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(63*a^5 + 385*a^4*b*x^2 + 990*a^3*b^2*x^4 + 1386*a^2*
b^3*x^6 + 1155*a*b^4*x^8 + 693*b^5*x^10))/(693*x^11*(a + b*x^2))
```

Maple [A] time = 0.17, size = 80, normalized size = 0.3

$$-\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,b^2a^3x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x)
```

```
[Out] -1/693*(693*b^5*x^10+1155*a*b^4*x^8+1386*a^2*b^3*x^6+990*a^3*b^2*x^4+385*a^
4*b*x^2+63*a^5)*((b*x^2+a)^2)^(5/2)/x^11/(b*x^2+a)^5
```

Maxima [A] time = 1.11278, size = 80, normalized size = 0.32

$$-\frac{693\, b^5 x^{10} + 1155\, ab^4 x^8 + 1386\, a^2 b^3 x^6 + 990\, a^3 b^2 x^4 + 385\, a^4 b x^2 + 63\, a^5}{693\, x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

Fricas [A] time = 1.1995, size = 144, normalized size = 0.57

$$-\frac{693\, b^5 x^{10} + 1155\, ab^4 x^8 + 1386\, a^2 b^3 x^6 + 990\, a^3 b^2 x^4 + 385\, a^4 b x^2 + 63\, a^5}{693\, x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**12, x)
```

Giac [A] time = 1.12671, size = 144, normalized size = 0.57

$$-\frac{693\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+1155\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+1386\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+990\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+385\,a^4bx^2\mathrm{s}}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/693*(693*b^5*x^10*sgn(b*x^2 + a) + 1155*a*b^4*x^8*sgn(b*x^2 + a) + 1386*
a^2*b^3*x^6*sgn(b*x^2 + a) + 990*a^3*b^2*x^4*sgn(b*x^2 + a) + 385*a^4*b*x^2
*sgn(b*x^2 + a) + 63*a^5*sgn(b*x^2 + a))/x^11
```

3.618 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{14}}\,dx$

Optimal. Leaf size=253

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{ab^4\sqrt{}}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(x^5*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*
x^2))
```

Rubi [A] time = 0.0601616, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{ab^4\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(x^5*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*
x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{14}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{14}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^{10}}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^4}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-10a^2$$

Mathematica [A] time = 0.0177823, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(12870a^2b^3x^6+10010a^3b^2x^4+4095a^4bx^2+693a^5+9009ab^4x^8+3003b^5x^{10}\right)}{9009x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(693*a^5 + 4095*a^4*b*x^2 + 10010*a^3*b^2*x^4 + 12870
*a^2*b^3*x^6 + 9009*a*b^4*x^8 + 3003*b^5*x^10))/(9009*x^13*(a + b*x^2))
```

Maple [A] time = 0.166, size = 80, normalized size = 0.3

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,b^2a^3x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x)
```

```
[Out] -1/9009*(3003*b^5*x^10+9009*a*b^4*x^8+12870*a^2*b^3*x^6+10010*a^3*b^2*x^4+4
095*a^4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/x^13/(b*x^2+a)^5
```

Maxima [A] time = 1.02601, size = 80, normalized size = 0.32

$$-\frac{3003\,b^{5}x^{10}+9009\,ab^{4}x^{8}+12870\,a^{2}b^{3}x^{6}+10010\,a^{3}b^{2}x^{4}+4095\,a^{4}bx^{2}+693\,a^{5}}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Fricas [A] time = 1.20966, size = 154, normalized size = 0.61

$$-\frac{3003\,b^{5}x^{10}+9009\,ab^{4}x^{8}+12870\,a^{2}b^{3}x^{6}+10010\,a^{3}b^{2}x^{4}+4095\,a^{4}bx^{2}+693\,a^{5}}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**14, x)
```

Giac [A] time = 1.13277, size = 144, normalized size = 0.57

$$-\frac{3003\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+9009\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+12870\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+10010\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+4095\,a}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(3003*b^5*x^10*sgn(b*x^2 + a) + 9009*a*b^4*x^8*sgn(b*x^2 + a) + 128
70*a^2*b^3*x^6*sgn(b*x^2 + a) + 10010*a^3*b^2*x^4*sgn(b*x^2 + a) + 4095*a^4
*b*x^2*sgn(b*x^2 + a) + 693*a^5*sgn(b*x^2 + a))/x^13
```

3.619 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5ab^4}{}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(7*x^7*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(
a + b*x^2))
```

Rubi [A] time = 0.0580007, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5ab^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(7*x^7*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(
a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{16}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{16}}+\frac{5a^4b^6}{x^{14}}+\frac{10a^3b^7}{x^{12}}+\frac{10a^2b^8}{x^{10}}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^6}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-10a^2$$

Mathematica [A] time = 0.0175777, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(50050a^2b^3x^6+40950a^3b^2x^4+17325a^4bx^2+3003a^5+32175ab^4x^8+9009b^5x^{10}\right)}{45045x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(3003*a^5 + 17325*a^4*b*x^2 + 40950*a^3*b^2*x^4 + 500
50*a^2*b^3*x^6 + 32175*a*b^4*x^8 + 9009*b^5*x^10))/(45045*x^15*(a + b*x^2))
```

Maple [A] time = 0.17, size = 80, normalized size = 0.3

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,b^2a^3x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x)
```

```
[Out] -1/45045*(9009*b^5*x^10+32175*a*b^4*x^8+50050*a^2*b^3*x^6+40950*a^3*b^2*x^4
+17325*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/x^15/(b*x^2+a)^5
```

Maxima [A] time = 1.00314, size = 80, normalized size = 0.31

$$-\frac{9009\,b^5x^{10} + 32175\,ab^4x^8 + 50050\,a^2b^3x^6 + 40950\,a^3b^2x^4 + 17325\,a^4bx^2 + 3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Fricas [A] time = 1.36302, size = 159, normalized size = 0.62

$$-\frac{9009\,b^5x^{10} + 32175\,ab^4x^8 + 50050\,a^2b^3x^6 + 40950\,a^3b^2x^4 + 17325\,a^4bx^2 + 3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**16, x)
```

Giac [A] time = 1.1232, size = 144, normalized size = 0.56

$$-\frac{9009\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+32175\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+50050\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+40950\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+1732}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(9009*b^5*x^10*sgn(b*x^2 + a) + 32175*a*b^4*x^8*sgn(b*x^2 + a) + 5
0050*a^2*b^3*x^6*sgn(b*x^2 + a) + 40950*a^3*b^2*x^4*sgn(b*x^2 + a) + 17325*
a^4*b*x^2*sgn(b*x^2 + a) + 3003*a^5*sgn(b*x^2 + a))/x^15
```

3.620 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{}}{9}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(9*x^9*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a
 + b*x^2))
```

Rubi [A] time = 0.0588274, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{}}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(9*x^9*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a
 + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{18}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{18}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{18}}+\frac{5a^4b^6}{x^{16}}+\frac{10a^3b^7}{x^{14}}+\frac{10a^2b^8}{x^{12}}+\frac{5ab^9}{x^{10}}+\frac{b^{10}}{x^8}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^2b}{}$$

Mathematica [A] time = 0.01785, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(139230a^2b^3x^6+117810a^3b^2x^4+51051a^4bx^2+9009a^5+85085ab^4x^8+21879b^5x^{10}\right)}{153153x^{17}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(9009*a^5 + 51051*a^4*b*x^2 + 117810*a^3*b^2*x^4 + 13
9230*a^2*b^3*x^6 + 85085*a*b^4*x^8 + 21879*b^5*x^10))/(153153*x^17*(a + b*x
^2))
```

Maple [A] time = 0.163, size = 80, normalized size = 0.3

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,b^2a^3x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x)
```

```
[Out] -1/153153*(21879*b^5*x^10+85085*a*b^4*x^8+139230*a^2*b^3*x^6+117810*a^3*b^2
*x^4+51051*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/x^17/(b*x^2+a)^5
```

Maxima [A] time = 1.01211, size = 80, normalized size = 0.31

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Fricas [A] time = 1.23473, size = 165, normalized size = 0.65

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{18}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**18,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**18, x)
```

Giac [A] time = 1.14061, size = 144, normalized size = 0.56

$$-\frac{21879\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+85085\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+139230\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+117810\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] -1/153153*(21879*b^5*x^10*sgn(b*x^2 + a) + 85085*a*b^4*x^8*sgn(b*x^2 + a) +
 139230*a^2*b^3*x^6*sgn(b*x^2 + a) + 117810*a^3*b^2*x^4*sgn(b*x^2 + a) + 51
051*a^4*b*x^2*sgn(b*x^2 + a) + 9009*a^5*sgn(b*x^2 + a))/x^17
```

3.621 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{20}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{}}{11}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (2*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(11*x^11*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9
*(a + b*x^2))
```

Rubi [A] time = 0.0582526, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (2*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(11*x^11*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9
*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{20}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{20}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{20}}+\frac{5a^4b^6}{x^{18}}+\frac{10a^3b^7}{x^{16}}+\frac{10a^2b^8}{x^{14}}+\frac{5ab^9}{x^{12}}+\frac{b^{10}}{x^{10}}\right)dx}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)} - \frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)} - \frac{10a^2b}{}$$

Mathematica [A] time = 0.0202025, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(319770a^2b^3x^6+277134a^3b^2x^4+122265a^4bx^2+21879a^5+188955ab^4x^8+46189b^5x^{10}\right)}{415701x^{19}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(21879*a^5 + 122265*a^4*b*x^2 + 277134*a^3*b^2*x^4 +
319770*a^2*b^3*x^6 + 188955*a*b^4*x^8 + 46189*b^5*x^10))/(415701*x^19*(a +
b*x^2))
```

Maple [A] time = 0.171, size = 80, normalized size = 0.3

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,b^2a^3x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x)
```

```
[Out] -1/415701*(46189*b^5*x^10+188955*a*b^4*x^8+319770*a^2*b^3*x^6+277134*a^3*b^
2*x^4+122265*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/x^19/(b*x^2+a)^5
```

Maxima [A] time = 0.99646, size = 80, normalized size = 0.31

$$-\frac{46189\,b^5x^{10} + 188955\,ab^4x^8 + 319770\,a^2b^3x^6 + 277134\,a^3b^2x^4 + 122265\,a^4bx^2 + 21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="maxima")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Fricas [A] time = 1.25834, size = 169, normalized size = 0.66

$$-\frac{46189\,b^5x^{10} + 188955\,ab^4x^8 + 319770\,a^2b^3x^6 + 277134\,a^3b^2x^4 + 122265\,a^4bx^2 + 21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{20}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**20,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**20, x)
```

Giac [A] time = 1.1368, size = 144, normalized size = 0.56

$$-\frac{46189\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+188955\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+319770\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+277134\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="giac")
```

```
[Out] -1/415701*(46189*b^5*x^10*sgn(b*x^2 + a) + 188955*a*b^4*x^8*sgn(b*x^2 + a)
+ 319770*a^2*b^3*x^6*sgn(b*x^2 + a) + 277134*a^3*b^2*x^4*sgn(b*x^2 + a) + 1
22265*a^4*b*x^2*sgn(b*x^2 + a) + 21879*a^5*sgn(b*x^2 + a))/x^19
```

3.622 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{22}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{}}{13}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(13*x^13*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^
11*(a + b*x^2))
```

Rubi [A] time = 0.0575609, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(13*x^13*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^
11*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{22}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{22}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{22}}+\frac{5a^4b^6}{x^{20}}+\frac{10a^3b^7}{x^{18}}+\frac{10a^2b^8}{x^{16}}+\frac{5ab^9}{x^{14}}+\frac{b^{10}}{x^{12}}\right)dx}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)} - \frac{2a^2b}{}$$

Mathematica [A] time = 0.0177407, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(646646a^2b^3x^6+570570a^3b^2x^4+255255a^4bx^2+46189a^5+373065ab^4x^8+88179b^5x^{10}\right)}{969969x^{21}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(46189*a^5 + 255255*a^4*b*x^2 + 570570*a^3*b^2*x^4 +
646646*a^2*b^3*x^6 + 373065*a*b^4*x^8 + 88179*b^5*x^10))/(969969*x^21*(a +
b*x^2))
```

Maple [A] time = 0.168, size = 80, normalized size = 0.3

$$-\frac{88179\,b^5x^{10}+373065\,ab^4x^8+646646\,a^2b^3x^6+570570\,b^2a^3x^4+255255\,a^4bx^2+46189\,a^5}{969969\,x^{21}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x)
```

```
[Out] -1/969969*(88179*b^5*x^10+373065*a*b^4*x^8+646646*a^2*b^3*x^6+570570*a^3*b^
2*x^4+255255*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2)/x^21/(b*x^2+a)^5
```

Maxima [A] time = 0.986076, size = 80, normalized size = 0.31

$$-\frac{88179\,b^5x^{10}+373065\,ab^4x^8+646646\,a^2b^3x^6+570570\,a^3b^2x^4+255255\,a^4bx^2+46189\,a^5}{969969\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] -1/969969*(88179*b^5*x^10 + 373065*a*b^4*x^8 + 646646*a^2*b^3*x^6 + 570570*
a^3*b^2*x^4 + 255255*a^4*b*x^2 + 46189*a^5)/x^21
```

Fricas [A] time = 1.29257, size = 169, normalized size = 0.66

$$-\frac{88179\,b^5x^{10}+373065\,ab^4x^8+646646\,a^2b^3x^6+570570\,a^3b^2x^4+255255\,a^4bx^2+46189\,a^5}{969969\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/969969*(88179*b^5*x^10 + 373065*a*b^4*x^8 + 646646*a^2*b^3*x^6 + 570570*
a^3*b^2*x^4 + 255255*a^4*b*x^2 + 46189*a^5)/x^21
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**22,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**22, x)
```

Giac [A] time = 1.1307, size = 144, normalized size = 0.56

$$-\frac{88179\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+373065\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+646646\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+570570\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+}{969969\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/969969*(88179*b^5*x^10*sgn(b*x^2 + a) + 373065*a*b^4*x^8*sgn(b*x^2 + a)
+ 646646*a^2*b^3*x^6*sgn(b*x^2 + a) + 570570*a^3*b^2*x^4*sgn(b*x^2 + a) + 2
55255*a^4*b*x^2*sgn(b*x^2 + a) + 46189*a^5*sgn(b*x^2 + a))/x^21
```

3.623 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}} dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)}-\frac{ab^4\sqrt{}}{3}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*x^23*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(3*x^15*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^1
3*(a + b*x^2))
```

Rubi [A] time = 0.0576959, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1112, 270}

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)}-\frac{ab^4\sqrt{}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*x^23*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(3*x^15*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^1
3*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{24}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{24}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{24}}+\frac{5a^4b^6}{x^{22}}+\frac{10a^3b^7}{x^{20}}+\frac{10a^2b^8}{x^{18}}+\frac{5ab^9}{x^{16}}+\frac{b^{10}}{x^{14}}\right)dx}{b^4\left(ab+b^2x^2\right)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^2}{}$$

Mathematica [A] time = 0.0182443, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(1193010a^2b^3x^6+1067430a^3b^2x^4+482885a^4bx^2+88179a^5+676039ab^4x^8+156009b^5x^{10}\right)}{2028117x^{23}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(88179*a^5 + 482885*a^4*b*x^2 + 1067430*a^3*b^2*x^4 +
 1193010*a^2*b^3*x^6 + 676039*a*b^4*x^8 + 156009*b^5*x^10))/(2028117*x^23*(
a + b*x^2))
```

Maple [A] time = 0.168, size = 80, normalized size = 0.3

$$-\frac{156009\,b^5x^{10}+676039\,ab^4x^8+1193010\,a^2b^3x^6+1067430\,b^2a^3x^4+482885\,a^4bx^2+88179\,a^5}{2028117\,x^{23}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x)
```

```
[Out] -1/2028117*(156009*b^5*x^10+676039*a*b^4*x^8+1193010*a^2*b^3*x^6+1067430*a^
3*b^2*x^4+482885*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/2)/x^23/(b*x^2+a)^5
```

Maxima [A] time = 1.00958, size = 80, normalized size = 0.31

$$-\frac{156009\,b^5x^{10}+676039\,ab^4x^8+1193010\,a^2b^3x^6+1067430\,a^3b^2x^4+482885\,a^4bx^2+88179\,a^5}{2028117\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1/2028117*(156009*b^5*x^10 + 676039*a*b^4*x^8 + 1193010*a^2*b^3*x^6 + 1067
430*a^3*b^2*x^4 + 482885*a^4*b*x^2 + 88179*a^5)/x^23
```

Fricas [A] time = 1.33954, size = 174, normalized size = 0.68

$$-\frac{156009\,b^5x^{10}+676039\,ab^4x^8+1193010\,a^2b^3x^6+1067430\,a^3b^2x^4+482885\,a^4bx^2+88179\,a^5}{2028117\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/2028117*(156009*b^5*x^10 + 676039*a*b^4*x^8 + 1193010*a^2*b^3*x^6 + 1067
430*a^3*b^2*x^4 + 482885*a^4*b*x^2 + 88179*a^5)/x^23
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{24}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**24,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**24, x)
```

Giac [A] time = 1.1528, size = 144, normalized size = 0.56

$$-\frac{156009\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+676039\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+1193010\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+1067430\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right.}{2028117\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="giac")
```

```
[Out] -1/2028117*(156009*b^5*x^10*sgn(b*x^2 + a) + 676039*a*b^4*x^8*sgn(b*x^2 + a
) + 1193010*a^2*b^3*x^6*sgn(b*x^2 + a) + 1067430*a^3*b^2*x^4*sgn(b*x^2 + a)
 + 482885*a^4*b*x^2*sgn(b*x^2 + a) + 88179*a^5*sgn(b*x^2 + a))/x^23
```

3.624 $\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=127

$$\frac{x^4(a+bx^2)}{4b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax^2(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2(a+bx^2)\log(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(a*x^2*(a + b*x^2))/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^4*(a + b*
x^2))/(4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^2*(a + b*x^2)*Log[a + b*x^
2])/(2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.101596, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{x^4(a+bx^2)}{4b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax^2(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2(a+bx^2)\log(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(a*x^2*(a + b*x^2))/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^4*(a + b*
x^2))/(4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^2*(a + b*x^2)*Log[a + b*x^
2])/(2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)\\
&= \frac{\left(ab+b^2x^2\right)\operatorname{Subst}\left(\int \frac{x^2}{ab+b^2x}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\left(ab+b^2x^2\right)\operatorname{Subst}\left(\int \left(-\frac{a}{b^3}+\frac{x}{b^2}+\frac{a^2}{b^3(a+bx)}\right)dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= -\frac{ax^2\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^4\left(a+bx^2\right)}{4b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.0240429, size = 55, normalized size = 0.43

$$\frac{\left(a+bx^2\right)\left(2a^2\log\left(a+bx^2\right)+bx^2\left(bx^2-2a\right)\right)}{4b^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(b*x^2*(-2*a + b*x^2) + 2*a^2*Log[a + b*x^2]))/(4*b^3*Sqrt[(a
+ b*x^2)^2])
```

Maple [A] time = 0.211, size = 52, normalized size = 0.4

$$\frac{\left(bx^2+a\right)\left(b^2x^4-2\,abx^2+2\,a^2\ln\left(bx^2+a\right)\right)}{4\,b^3}\frac{1}{\sqrt{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)*(b^2*x^4-2*a*b*x^2+2*a^2*ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/b^3
```

Maxima [A] time = 0.999292, size = 62, normalized size = 0.49

$$\frac{x^4}{4\sqrt{b^2}} - \frac{abx^2}{2\left(b^2\right)^{\frac{3}{2}}} + \frac{a^2b^2\log\left(x^2 + \frac{a}{b}\right)}{2\left(b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/sqrt(b^2) - 1/2*a*b*x^2/(b^2)^(3/2) + 1/2*a^2*b^2*log(x^2 + a/b)/(b
^2)^(5/2)
```

Fricas [A] time = 1.22679, size = 73, normalized size = 0.57

$$\frac{b^2x^4 - 2\,abx^2 + 2\,a^2\log\left(bx^2 + a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

Sympy [A] time = 0.327823, size = 32, normalized size = 0.25

$$\frac{a^2\log\left(a + bx^2\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a**2*log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)
```

Giac [A] time = 1.13911, size = 80, normalized size = 0.63

$$\frac{a^2 \log\left(\left|bx^2 + a\right|\right) \operatorname{sgn}\left(bx^2 + a\right)}{2\,b^3} + \frac{bx^4 \operatorname{sgn}\left(bx^2 + a\right) - 2\,ax^2 \operatorname{sgn}\left(bx^2 + a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(b*x^2 + a))*sgn(b*x^2 + a)/b^3 + 1/4*(b*x^4*sgn(b*x^2 + a)
- 2*a*x^2*sgn(b*x^2 + a))/b^2
```

3.625 $\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}} \, dx$

Optimal. Leaf size=75

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(2*b^2) - (a*(a + b*x^2)*Log[a + b*x^2])/(2
*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0559249, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1111, 640, 608, 31}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(2*b^2) - (a*(a + b*x^2)*Log[a + b*x^2])/(2
*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)}{2b}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{\left(a\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.0124745, size = 44, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left(bx^2-a\log\left(a+bx^2\right)\right)}{2b^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(b*x^2 - a*Log[a + b*x^2]))/(2*b^2*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.21, size = 41, normalized size = 0.6

$$-\frac{\left(bx^2+a\right)\left(-bx^2+a\ln\left(bx^2+a\right)\right)}{2\,b^2}\frac{1}{\sqrt{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(-b*x^2+a*ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/b^2
```

Maxima [A] time = 0.999998, size = 63, normalized size = 0.84

$$-\frac{a\sqrt{\frac{1}{b^2}}\log\left(x^2+\frac{a}{b}\right)}{2\,b}+\frac{\sqrt{b^2x^4+2\,abx^2+a^2}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*sqrt(b^(-2))*log(x^2 + a/b)/b + 1/2*sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)/
b^2
```

Fricas [A] time = 1.29094, size = 49, normalized size = 0.65

$$\frac{bx^2-a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

Sympy [A] time = 0.318036, size = 20, normalized size = 0.27

$$-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

Giac [A] time = 1.12577, size = 45, normalized size = 0.6

$$\frac{1}{2}\left(\frac{x^2}{b} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{b^2}\right)\operatorname{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(x^2/b - a*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

3.626 $\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0316934, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 608, 31}

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0073811, size = 35, normalized size = 0.8

$$\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.207, size = 32, normalized size = 0.7

$$\frac{\left(bx^2+a\right)\ln\left(bx^2+a\right)}{2\,b}\frac{1}{\sqrt{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*ln(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)
```

Maxima [A] time = 0.997002, size = 23, normalized size = 0.52

$$\frac{1}{2}\sqrt{\frac{1}{b^2}}\log\left(x^2+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^(-2))*log(x^2 + a/b)
```

Fricas [A] time = 1.18735, size = 30, normalized size = 0.68

$$\frac{\log\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

Sympy [A] time = 0.136152, size = 10, normalized size = 0.23

$$\frac{\log\left(a + bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] log(a + b*x**2)/(2*b)
```

Giac [A] time = 1.14841, size = 30, normalized size = 0.68

$$\frac{\log\left(\left|bx^2 + a\right|\right)\operatorname{sgn}\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))*sgn(b*x^2 + a)/b
```

3.627 $\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=80

$$\frac{\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log
[a + b*x^2])/(2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0337835, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1112, 266, 36, 29, 31}

$$\frac{\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log
[a + b*x^2])/(2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{x\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(a+bx^2\right)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0112525, size = 42, normalized size = 0.52

$$\frac{\left(a+bx^2\right)\left(2\log(x)-\log\left(a+bx^2\right)\right)}{2a\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(2*Log[x] - Log[a + b*x^2]))/(2*a*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.21, size = 39, normalized size = 0.5

$$\frac{(bx^2+a)\left(2\ln(x)-\ln(bx^2+a)\right)}{2a}\frac{1}{\sqrt{(bx^2+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(2*ln(x)-ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.24601, size = 49, normalized size = 0.61

$$-\frac{\log(bx^2+a)-2\log(x)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

Sympy [A] time = 0.222443, size = 15, normalized size = 0.19

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

Giac [A] time = 1.11977, size = 45, normalized size = 0.56

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a}-\frac{\log\left(\left|bx^2+a\right|\right)}{a}\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(log(x^2)/a - log(abs(b*x^2 + a))/a)*sgn(b*x^2 + a)
```

3.628 $\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=122

$$-\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\log(x)\left(a+bx^2\right)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(a + b*x^2)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*(a + b*x^2)*Log
[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*x^2)*Log[a + b*x^2])
/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0496018, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$-\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\log(x)\left(a+bx^2\right)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```


```
[Out] -(a + b*x^2)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*(a + b*x^2)*Log
[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*x^2)*Log[a + b*x^2])
/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{1}{x^3\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{\left(ab+b^2x^2\right)\operatorname{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{\left(ab+b^2x^2\right)\operatorname{Subst}\left(\int \left(\frac{1}{abx^2}-\frac{1}{a^2x}+\frac{b}{a^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\left(a+bx^2\right)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.0153979, size = 54, normalized size = 0.44

$$-\frac{\left(a+bx^2\right)\left(-bx^2\log\left(a+bx^2\right)+a+2bx^2\log(x)\right)}{2a^2x^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((a + b*x^2)*(a + 2*b*x^2*Log[x] - b*x^2*Log[a + b*x^2]))/(2*a^2*x^2*Sqrt[
(a + b*x^2)^2])
```

Maple [A] time = 0.214, size = 51, normalized size = 0.4

$$-\frac{\left(bx^2+a\right)\left(2\,b\ln(x)\,x^2-b\ln\left(bx^2+a\right)x^2+a\right)}{2\,a^2x^2}\frac{1}{\sqrt{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(2*b*ln(x)*x^2-b*ln(b*x^2+a)*x^2+a)/((b*x^2+a)^2)^(1/2)/x^2/
a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29492, size = 80, normalized size = 0.66

$$\frac{bx^2 \log\left(bx^2 + a\right) - 2\,bx^2 \log\left(x\right) - a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

Sympy [A] time = 0.464759, size = 31, normalized size = 0.25

$$-\frac{1}{2ax^2} - \frac{b\log\left(x\right)}{a^2} + \frac{b\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

Giac [A] time = 1.10707, size = 70, normalized size = 0.57

$$-\frac{1}{2}\left(\frac{b\log\left(x^2\right)}{a^2}-\frac{b\log\left(\left|bx^2+a\right|\right)}{a^2}-\frac{bx^2-a}{a^2x^2}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(x^2)/a^2 - b*log(abs(b*x^2 + a))/a^2 - (b*x^2 - a)/(a^2*x^2))*s
gn(b*x^2 + a)
```

3.629 $\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=129

$$\frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((a*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (x^3*(a + b*x^
2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/2)*(a + b*x^2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0460382, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 302, 205}

$$\frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((a*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (x^3*(a + b*x^
2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/2)*(a + b*x^2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{x^4}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{\left(ab+b^2x^2\right)\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2\left(ab+b^2x^2\right)}\right)dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a^2\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{b^2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.0250801, size = 66, normalized size = 0.51

$$\frac{\left(a+bx^2\right)\left(3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x\left(bx^2-3a\right)\right)}{3b^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x*(-3*a + b*x^2) + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.217, size = 63, normalized size = 0.5

$$\frac{bx^2+a}{3\,b^2}\left(\sqrt{ab}x^3b-3\,\sqrt{ab}xa+3\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{\left(bx^2+a\right)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((a*b)^(1/2)*x^3*b-3*(a*b)^(1/2)*x*a+3*a^2*arctan(b*x/(a*b)^(
1/2)))/((b*x^2+a)^2)^(1/2)/b^2/(a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34042, size = 217, normalized size = 1.68

$$\left[\frac{2\,bx^{3}+3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)-6\,ax}{6\,b^{2}},\ \frac{bx^{3}+3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,ax}{3\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*a*
x)/b^2]
```

Sympy [A] time = 0.351204, size = 80, normalized size = 0.62

$$-\frac{ax}{b^{2}}-\frac{\sqrt{-\frac{a^{3}}{b^{5}}}\log\left(x-\frac{b^{2}\sqrt{-\frac{a^{3}}{b^{5}}}}{a}\right)}{2}+\frac{\sqrt{-\frac{a^{3}}{b^{5}}}\log\left(x+\frac{b^{2}\sqrt{-\frac{a^{3}}{b^{5}}}}{a}\right)}{2}+\frac{x^{3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -a*x/b**2 - sqrt(-a**3/b**5)*log(x - b**2*sqrt(-a**3/b**5)/a)/2 + sqrt(-a**
3/b**5)*log(x + b**2*sqrt(-a**3/b**5)/a)/2 + x**3/(3*b)
```

Giac [A] time = 1.16783, size = 86, normalized size = 0.67

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right) \operatorname{sgn}\left(bx^2+a\right)}{\sqrt{ab}b^2} + \frac{b^2x^3 \operatorname{sgn}\left(bx^2+a\right) - 3\,abx \operatorname{sgn}\left(bx^2+a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3*sgn
(b*x^2 + a) - 3*a*b*x*sgn(b*x^2 + a))/b^3
```

3.630 $\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}} dx$

Optimal. Leaf size=89

$$\frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(a + b*x^2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.031588, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 321, 205}

$$\frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(a + b*x^2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{x^2}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{x\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{x\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.0136017, size = 54, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left(\sqrt{b}x-\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{b^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x - Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(b^(3/2)*Sq
rt[(a + b*x^2)^2])
```

Maple [A] time = 0.213, size = 48, normalized size = 0.5

$$\frac{bx^2+a}{b}\left(x\sqrt{ab}-a\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{\left(bx^2+a\right)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] (b*x^2+a)*(x*(a*b)^(1/2)-a*arctan(b*x/(a*b)^(1/2)))/((b*x^2+a)^2)^(1/2)/b/(
a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.284, size = 165, normalized size = 1.85

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

Sympy [A] time = 0.33345, size = 56, normalized size = 0.63

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

Giac [A] time = 1.13674, size = 57, normalized size = 0.64

$$-\frac{a \arctan\left(\frac{bx}{\sqrt{ab}}\right) \operatorname{sgn}\left(bx^2 + a\right)}{\sqrt{ab}b} + \frac{x \operatorname{sgn}\left(bx^2 + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/(sqrt(a*b)*b) + x*sgn(b*x^2 + a)/b
```

3.631 $\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0150325, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1088, 205}

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \, dx = \frac{\left(2ab + 2b^2x^2\right) \int \frac{1}{2ab+2b^2x^2} \, dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}}$$
$$= \frac{\left(a + bx^2\right) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Mathematica [A] time = 0.0117462, size = 44, normalized size = 0.83

$$\frac{\left(a + bx^2\right) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\left(a + bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[(a + b*x^2)
^2])
```

Maple [A] time = 0.167, size = 34, normalized size = 0.6

$$(bx^2 + a) \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{\left(bx^2 + a\right)^2}} \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.255, size = 151, normalized size = 2.85

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [A] time = 0.156914, size = 53, normalized size = 1.

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Giac [A] time = 1.12336, size = 31, normalized size = 0.58

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/sqrt(a*b)
```

3.632 $\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=92

$$-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((a + b*x^2)/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - (Sqrt[b]*(a + b*x^2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0333592, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 325, 205}

$$-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((a + b*x^2)/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - (Sqrt[b]*(a + b*x^2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{a\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.0135005, size = 56, normalized size = 0.61

$$-\frac{\left(a+bx^2\right)\left(\sqrt{b}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{a}\right)}{a^{3/2}x\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(((a + b*x^2)*(Sqrt[a] + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(a^(3/2)*
x*Sqrt[(a + b*x^2)^2]))
```

Maple [A] time = 0.171, size = 50, normalized size = 0.5

$$-\frac{bx^2+a}{ax}\left(b\arctan\left(bx\frac{1}{\sqrt{ab}}\right)x+\sqrt{ab}\right)\frac{1}{\sqrt{\left(bx^2+a\right)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -(b*x^2+a)*(b*arctan(b*x/(a*b)^(1/2))*x+(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a/
x/(a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35675, size = 173, normalized size = 1.88

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

Sympy [A] time = 0.363345, size = 65, normalized size = 0.71

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

Giac [A] time = 1.13449, size = 50, normalized size = 0.54

$$-\left(\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}+\frac{1}{ax}\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*x))*sgn(b*x^2 + a)
```

3.633 $\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=130

$$\frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(a + b*x^2)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*x^2))/(a
^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0434308, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 325, 205}

$$\frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(a + b*x^2)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*x^2))/(a
^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{a\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{a^2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.0232454, size = 70, normalized size = 0.54

$$-\frac{\left(a+bx^2\right)\left(\sqrt{a}\left(a-3bx^2\right)-3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{3a^{5/2}x^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((a + b*x^2)*(Sqrt[a]*(a - 3*b*x^2) - 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)/Sqr
t[a]]))/(3*a^(5/2)*x^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.225, size = 69, normalized size = 0.5

$$\frac{bx^2+a}{3\,a^2x^3}\left(3\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3+3\,bx^2\sqrt{ab}-a\sqrt{ab}\right)\frac{1}{\sqrt{\left(bx^2+a\right)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*(3*b^2*arctan(b*x/(a*b)^(1/2))*x^3+3*b*x^2*(a*b)^(1/2)-a*(a*b
)^(1/2))/((b*x^2+a)^2)^(1/2)/a^2/x^3/(a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2854, size = 234, normalized size = 1.8

$$\left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3},\ \frac{3\,bx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

Sympy [A] time = 0.421861, size = 87, normalized size = 0.67

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

Giac [A] time = 1.11542, size = 68, normalized size = 0.52

$$\frac{1}{3}\left(\frac{3\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{3\,bx^2-a}{a^2x^3}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^2 - a)/(a^2*x^3))
*sgn(b*x^2 + a)
```

3.634 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} dx$

Optimal. Leaf size=158

$$\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-3*a^2)/(2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^3/(4*b^4*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^2*(a + b*x^2))/(2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (3*a*(a + b*x^2)*Log[a + b*x^2])/(2*b^4*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rubi [A] time = 0.131324, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-3*a^2)/(2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^3/(4*b^4*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^2*(a + b*x^2))/(2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (3*a*(a + b*x^2)*Log[a + b*x^2])/(2*b^4*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
```

```
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^3}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \left(\frac{1}{b^6}-\frac{a^3}{b^6(a+bx)^3}+\frac{3a^2}{b^6(a+bx)^2}-\frac{3a}{b^6(a+bx)}\right) dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0295263, size = 81, normalized size = 0.51

$$\frac{-4a^2bx^2-5a^3+4ab^2x^4-6a\left(a+bx^2\right)^2\log\left(a+bx^2\right)+2b^3x^6}{4b^4\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*a^3 - 4*a^2*b*x^2 + 4*a*b^2*x^4 + 2*b^3*x^6 - 6*a*(a + b*x^2)^2*Log[a +
 b*x^2])/(4*b^4*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.227, size = 103, normalized size = 0.7

$$-\frac{\left(-2\,b^{3}x^{6}+6\,\ln\left(bx^{2}+a\right)x^{4}ab^{2}-4\,ax^{4}b^{2}+12\,\ln\left(bx^{2}+a\right)x^{2}a^{2}b+4\,a^{2}bx^{2}+6\,\ln\left(bx^{2}+a\right)a^{3}+5\,a^{3}\right)\left(bx^{2}+a\right)}{4\,b^{4}}\left(\left(bx^{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(-2*b^3*x^6+6*ln(b*x^2+a)*x^4*a*b^2-4*a*x^4*b^2+12*ln(b*x^2+a)*x^2*a^2
*b+4*a^2*b*x^2+6*ln(b*x^2+a)*a^3+5*a^3)*(b*x^2+a)/b^4/((b*x^2+a)^2)^(3/2)
```

Maxima [A] time = 1.32425, size = 198, normalized size = 1.25

$$\frac{x^{4}}{2\sqrt{b^{2}x^{4}+2\,abx^{2}+a^{2}}b^{2}}-\frac{3\,a^{2}x^{2}}{\left(b^{2}\right)^{\frac{5}{2}}\left(x^{2}+\frac{a}{b}\right)^{2}}-\frac{3\,a\log\left(x^{2}+\frac{a}{b}\right)}{2\left(b^{2}\right)^{\frac{3}{2}}b}-\frac{9\,a^{3}b}{4\left(b^{2}\right)^{\frac{7}{2}}\left(x^{2}+\frac{a}{b}\right)^{2}}+\frac{a^{2}}{\sqrt{b^{2}x^{4}+2\,abx^{2}+a^{2}}b^{4}}-\frac{a^{3}}{2\left(b^{2}\right)^{\frac{3}{2}}\left(x^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^4/(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*b^2) - 3*a^2*x^2/((b^2)^(5/2)*(x^2
 + a/b)^2) - 3/2*a*log(x^2 + a/b)/((b^2)^(3/2)*b) - 9/4*a^3*b/((b^2)^(7/2)*
(x^2 + a/b)^2) + a^2/(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*b^4) - 1/2*a^3/((b^2)
^(3/2)*(x^2 + a/b)^2*b^3)
```

Fricas [A] time = 1.34379, size = 186, normalized size = 1.18

$$\frac{2\,b^{3}x^{6}+4\,ab^{2}x^{4}-4\,a^{2}bx^{2}-5\,a^{3}-6\left(ab^{2}x^{4}+2\,a^{2}bx^{2}+a^{3}\right)\log\left(bx^{2}+a\right)}{4\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*b
*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)

[Out] Integral(x**7/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x
```

3.635 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] a/(b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^2/(4*b^3*(a + b*x^2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[a + b*x^2])/(2*b^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0983267, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] a/(b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^2/(4*b^3*(a + b*x^2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[a + b*x^2])/(2*b^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
```

```
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+b^2x\right)^3}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^5(a+bx)^3}-\frac{2a}{b^5(a+bx)^2}+\frac{1}{b^5(a+bx)}\right)dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.0216094, size = 61, normalized size = 0.54

$$\frac{a\left(3a+4bx^2\right)+2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4b^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a*(3*a + 4*b*x^2) + 2*(a + b*x^2)^2*Log[a + b*x^2])/(4*b^3*(a + b*x^2)*Sqr
t[(a + b*x^2)^2])
```

Maple [A] time = 0.237, size = 81, normalized size = 0.7

$$\frac{\left(2\,\ln\left(bx^2+a\right)x^4b^2+4\,\ln\left(bx^2+a\right)x^2ab+4\,abx^2+2\,a^2\ln\left(bx^2+a\right)+3\,a^2\right)\left(bx^2+a\right)}{4\,b^3}\left(\left(bx^2+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(2*ln(b*x^2+a)*x^4*b^2+4*ln(b*x^2+a)*x^2*a*b+4*a*b*x^2+2*a^2*ln(b*x^2+a
)+3*a^2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(3/2)
```

Maxima [A] time = 1.22592, size = 86, normalized size = 0.76

$$\frac{abx^2}{\left(b^2\right)^{\frac{5}{2}}\left(x^2+\frac{a}{b}\right)^2}+\frac{\log\left(x^2+\frac{a}{b}\right)}{2\left(b^2\right)^{\frac{3}{2}}}+\frac{3\,a^2b^2}{4\left(b^2\right)^{\frac{7}{2}}\left(x^2+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] a*b*x^2/((x^2 + a/b)^2*(b^2)^(5/2)) + 1/2*log(x^2 + a/b)/(b^2)^(3/2) + 3/4*
a^2*b^2/((b^2)^(7/2)*(x^2 + a/b)^2)
```

Fricas [A] time = 1.19541, size = 143, normalized size = 1.27

$$\frac{4\,abx^2+3\,a^2+2\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^5
*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.636 $\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{x^4}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] x^4/(4*a*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0501704, antiderivative size = 69, normalized size of antiderivative = 1.68, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 607}

$$\frac{a}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a/(4*b^2*(a + b*x^2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, x^2\right)}{2b}\\ &= -\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.0122646, size = 39, normalized size = 0.95

$$\frac{-a-2bx^2}{4b^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-a - 2*b*x^2)/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.175, size = 32, normalized size = 0.8

$$-\frac{\left(bx^2+a\right)\left(2\,bx^2+a\right)}{4\,b^2}\left(\left(bx^2+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(b*x^2+a)*(2*b*x^2+a)/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A] time = 1.01687, size = 65, normalized size = 1.59

$$-\frac{1}{2\sqrt{b^2x^4+2\,abx^2+a^2}b^2}+\frac{a}{4\,(b^2)^{\frac{3}{2}}\left(x^2+\frac{a}{b}\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2/(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*b^2) + 1/4*a/((b^2)^(3/2)*(x^2 + a/b)
^2*b)
```

Fricas [A] time = 1.23154, size = 73, normalized size = 1.78

$$-\frac{2\,bx^2+a}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.637 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0257146, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 607}

$$-\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)$$
$$=-\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0079719, size = 27, normalized size = 0.71

$$-\frac{a+bx^2}{4b\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -(a + b*x^2)/(4*b*((a + b*x^2)^2)^(3/2))
```

Maple [A] time = 0.164, size = 24, normalized size = 0.6

$$-\frac{bx^2+a}{4\,b}\left(\left(bx^2+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(b*x^2+a)/b/((b*x^2+a)^2)^(3/2)
```

Maxima [A] time = 1.01848, size = 24, normalized size = 0.63

$$-\frac{1}{4\left(b^2\right)^{\frac{3}{2}}\left(x^2+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4/((b^2)^(3/2)*(x^2 + a/b)^2)
```

Fricas [A] time = 1.18808, size = 51, normalized size = 1.34

$$-\frac{1}{4\left(b^3x^4 + 2\,ab^2x^2 + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.638 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*(a + b*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - ((a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi [A] time = 0.0826822, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*(a + b*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - ((a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log(x)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-$$

Mathematica [A] time = 0.0263441, size = 74, normalized size = 0.5

$$\frac{a\left(3a+2bx^2\right)+4\log(x)\left(a+bx^2\right)^2-2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^2) + 4*(a + b*x^2)^2*Log[x] - 2*(a + b*x^2)^2*Log[a + b*x^2
])/(4*a^3*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.231, size = 107, normalized size = 0.7

$$-\frac{\left(2\ln\left(bx^2+a\right)x^4b^2-4\ln\left(x\right)x^4b^2+4\ln\left(bx^2+a\right)x^2ab-8\ln\left(x\right)x^2ab-2abx^2+2a^2\ln\left(bx^2+a\right)-4a^2\ln\left(x\right)-3a}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(2*ln(b*x^2+a)*x^4*b^2-4*ln(x)*x^4*b^2+4*ln(b*x^2+a)*x^2*a*b-8*ln(x)*x
^2*a*b-2*a*b*x^2+2*a^2*ln(b*x^2+a)-4*a^2*ln(x)-3*a^2)*(b*x^2+a)/a^3/((b*x^2
+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26835, size = 196, normalized size = 1.33

$$\frac{2abx^2+3a^2-2\left(b^2x^4+2abx^2+a^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+2abx^2+a^2\right)\log\left(x\right)}{4\left(a^3b^2x^4+2a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a) + 4*(
b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**2)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.639 $\int \frac{1}{x^3 (a^2+2abx^2+b^2x^4)^{3/2}} \, dx$

Optimal. Leaf size=189

$$-\frac{b}{4a^2 (a+bx^2) \sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{a^3 \sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^3x^2 \sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b \log(x) (a+bx^2)}{a^4 \sqrt{a^2+2abx^2+b^2x^4}}+\frac{3}{}$$

```
[Out] -(b/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - b/(4*a^2*(a + b*x^2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) - (a + b*x^2)/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (3*b*(a + b*x^2)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
3*b*(a + b*x^2)*Log[a + b*x^2])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.095797, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$-\frac{b}{4a^2 (a+bx^2) \sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{a^3 \sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^3x^2 \sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b \log(x) (a+bx^2)}{a^4 \sqrt{a^2+2abx^2+b^2x^4}}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```


```
[Out] -(b/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - b/(4*a^2*(a + b*x^2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) - (a + b*x^2)/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (3*b*(a + b*x^2)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
3*b*(a + b*x^2)*Log[a + b*x^2])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x^3\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \left(\frac{1}{a^3b^3x^2}-\frac{3}{a^4b^2x}+\frac{1}{a^2b(a+bx)^3}+\frac{2}{a^3b(a+bx)^2}+\frac{3}{a^4b(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0386016, size = 97, normalized size = 0.51

$$\frac{-a\left(2a^2+9abx^2+6b^2x^4\right)-12bx^2\log(x)\left(a+bx^2\right)^2+6bx^2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^4x^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)) - 12*b*x^2*(a + b*x^2)^2*Log[x] + 6*b
*x^2*(a + b*x^2)^2*Log[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^
2])
```

Maple [A] time = 0.223, size = 133, normalized size = 0.7

$$-\frac{\left(12 b^{3} \ln \left(x\right) x^{6}-6 \ln \left(b x^{2}+a\right) x^{6} b^{3}+24 b^{2} a \ln \left(x\right) x^{4}-12 \ln \left(b x^{2}+a\right) x^{4} a b^{2}+6 a x^{4} b^{2}+12 b a^{2} \ln \left(x\right) x^{2}-6 \ln \left(b x^{2}\right.}{4 x^{2} a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(12*b^3*ln(x)*x^6-6*ln(b*x^2+a)*x^6*b^3+24*b^2*a*ln(x)*x^4-12*ln(b*x^2
+a)*x^4*a*b^2+6*a*x^4*b^2+12*b*a^2*ln(x)*x^2-6*ln(b*x^2+a)*x^2*a^2*b+9*a^2*
b*x^2+2*a^3)*(b*x^2+a)/x^2/a^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29989, size = 247, normalized size = 1.31

$$-\frac{6 a b^{2} x^{4}+9 a^{2} b x^{2}+2 a^{3}-6 \left(b^{3} x^{6}+2 a b^{2} x^{4}+a^{2} b x^{2}\right) \log \left(b x^{2}+a\right)+12 \left(b^{3} x^{6}+2 a b^{2} x^{4}+a^{2} b x^{2}\right) \log \left(x\right)}{4 \left(a^{4} b^{2} x^{6}+2 a^{5} b x^{4}+a^{6} x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.640 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-3*x)/(8*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(4*b*(a + b*x^2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*Sqrt[a]*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0526389, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 288, 205}

$$-\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-3*x)/(8*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(4*b*(a + b*x^2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*Sqrt[a]*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}$$

$$= -\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{8\sqrt{a}b^{5/2}\sqrt{a^2+2abx^2+b^2}$$

Mathematica [A] time = 0.0311322, size = 84, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}x\left(3a+5bx^2\right)}{8\sqrt{a}b^{5/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a + 5*b*x^2)) + 3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(8*Sqrt[a]*b^(5/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.224, size = 97, normalized size = 0.8

$$-\frac{bx^2+a}{8b^2}\left(-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4b^2+5\sqrt{ab}x^3b-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^2ab+3\sqrt{ab}xa-3a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{ab}}\left(\left(bx^2+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(-3*arctan(b*x/(a*b)^(1/2))*x^4*b^2+5*(a*b)^(1/2)*x^3*b-6*arctan(b*x/(
a*b)^(1/2))*x^2*a*b+3*(a*b)^(1/2)*x*a-3*a^2*arctan(b*x/(a*b)^(1/2)))*(b*x^2
+a)/(a*b)^(1/2)/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31183, size = 404, normalized size = 3.16

$$\left[-\frac{10\,ab^2x^3+6\,a^2bx+3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{16\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)},-\frac{5\,ab^2x^3+3\,a^2bx-3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ab}\,a}{8\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(10*a*b^2*x^3 + 6*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^5*x^4 + 2*a^2*b^4*x^2
+ a^3*b^3), -1/8*(5*a*b^2*x^3 + 3*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.641 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} dx$

Optimal. Leaf size=129

$$\frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] x/(8*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(4*b*(a + b*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)
*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0492152, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$\frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] x/(8*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(4*b*(a + b*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)
*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= -\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(ab+b^2x^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(ab+b^2x^2\right)\int \frac{1}{ab+b^2x^2}\,dx}{8ab\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.028382, size = 81, normalized size = 0.63

$$\frac{\sqrt{a}\sqrt{b}x\left(bx^2-a\right)+\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-a + b*x^2) + (a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*a^(3/2)*b^(3/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.225, size = 97, normalized size = 0.8

$$\frac{bx^2+a}{8\,ab}\left(\arctan\left(bx\frac{1}{\sqrt{ab}}\right)x^4b^2+\sqrt{ab}x^3b+2\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^2ab-\sqrt{ab}xa+a^2\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{ab}}\left(\left(bx^2+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(arctan(b*x/(a*b)^(1/2))*x^4*b^2+(a*b)^(1/2)*x^3*b+2*arctan(b*x/(a*b)^(
1/2))*x^2*a*b-(a*b)^(1/2)*x*a+a^2*arctan(b*x/(a*b)^(1/2)))*(b*x^2+a)/(a*b)^
(1/2)/b/a/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26358, size = 394, normalized size = 3.05

$$\left[\frac{2\,ab^2x^3-2\,a^2bx-\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{16\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)},\frac{ab^2x^3-a^2bx+\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a}\right.}{8\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 - 2*a^2*b*x - (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*log
((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 +
a^4*b^2), 1/8*(a*b^2*x^3 - a^2*b*x + (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.642 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{3\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] (x*(a + b*x^2))/(4*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*x*(a + b*x^2)^
2)/(8*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*(a + b*x^2)^3*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi [A] time = 0.0383879, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1088, 199, 205}

$$\frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{3\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```


```
[Out] (x*(a + b*x^2))/(4*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*x*(a + b*x^2)^
2)/(8*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*(a + b*x^2)^3*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```


Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(2ab+2b^2x^2\right)^3 \int \frac{1}{\left(2ab+2b^2x^2\right)^3}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

$$= \frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{\left(3\left(2ab+2b^2x^2\right)^3\right)\int \frac{1}{\left(2ab+2b^2x^2\right)^2}\,dx}{8ab\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

$$= \frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{\left(3\left(2ab+2b^2x^2\right)^3\right)\int \frac{1}{2ab+2b^2x^2}}{32a^2b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

$$= \frac{x\left(a+bx^2\right)}{4a\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{3x\left(a+bx^2\right)^2}{8a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{3\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Mathematica [A] time = 0.0246737, size = 83, normalized size = 0.61

$$\frac{\sqrt{a}\sqrt{b}x\left(5a+3bx^2\right)+3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(5*a + 3*b*x^2) + 3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(5/2)*Sqrt[b]*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.221, size = 97, normalized size = 0.7

$$\frac{bx^2+a}{8a^2}\left(3\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4b^2+3\sqrt{ab}\,x^3b+6\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^2ab+5\sqrt{ab}\,xa+3a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{ab}}\left(\left(bx^2+a\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(3*arctan(b*x/(a*b)^(1/2))*x^4*b^2+3*(a*b)^(1/2)*x^3*b+6*arctan(b*x/(a*
b)^(1/2))*x^2*a*b+5*(a*b)^(1/2)*x*a+3*a^2*arctan(b*x/(a*b)^(1/2)))*(b*x^2+a
)/(a*b)^(1/2)/a^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31085, size = 401, normalized size = 2.97

$$\left[\frac{6ab^2x^3+10a^2bx-3\left(b^2x^4+2abx^2+a^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{16\left(a^3b^3x^4+2a^4b^2x^2+a^5b\right)},\frac{3ab^2x^3+5a^2bx+3\left(b^2x^4+2abx^2+a^2\right)\sqrt{ab}\,\mathrm{arct}}{8\left(a^3b^3x^4+2a^4b^2x^2+a^5b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2
 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.643 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{15\left(a+bx^2\right)}{8a^3x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)}-\frac{15\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 5/(8*a^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x*(a + b*x^2)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) - (15*(a + b*x^2))/(8*a^3*x*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (15*Sqrt[b]*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0653457, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$-\frac{15\left(a+bx^2\right)}{8a^3x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)}-\frac{15\sqrt{b}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 5/(8*a^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x*(a + b*x^2)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) - (15*(a + b*x^2))/(8*a^3*x*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (15*Sqrt[b]*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(5b\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(15\left(ab+b^2x^2\right)\right)\int \frac{}{x^2}}{8a^2\sqrt{a^2+2abx^2}}\\
&= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15\left(a+bx^2\right)}{8a^3x\sqrt{a^2+2abx^2+b}}\\
&= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15\left(a+bx^2\right)}{8a^3x\sqrt{a^2+2abx^2+b}}\end{aligned}$$

Mathematica [A] time = 0.0322544, size = 93, normalized size = 0.55

$$\frac{-\sqrt{a}\left(8a^2+25abx^2+15b^2x^4\right)-15\sqrt{b}x\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)) - 15*Sqrt[b]*x*(a + b*x^2)^2*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.239, size = 119, normalized size = 0.7

$$-\frac{bx^2+a}{8\,xa^3}\left(15\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^5b^3+15\,\sqrt{ab}x^4b^2+30\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3ab^2+25\,\sqrt{ab}x^2ab+15\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)xa^2b+8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(15*arctan(b*x/(a*b)^(1/2))*x^5*b^3+15*(a*b)^(1/2)*x^4*b^2+30*arctan(b
*x/(a*b)^(1/2))*x^3*a*b^2+25*(a*b)^(1/2)*x^2*a*b+15*arctan(b*x/(a*b)^(1/2))
*x*a^2*b+8*(a*b)^(1/2)*a^2)*(b*x^2+a)/(a*b)^(1/2)/x/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32713, size = 428, normalized size = 2.53

$$\left[-\frac{30\,b^2x^4+50\,abx^2-15\,\left(b^2x^5+2\,abx^3+a^2x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+16\,a^2}{16\,\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)},-\frac{15\,b^2x^4+25\,abx^2+15\,\left(b^2x^5+2\,abx\right.}{8\,\left(a^3b^2x^5+2\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5 +
 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a*b*
x^3 + a^2*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*
x^3 + a^5*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.644 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{35b\left(a+bx^2\right)}{8a^4x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35\left(a+bx^2\right)}{24a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2}}$$

```
[Out] 7/(8*a^2*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x^3*(a + b*x^2)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*(a + b*x^2))/(24*a^3*x^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) + (35*b*(a + b*x^2))/(8*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (35*b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0791235, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$\frac{35b\left(a+bx^2\right)}{8a^4x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35\left(a+bx^2\right)}{24a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 7/(8*a^2*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x^3*(a + b*x^2)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*(a + b*x^2))/(24*a^3*x^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) + (35*b*(a + b*x^2))/(8*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (35*b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7b\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(35\left(ab+b^2x^2\right)\right)\int}{8a^2\sqrt{a^2+2abx}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35\left(a+bx^2\right)}{24a^3x^3\sqrt{a^2+2abx^2}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35\left(a+bx^2\right)}{24a^3x^3\sqrt{a^2+2abx^2}}$$

$$= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35\left(a+bx^2\right)}{24a^3x^3\sqrt{a^2+2abx^2}}$$

Mathematica [A] time = 0.0380871, size = 105, normalized size = 0.5

$$\frac{\sqrt{a}\left(56a^2bx^2-8a^3+175ab^2x^4+105b^3x^6\right)+105b^{3/2}x^3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24a^{9/2}x^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6) + 105*b^(3/2
)*x^3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(24*a^(9/2)*x^3*(a + b*x^2
)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.235, size = 139, normalized size = 0.7

$$\frac{bx^2+a}{24\,a^4x^3}\left(105\ \arctan\left(\frac{bx}{\sqrt{ab}}\right)x^7b^4+105\sqrt{ab}x^6b^3+210\ \arctan\left(\frac{bx}{\sqrt{ab}}\right)x^5ab^3+175\sqrt{ab}x^4ab^2+105\ \arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/24*(105*arctan(b*x/(a*b)^(1/2))*x^7*b^4+105*(a*b)^(1/2)*x^6*b^3+210*arcta
n(b*x/(a*b)^(1/2))*x^5*a*b^3+175*(a*b)^(1/2)*x^4*a*b^2+105*arctan(b*x/(a*b)
^(1/2))*x^3*a^2*b^2+56*(a*b)^(1/2)*x^2*a^2*b-8*(a*b)^(1/2)*a^3)*(b*x^2+a)/(
a*b)^(1/2)/x^3/a^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.23424, size = 504, normalized size = 2.41

$$\left[\frac{210\,b^3x^6+350\,ab^2x^4+112\,a^2bx^2-16\,a^3+105\left(b^3x^7+2\,ab^2x^5+a^2bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{48\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)},\frac{105\,b^3x^6+175\,ab^2x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105*(b^3*x^7
+ 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), 1/24*(105*b^3*x^6 + 175*
a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 + 105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**2)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.645 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=238

$$\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a^4}{6b^6\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5a^3}{2b^6\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{b^6\sqrt{a^2}}$$

```
[Out] (-5*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^5/(8*b^6*(a + b*x^2)^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a^4)/(6*b^6*(a + b*x^2)^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (5*a^3)/(2*b^6*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) + (x^2*(a + b*x^2))/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a*
(a + b*x^2)*Log[a + b*x^2])/(2*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.189469, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a^4}{6b^6\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5a^3}{2b^6\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{b^6\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-5*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^5/(8*b^6*(a + b*x^2)^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a^4)/(6*b^6*(a + b*x^2)^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (5*a^3)/(2*b^6*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) + (x^2*(a + b*x^2))/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a*
(a + b*x^2)*Log[a + b*x^2])/(2*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^5}{\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^{10}}-\frac{a^5}{b^{10}(a+bx)^5}+\frac{5a^4}{b^{10}(a+bx)^4}-\frac{10a^3}{b^{10}(a+bx)^3}+\frac{10a^2}{b^{10}(a+bx)^2}-\frac{5a}{b^{10}(a+bx)}\right)\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a^4}{6b^6\left(a+bx^2\right)^2\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.0382, size = 103, normalized size = 0.43

$$\frac{-48a^2b^3x^6-252a^3b^2x^4-248a^4bx^2-77a^5+48ab^4x^8-60a\left(a+bx^2\right)^4\log\left(a+bx^2\right)+12b^5x^{10}}{24b^6\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x^2 - 252*a^3*b^2*x^4 - 48*a^2*b^3*x^6 + 48*a*b^4*x^8
+ 12*b^5*x^10 - 60*a*(a + b*x^2)^4*Log[a + b*x^2])/(24*b^6*(a + b*x^2)^3*Sq
```

```
rt[(a + b*x^2)^2])
```

Maple [A] time = 0.238, size = 163, normalized size = 0.7

$$-\frac{\left(-12\,b^{5}x^{10}+60\,\ln\left(bx^{2}+a\right)x^{8}ab^{4}-48\,ab^{4}x^{8}+240\,\ln\left(bx^{2}+a\right)x^{6}a^{2}b^{3}+48\,a^{2}b^{3}x^{6}+360\,\ln\left(bx^{2}+a\right)x^{4}a^{3}b^{2}+252}{24\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(-12*b^5*x^10+60*ln(b*x^2+a)*x^8*a*b^4-48*a*b^4*x^8+240*ln(b*x^2+a)*x
^6*a^2*b^3+48*a^2*b^3*x^6+360*ln(b*x^2+a)*x^4*a^3*b^2+252*b^2*a^3*x^4+240*l
n(b*x^2+a)*x^2*a^4*b+248*a^4*b*x^2+60*ln(b*x^2+a)*a^5+77*a^5)*(b*x^2+a)/b^6
/((b*x^2+a)^2)^(5/2)
```

Maxima [A] time = 1.02443, size = 161, normalized size = 0.68

$$\frac{12\,b^{5}x^{10}+48\,ab^{4}x^{8}-48\,a^{2}b^{3}x^{6}-252\,a^{3}b^{2}x^{4}-248\,a^{4}bx^{2}-77\,a^{5}}{24\left(b^{10}x^{8}+4\,ab^{9}x^{6}+6\,a^{2}b^{8}x^{4}+4\,a^{3}b^{7}x^{2}+a^{4}b^{6}\right)}-\frac{5\,a\log\left(bx^{2}+a\right)}{2\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(12*b^5*x^10 + 48*a*b^4*x^8 - 48*a^2*b^3*x^6 - 252*a^3*b^2*x^4 - 248*a
^4*b*x^2 - 77*a^5)/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2
+ a^4*b^6) - 5/2*a*log(b*x^2 + a)/b^6
```

Fricas [A] time = 1.31697, size = 332, normalized size = 1.39

$$\frac{12\,b^{5}x^{10}+48\,ab^{4}x^{8}-48\,a^{2}b^{3}x^{6}-252\,a^{3}b^{2}x^{4}-248\,a^{4}bx^{2}-77\,a^{5}-60\left(ab^{4}x^{8}+4\,a^{2}b^{3}x^{6}+6\,a^{3}b^{2}x^{4}+4\,a^{4}bx^{2}+a^{5}\right)\log}{24\left(b^{10}x^{8}+4\,ab^{9}x^{6}+6\,a^{2}b^{8}x^{4}+4\,a^{3}b^{7}x^{2}+a^{4}b^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 48*a*b^4*x^8 - 48*a^2*b^3*x^6 - 252*a^3*b^2*x^4 - 248*a
^4*b*x^2 - 77*a^5 - 60*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b
*x^2 + a^5)*log(b*x^2 + a))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3
*b^7*x^2 + a^4*b^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**11/((a + b*x**2)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.646 $\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a^3}{3b^5\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a^2}{2b^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{b^5\sqrt{a}\ldots}$$

```
[Out] (2*a)/(b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^4/(8*b^5*(a + b*x^2)^3*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*a^3)/(3*b^5*(a + b*x^2)^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) - (3*a^2)/(2*b^5*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + ((a + b*x^2)*Log[a + b*x^2])/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.162137, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 43}

$$-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a^3}{3b^5\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a^2}{2b^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{b^5\sqrt{a}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a)/(b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^4/(8*b^5*(a + b*x^2)^3*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*a^3)/(3*b^5*(a + b*x^2)^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) - (3*a^2)/(2*b^5*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + ((a + b*x^2)*Log[a + b*x^2])/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
```

```
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^4}{\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{a^4}{b^9(a+bx)^5}-\frac{4a^3}{b^9(a+bx)^4}+\frac{6a^2}{b^9(a+bx)^3}-\frac{4a}{b^9(a+bx)^2}+\frac{1}{b^9(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a^3}{3b^5\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0292026, size = 83, normalized size = 0.42

$$\frac{a\left(88a^2bx^2+25a^3+108ab^2x^4+48b^3x^6\right)+12\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24b^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a*(25*a^3 + 88*a^2*b*x^2 + 108*a*b^2*x^4 + 48*b^3*x^6) + 12*(a + b*x^2)^4*
Log[a + b*x^2])/(24*b^5*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.23, size = 141, normalized size = 0.7

$$\frac{\left(12\ \ln\left(bx^{2}+a\right)x^{8}b^{4}+48\ \ln\left(bx^{2}+a\right)x^{6}ab^{3}+48\,ab^{3}x^{6}+72\ \ln\left(bx^{2}+a\right)x^{4}a^{2}b^{2}+108\,a^{2}b^{2}x^{4}+48\ \ln\left(bx^{2}+a\right)x^{2}a^{3}b}{24\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24*(12*ln(b*x^2+a)*x^8*b^4+48*ln(b*x^2+a)*x^6*a*b^3+48*a*b^3*x^6+72*ln(b*
x^2+a)*x^4*a^2*b^2+108*a^2*b^2*x^4+48*ln(b*x^2+a)*x^2*a^3*b+88*a^3*b*x^2+12
*ln(b*x^2+a)*a^4+25*a^4)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(5/2)
```

Maxima [A] time = 1.02962, size = 134, normalized size = 0.68

$$\frac{48\,ab^{3}x^{6}+108\,a^{2}b^{2}x^{4}+88\,a^{3}bx^{2}+25\,a^{4}}{24\left(b^{9}x^{8}+4\,ab^{8}x^{6}+6\,a^{2}b^{7}x^{4}+4\,a^{3}b^{6}x^{2}+a^{4}b^{5}\right)}+\frac{\log\left(bx^{2}+a\right)}{2\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4)/(b^9*x^8 + 4*
a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5) + 1/2*log(b*x^2 + a)/b
^5
```

Fricas [A] time = 1.26654, size = 282, normalized size = 1.44

$$\frac{48\,ab^{3}x^{6}+108\,a^{2}b^{2}x^{4}+88\,a^{3}bx^{2}+25\,a^{4}+12\left(b^{4}x^{8}+4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}+4\,a^{3}bx^{2}+a^{4}\right)\log\left(bx^{2}+a\right)}{24\left(b^{9}x^{8}+4\,ab^{8}x^{6}+6\,a^{2}b^{7}x^{4}+4\,a^{3}b^{6}x^{2}+a^{4}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4 + 12*(b^4*x^8
+ 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^9*x^8
```

```
+ 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**9/((a + b*x**2)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.647 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{x^8}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] x^8/(8*a*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0404306, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 646, 37}

$$\frac{x^8}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] x^8/(8*a*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^2\right)$$
$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^3}{\left(ab+b^2x\right)^5}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{x^8}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0170194, size = 61, normalized size = 1.49

$$\frac{-4a^2bx^2-a^3-6ab^2x^4-4b^3x^6}{8b^4\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a^3 - 4*a^2*b*x^2 - 6*a*b^2*x^4 - 4*b^3*x^6)/(8*b^4*(a + b*x^2)^3*Sqrt[(a
 + b*x^2)^2])
```

Maple [A] time = 0.173, size = 54, normalized size = 1.3

$$-\frac{\left(bx^2+a\right)\left(4\,b^3x^6+6\,ax^4b^2+4\,a^2bx^2+a^3\right)}{8\,b^4}\left(\left(bx^2+a\right)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/8*(b*x^2+a)*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)/b^4/((b*x^2+a)^2)^(5
/2)
```

Maxima [B] time = 0.995088, size = 197, normalized size = 4.8

$$-\frac{x^4}{2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}b^2}-\frac{a^2}{3\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}b^4}+\frac{a^2}{3\left(b^2\right)^{\frac{7}{2}}\left(x^2+\frac{a}{b}\right)^3}-\frac{a}{4\left(b^2\right)^{\frac{5}{2}}\left(x^2+\frac{a}{b}\right)^2b}-\frac{a^3b}{8\left(b^2\right)^{\frac{9}{2}}\left(x^2+\frac{a}{b}\right)^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^4/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*b^2) - 1/3*a^2/((b^2*x^4 + 2*a*
b*x^2 + a^2)^(3/2)*b^4) + 1/3*a^2/((b^2)^(7/2)*(x^2 + a/b)^3) - 1/4*a/((b^2
)^(5/2)*(x^2 + a/b)^2*b) - 1/8*a^3*b/((b^2)^(9/2)*(x^2 + a/b)^4) + 1/4*a^3/
((b^2)^(5/2)*(x^2 + a/b)^4*b^3)
```

Fricas [B] time = 1.30553, size = 159, normalized size = 3.88

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/(b^8*x^8 + 4*a*b^7*x^6 +
 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^7}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**7/((a + b*x**2)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.648 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] x^6/(24*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + x^6/(8*a*(a + b*x^2)*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi [A] time = 0.0158215, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1109}

$$\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] x^6/(24*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + x^6/(8*a*(a + b*x^2)*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(3/2))
```


Rule 1109

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(2*(d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(d*(m + 3)*(2*a + b*
x^2)), x] - Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*d*(m + 3)
*(p + 1)), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Mathematica [A] time = 0.0165833, size = 50, normalized size = 0.68

$$\frac{-a^2-4abx^2-6b^2x^4}{24b^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a^2 - 4*a*b*x^2 - 6*b^2*x^4)/(24*b^3*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.17, size = 43, normalized size = 0.6

$$-\frac{\left(bx^2+a\right)\left(6\,b^2x^4+4\,abx^2+a^2\right)}{24\,b^3}\left(\left(bx^2+a\right)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(b*x^2+a)*(6*b^2*x^4+4*a*b*x^2+a^2)/b^3/((b*x^2+a)^2)^(5/2)
```

Maxima [A] time = 0.975331, size = 85, normalized size = 1.15

$$-\frac{1}{4\left(b^2\right)^{\frac{5}{2}}\left(x^2+\frac{a}{b}\right)^2}+\frac{ab}{3\left(b^2\right)^{\frac{7}{2}}\left(x^2+\frac{a}{b}\right)^3}-\frac{a^2b^2}{8\left(b^2\right)^{\frac{9}{2}}\left(x^2+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4/((b^2)^(5/2)*(x^2 + a/b)^2) + 1/3*a*b/((b^2)^(7/2)*(x^2 + a/b)^3) - 1/
8*a^2*b^2/((b^2)^(9/2)*(x^2 + a/b)^4)
```

Fricas [A] time = 1.2508, size = 139, normalized size = 1.88

$$-\frac{6\,b^{2}x^{4}+4\,abx^{2}+a^{2}}{24\,\left(b^{7}x^{8}+4\,ab^{6}x^{6}+6\,a^{2}b^{5}x^{4}+4\,a^{3}b^{4}x^{2}+a^{4}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^2*b^5*x^4
+ 4*a^3*b^4*x^2 + a^4*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{5}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.649 $\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=69

$$\frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] -1/(6*b^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + a/(8*b^2*(a + b*x^2)*(a^2 +
2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi [A] time = 0.0537152, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1111, 640, 607}

$$\frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/(6*b^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + a/(8*b^2*(a + b*x^2)*(a^2 +
2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 1111

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^2\right)$$
$$= -\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^2\right)}{2b}$$
$$= -\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}} + \frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Mathematica [A] time = 0.0136127, size = 39, normalized size = 0.57

$$\frac{-a-4bx^2}{24b^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a - 4*b*x^2)/(24*b^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.167, size = 32, normalized size = 0.5

$$-\frac{\left(bx^2+a\right)\left(4\,bx^2+a\right)}{24\,b^2}\left(\left(bx^2+a\right)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(b*x^2+a)*(4*b*x^2+a)/b^2/((b*x^2+a)^2)^(5/2)
```

Maxima [A] time = 0.963772, size = 65, normalized size = 0.94

$$-\frac{1}{6\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{3}{2}}b^{2}}+\frac{a}{8\left(b^{2}\right)^{\frac{5}{2}}\left(x^{2}+\frac{a}{b}\right)^{4}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*b^2) + 1/8*a/((b^2)^(5/2)*(x^2 + a/
b)^4*b)
```

Fricas [A] time = 1.22864, size = 117, normalized size = 1.7

$$-\frac{4\,bx^{2}+a}{24\left(b^{6}x^{8}+4\,ab^{5}x^{6}+6\,a^{2}b^{4}x^{4}+4\,a^{3}b^{3}x^{2}+a^{4}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(4*b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x^2
+ a^4*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{3}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.650 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] -1/(8*b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi [A] time = 0.0251851, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1107, 607}

$$-\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/(8*b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^2\right)$$
$$= -\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Mathematica [A] time = 0.0106135, size = 27, normalized size = 0.71

$$-\frac{a+bx^2}{8b\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -(a + b*x^2)/(8*b*((a + b*x^2)^2)^(5/2))
```

Maple [A] time = 0.173, size = 24, normalized size = 0.6

$$-\frac{bx^2+a}{8\,b}\left(\left(bx^2+a\right)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/8*(b*x^2+a)/b/((b*x^2+a)^2)^(5/2)
```

Maxima [A] time = 0.967018, size = 24, normalized size = 0.63

$$-\frac{1}{8\left(b^2\right)^{\frac{5}{2}}\left(x^2+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8/((b^2)^(5/2)*(x^2 + a/b)^4)
```

Fricas [A] time = 1.23912, size = 95, normalized size = 2.5

$$-\frac{1}{8\left(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.651 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{1}{4a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^4\sqrt{a^2}}$$

```
[Out] 1/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*(a + b*x^2)^3*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) + 1/(6*a^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + 1/(4*a^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2
)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[a + b*x^
2])/(2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.120767, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$\frac{1}{4a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^4\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```


```
[Out] 1/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*(a + b*x^2)^3*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) + 1/(6*a^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + 1/(4*a^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2
)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[a + b*x^
2])/(2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^5}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int\left(\frac{1}{a^5b^5x}-\frac{1}{ab^4(a+bx)^5}-\frac{1}{a^2b^4(a+bx)^4}-\frac{1}{a^3b^4(a+bx)^3}-\frac{1}{a^4b^4(a+bx)^2}-\frac{1}{a^5b^4}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.039439, size = 96, normalized size = 0.43

$$\frac{a\left(52a^2bx^2+25a^3+42ab^2x^4+12b^3x^6\right)+24\log(x)\left(a+bx^2\right)^4-12\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24a^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (a*(25*a^3 + 52*a^2*b*x^2 + 42*a*b^2*x^4 + 12*b^3*x^6) + 24*(a + b*x^2)^4*L
og[x] - 12*(a + b*x^2)^4*Log[a + b*x^2])/(24*a^5*(a + b*x^2)^3*Sqrt[(a + b*
x^2)^2])
```

Maple [A] time = 0.23, size = 193, normalized size = 0.9

$$\left(24 \ln(x) x^8 b^4 - 12 \ln\left(bx^2 + a\right) x^8 b^4 + 96 \ln(x) x^6 a b^3 - 48 \ln\left(bx^2 + a\right) x^6 a b^3 + 12 a b^3 x^6 + 144 \ln(x) x^4 a^2 b^2 - 72 \ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24*(24*ln(x)*x^8*b^4-12*ln(b*x^2+a)*x^8*b^4+96*ln(x)*x^6*a*b^3-48*ln(b*x^
2+a)*x^6*a*b^3+12*a*b^3*x^6+144*ln(x)*x^4*a^2*b^2-72*ln(b*x^2+a)*x^4*a^2*b^
2+42*a^2*b^2*x^4+96*ln(x)*x^2*a^3*b-48*ln(b*x^2+a)*x^2*a^3*b+52*a^3*b*x^2+2
4*a^4*ln(x)-12*ln(b*x^2+a)*a^4+25*a^4)*(b*x^2+a)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33906, size = 378, normalized size = 1.7

$$\frac{12 a b^3 x^6 + 42 a^2 b^2 x^4 + 52 a^3 b x^2 + 25 a^4 - 12 \left(b^4 x^8 + 4 a b^3 x^6 + 6 a^2 b^2 x^4 + 4 a^3 b x^2 + a^4\right) \log\left(bx^2 + a\right) + 24 \left(b^4 x^8 + 4 ab \right.}{24 \left(a^5 b^4 x^8 + 4 a^6 b^3 x^6 + 6 a^7 b^2 x^4 + 4 a^8 b x^2 + a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*a*b^3*x^6 + 42*a^2*b^2*x^4 + 52*a^3*b*x^2 + 25*a^4 - 12*(b^4*x^8 +
 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b*x^2 + a) + 24*(b^4*
x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(x))/(a^5*b^4*x^8
```

```
+ 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*b*x^2 + a^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**2)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.652 $\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{3b}{4a^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{3a^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{8a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{a^5\sqrt{a}}$$

```
[Out] (-2*b)/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(8*a^2*(a + b*x^2)^3*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(3*a^3*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (3*b)/(4*a^4*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a
 + b*x^2)/(2*a^5*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*b*(a + b*x^2)*Lo
g[x])/(a^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*b*(a + b*x^2)*Log[a + b*x^
2])/(2*a^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.141949, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1112, 266, 44}

$$-\frac{3b}{4a^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{3a^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{8a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{a^5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*b)/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(8*a^2*(a + b*x^2)^3*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(3*a^3*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (3*b)/(4*a^4*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a
 + b*x^2)/(2*a^5*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*b*(a + b*x^2)*Lo
g[x])/(a^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*b*(a + b*x^2)*Log[a + b*x^
2])/(2*a^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x^3\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{x^2\left(ab+b^2x\right)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int\left(\frac{1}{a^5b^5x^2}-\frac{5}{a^6b^4x}+\frac{1}{a^2b^3(a+bx)^5}+\frac{2}{a^3b^3(a+bx)^4}+\frac{3}{a^4b^3(a+bx)^3}+\frac{4}{a^5b^3(a}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{8a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{3a^3\left(a+bx^2\right)^2\sqrt{a^2+}}$$

Mathematica [A] time = 0.0474817, size = 119, normalized size = 0.45

$$\frac{-a\left(260a^2b^2x^4+125a^3bx^2+12a^4+210ab^3x^6+60b^4x^8\right)-120bx^2\log(x)\left(a+bx^2\right)^4+60bx^2\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24a^6x^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^2 + 260*a^2*b^2*x^4 + 210*a*b^3*x^6 + 60*b^4*x^8
)) - 120*b*x^2*(a + b*x^2)^4*Log[x] + 60*b*x^2*(a + b*x^2)^4*Log[a + b*x^2]
```

```
)/(24*a^6*x^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.23, size = 219, normalized size = 0.8

$$\frac{\left(60 \ln\left(bx^2+a\right) x^{10} b^5 - 120 b^5 \ln(x) x^{10} + 240 \ln\left(bx^2+a\right) x^8 a b^4 - 480 a b^4 \ln(x) x^8 - 60 a b^4 x^8 + 360 \ln\left(bx^2+a\right) x^6 a^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24*(60*ln(b*x^2+a)*x^10*b^5-120*b^5*ln(x)*x^10+240*ln(b*x^2+a)*x^8*a*b^4-
480*a*b^4*ln(x)*x^8-60*a*b^4*x^8+360*ln(b*x^2+a)*x^6*a^2*b^3-720*a^2*b^3*ln
(x)*x^6-210*a^2*b^3*x^6+240*ln(b*x^2+a)*x^4*a^3*b^2-480*b^2*a^3*ln(x)*x^4-2
60*b^2*a^3*x^4+60*ln(b*x^2+a)*x^2*a^4*b-120*a^4*b*ln(x)*x^2-125*a^4*b*x^2-1
2*a^5)*(b*x^2+a)/x^2/a^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38873, size = 440, normalized size = 1.65

$$-\frac{60 a b^4 x^8 + 210 a^2 b^3 x^6 + 260 a^3 b^2 x^4 + 125 a^4 b x^2 + 12 a^5 - 60 \left(b^5 x^{10} + 4 a b^4 x^8 + 6 a^2 b^3 x^6 + 4 a^3 b^2 x^4 + a^4 b x^2\right) \log\left(bx^2 \cdots\right.}{24 \left(a^6 b^4 x^{10} + 4 a^7 b^3 x^8 + 6 a^8 b^2 x^6 + 4 a^9 b x^4 + a^{10} x^2 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(60*a*b^4*x^8 + 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 + 125*a^4*b*x^2 + 1
2*a^5 - 60*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*b*
x^2)*log(b*x^2 + a) + 120*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b
^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 +
 4*a^9*b*x^4 + a^10*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**2)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.653 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x}{64b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{128}$$

```
[Out] (5*x)/(128*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^5/(8*b*(a + b*x^2)^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*x^3)/(48*b^2*(a + b*x^2)^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) - (5*x)/(64*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(3/2)*b^(7/2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0845826, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$-\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x}{64b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{128}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (5*x)/(128*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^5/(8*b*(a + b*x^2)^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*x^3)/(48*b^2*(a + b*x^2)^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) - (5*x)/(64*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(3/2)*b^(7/2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b^2\left(ab+b^2x^2\right)\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\left(ab+b^2\cdots\right.\right.}{16\sqrt{a^2\cdots}}$$

$$= -\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{64b^3\left(a+b\cdots\right.}$$

$$= \frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+\cdots}}$$

$$= \frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+\cdots}}$$

Mathematica [A] time = 0.039236, size = 105, normalized size = 0.5

$$\frac{\sqrt{a}\sqrt{b}x\left(-55a^2bx^2-15a^3-73ab^2x^4+15b^3x^6\right)+15\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{3/2}b^{7/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-15*a^3 - 55*a^2*b*x^2 - 73*a*b^2*x^4 + 15*b^3*x^6) + 1
5*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(3/2)*b^(7/2)*(a + b*x^
2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.235, size = 172, normalized size = 0.8

$$\frac{bx^2+a}{384\,ab^3}\left(15\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^8b^4+15\,\sqrt{ab}x^7b^3+60\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^6ab^3-73\,\sqrt{ab}x^5ab^2+90\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4a^2b^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(15*arctan(b*x/(a*b)^(1/2))*x^8*b^4+15*(a*b)^(1/2)*x^7*b^3+60*arctan(
b*x/(a*b)^(1/2))*x^6*a*b^3-73*(a*b)^(1/2)*x^5*a*b^2+90*arctan(b*x/(a*b)^(1/
2))*x^4*a^2*b^2-55*(a*b)^(1/2)*x^3*a^2*b+60*arctan(b*x/(a*b)^(1/2))*x^2*a^3
*b-15*(a*b)^(1/2)*x*a^3+15*arctan(b*x/(a*b)^(1/2))*a^4)*(b*x^2+a)/(a*b)^(1/
2)/b^3/a/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57656, size = 684, normalized size = 3.24

$$\left[\frac{30\,ab^4x^7 - 146\,a^2b^3x^5 - 110\,a^3b^2x^3 - 30\,a^4bx - 15\left(b^4x^8 + 4\,ab^3x^6 + 6\,a^2b^2x^4 + 4\,a^3bx^2 + a^4\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{768\left(a^2b^8x^8 + 4\,a^3b^7x^6 + 6\,a^4b^6x^4 + 4\,a^5b^5x^2 + a^6b^4\right)}, \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 - 146*a^2*b^3*x^5 - 110*a^3*b^2*x^3 - 30*a^4*b*x - 15*
(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^8*x^8 + 4*a^3*b^7*x^6 + 6
*a^4*b^6*x^4 + 4*a^5*b^5*x^2 + a^6*b^4), 1/384*(15*a*b^4*x^7 - 73*a^2*b^3*x
^5 - 55*a^3*b^2*x^3 - 15*a^4*b*x + 15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^
4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^8*x^8 + 4*a^
3*b^7*x^6 + 6*a^4*b^6*x^4 + 4*a^5*b^5*x^2 + a^6*b^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**6/((a + b*x**2)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.654 $\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}} dx$

Optimal. Leaf size=212

$$-\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{64ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{16b^2\left(a+bx\right.}$$

```
[Out] (3*x)/(128*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(8*b*(a + b*x^2)^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(16*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) + x/(64*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0878467, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$-\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{64ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{16b^2\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (3*x)/(128*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(8*b*(a + b*x^2)^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(16*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) + x/(64*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{x^4}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3b^2\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right.}{16\sqrt{a^2+}}$$

$$= -\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{64ab^2\left(a+\right.}$$

$$= \frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

Mathematica [A] time = 0.04082, size = 105, normalized size = 0.5

$$\frac{\sqrt{a}\sqrt{b}x\left(-11a^2bx^2-3a^3+11ab^2x^4+3b^3x^6\right)+3\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-3*a^3 - 11*a^2*b*x^2 + 11*a*b^2*x^4 + 3*b^3*x^6) + 3*(
a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*(a + b*x^2)^
3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.228, size = 172, normalized size = 0.8

$$\frac{bx^2+a}{128\,b^2a^2}\left(3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^8b^4+3\,\sqrt{ab}x^7b^3+12\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^6ab^3+11\,\sqrt{ab}x^5ab^2+18\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4a^2b^2-11\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/128*(3*arctan(b*x/(a*b)^(1/2))*x^8*b^4+3*(a*b)^(1/2)*x^7*b^3+12*arctan(b*
x/(a*b)^(1/2))*x^6*a*b^3+11*(a*b)^(1/2)*x^5*a*b^2+18*arctan(b*x/(a*b)^(1/2)
)*x^4*a^2*b^2-11*(a*b)^(1/2)*x^3*a^2*b+12*arctan(b*x/(a*b)^(1/2))*x^2*a^3*b
-3*(a*b)^(1/2)*x*a^3+3*arctan(b*x/(a*b)^(1/2))*a^4)*(b*x^2+a)/(a*b)^(1/2)/a
^2/b^2/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52245, size = 674, normalized size = 3.18

$$\left[\frac{6\,ab^4x^7+22\,a^2b^3x^5-22\,a^3b^2x^3-6\,a^4bx-3\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{256\left(a^3b^7x^8+4\,a^4b^6x^6+6\,a^5b^5x^4+4\,a^6b^4x^2+a^7b^3\right)},\ \frac{3\,ab^4\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/256*(6*a*b^4*x^7 + 22*a^2*b^3*x^5 - 22*a^3*b^2*x^3 - 6*a^4*b*x - 3*(b^4*
x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log((b*x^
2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^7*x^8 + 4*a^4*b^6*x^6 + 6*a^5*
b^5*x^4 + 4*a^6*b^4*x^2 + a^7*b^3), 1/128*(3*a*b^4*x^7 + 11*a^2*b^3*x^5 - 1
1*a^3*b^2*x^3 - 3*a^4*b*x + 3*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^
3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^7*x^8 + 4*a^4*b^6*x^
6 + 6*a^5*b^5*x^4 + 4*a^6*b^4*x^2 + a^7*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.655 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{192a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{8b\left(a+b\right.}$$

```
[Out] (5*x)/(128*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(8*b*(a + b*x^2)^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + x/(48*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]) + (5*x)/(192*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0835851, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 288, 199, 205}

$$\frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{192a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{8b\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (5*x)/(128*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(8*b*(a + b*x^2)^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + x/(48*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]) + (5*x)/(192*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{x^2}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b\left(ab+b\right.\right.}{48a\sqrt{a^2}}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{192a^2b\left(a+\right.}$$

$$= \frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+\right.}$$

$$= \frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+\right.}$$

Mathematica [A] time = 0.0342345, size = 105, normalized size = 0.49

$$\frac{\sqrt{a}\sqrt{b}x\left(73a^2bx^2-15a^3+55ab^2x^4+15b^3x^6\right)+15\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{7/2}b^{3/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-15*a^3 + 73*a^2*b*x^2 + 55*a*b^2*x^4 + 15*b^3*x^6) + 1
5*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(7/2)*b^(3/2)*(a + b*x^
2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.225, size = 172, normalized size = 0.8

$$\frac{bx^2+a}{384\,a^3b}\left(15\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^8b^4+15\,\sqrt{ab}x^7b^3+60\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^6ab^3+55\,\sqrt{ab}x^5ab^2+90\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4a^2b^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(15*arctan(b*x/(a*b)^(1/2))*x^8*b^4+15*(a*b)^(1/2)*x^7*b^3+60*arctan(
b*x/(a*b)^(1/2))*x^6*a*b^3+55*(a*b)^(1/2)*x^5*a*b^2+90*arctan(b*x/(a*b)^(1/
2))*x^4*a^2*b^2+73*(a*b)^(1/2)*x^3*a^2*b+60*arctan(b*x/(a*b)^(1/2))*x^2*a^3
*b-15*(a*b)^(1/2)*x*a^3+15*arctan(b*x/(a*b)^(1/2))*a^4)*(b*x^2+a)/(a*b)^(1/
2)/b/a^3/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57314, size = 684, normalized size = 3.21

$$\left[\frac{30\,ab^4x^7+110\,a^2b^3x^5+146\,a^3b^2x^3-30\,a^4bx-15\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{768\left(a^4b^6x^8+4\,a^5b^5x^6+6\,a^6b^4x^4+4\,a^7b^3x^2+a^8b^2\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 + 110*a^2*b^3*x^5 + 146*a^3*b^2*x^3 - 30*a^4*b*x - 15*
(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^6*x^8 + 4*a^5*b^5*x^6 + 6
*a^6*b^4*x^4 + 4*a^7*b^3*x^2 + a^8*b^2), 1/384*(15*a*b^4*x^7 + 55*a^2*b^3*x
^5 + 73*a^3*b^2*x^3 - 15*a^4*b*x + 15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^
4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^6*x^8 + 4*a^
5*b^5*x^6 + 6*a^6*b^4*x^4 + 4*a^7*b^3*x^2 + a^8*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.656 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{35x\left(a+bx^2\right)^4}{128a^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

```
[Out] (x*(a + b*x^2))/(8*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (7*x*(a + b*x^2)^
2)/(48*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(a + b*x^2)^3)/(192*a
^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(a + b*x^2)^4)/(128*a^4*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*(a + b*x^2)^5*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(128*a^(9/2)*Sqrt[b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
```

Rubi [A] time = 0.07216, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1088, 199, 205}

$$\frac{35x\left(a+bx^2\right)^4}{128a^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

```
[Out] (x*(a + b*x^2))/(8*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (7*x*(a + b*x^2)^
2)/(48*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(a + b*x^2)^3)/(192*a
^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(a + b*x^2)^4)/(128*a^4*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*(a + b*x^2)^5*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(128*a^(9/2)*Sqrt[b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
```

Rule 1088

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(2ab+2b^2x^2\right)^5 \int \frac{1}{\left(2ab+2b^2x^2\right)^5}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{\left(7\left(2ab+2b^2x^2\right)^5\right)\int \frac{1}{\left(2ab+2b^2x^2\right)^4}\,dx}{16ab\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{\left(35\left(2ab+2b^2x^2\right)^5\right)\int \frac{}{(2ab+}}{192a^2b^2\left(a^2+2abx^2+b^2\right.}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b^2x^4\right)^{5/}}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b^2x^4\right)^{5/}}$$

$$= \frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b^2x^4\right)^{5/}}$$

Mathematica [A] time = 0.0382371, size = 105, normalized size = 0.49

$$\frac{\sqrt{a}\sqrt{b}x\left(511a^2bx^2+279a^3+385ab^2x^4+105b^3x^6\right)+105\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{9/2}\sqrt{b}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(279*a^3 + 511*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3*x^6)
+ 105*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(9/2)*Sqrt[b]*(a +
b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.22, size = 169, normalized size = 0.8

$$\frac{bx^2+a}{384\,a^4}\left(105\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^8b^4+105\,\sqrt{ab}x^7b^3+420\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^6ab^3+385\,\sqrt{ab}x^5ab^2+630\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(105*arctan(b*x/(a*b)^(1/2))*x^8*b^4+105*(a*b)^(1/2)*x^7*b^3+420*arct
an(b*x/(a*b)^(1/2))*x^6*a*b^3+385*(a*b)^(1/2)*x^5*a*b^2+630*arctan(b*x/(a*b
)^(1/2))*x^4*a^2*b^2+511*(a*b)^(1/2)*x^3*a^2*b+420*arctan(b*x/(a*b)^(1/2))*
x^2*a^3*b+279*(a*b)^(1/2)*x*a^3+105*arctan(b*x/(a*b)^(1/2))*a^4)*(b*x^2+a)/
(a*b)^(1/2)/a^4/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5553, size = 691, normalized size = 3.24

$$\left[\frac{210\,ab^4x^7+770\,a^2b^3x^5+1022\,a^3b^2x^3+558\,a^4bx-105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x}{bx^2+a}\right.}{768\left(a^5b^5x^8+4\,a^6b^4x^6+6\,a^7b^3x^4+4\,a^8b^2x^2+a^9b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(210*a*b^4*x^7 + 770*a^2*b^3*x^5 + 1022*a^3*b^2*x^3 + 558*a^4*b*x -
105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6
 + 6*a^7*b^3*x^4 + 4*a^8*b^2*x^2 + a^9*b), 1/384*(105*a*b^4*x^7 + 385*a^2*b
^3*x^5 + 511*a^3*b^2*x^3 + 279*a^4*b*x + 105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2
*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^5*x^8
 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8*b^2*x^2 + a^9*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.657 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{315\left(a+bx^2\right)}{128a^5x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21}{64a^3x\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)}+\frac{3}{16a^2x\sqrt{a^2+2abx^2}}$$

```
[Out] 105/(128*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x*(a + b*x^2)^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 3/(16*a^2*x*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + 21/(64*a^3*x*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (315*(a + b*x^2))/(128*a^5*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (315*Sq
rt[b]*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.109376, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$-\frac{315\left(a+bx^2\right)}{128a^5x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21}{64a^3x\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)}+\frac{3}{16a^2x\sqrt{a^2+2abx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 105/(128*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x*(a + b*x^2)^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 3/(16*a^2*x*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + 21/(64*a^3*x*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (315*(a + b*x^2))/(128*a^5*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (315*Sq
rt[b]*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3}{16a^2x\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21b^2\left(a\right.\right.}{16a}$$

$$= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3}{16a^2x\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{64a^3x\left(a\right.}$$

$$= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{16a^2x\left(a+bx^2\right)^2}$$

$$= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{16a^2x\left(a+bx^2\right)^2}$$

$$= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{16a^2x\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.0440012, size = 115, normalized size = 0.46

$$\frac{-\sqrt{a}\left(1533a^2b^2x^4+837a^3bx^2+128a^4+1155ab^3x^6+315b^4x^8\right)-315\sqrt{b}x\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}x\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-(Sqrt[a]*(128*a^4 + 837*a^3*b*x^2 + 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 + 3
15*b^4*x^8)) - 315*Sqrt[b]*x*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(12
8*a^(11/2)*x*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.233, size = 191, normalized size = 0.8

$$-\frac{bx^2+a}{128\,xa^5}\left(315\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^9b^5+315\sqrt{ab}x^8b^4+1260\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^7ab^4+1155\sqrt{ab}x^6ab^3+1890\arctan\left(\frac{bx}{\sqrt{ab}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/128*(315*arctan(b*x/(a*b)^(1/2))*x^9*b^5+315*(a*b)^(1/2)*x^8*b^4+1260*ar
ctan(b*x/(a*b)^(1/2))*x^7*a*b^4+1155*(a*b)^(1/2)*x^6*a*b^3+1890*arctan(b*x/
(a*b)^(1/2))*x^5*a^2*b^3+1533*(a*b)^(1/2)*x^4*a^2*b^2+1260*arctan(b*x/(a*b)
^(1/2))*x^3*a^3*b^2+837*(a*b)^(1/2)*x^2*a^3*b+315*arctan(b*x/(a*b)^(1/2))*x
*a^4*b+128*(a*b)^(1/2)*a^4)*(b*x^2+a)/(a*b)^(1/2)/x/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40395, size = 721, normalized size = 2.87

$$\left[-\frac{630\,b^4x^8+2310\,ab^3x^6+3066\,a^2b^2x^4+1674\,a^3bx^2+256\,a^4-315\left(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x\right)\sqrt{-\frac{b}{a}}\,\mathrm{l}}{256\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/256*(630*b^4*x^8 + 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 + 1674*a^3*b*x^2 +
 256*a^4 - 315*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x
```

```
)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^4*x^9
+ 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), -1/128*(315*b^4*x^8
 + 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 + 837*a^3*b*x^2 + 128*a^4 + 315*(b^4*x
^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*sqrt(b/a)*arctan(x*
sqrt(b/a)))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^
9*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.658 $\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=291

$$\frac{1155b\left(a+bx^2\right)}{128a^6x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{385\left(a+bx^2\right)}{128a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{33}{64a^3x^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{48a^2x^3\left(a+bx^2\right)^2}$$

```
[Out] 231/(128*a^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x^3*(a + b*x^2)^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 11/(48*a^2*x^3*(a + b*x^2)^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) + 33/(64*a^3*x^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (385*(a + b*x^2))/(128*a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (1155*b*(a + b*x^2))/(128*a^6*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (115
5*b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(13/2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.124814, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1112, 290, 325, 205}

$$\frac{1155b\left(a+bx^2\right)}{128a^6x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{385\left(a+bx^2\right)}{128a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{33}{64a^3x^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{48a^2x^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```



```
[Out] 231/(128*a^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x^3*(a + b*x^2)^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 11/(48*a^2*x^3*(a + b*x^2)^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) + 33/(64*a^3*x^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (385*(a + b*x^2))/(128*a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (1155*b*(a + b*x^2))/(128*a^6*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (115
5*b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(13/2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)^4}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(33b}{}$$

$$= \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{64a}$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48a^2x^3\left(a+bx\right.}$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48a^2x^3\left(a+bx\right.}$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48a^2x^3\left(a+bx\right.}$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48a^2x^3\left(a+bx\right.}$$

Mathematica [A] time = 0.0479967, size = 127, normalized size = 0.44

$$\frac{\sqrt{a}\left(16863a^2b^3x^6+9207a^3b^2x^4+1408a^4bx^2-128a^5+12705ab^4x^8+3465b^5x^{10}\right)+3465b^{3/2}x^3\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{13/2}x^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (Sqrt[a]*(-128*a^5 + 1408*a^4*b*x^2 + 9207*a^3*b^2*x^4 + 16863*a^2*b^3*x^6
+ 12705*a*b^4*x^8 + 3465*b^5*x^10) + 3465*b^(3/2)*x^3*(a + b*x^2)^4*ArcTan[
```

```
(Sqrt[b]*x)/Sqrt[a]])/(384*a^(13/2)*x^3*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.224, size = 211, normalized size = 0.7

$$\frac{bx^2+a}{384\,a^6x^3}\left(3465\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^{11}b^6+3465\sqrt{ab}x^{10}b^5+13860\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^9ab^5+12705\sqrt{ab}x^8ab^4+20790\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(3465*arctan(b*x/(a*b)^(1/2))*x^11*b^6+3465*(a*b)^(1/2)*x^10*b^5+1386
0*arctan(b*x/(a*b)^(1/2))*x^9*a*b^5+12705*(a*b)^(1/2)*x^8*a*b^4+20790*arcta
n(b*x/(a*b)^(1/2))*x^7*a^2*b^4+16863*(a*b)^(1/2)*x^6*a^2*b^3+13860*arctan(b
*x/(a*b)^(1/2))*x^5*a^3*b^3+9207*(a*b)^(1/2)*x^4*a^3*b^2+3465*arctan(b*x/(a
*b)^(1/2))*x^3*a^4*b^2+1408*(a*b)^(1/2)*x^2*a^4*b-128*(a*b)^(1/2)*a^5)*(b*x
^2+a)/(a*b)^(1/2)/x^3/a^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38203, size = 815, normalized size = 2.8

$$\left[\frac{6930\,b^5x^{10}+25410\,ab^4x^8+33726\,a^2b^3x^6+18414\,a^3b^2x^4+2816\,a^4bx^2-256\,a^5+3465\left(b^5x^{11}+4\,ab^4x^9+6\,a^2b^3x^7+4\right.}{768\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(6930*b^5*x^10 + 25410*a*b^4*x^8 + 33726*a^2*b^3*x^6 + 18414*a^3*b^2
*x^4 + 2816*a^4*b*x^2 - 256*a^5 + 3465*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*
x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x
^5 + a^10*x^3), 1/384*(3465*b^5*x^10 + 12705*a*b^4*x^8 + 16863*a^2*b^3*x^6
+ 9207*a^3*b^2*x^4 + 1408*a^4*b*x^2 - 128*a^5 + 3465*(b^5*x^11 + 4*a*b^4*x^
9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)
))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**2)**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.659 $\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=298

$$\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}+\frac{3\ldots}{5b\sqrt[3]{a^2\ldots}}$$

```
[Out] (3*x*(a + b*x^2))/(5*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)) + (3*3^(3/4)*Sqrt
[2 - Sqrt[3]]*a^2*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1
 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^
2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 -
 Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*(a^2 + 2*a*b*
x^2 + b^2*x^4)^(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 +
 (b*x^2)/a)^(1/3))^2)])
```

Rubi [A] time = 0.229479, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1113, 321, 236, 219}

$$\frac{3x\left(a+bx^2\right)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}}+\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-\ldots\right)}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2))/(5*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)) + (3*3^(3/4)*Sqrt
[2 - Sqrt[3]]*a^2*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1
 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^
```

```
2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 -
 Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*(a^2 + 2*a*b*
x^2 + b^2*x^4)^(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 +
 (b*x^2)/a)^(1/3))^2)])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{x^2}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3x\left(a+bx^2\right)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(3a\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3x\left(a+bx^2\right)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(9a^2\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right)}{10b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3x\left(a+bx^2\right)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}} + \frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{}}{\left(1-\sqrt{3}-\right.}}}$$

Mathematica [C] time = 0.0326251, size = 64, normalized size = 0.21

$$\frac{3x\left(-a\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\sqrt[3]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2,
-((b*x^2)/a)]))/(5*b*((a + b*x^2)^2)^(1/3))
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt[3]{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

```
[Out] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt[3]{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

3.660 $\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=256

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

```
[Out] -((3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(
1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3]
 - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a
)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a^2
 + 2*a*b*x^2 + b^2*x^4)^(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[
3] - (1 + (b*x^2)/a)^(1/3))^2)]))
```

Rubi [A] time = 0.12314, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1089, 236, 219}

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/3),x]
```

```
[Out] -((3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(
1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3]
 - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a
)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a^2
```

```
 + 2*a*b*x^2 + b^2*x^4)^(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[
3] - (1 + (b*x^2)/a)^(1/3))^2)]))
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(3a\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right)}{2bx\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}{1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}\right)\middle| -\right.}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

Mathematica [C] time = 0.0109351, size = 48, normalized size = 0.19

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/3),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/((
a + b*x^2)^2)^(1/3)
```

Maple [F] time = 0.169, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

```
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

3.661 $\int \frac{1}{x^2 \sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=289

$$\frac{\sqrt{2-\sqrt{3}}\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((a + b*x^2)/(a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3))) + (Sqrt[2 - Sqrt[3]]
*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a
)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*E
llipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (
b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^
(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/
3))^2)])
```

Rubi [A] time = 0.157868, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1113, 325, 236, 219}

$$\frac{\sqrt{2-\sqrt{3}}\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)),x]
```

```
[Out] -((a + b*x^2)/(a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3))) + (Sqrt[2 - Sqrt[3]]
*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a
)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*E
```

```
llipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (
b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^
(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/
3))^2)])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{x^2\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{3a\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right)}{2x\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{2-\sqrt{3}}\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}F}{\sqrt[4]{3}x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-}{\left(1-\sqrt{3}-\right.}}}$$

Mathematica [C] time = 0.0100523, size = 51, normalized size = 0.18

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, -((b*x^2)/a)])/(
x*((a + b*x^2)^2)^(1/3)))
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt[3]{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

```
[Out] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{2}{3}}}{b^2x^6 + 2\,abx^4 + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt[3]{\left(a + bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

3.662 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$

Optimal. Leaf size=618

$$-\frac{3\,3^{3/4}a^2\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{2}b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{\ldots}{2b\left(a^2+2abx^2\ldots\right.}$$

[Out] (-3*x*(a + b*x^2))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (9*a*x*(1 + (b*x^2)/a)^(4/3))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^2*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(4*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (3*3^(3/4)*a^2*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])

Rubi [A] time = 0.411956, antiderivative size = 618, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1113, 288, 235, 304, 219, 1879}

$$-\frac{9ax\left(\frac{bx^2}{a}+1\right)^{4/3}}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}+1-\sqrt{3}+1\right)}-\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{3\,3^{3/4}a^2\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\ldots}}{\sqrt{2}b^2x\left(a^2+2ab\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3),x]
```

```
[Out] (-3*x*(a + b*x^2))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (9*a*x*(1 + (b
*x^2)/a)^(4/3))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 +
(b*x^2)/a)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^2*(1 + (b*x^2)/a)^(4/3)
*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)
/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + S
qrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7
+ 4*Sqrt[3]])/(4*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (
b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (3*3^(3/4)*a^
2*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/
a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 +
(b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*(a^2 + 2*a*b*x^2 + b^2*
x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a
)^(1/3))^2)])
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
```

```
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{x^2}{\left(1+\frac{bx^2}{a}\right)^{4/3}}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= -\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(3a\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\int \frac{1}{\sqrt[3]{1+\frac{bx^2}{a}}}\,dx}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= -\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(9a^2\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right)}{4b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= -\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{\left(9a^2\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right)}{4b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= -\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{9ax\left(1+\frac{bx^2}{a}\right)^{4/3}}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)} + \frac{9\sqrt[4]{3}\sqrt{2}}{}$$

<u>Mathematica</u> [C] time = 0.0331036, size = 64, normalized size = 0.1

$$\frac{3x\left(a+bx^2\right)\left(\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-1\right)}{2b\left(\left(a+bx^2\right)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)*(-1 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/
2, -((b*x^2)/a)]))/(2*b*((a + b*x^2)^2)^(2/3))
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int x^{2} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{-\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

```
[Out] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{x^{2}}{{\left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right)}^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\left(a+bx^2\right)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

3.663 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$

Optimal. Leaf size=609

$$\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\left(\frac{bx^2}{a}+1\right)^{4/3}\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{2}bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}+\frac{3x}{2\left(a^2+2abx^2+b^2\right.}$$

```
[Out] (3*x*(a + b*x^2))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) + (3*x*(1 + (b*x^
2)/a)^(4/3))/(2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 + (b*x^
2)/a)^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a*(1 + (b*x^2)/a)^(4/3)*(1 - (
1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/
3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3]
- (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqr
t[3]])/(4*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)
^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) + (3^(3/4)*a*(1 + (b*x^2
)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1
 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[Arc
Sin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1
/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt
[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rubi [A] time = 0.366157, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1089, 199, 235, 304, 219, 1879}

$$\frac{3x\left(\frac{bx^2}{a}+1\right)^{4/3}}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}+1-\sqrt{3}+1\right)}+\frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}}+\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1+}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}{\sqrt{2}bx\left(a^2+2abx^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2/3),x]
```

```
[Out] (3*x*(a + b*x^2))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) + (3*x*(1 + (b*x^
2)/a)^(4/3))/(2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 + (b*x^
2)/a)^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a*(1 + (b*x^2)/a)^(4/3)*(1 - (
1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/
3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3]
- (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqr
t[3]])/(4*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)
^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) + (3^(3/4)*a*(1 + (b*x^2
)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1
 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[Arc
Sin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1
/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt
[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{4/3}}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= \frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{1}{\sqrt[3]{1+\frac{bx^2}{a}}}\,dx}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= \frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{\left(3a\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right)}{4bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= \frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(3a\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right)}{4bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}} -$$

$$= \frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{3x\left(1+\frac{bx^2}{a}\right)^{4/3}}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+}}{}$$

Mathematica [C] time = 0.0215587, size = 64, normalized size = 0.11

$$-\frac{x\left(a+bx^2\right)\left(\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-3\right)}{2a\left(\left(a+bx^2\right)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2/3),x]
```

```
[Out] -(x*(a + b*x^2)*(-3 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2
, -((b*x^2)/a)]))/(2*a*((a + b*x^2)^2)^(2/3))
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int \left(b^2 x^4 + 2\,ab x^2 + a^2\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

```
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2 x^4 + 2\,ab x^2 + a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(b^2 x^4 + 2\,ab x^2 + a^2\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2 + 2abx^2 + b^2 x^4\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

3.664 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$

Optimal. Leaf size=649

$$-\frac{5\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{2}\sqrt[4]{3}x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b \cdots\right.}$$

```
[Out] (3*(a + b*x^2))/(2*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*(a + b*x^2)^
2)/(2*a^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*b*x*(1 + (b*x^2)/a)^(4/
3))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 + (b*x^2)/a)^(
1/3))) + (5*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^
2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 -
Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b
*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(4
*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1
- Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (5*(1 + (b*x^2)/a)^(4/3)*(1 - (1
+ (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3)
)/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
(1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[
3]])/(Sqrt[2]*3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 +
(b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rubi [A] time = 0.444251, antiderivative size = 649, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1113, 290, 325, 235, 304, 219, 1879}

$$-\frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}}+\frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{5bx\left(\frac{bx^2}{a}+1\right)^{4/3}}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}-\frac{5\left(\frac{bx^2}{a} \cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)),x]
```

```
[Out] (3*(a + b*x^2))/(2*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*(a + b*x^2)^
2)/(2*a^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*b*x*(1 + (b*x^2)/a)^(4/
3))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 + (b*x^2)/a)^(
1/3))) + (5*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^
2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 -
Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b
*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(4
*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1
- Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (5*(1 + (b*x^2)/a)^(4/3)*(1 - (1
+ (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3)
)/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
(1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[
3]])/(Sqrt[2]*3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 +
(b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

<u>Rule 1113</u>

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

<u>Rule 290</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

<u>Rule 325</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

<u>Rule 235</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
```

```
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{4/3} \int \frac{1}{x^2\left(1+\frac{bx^2}{a}\right)^{4/3}}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= \frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(5\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\int \frac{1}{x^2\sqrt[3]{1+\frac{bx^2}{a}}}\,dx}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= \frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(5b\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\int \frac{1}{\sqrt[3]{1+}}}{6a\left(a^2+2abx^2+b^2x^4\right.}$$

$$= \frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(5\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Su}}{4x\left(a^2+\right.}$$

$$= \frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{\left(5\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Su}}{4x\left(a^2+\right.}$$

$$= \frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5bx\left(1+\right.}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/}}$$

Mathematica [C] time = 0.0143864, size = 61, normalized size = 0.09

$$-\frac{\left(a+bx^2\right)\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\left(\left(a+bx^2\right)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)),x]
```

```
[Out] -(((a + b*x^2)*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 4/3, 1/2, -((b
*x^2)/a)])/(a*x*((a + b*x^2)^2)^(2/3)))
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \left(b^2x^4 + 2\,abx^2 + a^2\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

```
[Out] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{2}{3}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}}{b^2x^6 + 2\,abx^4 + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)
```

3.665 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}$$

```
[Out] (2*a^2*(d*x)^(7/2))/(7*d) + (4*a*b*(d*x)^(11/2))/(11*d^3) + (2*b^2*(d*x)^(1
5/2))/(15*d^5)
```

Rubi [A] time = 0.0136195, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(7/2))/(7*d) + (4*a*b*(d*x)^(11/2))/(11*d^3) + (2*b^2*(d*x)^(1
5/2))/(15*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)dx &= \int \left(a^2(dx)^{5/2}+\frac{2ab(dx)^{9/2}}{d^2}+\frac{b^2(dx)^{13/2}}{d^4}\right)dx\\ &= \frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}\end{aligned}$$

Mathematica [A] time = 0.014221, size = 33, normalized size = 0.65

$$\frac{2x(dx)^{5/2}\left(165a^2+210abx^2+77b^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*(d*x)^(5/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

Maple [A] time = 0.046, size = 30, normalized size = 0.6

$$\frac{2\,x\left(77\,b^2x^4+210\,abx^2+165\,a^2\right)}{1155}\,(dx)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/1155*x*(77*b^2*x^4+210*a*b*x^2+165*a^2)*(d*x)^(5/2)
```

Maxima [A] time = 0.954908, size = 55, normalized size = 1.08

$$\frac{2\left(77\,(dx)^{\frac{15}{2}}\,b^2+210\,(dx)^{\frac{11}{2}}\,abd^2+165\,(dx)^{\frac{7}{2}}\,a^2d^4\right)}{1155\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 2/1155*(77*(d*x)^(15/2)*b^2 + 210*(d*x)^(11/2)*a*b*d^2 + 165*(d*x)^(7/2)*a^
2*d^4)/d^5
```

Fricas [A] time = 1.2434, size = 96, normalized size = 1.88

$$\frac{2}{1155}\left(77\,b^2d^2x^7+210\,abd^2x^5+165\,a^2d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^2*x^7 + 210*a*b*d^2*x^5 + 165*a^2*d^2*x^3)*sqrt(d*x)
```

Sympy [A] time = 2.58678, size = 49, normalized size = 0.96

$$\frac{2a^2d^{\frac{5}{2}}x^{\frac{7}{2}}}{7}+\frac{4abd^{\frac{5}{2}}x^{\frac{11}{2}}}{11}+\frac{2b^2d^{\frac{5}{2}}x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*d**(5/2)*x**(7/2)/7 + 4*a*b*d**(5/2)*x**(11/2)/11 + 2*b**2*d**(5/2)*
x**(15/2)/15
```

Giac [A] time = 1.11216, size = 65, normalized size = 1.27

$$\frac{2}{15}\sqrt{dx}b^2d^2x^7+\frac{4}{11}\sqrt{dx}abd^2x^5+\frac{2}{7}\sqrt{dx}a^2d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(d*x)*b^2*d^2*x^7 + 4/11*sqrt(d*x)*a*b*d^2*x^5 + 2/7*sqrt(d*x)*a^2
*d^2*x^3
```

3.666 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

```
[Out] (2*a^2*(d*x)^(5/2))/(5*d) + (4*a*b*(d*x)^(9/2))/(9*d^3) + (2*b^2*(d*x)^(13/
2))/(13*d^5)
```

Rubi [A] time = 0.0142823, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(5/2))/(5*d) + (4*a*b*(d*x)^(9/2))/(9*d^3) + (2*b^2*(d*x)^(13/
2))/(13*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)dx &= \int \left(a^2(dx)^{3/2}+\frac{2ab(dx)^{7/2}}{d^2}+\frac{b^2(dx)^{11/2}}{d^4}\right)dx\\ &= \frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}\end{aligned}$$

Mathematica [A] time = 0.0111555, size = 33, normalized size = 0.65

$$\frac{2}{585}x(dx)^{3/2}\left(117a^2+130abx^2+45b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*(d*x)^(3/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

Maple [A] time = 0.047, size = 30, normalized size = 0.6

$$\frac{2x\left(45b^2x^4+130abx^2+117a^2\right)}{585}(dx)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/585*x*(45*b^2*x^4+130*a*b*x^2+117*a^2)*(d*x)^(3/2)
```

Maxima [A] time = 0.954147, size = 55, normalized size = 1.08

$$\frac{2\left(45\,(dx)^{\frac{13}{2}}b^2+130\,(dx)^{\frac{9}{2}}abd^2+117\,(dx)^{\frac{5}{2}}a^2d^4\right)}{585\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 2/585*(45*(d*x)^(13/2)*b^2 + 130*(d*x)^(9/2)*a*b*d^2 + 117*(d*x)^(5/2)*a^2*
d^4)/d^5
```

Fricas [A] time = 1.21244, size = 86, normalized size = 1.69

$$\frac{2}{585}\left(45\,b^2dx^6+130\,abdx^4+117\,a^2dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d*x^6 + 130*a*b*d*x^4 + 117*a^2*d*x^2)*sqrt(d*x)
```

Sympy [A] time = 1.1575, size = 49, normalized size = 0.96

$$\frac{2a^2d^{\frac{3}{2}}x^{\frac{5}{2}}}{5} + \frac{4abd^{\frac{3}{2}}x^{\frac{9}{2}}}{9} + \frac{2b^2d^{\frac{3}{2}}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*d**(3/2)*x**(5/2)/5 + 4*a*b*d**(3/2)*x**(9/2)/9 + 2*b**2*d**(3/2)*x*
*(13/2)/13
```

Giac [A] time = 1.12954, size = 57, normalized size = 1.12

$$\frac{2}{13}\sqrt{dx}b^2dx^6 + \frac{4}{9}\sqrt{dx}abdx^4 + \frac{2}{5}\sqrt{dx}a^2dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 2/13*sqrt(d*x)*b^2*d*x^6 + 4/9*sqrt(d*x)*a*b*d*x^4 + 2/5*sqrt(d*x)*a^2*d*x^
2
```

3.667 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

```
[Out] (2*a^2*(d*x)^(3/2))/(3*d) + (4*a*b*(d*x)^(7/2))/(7*d^3) + (2*b^2*(d*x)^(11/
2))/(11*d^5)
```

Rubi [A] time = 0.0130496, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(3/2))/(3*d) + (4*a*b*(d*x)^(7/2))/(7*d^3) + (2*b^2*(d*x)^(11/
2))/(11*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx &= \int \left(a^2\sqrt{dx}+\frac{2ab(dx)^{5/2}}{d^2}+\frac{b^2(dx)^{9/2}}{d^4}\right)\,dx\\ &= \frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}\end{aligned}$$

Mathematica [A] time = 0.0086985, size = 33, normalized size = 0.65

$$\frac{2}{231}x\sqrt{dx}\left(77a^2+66abx^2+21b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*Sqrt[d*x]*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

Maple [A] time = 0.047, size = 30, normalized size = 0.6

$$\frac{2x\left(21\,b^2x^4+66\,abx^2+77\,a^2\right)}{231}\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x)
```

```
[Out] 2/231*x*(21*b^2*x^4+66*a*b*x^2+77*a^2)*(d*x)^(1/2)
```

Maxima [A] time = 0.974603, size = 55, normalized size = 1.08

$$\frac{2\left(21\,(dx)^{\frac{11}{2}}\,b^2+66\,(dx)^{\frac{7}{2}}\,abd^2+77\,(dx)^{\frac{3}{2}}\,a^2d^4\right)}{231\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*(d*x)^(11/2)*b^2 + 66*(d*x)^(7/2)*a*b*d^2 + 77*(d*x)^(3/2)*a^2*d^
4)/d^5
```

Fricas [A] time = 1.29525, size = 73, normalized size = 1.43

$$\frac{2}{231}\left(21\,b^2x^5+66\,abx^3+77\,a^2x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(d*x)
```

Sympy [A] time = 0.438113, size = 49, normalized size = 0.96

$$\frac{2a^2\sqrt{d}x^{\frac{3}{2}}}{3}+\frac{4ab\sqrt{d}x^{\frac{7}{2}}}{7}+\frac{2b^2\sqrt{d}x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)*(d*x)**(1/2),x)
```

```
[Out] 2*a**2*sqrt(d)*x**(3/2)/3 + 4*a*b*sqrt(d)*x**(7/2)/7 + 2*b**2*sqrt(d)*x**(1
1/2)/11
```

Giac [A] time = 1.11155, size = 61, normalized size = 1.2

$$\frac{2\left(21\sqrt{dx}b^2dx^5+66\sqrt{dx}abdx^3+77\sqrt{dx}a^2dx\right)}{231\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/231*(21*sqrt(d*x)*b^2*d*x^5 + 66*sqrt(d*x)*a*b*d*x^3 + 77*sqrt(d*x)*a^2*d
*x)/d
```

3.668 $\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx$

Optimal. Leaf size=49

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

```
[Out] (2*a^2*Sqrt[d*x])/d + (4*a*b*(d*x)^(5/2))/(5*d^3) + (2*b^2*(d*x)^(9/2))/(9*
d^5)
```

Rubi [A] time = 0.0131858, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

```
[Out] (2*a^2*Sqrt[d*x])/d + (4*a*b*(d*x)^(5/2))/(5*d^3) + (2*b^2*(d*x)^(9/2))/(9*
d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx &= \int \left(\frac{a^2}{\sqrt{dx}}+\frac{2ab(dx)^{3/2}}{d^2}+\frac{b^2(dx)^{7/2}}{d^4}\right)dx\\ &= \frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}\end{aligned}$$

Mathematica [A] time = 0.0103703, size = 33, normalized size = 0.67

$$\frac{2(45a^2x + 18abx^3 + 5b^2x^5)}{45\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

```
[Out] (2*(45*a^2*x + 18*a*b*x^3 + 5*b^2*x^5))/(45*Sqrt[d*x])
```

Maple [A] time = 0.047, size = 30, normalized size = 0.6

$$\frac{(10\,b^2x^4 + 36\,abx^2 + 90\,a^2)\,x}{45}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x)
```

```
[Out] 2/45*(5*b^2*x^4+18*a*b*x^2+45*a^2)*x/(d*x)^(1/2)
```

Maxima [A] time = 0.96012, size = 55, normalized size = 1.12

$$\frac{2\left(45\,\sqrt{dx}a^2 + \frac{5\,(dx)^{\frac{9}{2}}b^2}{d^4} + \frac{18\,(dx)^{\frac{5}{2}}ab}{d^2}\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(45*sqrt(d*x)*a^2 + 5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)/d
```

Fricas [A] time = 1.22918, size = 70, normalized size = 1.43

$$\frac{2\,(5\,b^2x^4 + 18\,abx^2 + 45\,a^2)\sqrt{dx}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(d*x)/d
```

Sympy [A] time = 0.596318, size = 48, normalized size = 0.98

$$\frac{2a^2\sqrt{x}}{\sqrt{d}} + \frac{4abx^{\frac{5}{2}}}{5\sqrt{d}} + \frac{2b^2x^{\frac{9}{2}}}{9\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x)/sqrt(d) + 4*a*b*x**(5/2)/(5*sqrt(d)) + 2*b**2*x**(9/2)/(9*sq
rt(d))
```

Giac [A] time = 1.11453, size = 55, normalized size = 1.12

$$\frac{2\left(5\sqrt{dx}b^2x^4 + 18\sqrt{dx}abx^2 + 45\sqrt{dx}a^2\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(5*sqrt(d*x)*b^2*x^4 + 18*sqrt(d*x)*a*b*x^2 + 45*sqrt(d*x)*a^2)/d
```

3.669 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

```
[Out] (-2*a^2)/(d*Sqrt[d*x]) + (4*a*b*(d*x)^(3/2))/(3*d^3) + (2*b^2*(d*x)^(7/2))/
(7*d^5)
```

Rubi [A] time = 0.0149061, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^2)/(d*Sqrt[d*x]) + (4*a*b*(d*x)^(3/2))/(3*d^3) + (2*b^2*(d*x)^(7/2))/
(7*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx &= \int \left(\frac{a^2}{(dx)^{3/2}}+\frac{2ab\sqrt{dx}}{d^2}+\frac{b^2(dx)^{5/2}}{d^4}\right)dx\\ &= -\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}\end{aligned}$$

Mathematica [A] time = 0.0114596, size = 33, normalized size = 0.67

$$\frac{2x\left(-21a^2+14abx^2+3b^2x^4\right)}{21(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-21*a^2 + 14*a*b*x^2 + 3*b^2*x^4))/(21*(d*x)^(3/2))
```

Maple [A] time = 0.049, size = 30, normalized size = 0.6

$$-\frac{\left(-6\,b^2x^4-28\,abx^2+42\,a^2\right)x}{21}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x)
```

```
[Out] -2/21*(-3*b^2*x^4-14*a*b*x^2+21*a^2)*x/(d*x)^(3/2)
```

Maxima [A] time = 0.972855, size = 59, normalized size = 1.2

$$-\frac{2\left(\frac{21\,a^2}{\sqrt{dx}}-\frac{3\,(dx)^{\frac{7}{2}}b^2+14\,(dx)^{\frac{3}{2}}abd^2}{d^4}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/21*(21*a^2/sqrt(d*x) - (3*(d*x)^(7/2)*b^2 + 14*(d*x)^(3/2)*a*b*d^2)/d^4)
/d
```

Fricas [A] time = 1.1699, size = 78, normalized size = 1.59

$$\frac{2\left(3\,b^2x^4+14\,abx^2-21\,a^2\right)\sqrt{dx}}{21\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)*sqrt(d*x)/(d^2*x)
```

Sympy [A] time = 0.623172, size = 48, normalized size = 0.98

$$-\frac{2a^2}{d^{\frac{3}{2}}\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3d^{\frac{3}{2}}}+\frac{2b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(3/2),x)
```

```
[Out] -2*a**2/(d**(3/2)*sqrt(x)) + 4*a*b*x**(3/2)/(3*d**(3/2)) + 2*b**2*x**(7/2)/
(7*d**(3/2))
```

Giac [A] time = 1.107, size = 69, normalized size = 1.41

$$-\frac{2\left(\frac{21\,a^2}{\sqrt{dx}}-\frac{3\,\sqrt{dx}b^2d^{27}x^3+14\,\sqrt{dx}abd^{27}x}{d^{28}}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/21*(21*a^2/sqrt(d*x) - (3*sqrt(d*x)*b^2*d^27*x^3 + 14*sqrt(d*x)*a*b*d^27
*x)/d^28)/d
```

3.670 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}} dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

```
[Out] (-2*a^2)/(3*d*(d*x)^(3/2)) + (4*a*b*Sqrt[d*x])/d^3 + (2*b^2*(d*x)^(5/2))/(5
*d^5)
```

Rubi [A] time = 0.0144974, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^2)/(3*d*(d*x)^(3/2)) + (4*a*b*Sqrt[d*x])/d^3 + (2*b^2*(d*x)^(5/2))/(5
*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}}\,dx &= \int \left(\frac{a^2}{(dx)^{5/2}}+\frac{2ab}{d^2\sqrt{dx}}+\frac{b^2(dx)^{3/2}}{d^4}\right) dx\\ &= -\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}\end{aligned}$$

Mathematica [A] time = 0.011717, size = 33, normalized size = 0.67

$$\frac{x\left(-10a^2+60abx^2+6b^2x^4\right)}{15(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

```
[Out] (x*(-10*a^2 + 60*a*b*x^2 + 6*b^2*x^4))/(15*(d*x)^(5/2))
```

Maple [A] time = 0.046, size = 30, normalized size = 0.6

$$-\frac{\left(-6\,b^2x^4-60\,abx^2+10\,a^2\right)x}{15}\,(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x)
```

```
[Out] -2/15*(-3*b^2*x^4-30*a*b*x^2+5*a^2)*x/(d*x)^(5/2)
```

Maxima [A] time = 0.975606, size = 58, normalized size = 1.18

$$-\frac{2\left(\frac{5\,a^2}{(dx)^{\frac{3}{2}}}-\frac{3\left((dx)^{\frac{5}{2}}b^2+10\,\sqrt{dx}abd^2\right)}{d^4}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*(5*a^2/(d*x)^(3/2) - 3*((d*x)^(5/2)*b^2 + 10*sqrt(d*x)*a*b*d^2)/d^4)/
d
```

Fricas [A] time = 1.30104, size = 80, normalized size = 1.63

$$\frac{2\left(3\,b^2x^4+30\,abx^2-5\,a^2\right)\sqrt{dx}}{15\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)*sqrt(d*x)/(d^3*x^2)
```

Sympy [A] time = 0.880359, size = 48, normalized size = 0.98

$$-\frac{2a^2}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{4ab\sqrt{x}}{d^{\frac{5}{2}}}+\frac{2b^2x^{\frac{5}{2}}}{5d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(5/2),x)
```

```
[Out] -2*a**2/(3*d**(5/2)*x**(3/2)) + 4*a*b*sqrt(x)/d**(5/2) + 2*b**2*x**(5/2)/(5
*d**(5/2))
```

Giac [A] time = 1.12469, size = 72, normalized size = 1.47

$$-\frac{2\left(\frac{5\,a^2d}{\sqrt{dx}x}-\frac{3\left(\sqrt{dx}b^2d^{10}x^2+10\,\sqrt{dx}abd^{10}\right)}{d^{10}}\right)}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*a^2*d/(sqrt(d*x)*x) - 3*(sqrt(d*x)*b^2*d^10*x^2 + 10*sqrt(d*x)*a*b
*d^10)/d^10)/d^3
```

3.671 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

```
[Out] (-2*a^2)/(5*d*(d*x)^(5/2)) - (4*a*b)/(d^3*Sqrt[d*x]) + (2*b^2*(d*x)^(3/2))/
(3*d^5)
```

Rubi [A] time = 0.0130246, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^2)/(5*d*(d*x)^(5/2)) - (4*a*b)/(d^3*Sqrt[d*x]) + (2*b^2*(d*x)^(3/2))/
(3*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx &= \int \left(\frac{a^2}{(dx)^{7/2}}+\frac{2ab}{d^2(dx)^{3/2}}+\frac{b^2\sqrt{dx}}{d^4}\right)dx\\ &= -\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}\end{aligned}$$

Mathematica [A] time = 0.0127884, size = 38, normalized size = 0.78

$$\frac{2\sqrt{dx}\left(-3a^2-30abx^2+5b^2x^4\right)}{15d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*d^4*x^3)
```

Maple [A] time = 0.046, size = 30, normalized size = 0.6

$$-\frac{\left(-10\,b^2x^4+60\,abx^2+6\,a^2\right)x}{15}(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x)
```

```
[Out] -2/15*(-5*b^2*x^4+30*a*b*x^2+3*a^2)*x/(d*x)^(7/2)
```

Maxima [A] time = 0.968787, size = 63, normalized size = 1.29

$$\frac{2\left(\frac{5\,(dx)^{\frac{3}{2}}b^2}{d^4}-\frac{3\left(10\,abd^2x^2+a^2d^2\right)}{(dx)^{\frac{5}{2}}d^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*(d*x)^(3/2)*b^2/d^4 - 3*(10*a*b*d^2*x^2 + a^2*d^2)/((d*x)^(5/2)*d^2
))/d
```

Fricas [A] time = 1.2439, size = 80, normalized size = 1.63

$$\frac{2\left(5\,b^2x^4-30\,abx^2-3\,a^2\right)\sqrt{dx}}{15\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)*sqrt(d*x)/(d^4*x^3)
```

Sympy [A] time = 1.80933, size = 48, normalized size = 0.98

$$-\frac{2a^2}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{4ab}{d^{\frac{7}{2}}\sqrt{x}}+\frac{2b^2x^{\frac{3}{2}}}{3d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(7/2),x)
```

```
[Out] -2*a**2/(5*d**(7/2)*x**(5/2)) - 4*a*b/(d**(7/2)*sqrt(x)) + 2*b**2*x**(3/2)/
(3*d**(7/2))
```

Giac [A] time = 1.13409, size = 65, normalized size = 1.33

$$\frac{2\left(5\,\sqrt{dx}b^2x-\frac{3\left(10\,abd^3x^2+a^2d^3\right)}{\sqrt{dx}d^2x^2}\right)}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*sqrt(d*x)*b^2*x - 3*(10*a*b*d^3*x^2 + a^2*d^3)/(sqrt(d*x)*d^2*x^2))
/d^4
```

3.672 $\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{2a^4(dx)^{7/2}}{7d}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

```
[Out] (2*a^4*(d*x)^(7/2))/(7*d) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (4*a^2*b^2*(d
*x)^(15/2))/(5*d^5) + (8*a*b^3*(d*x)^(19/2))/(19*d^7) + (2*b^4*(d*x)^(23/2)
)/(23*d^9)
```

Rubi [A] time = 0.0448823, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{2a^4(dx)^{7/2}}{7d}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(7/2))/(7*d) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (4*a^2*b^2*(d
*x)^(15/2))/(5*d^5) + (8*a*b^3*(d*x)^(19/2))/(19*d^7) + (2*b^4*(d*x)^(23/2)
)/(23*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\int (dx)^{5/2}\left(ab+b^2x^2\right)^4\,dx}{b^4}$$
$$=\frac{\int \left(a^4b^4(dx)^{5/2}+\frac{4a^3b^5(dx)^{9/2}}{d^2}+\frac{6a^2b^6(dx)^{13/2}}{d^4}+\frac{4ab^7(dx)^{17/2}}{d^6}+\frac{b^8(dx)^{21/2}}{d^8}\right)dx}{b^4}$$
$$=\frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

Mathematica [A] time = 0.0213344, size = 55, normalized size = 0.6

$$\frac{2x(dx)^{5/2}\left(67298a^2b^2x^4+61180a^3bx^2+24035a^4+35420ab^3x^6+7315b^4x^8\right)}{168245}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(24035*a^4 + 61180*a^3*b*x^2 + 67298*a^2*b^2*x^4 + 35420*a
*b^3*x^6 + 7315*b^4*x^8))/168245
```

Maple [A] time = 0.048, size = 52, normalized size = 0.6

$$\frac{2\,x\left(7315\,b^4x^8+35420\,ab^3x^6+67298\,a^2b^2x^4+61180\,a^3bx^2+24035\,a^4\right)}{168245}(dx)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/168245*x*(7315*b^4*x^8+35420*a*b^3*x^6+67298*a^2*b^2*x^4+61180*a^3*b*x^2+
24035*a^4)*(d*x)^(5/2)
```

Maxima [A] time = 1.08328, size = 99, normalized size = 1.09

$$\frac{2\left(7315\,(dx)^{\frac{23}{2}}\,b^4+35420\,(dx)^{\frac{19}{2}}\,ab^3d^2+67298\,(dx)^{\frac{15}{2}}\,a^2b^2d^4+61180\,(dx)^{\frac{11}{2}}\,a^3bd^6+24035\,(dx)^{\frac{7}{2}}\,a^4d^8\right)}{168245\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/168245*(7315*(d*x)^(23/2)*b^4 + 35420*(d*x)^(19/2)*a*b^3*d^2 + 67298*(d*x
)^(15/2)*a^2*b^2*d^4 + 61180*(d*x)^(11/2)*a^3*b*d^6 + 24035*(d*x)^(7/2)*a^4
*d^8)/d^9
```

Fricas [A] time = 1.22778, size = 173, normalized size = 1.9

$$\frac{2}{168245}\left(7315\,b^4d^2x^{11}+35420\,ab^3d^2x^9+67298\,a^2b^2d^2x^7+61180\,a^3bd^2x^5+24035\,a^4d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*b^4*d^2*x^11 + 35420*a*b^3*d^2*x^9 + 67298*a^2*b^2*d^2*x^7 +
 61180*a^3*b*d^2*x^5 + 24035*a^4*d^2*x^3)*sqrt(d*x)
```

Sympy [A] time = 5.3947, size = 90, normalized size = 0.99

$$\frac{2a^4d^{\frac{5}{2}}x^{\frac{7}{2}}}{7}+\frac{8a^3bd^{\frac{5}{2}}x^{\frac{11}{2}}}{11}+\frac{4a^2b^2d^{\frac{5}{2}}x^{\frac{15}{2}}}{5}+\frac{8ab^3d^{\frac{5}{2}}x^{\frac{19}{2}}}{19}+\frac{2b^4d^{\frac{5}{2}}x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*d**(5/2)*x**(7/2)/7 + 8*a**3*b*d**(5/2)*x**(11/2)/11 + 4*a**2*b**2*d
**(5/2)*x**(15/2)/5 + 8*a*b**3*d**(5/2)*x**(19/2)/19 + 2*b**4*d**(5/2)*x**(
23/2)/23
```

Giac [A] time = 1.11827, size = 116, normalized size = 1.27

$$\frac{2}{23}\sqrt{dx}b^4d^2x^{11}+\frac{8}{19}\sqrt{dx}ab^3d^2x^9+\frac{4}{5}\sqrt{dx}a^2b^2d^2x^7+\frac{8}{11}\sqrt{dx}a^3bd^2x^5+\frac{2}{7}\sqrt{dx}a^4d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/23*sqrt(d*x)*b^4*d^2*x^11 + 8/19*sqrt(d*x)*a*b^3*d^2*x^9 + 4/5*sqrt(d*x)*
a^2*b^2*d^2*x^7 + 8/11*sqrt(d*x)*a^3*b*d^2*x^5 + 2/7*sqrt(d*x)*a^4*d^2*x^3
```

3.673 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=91

$$\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{2a^4(dx)^{5/2}}{5d}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

```
[Out] (2*a^4*(d*x)^(5/2))/(5*d) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (12*a^2*b^2*(d*
x)^(13/2))/(13*d^5) + (8*a*b^3*(d*x)^(17/2))/(17*d^7) + (2*b^4*(d*x)^(21/2)
)/(21*d^9)
```

Rubi [A] time = 0.0439847, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{2a^4(dx)^{5/2}}{5d}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(5/2))/(5*d) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (12*a^2*b^2*(d*
x)^(13/2))/(13*d^5) + (8*a*b^3*(d*x)^(17/2))/(17*d^7) + (2*b^4*(d*x)^(21/2)
)/(21*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2 dx = \frac{\int (dx)^{3/2}\left(ab+b^2x^2\right)^4 dx}{b^4}$$
$$= \frac{\int \left(a^4b^4(dx)^{3/2}+\frac{4a^3b^5(dx)^{7/2}}{d^2}+\frac{6a^2b^6(dx)^{11/2}}{d^4}+\frac{4ab^7(dx)^{15/2}}{d^6}+\frac{b^8(dx)^{19/2}}{d^8}\right) dx}{b^4}$$
$$= \frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

Mathematica [A] time = 0.0178492, size = 55, normalized size = 0.6

$$\frac{2x(dx)^{3/2}\left(32130a^2b^2x^4+30940a^3bx^2+13923a^4+16380ab^3x^6+3315b^4x^8\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(13923*a^4 + 30940*a^3*b*x^2 + 32130*a^2*b^2*x^4 + 16380*a
*b^3*x^6 + 3315*b^4*x^8))/69615
```

Maple [A] time = 0.049, size = 52, normalized size = 0.6

$$\frac{2\,x\left(3315\,b^4x^8+16380\,ab^3x^6+32130\,a^2b^2x^4+30940\,a^3bx^2+13923\,a^4\right)}{69615}(dx)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/69615*x*(3315*b^4*x^8+16380*a*b^3*x^6+32130*a^2*b^2*x^4+30940*a^3*b*x^2+1
3923*a^4)*(d*x)^(3/2)
```

Maxima [A] time = 1.00658, size = 99, normalized size = 1.09

$$\frac{2\left(3315\,(dx)^{\frac{21}{2}}b^4+16380\,(dx)^{\frac{17}{2}}ab^3d^2+32130\,(dx)^{\frac{13}{2}}a^2b^2d^4+30940\,(dx)^{\frac{9}{2}}a^3bd^6+13923\,(dx)^{\frac{5}{2}}a^4d^8\right)}{69615\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/69615*(3315*(d*x)^(21/2)*b^4 + 16380*(d*x)^(17/2)*a*b^3*d^2 + 32130*(d*x)
^(13/2)*a^2*b^2*d^4 + 30940*(d*x)^(9/2)*a^3*b*d^6 + 13923*(d*x)^(5/2)*a^4*d
^8)/d^9
```

Fricas [A] time = 1.32095, size = 158, normalized size = 1.74

$$\frac{2}{69615}\left(3315\,b^4dx^{10}+16380\,ab^3dx^8+32130\,a^2b^2dx^6+30940\,a^3bdx^4+13923\,a^4dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^4*d*x^10 + 16380*a*b^3*d*x^8 + 32130*a^2*b^2*d*x^6 + 30940*
a^3*b*d*x^4 + 13923*a^4*d*x^2)*sqrt(d*x)
```

Sympy [A] time = 2.77763, size = 90, normalized size = 0.99

$$\frac{2a^4d^{\frac{3}{2}}x^{\frac{5}{2}}}{5}+\frac{8a^3bd^{\frac{3}{2}}x^{\frac{9}{2}}}{9}+\frac{12a^2b^2d^{\frac{3}{2}}x^{\frac{13}{2}}}{13}+\frac{8ab^3d^{\frac{3}{2}}x^{\frac{17}{2}}}{17}+\frac{2b^4d^{\frac{3}{2}}x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*d**(3/2)*x**(5/2)/5 + 8*a**3*b*d**(3/2)*x**(9/2)/9 + 12*a**2*b**2*d*
*(3/2)*x**(13/2)/13 + 8*a*b**3*d**(3/2)*x**(17/2)/17 + 2*b**4*d**(3/2)*x**(
21/2)/21
```

Giac [A] time = 1.11493, size = 103, normalized size = 1.13

$$\frac{2}{21}\sqrt{dx}b^4dx^{10}+\frac{8}{17}\sqrt{dx}ab^3dx^8+\frac{12}{13}\sqrt{dx}a^2b^2dx^6+\frac{8}{9}\sqrt{dx}a^3bdx^4+\frac{2}{5}\sqrt{dx}a^4dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/21*sqrt(d*x)*b^4*d*x^10 + 8/17*sqrt(d*x)*a*b^3*d*x^8 + 12/13*sqrt(d*x)*a^
2*b^2*d*x^6 + 8/9*sqrt(d*x)*a^3*b*d*x^4 + 2/5*sqrt(d*x)*a^4*d*x^2
```

3.674 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{2a^4(dx)^{3/2}}{3d}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

```
[Out] (2*a^4*(d*x)^(3/2))/(3*d) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (12*a^2*b^2*(d*
x)^(11/2))/(11*d^5) + (8*a*b^3*(d*x)^(15/2))/(15*d^7) + (2*b^4*(d*x)^(19/2)
)/(19*d^9)
```

Rubi [A] time = 0.040864, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{2a^4(dx)^{3/2}}{3d}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(3/2))/(3*d) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (12*a^2*b^2*(d*
x)^(11/2))/(11*d^5) + (8*a*b^3*(d*x)^(15/2))/(15*d^7) + (2*b^4*(d*x)^(19/2)
)/(19*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx = \frac{\int \sqrt{dx}\left(ab+b^2x^2\right)^4\,dx}{b^4}$$
$$= \frac{\int \left(a^4b^4\sqrt{dx}+\frac{4a^3b^5(dx)^{5/2}}{d^2}+\frac{6a^2b^6(dx)^{9/2}}{d^4}+\frac{4ab^7(dx)^{13/2}}{d^6}+\frac{b^8(dx)^{17/2}}{d^8}\right)dx}{b^4}$$
$$= \frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

Mathematica [A] time = 0.0139412, size = 55, normalized size = 0.6

$$\frac{2x\sqrt{dx}\left(11970a^2b^2x^4+12540a^3bx^2+7315a^4+5852ab^3x^6+1155b^4x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*Sqrt[d*x]*(7315*a^4 + 12540*a^3*b*x^2 + 11970*a^2*b^2*x^4 + 5852*a*b^3
*x^6 + 1155*b^4*x^8))/21945
```

Maple [A] time = 0.049, size = 52, normalized size = 0.6

$$\frac{2\,x\left(1155\,b^4x^8+5852\,ab^3x^6+11970\,a^2b^2x^4+12540\,a^3bx^2+7315\,a^4\right)}{21945}\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x)
```

```
[Out] 2/21945*x*(1155*b^4*x^8+5852*a*b^3*x^6+11970*a^2*b^2*x^4+12540*a^3*b*x^2+73
15*a^4)*(d*x)^(1/2)
```

Maxima [A] time = 0.983754, size = 99, normalized size = 1.09

$$\frac{2\left(1155\,(dx)^{\frac{19}{2}}\,b^4+5852\,(dx)^{\frac{15}{2}}\,ab^3d^2+11970\,(dx)^{\frac{11}{2}}\,a^2b^2d^4+12540\,(dx)^{\frac{7}{2}}\,a^3bd^6+7315\,(dx)^{\frac{3}{2}}\,a^4d^8\right)}{21945\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21945*(1155*(d*x)^(19/2)*b^4 + 5852*(d*x)^(15/2)*a*b^3*d^2 + 11970*(d*x)^
(11/2)*a^2*b^2*d^4 + 12540*(d*x)^(7/2)*a^3*b*d^6 + 7315*(d*x)^(3/2)*a^4*d^8
)/d^9
```

Fricas [A] time = 1.30301, size = 138, normalized size = 1.52

$$\frac{2}{21945}\left(1155\,b^4x^9 + 5852\,ab^3x^7 + 11970\,a^2b^2x^5 + 12540\,a^3bx^3 + 7315\,a^4x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^4*x^9 + 5852*a*b^3*x^7 + 11970*a^2*b^2*x^5 + 12540*a^3*b*x^
3 + 7315*a^4*x)*sqrt(d*x)
```

Sympy [A] time = 1.24744, size = 90, normalized size = 0.99

$$\frac{2a^4\sqrt{d}x^{\frac{3}{2}}}{3} + \frac{8a^3b\sqrt{d}x^{\frac{7}{2}}}{7} + \frac{12a^2b^2\sqrt{d}x^{\frac{11}{2}}}{11} + \frac{8ab^3\sqrt{d}x^{\frac{15}{2}}}{15} + \frac{2b^4\sqrt{d}x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2*(d*x)**(1/2),x)
```

```
[Out] 2*a**4*sqrt(d)*x**(3/2)/3 + 8*a**3*b*sqrt(d)*x**(7/2)/7 + 12*a**2*b**2*sqrt
(d)*x**(11/2)/11 + 8*a*b**3*sqrt(d)*x**(15/2)/15 + 2*b**4*sqrt(d)*x**(19/2)
/19
```

Giac [A] time = 1.11738, size = 107, normalized size = 1.18

$$\frac{2\left(1155\,\sqrt{dx}b^4dx^9 + 5852\,\sqrt{dx}ab^3dx^7 + 11970\,\sqrt{dx}a^2b^2dx^5 + 12540\,\sqrt{dx}a^3bdx^3 + 7315\,\sqrt{dx}a^4dx\right)}{21945\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/21945*(1155*sqrt(d*x)*b^4*d*x^9 + 5852*sqrt(d*x)*a*b^3*d*x^7 + 11970*sqrt
(d*x)*a^2*b^2*d*x^5 + 12540*sqrt(d*x)*a^3*b*d*x^3 + 7315*sqrt(d*x)*a^4*d*x)
/d
```

3.675 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}} dx$

Optimal. Leaf size=89

$$\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{2a^4\sqrt{dx}}{d}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

```
[Out] (2*a^4*Sqrt[d*x])/d + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (4*a^2*b^2*(d*x)^(9/2
))/(3*d^5) + (8*a*b^3*(d*x)^(13/2))/(13*d^7) + (2*b^4*(d*x)^(17/2))/(17*d^9
)
```

Rubi [A] time = 0.0413977, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{2a^4\sqrt{dx}}{d}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*a^4*Sqrt[d*x])/d + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (4*a^2*b^2*(d*x)^(9/2
))/(3*d^5) + (8*a*b^3*(d*x)^(13/2))/(13*d^7) + (2*b^4*(d*x)^(17/2))/(17*d^9
)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{\sqrt{dx}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{\sqrt{dx}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{\sqrt{dx}} + \frac{4a^3b^5(dx)^{3/2}}{d^2} + \frac{6a^2b^6(dx)^{7/2}}{d^4} + \frac{4ab^7(dx)^{11/2}}{d^6} + \frac{b^8(dx)^{15/2}}{d^8}\right)dx}{b^4}$$
$$= \frac{2a^4\sqrt{dx}}{d} + \frac{8a^3b(dx)^{5/2}}{5d^3} + \frac{4a^2b^2(dx)^{9/2}}{3d^5} + \frac{8ab^3(dx)^{13/2}}{13d^7} + \frac{2b^4(dx)^{17/2}}{17d^9}$$

Mathematica [A] time = 0.0153181, size = 55, normalized size = 0.62

$$\frac{2\left(2210a^2b^2x^5+2652a^3bx^3+3315a^4x+1020ab^3x^7+195b^4x^9\right)}{3315\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*(3315*a^4*x + 2652*a^3*b*x^3 + 2210*a^2*b^2*x^5 + 1020*a*b^3*x^7 + 195*b
^4*x^9))/(3315*Sqrt[d*x])
```

Maple [A] time = 0.049, size = 52, normalized size = 0.6

$$\frac{\left(390\,b^4x^8+2040\,ab^3x^6+4420\,a^2b^2x^4+5304\,a^3bx^2+6630\,a^4\right)x}{3315}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x)
```

```
[Out] 2/3315*(195*b^4*x^8+1020*a*b^3*x^6+2210*a^2*b^2*x^4+2652*a^3*b*x^2+3315*a^4
)*x/(d*x)^(1/2)
```

Maxima [A] time = 0.978911, size = 122, normalized size = 1.37

$$\frac{2\left(9945\,\sqrt{dx}a^4+\frac{585\,(dx)^{\frac{17}{2}}b^4}{d^8}+\frac{3060\,(dx)^{\frac{13}{2}}ab^3}{d^6}+\frac{4420\,(dx)^{\frac{9}{2}}a^2b^2}{d^4}+442\left(\frac{5\,(dx)^{\frac{9}{2}}b^2}{d^4}+\frac{18\,(dx)^{\frac{5}{2}}ab}{d^2}\right)a^2\right)}{9945\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9945*(9945*sqrt(d*x)*a^4 + 585*(d*x)^(17/2)*b^4/d^8 + 3060*(d*x)^(13/2)*a
*b^3/d^6 + 4420*(d*x)^(9/2)*a^2*b^2/d^4 + 442*(5*(d*x)^(9/2)*b^2/d^4 + 18*(
d*x)^(5/2)*a*b/d^2)*a^2)/d
```

Fricas [A] time = 1.23957, size = 132, normalized size = 1.48

$$\frac{2\left(195\,b^4x^8 + 1020\,ab^3x^6 + 2210\,a^2b^2x^4 + 2652\,a^3bx^2 + 3315\,a^4\right)\sqrt{dx}}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*b^4*x^8 + 1020*a*b^3*x^6 + 2210*a^2*b^2*x^4 + 2652*a^3*b*x^2 +
3315*a^4)*sqrt(d*x)/d
```

Sympy [A] time = 1.70209, size = 88, normalized size = 0.99

$$\frac{2a^4\sqrt{x}}{\sqrt{d}} + \frac{8a^3bx^{\frac{5}{2}}}{5\sqrt{d}} + \frac{4a^2b^2x^{\frac{9}{2}}}{3\sqrt{d}} + \frac{8ab^3x^{\frac{13}{2}}}{13\sqrt{d}} + \frac{2b^4x^{\frac{17}{2}}}{17\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] 2*a**4*sqrt(x)/sqrt(d) + 8*a**3*b*x**(5/2)/(5*sqrt(d)) + 4*a**2*b**2*x**(9/
2)/(3*sqrt(d)) + 8*a*b**3*x**(13/2)/(13*sqrt(d)) + 2*b**4*x**(17/2)/(17*sqr
t(d))
```

Giac [A] time = 1.12595, size = 99, normalized size = 1.11

$$\frac{2\left(195\,\sqrt{dx}b^4x^8 + 1020\,\sqrt{dx}ab^3x^6 + 2210\,\sqrt{dx}a^2b^2x^4 + 2652\,\sqrt{dx}a^3bx^2 + 3315\,\sqrt{dx}a^4\right)}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3315*(195*sqrt(d*x)*b^4*x^8 + 1020*sqrt(d*x)*a*b^3*x^6 + 2210*sqrt(d*x)*a
^2*b^2*x^4 + 2652*sqrt(d*x)*a^3*b*x^2 + 3315*sqrt(d*x)*a^4)/d
```

3.676 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{3/2}} dx$

Optimal. Leaf size=89

$$\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8a^3b(dx)^{3/2}}{3d^3}-\frac{2a^4}{d\sqrt{dx}}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

```
[Out] (-2*a^4)/(d*Sqrt[d*x]) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (12*a^2*b^2*(d*x)^
(7/2))/(7*d^5) + (8*a*b^3*(d*x)^(11/2))/(11*d^7) + (2*b^4*(d*x)^(15/2))/(15
*d^9)
```

Rubi [A] time = 0.0420059, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8a^3b(dx)^{3/2}}{3d^3}-\frac{2a^4}{d\sqrt{dx}}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (-2*a^4)/(d*Sqrt[d*x]) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (12*a^2*b^2*(d*x)^
(7/2))/(7*d^5) + (8*a*b^3*(d*x)^(11/2))/(11*d^7) + (2*b^4*(d*x)^(15/2))/(15
*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{(dx)^{3/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{3/2}} + \frac{4a^3b^5\sqrt{dx}}{d^2} + \frac{6a^2b^6(dx)^{5/2}}{d^4} + \frac{4ab^7(dx)^{9/2}}{d^6} + \frac{b^8(dx)^{13/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{d\sqrt{dx}} + \frac{8a^3b(dx)^{3/2}}{3d^3} + \frac{12a^2b^2(dx)^{7/2}}{7d^5} + \frac{8ab^3(dx)^{11/2}}{11d^7} + \frac{2b^4(dx)^{15/2}}{15d^9}$$

Mathematica [A] time = 0.0160557, size = 55, normalized size = 0.62

$$\frac{2x\left(990a^2b^2x^4 + 1540a^3bx^2 - 1155a^4 + 420ab^3x^6 + 77b^4x^8\right)}{1155(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-1155*a^4 + 1540*a^3*b*x^2 + 990*a^2*b^2*x^4 + 420*a*b^3*x^6 + 77*b^4
*x^8))/(1155*(d*x)^(3/2))
```

Maple [A] time = 0.049, size = 52, normalized size = 0.6

$$-\frac{\left(-154\,b^4x^8 - 840\,ab^3x^6 - 1980\,a^2b^2x^4 - 3080\,a^3bx^2 + 2310\,a^4\right)x}{1155}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x)
```

```
[Out] -2/1155*(-77*b^4*x^8-420*a*b^3*x^6-990*a^2*b^2*x^4-1540*a^3*b*x^2+1155*a^4)
*x/(d*x)^(3/2)
```

Maxima [A] time = 0.983885, size = 103, normalized size = 1.16

$$-\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}} - \frac{77\,(dx)^{\frac{15}{2}}b^4 + 420\,(dx)^{\frac{11}{2}}ab^3d^2 + 990\,(dx)^{\frac{7}{2}}a^2b^2d^4 + 1540\,(dx)^{\frac{3}{2}}a^3bd^6}{d^8}\right)}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(1155*a^4/sqrt(d*x) - (77*(d*x)^(15/2)*b^4 + 420*(d*x)^(11/2)*a*b^3
*d^2 + 990*(d*x)^(7/2)*a^2*b^2*d^4 + 1540*(d*x)^(3/2)*a^3*b*d^6)/d^8)/d
```

Fricas [A] time = 1.19973, size = 136, normalized size = 1.53

$$\frac{2\left(77\,b^4x^8+420\,ab^3x^6+990\,a^2b^2x^4+1540\,a^3bx^2-1155\,a^4\right)\sqrt{dx}}{1155\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^4*x^8 + 420*a*b^3*x^6 + 990*a^2*b^2*x^4 + 1540*a^3*b*x^2 - 115
5*a^4)*sqrt(d*x)/(d^2*x)
```

Sympy [A] time = 1.3449, size = 88, normalized size = 0.99

$$-\frac{2a^4}{d^{\frac{3}{2}}\sqrt{x}}+\frac{8a^3bx^{\frac{3}{2}}}{3d^{\frac{3}{2}}}+\frac{12a^2b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}+\frac{8ab^3x^{\frac{11}{2}}}{11d^{\frac{3}{2}}}+\frac{2b^4x^{\frac{15}{2}}}{15d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(3/2),x)
```

```
[Out] -2*a**4/(d**(3/2)*sqrt(x)) + 8*a**3*b*x**(3/2)/(3*d**(3/2)) + 12*a**2*b**2*
x**(7/2)/(7*d**(3/2)) + 8*a*b**3*x**(11/2)/(11*d**(3/2)) + 2*b**4*x**(15/2)
/(15*d**(3/2))
```

Giac [A] time = 1.12669, size = 120, normalized size = 1.35

$$-\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}}-\frac{77\,\sqrt{dx}b^4d^{119}x^7+420\,\sqrt{dx}ab^3d^{119}x^5+990\,\sqrt{dx}a^2b^2d^{119}x^3+1540\,\sqrt{dx}a^3bd^{119}x}{d^{120}}\right)}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/1155*(1155*a^4/sqrt(d*x) - (77*sqrt(d*x)*b^4*d^119*x^7 + 420*sqrt(d*x)*a
*b^3*d^119*x^5 + 990*sqrt(d*x)*a^2*b^2*d^119*x^3 + 1540*sqrt(d*x)*a^3*b*d^1
19*x)/d^120)/d
```

3.677 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8a^3b\sqrt{dx}}{d^3}-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

```
[Out] (-2*a^4)/(3*d*(d*x)^(3/2)) + (8*a^3*b*Sqrt[d*x])/d^3 + (12*a^2*b^2*(d*x)^(5
/2))/(5*d^5) + (8*a*b^3*(d*x)^(9/2))/(9*d^7) + (2*b^4*(d*x)^(13/2))/(13*d^9
)
```

Rubi [A] time = 0.0418703, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8a^3b\sqrt{dx}}{d^3}-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (-2*a^4)/(3*d*(d*x)^(3/2)) + (8*a^3*b*Sqrt[d*x])/d^3 + (12*a^2*b^2*(d*x)^(5
/2))/(5*d^5) + (8*a*b^3*(d*x)^(9/2))/(9*d^7) + (2*b^4*(d*x)^(13/2))/(13*d^9
)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{5/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^4}{(dx)^{5/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{5/2}}+\frac{4a^3b^5}{d^2\sqrt{dx}}+\frac{6a^2b^6(dx)^{3/2}}{d^4}+\frac{4ab^7(dx)^{7/2}}{d^6}+\frac{b^8(dx)^{11/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{3d(dx)^{3/2}}+\frac{8a^3b\sqrt{dx}}{d^3}+\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

Mathematica [A] time = 0.0161114, size = 55, normalized size = 0.62

$$\frac{x\left(1404a^2b^2x^4+4680a^3bx^2-390a^4+520ab^3x^6+90b^4x^8\right)}{585(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (x*(-390*a^4 + 4680*a^3*b*x^2 + 1404*a^2*b^2*x^4 + 520*a*b^3*x^6 + 90*b^4*x
^8))/(585*(d*x)^(5/2))
```

Maple [A] time = 0.049, size = 52, normalized size = 0.6

$$-\frac{\left(-90\,b^4x^8-520\,ab^3x^6-1404\,a^2b^2x^4-4680\,a^3bx^2+390\,a^4\right)x}{585}(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x)
```

```
[Out] -2/585*(-45*b^4*x^8-260*a*b^3*x^6-702*a^2*b^2*x^4-2340*a^3*b*x^2+195*a^4)*x
/(d*x)^(5/2)
```

Maxima [A] time = 0.991041, size = 103, normalized size = 1.16

$$-\frac{2\left(\frac{195\,a^4}{(dx)^{\frac{3}{2}}}-\frac{45\,(dx)^{\frac{13}{2}}b^4+260\,(dx)^{\frac{9}{2}}ab^3d^2+702\,(dx)^{\frac{5}{2}}a^2b^2d^4+2340\,\sqrt{dx}a^3bd^6}{d^8}\right)}{585\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/585*(195*a^4/(d*x)^(3/2) - (45*(d*x)^(13/2)*b^4 + 260*(d*x)^(9/2)*a*b^3*
d^2 + 702*(d*x)^(5/2)*a^2*b^2*d^4 + 2340*sqrt(d*x)*a^3*b*d^6)/d^8)/d
```

Fricas [A] time = 1.22907, size = 136, normalized size = 1.53

$$\frac{2\left(45\,b^4x^8+260\,ab^3x^6+702\,a^2b^2x^4+2340\,a^3bx^2-195\,a^4\right)\sqrt{dx}}{585\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^4*x^8 + 260*a*b^3*x^6 + 702*a^2*b^2*x^4 + 2340*a^3*b*x^2 - 195*
a^4)*sqrt(d*x)/(d^3*x^2)
```

Sympy [A] time = 1.69748, size = 88, normalized size = 0.99

$$-\frac{2a^4}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{8a^3b\sqrt{x}}{d^{\frac{5}{2}}}+\frac{12a^2b^2x^{\frac{5}{2}}}{5d^{\frac{5}{2}}}+\frac{8ab^3x^{\frac{9}{2}}}{9d^{\frac{5}{2}}}+\frac{2b^4x^{\frac{13}{2}}}{13d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(5/2),x)
```

```
[Out] -2*a**4/(3*d**(5/2)*x**(3/2)) + 8*a**3*b*sqrt(x)/d**(5/2) + 12*a**2*b**2*x*
*(5/2)/(5*d**(5/2)) + 8*a*b**3*x**(9/2)/(9*d**(5/2)) + 2*b**4*x**(13/2)/(13
*d**(5/2))
```

Giac [A] time = 1.14139, size = 124, normalized size = 1.39

$$-\frac{2\left(\frac{195\,a^4d}{\sqrt{dx}x}-\frac{45\,\sqrt{dx}b^4d^{78}x^6+260\,\sqrt{dx}ab^3d^{78}x^4+702\,\sqrt{dx}a^2b^2d^{78}x^2+2340\,\sqrt{dx}a^3bd^{78}}{d^{78}}\right)}{585\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/585*(195*a^4*d/(sqrt(d*x)*x) - (45*sqrt(d*x)*b^4*d^78*x^6 + 260*sqrt(d*x
)*a*b^3*d^78*x^4 + 702*sqrt(d*x)*a^2*b^2*d^78*x^2 + 2340*sqrt(d*x)*a^3*b*d^
78)/d^78)/d^3
```

3.678 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=87

$$\frac{4a^2b^2(dx)^{3/2}}{d^5}-\frac{8a^3b}{d^3\sqrt{dx}}-\frac{2a^4}{5d(dx)^{5/2}}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

```
[Out] (-2*a^4)/(5*d*(d*x)^(5/2)) - (8*a^3*b)/(d^3*Sqrt[d*x]) + (4*a^2*b^2*(d*x)^(
3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7) + (2*b^4*(d*x)^(11/2))/(11*d^9)
```

Rubi [A] time = 0.0415425, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{4a^2b^2(dx)^{3/2}}{d^5}-\frac{8a^3b}{d^3\sqrt{dx}}-\frac{2a^4}{5d(dx)^{5/2}}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```



```
[Out] (-2*a^4)/(5*d*(d*x)^(5/2)) - (8*a^3*b)/(d^3*Sqrt[d*x]) + (4*a^2*b^2*(d*x)^(
3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7) + (2*b^4*(d*x)^(11/2))/(11*d^9)
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{(dx)^{7/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{7/2}} + \frac{4a^3b^5}{d^2(dx)^{3/2}} + \frac{6a^2b^6\sqrt{dx}}{d^4} + \frac{4ab^7(dx)^{5/2}}{d^6} + \frac{b^8(dx)^{9/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{5d(dx)^{5/2}} - \frac{8a^3b}{d^3\sqrt{dx}} + \frac{4a^2b^2(dx)^{3/2}}{d^5} + \frac{8ab^3(dx)^{7/2}}{7d^7} + \frac{2b^4(dx)^{11/2}}{11d^9}$$

Mathematica [A] time = 0.0193988, size = 60, normalized size = 0.69

$$\frac{2\sqrt{dx}\left(770a^2b^2x^4 - 1540a^3bx^2 - 77a^4 + 220ab^3x^6 + 35b^4x^8\right)}{385d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-77*a^4 - 1540*a^3*b*x^2 + 770*a^2*b^2*x^4 + 220*a*b^3*x^6 +
35*b^4*x^8))/(385*d^4*x^3)
```

Maple [A] time = 0.047, size = 52, normalized size = 0.6

$$-\frac{\left(-70\,b^4x^8 - 440\,ab^3x^6 - 1540\,a^2b^2x^4 + 3080\,a^3bx^2 + 154\,a^4\right)x}{385}(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x)
```

```
[Out] -2/385*(-35*b^4*x^8-220*a*b^3*x^6-770*a^2*b^2*x^4+1540*a^3*b*x^2+77*a^4)*x/
(d*x)^(7/2)
```

Maxima [A] time = 0.983806, size = 111, normalized size = 1.28

$$-\frac{2\left(\frac{77\left(20\,a^3bd^2x^2+a^4d^2\right)}{(dx)^{\frac{5}{2}}d^2} - \frac{5\left(7\,(dx)^{\frac{11}{2}}b^4+44\,(dx)^{\frac{7}{2}}ab^3d^2+154\,(dx)^{\frac{3}{2}}a^2b^2d^4\right)}{d^8}\right)}{385\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/385*(77*(20*a^3*b*d^2*x^2 + a^4*d^2)/((d*x)^(5/2)*d^2) - 5*(7*(d*x)^(11/
2)*b^4 + 44*(d*x)^(7/2)*a*b^3*d^2 + 154*(d*x)^(3/2)*a^2*b^2*d^4)/d^8)/d
```

Fricas [A] time = 1.21486, size = 135, normalized size = 1.55

$$\frac{2\left(35\,b^4x^8+220\,ab^3x^6+770\,a^2b^2x^4-1540\,a^3bx^2-77\,a^4\right)\sqrt{dx}}{385\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/385*(35*b^4*x^8 + 220*a*b^3*x^6 + 770*a^2*b^2*x^4 - 1540*a^3*b*x^2 - 77*a
^4)*sqrt(d*x)/(d^4*x^3)
```

Sympy [A] time = 2.30402, size = 87, normalized size = 1.

$$-\frac{2a^4}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{8a^3b}{d^{\frac{7}{2}}\sqrt{x}}+\frac{4a^2b^2x^{\frac{3}{2}}}{d^{\frac{7}{2}}}+\frac{8ab^3x^{\frac{7}{2}}}{7d^{\frac{7}{2}}}+\frac{2b^4x^{\frac{11}{2}}}{11d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(7/2),x)
```

```
[Out] -2*a**4/(5*d**(7/2)*x**(5/2)) - 8*a**3*b/(d**(7/2)*sqrt(x)) + 4*a**2*b**2*x
**(3/2)/d**(7/2) + 8*a*b**3*x**(7/2)/(7*d**(7/2)) + 2*b**4*x**(11/2)/(11*d*
*(7/2))
```

Giac [A] time = 1.11246, size = 128, normalized size = 1.47

$$-\frac{2\left(\frac{77\left(20\,a^3bd^3x^2+a^4d^3\right)}{\sqrt{dx}d^2x^2}-\frac{5\left(7\,\sqrt{dx}b^4d^{55}x^5+44\,\sqrt{dx}ab^3d^{55}x^3+154\,\sqrt{dx}a^2b^2d^{55}x\right)}{d^{55}}\right)}{385\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/385*(77*(20*a^3*b*d^3*x^2 + a^4*d^3)/(sqrt(d*x)*d^2*x^2) - 5*(7*sqrt(d*x
)*b^4*d^55*x^5 + 44*sqrt(d*x)*a*b^3*d^55*x^3 + 154*sqrt(d*x)*a^2*b^2*d^55*x
)/d^55)/d^4
```

3.679 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=129

$$\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^6(dx)^{7/2}}{7d}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

```
[Out] (2*a^6*(d*x)^(7/2))/(7*d) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (2*a^4*b^2*(
d*x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) + (30*a^2*b^4*(d*x)^(
23/2))/(23*d^9) + (4*a*b^5*(d*x)^(27/2))/(9*d^11) + (2*b^6*(d*x)^(31/2))/(3
1*d^13)
```

Rubi [A] time = 0.066312, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^6(dx)^{7/2}}{7d}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(7/2))/(7*d) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (2*a^4*b^2*(
d*x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) + (30*a^2*b^4*(d*x)^(
23/2))/(23*d^9) + (4*a*b^5*(d*x)^(27/2))/(9*d^11) + (2*b^6*(d*x)^(31/2))/(3
1*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx = \frac{\int (dx)^{5/2}\left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$= \frac{\int \left(a^6b^6(dx)^{5/2}+\frac{6a^5b^7(dx)^{9/2}}{d^2}+\frac{15a^4b^8(dx)^{13/2}}{d^4}+\frac{20a^3b^9(dx)^{17/2}}{d^6}+\frac{15a^2b^{10}(dx)^{21/2}}{d^8}+\frac{6ab^{11}(dx)^{25/2}}{d^{10}}\right.}{b^6}$$
$$= \frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{4a}{}$$

Mathematica [A] time = 0.0286528, size = 77, normalized size = 0.6

$$\frac{2x(dx)^{5/2}\left(6122655a^2b^4x^8+9882180a^3b^3x^6+9388071a^4b^2x^4+5120766a^5bx^2+1341153a^6+2086238ab^5x^{10}+302841b\right.}{9388071}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(1341153*a^6 + 5120766*a^5*b*x^2 + 9388071*a^4*b^2*x^4 + 9
882180*a^3*b^3*x^6 + 6122655*a^2*b^4*x^8 + 2086238*a*b^5*x^10 + 302841*b^6*
x^12))/9388071
```

Maple [A] time = 0.049, size = 74, normalized size = 0.6

$$\frac{2\,x\left(302841\,b^6x^{12}+2086238\,ab^5x^{10}+6122655\,a^2b^4x^8+9882180\,a^3b^3x^6+9388071\,a^4b^2x^4+5120766\,a^5bx^2+1341153\,a\right.}{9388071}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/9388071*x*(302841*b^6*x^12+2086238*a*b^5*x^10+6122655*a^2*b^4*x^8+9882180
*a^3*b^3*x^6+9388071*a^4*b^2*x^4+5120766*a^5*b*x^2+1341153*a^6)*(d*x)^(5/2)
```

Maxima [A] time = 0.95892, size = 142, normalized size = 1.1

$$\frac{2\left(302841\,(dx)^{\frac{31}{2}}\,b^6+2086238\,(dx)^{\frac{27}{2}}\,ab^5d^2+6122655\,(dx)^{\frac{23}{2}}\,a^2b^4d^4+9882180\,(dx)^{\frac{19}{2}}\,a^3b^3d^6+9388071\,(dx)^{\frac{15}{2}}\,a^4b^2d^8\right.}{9388071\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/9388071*(302841*(d*x)^(31/2)*b^6 + 2086238*(d*x)^(27/2)*a*b^5*d^2 + 61226
55*(d*x)^(23/2)*a^2*b^4*d^4 + 9882180*(d*x)^(19/2)*a^3*b^3*d^6 + 9388071*(d
*x)^(15/2)*a^4*b^2*d^8 + 5120766*(d*x)^(11/2)*a^5*b*d^10 + 1341153*(d*x)^(7
/2)*a^6*d^12)/d^13
```

Fricas [A] time = 1.22685, size = 261, normalized size = 2.02

$$\frac{2}{9388071}\left(302841\,b^6d^2x^{15} + 2086238\,ab^5d^2x^{13} + 6122655\,a^2b^4d^2x^{11} + 9882180\,a^3b^3d^2x^9 + 9388071\,a^4b^2d^2x^7 + 512076\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/9388071*(302841*b^6*d^2*x^15 + 2086238*a*b^5*d^2*x^13 + 6122655*a^2*b^4*d
^2*x^11 + 9882180*a^3*b^3*d^2*x^9 + 9388071*a^4*b^2*d^2*x^7 + 5120766*a^5*b
*d^2*x^5 + 1341153*a^6*d^2*x^3)*sqrt(d*x)
```

Sympy [A] time = 12.5737, size = 129, normalized size = 1.

$$\frac{2a^6d^{\frac{5}{2}}x^{\frac{7}{2}}}{7} + \frac{12a^5bd^{\frac{5}{2}}x^{\frac{11}{2}}}{11} + 2a^4b^2d^{\frac{5}{2}}x^{\frac{15}{2}} + \frac{40a^3b^3d^{\frac{5}{2}}x^{\frac{19}{2}}}{19} + \frac{30a^2b^4d^{\frac{5}{2}}x^{\frac{23}{2}}}{23} + \frac{4ab^5d^{\frac{5}{2}}x^{\frac{27}{2}}}{9} + \frac{2b^6d^{\frac{5}{2}}x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*d**(5/2)*x**(7/2)/7 + 12*a**5*b*d**(5/2)*x**(11/2)/11 + 2*a**4*b**2*
d**(5/2)*x**(15/2) + 40*a**3*b**3*d**(5/2)*x**(19/2)/19 + 30*a**2*b**4*d**(
5/2)*x**(23/2)/23 + 4*a*b**5*d**(5/2)*x**(27/2)/9 + 2*b**6*d**(5/2)*x**(31/
2)/31
```

Giac [A] time = 1.1135, size = 167, normalized size = 1.29

$$\frac{2}{31}\sqrt{dx}b^6d^2x^{15}+\frac{4}{9}\sqrt{dx}ab^5d^2x^{13}+\frac{30}{23}\sqrt{dx}a^2b^4d^2x^{11}+\frac{40}{19}\sqrt{dx}a^3b^3d^2x^9+2\sqrt{dx}a^4b^2d^2x^7+\frac{12}{11}\sqrt{dx}a^5bd^2x^5+\frac{2}{7}\sqrt{dx}a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/31*sqrt(d*x)*b^6*d^2*x^15 + 4/9*sqrt(d*x)*a*b^5*d^2*x^13 + 30/23*sqrt(d*x
)*a^2*b^4*d^2*x^11 + 40/19*sqrt(d*x)*a^3*b^3*d^2*x^9 + 2*sqrt(d*x)*a^4*b^2*
d^2*x^7 + 12/11*sqrt(d*x)*a^5*b*d^2*x^5 + 2/7*sqrt(d*x)*a^6*d^2*x^3
```

3.680 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=131

$$\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{2a^6(dx)^{5/2}}{5d}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

```
[Out] (2*a^6*(d*x)^(5/2))/(5*d) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (30*a^4*b^2*(d*
x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7) + (10*a^2*b^4*(d*x
)^(21/2))/(7*d^9) + (12*a*b^5*(d*x)^(25/2))/(25*d^11) + (2*b^6*(d*x)^(29/2)
)/(29*d^13)
```

Rubi [A] time = 0.0613499, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{2a^6(dx)^{5/2}}{5d}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```



```
[Out] (2*a^6*(d*x)^(5/2))/(5*d) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (30*a^4*b^2*(d*
x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7) + (10*a^2*b^4*(d*x
)^(21/2))/(7*d^9) + (12*a*b^5*(d*x)^(25/2))/(25*d^11) + (2*b^6*(d*x)^(29/2)
)/(29*d^13)
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^3 dx = \frac{\int (dx)^{3/2} \left(ab+b^2x^2\right)^6 dx}{b^6}$$
$$= \frac{\int \left(a^6b^6(dx)^{3/2} + \frac{6a^5b^7(dx)^{7/2}}{d^2} + \frac{15a^4b^8(dx)^{11/2}}{d^4} + \frac{20a^3b^9(dx)^{15/2}}{d^6} + \frac{15a^2b^{10}(dx)^{19/2}}{d^8} + \frac{6ab^{11}(dx)^{23/2}}{d^{10}}\right.}{b^6}$$
$$= \frac{2a^6(dx)^{5/2}}{5d} + \frac{4a^5b(dx)^{9/2}}{3d^3} + \frac{30a^4b^2(dx)^{13/2}}{13d^5} + \frac{40a^3b^3(dx)^{17/2}}{17d^7} + \frac{10a^2b^4(dx)^{21/2}}{7d^9} + \frac{12a}{}$$

Mathematica [A] time = 0.0247262, size = 77, normalized size = 0.59

$$\frac{2x(dx)^{3/2}\left(2403375a^2b^4x^8 + 3958500a^3b^3x^6 + 3882375a^4b^2x^4 + 2243150a^5bx^2 + 672945a^6 + 807534ab^5x^{10} + 116025b^6\right.}{3364725}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(672945*a^6 + 2243150*a^5*b*x^2 + 3882375*a^4*b^2*x^4 + 39
58500*a^3*b^3*x^6 + 2403375*a^2*b^4*x^8 + 807534*a*b^5*x^10 + 116025*b^6*x^
12))/3364725
```

Maple [A] time = 0.05, size = 74, normalized size = 0.6

$$\frac{2x\left(116025b^6x^{12} + 807534ab^5x^{10} + 2403375a^2b^4x^8 + 3958500a^3b^3x^6 + 3882375a^4b^2x^4 + 2243150a^5bx^2 + 672945a^6\right)}{3364725}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/3364725*x*(116025*b^6*x^12+807534*a*b^5*x^10+2403375*a^2*b^4*x^8+3958500*
a^3*b^3*x^6+3882375*a^4*b^2*x^4+2243150*a^5*b*x^2+672945*a^6)*(d*x)^(3/2)
```

Maxima [A] time = 0.982645, size = 142, normalized size = 1.08

$$\frac{2\left(116025\,(dx)^{\frac{29}{2}}\,b^6 + 807534\,(dx)^{\frac{25}{2}}\,ab^5d^2 + 2403375\,(dx)^{\frac{21}{2}}\,a^2b^4d^4 + 3958500\,(dx)^{\frac{17}{2}}\,a^3b^3d^6 + 3882375\,(dx)^{\frac{13}{2}}\,a^4b^2d^8\right.}{3364725\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/3364725*(116025*(d*x)^(29/2)*b^6 + 807534*(d*x)^(25/2)*a*b^5*d^2 + 240337
5*(d*x)^(21/2)*a^2*b^4*d^4 + 3958500*(d*x)^(17/2)*a^3*b^3*d^6 + 3882375*(d*
x)^(13/2)*a^4*b^2*d^8 + 2243150*(d*x)^(9/2)*a^5*b*d^10 + 672945*(d*x)^(5/2)
*a^6*d^12)/d^13
```

Fricas [A] time = 1.24482, size = 239, normalized size = 1.82

$$\frac{2}{3364725}\left(116025\, b^6 d x^{14} + 807534\, a b^5 d x^{12} + 2403375\, a^2 b^4 d x^{10} + 3958500\, a^3 b^3 d x^8 + 3882375\, a^4 b^2 d x^6 + 2243150\, a^5 b d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/3364725*(116025*b^6*d*x^14 + 807534*a*b^5*d*x^12 + 2403375*a^2*b^4*d*x^10
 + 3958500*a^3*b^3*d*x^8 + 3882375*a^4*b^2*d*x^6 + 2243150*a^5*b*d*x^4 + 67
2945*a^6*d*x^2)*sqrt(d*x)
```

Sympy [A] time = 5.61801, size = 131, normalized size = 1.

$$\frac{2a^6 d^{\frac{3}{2}} x^{\frac{5}{2}}}{5} + \frac{4a^5 b d^{\frac{3}{2}} x^{\frac{9}{2}}}{3} + \frac{30a^4 b^2 d^{\frac{3}{2}} x^{\frac{13}{2}}}{13} + \frac{40a^3 b^3 d^{\frac{3}{2}} x^{\frac{17}{2}}}{17} + \frac{10a^2 b^4 d^{\frac{3}{2}} x^{\frac{21}{2}}}{7} + \frac{12a b^5 d^{\frac{3}{2}} x^{\frac{25}{2}}}{25} + \frac{2b^6 d^{\frac{3}{2}} x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*d**(3/2)*x**(5/2)/5 + 4*a**5*b*d**(3/2)*x**(9/2)/3 + 30*a**4*b**2*d*
*(3/2)*x**(13/2)/13 + 40*a**3*b**3*d**(3/2)*x**(17/2)/17 + 10*a**2*b**4*d**
(3/2)*x**(21/2)/7 + 12*a*b**5*d**(3/2)*x**(25/2)/25 + 2*b**6*d**(3/2)*x**(2
9/2)/29
```

Giac [A] time = 1.15185, size = 149, normalized size = 1.14

$$\frac{2}{29}\sqrt{dx}b^6dx^{14}+\frac{12}{25}\sqrt{dx}ab^5dx^{12}+\frac{10}{7}\sqrt{dx}a^2b^4dx^{10}+\frac{40}{17}\sqrt{dx}a^3b^3dx^8+\frac{30}{13}\sqrt{dx}a^4b^2dx^6+\frac{4}{3}\sqrt{dx}a^5bdx^4+\frac{2}{5}\sqrt{dx}a^6dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/29*sqrt(d*x)*b^6*d*x^14 + 12/25*sqrt(d*x)*a*b^5*d*x^12 + 10/7*sqrt(d*x)*a
^2*b^4*d*x^10 + 40/17*sqrt(d*x)*a^3*b^3*d*x^8 + 30/13*sqrt(d*x)*a^4*b^2*d*x
^6 + 4/3*sqrt(d*x)*a^5*b*d*x^4 + 2/5*sqrt(d*x)*a^6*d*x^2
```

3.681 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=131

$$\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{2a^6(dx)^{3/2}}{3d}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

```
[Out] (2*a^6*(d*x)^(3/2))/(3*d) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (30*a^4*b^2*(d
*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7) + (30*a^2*b^4*(d*x)
^(19/2))/(19*d^9) + (12*a*b^5*(d*x)^(23/2))/(23*d^11) + (2*b^6*(d*x)^(27/2)
)/(27*d^13)
```

Rubi [A] time = 0.0610318, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{2a^6(dx)^{3/2}}{3d}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(3/2))/(3*d) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (30*a^4*b^2*(d
*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7) + (30*a^2*b^4*(d*x)
^(19/2))/(19*d^9) + (12*a*b^5*(d*x)^(23/2))/(23*d^11) + (2*b^6*(d*x)^(27/2)
)/(27*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{\int \sqrt{dx}\left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$=\frac{\int \left(a^6b^6\sqrt{dx}+\frac{6a^5b^7(dx)^{5/2}}{d^2}+\frac{15a^4b^8(dx)^{9/2}}{d^4}+\frac{20a^3b^9(dx)^{13/2}}{d^6}+\frac{15a^2b^{10}(dx)^{17/2}}{d^8}+\frac{6ab^{11}(dx)^{21/2}}{d^{10}}+\frac{b^{1}\cdots}{}\right.}{b^6}$$
$$=\frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{12ab\cdots}{2\cdots}$$

Mathematica [A] time = 0.0194856, size = 77, normalized size = 0.59

$$\frac{2x\sqrt{dx}\left(717255a^2b^4x^8+1211364a^3b^3x^6+1238895a^4b^2x^4+778734a^5bx^2+302841a^6+237006ab^5x^{10}+33649b^6x^{12}\right)}{908523}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*Sqrt[d*x]*(302841*a^6 + 778734*a^5*b*x^2 + 1238895*a^4*b^2*x^4 + 12113
64*a^3*b^3*x^6 + 717255*a^2*b^4*x^8 + 237006*a*b^5*x^10 + 33649*b^6*x^12))/
908523
```

Maple [A] time = 0.05, size = 74, normalized size = 0.6

$$\frac{2\,x\left(33649\,b^6x^{12}+237006\,ab^5x^{10}+717255\,a^2b^4x^8+1211364\,a^3b^3x^6+1238895\,a^4b^2x^4+778734\,a^5bx^2+302841\,a^6\right)}{908523}\sqrt{d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x)
```

```
[Out] 2/908523*x*(33649*b^6*x^12+237006*a*b^5*x^10+717255*a^2*b^4*x^8+1211364*a^3
*b^3*x^6+1238895*a^4*b^2*x^4+778734*a^5*b*x^2+302841*a^6)*(d*x)^(1/2)
```

Maxima [A] time = 0.966889, size = 142, normalized size = 1.08

$$\frac{2\left(33649\,(dx)^{\frac{27}{2}}\,b^6+237006\,(dx)^{\frac{23}{2}}\,ab^5d^2+717255\,(dx)^{\frac{19}{2}}\,a^2b^4d^4+1211364\,(dx)^{\frac{15}{2}}\,a^3b^3d^6+1238895\,(dx)^{\frac{11}{2}}\,a^4b^2d^8+7\cdots\right.}{908523\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/908523*(33649*(d*x)^(27/2)*b^6 + 237006*(d*x)^(23/2)*a*b^5*d^2 + 717255*(
d*x)^(19/2)*a^2*b^4*d^4 + 1211364*(d*x)^(15/2)*a^3*b^3*d^6 + 1238895*(d*x)^
(11/2)*a^4*b^2*d^8 + 778734*(d*x)^(7/2)*a^5*b*d^10 + 302841*(d*x)^(3/2)*a^6
*d^12)/d^13
```

Fricas [A] time = 1.25841, size = 211, normalized size = 1.61

$$\frac{2}{908523}\left(33649\,b^6x^{13} + 237006\,ab^5x^{11} + 717255\,a^2b^4x^9 + 1211364\,a^3b^3x^7 + 1238895\,a^4b^2x^5 + 778734\,a^5bx^3 + 302841\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/908523*(33649*b^6*x^13 + 237006*a*b^5*x^11 + 717255*a^2*b^4*x^9 + 1211364
*a^3*b^3*x^7 + 1238895*a^4*b^2*x^5 + 778734*a^5*b*x^3 + 302841*a^6*x)*sqrt(
d*x)
```

Sympy [A] time = 2.85401, size = 131, normalized size = 1.

$$\frac{2a^6\sqrt{d}x^{\frac{3}{2}}}{3} + \frac{12a^5b\sqrt{d}x^{\frac{7}{2}}}{7} + \frac{30a^4b^2\sqrt{d}x^{\frac{11}{2}}}{11} + \frac{8a^3b^3\sqrt{d}x^{\frac{15}{2}}}{3} + \frac{30a^2b^4\sqrt{d}x^{\frac{19}{2}}}{19} + \frac{12ab^5\sqrt{d}x^{\frac{23}{2}}}{23} + \frac{2b^6\sqrt{d}x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3*(d*x)**(1/2),x)
```

```
[Out] 2*a**6*sqrt(d)*x**(3/2)/3 + 12*a**5*b*sqrt(d)*x**(7/2)/7 + 30*a**4*b**2*sqr
t(d)*x**(11/2)/11 + 8*a**3*b**3*sqrt(d)*x**(15/2)/3 + 30*a**2*b**4*sqrt(d)*
x**(19/2)/19 + 12*a*b**5*sqrt(d)*x**(23/2)/23 + 2*b**6*sqrt(d)*x**(27/2)/27
```

Giac [A] time = 1.12483, size = 153, normalized size = 1.17

$$\frac{2\left(33649\,\sqrt{dx}b^6dx^{13} + 237006\,\sqrt{dx}ab^5dx^{11} + 717255\,\sqrt{dx}a^2b^4dx^9 + 1211364\,\sqrt{dx}a^3b^3dx^7 + 1238895\,\sqrt{dx}a^4b^2dx^5 + 77\right.}{908523\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/908523*(33649*sqrt(d*x)*b^6*d*x^13 + 237006*sqrt(d*x)*a*b^5*d*x^11 + 7172
55*sqrt(d*x)*a^2*b^4*d*x^9 + 1211364*sqrt(d*x)*a^3*b^3*d*x^7 + 1238895*sqrt
(d*x)*a^4*b^2*d*x^5 + 778734*sqrt(d*x)*a^5*b*d*x^3 + 302841*sqrt(d*x)*a^6*d
*x)/d
```

3.682 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}\,dx$

Optimal. Leaf size=129

$$\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{2a^6\sqrt{dx}}{d}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

```
[Out] (2*a^6*Sqrt[d*x])/d + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (10*a^4*b^2*(d*x)^(9
/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30*a^2*b^4*(d*x)^(17/2
))/(17*d^9) + (4*a*b^5*(d*x)^(21/2))/(7*d^11) + (2*b^6*(d*x)^(25/2))/(25*d^
13)
```

Rubi [A] time = 0.0597216, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{2a^6\sqrt{dx}}{d}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```


```
[Out] (2*a^6*Sqrt[d*x])/d + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (10*a^4*b^2*(d*x)^(9
/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30*a^2*b^4*(d*x)^(17/2
))/(17*d^9) + (4*a*b^5*(d*x)^(21/2))/(7*d^11) + (2*b^6*(d*x)^(25/2))/(25*d^
13)
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{\sqrt{dx}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{\sqrt{dx}}\,dx}{b^6}$$

$$= \frac{\int \left(\frac{a^6b^6}{\sqrt{dx}} + \frac{6a^5b^7(dx)^{3/2}}{d^2} + \frac{15a^4b^8(dx)^{7/2}}{d^4} + \frac{20a^3b^9(dx)^{11/2}}{d^6} + \frac{15a^2b^{10}(dx)^{15/2}}{d^8} + \frac{6ab^{11}(dx)^{19/2}}{d^{10}} + \frac{b^{12}(dx)^{23/2}}{d^{12}}\right) d}{b^6}$$

$$= \frac{2a^6\sqrt{dx}}{d} + \frac{12a^5b(dx)^{5/2}}{5d^3} + \frac{10a^4b^2(dx)^{9/2}}{3d^5} + \frac{40a^3b^3(dx)^{13/2}}{13d^7} + \frac{30a^2b^4(dx)^{17/2}}{17d^9} + \frac{4ab^5(dx)^{21/2}}{7d^{11}}$$

Mathematica [A] time = 0.0216615, size = 77, normalized size = 0.6

$$\frac{2\left(102375a^2b^4x^9+178500a^3b^3x^7+193375a^4b^2x^5+139230a^5bx^3+116025a^6x+33150ab^5x^{11}+4641b^6x^{13}\right)}{116025\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

```
[Out] (2*(116025*a^6*x + 139230*a^5*b*x^3 + 193375*a^4*b^2*x^5 + 178500*a^3*b^3*x
^7 + 102375*a^2*b^4*x^9 + 33150*a*b^5*x^11 + 4641*b^6*x^13))/(116025*Sqrt[d
*x])
```

Maple [A] time = 0.049, size = 74, normalized size = 0.6

$$\frac{\left(9282\,b^6x^{12}+66300\,ab^5x^{10}+204750\,a^2b^4x^8+357000\,a^3b^3x^6+386750\,a^4b^2x^4+278460\,a^5bx^2+232050\,a^6\right)x}{116025}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x)
```

```
[Out] 2/116025*(4641*b^6*x^12+33150*a*b^5*x^10+102375*a^2*b^4*x^8+178500*a^3*b^3*
x^6+193375*a^4*b^2*x^4+139230*a^5*b*x^2+116025*a^6)*x/(d*x)^(1/2)
```

Maxima [A] time = 0.987749, size = 209, normalized size = 1.62

$$\frac{2\left(116025\sqrt{dx}a^6+\frac{4641(dx)^{\frac{25}{2}}b^6}{d^{12}}+\frac{33150(dx)^{\frac{21}{2}}ab^5}{d^{10}}+\frac{81900(dx)^{\frac{17}{2}}a^2b^4}{d^8}+\frac{71400(dx)^{\frac{13}{2}}a^3b^3}{d^6}+7735\left(\frac{5(dx)^{\frac{9}{2}}b^2}{d^4}+\frac{18(dx)^{\frac{5}{2}}ab}{d^2}\right)a^4+175\left(\right.$$
$$116025\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/116025*(116025*sqrt(d*x)*a^6 + 4641*(d*x)^(25/2)*b^6/d^12 + 33150*(d*x)^(
21/2)*a*b^5/d^10 + 81900*(d*x)^(17/2)*a^2*b^4/d^8 + 71400*(d*x)^(13/2)*a^3*
b^3/d^6 + 7735*(5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)*a^4 + 175*(
117*(d*x)^(17/2)*b^4/d^8 + 612*(d*x)^(13/2)*a*b^3/d^6 + 884*(d*x)^(9/2)*a^2
*b^2/d^4)*a^2)/d
```

Fricas [A] time = 1.27325, size = 205, normalized size = 1.59

$$\frac{2\left(4641\,b^6x^{12}+33150\,ab^5x^{10}+102375\,a^2b^4x^8+178500\,a^3b^3x^6+193375\,a^4b^2x^4+139230\,a^5bx^2+116025\,a^6\right)\sqrt{dx}}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*b^6*x^12 + 33150*a*b^5*x^10 + 102375*a^2*b^4*x^8 + 178500*a^
3*b^3*x^6 + 193375*a^4*b^2*x^4 + 139230*a^5*b*x^2 + 116025*a^6)*sqrt(d*x)/d
```

Sympy [A] time = 2.69479, size = 129, normalized size = 1.

$$\frac{2a^6\sqrt{x}}{\sqrt{d}}+\frac{12a^5bx^{\frac{5}{2}}}{5\sqrt{d}}+\frac{10a^4b^2x^{\frac{9}{2}}}{3\sqrt{d}}+\frac{40a^3b^3x^{\frac{13}{2}}}{13\sqrt{d}}+\frac{30a^2b^4x^{\frac{17}{2}}}{17\sqrt{d}}+\frac{4ab^5x^{\frac{21}{2}}}{7\sqrt{d}}+\frac{2b^6x^{\frac{25}{2}}}{25\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

```
[Out] 2*a**6*sqrt(x)/sqrt(d) + 12*a**5*b*x**(5/2)/(5*sqrt(d)) + 10*a**4*b**2*x**(
9/2)/(3*sqrt(d)) + 40*a**3*b**3*x**(13/2)/(13*sqrt(d)) + 30*a**2*b**4*x**(1
```

```
7/2)/(17*sqrt(d)) + 4*a*b**5*x**(21/2)/(7*sqrt(d)) + 2*b**6*x**(25/2)/(25*s
qrt(d))
```

Giac [A] time = 1.1185, size = 142, normalized size = 1.1

$$\frac{2\left(4641\sqrt{dx}b^6x^{12} + 33150\sqrt{dx}ab^5x^{10} + 102375\sqrt{dx}a^2b^4x^8 + 178500\sqrt{dx}a^3b^3x^6 + 193375\sqrt{dx}a^4b^2x^4 + 139230\sqrt{dx}a^5b\right.}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/116025*(4641*sqrt(d*x)*b^6*x^12 + 33150*sqrt(d*x)*a*b^5*x^10 + 102375*sqr
t(d*x)*a^2*b^4*x^8 + 178500*sqrt(d*x)*a^3*b^3*x^6 + 193375*sqrt(d*x)*a^4*b^
2*x^4 + 139230*sqrt(d*x)*a^5*b*x^2 + 116025*sqrt(d*x)*a^6)/d
```

3.683 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=125

$$\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{4a^5b(dx)^{3/2}}{d^3}-\frac{2a^6}{d\sqrt{dx}}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

```
[Out] (-2*a^6)/(d*Sqrt[d*x]) + (4*a^5*b*(d*x)^(3/2))/d^3 + (30*a^4*b^2*(d*x)^(7/2
))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^2*b^4*(d*x)^(15/2))/
d^9 + (12*a*b^5*(d*x)^(19/2))/(19*d^11) + (2*b^6*(d*x)^(23/2))/(23*d^13)
```

Rubi [A] time = 0.0598052, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{4a^5b(dx)^{3/2}}{d^3}-\frac{2a^6}{d\sqrt{dx}}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (-2*a^6)/(d*Sqrt[d*x]) + (4*a^5*b*(d*x)^(3/2))/d^3 + (30*a^4*b^2*(d*x)^(7/2
))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^2*b^4*(d*x)^(15/2))/
d^9 + (12*a*b^5*(d*x)^(19/2))/(19*d^11) + (2*b^6*(d*x)^(23/2))/(23*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{3/2}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{(dx)^{3/2}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{3/2}} + \frac{6a^5b^7\sqrt{dx}}{d^2} + \frac{15a^4b^8(dx)^{5/2}}{d^4} + \frac{20a^3b^9(dx)^{9/2}}{d^6} + \frac{15a^2b^{10}(dx)^{13/2}}{d^8} + \frac{6ab^{11}(dx)^{17/2}}{d^{10}} + \frac{b^{12}(dx)^{21/2}}{d^{12}}\right) dx}{b^6}$$
$$= -\frac{2a^6}{d\sqrt{dx}} + \frac{4a^5b(dx)^{3/2}}{d^3} + \frac{30a^4b^2(dx)^{7/2}}{7d^5} + \frac{40a^3b^3(dx)^{11/2}}{11d^7} + \frac{2a^2b^4(dx)^{15/2}}{d^9} + \frac{12ab^5(dx)^{19/2}}{19d^{11}}$$

Mathematica [A] time = 0.0216665, size = 77, normalized size = 0.62

$$\frac{2x\left(33649a^2b^4x^8 + 61180a^3b^3x^6 + 72105a^4b^2x^4 + 67298a^5bx^2 - 33649a^6 + 10626ab^5x^{10} + 1463b^6x^{12}\right)}{33649(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-33649*a^6 + 67298*a^5*b*x^2 + 72105*a^4*b^2*x^4 + 61180*a^3*b^3*x^6
+ 33649*a^2*b^4*x^8 + 10626*a*b^5*x^10 + 1463*b^6*x^12))/(33649*(d*x)^(3/2)
)
```

Maple [A] time = 0.048, size = 74, normalized size = 0.6

$$-\frac{\left(-2926\,b^6x^{12} - 21252\,ab^5x^{10} - 67298\,a^2b^4x^8 - 122360\,a^3b^3x^6 - 144210\,a^4b^2x^4 - 134596\,a^5bx^2 + 67298\,a^6\right)x}{33649}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x)
```

```
[Out] -2/33649*(-1463*b^6*x^12-10626*a*b^5*x^10-33649*a^2*b^4*x^8-61180*a^3*b^3*x
^6-72105*a^4*b^2*x^4-67298*a^5*b*x^2+33649*a^6)*x/(d*x)^(3/2)
```

Maxima [A] time = 0.974442, size = 146, normalized size = 1.17

$$-\frac{2\left(\frac{33649\,a^6}{\sqrt{dx}} - \frac{1463\,(dx)^{\frac{23}{2}}b^6 + 10626\,(dx)^{\frac{19}{2}}ab^5d^2 + 33649\,(dx)^{\frac{15}{2}}a^2b^4d^4 + 61180\,(dx)^{\frac{11}{2}}a^3b^3d^6 + 72105\,(dx)^{\frac{7}{2}}a^4b^2d^8 + 67298\,(dx)^{\frac{3}{2}}a^5bd^{10}}{d^{12}}\right)}{33649\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/33649*(33649*a^6/sqrt(d*x) - (1463*(d*x)^(23/2)*b^6 + 10626*(d*x)^(19/2)
*a*b^5*d^2 + 33649*(d*x)^(15/2)*a^2*b^4*d^4 + 61180*(d*x)^(11/2)*a^3*b^3*d^
6 + 72105*(d*x)^(7/2)*a^4*b^2*d^8 + 67298*(d*x)^(3/2)*a^5*b*d^10)/d^12)/d
```

Fricas [A] time = 1.20081, size = 205, normalized size = 1.64

$$\frac{2\left(1463\,b^6x^{12} + 10626\,ab^5x^{10} + 33649\,a^2b^4x^8 + 61180\,a^3b^3x^6 + 72105\,a^4b^2x^4 + 67298\,a^5bx^2 - 33649\,a^6\right)\sqrt{dx}}{33649\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/33649*(1463*b^6*x^12 + 10626*a*b^5*x^10 + 33649*a^2*b^4*x^8 + 61180*a^3*b
^3*x^6 + 72105*a^4*b^2*x^4 + 67298*a^5*b*x^2 - 33649*a^6)*sqrt(d*x)/(d^2*x)
```

Sympy [A] time = 2.776, size = 126, normalized size = 1.01

$$-\frac{2a^6}{d^{\frac{3}{2}}\sqrt{x}} + \frac{4a^5bx^{\frac{3}{2}}}{d^{\frac{3}{2}}} + \frac{30a^4b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}} + \frac{40a^3b^3x^{\frac{11}{2}}}{11d^{\frac{3}{2}}} + \frac{2a^2b^4x^{\frac{15}{2}}}{d^{\frac{3}{2}}} + \frac{12ab^5x^{\frac{19}{2}}}{19d^{\frac{3}{2}}} + \frac{2b^6x^{\frac{23}{2}}}{23d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(3/2),x)
```

```
[Out] -2*a**6/(d**(3/2)*sqrt(x)) + 4*a**5*b*x**(3/2)/d**(3/2) + 30*a**4*b**2*x**(
7/2)/(7*d**(3/2)) + 40*a**3*b**3*x**(11/2)/(11*d**(3/2)) + 2*a**2*b**4*x**(
15/2)/d**(3/2) + 12*a*b**5*x**(19/2)/(19*d**(3/2)) + 2*b**6*x**(23/2)/(23*d
**(3/2))
```

Giac [A] time = 1.15905, size = 171, normalized size = 1.37

$$-\frac{2\left(\frac{33649\,a^6}{\sqrt{dx}} - \frac{1463\,\sqrt{dx}b^6d^{275}x^{11} + 10626\,\sqrt{dx}ab^5d^{275}x^9 + 33649\,\sqrt{dx}a^2b^4d^{275}x^7 + 61180\,\sqrt{dx}a^3b^3d^{275}x^5 + 72105\,\sqrt{dx}a^4b^2d^{275}x^3 + 67298\,\sqrt{dx}a^5bd^{275}x}{d^{276}}\right)}{33649\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/33649*(33649*a^6/sqrt(d*x) - (1463*sqrt(d*x)*b^6*d^275*x^11 + 10626*sqrt
(d*x)*a*b^5*d^275*x^9 + 33649*sqrt(d*x)*a^2*b^4*d^275*x^7 + 61180*sqrt(d*x)
*a^3*b^3*d^275*x^5 + 72105*sqrt(d*x)*a^4*b^2*d^275*x^3 + 67298*sqrt(d*x)*a^
5*b*d^275*x)/d^276)/d
```

3.684 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=127

$$\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{12a^5b\sqrt{dx}}{d^3}-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

```
[Out] (-2*a^6)/(3*d*(d*x)^(3/2)) + (12*a^5*b*Sqrt[d*x])/d^3 + (6*a^4*b^2*(d*x)^(5
/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b^4*(d*x)^(13/2))/(13
*d^9) + (12*a*b^5*(d*x)^(17/2))/(17*d^11) + (2*b^6*(d*x)^(21/2))/(21*d^13)
```

Rubi [A] time = 0.0621556, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{12a^5b\sqrt{dx}}{d^3}-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```


```
[Out] (-2*a^6)/(3*d*(d*x)^(3/2)) + (12*a^5*b*Sqrt[d*x])/d^3 + (6*a^4*b^2*(d*x)^(5
/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b^4*(d*x)^(13/2))/(13
*d^9) + (12*a*b^5*(d*x)^(17/2))/(17*d^11) + (2*b^6*(d*x)^(21/2))/(21*d^13)
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{5/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{(dx)^{5/2}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{5/2}} + \frac{6a^5b^7}{d^2\sqrt{dx}} + \frac{15a^4b^8(dx)^{3/2}}{d^4} + \frac{20a^3b^9(dx)^{7/2}}{d^6} + \frac{15a^2b^{10}(dx)^{11/2}}{d^8} + \frac{6ab^{11}(dx)^{15/2}}{d^{10}} + \frac{b^{12}(dx)^{19/2}}{d^{12}}\right)dx}{b^6}$$
$$= -\frac{2a^6}{3d(dx)^{3/2}} + \frac{12a^5b\sqrt{dx}}{d^3} + \frac{6a^4b^2(dx)^{5/2}}{d^5} + \frac{40a^3b^3(dx)^{9/2}}{9d^7} + \frac{30a^2b^4(dx)^{13/2}}{13d^9} + \frac{12ab^5(dx)^{17/2}}{17d^{11}}$$

Mathematica [A] time = 0.0232515, size = 77, normalized size = 0.61

$$\frac{2x\left(16065a^2b^4x^8+30940a^3b^3x^6+41769a^4b^2x^4+83538a^5bx^2-4641a^6+4914ab^5x^{10}+663b^6x^{12}\right)}{13923(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

```
[Out] (2*x*(-4641*a^6 + 83538*a^5*b*x^2 + 41769*a^4*b^2*x^4 + 30940*a^3*b^3*x^6 +
 16065*a^2*b^4*x^8 + 4914*a*b^5*x^10 + 663*b^6*x^12))/(13923*(d*x)^(5/2))
```

Maple [A] time = 0.049, size = 74, normalized size = 0.6

$$-\frac{\left(-1326\,b^6x^{12}-9828\,ab^5x^{10}-32130\,a^2b^4x^8-61880\,a^3b^3x^6-83538\,a^4b^2x^4-167076\,a^5bx^2+9282\,a^6\right)x}{13923}(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x)
```

```
[Out] -2/13923*(-663*b^6*x^12-4914*a*b^5*x^10-16065*a^2*b^4*x^8-30940*a^3*b^3*x^6
-41769*a^4*b^2*x^4-83538*a^5*b*x^2+4641*a^6)*x/(d*x)^(5/2)
```

Maxima [A] time = 1.01702, size = 146, normalized size = 1.15

$$-\frac{2\left(\frac{4641\,a^6}{(dx)^{\frac{3}{2}}}-\frac{663\,(dx)^{\frac{21}{2}}b^6+4914\,(dx)^{\frac{17}{2}}ab^5d^2+16065\,(dx)^{\frac{13}{2}}a^2b^4d^4+30940\,(dx)^{\frac{9}{2}}a^3b^3d^6+41769\,(dx)^{\frac{5}{2}}a^4b^2d^8+83538\,\sqrt{dx}a^5bd^{10}}{d^{12}}\right)}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/13923*(4641*a^6/(d*x)^(3/2) - (663*(d*x)^(21/2)*b^6 + 4914*(d*x)^(17/2)*
a*b^5*d^2 + 16065*(d*x)^(13/2)*a^2*b^4*d^4 + 30940*(d*x)^(9/2)*a^3*b^3*d^6
+ 41769*(d*x)^(5/2)*a^4*b^2*d^8 + 83538*sqrt(d*x)*a^5*b*d^10)/d^12)/d
```

Fricas [A] time = 1.30171, size = 204, normalized size = 1.61

$$\frac{2\left(663\,b^6x^{12}+4914\,ab^5x^{10}+16065\,a^2b^4x^8+30940\,a^3b^3x^6+41769\,a^4b^2x^4+83538\,a^5bx^2-4641\,a^6\right)\sqrt{dx}}{13923\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^6*x^12 + 4914*a*b^5*x^10 + 16065*a^2*b^4*x^8 + 30940*a^3*b^3
*x^6 + 41769*a^4*b^2*x^4 + 83538*a^5*b*x^2 - 4641*a^6)*sqrt(d*x)/(d^3*x^2)
```

Sympy [A] time = 3.33922, size = 128, normalized size = 1.01

$$-\frac{2a^6}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{12a^5b\sqrt{x}}{d^{\frac{5}{2}}}+\frac{6a^4b^2x^{\frac{5}{2}}}{d^{\frac{5}{2}}}+\frac{40a^3b^3x^{\frac{9}{2}}}{9d^{\frac{5}{2}}}+\frac{30a^2b^4x^{\frac{13}{2}}}{13d^{\frac{5}{2}}}+\frac{12ab^5x^{\frac{17}{2}}}{17d^{\frac{5}{2}}}+\frac{2b^6x^{\frac{21}{2}}}{21d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(5/2),x)
```

```
[Out] -2*a**6/(3*d**(5/2)*x**(3/2)) + 12*a**5*b*sqrt(x)/d**(5/2) + 6*a**4*b**2*x*
*(5/2)/d**(5/2) + 40*a**3*b**3*x**(9/2)/(9*d**(5/2)) + 30*a**2*b**4*x**(13/
2)/(13*d**(5/2)) + 12*a*b**5*x**(17/2)/(17*d**(5/2)) + 2*b**6*x**(21/2)/(21
*d**(5/2))
```

Giac [A] time = 1.12439, size = 176, normalized size = 1.39

$$-\frac{2\left(\frac{4641\,a^6d}{\sqrt{dx}x}-\frac{663\,\sqrt{dx}b^6d^{210}x^{10}+4914\,\sqrt{dx}ab^5d^{210}x^8+16065\,\sqrt{dx}a^2b^4d^{210}x^6+30940\,\sqrt{dx}a^3b^3d^{210}x^4+41769\,\sqrt{dx}a^4b^2d^{210}x^2+83538\,\sqrt{dx}a^5bd^{210}}{d^{210}}\right)}{13923\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/13923*(4641*a^6*d/(sqrt(d*x)*x) - (663*sqrt(d*x)*b^6*d^210*x^10 + 4914*s
qrt(d*x)*a*b^5*d^210*x^8 + 16065*sqrt(d*x)*a^2*b^4*d^210*x^6 + 30940*sqrt(d
*x)*a^3*b^3*d^210*x^4 + 41769*sqrt(d*x)*a^4*b^2*d^210*x^2 + 83538*sqrt(d*x)
*a^5*b*d^210)/d^210)/d^3
```

3.685 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=127

$$\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{10a^4b^2(dx)^{3/2}}{d^5}-\frac{12a^5b}{d^3\sqrt{dx}}-\frac{2a^6}{5d(dx)^{5/2}}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

```
[Out] (-2*a^6)/(5*d*(d*x)^(5/2)) - (12*a^5*b)/(d^3*Sqrt[d*x]) + (10*a^4*b^2*(d*x)
^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7) + (30*a^2*b^4*(d*x)^(11/2))/
(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11) + (2*b^6*(d*x)^(19/2))/(19*d^13)
```

Rubi [A] time = 0.0637789, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 270}

$$\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{10a^4b^2(dx)^{3/2}}{d^5}-\frac{12a^5b}{d^3\sqrt{dx}}-\frac{2a^6}{5d(dx)^{5/2}}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (-2*a^6)/(5*d*(d*x)^(5/2)) - (12*a^5*b)/(d^3*Sqrt[d*x]) + (10*a^4*b^2*(d*x)
^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7) + (30*a^2*b^4*(d*x)^(11/2))/
(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11) + (2*b^6*(d*x)^(19/2))/(19*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{7/2}}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{(dx)^{7/2}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{7/2}} + \frac{6a^5b^7}{d^2(dx)^{3/2}} + \frac{15a^4b^8\sqrt{dx}}{d^4} + \frac{20a^3b^9(dx)^{5/2}}{d^6} + \frac{15a^2b^{10}(dx)^{9/2}}{d^8} + \frac{6ab^{11}(dx)^{13/2}}{d^{10}} + \frac{b^{12}(dx)^{17/2}}{d^{12}}\right)dx}{b^6}$$
$$= -\frac{2a^6}{5d(dx)^{5/2}} - \frac{12a^5b}{d^3\sqrt{dx}} + \frac{10a^4b^2(dx)^{3/2}}{d^5} + \frac{40a^3b^3(dx)^{7/2}}{7d^7} + \frac{30a^2b^4(dx)^{11/2}}{11d^9} + \frac{4ab^5(dx)^{15/2}}{5d^{11}} +$$

Mathematica [A] time = 0.0245415, size = 82, normalized size = 0.65

$$\frac{2\sqrt{dx}\left(9975a^2b^4x^8+20900a^3b^3x^6+36575a^4b^2x^4-43890a^5bx^2-1463a^6+2926ab^5x^{10}+385b^6x^{12}\right)}{7315d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-1463*a^6 - 43890*a^5*b*x^2 + 36575*a^4*b^2*x^4 + 20900*a^3*b
^3*x^6 + 9975*a^2*b^4*x^8 + 2926*a*b^5*x^10 + 385*b^6*x^12))/(7315*d^4*x^3)
```

Maple [A] time = 0.05, size = 74, normalized size = 0.6

$$-\frac{\left(-770\,b^6x^{12}-5852\,ab^5x^{10}-19950\,a^2b^4x^8-41800\,a^3b^3x^6-73150\,a^4b^2x^4+87780\,a^5bx^2+2926\,a^6\right)x}{7315}(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x)
```

```
[Out] -2/7315*(-385*b^6*x^12-2926*a*b^5*x^10-9975*a^2*b^4*x^8-20900*a^3*b^3*x^6-3
6575*a^4*b^2*x^4+43890*a^5*b*x^2+1463*a^6)*x/(d*x)^(7/2)
```

Maxima [A] time = 0.991902, size = 154, normalized size = 1.21

$$-\frac{2\left(\frac{1463\left(30\,a^5bd^2x^2+a^6d^2\right)}{(dx)^{\frac{5}{2}}d^2}-\frac{385\,(dx)^{\frac{19}{2}}b^6+2926\,(dx)^{\frac{15}{2}}ab^5d^2+9975\,(dx)^{\frac{11}{2}}a^2b^4d^4+20900\,(dx)^{\frac{7}{2}}a^3b^3d^6+36575\,(dx)^{\frac{3}{2}}a^4b^2d^8}{d^{12}}\right)}{7315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/7315*(1463*(30*a^5*b*d^2*x^2 + a^6*d^2)/((d*x)^(5/2)*d^2) - (385*(d*x)^(
19/2)*b^6 + 2926*(d*x)^(15/2)*a*b^5*d^2 + 9975*(d*x)^(11/2)*a^2*b^4*d^4 + 2
0900*(d*x)^(7/2)*a^3*b^3*d^6 + 36575*(d*x)^(3/2)*a^4*b^2*d^8)/d^12)/d
```

Fricas [A] time = 1.19204, size = 201, normalized size = 1.58

$$\frac{2\left(385\,b^{6}x^{12}+2926\,ab^{5}x^{10}+9975\,a^{2}b^{4}x^{8}+20900\,a^{3}b^{3}x^{6}+36575\,a^{4}b^{2}x^{4}-43890\,a^{5}bx^{2}-1463\,a^{6}\right)\sqrt{dx}}{7315\,d^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^6*x^12 + 2926*a*b^5*x^10 + 9975*a^2*b^4*x^8 + 20900*a^3*b^3*x
^6 + 36575*a^4*b^2*x^4 - 43890*a^5*b*x^2 - 1463*a^6)*sqrt(d*x)/(d^4*x^3)
```

Sympy [A] time = 5.08522, size = 128, normalized size = 1.01

$$-\frac{2a^{6}}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{12a^{5}b}{d^{\frac{7}{2}}\sqrt{x}}+\frac{10a^{4}b^{2}x^{\frac{3}{2}}}{d^{\frac{7}{2}}}+\frac{40a^{3}b^{3}x^{\frac{7}{2}}}{7d^{\frac{7}{2}}}+\frac{30a^{2}b^{4}x^{\frac{11}{2}}}{11d^{\frac{7}{2}}}+\frac{4ab^{5}x^{\frac{15}{2}}}{5d^{\frac{7}{2}}}+\frac{2b^{6}x^{\frac{19}{2}}}{19d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(7/2),x)
```

```
[Out] -2*a**6/(5*d**(7/2)*x**(5/2)) - 12*a**5*b/(d**(7/2)*sqrt(x)) + 10*a**4*b**2
*x**(3/2)/d**(7/2) + 40*a**3*b**3*x**(7/2)/(7*d**(7/2)) + 30*a**2*b**4*x**(
11/2)/(11*d**(7/2)) + 4*a*b**5*x**(15/2)/(5*d**(7/2)) + 2*b**6*x**(19/2)/(1
9*d**(7/2))
```

Giac [A] time = 1.12686, size = 180, normalized size = 1.42

$$-\frac{2\left(\frac{1463\left(30\,a^5bd^3x^2+a^6d^3\right)}{\sqrt{dx}d^2x^2}-\frac{385\,\sqrt{dx}b^6d^{171}x^9+2926\,\sqrt{dx}ab^5d^{171}x^7+9975\,\sqrt{dx}a^2b^4d^{171}x^5+20900\,\sqrt{dx}a^3b^3d^{171}x^3+36575\,\sqrt{dx}a^4b^2d^{171}x}{d^{171}}\right)}{7315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/7315*(1463*(30*a^5*b*d^3*x^2 + a^6*d^3)/(sqrt(d*x)*d^2*x^2) - (385*sqrt(
d*x)*b^6*d^171*x^9 + 2926*sqrt(d*x)*a*b^5*d^171*x^7 + 9975*sqrt(d*x)*a^2*b^
4*d^171*x^5 + 20900*sqrt(d*x)*a^3*b^3*d^171*x^3 + 36575*sqrt(d*x)*a^4*b^2*d
^171*x)/d^171)/d^4
```

3.686 $\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=316

$$-\frac{9a^{5/4}d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{13/4}}+\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{13/4}}-\frac{9a^{5/4}d^{11/2}\text{ta}}{4}$$

```
[Out] (-9*a*d^5*Sqrt[d*x])/(2*b^3) + (9*d^3*(d*x)^(5/2))/(10*b^2) - (d*(d*x)^(9/2
))/(2*b*(a + b*x^2)) - (9*a^(5/4)*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4))
- (9*a^(5/4)*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*S
qrt[2]*b^(13/4))
```

Rubi [A] time = 0.383407, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{9a^{5/4}d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{13/4}}+\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{13/4}}-\frac{9a^{5/4}d^{11/2}\text{ta}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```


```
[Out] (-9*a*d^5*Sqrt[d*x])/(2*b^3) + (9*d^3*(d*x)^(5/2))/(10*b^2) - (d*(d*x)^(9/2
))/(2*b*(a + b*x^2)) - (9*a^(5/4)*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4))
- (9*a^(5/4)*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*S
qrt[2]*b^(13/4))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{1}{4}\left(9d^2\right)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx$$

$$= \frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}-\frac{\left(9ad^4\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{4b}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{\left(9a^2d^6\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4b^2}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{\left(9a^2d^5\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{2b^2}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{\left(9a^{3/2}d^4\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4b^2}+\frac{\left(9a^{3/2}d\right.}{}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}-\frac{\left(9a^{5/4}d^{11/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{dx}\right)}{8\sqrt{2}b^{13/4}}$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}-\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}b^{13/4}}+$$

$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}-\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{13/4}}+\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(\right.}{4\sqrt{2}b}$$

Mathematica [A] time = 0.319475, size = 235, normalized size = 0.74

$$\frac{d^5\sqrt{dx}\left(\frac{8\sqrt[4]{b}\sqrt{x}\left(-45a^2-36abx^2+4b^2x^4\right)}{a+bx^2}-45\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+45\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)\right.}{80b^{13/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^5*Sqrt[d*x]*((8*b^(1/4)*Sqrt[x]*(-45*a^2 - 36*a*b*x^2 + 4*b^2*x^4))/(a +
 b*x^2) - 90*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 90*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 45*Sqr
t[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 4
5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
]))/(80*b^(13/4)*Sqrt[x])
```

Maple [A] time = 0.088, size = 242, normalized size = 0.8

$$\frac{2\,d^3}{5\,b^2}\,(dx)^{\frac{5}{2}}-4\,\frac{ad^5\sqrt{dx}}{b^3}-\frac{d^7a^2}{2\,b^3\left(bd^2x^2+ad^2\right)}\sqrt{dx}+\frac{9\,ad^5\sqrt{2}}{16\,b^3}\sqrt[4]{\frac{ad^2}{b}}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/5*d^3*(d*x)^(5/2)/b^2-4*a*d^5*(d*x)^(1/2)/b^3-1/2*d^7/b^3*a^2*(d*x)^(1/2)
/(b*d^2*x^2+a*d^2)+9/16*d^5/b^3*a*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(a*d^2/b)^(1/2)))+9/8*d^5/b^3*a*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+9/8*d^5/b^3*a*(a*d^2/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40033, size = 636, normalized size = 2.01

$$\frac{180\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^4x^2+ab^3\right)\arctan\left(-\frac{\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}ab^{10}d^5-\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{a^2d^{11}x+\sqrt{-\frac{a^5d^{22}}{b^{13}}}b^6}b^{10}}{a^5d^{22}}\right)+45\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^4x^2+ab^3\right)\log\left(9\right.}{40(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/40*(180*(-a^5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*arctan(-((-a^5*d^22/b^13
)^(3/4)*sqrt(d*x)*a*b^10*d^5 - (-a^5*d^22/b^13)^(3/4)*sqrt(a^2*d^11*x + sqr
t(-a^5*d^22/b^13)*b^6)*b^10)/(a^5*d^22)) + 45*(-a^5*d^22/b^13)^(1/4)*(b^4*x
^2 + a*b^3)*log(9*sqrt(d*x)*a*d^5 + 9*(-a^5*d^22/b^13)^(1/4)*b^3) - 45*(-a^
5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*log(9*sqrt(d*x)*a*d^5 - 9*(-a^5*d^22/b
^13)^(1/4)*b^3) + 4*(4*b^2*d^5*x^4 - 36*a*b*d^5*x^2 - 45*a^2*d^5)*sqrt(d*x)
)/(b^4*x^2 + a*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{11}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(11/2)/(a + b*x**2)**2, x)
```

Giac [A] time = 1.16369, size = 406, normalized size = 1.28

$$-\frac{1}{80}\left(\frac{40\,\sqrt{dx}a^2d^3}{\left(bd^2x^2+ad^2\right)b^3}-\frac{90\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{90\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{}\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/80*(40*sqrt(d*x)*a^2*d^3/((b*d^2*x^2 + a*d^2)*b^3) - 90*sqrt(2)*(a*b^3*d
^2)^(1/4)*a*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a
*d^2/b)^(1/4))/b^4 - 90*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^4 - 45*sqrt(2)*(a
*b^3*d^2)^(1/4)*a*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^
2/b))/b^4 + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*log(d*x - sqrt(2)*(a*d^2/b)^(1
/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^4 - 32*(sqrt(d*x)*b^8*d^6*x^2 - 10*sqrt(d*
x)*a*b^7*d^6)/(b^10*d^5))*d^4
```

3.687 $\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=298

$$-\frac{7a^{3/4}d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{11/4}}+\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{11/4}}+\frac{7a^{3/4}d^{9/2}\tan^{-1}}{4\sqrt{2}b}$$

```
[Out] (7*d^3*(d*x)^(3/2))/(6*b^2) - (d*(d*x)^(7/2))/(2*b*(a + b*x^2)) + (7*a^(3/4
)*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqr
t[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]
*b^(11/4)) + (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(11/4))
```

Rubi [A] time = 0.299031, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7a^{3/4}d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{11/4}}+\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{11/4}}+\frac{7a^{3/4}d^{9/2}\tan^{-1}}{4\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```



```
[Out] (7*d^3*(d*x)^(3/2))/(6*b^2) - (d*(d*x)^(7/2))/(2*b*(a + b*x^2)) + (7*a^(3/4
)*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqr
t[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]
*b^(11/4)) + (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(11/4))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^2}\,dx \\
&= -\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}+\frac{1}{4}\left(7d^2\right)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx \\
&= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}-\frac{\left(7ad^4\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4b} \\
&= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}-\frac{\left(7ad^3\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{2b} \\
&= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}+\frac{\left(7ad^3\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4b^{3/2}}-\frac{\left(7ad^3\right)\text{Subst}\left(\int \frac{\sqrt{a}d+\cdots}{ab+\cdots}\cdots\right)}{4b^{3/2}} \\
&= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}-\frac{\left(7a^{3/4}d^{9/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{dx}\right)}{8\sqrt{2}b^{11/4}}-\frac{\left(7a^{3/4}d^{9/2}\right)\cdots}{\cdots} \\
&= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}-\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}b^{11/4}}+\frac{7a^{3/4}d^{9/2}\log\cdots}{\cdots} \\
&= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}+\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{11/4}}-\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{11/4}}\cdots
\end{aligned}$$

Mathematica [C] time = 0.0183977, size = 63, normalized size = 0.21

$$-\frac{2d^4x\sqrt{dx}\left(7\left(a+bx^2\right){}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)-7a-bx^2\right)}{3b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-2*d^4*x*Sqrt[d*x]*(-7*a - b*x^2 + 7*(a + b*x^2)*Hypergeometric2F1[3/4, 2,
 7/4, -((b*x^2)/a)]))/(3*b^2*(a + b*x^2))
```

Maple [A] time = 0.062, size = 226, normalized size = 0.8

$$\frac{2d^3}{3b^2}(dx)^{\frac{3}{2}}+\frac{d^5a}{2b^2\left(bd^2x^2+ad^2\right)}(dx)^{\frac{3}{2}}-\frac{7d^5a\sqrt{2}}{16b^3}\ln\left(\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/3*d^3*(d*x)^(3/2)/b^2+1/2*d^5*a/b^2*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)-7/16*d^
5*a/b^3/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
)-7/8*d^5*a/b^3/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x
)^(1/2)+1)-7/8*d^5*a/b^3/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(
1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40596, size = 640, normalized size = 2.15

$$84\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\arctan\left(-\frac{\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\sqrt{dx}a^2b^3d^{13}-\sqrt{a^4d^{27}x-\sqrt{-\frac{a^3d^{18}}{b^{11}}}a^3b^5d^{18}}\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}b^3}{a^3d^{18}}\right)-21\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\log$$

24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(84*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((-a^3*d^18/b^11)
^(1/4)*sqrt(d*x)*a^2*b^3*d^13 - sqrt(a^4*d^27*x - sqrt(-a^3*d^18/b^11)*a^3*
b^5*d^18)*(-a^3*d^18/b^11)^(1/4)*b^3)/(a^3*d^18)) - 21*(-a^3*d^18/b^11)^(1/
4)*(b^3*x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*d^13 + 343*(-a^3*d^18/b^11)^(3/4
)*b^8) + 21*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*
d^13 - 343*(-a^3*d^18/b^11)^(3/4)*b^8) + 4*(4*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d
*x))/(b^3*x^2 + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{9}{2}}}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(9/2)/(a + b*x**2)**2, x)
```

Giac [A] time = 1.14137, size = 359, normalized size = 1.2

$$\frac{1}{48}\left(\frac{24\sqrt{dx}ad^3x}{\left(bd^2x^2 + ad^2\right)b^2} + \frac{32\sqrt{dx}dx}{b^2} - \frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}} + 2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5} - \frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right.\right.}{2\left(\frac{ad}{b}\right.}\right.}{b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/48*(24*sqrt(d*x)*a*d^3*x/((b*d^2*x^2 + a*d^2)*b^2) + 32*sqrt(d*x)*d*x/b^2
 - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
 + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 +
 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/b^5 - 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5)*d^3
```

3.688 $\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=298

$$\frac{5\sqrt[4]{a}d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{a}d^{7/2}\tan^{-1}\left(1\right.}{4\sqrt{2}b^{9}}$$

```
[Out] (5*d^3*Sqrt[d*x])/(2*b^2) - (d*(d*x)^(5/2))/(2*b*(a + b*x^2)) + (5*a^(1/4)*
d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[
2]*b^(9/4)) - (5*a^(1/4)*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(9/4)) + (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(
9/4)) - (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(9/4))
```

Rubi [A] time = 0.294855, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5\sqrt[4]{a}d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{a}d^{7/2}\tan^{-1}\left(1\right.}{4\sqrt{2}b^{9}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```


```
[Out] (5*d^3*Sqrt[d*x])/(2*b^2) - (d*(d*x)^(5/2))/(2*b*(a + b*x^2)) + (5*a^(1/4)*
d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[
2]*b^(9/4)) - (5*a^(1/4)*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(9/4)) + (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(
9/4)) - (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(9/4))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{1}{4}\left(5d^2\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5ad^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4b}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5ad^3\right)\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{2b}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5\sqrt{a}d^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4b}-\frac{\left(5\sqrt{a}d^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}d+}{ab+\frac{b}{}}\right.}{4b}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{\left(5\sqrt[4]{a}d^{7/2}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{dx}\right)}{8\sqrt{2}b^{9/4}}+\frac{\left(5\sqrt[4]{a}d^{7/2}\right)\mathrm{Subs}}{}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt[4]{a}d^{7/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}d^{7/2}\log\left(\sqrt{a}\right.}{}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt[4]{a}d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}d^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{9/4}}+5$$

Mathematica [A] time = 0.154868, size = 244, normalized size = 0.82

$$\frac{d^3\sqrt{dx}\left(\frac{32b^{5/4}x^2}{a+bx^2}+\frac{40a\sqrt[4]{b}}{a+bx^2}+\frac{5\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}-\frac{5\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}+\frac{10\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{x}}-\frac{10\sqrt{2}\sqrt[4]{a}}{}\right.}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^3*Sqrt[d*x]*((40*a*b^(1/4))/(a + b*x^2) + (32*b^(5/4)*x^2)/(a + b*x^2) +
 (10*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[x]
 - (10*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[
x] + (5*Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/Sqrt[x] - (5*Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/Sqrt[x]))/(16*b^(9/4))
```

Maple [A] time = 0.059, size = 223, normalized size = 0.8

$$2\,\frac{d^3\sqrt{dx}}{b^2}+\frac{d^5a}{2\,b^2\left(bd^2x^2+ad^2\right)}\sqrt{dx}-\frac{5\,d^3\sqrt{2}}{16\,b^2}\sqrt[4]{\frac{ad^2}{b}}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2*d^3*(d*x)^(1/2)/b^2+1/2*d^5/b^2*a*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)-5/16*d^3/
b^2*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))-5/
8*d^3/b^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2
)+1)-5/8*d^3/b^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*
x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38641, size = 549, normalized size = 1.84

$$-\frac{20\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\arctan\left(-\frac{\left(-\frac{ad^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{dx}b^7d^3-\sqrt{d^7x+\sqrt{-\frac{ad^{14}}{b^9}}b^4}\left(-\frac{ad^{14}}{b^9}\right)^{\frac{3}{4}}b^7}{ad^{14}}\right)+5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\log\left(5\sqrt{dx}d^3+5\right.}{8\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(20*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((-a*d^14/b^9)^(3/4)
*sqrt(d*x)*b^7*d^3 - sqrt(d^7*x + sqrt(-a*d^14/b^9)*b^4)*(-a*d^14/b^9)^(3/4
)*b^7)/(a*d^14)) + 5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*sqrt(d*x)*
d^3 + 5*(-a*d^14/b^9)^(1/4)*b^2) - 5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*
log(5*sqrt(d*x)*d^3 - 5*(-a*d^14/b^9)^(1/4)*b^2) - 4*(4*b*d^3*x^2 + 5*a*d^3
)*sqrt(d*x))/(b^3*x^2 + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**2, x)
```

Giac [A] time = 1.15487, size = 362, normalized size = 1.21

$$\frac{1}{16}\left(\frac{8\sqrt{dx}ad^3}{(bd^2x^2+ad^2)b^2}-\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}-\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/16*(8*sqrt(d*x)*a*d^3/((b*d^2*x^2 + a*d^2)*b^2) - 10*sqrt(2)*(a*b^3*d^2)^
(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/b^3 - 10*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 - 5*sqrt(2)*(a*b^3*d^2)
^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 +
 5*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/b^3 + 32*sqrt(d*x)*d/b^2)*d^2
```

3.689 $\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=281

$$\frac{3d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

```
[Out] -(d*(d*x)^(3/2))/(2*b*(a + b*x^2)) - (3*d^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*d^(5/2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)
*b^(7/4)) + (3*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*d^(5/2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sq
rt[2]*a^(1/4)*b^(7/4))
```

Rubi [A] time = 0.279858, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -(d*(d*x)^(3/2))/(2*b*(a + b*x^2)) - (3*d^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*d^(5/2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)
*b^(7/4)) + (3*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*d^(5/2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sq
rt[2]*a^(1/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{1}{4}\left(3d^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{1}{2}(3d)\,\mathrm{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} - \frac{(3d)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{b}} + \frac{(3d)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}d+\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{b}}$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{\left(3d^{5/2}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{\left(3d^{5/2}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}\cdots}{-\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{b}}\cdots}\cdots\right.}{8\sqrt{2}\sqrt[4]{a}b^{7/4}\cdots}$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} + \frac{3d^{5/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}} - \frac{3d^{5/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x+\cdots\right.}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

$$= -\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)} - \frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{3d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{3d^{5/2}\log\left(\sqrt{a}\sqrt{d}\cdots\right.}{8\cdots}$$

Mathematica [C] time = 0.0152097, size = 54, normalized size = 0.19

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right){}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)-a\right)}{ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a + (a + b*x^2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)
/a)]))/(a*b*(a + b*x^2))
```

Maple [A] time = 0.06, size = 209, normalized size = 0.7

$$-\frac{d^3}{2b\left(bd^2x^2+ad^2\right)}(dx)^{\frac{3}{2}}+\frac{3d^3\sqrt{2}}{16b^2}\ln\left(\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{3d^3\sqrt{}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*d^3/b*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+3/16*d^3/b^2/(a*d^2/b)^(1/4)*2^(1/
2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+3/8*d^3/b^2/(a*d^2/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+3/8*d^3/b^2/(a*d^2/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37561, size = 552, normalized size = 1.96

$$-\frac{4\sqrt{dx}d^2x+12\left(b^2x^2+ab\right)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\sqrt{dx}b^2d^7-\sqrt{d^{15}x-\sqrt{-\frac{d^{10}}{ab^7}}ab^3d^{10}}\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}b^2}{d^{10}}\right)-3\left(b^2x^2+ab\right)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\log\left(27\cdots\right.}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*sqrt(d*x)*d^2*x + 12*(b^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*arctan(-
((-d^10/(a*b^7))^(1/4)*sqrt(d*x)*b^2*d^7 - sqrt(d^15*x - sqrt(-d^10/(a*b^7)
)*a*b^3*d^10)*(-d^10/(a*b^7))^(1/4)*b^2)/d^10) - 3*(b^2*x^2 + a*b)*(-d^10/(
a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 + 27*(-d^10/(a*b^7))^(3/4)*a*b^5) + 3*(b
^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 - 27*(-d^10/(a*b^7
))^(3/4)*a*b^5))/(b^2*x^2 + a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**2, x)
```

Giac [A] time = 1.20044, size = 355, normalized size = 1.26

$$-\frac{1}{16}\left(\frac{8\sqrt{dx}d^3x}{(bd^2x^2+ad^2)b}-\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}-\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}+\frac{3\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*(8*sqrt(d*x)*d^3*x/((b*d^2*x^2 + a*d^2)*b) - 6*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1
/4))/(a*b^4) - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4) + 3*sqrt(2)*(a*b^3*d^2
)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4
) - 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a*b^4))*d
```

3.690 $\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=281

$$-\frac{d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+\frac{d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{3/4}b^{5/4}}-\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

```
[Out] -(d*Sqrt[d*x])/(2*b*(a + b*x^2)) - (d^(3/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/
4)) - (d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3
/4)*b^(5/4))
```

Rubi [A] time = 0.260389, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+\frac{d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{3/4}b^{5/4}}-\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```


```
[Out] -(d*Sqrt[d*x])/(2*b*(a + b*x^2)) - (d^(3/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/
4)) - (d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3
/4)*b^(5/4))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} + \frac{1}{4}d^2\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}d+\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{a}}$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} - \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}a^{3/4}b^{5/4}} - \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-x^2}\,d\ldots}{8\sqrt{2}a^{3/4}b^{5/4}}$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} - \frac{d^{3/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}a^{3/4}b^{5/4}} + \frac{d^{3/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}+\sqrt{\ldots}\right.}{8\sqrt{2}a^{3/4}b^{5/4}}$$

$$= -\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} - \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \frac{d^{3/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{\ldots}\right.}{8\sqrt{2}\ldots}$$

Mathematica [A] time = 0.166154, size = 210, normalized size = 0.75

$$\frac{(dx)^{3/2}\left(-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-\frac{8\sqrt[4]{b}\sqrt{x}}{a+bx^2}\right)}{16b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] ((d*x)^(3/2)*((-8*b^(1/4)*Sqrt[x])/(a + b*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4)))/(16*b^(5/4)*x^(3/2))
```

Maple [A] time = 0.061, size = 212, normalized size = 0.8

$$-\frac{d^3}{2b\left(bd^2x^2+ad^2\right)}\sqrt{dx}+\frac{d\sqrt{2}}{16ab}\sqrt[4]{\frac{ad^2}{b}}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+\frac{d\sqrt{2}}{8ab}\sqrt[4]{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*d^3/b*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)+1/16*d/b*(a*d^2/b)^(1/4)/a*2^(1/2)
*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+1/8*d/b*(a*d^2/b)^(1/4)/a*2^(
1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+1/8*d/b*(a*d^2/b)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37143, size = 525, normalized size = 1.87

$$\frac{4\left(b^2x^2+ab\right)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^2b^4d\left(-\frac{d^6}{a^3b^5}\right)^{\frac{3}{4}}-\sqrt{a^2b^2\sqrt{-\frac{d^6}{a^3b^5}}+d^3x}a^2b^4\left(-\frac{d^6}{a^3b^5}\right)^{\frac{3}{4}}}{d^6}\right)+\left(b^2x^2+ab\right)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}+\right.}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^2*b^4*d*
(-d^6/(a^3*b^5))^(3/4) - sqrt(a^2*b^2*sqrt(-d^6/(a^3*b^5)) + d^3*x)*a^2*b^4
*(-d^6/(a^3*b^5))^(3/4))/d^6) + (b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*log(
a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - (b^2*x^2 + a*b)*(-d^6/(a^3*b^5)
)^(1/4)*log(-a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - 4*sqrt(d*x)*d)/(b^
2*x^2 + a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**2, x)
```

Giac [A] time = 1.20026, size = 355, normalized size = 1.26

$$-\frac{\sqrt{dx}d^3}{2\left(bd^2x^2+ad^2\right)b}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8ab^2}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8ab^2}+\frac{\sqrt{2}(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(d*x)*d^3/((b*d^2*x^2 + a*d^2)*b) + 1/8*sqrt(2)*(a*b^3*d^2)^(1/4)*
d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4
))/(a*b^2) + 1/8*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^2) + 1/16*sqrt(2)*(a*b^
3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a*b^2) - 1/16*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^2)
```

3.691 $\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\cdots$$

```
[Out] (d*x)^(3/2)/(2*a*d*(a + b*x^2)) - (Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4)) + (Sqrt[d]*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4
)) + (Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - (Sqrt[d]*Log[Sqrt[a]*Sqrt[d]
 + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/
4)*b^(3/4))
```

Rubi [A] time = 0.271493, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```


```
[Out] (d*x)^(3/2)/(2*a*d*(a + b*x^2)) - (Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4)) + (Sqrt[d]*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4
)) + (Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - (Sqrt[d]*Log[Sqrt[a]*Sqrt[d]
 + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/
4)*b^(3/4))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}+\frac{b\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4a}$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}+\frac{b\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{2ad}$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}-\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4ad}+\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}d+\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4ad}$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}+\frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{dx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}+\frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-x^2}\,dx\right.}{8\sqrt{2}a^{5/4}b^{3/4}}$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}+\frac{\sqrt{d}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x+\sqrt{2}\right.}{8\sqrt{2}a^{5/4}b^{3/4}}$$

$$= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{\sqrt{d}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\right.}{8\sqrt{2}a}$$

Mathematica [C] time = 0.0059782, size = 32, normalized size = 0.11

$$\frac{2x\sqrt{dx}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*Sqrt[d*x]*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^2)
```

Maple [A] time = 0.056, size = 210, normalized size = 0.7

$$\frac{d}{2\,a\left(bd^2x^2+ad^2\right)}(dx)^{\frac{3}{2}}+\frac{d\sqrt{2}}{16\,ab}\ln\left(\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{d\sqrt{2}}{8\,ab}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*d*(d*x)^(3/2)/a/(b*d^2*x^2+a*d^2)+1/16*d/a/b/(a*d^2/b)^(1/4)*2^(1/2)*ln
((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+1/8*d/a/b/(a*d^2/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+1/8*d/a/b/(a*d^2/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38718, size = 531, normalized size = 1.88

$$-\frac{4\left(abx^2+a^2\right)\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}abd\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}-\sqrt{-a^3bd^2\sqrt{-\frac{d^2}{a^5b^3}}+d^3x}ab\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}}{d^2}\right)-\left(abx^2+a^2\right)\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\left(-\frac{d^2}{a^5b^3}\right)^{\frac{3}{4}}\right.}{8\left(abx^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*arctan(-(sqrt(d*x)*a*b*d*(-d
^2/(a^5*b^3))^(1/4) - sqrt(-a^3*b*d^2*sqrt(-d^2/(a^5*b^3)) + d^3*x)*a*b*(-d
^2/(a^5*b^3))^(1/4))/d^2) - (a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*log(a^4*
b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) + (a*b*x^2 + a^2)*(-d^2/(a^5*b^3)
)^(1/4)*log(-a^4*b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) - 4*sqrt(d*x)*x)
/(a*b*x^2 + a^2)
```

Sympy [A] time = 6.39636, size = 78, normalized size = 0.28

$$\frac{2d^3\left(dx\right)^{\frac{3}{2}}}{4a^2d^4+4abd^4x^2}+2d^3\operatorname{RootSum}\left(65536t^4a^5b^3d^{10}+1,\left(t\mapsto t\log\left(4096t^3a^4b^2d^8+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*d**3*(d*x)**(3/2)/(4*a**2*d**4 + 4*a*b*d**4*x**2) + 2*d**3*RootSum(65536*
_t**4*a**5*b**3*d**10 + 1, Lambda(_t, _t*log(4096*_t**3*a**4*b**2*d**8 + sq
rt(d*x))))
```

Giac [A] time = 1.25684, size = 367, normalized size = 1.3

$$\frac{\sqrt{dx}d^2x}{2\left(bd^2x^2+ad^2\right)a}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^3d}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^3d}-\frac{\sqrt{2}\left(ab^3d^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*d^2*x/((b*d^2*x^2 + a*d^2)*a) + 1/8*sqrt(2)*(a*b^3*d^2)^(3/4)
*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4)
)/(a^2*b^3*d) + 1/8*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3*d) - 1/16*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a^2*b^3*d) + 1/16*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3*d)
```

3.692 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=283

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\tan$$

```
[Out] Sqrt[d*x]/(2*a*d*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt
[d]) - (3*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)
*b^(1/4)*Sqrt[d])
```

Rubi [A] time = 0.264979, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\tan$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```


```
[Out] Sqrt[d*x]/(2*a*d*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt
[d]) - (3*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)
*b^(1/4)*Sqrt[d])
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{(3b)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2ad}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{(3b)\,\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4a^{3/2}d^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{\sqrt{a}d+\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{}\right.}{4a^{3/2}d^2}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{dx}\right)}{8a^{3/2}\sqrt{b}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}+x^2}\right.}{8a^{3/2}\sqrt{b}}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} - \frac{3\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}} + \frac{3\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}+\right.}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

$$= \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}} - \frac{3\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\right.}{8\sqrt{2}a}$$

Mathematica [A] time = 0.160656, size = 211, normalized size = 0.75

$$\frac{\sqrt{x}\left(\frac{8a^{3/4}\sqrt{x}}{a+bx^2}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}\right)}{16a^{7/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (Sqrt[x]*((8*a^(3/4)*Sqrt[x])/(a + b*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/b^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/(16*a^(7/4)*Sqrt[d*x])
```

Maple [A] time = 0.055, size = 207, normalized size = 0.7

$$\frac{d}{2\,a\left(bd^2x^2+ad^2\right)}\sqrt{dx}+\frac{3\sqrt{2}}{16\,a^2d}\sqrt[4]{\frac{ad^2}{b}}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+\frac{3\sqrt{2}}{8\,a^2d}\sqrt[4]{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x)
```

```
[Out] 1/2*d*(d*x)^(1/2)/a/(b*d^2*x^2+a*d^2)+3/16/d/a^2*(a*d^2/b)^(1/4)*2^(1/2)*ln
((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+3/8/d/a^2*(a*d^2/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+3/8/d/a^2*(a*d^2/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38892, size = 536, normalized size = 1.89

$$\frac{12\left(abdx^2+a^2d\right)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4d^2\sqrt{-\frac{1}{a^7bd^2}}+dx}\,a^5bd\left(-\frac{1}{a^7bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^5bd\left(-\frac{1}{a^7bd^2}\right)^{\frac{3}{4}}\right)+3\left(abdx^2+a^2d\right)\left(-\frac{1}{a^7bd^2}\right.}{8\left(abdx^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(12*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*arctan(sqrt(a^4*d^2*sqrt
(-1/(a^7*b*d^2)) + d*x)*a^5*b*d*(-1/(a^7*b*d^2))^(3/4) - sqrt(d*x)*a^5*b*d*
(-1/(a^7*b*d^2))^(3/4)) + 3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*log(
a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x)) - 3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*
b*d^2))^(1/4)*log(-a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x)) + 4*sqrt(d*x))
/(a*b*d*x^2 + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**2), x)
```

Giac [A] time = 1.25347, size = 363, normalized size = 1.28

$$\frac{\sqrt{dx}d}{2\left(bd^2x^2+ad^2\right)a}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^2bd}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^2bd}+\frac{3\sqrt{2}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*d/((b*d^2*x^2 + a*d^2)*a) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a
^2*b*d) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b*d) + 3/16*sqrt(2)*(a*b^3*d
^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2
*b*d) - 3/16*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a^2*b*d)
```

3.693 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=300

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}a^{9/4}d^{3/2}}$$

```
[Out] -5/(2*a^2*d*Sqrt[d*x]) + 1/(2*a*d*Sqrt[d*x]*(a + b*x^2)) + (5*b^(1/4)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(9/4)*d^
(3/2)) - (5*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(4*Sqrt[2]*a^(9/4)*d^(3/2)) - (5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*
Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
 + (5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
```

Rubi [A] time = 0.323169, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}a^{9/4}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```


```
[Out] -5/(2*a^2*d*Sqrt[d*x]) + 1/(2*a*d*Sqrt[d*x]*(a + b*x^2)) + (5*b^(1/4)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(9/4)*d^
(3/2)) - (5*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(4*Sqrt[2]*a^(9/4)*d^(3/2)) - (5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*
Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
 + (5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}dx = b^2\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^2}dx$$

$$= \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} + \frac{(5b)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}dx}{4a}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} - \frac{\left(5b^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}dx}{4a^2d^2}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}dx, x, \sqrt{dx}\right)}{2a^2d^3}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} + \frac{\left(5b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}dx, x, \sqrt{dx}\right)}{4a^2d^3} - \frac{\left(5b^{3/2}\right)}{}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} - \frac{\left(5\sqrt[4]{b}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-x^2}dx, x, \sqrt{dx}\right)}{8\sqrt{2}a^{9/4}d^{3/2}} -$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} - \frac{5\sqrt[4]{b}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}a^{9/4}d^{3/2}} + \frac{5}{}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} + \frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{9/4}d^{3/2}} - \frac{5\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{4\sqrt{2}a^{9/4}d^{3/2}}$$

Mathematica [C] time = 0.0105859, size = 30, normalized size = 0.1

$$-\frac{2x\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^2(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-1/4, 2, 3/4, -((b*x^2)/a)])/(a^2*(d*x)^(3/2))
```

Maple [A] time = 0.109, size = 223, normalized size = 0.7

$$-2\,\frac{1}{a^2d\sqrt{dx}}-\frac{b}{2\,a^2d\left(bd^2x^2+ad^2\right)}(dx)^{\frac{3}{2}}-\frac{5\,\sqrt{2}}{16\,a^2d}\ln\left(\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -2/a^2/d/(d*x)^(1/2)-1/2/d*b/a^2*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)-5/16/d/a^2/(
a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))-5/8/d/a
^2/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-5/
8/d/a^2/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-
1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42796, size = 655, normalized size = 2.18

$$\frac{20\left(a^2bd^2x^3+a^3d^2x\right)\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,\sqrt{dx}a^2bd\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^5bd^4\sqrt{-\frac{b}{a^9d^6}}+15625\,b^2dxa^2d\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}}}{125\,b}\right)-5\left(a^2bd^2x^3+a^3d^2x\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/8*(20*(a^2*b*d^2*x^3 + a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*arctan(-1/125*(125
*sqrt(d*x)*a^2*b*d*(-b/(a^9*d^6))^(1/4) - sqrt(-15625*a^5*b*d^4*sqrt(-b/(a^
9*d^6)) + 15625*b^2*d*x)*a^2*d*(-b/(a^9*d^6))^(1/4))/b) - 5*(a^2*b*d^2*x^3
+ a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*log(125*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 12
5*sqrt(d*x)*b) + 5*(a^2*b*d^2*x^3 + a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*log(-12
5*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 125*sqrt(d*x)*b) - 4*(5*b*x^2 + 4*a)*sqrt(
d*x))/(a^2*b*d^2*x^3 + a^3*d^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**2), x)
```

Giac [A] time = 1.29082, size = 397, normalized size = 1.32

$$-\frac{\frac{8\left(5\,bd^2x^2+4\,ad^2\right)}{\left(\sqrt{dx}bd^2x^2+\sqrt{dx}ad^2\right)a^2}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2d^2}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2d^2}-\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\cdots\right.}{a^3b^2d^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*(8*(5*b*d^2*x^2 + 4*a*d^2)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)*a
^2) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2*d^2) + 10*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^3*b^2*d^2) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2*d^2) + 5*sqrt(2)*(a*b^3*d^2)
^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^
```

```
2*d^2))/d
```

3.694 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=300

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{11/4}d^{5/2}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{11/4}d^{5/2}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdot}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}a^{11/4}d^{5/2}}$$

```
[Out] -7/(6*a^2*d*(d*x)^(3/2)) + 1/(2*a*d*(d*x)^(3/2)*(a + b*x^2)) + (7*b^(3/4)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(11/
4)*d^(5/2)) - (7*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])])/(4*Sqrt[2]*a^(11/4)*d^(5/2)) + (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sq
rt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d
^(5/2)) - (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d^(5/2))
```

Rubi [A] time = 0.299124, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{11/4}d^{5/2}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{11/4}d^{5/2}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdot}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}a^{11/4}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -7/(6*a^2*d*(d*x)^(3/2)) + 1/(2*a*d*(d*x)^(3/2)*(a + b*x^2)) + (7*b^(3/4)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(11/
4)*d^(5/2)) - (7*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])])/(4*Sqrt[2]*a^(11/4)*d^(5/2)) + (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sq
rt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d
^(5/2)) - (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{(7b)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} - \frac{\left(7b^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{4a^2d^2}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2a^2d^3}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4a^{5/2}d^4} - \frac{\left(7b^2\right)}{}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{\left(7b^{3/4}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}a^{11/4}d^{5/2}}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{7b^{3/4}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}a^{11/4}d^{5/2}}$$

$$= -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)} + \frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{11/4}d^{5/2}} - \frac{7b^{3/4}\tan^{-1}\left(1+\right.}{4\sqrt{2}a^{11/4}}$$

Mathematica [C] time = 0.0122955, size = 32, normalized size = 0.11

$$-\frac{2x\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^2(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-3/4, 2, 1/4, -((b*x^2)/a)])/(3*a^2*(d*x)^(5/2))
```

Maple [A] time = 0.064, size = 226, normalized size = 0.8

$$-\frac{2}{3a^2d}(dx)^{-\frac{3}{2}}-\frac{b}{2a^2d\left(bd^2x^2+ad^2\right)}\sqrt{dx}-\frac{7b\sqrt{2}}{16a^3d^3}\sqrt[4]{\frac{ad^2}{b}}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -2/3/a^2/d/(d*x)^(3/2)-1/2/d/a^2*b*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)-7/16/d^3/a
^3*b*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))-7
/8/d^3/a^3*b*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(
1/2)+1)-7/8/d^3/a^3*b*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4
)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4402, size = 678, normalized size = 2.26

$$-\frac{84\left(a^2bd^3x^4+a^3d^3x^2\right)\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^8bd^7\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^6d^6\sqrt{-\frac{b^3}{a^{11}d^{10}}}+b^2dxa^8d^7\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{3}{4}}}}{b^3}\right)+21\left(a^2bd^3x^4+a^3d^3x^2\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/24*(84*(a^2*b*d^3*x^4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*arctan(-(s
qrt(d*x)*a^8*b*d^7*(-b^3/(a^11*d^10))^(3/4) - sqrt(a^6*d^6*sqrt(-b^3/(a^11*
d^10)) + b^2*d*x)*a^8*d^7*(-b^3/(a^11*d^10))^(3/4))/b^3) + 21*(a^2*b*d^3*x^
4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*log(7*a^3*d^3*(-b^3/(a^11*d^10))^
(1/4) + 7*sqrt(d*x)*b) - 21*(a^2*b*d^3*x^4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10)
)^(1/4)*log(-7*a^3*d^3*(-b^3/(a^11*d^10))^(1/4) + 7*sqrt(d*x)*b) + 4*(7*b*x
^2 + 4*a)*sqrt(d*x))/(a^2*b*d^3*x^4 + a^3*d^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*(a + b*x**2)**2), x)
```

Giac [A] time = 1.26881, size = 373, normalized size = 1.24

$$-\frac{\sqrt{dx}b}{2\left(bd^2x^2+ad^2\right)a^2d}-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^3d^3}-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^3d^3}-\frac{7\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(d*x)*b/((b*d^2*x^2 + a*d^2)*a^2*d) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/4
)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4
))/(a^3*d^3) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*d^3) - 7/16*sqrt(2)*(a*
b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
```

```
/(a^3*d^3) + 7/16*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*d^3) - 2/3/(sqrt(d*x)*a^2*d^2*x)
```

3.695 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=318

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{13/4}d^{7/2}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{13/4}d^{7/2}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}a^{13/4}d^{7/2}}$$

```
[Out] -9/(10*a^2*d*(d*x)^(5/2)) + (9*b)/(2*a^3*d^3*Sqrt[d*x]) + 1/(2*a*d*(d*x)^(5
/2)*(a + b*x^2)) - (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b
^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(8*Sqrt[2]*a^(13/4)*d^(7/2)) - (9*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(13/4)*
d^(7/2))
```

Rubi [A] time = 0.342626, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{13/4}d^{7/2}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{13/4}d^{7/2}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}a^{13/4}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```


```
[Out] -9/(10*a^2*d*(d*x)^(5/2)) + (9*b)/(2*a^3*d^3*Sqrt[d*x]) + 1/(2*a*d*(d*x)^(5
/2)*(a + b*x^2)) - (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b
^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(8*Sqrt[2]*a^(13/4)*d^(7/2)) - (9*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(13/4)*
d^(7/2))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{(9b)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)}\,dx}{4a}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{\left(9b^2\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{4a^2d^2}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^3\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4a^3d^4}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^3\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x,\right.}{2a^3d^5}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{\left(9b^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx\right.}{4a^3d^5}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^{5/4}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2}{-\frac{\sqrt{ad}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{}}{\sqrt[4]{b}}}\right.}{8\sqrt{2}a^{13/4}d^{7/2}}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{9b^{5/4}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\right.}{8\sqrt{2}a^{13/4}d^{7/2}}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{13/4}d^{7/2}} +$$

Mathematica [C] time = 0.0112613, size = 37, normalized size = 0.12

$$-\frac{2\sqrt{dx}\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^2d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (-2*Sqrt[d*x]*Hypergeometric2F1[-5/4, 2, -1/4, -((b*x^2)/a)])/(5*a^2*d^4*x^
3)
```

Maple [A] time = 0.063, size = 242, normalized size = 0.8

$$-\frac{2}{5\,a^2d}\,(dx)^{-\frac{5}{2}}+4\,\frac{b}{a^3d^3\sqrt{dx}}+\frac{b^2}{2\,a^3d^3\left(bd^2x^2+ad^2\right)}\,(dx)^{\frac{3}{2}}+\frac{9\,b\sqrt{2}}{16\,a^3d^3}\ln\left(\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -2/5/a^2/d/(d*x)^(5/2)+4*b/a^3/d^3/(d*x)^(1/2)+1/2/d^3*b^2/a^3*(d*x)^(3/2)/
(b*d^2*x^2+a*d^2)+9/16/d^3*b/a^3/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2)))+9/8/d^3*b/a^3/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+9/8/d^3*b/a^3/(a*d^2/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3864, size = 767, normalized size = 2.41

$$-\frac{180\left(a^3bd^4x^5+a^4d^4x^3\right)\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729\sqrt{dx}a^3b^4d^3\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-531441\,a^7b^5d^8\sqrt{-\frac{b^5}{a^{13}d^{14}}}+531441\,b^8dxa^3d^3\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}}}{729\,b^5}\right)-45\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/40*(180*(a^3*b*d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*arctan(-1
/729*(729*sqrt(d*x)*a^3*b^4*d^3*(-b^5/(a^13*d^14))^(1/4) - sqrt(-531441*a^7
*b^5*d^8*sqrt(-b^5/(a^13*d^14)) + 531441*b^8*d*x)*a^3*d^3*(-b^5/(a^13*d^14)
)^(1/4))/b^5) - 45*(a^3*b*d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*l
og(729*a^10*d^11*(-b^5/(a^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) + 45*(a^3*b*
d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*log(-729*a^10*d^11*(-b^5/(a
^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) - 4*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)
*sqrt(d*x))/(a^3*b*d^4*x^5 + a^4*d^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{7}{2}}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(7/2)*(a + b*x**2)**2), x)
```

Giac [A] time = 1.25903, size = 414, normalized size = 1.3

$$\frac{\sqrt{dx}b^2x}{2\left(bd^2x^2+ad^2\right)a^3d^2}+\frac{9\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^4bd^5}+\frac{9\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^4bd^5}-\frac{9\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*b^2*x/((b*d^2*x^2 + a*d^2)*a^3*d^2) + 9/8*sqrt(2)*(a*b^3*d^2)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)
```

```
^(1/4))/(a^4*b*d^5) + 9/8*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d^5) - 9/16*sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^4*b*d^5) + 9/16*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d^5) + 2/5*(10*b*d^2*x^2 -
a*d^2)/(sqrt(d*x)*a^3*d^5*x^2)
```

3.696 $\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=368

$$-\frac{663a^{5/4}d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{21/4}}+\frac{663a^{5/4}d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{21/4}}-\frac{663a^{5/4}}{}$$

```
[Out] (-663*a*d^9*Sqrt[d*x])/(64*b^5) + (663*d^7*(d*x)^(5/2))/(320*b^4) - (d*(d*x
)^(17/2))/(6*b*(a + b*x^2)^3) - (17*d^3*(d*x)^(13/2))/(48*b^2*(a + b*x^2)^2
) - (221*d^5*(d*x)^(9/2))/(192*b^3*(a + b*x^2)) - (663*a^(5/4)*d^(19/2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(21/
4)) + (663*a^(5/4)*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(128*Sqrt[2]*b^(21/4)) - (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*
b^(21/4)) + (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(21/4))
```

Rubi [A] time = 0.418214, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{663a^{5/4}d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{21/4}}+\frac{663a^{5/4}d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{21/4}}-\frac{663a^{5/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```


```
[Out] (-663*a*d^9*Sqrt[d*x])/(64*b^5) + (663*d^7*(d*x)^(5/2))/(320*b^4) - (d*(d*x
)^(17/2))/(6*b*(a + b*x^2)^3) - (17*d^3*(d*x)^(13/2))/(48*b^2*(a + b*x^2)^2
) - (221*d^5*(d*x)^(9/2))/(192*b^3*(a + b*x^2)) - (663*a^(5/4)*d^(19/2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(21/
4)) + (663*a^(5/4)*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(128*Sqrt[2]*b^(21/4)) - (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*
b^(21/4)) + (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(21/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(17b^2d^2\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{96}\left(221d^4\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} + \frac{\left(663d^6\right)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} - \frac{\left(663ad^8\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{128b^3}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} + \frac{(663}{}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} + \frac{(663}{}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} + \frac{(663}{}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} - \frac{(663}{}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} - \frac{663a}{}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5} + \frac{663d^7(dx)^{5/2}}{320b^4} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} - \frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2} - \frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)} - \frac{663a}{}$$

Mathematica [A] time = 0.226318, size = 347, normalized size = 0.94

$$\frac{d^9\sqrt{dx}\left(\frac{-1584128a^2b^{9/4}x^{9/2}-2036736a^3b^{5/4}x^{5/2}+185640a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+106080a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-69615\sqrt{2}a^{5/4}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+}{(a-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^9*Sqrt[d*x]*(-139230*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] + (-848640*a^4*b^(1/4)*Sqrt[x] - 2036736*a^3*b^(5/4)*x^(5/2) - 1
584128*a^2*b^(9/4)*x^(9/2) - 365568*a*b^(13/4)*x^(13/2) + 21504*b^(17/4)*x^
(17/2) + 106080*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2) + 185640*a^2*b^(1/4)*Sqrt[x
]*(a + b*x^2)^2 + 139230*Sqrt[2]*a^(5/4)*(a + b*x^2)^3*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)] - 69615*Sqrt[2]*a^(5/4)*(a + b*x^2)^3*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 69615*Sqrt[2]*a^(5/4)*(a
+ b*x^2)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a +
 b*x^2)^3))/(53760*b^(21/4)*Sqrt[x])
```

Maple [A] time = 0.065, size = 306, normalized size = 0.8

$$\frac{2d^7}{5b^4}(dx)^{\frac{5}{2}}-8\frac{ad^9\sqrt{dx}}{b^5}-\frac{617d^{11}a^2}{192b^3\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{9}{2}}-\frac{173d^{13}a^3}{32b^4\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{5}{2}}-\frac{151d^{15}a^4}{64b^5\left(bd^2x^2+ad^2\right)^3}\sqrt{dx}+\frac{663a}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/5*d^7*(d*x)^(5/2)/b^4-8*a*d^9*(d*x)^(1/2)/b^5-617/192*d^11/b^3*a^2/(b*d^2
*x^2+a*d^2)^3*(d*x)^(9/2)-173/32*d^13/b^4*a^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(5/
2)-151/64*d^15/b^5*a^4/(b*d^2*x^2+a*d^2)^3*(d*x)^(1/2)+663/512*d^9/b^5*a*(a
*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+663/256*
d^9/b^5*a*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2
)+1)+663/256*d^9/b^5*a*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/
4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40898, size = 905, normalized size = 2.46

$$39780\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)\arctan\left(-\frac{\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{3}{4}}\sqrt{dx}ab^{16}d^9-\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{3}{4}}\sqrt{a^2d^{19}x+\sqrt{-\frac{a^5d^{38}}{b^{21}}}b^{10}}b^{16}}{a^5d^{38}}\right)+9945\,\Big(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3840*(39780*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2
 + a^3*b^5)*arctan(-((-a^5*d^38/b^21)^(3/4)*sqrt(d*x)*a*b^16*d^9 - (-a^5*d^
38/b^21)^(3/4)*sqrt(a^2*d^19*x + sqrt(-a^5*d^38/b^21)*b^10)*b^16)/(a^5*d^38
)) + 9945*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a
^3*b^5)*log(663*sqrt(d*x)*a*d^9 + 663*(-a^5*d^38/b^21)^(1/4)*b^5) - 9945*(-
a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)*log(
663*sqrt(d*x)*a*d^9 - 663*(-a^5*d^38/b^21)^(1/4)*b^5) + 4*(384*b^4*d^9*x^8
- 6528*a*b^3*d^9*x^6 - 24973*a^2*b^2*d^9*x^4 - 27846*a^3*b*d^9*x^2 - 9945*a
^4*d^9)*sqrt(d*x))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25187, size = 459, normalized size = 1.25

$$\frac{1}{7680}d^8\left(\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}+\frac{9\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/7680*d^8*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^6 + 19890*sqrt(2)*(a*b^3
*d^2)^(1/4)*a*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/b^6 + 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*log(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^6 - 9945*sqrt(2)*(a*b^3*d^2)
^(1/4)*a*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^6
 - 40*(617*sqrt(d*x)*a^2*b^2*d^7*x^4 + 1038*sqrt(d*x)*a^3*b*d^7*x^2 + 453*s
qrt(d*x)*a^4*d^7)/((b*d^2*x^2 + a*d^2)^3*b^5) + 3072*(sqrt(d*x)*b^16*d^6*x^
2 - 20*sqrt(d*x)*a*b^15*d^6)/(b^20*d^5))
```

3.697 $\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=350

$$-\frac{385a^{3/4}d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{19/4}}+\frac{385a^{3/4}d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{19/4}}+\frac{385a^{3/4}d}{}$$

```
[Out] (385*d^7*(d*x)^(3/2))/(192*b^4) - (d*(d*x)^(15/2))/(6*b*(a + b*x^2)^3) - (5
*d^3*(d*x)^(11/2))/(16*b^2*(a + b*x^2)^2) - (55*d^5*(d*x)^(7/2))/(64*b^3*(a
 + b*x^2)) + (385*a^(3/4)*d^(17/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) - (385*a^(3/4)*d^(17/2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) -
(385*a^(3/4)*d^(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(19/4)) + (385*a^(3/4)*d^(17/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(256*Sqrt[2]*b^(19/4))
```

Rubi [A] time = 0.392945, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{385a^{3/4}d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{19/4}}+\frac{385a^{3/4}d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{19/4}}+\frac{385a^{3/4}d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```



```
[Out] (385*d^7*(d*x)^(3/2))/(192*b^4) - (d*(d*x)^(15/2))/(6*b*(a + b*x^2)^3) - (5
*d^3*(d*x)^(11/2))/(16*b^2*(a + b*x^2)^2) - (55*d^5*(d*x)^(7/2))/(64*b^3*(a
 + b*x^2)) + (385*a^(3/4)*d^(17/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) - (385*a^(3/4)*d^(17/2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) -
(385*a^(3/4)*d^(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(19/4)) + (385*a^(3/4)*d^(17/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(256*Sqrt[2]*b^(19/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^4}\,dx$$
$$= -\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{4}\left(5b^2d^2\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^3}\,dx$$
$$= -\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2}+\frac{1}{32}\left(55d^4\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= -\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2}-\frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)}+\frac{\left(385d^6\right)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx}{128b^2}$$
$$= \frac{385d^7(dx)^{3/2}}{192b^4}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2}-\frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)}-\frac{\left(385ad^8\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,d}{128b^3}$$
$$= \frac{385d^7(dx)^{3/2}}{192b^4}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2}-\frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)}-\frac{\left(385ad^7\right)\text{Subst}\left(\int \right.}{64b}$$
$$= \frac{385d^7(dx)^{3/2}}{192b^4}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2}-\frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)}+\frac{\left(385ad^7\right)\text{Subst}\left(\int \right.}{128}$$
$$= \frac{385d^7(dx)^{3/2}}{192b^4}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2}-\frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)}-\frac{\left(385a^{3/4}d^{17/2}\right)\text{Subst}}{}$$
$$= \frac{385d^7(dx)^{3/2}}{192b^4}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2}-\frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)}-\frac{385a^{3/4}d^{17/2}\log\left(\sqrt{a}\right.}{2}$$
$$= \frac{385d^7(dx)^{3/2}}{192b^4}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3(dx)^{11/2}}{16b^2\left(a+bx^2\right)^2}-\frac{55d^5(dx)^{7/2}}{64b^3\left(a+bx^2\right)}+\frac{385a^{3/4}d^{17/2}\tan^{-1}\left(1\right.}{128\sqrt{2}b^{19}}$$

Mathematica [C] time = 0.0289215, size = 87, normalized size = 0.25

$$-\frac{2d^8x\sqrt{dx}\left(-99a^2bx^2-77a^3-45ab^2x^4+77\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-3b^3x^6\right)}{9b^4\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-2*d^8*x*Sqrt[d*x]*(-77*a^3 - 99*a^2*b*x^2 - 45*a*b^2*x^4 - 3*b^3*x^6 + 77
*(a + b*x^2)^3*Hypergeometric2F1[3/4, 4, 7/4, -((b*x^2)/a)]))/(9*b^4*(a + b
*x^2)^3)
```

Maple [A] time = 0.066, size = 290, normalized size = 0.8

$$\frac{2d^7}{3b^4}(dx)^{\frac{3}{2}}+\frac{127d^9a}{64b^2\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{11}{2}}+\frac{101d^{11}a^2}{32b^3\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{7}{2}}+\frac{257d^{13}a^3}{192b^4\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{3}{2}}-\frac{385d^9a\sqrt{2}}{512b^5}\ln\Bigg(\bigg(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/3*d^7*(d*x)^(3/2)/b^4+127/64*d^9*a/b^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(11/2)+1
01/32*d^11*a^2/b^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(7/2)+257/192*d^13*a^3/b^4/(b*
d^2*x^2+a*d^2)^3*(d*x)^(3/2)-385/512*d^9*a/b^5/(a*d^2/b)^(1/4)*2^(1/2)*ln((
d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))-385/256*d^9*a/b^5/(a*d^2/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-385/256*d^9*a/b^5/(a*
d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45145, size = 917, normalized size = 2.62

$$\frac{4620\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)\arctan\left(-\frac{\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\sqrt{dx}a^2b^5d^{25}-\sqrt{a^4d^{51}x-\sqrt{-\frac{a^3d^{34}}{b^{19}}}a^3b^9d^{34}}\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}b^5}{a^3d^{34}}\right)-115}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(4620*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 +
 a^3*b^4)*arctan(-((-a^3*d^34/b^19)^(1/4)*sqrt(d*x)*a^2*b^5*d^25 - sqrt(a^4
*d^51*x - sqrt(-a^3*d^34/b^19)*a^3*b^9*d^34)*(-a^3*d^34/b^19)^(1/4)*b^5)/(a
^3*d^34)) - 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*
x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 + 57066625*(-a^3*d^34/b^19)^
(3/4)*b^14) + 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^
5*x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 - 57066625*(-a^3*d^34/b^19
)^(3/4)*b^14) + 4*(128*b^3*d^8*x^7 + 765*a*b^2*d^8*x^5 + 990*a^2*b*d^8*x^3
+ 385*a^3*d^8*x)*sqrt(d*x))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^
4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31688, size = 412, normalized size = 1.18

$$\frac{1}{1536}d^7\left(\frac{1024\sqrt{dx}dx}{b^4}-\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}-\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^7*(1024*sqrt(d*x)*d*x/b^4 - 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^7 -
2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
 - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^7 + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d
*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 1155*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/b^7 + 8*(381*sqrt(d*x)*a*b^2*d^7*x^5 + 606*sqrt(d*x)*a^2*b*d^7*x^3 + 25
7*sqrt(d*x)*a^3*d^7*x)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

3.698 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=350

$$-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}+\frac{195\sqrt[4]{a}d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{17/4}}-\frac{195\sqrt[4]{a}d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{256\sqrt{2}}$$

```
[Out] (195*d^7*Sqrt[d*x])/(64*b^4) - (d*(d*x)^(13/2))/(6*b*(a + b*x^2)^3) - (13*d
^3*(d*x)^(9/2))/(48*b^2*(a + b*x^2)^2) - (39*d^5*(d*x)^(5/2))/(64*b^3*(a +
b*x^2)) + (195*a^(1/4)*d^(15/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) + (19
5*a^(1/4)*d^(15/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(25
6*Sqrt[2]*b^(17/4))
```

Rubi [A] time = 0.381998, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}+\frac{195\sqrt[4]{a}d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{17/4}}-\frac{195\sqrt[4]{a}d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{256\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (195*d^7*Sqrt[d*x])/(64*b^4) - (d*(d*x)^(13/2))/(6*b*(a + b*x^2)^3) - (13*d
^3*(d*x)^(9/2))/(48*b^2*(a + b*x^2)^2) - (39*d^5*(d*x)^(5/2))/(64*b^3*(a +
b*x^2)) + (195*a^(1/4)*d^(15/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) + (19
5*a^(1/4)*d^(15/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(25
6*Sqrt[2]*b^(17/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{12}\left(13b^2d^2\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}+\frac{1}{32}\left(39d^4\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}+\frac{\left(195d^6\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{\left(195ad^8\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,d}{128b^3}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{\left(195ad^7\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2}{a}}\right.}{64b^3}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{\left(195\sqrt{a}d^6\right)\text{Subst}\left(\int \frac{\sqrt{a}}{ab}\right.}{128b^3}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}+\frac{\left(195\sqrt[4]{a}d^{15/2}\right)\text{Subst}\left(\int\right.}{256}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}+\frac{195\sqrt[4]{a}d^{15/2}\log\left(\sqrt{a}\sqrt{d}-\right.}{256}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}+\frac{195\sqrt[4]{a}d^{15/2}\tan^{-1}\left(1-\right.}{128\sqrt{2}b^{17/4}}$$

Mathematica [A] time = 0.144782, size = 324, normalized size = 0.93

$$\frac{d^7\sqrt{dx}\left(\frac{119808a^2b^{5/4}x^2}{(a+bx^2)^3}-\frac{6240a^2\sqrt[4]{b}}{(a+bx^2)^2}+\frac{49920a^3\sqrt[4]{b}}{(a+bx^2)^3}+\frac{21504b^{13/4}x^6}{(a+bx^2)^3}+\frac{93184ab^{9/4}x^4}{(a+bx^2)^3}-\frac{10920a\sqrt[4]{b}}{a+bx^2}+\frac{4095\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}-\cdots\right.}{10752b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^7*Sqrt[d*x]*((49920*a^3*b^(1/4))/(a + b*x^2)^3 + (119808*a^2*b^(5/4)*x^2
)/(a + b*x^2)^3 + (93184*a*b^(9/4)*x^4)/(a + b*x^2)^3 + (21504*b^(13/4)*x^6
)/(a + b*x^2)^3 - (6240*a^2*b^(1/4))/(a + b*x^2)^2 - (10920*a*b^(1/4))/(a +
 b*x^2) + (8190*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/Sqrt[x] - (8190*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/Sqrt[x] + (4095*Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x] - (4095*Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x]))/(10752*b^(17/4))
```

Maple [A] time = 0.067, size = 287, normalized size = 0.8

$$2\,\frac{d^7\sqrt{dx}}{b^4}+\frac{317\,d^9a}{192\,b^2\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{9}{2}}+\frac{81\,d^{11}a^2}{32\,b^3\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{5}{2}}+\frac{67\,d^{13}a^3}{64\,b^4\left(bd^2x^2+ad^2\right)^3}\sqrt{dx}-\frac{195\,d^7\sqrt{2}}{512\,b^4}\sqrt[4]{\frac{ad^2}{b}}\ln\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2*d^7*(d*x)^(1/2)/b^4+317/192*d^9/b^2*a/(b*d^2*x^2+a*d^2)^3*(d*x)^(9/2)+81/
32*d^11/b^3*a^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(5/2)+67/64*d^13/b^4*a^3/(b*d^2*x
^2+a*d^2)^3*(d*x)^(1/2)-195/512*d^7/b^4*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))-195/256*d^7/b^4*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-195/256*d^7/b^4*(a*d^2/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40936, size = 826, normalized size = 2.36

$$-\frac{2340\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)\arctan\left(-\frac{\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}b^{13}d^7-\sqrt{d^{15}x+\sqrt{-\frac{ad^{30}}{b^{17}}}b^8}\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{3}{4}}b^{13}}{ad^{30}}\right)+585\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(2340*(-a*d^30/b^17)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 +
a^3*b^4)*arctan(-((-a*d^30/b^17)^(3/4)*sqrt(d*x)*b^13*d^7 - sqrt(d^15*x + s
qrt(-a*d^30/b^17)*b^8)*(-a*d^30/b^17)^(3/4)*b^13)/(a*d^30)) + 585*(-a*d^30/
b^17)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sqrt(
d*x)*d^7 + 195*(-a*d^30/b^17)^(1/4)*b^4) - 585*(-a*d^30/b^17)^(1/4)*(b^7*x^
6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sqrt(d*x)*d^7 - 195*(-a*
d^30/b^17)^(1/4)*b^4) - 4*(384*b^3*d^7*x^6 + 1469*a*b^2*d^7*x^4 + 1638*a^2*
b*d^7*x^2 + 585*a^3*d^7)*sqrt(d*x))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2
+ a^3*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31077, size = 414, normalized size = 1.18

$$-\frac{1}{1536}d^6\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5}+\frac{585\,\sqrt{}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*d^6*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 + 1170*sqrt(2)*(a*b^3*d^
2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/b^5 + 585*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5 - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*d*
log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5 - 3072*sqr
t(d*x)*d/b^4 - 8*(317*sqrt(d*x)*a*b^2*d^7*x^4 + 486*sqrt(d*x)*a^2*b*d^7*x^2
 + 201*sqrt(d*x)*a^3*d^7)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

3.699 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=333

$$-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}+\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{\right.}}{256\sqrt{2}\sqrt[4]{a}b^{15/4}}$$

```
[Out] -(d*(d*x)^(11/2))/(6*b*(a + b*x^2)^3) - (11*d^3*(d*x)^(7/2))/(48*b^2*(a + b
*x^2)^2) - (77*d^5*(d*x)^(3/2))/(192*b^3*(a + b*x^2)) - (77*d^(13/2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(1/4)*b
^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(128*Sqrt[2]*a^(1/4)*b^(15/4)) + (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/
4)*b^(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/4)*b^(15/4))
```

Rubi [A] time = 0.345098, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}+\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{\right.}}{256\sqrt{2}\sqrt[4]{a}b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```



```
[Out] -(d*(d*x)^(11/2))/(6*b*(a + b*x^2)^3) - (11*d^3*(d*x)^(7/2))/(48*b^2*(a + b
*x^2)^2) - (77*d^5*(d*x)^(3/2))/(192*b^3*(a + b*x^2)) - (77*d^(13/2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(1/4)*b
^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(128*Sqrt[2]*a^(1/4)*b^(15/4)) + (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/
4)*b^(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/4)*b^(15/4))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{12}\left(11b^2d^2\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}+\frac{1}{96}\left(77d^4\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}+\frac{\left(77d^6\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}+\frac{\left(77d^5\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{d\right.}}{64b^2}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}-\frac{\left(77d^5\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\right.}{128b^{5/2}}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}+\frac{\left(77d^{13/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-\right.}{256\sqrt{2}\sqrt[4]{a}b^{15/4}}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}+\frac{77d^{13/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{\right.}}{256\sqrt{2}\sqrt[4]{a}b^{15/4}}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}-\frac{77d^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{77}{}$$

Mathematica [C] time = 0.0256973, size = 83, normalized size = 0.25

$$\frac{2d^6x\sqrt{dx}\left(77\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(77a^2+99abx^2+45b^2x^4\right)\right)}{45ab^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*d^6*x*Sqrt[d*x]*(-(a*(77*a^2 + 99*a*b*x^2 + 45*b^2*x^4)) + 77*(a + b*x^2
)^3*Hypergeometric2F1[3/4, 4, 7/4, -((b*x^2)/a)]))/(45*a*b^3*(a + b*x^2)^3)
```

Maple [A] time = 0.066, size = 271, normalized size = 0.8

$$-\frac{51\,d^7}{64\left(bd^2x^2+ad^2\right)^3 b}(dx)^{\frac{11}{2}}-\frac{33\,d^9a}{32\left(bd^2x^2+ad^2\right)^3 b^2}(dx)^{\frac{7}{2}}-\frac{77\,d^{11}a^2}{192\left(bd^2x^2+ad^2\right)^3 b^3}(dx)^{\frac{3}{2}}+\frac{77\,d^7\sqrt{2}}{512\,b^4}\ln\left(\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -51/64*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(11/2)-33/32*d^9/(b*d^2*x^2+a*d^2)^3
/b^2*a*(d*x)^(7/2)-77/192*d^11/(b*d^2*x^2+a*d^2)^3/b^3*a^2*(d*x)^(3/2)+77/5
12*d^7/b^4/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))+77/256*d^7/b^4/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*
(d*x)^(1/2)+1)+77/256*d^7/b^4/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2
/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41557, size = 845, normalized size = 2.54

$$-\frac{924\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\sqrt{dx}b^4d^{19}-\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{ab^{15}}}ab^7d^{26}}\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}b^4}{d^{26}}\right)-231\left(b^6x^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(924*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^1
5))^(1/4)*arctan(-((-d^26/(a*b^15))^(1/4)*sqrt(d*x)*b^4*d^19 - sqrt(d^39*x
- sqrt(-d^26/(a*b^15))*a*b^7*d^26)*(-d^26/(a*b^15))^(1/4)*b^4)/d^26) - 231*
(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^15))^(1/4)*lo
g(456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a*b^15))^(3/4)*a*b^11) + 231*(b^6*
x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^15))^(1/4)*log(456
533*sqrt(d*x)*d^19 - 456533*(-d^26/(a*b^15))^(3/4)*a*b^11) + 4*(153*b^2*d^6
*x^5 + 198*a*b*d^6*x^3 + 77*a^2*d^6*x)*sqrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 +
3*a^2*b^4*x^2 + a^3*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24082, size = 408, normalized size = 1.23

$$\frac{1}{1536}d^5\left(\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6}+\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6}-\frac{231\sqrt{2}\left(ab^3d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^5*(462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6) + 462*sqrt(2)*(a*b^3*d^2
)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a*b^6) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/
b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6) + 231*sqrt(2)*(a*b^3*d^2)^(3/4)
*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6) - 8*(
153*sqrt(d*x)*b^2*d^7*x^5 + 198*sqrt(d*x)*a*b*d^7*x^3 + 77*sqrt(d*x)*a^2*d^
7*x)/((b*d^2*x^2 + a*d^2)^3*b^3))
```

3.700 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=333

$$-\frac{15d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{3/4}b^{13/4}}+\frac{15d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{3/4}b^{13/4}}-\frac{15d^{11/2}\tan^{-1}\left(1-\right.}{128\sqrt{2}a^{3/4}}$$

```
[Out] -(d*(d*x)^(9/2))/(6*b*(a + b*x^2)^3) - (3*d^3*(d*x)^(5/2))/(16*b^2*(a + b*x
^2)^2) - (15*d^5*Sqrt[d*x])/(64*b^3*(a + b*x^2)) - (15*d^(11/2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(3/4)*b^(13/
4)) + (15*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(128*Sqrt[2]*a^(3/4)*b^(13/4)) - (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt
[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^
(13/4)) + (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rubi [A] time = 0.344579, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{15d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{3/4}b^{13/4}}+\frac{15d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{3/4}b^{13/4}}-\frac{15d^{11/2}\tan^{-1}\left(1-\right.}{128\sqrt{2}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```


```
[Out] -(d*(d*x)^(9/2))/(6*b*(a + b*x^2)^3) - (3*d^3*(d*x)^(5/2))/(16*b^2*(a + b*x
^2)^2) - (15*d^5*Sqrt[d*x])/(64*b^3*(a + b*x^2)) - (15*d^(11/2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(3/4)*b^(13/
4)) + (15*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(128*Sqrt[2]*a^(3/4)*b^(13/4)) - (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt
[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^
(13/4)) + (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^(13/4))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{4}\left(3b^2d^2\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}+\frac{1}{32}\left(15d^4\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}+\frac{\left(15d^6\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{128b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}+\frac{\left(15d^5\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{64b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}+\frac{\left(15d^4\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right.}{128\sqrt{a}b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{\left(15d^{11/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-x^2}\right.}{256\sqrt{2}a^{3/4}b^{13/4}}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{15d^{11/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{}\right.}{256\sqrt{2}a^{3/4}b^{13/4}}$$

$$= -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{15d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{3/4}b^{13/4}}+\frac{15d^{1}}{}$$

Mathematica [A] time = 0.131344, size = 299, normalized size = 0.9

$$\frac{d^5\sqrt{dx}\left(-\frac{3840a^2\sqrt[4]{b}}{\left(a+bx^2\right)^3}-\frac{315\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}+\frac{315\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}-\frac{630\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}\sqrt{x}}+\frac{630\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{a^{3/4}\sqrt{x}}\right.}{10752b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^5*Sqrt[d*x]*((-3840*a^2*b^(1/4))/(a + b*x^2)^3 - (9216*a*b^(5/4)*x^2)/(a
 + b*x^2)^3 - (7168*b^(9/4)*x^4)/(a + b*x^2)^3 + (480*a*b^(1/4))/(a + b*x^2
)^2 + (840*b^(1/4))/(a + b*x^2) - (630*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(a^(3/4)*Sqrt[x]) + (630*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sqrt[x]) - (315*Sqrt[2]*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x]) + (315*Sqrt[2]*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x
])))/(10752*b^(13/4))
```

Maple [A] time = 0.064, size = 280, normalized size = 0.8

$$-\frac{113\,d^7}{192\left(bd^2x^2+ad^2\right)^3 b}(dx)^{\frac{9}{2}}-\frac{21\,d^9a}{32\left(bd^2x^2+ad^2\right)^3 b^2}(dx)^{\frac{5}{2}}-\frac{15\,d^{11}a^2}{64\left(bd^2x^2+ad^2\right)^3 b^3}\sqrt{dx}+\frac{15\,d^5\sqrt{2}}{512\,ab^3}\sqrt[4]{\frac{ad^2}{b}}\ln\left(\left(dx+\sqrt[4]{\frac{a}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -113/192*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(9/2)-21/32*d^9/(b*d^2*x^2+a*d^2)^
3/b^2*a*(d*x)^(5/2)-15/64*d^11/(b*d^2*x^2+a*d^2)^3/b^3*a^2*(d*x)^(1/2)+15/5
12*d^5/b^3*(a*d^2/b)^(1/4)/a*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))+15/256*d^5/b^3*(a*d^2/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1
/4)*(d*x)^(1/2)+1)+15/256*d^5/b^3*(a*d^2/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42012, size = 841, normalized size = 2.53

$$\frac{180\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}a^2b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{d^{22}}{a^3b^{13}}}a^2b^6}\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{3}{4}}a^2b^{10}}{d^{22}}\right)+45\left(b^6x^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(180*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^
13))^(1/4)*arctan(-((-d^22/(a^3*b^13))^(3/4)*sqrt(d*x)*a^2*b^10*d^5 - sqrt(
d^11*x + sqrt(-d^22/(a^3*b^13))*a^2*b^6)*(-d^22/(a^3*b^13))^(3/4)*a^2*b^10)
/d^22) + 45*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b
^13))^(1/4)*log(15*sqrt(d*x)*d^5 + 15*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 45*
(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^13))^(1/4)*
log(15*sqrt(d*x)*d^5 - 15*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 4*(113*b^2*d^5*
x^4 + 126*a*b*d^5*x^2 + 45*a^2*d^5)*sqrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 + 3*a
^2*b^4*x^2 + a^3*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32454, size = 412, normalized size = 1.24

$$\frac{1}{1536}d^4\left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}+\frac{45\sqrt{2}\left(ab^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^4*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4) + 90*sqrt(2)*(a*b^3*d^2
)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^
2/b)^(1/4))/(a*b^4) + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) - 45*sqrt(2)*(a*b^3*d^2)^(1/
4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) -
 8*(113*sqrt(d*x)*b^2*d^7*x^4 + 126*sqrt(d*x)*a*b*d^7*x^2 + 45*sqrt(d*x)*a^
2*d^7)/((b*d^2*x^2 + a*d^2)^3*b^3))
```

3.701 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=336

$$\frac{7d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{5/4}b^{11/4}}-\frac{7d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{5/4}b^{11/4}}-\frac{7d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{128\sqrt{2}a^{5/4}b^{11/4}}$$

```
[Out] -(d*(d*x)^(7/2))/(6*b*(a + b*x^2)^3) - (7*d^3*(d*x)^(3/2))/(48*b^2*(a + b*x
^2)^2) + (7*d^3*(d*x)^(3/2))/(64*a*b^2*(a + b*x^2)) - (7*d^(9/2)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(5/4)*b^(11
/4)) + (7*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(128*Sqrt[2]*a^(5/4)*b^(11/4)) + (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11
/4)) - (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rubi [A] time = 0.351459, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{7d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{5/4}b^{11/4}}-\frac{7d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{5/4}b^{11/4}}-\frac{7d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{128\sqrt{2}a^{5/4}b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```


```
[Out] -(d*(d*x)^(7/2))/(6*b*(a + b*x^2)^3) - (7*d^3*(d*x)^(3/2))/(48*b^2*(a + b*x
^2)^2) + (7*d^3*(d*x)^(3/2))/(64*a*b^2*(a + b*x^2)) - (7*d^(9/2)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(5/4)*b^(11
/4)) + (7*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(128*Sqrt[2]*a^(5/4)*b^(11/4)) + (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11
/4)) - (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11/4))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(7b^2d^2\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{32}\left(7d^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{\left(7d^4\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128ab}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right.}{64ab}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} - \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{\right.}{128ab^{3/2}}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{\left(7d^{9/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-x^2}\right.}{256\sqrt{2}a^{5/4}b^{11/4}}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{7d^{9/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{2}\right.}{256\sqrt{2}a^{5/4}b^{11/4}}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} - \frac{7d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{5/4}b^{11/4}} + \frac{7d^{9/}}{}$$

Mathematica [C] time = 0.0221034, size = 74, normalized size = 0.22

$$\frac{2d^4x\sqrt{dx}\left(7\left(a+bx^2\right)^3 {}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\left(7a+9bx^2\right)\right)}{45a^2b^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*d^4*x*Sqrt[d*x]*(-(a^2*(7*a + 9*b*x^2)) + 7*(a + b*x^2)^3*Hypergeometric
2F1[3/4, 4, 7/4, -((b*x^2)/a)]))/(45*a^2*b^2*(a + b*x^2)^3)
```

Maple [A] time = 0.065, size = 277, normalized size = 0.8

$$\frac{7 d^5}{64 \left(b d^2 x^2+a d^2\right)^3 a}(dx)^{\frac{11}{2}}-\frac{3 d^7}{32 \left(b d^2 x^2+a d^2\right)^3 b}(dx)^{\frac{7}{2}}-\frac{7 d^9 a}{192 \left(b d^2 x^2+a d^2\right)^3 b^2}(dx)^{\frac{3}{2}}+\frac{7 d^5\sqrt{2}}{512\, a b^3}\ln\left(\left(dx-\sqrt[4]{\frac{a d^2}{b}}\sqrt{dx}\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 7/64*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(11/2)-3/32*d^7/(b*d^2*x^2+a*d^2)^3/b*
(d*x)^(7/2)-7/192*d^9/(b*d^2*x^2+a*d^2)^3/b^2*a*(d*x)^(3/2)+7/512*d^5/a/b^3
/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2
/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+7/256
*d^5/a/b^3/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/
2)+1)+7/256*d^5/a/b^3/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4
)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42258, size = 875, normalized size = 2.6

$$-\frac{84\left(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\sqrt{dx}ab^3d^{13}-\sqrt{d^{27}x-\sqrt{-\frac{d^{18}}{a^5b^{11}}}a^3b^5d^{18}}\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}ab^3}{d^{18}}\right)-21\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(84*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^18/(a^
5*b^11))^(1/4)*arctan(-((-d^18/(a^5*b^11))^(1/4)*sqrt(d*x)*a*b^3*d^13 - sqr
t(d^27*x - sqrt(-d^18/(a^5*b^11))*a^3*b^5*d^18)*(-d^18/(a^5*b^11))^(1/4)*a*
b^3)/d^18) - 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^1
8/(a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 + 343*(-d^18/(a^5*b^11))^(3/4)*a
^4*b^8) + 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^18/(
a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 - 343*(-d^18/(a^5*b^11))^(3/4)*a^4*
b^8) - 4*(21*b^2*d^4*x^5 - 18*a*b*d^4*x^3 - 7*a^2*d^4*x)*sqrt(d*x))/(a*b^5*
x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{9}{2}}}{\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(9/2)/(a + b*x**2)**4, x)
```

Giac [A] time = 1.37149, size = 412, normalized size = 1.23

$$\frac{1}{1536}d^3\left(\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}+\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}-\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^3*(42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5) + 42*sqrt(2)*(a*b^3*d^2
)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a^2*b^5) - 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5) + 21*sqrt(2)*(a*b^3*d^2)^(3/
4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5) +
 8*(21*sqrt(d*x)*b^2*d^7*x^5 - 18*sqrt(d*x)*a*b*d^7*x^3 - 7*sqrt(d*x)*a^2*d
^7*x)/((b*d^2*x^2 + a*d^2)^3*a*b^2))
```

3.702 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=336

$$-\frac{5d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{7/4}b^{9/4}}-\frac{5d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.}{128\sqrt{2}a^{7/4}b^{9/4}}$$

```
[Out] -(d*(d*x)^(5/2))/(6*b*(a + b*x^2)^3) - (5*d^3*Sqrt[d*x])/(48*b^2*(a + b*x^2
)^2) + (5*d^3*Sqrt[d*x])/(192*a*b^2*(a + b*x^2)) - (5*d^(7/2)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(7/4)*b^(9/4))
 + (5*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(1
28*Sqrt[2]*a^(7/4)*b^(9/4)) - (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4)) +
 (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rubi [A] time = 0.341019, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{7/4}b^{9/4}}-\frac{5d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.}{128\sqrt{2}a^{7/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(5/2))/(6*b*(a + b*x^2)^3) - (5*d^3*Sqrt[d*x])/(48*b^2*(a + b*x^2
)^2) + (5*d^3*Sqrt[d*x])/(192*a*b^2*(a + b*x^2)) - (5*d^(7/2)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(7/4)*b^(9/4))
 + (5*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(1
28*Sqrt[2]*a^(7/4)*b^(9/4)) - (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4)) +
 (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(5b^2d^2\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{96}\left(5d^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} + \frac{\left(5d^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{128ab}$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} + \frac{\left(5d^3\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right.}{64ab}$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} + \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{d}\right.}{128a^{3/2}b}$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} - \frac{\left(5d^{7/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-x^2}\right.}{256\sqrt{2}a^{7/4}b^{9/4}}$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} - \frac{5d^{7/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{\right.}{256\sqrt{2}a^{7/4}b^{9/4}}$$

$$= -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3} - \frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)} - \frac{5d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{7/4}b^{9/4}} + \frac{5d^{7/2}}{}$$

Mathematica [A] time = 0.163696, size = 279, normalized size = 0.83

$$\frac{d^3\sqrt{dx}\left(\frac{280\sqrt[4]{b}}{a^2+abx^2} - \frac{105\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}} + \frac{105\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}} - \frac{210\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}\sqrt{x}} + \frac{210\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+\right.}{a^{7/4}\sqrt{x}}\right.}{10752b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^3*Sqrt[d*x]*((-1280*a*b^(1/4))/(a + b*x^2)^3 - (3072*b^(5/4)*x^2)/(a + b
*x^2)^3 + (160*b^(1/4))/(a + b*x^2)^2 + (280*b^(1/4))/(a^2 + a*b*x^2) - (21
0*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*Sqrt[x])
+ (210*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*Sqrt
[x]) - (105*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(a^(7/4)*Sqrt[x]) + (105*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x])))/(10752*b^(9/4))
```

Maple [A] time = 0.063, size = 277, normalized size = 0.8

$$\frac{5\,d^5}{192\left(bd^2x^2+ad^2\right)^3a}(dx)^{\frac{9}{2}}-\frac{7\,d^7}{32\left(bd^2x^2+ad^2\right)^3b}(dx)^{\frac{5}{2}}-\frac{5\,d^9a}{64\left(bd^2x^2+ad^2\right)^3b^2}\sqrt{dx}+\frac{5\,d^3\sqrt{2}}{512\,b^2a^2}\sqrt[4]{\frac{ad^2}{b}}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 5/192*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(9/2)-7/32*d^7/(b*d^2*x^2+a*d^2)^3/b*
(d*x)^(5/2)-5/64*d^9/(b*d^2*x^2+a*d^2)^3/b^2*a*(d*x)^(1/2)+5/512*d^3/a^2/b^
2*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^
2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+5/25
6*d^3/a^2/b^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^
(1/2)+1)+5/256*d^3/a^2/b^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)
^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.464, size = 844, normalized size = 2.51

$$\frac{60\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^5b^7d^3\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{3}{4}}-\sqrt{a^4b^4\sqrt{-\frac{d^{14}}{a^7b^9}}+d^7x}a^5b^7\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{3}{4}}}{d^{14}}\right)+15\left(ab^5x^6+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(60*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7
*b^9))^(1/4)*arctan(-(sqrt(d*x)*a^5*b^7*d^3*(-d^14/(a^7*b^9))^(3/4) - sqrt(
a^4*b^4*sqrt(-d^14/(a^7*b^9)) + d^7*x)*a^5*b^7*(-d^14/(a^7*b^9))^(3/4))/d^1
4) + 15*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7*b
^9))^(1/4)*log(5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3) - 15*(a
*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7*b^9))^(1/4)
*log(-5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3) + 4*(5*b^2*d^3*x
^4 - 42*a*b*d^3*x^2 - 15*a^2*d^3)*sqrt(d*x))/(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3
*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**4, x)
```

Giac [A] time = 1.28189, size = 416, normalized size = 1.24

$$\frac{1}{1536}d^2\left(\frac{30\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}+\frac{30\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}+\frac{15\sqrt{2}\left(ab^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^2*(30*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) + 30*sqrt(2)*(a*b^3*d
^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^2*b^3) + 15*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3) - 15*sqrt(2)*(a*b^3*d^
2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
2*b^3) + 8*(5*sqrt(d*x)*b^2*d^7*x^4 - 42*sqrt(d*x)*a*b*d^7*x^2 - 15*sqrt(d*
x)*a^2*d^7)/((b*d^2*x^2 + a*d^2)^3*a*b^2))
```

3.703 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{5d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{9/4}b^{7/4}}-\frac{5d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{9/4}b^{7/4}}-\frac{5d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{128\sqrt{2}a^{9/4}b^{7/4}}$$

```
[Out] -(d*(d*x)^(3/2))/(6*b*(a + b*x^2)^3) + (d*(d*x)^(3/2))/(16*a*b*(a + b*x^2)^
2) + (5*d*(d*x)^(3/2))/(64*a^2*b*(a + b*x^2)) - (5*d^(5/2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(9/4)*b^(7/4)) +
(5*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*
Sqrt[2]*a^(9/4)*b^(7/4)) + (5*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4)) - (5
*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rubi [A] time = 0.348355, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{9/4}b^{7/4}}-\frac{5d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{9/4}b^{7/4}}-\frac{5d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{128\sqrt{2}a^{9/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```


```
[Out] -(d*(d*x)^(3/2))/(6*b*(a + b*x^2)^3) + (d*(d*x)^(3/2))/(16*a*b*(a + b*x^2)^
2) + (5*d*(d*x)^(3/2))/(64*a^2*b*(a + b*x^2)) - (5*d^(5/2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(9/4)*b^(7/4)) +
(5*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*
Sqrt[2]*a^(9/4)*b^(7/4)) + (5*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4)) - (5
*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{4}\left(b^2d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{\left(5bd^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{32a}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} + \frac{\left(5d^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128a^2}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} + \frac{(5d)\,\mathrm{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right.}{64a^2}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} - \frac{(5d)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{d\right.}{128a^2\sqrt{b}}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} + \frac{\left(5d^{5/2}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{ad}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-x^2}\right.}{256\sqrt{2}a^{9/4}b^{7/4}}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} + \frac{5d^{5/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{2}\right.}{256\sqrt{2}a^{9/4}b^{7/4}}$$

$$= -\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3} + \frac{d(dx)^{3/2}}{16ab\left(a+bx^2\right)^2} + \frac{5d(dx)^{3/2}}{64a^2b\left(a+bx^2\right)} - \frac{5d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{9/4}b^{7/4}} + \frac{5d^{5}}{}$$

Mathematica [C] time = 0.020821, size = 60, normalized size = 0.18

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right)^3\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)-a^3\right)}{9a^3b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a^3 + (a + b*x^2)^3*Hypergeometric2F1[3/4, 4, 7/4, -((b*
x^2)/a)]))/(9*a^3*b*(a + b*x^2)^3)
```

Maple [A] time = 0.065, size = 277, normalized size = 0.8

$$\frac{5\,d^3 b}{64\left(bd^2x^2+ad^2\right)^3 a^2}(dx)^{\frac{11}{2}}+\frac{7\,d^5}{32\left(bd^2x^2+ad^2\right)^3 a}(dx)^{\frac{7}{2}}-\frac{5\,d^7}{192\left(bd^2x^2+ad^2\right)^3 b}(dx)^{\frac{3}{2}}+\frac{5\,d^3\sqrt{2}}{512\,b^2a^2}\ln\left(\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 5/64*d^3/(b*d^2*x^2+a*d^2)^3/a^2*b*(d*x)^(11/2)+7/32*d^5/(b*d^2*x^2+a*d^2)^
3/a*(d*x)^(7/2)-5/192*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(3/2)+5/512*d^3/a^2/b
^2/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+5/2
56*d^3/a^2/b^2/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)
^(1/2)+1)+5/256*d^3/a^2/b^2/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b
)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41831, size = 898, normalized size = 2.68

$$-\frac{60\left(a^2b^4x^6+3a^3b^3x^4+3a^4b^2x^2+a^5b\right)\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\sqrt{dx}a^2b^2d^7\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}-\sqrt{-15625a^5b^3d^{10}\sqrt{-\frac{d^{10}}{a^9b^7}}+15625d^{15}x}a^2b^2\left(-\frac{d^{10}}{a^9b^7}\right.}{125d^{10}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(60*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^10/(a^
9*b^7))^(1/4)*arctan(-1/125*(125*sqrt(d*x)*a^2*b^2*d^7*(-d^10/(a^9*b^7))^(1
/4) - sqrt(-15625*a^5*b^3*d^10*sqrt(-d^10/(a^9*b^7)) + 15625*d^15*x)*a^2*b^
2*(-d^10/(a^9*b^7))^(1/4))/d^10) - 15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*
b^2*x^2 + a^5*b)*(-d^10/(a^9*b^7))^(1/4)*log(125*a^7*b^5*(-d^10/(a^9*b^7))^
(3/4) + 125*sqrt(d*x)*d^7) + 15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^
2 + a^5*b)*(-d^10/(a^9*b^7))^(1/4)*log(-125*a^7*b^5*(-d^10/(a^9*b^7))^(3/4)
 + 125*sqrt(d*x)*d^7) - 4*(15*b^2*d^2*x^5 + 42*a*b*d^2*x^3 - 5*a^2*d^2*x)*s
qrt(d*x))/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**4, x)
```

Giac [A] time = 1.28677, size = 409, normalized size = 1.22

$$\frac{1}{1536}\,d\left(\frac{30\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}+\frac{30\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}-\frac{15\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\,\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d*(30*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4) + 30*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^3*b^4) - 15*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4) + 15*sqrt(2)*(a*b^3*d^2)^(3/4)
*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4) + 8
*(15*sqrt(d*x)*b^2*d^7*x^5 + 42*sqrt(d*x)*a*b*d^7*x^3 - 5*sqrt(d*x)*a^2*d^7
*x)/((b*d^2*x^2 + a*d^2)^3*a^2*b))
```

3.704 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$-\frac{7d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{11/4}b^{5/4}}+\frac{7d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{11/4}b^{5/4}}-\frac{7d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{128\sqrt{2}a^{11/4}b^{5/4}}$$

```
[Out] -(d*Sqrt[d*x])/(6*b*(a + b*x^2)^3) + (d*Sqrt[d*x])/(48*a*b*(a + b*x^2)^2) +
 (7*d*Sqrt[d*x])/(192*a^2*b*(a + b*x^2)) - (7*d^(3/2)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + (7*d
^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt
[2]*a^(11/4)*b^(5/4)) - (7*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4)) + (7*d
^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi [A] time = 0.346679, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{7d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{11/4}b^{5/4}}+\frac{7d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{11/4}b^{5/4}}-\frac{7d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{128\sqrt{2}a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```


```
[Out] -(d*Sqrt[d*x])/(6*b*(a + b*x^2)^3) + (d*Sqrt[d*x])/(48*a*b*(a + b*x^2)^2) +
 (7*d*Sqrt[d*x])/(192*a^2*b*(a + b*x^2)) - (7*d^(3/2)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + (7*d
^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt
[2]*a^(11/4)*b^(5/4)) - (7*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4)) + (7*d
^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(b^2d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{\left(7bd^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{96a}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} + \frac{\left(7d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{128a^2}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} + \frac{(7d)\,\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{64a^2}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} + \frac{7\,\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{128a^{5/2}}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} - \frac{\left(7d^{3/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-x^2}\right.}{256\sqrt{2}a^{11/4}b^{5/4}}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} - \frac{7d^{3/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{2}\right.}{256\sqrt{2}a^{11/4}b^{5/4}}$$

$$= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3} + \frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} + \frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)} - \frac{7d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{11/4}b^{5/4}} + \frac{7d^{3/}}{}$$

Mathematica [A] time = 0.132366, size = 260, normalized size = 0.78

$$\frac{d\sqrt{dx}\left(\frac{56\sqrt[4]{b}}{a^2\left(a+bx^2\right)} - \frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}} + \frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}} - \frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}\sqrt{x}} + \frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{11/4}\sqrt{x}} \right.}{1536b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d*Sqrt[d*x]*((-256*b^(1/4))/(a + b*x^2)^3 + (32*b^(1/4))/(a*(a + b*x^2)^2)
 + (56*b^(1/4))/(a^2*(a + b*x^2)) - (42*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) + (42*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) - (21*Sqrt[2]*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt[x]) + (21*Sqrt[2]
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt
[x])))/(1536*b^(5/4))
```

Maple [A] time = 0.063, size = 271, normalized size = 0.8

$$\frac{7\,d^3 b}{192\left(bd^2x^2+ad^2\right)^3 a^2}(dx)^{\frac{9}{2}}+\frac{3\,d^5}{32\left(bd^2x^2+ad^2\right)^3 a}(dx)^{\frac{5}{2}}-\frac{7\,d^7}{64\left(bd^2x^2+ad^2\right)^3 b}\sqrt{dx}+\frac{7\,d\sqrt{2}}{512\,a^3 b}\sqrt[4]{\frac{ad^2}{b}}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 7/192*d^3/(b*d^2*x^2+a*d^2)^3/a^2*b*(d*x)^(9/2)+3/32*d^5/(b*d^2*x^2+a*d^2)^
3/a*(d*x)^(5/2)-7/64*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(1/2)+7/512*d/a^3/b*(a
*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+7/256*d/
a^3/b*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)
+7/256*d/a^3/b*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)
^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36485, size = 821, normalized size = 2.45

$$\frac{84\left(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b\right)\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac14}\arctan\left(-\frac{\sqrt{dx}a^8b^4d\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac34}-\sqrt{a^6b^2\sqrt{-\frac{d^6}{a^{11}b^5}}+d^3x}a^8b^4\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac34}}{d^6}\right)+21\left(a^2b^4x^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(84*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11
*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^8*b^4*d*(-d^6/(a^11*b^5))^(3/4) - sqrt(a^
6*b^2*sqrt(-d^6/(a^11*b^5)) + d^3*x)*a^8*b^4*(-d^6/(a^11*b^5))^(3/4))/d^6)
+ 21*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11*b^5)
)^(1/4)*log(7*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) - 21*(a^2*b^4*
x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11*b^5))^(1/4)*log(-7
*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) + 4*(7*b^2*d*x^4 + 18*a*b*d
*x^2 - 21*a^2*d)*sqrt(d*x))/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 +
a^5*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^{\frac32}}{\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**4, x)
```

Giac [A] time = 1.25594, size = 409, normalized size = 1.22

$$\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^3b^2}+\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^3b^2}+\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\log\left(dx\right.}{51}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 7/256*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2) + 7/256*sqrt(2)*(a*b^3*d^2)^(1
/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^3*b^2) + 7/512*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*
d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2) - 7/512*sqrt(2)*(a*b^3*d^
2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
3*b^2) + 1/192*(7*sqrt(d*x)*b^2*d^7*x^4 + 18*sqrt(d*x)*a*b*d^7*x^2 - 21*sqr
t(d*x)*a^2*d^7)/((b*d^2*x^2 + a*d^2)^3*a^2*b)
```

3.705 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{15\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}\sqrt{d}}\cdots\right.}{128\sqrt{2}a^{13/4}b^{3/4}}$$

```
[Out] (d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*(d*x)^(3/2))/(16*a^2*d*(a + b*x^2)^2
) + (15*(d*x)^(3/2))/(64*a^3*d*(a + b*x^2)) - (15*Sqrt[d]*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(13/4)*b^(3/4)) +
(15*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128
*Sqrt[2]*a^(13/4)*b^(3/4)) + (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
- (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rubi [A] time = 0.351836, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{15\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}\sqrt{d}}\cdots\right.}{128\sqrt{2}a^{13/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*(d*x)^(3/2))/(16*a^2*d*(a + b*x^2)^2
) + (15*(d*x)^(3/2))/(64*a^3*d*(a + b*x^2)) - (15*Sqrt[d]*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(13/4)*b^(3/4)) +
(15*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128
*Sqrt[2]*a^(13/4)*b^(3/4)) + (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
- (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{\left(3b^3\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{4a}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{\left(15b^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{32a^2}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{(15b)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128a^3}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{(15b)\,\mathrm{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{d}\right.}{64a^3d}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} - \frac{\left(15\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x\right.}{128a^3d}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{\left(15\sqrt{d}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-\right.}{256\sqrt{2}a^{13/4}b^{3/4}}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{15\sqrt{d}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{\right.}}{256\sqrt{2}a^{13/4}b^{3/4}}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} - \frac{15\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{13/4}b^{3/4}} + \frac{15}{}$$

Mathematica [C] time = 0.0073452, size = 32, normalized size = 0.1

$$\frac{2x\sqrt{dx}\,{}_2F_1\left(\frac{3}{4},4;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*Sqrt[d*x]*Hypergeometric2F1[3/4, 4, 7/4, -((b*x^2)/a)])/(3*a^4)
```

Maple [A] time = 0.061, size = 272, normalized size = 0.8

$$\frac{15\,b^2d}{64\,\left(bd^2x^2+ad^2\right)^3a^3}(dx)^{\frac{11}{2}}+\frac{21\,d^3b}{32\,\left(bd^2x^2+ad^2\right)^3a^2}(dx)^{\frac{7}{2}}+\frac{113\,d^5}{192\,\left(bd^2x^2+ad^2\right)^3a}(dx)^{\frac{3}{2}}+\frac{15\,d\sqrt{2}}{512\,a^3b}\ln\left(\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 15/64*d/(b*d^2*x^2+a*d^2)^3/a^3*b^2*(d*x)^(11/2)+21/32*d^3/(b*d^2*x^2+a*d^2
)^3/a^2*b*(d*x)^(7/2)+113/192*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(3/2)+15/512*
d/a^3/b/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
)+15/256*d/a^3/b/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*
x)^(1/2)+1)+15/256*d/a^3/b/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)
^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41322, size = 864, normalized size = 2.58

$$-\frac{180\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{3375\,\sqrt{dx}a^3bd\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}-\sqrt{-11390625\,a^7bd^2\sqrt{-\frac{d^2}{a^{13}b^3}}+11390625\,d^3xa^3b\left(-\frac{}{a^{1}}\right.}}{3375\,d^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(180*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-d^2/(a^13*b
^3))^(1/4)*arctan(-1/3375*(3375*sqrt(d*x)*a^3*b*d*(-d^2/(a^13*b^3))^(1/4) -
 sqrt(-11390625*a^7*b*d^2*sqrt(-d^2/(a^13*b^3)) + 11390625*d^3*x)*a^3*b*(-d
^2/(a^13*b^3))^(1/4))/d^2) - 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2
+ a^6)*(-d^2/(a^13*b^3))^(1/4)*log(3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) +
3375*sqrt(d*x)*d) + 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-
d^2/(a^13*b^3))^(1/4)*log(-3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) + 3375*sqr
t(d*x)*d) - 4*(45*b^2*x^5 + 126*a*b*x^3 + 113*a^2*x)*sqrt(d*x))/(a^3*b^3*x^
6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

Sympy [A] time = 42.8826, size = 252, normalized size = 0.75

$$\frac{226a^2d^{11}(dx)^{\frac{3}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+\frac{252abd^9(dx)^{\frac{7}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 226*a**2*d**11*(d*x)**(3/2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x**2 + 1152
*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6) + 252*a*b*d**9*(d*x)**(7/
2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x**2 + 1152*a**4*b**2*d**12*x**4 + 3
84*a**3*b**3*d**12*x**6) + 90*b**2*d**7*(d*x)**(11/2)/(384*a**6*d**12 + 115
2*a**5*b*d**12*x**2 + 1152*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6)
 + 2*d**7*RootSum(68719476736*_t**4*a**13*b**3*d**26 + 50625, Lambda(_t, _t
*log(134217728*_t**3*a**10*b**2*d**20/3375 + sqrt(d*x))))
```

Giac [A] time = 1.31688, size = 417, normalized size = 1.24

$$\frac{45\sqrt{dx}b^2d^6x^5+126\sqrt{dx}abd^6x^3+113\sqrt{dx}a^2d^6x}{192\left(bd^2x^2+ad^2\right)^3a^3}+\frac{15\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256a^4b^3d}+\frac{15\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\text{arct}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/192*(45*sqrt(d*x)*b^2*d^6*x^5 + 126*sqrt(d*x)*a*b*d^6*x^3 + 113*sqrt(d*x)
*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*a^3) + 15/256*sqrt(2)*(a*b^3*d^2)^(3/4)*
arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))
/(a^4*b^3*d) + 15/256*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3*d) - 15/512*sqrt
(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*
d^2/b))/(a^4*b^3*d) + 15/512*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3*d)
```

3.706 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)}+\frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2}-\frac{77\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{256\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}$$

```
[Out] Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*Sqrt[d*x])/(48*a^2*d*(a + b*x^2)^2) +
 (77*Sqrt[d*x])/(192*a^3*d*(a + b*x^2)) - (77*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (77*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(
15/4)*b^(1/4)*Sqrt[d]) - (77*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (7
7*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d])
```

Rubi [A] time = 0.347814, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)}+\frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2}-\frac{77\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\right.}{256\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*Sqrt[d*x])/(48*a^2*d*(a + b*x^2)^2) +
 (77*Sqrt[d*x])/(192*a^3*d*(a + b*x^2)) - (77*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (77*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(
15/4)*b^(1/4)*Sqrt[d]) - (77*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (7
7*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{\left(11b^3\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx}{12a}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{\left(77b^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{96a^2}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{128a^3}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\,\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x\right.}{64a^3d}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx\right.}{128a^{7/2}d^2}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{77\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}+x^2}\right.}{256a^{7/2}\sqrt{b}}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} - \frac{77\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\right.}{256\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} - \frac{77\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}} + \cdots$$

Mathematica [A] time = 0.115382, size = 253, normalized size = 0.76

$$\frac{\sqrt{x}\left(\frac{256a^{11/4}\sqrt{x}}{\left(a+bx^2\right)^3} + \frac{352a^{7/4}\sqrt{x}}{\left(a+bx^2\right)^2} + \frac{616a^{3/4}\sqrt{x}}{a+bx^2} - \frac{231\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}} + \frac{231\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}} - \frac{462\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}\right.}{1536a^{15/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (Sqrt[x]*((256*a^(11/4)*Sqrt[x])/(a + b*x^2)^3 + (352*a^(7/4)*Sqrt[x])/(a +
 b*x^2)^2 + (616*a^(3/4)*Sqrt[x])/(a + b*x^2) - (462*Sqrt[2]*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (462*Sqrt[2]*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (231*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (231*Sqrt[2]*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/(1536*a^(15/4)*Sqrt[d
*x])
```

Maple [A] time = 0.062, size = 269, normalized size = 0.8

$$\frac{77\,b^2d}{192\,\left(bd^2x^2+ad^2\right)^3a^3}(dx)^{\frac{9}{2}}+\frac{33\,d^3b}{32\,\left(bd^2x^2+ad^2\right)^3a^2}(dx)^{\frac{5}{2}}+\frac{51\,d^5}{64\,\left(bd^2x^2+ad^2\right)^3a}\sqrt{dx}+\frac{77\,\sqrt{2}}{512\,da^4}\sqrt[4]{\frac{ad^2}{b}}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x)
```

```
[Out] 77/192*d/(b*d^2*x^2+a*d^2)^3/a^3*b^2*(d*x)^(9/2)+33/32*d^3/(b*d^2*x^2+a*d^2
)^3/a^2*b*(d*x)^(5/2)+51/64*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(1/2)+77/512/d/
a^4*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+77
/256/d/a^4*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/
2)+1)+77/256/d/a^4*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(
d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38059, size = 814, normalized size = 2.43

$$\frac{924\left(a^3b^3dx^6+3\,a^4b^2dx^4+3\,a^5bdx^2+a^6d\right)\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^8d^2\sqrt{-\frac{1}{a^{15}bd^2}}+dx}a^{11}bd\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}a^{11}bd\left(-\frac{1}{a^{15}b\cdots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(924*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a
^15*b*d^2))^(1/4)*arctan(sqrt(a^8*d^2*sqrt(-1/(a^15*b*d^2)) + d*x)*a^11*b*d
*(-1/(a^15*b*d^2))^(3/4) - sqrt(d*x)*a^11*b*d*(-1/(a^15*b*d^2))^(3/4)) + 23
1*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a^15*b*d^2
))^(1/4)*log(a^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) - 231*(a^3*b^3*d*x^
6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a^15*b*d^2))^(1/4)*log(-a
^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) + 4*(77*b^2*x^4 + 198*a*b*x^2 + 1
53*a^2)*sqrt(d*x))/(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**4), x)
```

Giac [A] time = 1.2907, size = 416, normalized size = 1.24

$$\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\cdots\right.}{512\,a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 77/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d) + 77/256*sqrt(2)*(a*b^3*d^2)^(1
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^4*b*d) + 77/512*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d) - 77/512*sqrt(2)*(a*b^3*d^2)
^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*
d) + 1/192*(77*sqrt(d*x)*b^2*d^5*x^4 + 198*sqrt(d*x)*a*b*d^5*x^2 + 153*sqrt
(d*x)*a^2*d^5)/((b*d^2*x^2 + a*d^2)^3*a^3)
```

3.707 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=352

$$-\frac{195\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\tan^{-1}\left(1-\right.}{128\sqrt{2}a^{17/4}d}$$

```
[Out] -195/(64*a^4*d*Sqrt[d*x]) + 1/(6*a*d*Sqrt[d*x]*(a + b*x^2)^3) + 13/(48*a^2*
d*Sqrt[d*x]*(a + b*x^2)^2) + 39/(64*a^3*d*Sqrt[d*x]*(a + b*x^2)) + (195*b^(
1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2
]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sq
rt[2]*a^(17/4)*d^(3/2)) + (195*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(17/4)*d^(3/2))
```

Rubi [A] time = 0.402453, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{195\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\tan^{-1}\left(1-\right.}{128\sqrt{2}a^{17/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```


```
[Out] -195/(64*a^4*d*Sqrt[d*x]) + 1/(6*a*d*Sqrt[d*x]*(a + b*x^2)^3) + 13/(48*a^2*
d*Sqrt[d*x]*(a + b*x^2)^2) + 39/(64*a^3*d*Sqrt[d*x]*(a + b*x^2)) + (195*b^(
1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2
]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sq
rt[2]*a^(17/4)*d^(3/2)) + (195*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(17/4)*d^(3/2))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{\left(13b^3\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^3}\,dx}{12a}$$

$$= \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{\left(39b^2\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^2}\,dx}{32a^2}$$

$$= \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\left(a+bx^2\right)} + \frac{(195b)\int \frac{}{(dx}}{12}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\left(a+bx^2\right)}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\left(a+bx^2\right)}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\left(a+bx^2\right)}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\left(a+bx^2\right)}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\left(a+bx^2\right)}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\left(a+bx^2\right)}$$

Mathematica [C] time = 0.0118885, size = 30, normalized size = 0.09

$$-\frac{2x\,{}_2F_1\left(-\frac{1}{4},4;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^4(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-1/4, 4, 3/4, -((b*x^2)/a)])/(a^4*(d*x)^(3/2))
```

Maple [A] time = 0.07, size = 285, normalized size = 0.8

$$-2\frac{1}{a^4d\sqrt{dx}}-\frac{67\,b^3}{64\,a^4d\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{11}{2}}-\frac{81\,b^2d}{32\,a^3\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{7}{2}}-\frac{317\,d^3b}{192\,a^2\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{3}{2}}-\frac{195\sqrt{2}}{512\,a^4d}\ln\left(\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -2/a^4/d/(d*x)^(1/2)-67/64/d*b^3/a^4/(b*d^2*x^2+a*d^2)^3*(d*x)^(11/2)-81/32
*d*b^2/a^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(7/2)-317/192*d^3*b/a^2/(b*d^2*x^2+a*d
^2)^3*(d*x)^(3/2)-195/512/d/a^4/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2)))-195/256/d/a^4/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-195/256/d/a^4/(a*d^2/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5407, size = 1008, normalized size = 2.86

$$2340\left(a^4b^3d^2x^7+3\,a^5b^2d^2x^5+3\,a^6bd^2x^3+a^7d^2x\right)\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{7414875\,\sqrt{dx}a^4bd\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}-\sqrt{-54980371265625\,a^9bd^4\sqrt{-\frac{}{a^1}}}}{7414875\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(2340*(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^
2*x)*(-b/(a^17*d^6))^(1/4)*arctan(-1/7414875*(7414875*sqrt(d*x)*a^4*b*d*(-b
/(a^17*d^6))^(1/4) - sqrt(-54980371265625*a^9*b*d^4*sqrt(-b/(a^17*d^6)) + 5
4980371265625*b^2*d*x)*a^4*d*(-b/(a^17*d^6))^(1/4))/b) - 585*(a^4*b^3*d^2*x
^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)*(-b/(a^17*d^6))^(1/4)
*log(7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x)*b) + 585*(a
^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)*(-b/(a^17
*d^6))^(1/4)*log(-7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x
)*b) - 4*(585*b^3*x^6 + 1638*a*b^2*x^4 + 1469*a^2*b*x^2 + 384*a^3)*sqrt(d*x
))/(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**4), x)
```

Giac [A] time = 1.22216, size = 441, normalized size = 1.25

$$-\frac{\frac{3072}{\sqrt{dx}a^4}+\frac{8\left(201\sqrt{dx}b^3d^5x^5+486\sqrt{dx}ab^2d^5x^3+317\sqrt{dx}a^2bd^5x\right)}{\left(bd^2x^2+ad^2\right)^3a^4}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2d^2}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a\cdots}{}\right.\right.}{2\left(\cdots\right.}\right.}{a^5b^2d^2}\cdots}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*(3072/(sqrt(d*x)*a^4) + 8*(201*sqrt(d*x)*b^3*d^5*x^5 + 486*sqrt(d*x
)*a*b^2*d^5*x^3 + 317*sqrt(d*x)*a^2*b*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^4) +
1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
+ 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2*d^2) + 1170*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^5*b^2*d^2) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2*d^2) + 585*sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*
b^2*d^2))/d
```

3.708 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=352

$$\frac{385b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{19/4}d^{5/2}}-\frac{385b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{19/4}d^{5/2}}+\frac{385b^{3/4}\tan^{-1}\left(1-\right.}{128\sqrt{2}a^{19/4}}$$

```
[Out] -385/(192*a^4*d*(d*x)^(3/2)) + 1/(6*a*d*(d*x)^(3/2)*(a + b*x^2)^3) + 5/(16*
a^2*d*(d*x)^(3/2)*(a + b*x^2)^2) + 55/(64*a^3*d*(d*x)^(3/2)*(a + b*x^2)) +
(385*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(12
8*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(19/4)*d^(5/2)) + (385*b^(3/4)*Lo
g[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(256*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(19/4)*d^(
5/2))
```

Rubi [A] time = 0.387744, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{385b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{19/4}d^{5/2}}-\frac{385b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{19/4}d^{5/2}}+\frac{385b^{3/4}\tan^{-1}\left(1-\right.}{128\sqrt{2}a^{19/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```


```
[Out] -385/(192*a^4*d*(d*x)^(3/2)) + 1/(6*a*d*(d*x)^(3/2)*(a + b*x^2)^3) + 5/(16*
a^2*d*(d*x)^(3/2)*(a + b*x^2)^2) + 55/(64*a^3*d*(d*x)^(3/2)*(a + b*x^2)) +
(385*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(12
8*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(19/4)*d^(5/2)) + (385*b^(3/4)*Lo
g[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(256*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(19/4)*d^(
5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{\left(5b^3\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^3}\,dx}{4a}$$

$$= \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{\left(55b^2\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^2}\,dx}{32a^2}$$

$$= \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2}\left(a+bx^2\right)} + \frac{(385b)}{}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2}\left(\right.}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2}\left(\right.}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2}\left(\right.}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2}\left(\right.}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2}\left(\right.}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2}\left(\right.}$$

Mathematica [C] time = 0.0130759, size = 32, normalized size = 0.09

$$-\frac{2x\,{}_2F_1\left(-\frac{3}{4},4;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^4(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-3/4, 4, 1/4, -((b*x^2)/a)])/(3*a^4*(d*x)^(5/2))
```

Maple [A] time = 0.069, size = 288, normalized size = 0.8

$$-\frac{2}{3\,a^4d}(dx)^{-\frac{3}{2}}-\frac{257\,b^3}{192\,a^4d\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{9}{2}}-\frac{101\,b^2d}{32\,a^3\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{5}{2}}-\frac{127\,d^3b}{64\,a^2\left(bd^2x^2+ad^2\right)^3}\sqrt{dx}-\frac{385\,b\sqrt{2}}{512\,d^3a^5}\sqrt[4]{a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -2/3/a^4/d/(d*x)^(3/2)-257/192/d/a^4*b^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(9/2)-10
1/32*d/a^3*b^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(5/2)-127/64*d^3/a^2*b/(b*d^2*x^2+
a*d^2)^3*(d*x)^(1/2)-385/512/d^3/a^5*b*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))-385/256/d^3/a^5*b*(a*d^2/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-385/256/d^3/a^5*b*(a*d^2/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41582, size = 972, normalized size = 2.76

$$-\frac{4620\left(a^4b^3d^3x^8+3\,a^5b^2d^3x^6+3\,a^6bd^3x^4+a^7d^3x^2\right)\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{14}bd^7\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^{10}d^6\sqrt{-\frac{b^3}{a^{19}d^{10}}}+b^2dxa^{14}d^7\left(-\frac{b^3}{a^{19}\cdots}\right.}}{b^3}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(4620*(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d
^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*arctan(-(sqrt(d*x)*a^14*b*d^7*(-b^3/(a^19*
d^10))^(3/4) - sqrt(a^10*d^6*sqrt(-b^3/(a^19*d^10)) + b^2*d*x)*a^14*d^7*(-b
^3/(a^19*d^10))^(3/4))/b^3) + 1155*(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3
*a^6*b*d^3*x^4 + a^7*d^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*log(385*a^5*d^3*(-b^
3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) - 1155*(a^4*b^3*d^3*x^8 + 3*a^5*b^2
*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*log(-385
*a^5*d^3*(-b^3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) + 4*(385*b^3*x^6 + 990
*a*b^2*x^4 + 765*a^2*b*x^2 + 128*a^3)*sqrt(d*x))/(a^4*b^3*d^3*x^8 + 3*a^5*b
^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(5/2)*(a + b*x**2)**4), x)
```

Giac [A] time = 1.33336, size = 416, normalized size = 1.18

$$-\frac{385\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^5d^3}-\frac{385\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^5d^3}-\frac{385\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -385/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*d^3) - 385/256*sqrt(2)*(a*b^3*d^2)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/(a^5*d^3) - 385/512*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*d^3) + 385/512*sqrt(2)*(a*b^3
*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
^5*d^3) - 1/192*(385*b^3*d^6*x^6 + 990*a*b^2*d^6*x^4 + 765*a^2*b*d^6*x^2 +
128*a^3*d^6)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)^3*a^4*d)
```

3.709 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=370

$$\frac{663b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{21/4}d^{7/2}}-\frac{663b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{21/4}d^{7/2}}-\frac{663b^{5/4}\tan^{-1}\left(1-\right.}{128\sqrt{2}a^{21/4}d^{7}}$$

```
[Out] -663/(320*a^4*d*(d*x)^(5/2)) + (663*b)/(64*a^5*d^3*Sqrt[d*x]) + 1/(6*a*d*(d
*x)^(5/2)*(a + b*x^2)^3) + 17/(48*a^2*d*(d*x)^(5/2)*(a + b*x^2)^2) + 221/(1
92*a^3*d*(d*x)^(5/2)*(a + b*x^2)) - (663*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(21/4)*d^(7/2)) + (663*b^(5
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]
*a^(21/4)*d^(7/2)) + (663*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2)) - (663*
b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2))
```

Rubi [A] time = 0.446997, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{663b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{21/4}d^{7/2}}-\frac{663b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{21/4}d^{7/2}}-\frac{663b^{5/4}\tan^{-1}\left(1-\right.}{128\sqrt{2}a^{21/4}d^{7}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```


```
[Out] -663/(320*a^4*d*(d*x)^(5/2)) + (663*b)/(64*a^5*d^3*Sqrt[d*x]) + 1/(6*a*d*(d
*x)^(5/2)*(a + b*x^2)^3) + 17/(48*a^2*d*(d*x)^(5/2)*(a + b*x^2)^2) + 221/(1
92*a^3*d*(d*x)^(5/2)*(a + b*x^2)) - (663*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(21/4)*d^(7/2)) + (663*b^(5
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]
*a^(21/4)*d^(7/2)) + (663*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2)) - (663*
b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{\left(17b^3\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^3}\,dx}{12a}$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{\left(221b^2\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^2}\,dx}{96a^2}$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{221}{192a^3d(dx)^{5/2}\left(a+bx^2\right)} + \frac{(663}{}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{22}{192a^3d(dx)^5}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2}$$

Mathematica [C] time = 0.0121173, size = 37, normalized size = 0.1

$$-\frac{2\sqrt{dx}\,{}_2F_1\left(-\frac{5}{4},4;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^4d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (-2*Sqrt[d*x]*Hypergeometric2F1[-5/4, 4, -1/4, -((b*x^2)/a)])/(5*a^4*d^4*x^
3)
```

Maple [A] time = 0.073, size = 304, normalized size = 0.8

$$-\frac{2}{5\,a^4d}(dx)^{-\frac{5}{2}}+8\,\frac{b}{a^5d^3\sqrt{dx}}+\frac{151\,b^4}{64\,a^5d^3\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{11}{2}}+\frac{173\,b^3}{32\,a^4d\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{7}{2}}+\frac{617\,b^2d}{192\,a^3\left(bd^2x^2+ad^2\right)^3}(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -2/5/a^4/d/(d*x)^(5/2)+8*b/a^5/d^3/(d*x)^(1/2)+151/64/d^3*b^4/a^5/(b*d^2*x^
2+a*d^2)^3*(d*x)^(11/2)+173/32/d*b^3/a^4/(b*d^2*x^2+a*d^2)^3*(d*x)^(7/2)+61
7/192*d*b^2/a^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(3/2)+663/512/d^3*b/a^5/(a*d^2/b)
^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+663/256/d^3*b/a
^5/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+66
3/256/d^3*b/a^5/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x
)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54075, size = 1135, normalized size = 3.07

$$-\frac{39780\left(a^5b^3d^4x^9+3\,a^6b^2d^4x^7+3\,a^7bd^4x^5+a^8d^4x^3\right)\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{291434247\sqrt{dx}a^5b^4d^3\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-84933920324457\cdots}}{29\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3840*(39780*(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5 + a^8
*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*arctan(-1/291434247*(291434247*sqrt(d*x)
*a^5*b^4*d^3*(-b^5/(a^21*d^14))^(1/4) - sqrt(-84933920324457009*a^11*b^5*d^
8*sqrt(-b^5/(a^21*d^14)) + 84933920324457009*b^8*d*x)*a^5*d^3*(-b^5/(a^21*d
^14))^(1/4))/b^5) - 9945*(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4
*x^5 + a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*log(291434247*a^16*d^11*(-b^5/
(a^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4) + 9945*(a^5*b^3*d^4*x^9 + 3*a
^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5 + a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*lo
g(-291434247*a^16*d^11*(-b^5/(a^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4)
- 4*(9945*b^4*x^8 + 27846*a*b^3*x^6 + 24973*a^2*b^2*x^4 + 6528*a^3*b*x^2 -
384*a^4)*sqrt(d*x))/(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5
+ a^8*d^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{7}{2}}\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(7/2)*(a + b*x**2)**4), x)
```

Giac [A] time = 1.30241, size = 471, normalized size = 1.27

$$\frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^6bd^5}+\frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^6bd^5}-\frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(d\right.}{51}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 663/256*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d^5) + 663/256*sqrt(2)*(a*b^3*d^2
)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a^6*b*d^5) - 663/512*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b*d^5) + 663/512*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^6*b*d^5) + 1/192*(453*sqrt(d*x)*b^4*d^5*x^5 + 1038*sqrt(d*x)*a*b^3*d^
5*x^3 + 617*sqrt(d*x)*a^2*b^2*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^5*d^3) + 2/5*
(20*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^5*d^5*x^2)
```

3.710 $\int \frac{(dx)^{27/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=420

$$-\frac{69615a^{5/4}d^{27/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{29/4}}-\frac{69}{}$$

```
[Out] (-69615*a*d^13*Sqrt[d*x])/(4096*b^7) + (13923*d^11*(d*x)^(5/2))/(4096*b^6)
- (d*(d*x)^(25/2))/(10*b*(a + b*x^2)^5) - (5*d^3*(d*x)^(21/2))/(32*b^2*(a +
 b*x^2)^4) - (35*d^5*(d*x)^(17/2))/(128*b^3*(a + b*x^2)^3) - (595*d^7*(d*x)
^(13/2))/(1024*b^4*(a + b*x^2)^2) - (7735*d^9*(d*x)^(9/2))/(4096*b^5*(a + b
*x^2)) - (69615*a^(5/4)*d^(27/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(27/2)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29/4))
- (69615*a^(5/4)*d^(27/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(2
7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*b^(29/4))
```

Rubi [A] time = 0.529127, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{69615a^{5/4}d^{27/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{29/4}}-\frac{69}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] (-69615*a*d^13*Sqrt[d*x])/(4096*b^7) + (13923*d^11*(d*x)^(5/2))/(4096*b^6)
- (d*(d*x)^(25/2))/(10*b*(a + b*x^2)^5) - (5*d^3*(d*x)^(21/2))/(32*b^2*(a +
 b*x^2)^4) - (35*d^5*(d*x)^(17/2))/(128*b^3*(a + b*x^2)^3) - (595*d^7*(d*x)
^(13/2))/(1024*b^4*(a + b*x^2)^2) - (7735*d^9*(d*x)^(9/2))/(4096*b^5*(a + b
*x^2)) - (69615*a^(5/4)*d^(27/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(27/2)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29/4))
```

```
- (69615*a^(5/4)*d^(27/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(2
7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*b^(29/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{27/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{27/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{4}\left(5b^4d^2\right)\int \frac{(dx)^{23/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(105b^2d^4\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(595d^6\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} - \frac{595d^7(dx)^{13/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{\left(7735d^8\right)\int}{204}$$

$$= -\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} - \frac{595d^7(dx)^{13/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{7735d^9(dx}{4096b^5\left(a+\right.}$$

$$= \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3} - \frac{595d^7(dx)^{13/2}}{1024b^4\left(a+bx^2\right.}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.186116, size = 432, normalized size = 1.03

$$d^{13}\sqrt{dx}\Big(-126156800a^2b^{17/4}x^{17/2}-306380800a^3b^{13/4}x^{13/2}-362086400a^4b^{9/4}x^{9/2}-217251840a^5b^{5/4}x^{5/2}+10210200a^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^13*Sqrt[d*x]*(-54312960*a^6*b^(1/4)*Sqrt[x] - 217251840*a^5*b^(5/4)*x^(5
/2) - 362086400*a^4*b^(9/4)*x^(9/2) - 306380800*a^3*b^(13/4)*x^(13/2) - 126
156800*a^2*b^(17/4)*x^(17/2) - 18022400*a*b^(21/4)*x^(21/2) + 720896*b^(25/
4)*x^(25/2) + 3394560*a^5*b^(1/4)*Sqrt[x]*(a + b*x^2) + 4243200*a^4*b^(1/4)
*Sqrt[x]*(a + b*x^2)^2 + 5834400*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 + 102102
00*a^2*b^(1/4)*Sqrt[x]*(a + b*x^2)^4 - 7657650*Sqrt[2]*a^(5/4)*(a + b*x^2)^
5*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 7657650*Sqrt[2]*a^(5/4)*(
a + b*x^2)^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 3828825*Sqrt[2
]*a^(5/4)*(a + b*x^2)^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x] + 3828825*Sqrt[2]*a^(5/4)*(a + b*x^2)^5*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(1802240*b^(29/4)*Sqrt[x]*(a + b*x^2)^5)
```

Maple [A] time = 0.122, size = 370, normalized size = 0.9

$$\frac{2\,d^{11}}{5\,b^{6}}(dx)^{\frac{5}{2}}-12\,\frac{ad^{13}\sqrt{dx}}{b^{7}}-\frac{20463\,d^{23}a^{6}}{4096\,b^{7}\left(bd^{2}x^{2}+ad^{2}\right)^{5}}\sqrt{dx}-\frac{56269\,d^{21}a^{5}}{2560\,b^{6}\left(bd^{2}x^{2}+ad^{2}\right)^{5}}(dx)^{\frac{5}{2}}-\frac{75471\,d^{19}a^{4}}{2048\,b^{5}\left(bd^{2}x^{2}+ad^{2}\right)^{5}}(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/5*d^11*(d*x)^(5/2)/b^6-12*a*d^13*(d*x)^(1/2)/b^7-20463/4096*d^23/b^7*a^6/
(b*d^2*x^2+a*d^2)^5*(d*x)^(1/2)-56269/2560*d^21/b^6*a^5/(b*d^2*x^2+a*d^2)^5
*(d*x)^(5/2)-75471/2048*d^19/b^5*a^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(9/2)-3597/1
28*d^17/b^4*a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(13/2)-34139/4096*d^15/b^3*a^2/(b
*d^2*x^2+a*d^2)^5*(d*x)^(17/2)+69615/32768*d^13/b^7*a*(a*d^2/b)^(1/4)*2^(1/
2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+69615/16384*d^13/b^7*a*(a*d
^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+69615/163
84*d^13/b^7*a*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^
```

(1/2)-1)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69083, size = 1229, normalized size = 2.93

$$1392300\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)\arctan\left(-\frac{\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{3}{4}}\sqrt{dx}ab^{22}d^{13}-\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{3}{4}}\cdots}{a^5d^{54}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(1392300*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*
b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*arctan(-((-a^5*d^54/b^
29)^(3/4)*sqrt(d*x)*a*b^22*d^13 - (-a^5*d^54/b^29)^(3/4)*sqrt(a^2*d^27*x +
sqrt(-a^5*d^54/b^29)*b^14)*b^22)/(a^5*d^54)) + 348075*(-a^5*d^54/b^29)^(1/4
)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*
x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13 + 69615*(-a^5*d^54/b^29)^(1/4)*b^
7) - 348075*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*
x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13
- 69615*(-a^5*d^54/b^29)^(1/4)*b^7) + 4*(8192*b^6*d^13*x^12 - 204800*a*b^5*
d^13*x^10 - 1317575*a^2*b^4*d^13*x^8 - 2951200*a^3*b^3*d^13*x^6 - 3171350*a
^4*b^2*d^13*x^4 - 1670760*a^5*b*d^13*x^2 - 348075*a^6*d^13)*sqrt(d*x))/(b^1
2*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 +
a^5*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(27/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27497, size = 510, normalized size = 1.21

$$\frac{1}{163840}d^{12}\left(\frac{696150\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8}+\frac{696150\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{b^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^12*(696150*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^8 + 696150*sqrt(2)*(
a*b^3*d^2)^(1/4)*a*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(
d*x))/(a*d^2/b)^(1/4))/b^8 + 348075*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*log(d*x +
 sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^8 - 348075*sqrt(2)*(a
*b^3*d^2)^(1/4)*a*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^
2/b))/b^8 - 8*(170695*sqrt(d*x)*a^2*b^4*d^11*x^8 + 575520*sqrt(d*x)*a^3*b^3
*d^11*x^6 + 754710*sqrt(d*x)*a^4*b^2*d^11*x^4 + 450152*sqrt(d*x)*a^5*b*d^11
*x^2 + 102315*sqrt(d*x)*a^6*d^11)/((b*d^2*x^2 + a*d^2)^5*b^7) + 65536*(sqrt
(d*x)*b^24*d^6*x^2 - 30*sqrt(d*x)*a*b^23*d^6)/(b^30*d^5))
```

3.711 $\int \frac{(dx)^{25/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=402

$$-\frac{33649a^{3/4}d^{25/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{27/4}}+\frac{336}{}$$

```
[Out] (33649*d^11*(d*x)^(3/2))/(12288*b^6) - (d*(d*x)^(23/2))/(10*b*(a + b*x^2)^5
) - (23*d^3*(d*x)^(19/2))/(160*b^2*(a + b*x^2)^4) - (437*d^5*(d*x)^(15/2))/
(1920*b^3*(a + b*x^2)^3) - (437*d^7*(d*x)^(11/2))/(1024*b^4*(a + b*x^2)^2)
- (4807*d^9*(d*x)^(7/2))/(4096*b^5*(a + b*x^2)) + (33649*a^(3/4)*d^(25/2)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(
27/4)) - (33649*a^(3/4)*d^(25/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(27/4)) - (33649*a^(3/4)*d^(25/2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(1638
4*Sqrt[2]*b^(27/4)) + (33649*a^(3/4)*d^(25/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(27/4))
```

Rubi [A] time = 0.468539, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{33649a^{3/4}d^{25/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{27/4}}+\frac{336}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] (33649*d^11*(d*x)^(3/2))/(12288*b^6) - (d*(d*x)^(23/2))/(10*b*(a + b*x^2)^5
) - (23*d^3*(d*x)^(19/2))/(160*b^2*(a + b*x^2)^4) - (437*d^5*(d*x)^(15/2))/
(1920*b^3*(a + b*x^2)^3) - (437*d^7*(d*x)^(11/2))/(1024*b^4*(a + b*x^2)^2)
- (4807*d^9*(d*x)^(7/2))/(4096*b^5*(a + b*x^2)) + (33649*a^(3/4)*d^(25/2)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(
27/4)) - (33649*a^(3/4)*d^(25/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(27/4)) - (33649*a^(3/4)*d^(25/2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(1638
```

```
4*Sqrt[2]*b^(27/4)) + (33649*a^(3/4)*d^(25/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(27/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{25/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{25/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(23b^4d^2\right)\int \frac{(dx)^{21/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(437b^2d^4\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(437d^6\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^3}$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{\left(4807d^{8}\right.}{}$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{4807d}{4096b^5}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx}{1024b^4\left(a+\right.}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx}{1024b^4\left(a+\right.}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx}{1024b^4\left(a+\right.}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx}{1024b^4\left(a+\right.}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx}{1024b^4\left(a+\right.}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5} - \frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{437d^7(dx}{1024b^4\left(a+\right.}$$

Mathematica [C] time = 0.0351908, size = 109, normalized size = 0.27

$$-\frac{2d^{12}x\sqrt{dx}\left(-289731a^2b^3x^6-482885a^3b^2x^4-408595a^4bx^2-168245a^5-76245ab^4x^8+168245\left(a+bx^2\right)^5\,{}_2F_1\left(\frac{3}{4},6;\frac{7}{4}\right.\right.}{9945b^6\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-2*d^12*x*Sqrt[d*x]*(-168245*a^5 - 408595*a^4*b*x^2 - 482885*a^3*b^2*x^4 -
 289731*a^2*b^3*x^6 - 76245*a*b^4*x^8 - 3315*b^5*x^10 + 168245*(a + b*x^2)^
5*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)]))/(9945*b^6*(a + b*x^2)^5)
```

Maple [A] time = 0.074, size = 354, normalized size = 0.9

$$\frac{2\,d^{11}}{3\,b^6}(dx)^{\frac{3}{2}}+\frac{25457\,d^{21}a^5}{12288\,b^6\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{3}{2}}+\frac{3527\,d^{19}a^4}{384\,b^5\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{7}{2}}+\frac{95821\,d^{17}a^3}{6144\,b^4\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{11}{2}}+\frac{3114}{2560\,b^3\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/3*d^11*(d*x)^(3/2)/b^6+25457/12288*d^21*a^5/b^6/(b*d^2*x^2+a*d^2)^5*(d*x)
^(3/2)+3527/384*d^19*a^4/b^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(7/2)+95821/6144*d^1
7*a^3/b^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(11/2)+31149/2560*d^15*a^2/b^3/(b*d^2*x
^2+a*d^2)^5*(d*x)^(15/2)+15503/4096*d^13*a/b^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(1
9/2)-33649/32768*d^13*a/b^7/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2)))-33649/16384*d^13*a/b^7/(a*d^2/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-33649/16384*d^13*a/b^7/(a*d^2/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73702, size = 1258, normalized size = 3.13

$$2018940\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)\arctan\left(-\frac{\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{1}{4}}\sqrt{dx}a^2b^7d^{37}-\sqrt{a^4d^{75}x-}}{a^3d^{50}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(2018940*(-a^3*d^50/b^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2
*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*arctan(-((-a^3*d^50/b^
27)^(1/4)*sqrt(d*x)*a^2*b^7*d^37 - sqrt(a^4*d^75*x - sqrt(-a^3*d^50/b^27)*a
^3*b^13*d^50)*(-a^3*d^50/b^27)^(1/4)*b^7)/(a^3*d^50)) - 504735*(-a^3*d^50/b
^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*
a^4*b^7*x^2 + a^5*b^6)*log(38099255258449*sqrt(d*x)*a^2*d^37 + 380992552584
49*(-a^3*d^50/b^27)^(3/4)*b^20) + 504735*(-a^3*d^50/b^27)^(1/4)*(b^11*x^10
+ 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
*log(38099255258449*sqrt(d*x)*a^2*d^37 - 38099255258449*(-a^3*d^50/b^27)^(3
/4)*b^20) + 4*(40960*b^5*d^12*x^11 + 437345*a*b^4*d^12*x^9 + 1157176*a^2*b^
3*d^12*x^7 + 1367810*a^3*b^2*d^12*x^5 + 769120*a^4*b*d^12*x^3 + 168245*a^5*
d^12*x)*sqrt(d*x))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*
x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(25/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30389, size = 463, normalized size = 1.15

$$\frac{1}{491520}d^{11}\left(\frac{327680\sqrt{dx}dx}{b^6}-\frac{1009470\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^9}-\frac{1009470\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{}\right.}{b^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^11*(327680*sqrt(d*x)*d*x/b^6 - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)
*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4)
)/b^9 - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^9 + 504735*sqrt(2)*(a*b^3*d^2
)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^9 -
504735*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/b^9 + 8*(232545*sqrt(d*x)*a*b^4*d^11*x^9 + 747576*sqrt(d
*x)*a^2*b^3*d^11*x^7 + 958210*sqrt(d*x)*a^3*b^2*d^11*x^5 + 564320*sqrt(d*x)
*a^4*b*d^11*x^3 + 127285*sqrt(d*x)*a^5*d^11*x)/((b*d^2*x^2 + a*d^2)^5*b^6))
```

3.712 $\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=402

$$-\frac{13923d^9(dx)^{5/2}}{20480b^5\left(a+bx^2\right)}-\frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2}-\frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3}-\frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4}+\frac{13923\sqrt[4]{a}d^{23/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right.}{16384\sqrt{2}b^{25/4}}$$

```
[Out] (13923*d^11*Sqrt[d*x])/(4096*b^6) - (d*(d*x)^(21/2))/(10*b*(a + b*x^2)^5) -
 (21*d^3*(d*x)^(17/2))/(160*b^2*(a + b*x^2)^4) - (119*d^5*(d*x)^(13/2))/(64
0*b^3*(a + b*x^2)^3) - (1547*d^7*(d*x)^(9/2))/(5120*b^4*(a + b*x^2)^2) - (1
3923*d^9*(d*x)^(5/2))/(20480*b^5*(a + b*x^2)) + (13923*a^(1/4)*d^(23/2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25
/4)) - (13923*a^(1/4)*d^(23/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25/4)) + (13923*a^(1/4)*d^(23/2)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*
Sqrt[2]*b^(25/4)) - (13923*a^(1/4)*d^(23/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*S
qrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(25/4))
```

Rubi [A] time = 0.485538, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{13923d^9(dx)^{5/2}}{20480b^5\left(a+bx^2\right)}-\frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2}-\frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3}-\frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4}+\frac{13923\sqrt[4]{a}d^{23/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right.}{16384\sqrt{2}b^{25/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] (13923*d^11*Sqrt[d*x])/(4096*b^6) - (d*(d*x)^(21/2))/(10*b*(a + b*x^2)^5) -
 (21*d^3*(d*x)^(17/2))/(160*b^2*(a + b*x^2)^4) - (119*d^5*(d*x)^(13/2))/(64
0*b^3*(a + b*x^2)^3) - (1547*d^7*(d*x)^(9/2))/(5120*b^4*(a + b*x^2)^2) - (1
3923*d^9*(d*x)^(5/2))/(20480*b^5*(a + b*x^2)) + (13923*a^(1/4)*d^(23/2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25
/4)) - (13923*a^(1/4)*d^(23/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25/4)) + (13923*a^(1/4)*d^(23/2)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*
```

```
Sqrt[2]*b^(25/4)) - (13923*a^(1/4)*d^(23/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*S
qrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(25/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{23/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(21b^4d^2\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(357b^2d^4\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} + \frac{\left(1547d^6\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^3}\,dx}{1280}$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2} + \frac{\left(13923d^8\right)}{10}$$

$$= -\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2} - \frac{13923d^9(}{20480b^5\left(a\right.}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)}$$

Mathematica [A] time = 0.296907, size = 408, normalized size = 1.01

$$\frac{d^{11}\sqrt{dx}\left(\frac{61276160a^2b^{13/4}x^{13/2}+72417280a^3b^{9/4}x^{9/2}+43450368a^4b^{5/4}x^{5/2}-1166880a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3-848640a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2-678912a^4\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+1}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^11*Sqrt[d*x]*(1531530*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)] + (10862592*a^5*b^(1/4)*Sqrt[x] + 43450368*a^4*b^(5/4)*x^(5/2)
+ 72417280*a^3*b^(9/4)*x^(9/2) + 61276160*a^2*b^(13/4)*x^(13/2) + 25231360*
a*b^(17/4)*x^(17/2) + 3604480*b^(21/4)*x^(21/2) - 678912*a^4*b^(1/4)*Sqrt[x
]*(a + b*x^2) - 848640*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 - 1166880*a^2*b^(1
/4)*Sqrt[x]*(a + b*x^2)^3 - 2042040*a*b^(1/4)*Sqrt[x]*(a + b*x^2)^4 - 15315
30*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)] + 765765*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x] - 765765*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a + b*x^2)^5))/(1802240
*b^(25/4)*Sqrt[x])
```

Maple [A] time = 0.072, size = 351, normalized size = 0.9

$$2\,\frac{d^{11}\sqrt{dx}}{b^6}+\frac{5731\,d^{21}a^5}{4096\,b^6\left(bd^2x^2+ad^2\right)^5}\sqrt{dx}+\frac{16169\,d^{19}a^4}{2560\,b^5\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{5}{2}}+\frac{22467\,d^{17}a^3}{2048\,b^4\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{9}{2}}+\frac{1129\,d}{128\,b^3\left(bd^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2*d^11*(d*x)^(1/2)/b^6+5731/4096*d^21/b^6*a^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(1/
2)+16169/2560*d^19/b^5*a^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(5/2)+22467/2048*d^17/
b^4*a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(9/2)+1129/128*d^15/b^3*a^2/(b*d^2*x^2+a*
d^2)^5*(d*x)^(13/2)+11743/4096*d^13/b^2*a/(b*d^2*x^2+a*d^2)^5*(d*x)^(17/2)-
13923/32768*d^11/b^6*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2)))-13923/16384*d^11/b^6*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-13923/16384*d^11/b^6*(a*d^2/b)^(1/4)*2^(1/2)
```

```
*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69568, size = 1141, normalized size = 2.84

$$-\frac{278460\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5\,ab^{10}x^{8}+10\,a^{2}b^{9}x^{6}+10\,a^{3}b^{8}x^{4}+5\,a^{4}b^{7}x^{2}+a^{5}b^{6}\right)\arctan\left(-\frac{\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{3}{4}}\sqrt{dx}b^{19}d^{11}-\sqrt{d^{23}x+\sqrt{-\frac{ad^{46}}{b^{25}}}\cdots}}{ad^{46}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")

[Out] -1/81920*(278460*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^
9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*arctan(-((-a*d^46/b^25)^(
3/4)*sqrt(d*x)*b^19*d^11 - sqrt(d^23*x + sqrt(-a*d^46/b^25)*b^12)*(-a*d^46/
b^25)^(3/4)*b^19)/(a*d^46)) + 69615*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b
^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*log(13
923*sqrt(d*x)*d^11 + 13923*(-a*d^46/b^25)^(1/4)*b^6) - 69615*(-a*d^46/b^25)
^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*
b^7*x^2 + a^5*b^6)*log(13923*sqrt(d*x)*d^11 - 13923*(-a*d^46/b^25)^(1/4)*b^
6) - 4*(40960*b^5*d^11*x^10 + 263515*a*b^4*d^11*x^8 + 590240*a^2*b^3*d^11*x
^6 + 634270*a^3*b^2*d^11*x^4 + 334152*a^4*b*d^11*x^2 + 69615*a^5*d^11)*sqrt
(d*x))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*
b^7*x^2 + a^5*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33164, size = 466, normalized size = 1.16

$$-\frac{1}{163840}d^{10}\left(\frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}+\frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/163840*d^10*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt
(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^7 + 139230*sqrt(2)*(a
*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x
))/(a*d^2/b)^(1/4))/b^7 + 69615*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 69615*sqrt(2)*(a*b^3*d^
2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7
 - 327680*sqrt(d*x)*d/b^6 - 8*(58715*sqrt(d*x)*a*b^4*d^11*x^8 + 180640*sqrt
(d*x)*a^2*b^3*d^11*x^6 + 224670*sqrt(d*x)*a^3*b^2*d^11*x^4 + 129352*sqrt(d*
x)*a^4*b*d^11*x^2 + 28655*sqrt(d*x)*a^5*d^11)/((b*d^2*x^2 + a*d^2)^5*b^6))
```

3.713 $\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=385

$$-\frac{1463d^9(dx)^{3/2}}{4096b^5\left(a+bx^2\right)}-\frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3}-\frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4}+\frac{4389d^{21/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}\right.}{16384\sqrt{2}\sqrt[4]{a}b^{23/4}}$$

```
[Out] -(d*(d*x)^(19/2))/(10*b*(a + b*x^2)^5) - (19*d^3*(d*x)^(15/2))/(160*b^2*(a
+ b*x^2)^4) - (19*d^5*(d*x)^(11/2))/(128*b^3*(a + b*x^2)^3) - (209*d^7*(d*x
)^(7/2))/(1024*b^4*(a + b*x^2)^2) - (1463*d^9*(d*x)^(3/2))/(4096*b^5*(a + b
*x^2)) - (4389*d^(21/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) + (4389*d^(21/2)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) +
(4389*d^(21/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1/4)*b^(23/4)) - (4389*d^(21/2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(163
84*Sqrt[2]*a^(1/4)*b^(23/4))
```

Rubi [A] time = 0.448027, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1463d^9(dx)^{3/2}}{4096b^5\left(a+bx^2\right)}-\frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3}-\frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4}+\frac{4389d^{21/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}\right.}{16384\sqrt{2}\sqrt[4]{a}b^{23/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(19/2))/(10*b*(a + b*x^2)^5) - (19*d^3*(d*x)^(15/2))/(160*b^2*(a
+ b*x^2)^4) - (19*d^5*(d*x)^(11/2))/(128*b^3*(a + b*x^2)^3) - (209*d^7*(d*x
)^(7/2))/(1024*b^4*(a + b*x^2)^2) - (1463*d^9*(d*x)^(3/2))/(4096*b^5*(a + b
*x^2)) - (4389*d^(21/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) + (4389*d^(21/2)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) +
(4389*d^(21/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1/4)*b^(23/4)) - (4389*d^(21/2)*Log[Sqr
```

```
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(163
84*Sqrt[2]*a^(1/4)*b^(23/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{21/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(19b^4d^2\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(57b^2d^4\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(209d^6\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^3}\,a$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{\left(1463d^8\right)}{2}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{1463d^9(}{4096b^5\left(a\right.}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{1463d^9(}{4096b^5\left(a\right.}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{1463d^9(}{4096b^5\left(a\right.}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{1463d^9(}{4096b^5\left(a\right.}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{1463d^9(}{4096b^5\left(a\right.}$$

$$= -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{1463d^9(}{4096b^5\left(a\right.}$$

Mathematica [C] time = 0.0368045, size = 104, normalized size = 0.27

$$\frac{2d^9(dx)^{3/2}\left(7315\left(a+bx^2\right)^5 {}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(20995a^2b^2x^4+17765a^3bx^2+7315a^4+12597ab^3x^6+3315b^4x^8\right)\right)}{3315ab^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^9*(d*x)^(3/2)*(-(a*(7315*a^4 + 17765*a^3*b*x^2 + 20995*a^2*b^2*x^4 + 1
2597*a*b^3*x^6 + 3315*b^4*x^8)) + 7315*(a + b*x^2)^5*Hypergeometric2F1[3/4,
 6, 7/4, -((b*x^2)/a)]))/(3315*a*b^5*(a + b*x^2)^5)
```

Maple [A] time = 0.072, size = 335, normalized size = 0.9

$$-\frac{1463\,d^{19}a^4}{4096\left(bd^2x^2+ad^2\right)^5b^5}(dx)^{\frac{3}{2}}-\frac{209\,d^{17}a^3}{128\left(bd^2x^2+ad^2\right)^5b^4}(dx)^{\frac{7}{2}}-\frac{5947\,d^{15}a^2}{2048\left(bd^2x^2+ad^2\right)^5b^3}(dx)^{\frac{11}{2}}-\frac{6289\,d^{13}a}{2560\left(bd^2x^2+ad^2\right)^5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1463/4096*d^19/(b*d^2*x^2+a*d^2)^5/b^5*a^4*(d*x)^(3/2)-209/128*d^17/(b*d^2
*x^2+a*d^2)^5/b^4*a^3*(d*x)^(7/2)-5947/2048*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^
2*(d*x)^(11/2)-6289/2560*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(15/2)-3803/4
096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(19/2)+4389/32768*d^11/b^6/(a*d^2/b)^(
1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(
d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+4389/16384*d^11/b
^6/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+43
89/16384*d^11/b^6/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d
*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74617, size = 1158, normalized size = 3.01

$$-\frac{87780\left(b^{10}x^{10}+5\,ab^{9}x^{8}+10\,a^{2}b^{8}x^{6}+10\,a^{3}b^{7}x^{4}+5\,a^{4}b^{6}x^{2}+a^{5}b^{5}\right)\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}\sqrt{dx}b^{6}d^{31}-\sqrt{d^{63}x-\sqrt{-\frac{d^{42}}{ab^{23}}}ab}}{d^{42}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(87780*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4
+ 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*arctan(-((-d^42/(a*b^23))
^(1/4)*sqrt(d*x)*b^6*d^31 - sqrt(d^63*x - sqrt(-d^42/(a*b^23))*a*b^11*d^42)
*(-d^42/(a*b^23))^(1/4)*b^6)/d^42) - 21945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^
2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4
)*log(84546715869*sqrt(d*x)*d^31 + 84546715869*(-d^42/(a*b^23))^(3/4)*a*b^1
7) + 21945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a
^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*log(84546715869*sqrt(d*x)*d^31
 - 84546715869*(-d^42/(a*b^23))^(3/4)*a*b^17) + 4*(19015*b^4*d^10*x^9 + 503
12*a*b^3*d^10*x^7 + 59470*a^2*b^2*d^10*x^5 + 33440*a^3*b*d^10*x^3 + 7315*a^
4*d^10*x)*sqrt(d*x))/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7
*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17143, size = 459, normalized size = 1.19

$$\frac{1}{163840}d^9\left(\frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^8}+\frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^8}-\frac{21945}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^9*(43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^8) + 43890*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a*b^8) - 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^8) + 21945*sqrt(2)*(a*b^3*
d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*
b^8) - 8*(19015*sqrt(d*x)*b^4*d^11*x^9 + 50312*sqrt(d*x)*a*b^3*d^11*x^7 + 5
9470*sqrt(d*x)*a^2*b^2*d^11*x^5 + 33440*sqrt(d*x)*a^3*b*d^11*x^3 + 7315*sqr
t(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

3.714 $\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=385

$$-\frac{663d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}+\frac{663d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}-\frac{663d^{19/2}\tan^{-1}}{8192\sqrt{2}}$$

```
[Out] -(d*(d*x)^(17/2))/(10*b*(a + b*x^2)^5) - (17*d^3*(d*x)^(13/2))/(160*b^2*(a
+ b*x^2)^4) - (221*d^5*(d*x)^(9/2))/(1920*b^3*(a + b*x^2)^3) - (663*d^7*(d*
x)^(5/2))/(5120*b^4*(a + b*x^2)^2) - (663*d^9*Sqrt[d*x])/(4096*b^5*(a + b*x
^2)) - (663*d^(19/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) - (663
*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqr
t[2]*a^(3/4)*b^(21/4))
```

Rubi [A] time = 0.445966, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{663d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}+\frac{663d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}-\frac{663d^{19/2}\tan^{-1}}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(17/2))/(10*b*(a + b*x^2)^5) - (17*d^3*(d*x)^(13/2))/(160*b^2*(a
+ b*x^2)^4) - (221*d^5*(d*x)^(9/2))/(1920*b^3*(a + b*x^2)^3) - (663*d^7*(d*
x)^(5/2))/(5120*b^4*(a + b*x^2)^2) - (663*d^9*Sqrt[d*x])/(4096*b^5*(a + b*x
^2)) - (663*d^(19/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) - (663
*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*Log[Sqrt[a]*S
```

```
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqr
t[2]*a^(3/4)*b^(21/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6 \int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(17b^4d^2\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(221b^2d^4\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{\left(663d^6\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^3}\,dx}{1280}$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} + \frac{\left(663d^8\right)\int \cdots}{20\cdots}$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} - \frac{663d^9\cdots}{4096b^5\left(a\cdots\right.}$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} - \frac{663d^9\cdots}{4096b^5\left(a\cdots\right.}$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} - \frac{663d^9\cdots}{4096b^5\left(a\cdots\right.}$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} - \frac{663d^9\cdots}{4096b^5\left(a\cdots\right.}$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} - \frac{663d^9\cdots}{4096b^5\left(a\cdots\right.}$$

$$= -\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\left(a+bx^2\right)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\left(a+bx^2\right)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\left(a+bx^2\right)^2} - \frac{663d^9\cdots}{4096b^5\left(a\cdots\right.}$$

Mathematica [A] time = 0.204056, size = 381, normalized size = 0.99

$$d^9\sqrt{dx}\left(-\frac{72417280a^2b^{9/4}x^4}{\left(a+bx^2\right)^5}-\frac{43450368a^3b^{5/4}x^2}{\left(a+bx^2\right)^5}+\frac{848640a^2\sqrt[4]{b}}{\left(a+bx^2\right)^3}+\frac{678912a^3\sqrt[4]{b}}{\left(a+bx^2\right)^4}-\frac{10862592a^4\sqrt[4]{b}}{\left(a+bx^2\right)^5}-\frac{765765\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}+\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^9*Sqrt[d*x]*((-10862592*a^4*b^(1/4))/(a + b*x^2)^5 - (43450368*a^3*b^(5/
4)*x^2)/(a + b*x^2)^5 - (72417280*a^2*b^(9/4)*x^4)/(a + b*x^2)^5 - (6127616
0*a*b^(13/4)*x^6)/(a + b*x^2)^5 - (25231360*b^(17/4)*x^8)/(a + b*x^2)^5 + (
678912*a^3*b^(1/4))/(a + b*x^2)^4 + (848640*a^2*b^(1/4))/(a + b*x^2)^3 + (1
166880*a*b^(1/4))/(a + b*x^2)^2 + (2042040*b^(1/4))/(a + b*x^2) - (1531530*
Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sqrt[x]) +
(1531530*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sq
rt[x]) - (765765*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(a^(3/4)*Sqrt[x]) + (765765*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x])))/(37847040*b^(21/4))
```

Maple [A] time = 0.071, size = 344, normalized size = 0.9

$$-\frac{663\,d^{19}a^4}{4096\,\left(bd^2x^2+ad^2\right)^5b^5}\sqrt{dx}-\frac{1989\,d^{17}a^3}{2560\,\left(bd^2x^2+ad^2\right)^5b^4}(dx)^{\frac{5}{2}}-\frac{9061\,d^{15}a^2}{6144\,\left(bd^2x^2+ad^2\right)^5b^3}(dx)^{\frac{9}{2}}-\frac{527\,d^{13}a}{384\,\left(bd^2x^2+ad^2\right)^5b^2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -663/4096*d^19/(b*d^2*x^2+a*d^2)^5/b^5*a^4*(d*x)^(1/2)-1989/2560*d^17/(b*d^
2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(5/2)-9061/6144*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a
^2*(d*x)^(9/2)-527/384*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(13/2)-7529/122
88*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(17/2)+663/32768*d^9/b^5*(a*d^2/b)^(1/4
)/a*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d
*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+663/16384*d^9/b^5*
(a*d^2/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+663
/16384*d^9/b^5*(a*d^2/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*
x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71322, size = 1127, normalized size = 2.93

$$39780\left(b^{10}x^{10}+5\,ab^{9}x^{8}+10\,a^{2}b^{8}x^{6}+10\,a^{3}b^{7}x^{4}+5\,a^{4}b^{6}x^{2}+a^{5}b^{5}\right)\left(-\frac{d^{38}}{a^{3}b^{21}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{38}}{a^{3}b^{21}}\right)^{\frac{3}{4}}\sqrt{dx}a^{2}b^{16}d^{9}-\sqrt{d^{19}x+\sqrt{-\frac{d^{38}}{a^{3}b^{21}}}}}{d^{38}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(39780*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4
+ 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*arctan(-((-d^38/(a^3*b^
21))^(3/4)*sqrt(d*x)*a^2*b^16*d^9 - sqrt(d^19*x + sqrt(-d^38/(a^3*b^21))*a^
2*b^10)*(-d^38/(a^3*b^21))^(3/4)*a^2*b^16)/d^38) + 9945*(b^10*x^10 + 5*a*b^
9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(
a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 + 663*(-d^38/(a^3*b^21))^(1/4)*a*b^5
) - 9945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4
*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 - 663*(-
d^38/(a^3*b^21))^(1/4)*a*b^5) - 4*(37645*b^4*d^9*x^8 + 84320*a*b^3*d^9*x^6
+ 90610*a^2*b^2*d^9*x^4 + 47736*a^3*b*d^9*x^2 + 9945*a^4*d^9)*sqrt(d*x))/(b
^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 +
a^5*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17475, size = 463, normalized size = 1.2

$$\frac{1}{491520}d^8\left(\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6}+\frac{9\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^8*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6) + 19890*sqrt(2)*(a
*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x
))/(a*d^2/b)^(1/4))/(a*b^6) + 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6) - 9945*sqrt(2)*(a*
b^3*d^2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a*b^6) - 8*(37645*sqrt(d*x)*b^4*d^11*x^8 + 84320*sqrt(d*x)*a*b^3*d^11*x
^6 + 90610*sqrt(d*x)*a^2*b^2*d^11*x^4 + 47736*sqrt(d*x)*a^3*b*d^11*x^2 + 99
45*sqrt(d*x)*a^4*d^11)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

3.715 $\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=388

$$\frac{231d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{5/4}b^{19/4}}-\frac{231d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{5/4}b^{19/4}}-\frac{231d^{17/2}\tan^{-1}\left(1\right.}{8192\sqrt{2}a^{5/4}}$$

```
[Out] -(d*(d*x)^(15/2))/(10*b*(a + b*x^2)^5) - (3*d^3*(d*x)^(11/2))/(32*b^2*(a +
b*x^2)^4) - (11*d^5*(d*x)^(7/2))/(128*b^3*(a + b*x^2)^3) - (77*d^7*(d*x)^(3
/2))/(1024*b^4*(a + b*x^2)^2) + (231*d^7*(d*x)^(3/2))/(4096*a*b^4*(a + b*x^
2)) - (231*d^(17/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*d^(17/2)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*
d^(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(16384*Sqrt[2]*a^(5/4)*b^(19/4)) - (231*d^(17/2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt
[2]*a^(5/4)*b^(19/4))
```

Rubi [A] time = 0.448817, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{231d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{5/4}b^{19/4}}-\frac{231d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{5/4}b^{19/4}}-\frac{231d^{17/2}\tan^{-1}\left(1\right.}{8192\sqrt{2}a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] -(d*(d*x)^(15/2))/(10*b*(a + b*x^2)^5) - (3*d^3*(d*x)^(11/2))/(32*b^2*(a +
b*x^2)^4) - (11*d^5*(d*x)^(7/2))/(128*b^3*(a + b*x^2)^3) - (77*d^7*(d*x)^(3
/2))/(1024*b^4*(a + b*x^2)^2) + (231*d^7*(d*x)^(3/2))/(4096*a*b^4*(a + b*x^
2)) - (231*d^(17/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*d^(17/2)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*
d^(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(16384*Sqrt[2]*a^(5/4)*b^(19/4)) - (231*d^(17/2)*Log[Sqrt[a]*Sq
```

```
rt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt
[2]*a^(5/4)*b^(19/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6 \int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{4}\left(3b^4d^2\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(33b^2d^4\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(77d^6\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{\left(231d^8\right)\int}{20}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{231d^7(}{4096ab^4\left(\right.}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{231d^7(}{4096ab^4\left(\right.}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{231d^7(}{4096ab^4\left(\right.}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{231d^7(}{4096ab^4\left(\right.}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{231d^7(}{4096ab^4\left(\right.}$$

$$= -\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2} + \frac{231d^7(}{4096ab^4\left(\right.}$$

Mathematica [C] time = 0.0314192, size = 96, normalized size = 0.25

$$\frac{2d^8x\sqrt{dx}\left(385\left(a+bx^2\right)^5\,{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\left(935a^2bx^2+385a^3+1105ab^2x^4+663b^3x^6\right)\right)}{3315a^2b^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^8*x*Sqrt[d*x]*(-(a^2*(385*a^3 + 935*a^2*b*x^2 + 1105*a*b^2*x^4 + 663*b
^3*x^6)) + 385*(a + b*x^2)^5*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)]))
/(3315*a^2*b^4*(a + b*x^2)^5)
```

Maple [A] time = 0.069, size = 341, normalized size = 0.9

$$-\frac{77\,d^{17}a^3}{4096\left(bd^2x^2+ad^2\right)^5b^4}(dx)^{\frac{3}{2}}-\frac{11\,d^{15}a^2}{128\left(bd^2x^2+ad^2\right)^5b^3}(dx)^{\frac{7}{2}}-\frac{313\,d^{13}a}{2048\left(bd^2x^2+ad^2\right)^5b^2}(dx)^{\frac{11}{2}}-\frac{331\,d^{11}}{2560\left(bd^2x^2+ad^2\right)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/4096*d^17/(b*d^2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(3/2)-11/128*d^15/(b*d^2*x^
2+a*d^2)^5/b^3*a^2*(d*x)^(7/2)-313/2048*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x
)^(11/2)-331/2560*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(15/2)+231/4096*d^9/(b*d
^2*x^2+a*d^2)^5/a*(d*x)^(19/2)+231/32768*d^9/a/b^5/(a*d^2/b)^(1/4)*2^(1/2)*
ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+231/16384*d^9/a/b^5/(a*d^2/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+231/16384*d^9/a
/b^5/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45232, size = 1175, normalized size = 3.03

$$-\frac{4620\left(ab^9x^{10}+5a^2b^8x^8+10a^3b^7x^6+10a^4b^6x^4+5a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{1}{4}}\sqrt{dx}ab^5d^{25}-\sqrt{d^{51}x-\sqrt{-\frac{d^{3}}{a^5b}}}}{d^{34}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(4620*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^
4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*arctan(-((-d^34/(a^5*
b^19))^(1/4)*sqrt(d*x)*a*b^5*d^25 - sqrt(d^51*x - sqrt(-d^34/(a^5*b^19))*a^
3*b^9*d^34)*(-d^34/(a^5*b^19))^(1/4)*a*b^5)/d^34) - 1155*(a*b^9*x^10 + 5*a^
2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^
34/(a^5*b^19))^(1/4)*log(12326391*sqrt(d*x)*d^25 + 12326391*(-d^34/(a^5*b^1
9))^(3/4)*a^4*b^14) + 1155*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 1
0*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*log(12326
391*sqrt(d*x)*d^25 - 12326391*(-d^34/(a^5*b^19))^(3/4)*a^4*b^14) - 4*(1155*
b^4*d^8*x^9 - 2648*a*b^3*d^8*x^7 - 3130*a^2*b^2*d^8*x^5 - 1760*a^3*b*d^8*x^
3 - 385*a^4*d^8*x)*sqrt(d*x))/(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6
+ 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17569, size = 463, normalized size = 1.19

$$\frac{1}{163840}d^7\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^7}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^7}-\frac{1155\sqrt{2}\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^7*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^7) + 2310*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^2*b^7) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^7) + 1155*sqrt(2)*(a*b^
3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^2*b^7) + 8*(1155*sqrt(d*x)*b^4*d^11*x^9 - 2648*sqrt(d*x)*a*b^3*d^11*x^7 -
 3130*sqrt(d*x)*a^2*b^2*d^11*x^5 - 1760*sqrt(d*x)*a^3*b*d^11*x^3 - 385*sqrt
(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a*b^4))
```

3.716 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=388

$$-\frac{117d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{7/4}b^{17/4}}+\frac{117d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{7/4}b^{17/4}}-\frac{117d^{15/2}\tan^{-1}(}{8192\sqrt{2}a}$$

```
[Out] -(d*(d*x)^(13/2))/(10*b*(a + b*x^2)^5) - (13*d^3*(d*x)^(9/2))/(160*b^2*(a +
 b*x^2)^4) - (39*d^5*(d*x)^(5/2))/(640*b^3*(a + b*x^2)^3) - (39*d^7*Sqrt[d*
x])/(1024*b^4*(a + b*x^2)^2) + (39*d^7*Sqrt[d*x])/(4096*a*b^4*(a + b*x^2))
- (117*d^(15/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(8192*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(7/4)*b^(17/4)) - (117*d^(1
5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*
a^(7/4)*b^(17/4))
```

Rubi [A] time = 0.445878, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{117d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{7/4}b^{17/4}}+\frac{117d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{7/4}b^{17/4}}-\frac{117d^{15/2}\tan^{-1}(}{8192\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] -(d*(d*x)^(13/2))/(10*b*(a + b*x^2)^5) - (13*d^3*(d*x)^(9/2))/(160*b^2*(a +
 b*x^2)^4) - (39*d^5*(d*x)^(5/2))/(640*b^3*(a + b*x^2)^3) - (39*d^7*Sqrt[d*
x])/(1024*b^4*(a + b*x^2)^2) + (39*d^7*Sqrt[d*x])/(4096*a*b^4*(a + b*x^2))
- (117*d^(15/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(8192*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(7/4)*b^(17/4)) - (117*d^(1
5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*Log[Sqrt[a]*Sqrt[d
```

```
] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*
a^(7/4)*b^(17/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(13b^4d^2\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(117b^2d^4\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(39d^6\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{\left(39d^8\right)\int -}{20}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{39d^7}{4096ab^4\left(a\right.}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{39d^7}{4096ab^4\left(a\right.}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{39d^7}{4096ab^4\left(a\right.}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{39d^7}{4096ab^4\left(a\right.}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{39d^7}{4096ab^4\left(a\right.}$$

$$= -\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2} + \frac{39d^7}{4096ab^4\left(a\right.}$$

Mathematica [A] time = 0.2467, size = 359, normalized size = 0.93

$$\frac{d^7\sqrt{dx}\left(-\frac{2555904a^2b^{5/4}x^2}{\left(a+bx^2\right)^5}+\frac{120120\sqrt[4]{b}}{a^2+abx^2}+\frac{39936a^2\sqrt[4]{b}}{\left(a+bx^2\right)^4}-\frac{638976a^3\sqrt[4]{b}}{\left(a+bx^2\right)^5}-\frac{45045\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}+\frac{45045\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\cdots\right.}{a^{7/4}\sqrt{x}}\right.}{12615680b^{17/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^7*Sqrt[d*x]*((-638976*a^3*b^(1/4))/(a + b*x^2)^5 - (2555904*a^2*b^(5/4)*
x^2)/(a + b*x^2)^5 - (4259840*a*b^(9/4)*x^4)/(a + b*x^2)^5 - (3604480*b^(13
/4)*x^6)/(a + b*x^2)^5 + (39936*a^2*b^(1/4))/(a + b*x^2)^4 + (49920*a*b^(1/
4))/(a + b*x^2)^3 + (68640*b^(1/4))/(a + b*x^2)^2 + (120120*b^(1/4))/(a^2 +
 a*b*x^2) - (90090*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
a^(7/4)*Sqrt[x]) + (90090*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(a^(7/4)*Sqrt[x]) - (45045*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x]) + (45045*Sqrt[2]*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x])))/(12615680
*b^(17/4))
```

Maple [A] time = 0.067, size = 341, normalized size = 0.9

$$-\frac{117\,d^{17}a^3}{4096\left(bd^2x^2+ad^2\right)^5b^4}\sqrt{dx}-\frac{351\,d^{15}a^2}{2560\left(bd^2x^2+ad^2\right)^5b^3}(dx)^{\frac{5}{2}}-\frac{533\,d^{13}a}{2048\left(bd^2x^2+ad^2\right)^5b^2}(dx)^{\frac{9}{2}}-\frac{31\,d^{11}}{128\left(bd^2x^2+ad^2\right)^5b}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -117/4096*d^17/(b*d^2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(1/2)-351/2560*d^15/(b*d^2
*x^2+a*d^2)^5/b^3*a^2*(d*x)^(5/2)-533/2048*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(
d*x)^(9/2)-31/128*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(13/2)+39/4096*d^9/(b*d^
2*x^2+a*d^2)^5/a*(d*x)^(17/2)+117/32768*d^7/a^2/b^4*(a*d^2/b)^(1/4)*2^(1/2)
*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+117/16384*d^7/a^2/b^4*(a*d^2/
b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+117/16384*d^
7/a^2/b^4*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2
)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48943, size = 1134, normalized size = 2.92

$$\frac{2340\left(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}a^5b^{13}d^7-\sqrt{d^{15}x+\sqrt{-\frac{d^{30}}{a^7b^{17}}}}}{d^{30}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(2340*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4
 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*arctan(-((-d^30/(a^7*b
^17))^(3/4)*sqrt(d*x)*a^5*b^13*d^7 - sqrt(d^15*x + sqrt(-d^30/(a^7*b^17))*a
^4*b^8)*(-d^30/(a^7*b^17))^(3/4)*a^5*b^13)/d^30) + 585*(a*b^9*x^10 + 5*a^2*
b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30
/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 + 117*(-d^30/(a^7*b^17))^(1/4)*a^2
*b^4) - 585*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 +
 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 -
117*(-d^30/(a^7*b^17))^(1/4)*a^2*b^4) + 4*(195*b^4*d^7*x^8 - 4960*a*b^3*d^7
*x^6 - 5330*a^2*b^2*d^7*x^4 - 2808*a^3*b*d^7*x^2 - 585*a^4*d^7)*sqrt(d*x))/
(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x
^2 + a^6*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18039, size = 467, normalized size = 1.2

$$\frac{1}{163840}d^6\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}+\frac{585}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^6*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5) + 1170*sqrt(2)*(a
*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x
))/(a*d^2/b)^(1/4))/(a^2*b^5) + 585*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5) - 585*sqrt(2)*(
a*b^3*d^2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^2*b^5) + 8*(195*sqrt(d*x)*b^4*d^11*x^8 - 4960*sqrt(d*x)*a*b^3*d^11*
x^6 - 5330*sqrt(d*x)*a^2*b^2*d^11*x^4 - 2808*sqrt(d*x)*a^3*b*d^11*x^2 - 585
*sqrt(d*x)*a^4*d^11)/((b*d^2*x^2 + a*d^2)^5*a*b^4))
```

3.717 $\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=391

$$\frac{77d^5(dx)^{3/2}}{4096a^2b^3(a+bx^2)}+\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{9/4}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{9/4}b^{15/4}}$$

```
[Out] -(d*(d*x)^(11/2))/(10*b*(a + b*x^2)^5) - (11*d^3*(d*x)^(7/2))/(160*b^2*(a +
 b*x^2)^4) - (77*d^5*(d*x)^(3/2))/(1920*b^3*(a + b*x^2)^3) + (77*d^5*(d*x)^
(3/2))/(5120*a*b^3*(a + b*x^2)^2) + (77*d^5*(d*x)^(3/2))/(4096*a^2*b^3*(a +
 b*x^2)) - (77*d^(13/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (7
7*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(9/4)*b^(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqr
t[2]*a^(9/4)*b^(15/4))
```

Rubi [A] time = 0.479026, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{77d^5(dx)^{3/2}}{4096a^2b^3(a+bx^2)}+\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{9/4}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{9/4}b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] -(d*(d*x)^(11/2))/(10*b*(a + b*x^2)^5) - (11*d^3*(d*x)^(7/2))/(160*b^2*(a +
 b*x^2)^4) - (77*d^5*(d*x)^(3/2))/(1920*b^3*(a + b*x^2)^3) + (77*d^5*(d*x)^
(3/2))/(5120*a*b^3*(a + b*x^2)^2) + (77*d^5*(d*x)^(3/2))/(4096*a^2*b^3*(a +
 b*x^2)) - (77*d^(13/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (7
7*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(9/4)*b^(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*S
```

```
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqr
t[2]*a^(9/4)*b^(15/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(11b^4d^2\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(77b^2d^4\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{\left(77d^6\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{1280}$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \frac{\left(77d^6\right)}{}$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \frac{77}{4096a}$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \frac{77}{4096a}$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \frac{77}{4096a}$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \frac{77}{4096a}$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \frac{77}{4096a}$$

$$= -\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\left(a+bx^2\right)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\left(a+bx^2\right)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\left(a+bx^2\right)^2} + \frac{77}{4096a}$$

Mathematica [C] time = 0.030944, size = 85, normalized size = 0.22

$$\frac{2d^6x\sqrt{dx}\left(77\left(a+bx^2\right)^5{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)-a^3\left(77a^2+187abx^2+221b^2x^4\right)\right)}{1989a^3b^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^6*x*Sqrt[d*x]*(-(a^3*(77*a^2 + 187*a*b*x^2 + 221*b^2*x^4)) + 77*(a + b
*x^2)^5*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)]))/(1989*a^3*b^3*(a + b
*x^2)^5)
```

Maple [A] time = 0.073, size = 339, normalized size = 0.9

$$-\frac{77\,d^{15}a^2}{12288\left(bd^2x^2+ad^2\right)^5b^3}(dx)^{\frac{3}{2}}-\frac{11\,d^{13}a}{384\left(bd^2x^2+ad^2\right)^5b^2}(dx)^{\frac{7}{2}}-\frac{313\,d^{11}}{6144\left(bd^2x^2+ad^2\right)^5b}(dx)^{\frac{11}{2}}+\frac{231\,d^9}{2560\left(bd^2x^2+ad^2\right)^5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/12288*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(3/2)-11/384*d^13/(b*d^2*x
^2+a*d^2)^5/b^2*a*(d*x)^(7/2)-313/6144*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(11
/2)+231/2560*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(15/2)+77/4096*d^7/(b*d^2*x^2+
a*d^2)^5/a^2*b*(d*x)^(19/2)+77/32768*d^7/a^2/b^4/(a*d^2/b)^(1/4)*2^(1/2)*ln
((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+77/16384*d^7/a^2/b^4/(a*d^2/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+77/16384*d^7/a^2
/b^4/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49084, size = 1181, normalized size = 3.02

$$-\frac{4620\left(a^{2}b^{8}x^{10}+5\,a^{3}b^{7}x^{8}+10\,a^{4}b^{6}x^{6}+10\,a^{5}b^{5}x^{4}+5\,a^{6}b^{4}x^{2}+a^{7}b^{3}\right)\left(-\frac{d^{26}}{a^{9}b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{a^{9}b^{15}}\right)^{\frac{1}{4}}\sqrt{dx}a^{2}b^{4}d^{19}-\sqrt{d^{39}x-\sqrt{-}}}{d^{26}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(4620*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5
*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*arctan(-((-d^26/(a
^9*b^15))^(1/4)*sqrt(d*x)*a^2*b^4*d^19 - sqrt(d^39*x - sqrt(-d^26/(a^9*b^15
))*a^5*b^7*d^26)*(-d^26/(a^9*b^15))^(1/4)*a^2*b^4)/d^26) - 1155*(a^2*b^8*x^
10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*
b^3)*(-d^26/(a^9*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a^
9*b^15))^(3/4)*a^7*b^11) + 1155*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*
x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*lo
g(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(a^9*b^15))^(3/4)*a^7*b^11) - 4*(11
55*b^4*d^6*x^9 + 5544*a*b^3*d^6*x^7 - 3130*a^2*b^2*d^6*x^5 - 1760*a^3*b*d^6
*x^3 - 385*a^4*d^6*x)*sqrt(d*x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6
*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19677, size = 463, normalized size = 1.18

$$\frac{1}{491520}d^5\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^6}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^6}-\frac{1155\sqrt{2}\left(\cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^5*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^6) + 2310*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^3*b^6) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^6) + 1155*sqrt(2)*(a*b^
3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^3*b^6) + 8*(1155*sqrt(d*x)*b^4*d^11*x^9 + 5544*sqrt(d*x)*a*b^3*d^11*x^7 -
 3130*sqrt(d*x)*a^2*b^2*d^11*x^5 - 1760*sqrt(d*x)*a^3*b*d^11*x^3 - 385*sqrt
(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

3.718 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=391

$$\frac{21d^5\sqrt{dx}}{4096a^2b^3\left(a+bx^2\right)}-\frac{63d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{11/4}b^{13/4}}+\frac{63d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{11/4}b^{13/4}}$$

```
[Out] -(d*(d*x)^(9/2))/(10*b*(a + b*x^2)^5) - (9*d^3*(d*x)^(5/2))/(160*b^2*(a + b
*x^2)^4) - (3*d^5*Sqrt[d*x])/(128*b^3*(a + b*x^2)^3) + (3*d^5*Sqrt[d*x])/(1
024*a*b^3*(a + b*x^2)^2) + (21*d^5*Sqrt[d*x])/(4096*a^2*b^3*(a + b*x^2)) -
(63*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(81
92*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(11/4)*b^(13/4)) - (63*d^(11/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(16384*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(
11/4)*b^(13/4))
```

Rubi [A] time = 0.473032, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{21d^5\sqrt{dx}}{4096a^2b^3\left(a+bx^2\right)}-\frac{63d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{11/4}b^{13/4}}+\frac{63d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{11/4}b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(9/2))/(10*b*(a + b*x^2)^5) - (9*d^3*(d*x)^(5/2))/(160*b^2*(a + b
*x^2)^4) - (3*d^5*Sqrt[d*x])/(128*b^3*(a + b*x^2)^3) + (3*d^5*Sqrt[d*x])/(1
024*a*b^3*(a + b*x^2)^2) + (21*d^5*Sqrt[d*x])/(4096*a^2*b^3*(a + b*x^2)) -
(63*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(81
92*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(11/4)*b^(13/4)) - (63*d^(11/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(16384*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*Log[Sqrt[a]*Sqrt[d] +
```

```
 Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(
11/4)*b^(13/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(9b^4d^2\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(9b^2d^4\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{1}{256}\left(3d^6\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \frac{\left(21d^6\right)\int}{2}$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \frac{21d^5}{4096a^2b^3}$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \frac{21d^5}{4096a^2b^3}$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \frac{21d^5}{4096a^2b^3}$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \frac{21d^5}{4096a^2b^3}$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \frac{21d^5}{4096a^2b^3}$$

$$= -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \frac{21d^5}{4096a^2b^3}$$

Mathematica [A] time = 0.18397, size = 337, normalized size = 0.86

$$\frac{d^5\sqrt{dx}\left(\frac{9240\sqrt[4]{b}}{a^2(a+bx^2)}-\frac{49152a^2\sqrt[4]{b}}{(a+bx^2)^5}-\frac{3465\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}+\frac{3465\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}-\frac{6930\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}\sqrt{x}}+\cdots\right)}{1802240b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^5*Sqrt[d*x]*((-49152*a^2*b^(1/4))/(a + b*x^2)^5 - (196608*a*b^(5/4)*x^2)
/(a + b*x^2)^5 - (327680*b^(9/4)*x^4)/(a + b*x^2)^5 + (3072*a*b^(1/4))/(a +
 b*x^2)^4 + (3840*b^(1/4))/(a + b*x^2)^3 + (5280*b^(1/4))/(a*(a + b*x^2)^2)
 + (9240*b^(1/4))/(a^2*(a + b*x^2)) - (6930*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) + (6930*Sqrt[2]*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) - (3465*Sqrt[2]*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt[x]) + (34
65*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(
11/4)*Sqrt[x])))/(1802240*b^(13/4))
```

Maple [A] time = 0.069, size = 339, normalized size = 0.9

$$-\frac{63\,d^{15}a^2}{4096\,\left(bd^2x^2+ad^2\right)^5b^3}\sqrt{dx}-\frac{189\,d^{13}a}{2560\,\left(bd^2x^2+ad^2\right)^5b^2}(dx)^{\frac{5}{2}}-\frac{287\,d^{11}}{2048\,\left(bd^2x^2+ad^2\right)^5b}(dx)^{\frac{9}{2}}+\frac{3\,d^9}{128\,\left(bd^2x^2+ad^2\right)^5a}(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -63/4096*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(1/2)-189/2560*d^13/(b*d^2*
x^2+a*d^2)^5/b^2*a*(d*x)^(5/2)-287/2048*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(9
/2)+3/128*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(13/2)+21/4096*d^7/(b*d^2*x^2+a*d
^2)^5/a^2*b*(d*x)^(17/2)+63/32768*d^5/a^3/b^3*(a*d^2/b)^(1/4)*2^(1/2)*ln((d
*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+63/16384*d^5/a^3/b^3*(a*d^2/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+63/16384*d^5/a^3/b^
3*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4187, size = 1150, normalized size = 2.94

$$\frac{1260\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^8b^{10}d^5\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{3}{4}}-\sqrt{a^6b^6\sqrt{-\frac{d^{22}}{a^{11}b^{13}}}}}{d^{22}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(1260*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x
^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*arctan(-(sqrt(d*x)*
a^8*b^10*d^5*(-d^22/(a^11*b^13))^(3/4) - sqrt(a^6*b^6*sqrt(-d^22/(a^11*b^13
)) + d^11*x)*a^8*b^10*(-d^22/(a^11*b^13))^(3/4))/d^22) + 315*(a^2*b^8*x^10
+ 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3
)*(-d^22/(a^11*b^13))^(1/4)*log(63*a^3*b^3*(-d^22/(a^11*b^13))^(1/4) + 63*s
qrt(d*x)*d^5) - 315*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5
*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*log(-63*a^3*b
^3*(-d^22/(a^11*b^13))^(1/4) + 63*sqrt(d*x)*d^5) + 4*(105*b^4*d^5*x^8 + 480
*a*b^3*d^5*x^6 - 2870*a^2*b^2*d^5*x^4 - 1512*a^3*b*d^5*x^2 - 315*a^4*d^5)*s
qrt(d*x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 +
 5*a^6*b^4*x^2 + a^7*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17385, size = 467, normalized size = 1.19

$$\frac{1}{163840}d^4\left(\frac{630\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}+\frac{630\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4}+\frac{315\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^4*(630*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4) + 630*sqrt(2)*(a*b
^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^3*b^4) + 315*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqr
t(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4) - 315*sqrt(2)*(a*
b^3*d^2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^3*b^4) + 8*(105*sqrt(d*x)*b^4*d^11*x^8 + 480*sqrt(d*x)*a*b^3*d^11*x^6
 - 2870*sqrt(d*x)*a^2*b^2*d^11*x^4 - 1512*sqrt(d*x)*a^3*b*d^11*x^2 - 315*sq
rt(d*x)*a^4*d^11)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

3.719 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=394

$$\frac{63d^3(dx)^{3/2}}{4096a^3b^2\left(a+bx^2\right)}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}+\frac{63d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{13/4}b^{11/4}}-\frac{63d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right.}{16384\sqrt{2}a^{1}}$$

```
[Out] -(d*(d*x)^(7/2))/(10*b*(a + b*x^2)^5) - (7*d^3*(d*x)^(3/2))/(160*b^2*(a + b
*x^2)^4) + (7*d^3*(d*x)^(3/2))/(640*a*b^2*(a + b*x^2)^3) + (63*d^3*(d*x)^(3
/2))/(5120*a^2*b^2*(a + b*x^2)^2) + (63*d^3*(d*x)^(3/2))/(4096*a^3*b^2*(a +
 b*x^2)) - (63*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (63*d^(9/2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (6
3*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(13/4)*b^(11/4)) - (63*d^(9/2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt
[2]*a^(13/4)*b^(11/4))
```

Rubi [A] time = 0.461055, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{63d^3(dx)^{3/2}}{4096a^3b^2\left(a+bx^2\right)}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2}+\frac{63d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{13/4}b^{11/4}}-\frac{63d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right.}{16384\sqrt{2}a^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] -(d*(d*x)^(7/2))/(10*b*(a + b*x^2)^5) - (7*d^3*(d*x)^(3/2))/(160*b^2*(a + b
*x^2)^4) + (7*d^3*(d*x)^(3/2))/(640*a*b^2*(a + b*x^2)^3) + (63*d^3*(d*x)^(3
/2))/(5120*a^2*b^2*(a + b*x^2)^2) + (63*d^3*(d*x)^(3/2))/(4096*a^3*b^2*(a +
 b*x^2)) - (63*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (63*d^(9/2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (6
3*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(13/4)*b^(11/4)) - (63*d^(9/2)*Log[Sqrt[a]*Sq
```

```
rt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt
[2]*a^(13/4)*b^(11/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(7b^4d^2\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{1}{320}\left(21b^2d^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{\left(63bd^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{1280a}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} + \frac{\left(63d^4\right.}{}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} + \frac{6}{4096}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} + \frac{6}{4096}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} + \frac{6}{4096}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} + \frac{6}{4096}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} + \frac{6}{4096}$$

$$= -\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5} - \frac{7d^3(dx)^{3/2}}{160b^2\left(a+bx^2\right)^4} + \frac{7d^3(dx)^{3/2}}{640ab^2\left(a+bx^2\right)^3} + \frac{63d^3(dx)^{3/2}}{5120a^2b^2\left(a+bx^2\right)^2} + \frac{6}{4096}$$

Mathematica [C] time = 0.0270838, size = 61, normalized size = 0.15

$$\frac{2d^4x\sqrt{dx}\left(\frac{7\,{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)}{a^4}+\frac{-7a-17bx^2}{\left(a+bx^2\right)^5}\right)}{221b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d^4*x*Sqrt[d*x]*((-7*a - 17*b*x^2)/(a + b*x^2)^5 + (7*Hypergeometric2F1[
3/4, 6, 7/4, -((b*x^2)/a)])/a^4))/(221*b^2)
```

Maple [A] time = 0.07, size = 339, normalized size = 0.9

$$-\frac{21\,d^{13}a}{4096\left(bd^2x^2+ad^2\right)^5b^2}(dx)^{\frac{3}{2}}-\frac{3\,d^{11}}{128\left(bd^2x^2+ad^2\right)^5b}(dx)^{\frac{7}{2}}+\frac{287\,d^9}{2048\left(bd^2x^2+ad^2\right)^5a}(dx)^{\frac{11}{2}}+\frac{189\,d^7b}{2560\left(bd^2x^2+ad^2\right)^5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -21/4096*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(3/2)-3/128*d^11/(b*d^2*x^2+a
*d^2)^5/b*(d*x)^(7/2)+287/2048*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(11/2)+189/2
560*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(15/2)+63/4096*d^5/(b*d^2*x^2+a*d^2
)^5/a^3*b^2*(d*x)^(19/2)+63/32768*d^5/a^3/b^3/(a*d^2/b)^(1/4)*2^(1/2)*ln((d
*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+63/16384*d^5/a^3/b^3/(a*d^2/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+63/16384*d^5/a^3/b^
3/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4876, size = 1242, normalized size = 3.15

$$-\frac{1260\left(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2\right)\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{250047\,\sqrt{dx}a^3b^3d^{13}\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}-\sqrt{-}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(1260*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*
x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*arctan(-1/250047*(
250047*sqrt(d*x)*a^3*b^3*d^13*(-d^18/(a^13*b^11))^(1/4) - sqrt(-62523502209
*a^7*b^5*d^18*sqrt(-d^18/(a^13*b^11)) + 62523502209*d^27*x)*a^3*b^3*(-d^18/
(a^13*b^11))^(1/4))/d^18) - 315*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*
x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*l
og(250047*a^10*b^8*(-d^18/(a^13*b^11))^(3/4) + 250047*sqrt(d*x)*d^13) + 315
*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^
3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*log(-250047*a^10*b^8*(-d^18/(a^1
3*b^11))^(3/4) + 250047*sqrt(d*x)*d^13) - 4*(315*b^4*d^4*x^9 + 1512*a*b^3*d
^4*x^7 + 2870*a^2*b^2*d^4*x^5 - 480*a^3*b*d^4*x^3 - 105*a^4*d^4*x)*sqrt(d*x
))/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*
b^3*x^2 + a^8*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

[Out] Timed out

Giac [A] time = 1.26712, size = 463, normalized size = 1.18

$$\frac{1}{163840}d^3\left(\frac{630\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5}+\frac{630\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5}-\frac{315\sqrt{2}\left(ab^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^3*(630*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^5) + 630*sqrt(2)*(a*b^3
*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^4*b^5) - 315*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(
a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^5) + 315*sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*
b^5) + 8*(315*sqrt(d*x)*b^4*d^11*x^9 + 1512*sqrt(d*x)*a*b^3*d^11*x^7 + 2870
*sqrt(d*x)*a^2*b^2*d^11*x^5 - 480*sqrt(d*x)*a^3*b*d^11*x^3 - 105*sqrt(d*x)*
a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))
```

3.720 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=394

$$\frac{77d^3\sqrt{dx}}{12288a^3b^2\left(a+bx^2\right)}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}-\frac{77d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{16384\sqrt{}}$$

```
[Out] -(d*(d*x)^(5/2))/(10*b*(a + b*x^2)^5) - (d^3*Sqrt[d*x])/(32*b^2*(a + b*x^2)
^4) + (d^3*Sqrt[d*x])/(384*a*b^2*(a + b*x^2)^3) + (11*d^3*Sqrt[d*x])/(3072*
a^2*b^2*(a + b*x^2)^2) + (77*d^3*Sqrt[d*x])/(12288*a^3*b^2*(a + b*x^2)) - (
77*d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192
*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(15/4)*b^(9/4)) - (77*d^(7/2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(15/4)*b^
(9/4))
```

Rubi [A] time = 0.448744, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77d^3\sqrt{dx}}{12288a^3b^2\left(a+bx^2\right)}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}-\frac{77d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{16384\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(5/2))/(10*b*(a + b*x^2)^5) - (d^3*Sqrt[d*x])/(32*b^2*(a + b*x^2)
^4) + (d^3*Sqrt[d*x])/(384*a*b^2*(a + b*x^2)^3) + (11*d^3*Sqrt[d*x])/(3072*
a^2*b^2*(a + b*x^2)^2) + (77*d^3*Sqrt[d*x])/(12288*a^3*b^2*(a + b*x^2)) - (
77*d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192
*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(15/4)*b^(9/4)) - (77*d^(7/2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
```

```
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(15/4)*b^
(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{4}\left(b^4d^2\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{1}{64}\left(b^2d^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{\left(11bd^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx}{768a}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} + \frac{\left(77d^4\right)\int}{}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} + \frac{77}{12288a^3}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} + \frac{77}{12288a^3}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} + \frac{77}{12288a^3}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} + \frac{77}{12288a^3}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} + \frac{77}{12288a^3}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5} - \frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4} + \frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3} + \frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2} + \frac{77}{12288a^3}$$

Mathematica [A] time = 0.175644, size = 317, normalized size = 0.8

$$\frac{d^3\sqrt{dx}\left(\frac{3080\sqrt[4]{b}}{a^3(a+bx^2)}+\frac{1760\sqrt[4]{b}}{a^2(a+bx^2)^2}-\frac{1155\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{15/4}\sqrt{x}}+\frac{1155\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{15/4}\sqrt{x}}-\frac{2310\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{15/4}\sqrt{x}}+\cdots}{491520b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^3*Sqrt[d*x]*((-16384*a*b^(1/4))/(a + b*x^2)^5 - (65536*b^(5/4)*x^2)/(a +
 b*x^2)^5 + (1024*b^(1/4))/(a + b*x^2)^4 + (1280*b^(1/4))/(a*(a + b*x^2)^3)
 + (1760*b^(1/4))/(a^2*(a + b*x^2)^2) + (3080*b^(1/4))/(a^3*(a + b*x^2)) -
(2310*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(15/4)*Sqrt
[x]) + (2310*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(15/
4)*Sqrt[x]) - (1155*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(a^(15/4)*Sqrt[x]) + (1155*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(15/4)*Sqrt[x])))/(491520*b^(9/4))
```

Maple [A] time = 0.066, size = 339, normalized size = 0.9

$$-\frac{77\,d^{13}a}{4096\,\left(bd^2x^2+ad^2\right)^5 b^2}\sqrt{dx}-\frac{231\,d^{11}}{2560\,\left(bd^2x^2+ad^2\right)^5 b}(dx)^{\frac{5}{2}}+\frac{313\,d^9}{6144\,\left(bd^2x^2+ad^2\right)^5 a}(dx)^{\frac{9}{2}}+\frac{11\,d^7b}{384\,\left(bd^2x^2+ad^2\right)^5 a^2}(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/4096*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(1/2)-231/2560*d^11/(b*d^2*x^
2+a*d^2)^5/b*(d*x)^(5/2)+313/6144*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(9/2)+11/
384*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(13/2)+77/12288*d^5/(b*d^2*x^2+a*d^
2)^5/a^3*b^2*(d*x)^(17/2)+77/32768*d^3/a^4/b^2*(a*d^2/b)^(1/4)*2^(1/2)*ln((
d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+77/16384*d^3/a^4/b^2*(a*d^2/b)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+77/16384*d^3/a^4/b
^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50409, size = 1145, normalized size = 2.91

$$\frac{4620\left(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2\right)\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{11}b^7d^3\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac{3}{4}}-\sqrt{a^8b^4\sqrt{-\frac{d^{14}}{a^{15}b^9}}}}{d^{14}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(4620*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*
x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*arctan(-(sqrt(d*x)*
a^11*b^7*d^3*(-d^14/(a^15*b^9))^(3/4) - sqrt(a^8*b^4*sqrt(-d^14/(a^15*b^9))
 + d^7*x)*a^11*b^7*(-d^14/(a^15*b^9))^(3/4))/d^14) + 1155*(a^3*b^7*x^10 + 5
*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(
-d^14/(a^15*b^9))^(1/4)*log(77*a^4*b^2*(-d^14/(a^15*b^9))^(1/4) + 77*sqrt(d
*x)*d^3) - 1155*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4
*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*log(-77*a^4*b^2*(-
d^14/(a^15*b^9))^(1/4) + 77*sqrt(d*x)*d^3) + 4*(385*b^4*d^3*x^8 + 1760*a*b^
3*d^3*x^6 + 3130*a^2*b^2*d^3*x^4 - 5544*a^3*b*d^3*x^2 - 1155*a^4*d^3)*sqrt(
d*x))/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a
^7*b^3*x^2 + a^8*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26736, size = 467, normalized size = 1.19

$$\frac{1}{491520}d^2\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3}+\frac{1155}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^2*(2310*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 2310*sqrt(2)*(a
*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x
))/(a*d^2/b)^(1/4))/(a^4*b^3) + 1155*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x +
sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) - 1155*sqrt(2)
*(a*b^3*d^2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d
^2/b))/(a^4*b^3) + 8*(385*sqrt(d*x)*b^4*d^11*x^8 + 1760*sqrt(d*x)*a*b^3*d^1
1*x^6 + 3130*sqrt(d*x)*a^2*b^2*d^11*x^4 - 5544*sqrt(d*x)*a^3*b*d^11*x^2 - 1
155*sqrt(d*x)*a^4*d^11)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))
```

3.721 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=389

$$\frac{117d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{17/4}b^{7/4}}-\frac{117d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{17/4}b^{7/4}}-\frac{117d^{5/2}\tan^{-1}\left(1-\right.}{8192\sqrt{2}a^{17/4}b^{}}$$

```
[Out] -(d*(d*x)^(3/2))/(10*b*(a + b*x^2)^5) + (3*d*(d*x)^(3/2))/(160*a*b*(a + b*x
^2)^4) + (13*d*(d*x)^(3/2))/(640*a^2*b*(a + b*x^2)^3) + (117*d*(d*x)^(3/2))
/(5120*a^3*b*(a + b*x^2)^2) + (117*d*(d*x)^(3/2))/(4096*a^4*b*(a + b*x^2))
- (117*d^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(16384*Sqrt[2]*a^(17/4)*b^(7/4)) - (117*d^(5/2)*Log[Sqrt[a]*Sqrt[d] +
Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1
7/4)*b^(7/4))
```

Rubi [A] time = 0.455239, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{17/4}b^{7/4}}-\frac{117d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{17/4}b^{7/4}}-\frac{117d^{5/2}\tan^{-1}\left(1-\right.}{8192\sqrt{2}a^{17/4}b^{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(3/2))/(10*b*(a + b*x^2)^5) + (3*d*(d*x)^(3/2))/(160*a*b*(a + b*x
^2)^4) + (13*d*(d*x)^(3/2))/(640*a^2*b*(a + b*x^2)^3) + (117*d*(d*x)^(3/2))
/(5120*a^3*b*(a + b*x^2)^2) + (117*d*(d*x)^(3/2))/(4096*a^4*b*(a + b*x^2))
- (117*d^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(16384*Sqrt[2]*a^(17/4)*b^(7/4)) - (117*d^(5/2)*Log[Sqrt[a]*Sqrt[d] +
```

```
Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1
7/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6 \int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(3b^4d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{\left(39b^3d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{320a}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{\left(117b^2d^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{1280a^2}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} + \frac{\left(117b\right.}{}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} + \frac{11}{4096a}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} + \frac{11}{4096a}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} + \frac{11}{4096a}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} + \frac{11}{4096a}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} + \frac{11}{4096a}$$

$$= -\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5} + \frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4} + \frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2} + \frac{11}{4096a}$$

Mathematica [C] time = 0.0211441, size = 48, normalized size = 0.12

$$\frac{2d(dx)^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)}{a^5}-\frac{1}{(a+bx^2)^5}\right)}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-(a + b*x^2)^(-5) + Hypergeometric2F1[3/4, 6, 7/4, -((b*x
^2)/a)]/a^5))/(17*b)
```

Maple [A] time = 0.066, size = 341, normalized size = 0.9

$$-\frac{39\,d^{11}}{4096\,\left(bd^2x^2+ad^2\right)^5 b}(dx)^{\frac{3}{2}}+\frac{31\,d^9}{128\,\left(bd^2x^2+ad^2\right)^5 a}(dx)^{\frac{7}{2}}+\frac{533\,d^7b}{2048\,\left(bd^2x^2+ad^2\right)^5 a^2}(dx)^{\frac{11}{2}}+\frac{351\,d^5b^2}{2560\,\left(bd^2x^2+ad^2\right)^5 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -39/4096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(3/2)+31/128*d^9/(b*d^2*x^2+a*d^2
)^5/a*(d*x)^(7/2)+533/2048*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(11/2)+351/2
560*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(15/2)+117/4096*d^3/(b*d^2*x^2+a*
d^2)^5/a^4*b^3*(d*x)^(19/2)+117/32768*d^3/a^4/b^2/(a*d^2/b)^(1/4)*2^(1/2)*l
n((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+117/16384*d^3/a^4/b^2/(a*d^2/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+117/16384*d^3/
a^4/b^2/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-
1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4689, size = 1231, normalized size = 3.16

$$-\frac{2340\left(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b\right)\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613\,\sqrt{dx}a^4b^2d^7\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{1}{4}}-\sqrt{-25}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(2340*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*
x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*arctan(-1/1601613*(16
01613*sqrt(d*x)*a^4*b^2*d^7*(-d^10/(a^17*b^7))^(1/4) - sqrt(-2565164201769*
a^9*b^3*d^10*sqrt(-d^10/(a^17*b^7)) + 2565164201769*d^15*x)*a^4*b^2*(-d^10/
(a^17*b^7))^(1/4))/d^10) - 585*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x
^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(1
601613*a^13*b^5*(-d^10/(a^17*b^7))^(3/4) + 1601613*sqrt(d*x)*d^7) + 585*(a^
4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^
2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(-1601613*a^13*b^5*(-d^10/(a^17*b^7)
)^(3/4) + 1601613*sqrt(d*x)*d^7) - 4*(585*b^4*d^2*x^9 + 2808*a*b^3*d^2*x^7
+ 5330*a^2*b^2*d^2*x^5 + 4960*a^3*b*d^2*x^3 - 195*a^4*d^2*x)*sqrt(d*x))/(a^
4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^
2 + a^9*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20988, size = 460, normalized size = 1.18

$$\frac{1}{163840}d\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4}-\frac{585\sqrt{2}\left(ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d*(1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^4) + 1170*sqrt(2)*(a*b^3
*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^5*b^4) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(
a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^4) + 585*sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*
b^4) + 8*(585*sqrt(d*x)*b^4*d^11*x^9 + 2808*sqrt(d*x)*a*b^3*d^11*x^7 + 5330
*sqrt(d*x)*a^2*b^2*d^11*x^5 + 4960*sqrt(d*x)*a^3*b*d^11*x^3 - 195*sqrt(d*x)
*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a^4*b))
```

3.722 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=389

$$-\frac{231d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{19/4}b^{5/4}}+\frac{231d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{19/4}b^{5/4}}-\frac{231d^{3/2}\tan^{-1}\left(1\right.}{8192\sqrt{2}a^{19}}$$

```
[Out] -(d*Sqrt[d*x])/(10*b*(a + b*x^2)^5) + (d*Sqrt[d*x])/(160*a*b*(a + b*x^2)^4)
 + (d*Sqrt[d*x])/(128*a^2*b*(a + b*x^2)^3) + (11*d*Sqrt[d*x])/(1024*a^3*b*(
a + b*x^2)^2) + (77*d*Sqrt[d*x])/(4096*a^4*b*(a + b*x^2)) - (231*d^(3/2)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1
9/4)*b^(5/4)) + (231*d^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4
)*Sqrt[d])])/(8192*Sqrt[2]*a^(19/4)*b^(5/4)) - (231*d^(3/2)*Log[Sqrt[a]*Sqr
t[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[
2]*a^(19/4)*b^(5/4)) + (231*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(19/4)*b^(5/4))
```

Rubi [A] time = 0.498721, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{231d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{19/4}b^{5/4}}+\frac{231d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{19/4}b^{5/4}}-\frac{231d^{3/2}\tan^{-1}\left(1\right.}{8192\sqrt{2}a^{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] -(d*Sqrt[d*x])/(10*b*(a + b*x^2)^5) + (d*Sqrt[d*x])/(160*a*b*(a + b*x^2)^4)
 + (d*Sqrt[d*x])/(128*a^2*b*(a + b*x^2)^3) + (11*d*Sqrt[d*x])/(1024*a^3*b*(
a + b*x^2)^2) + (77*d*Sqrt[d*x])/(4096*a^4*b*(a + b*x^2)) - (231*d^(3/2)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1
9/4)*b^(5/4)) + (231*d^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4
)*Sqrt[d])])/(8192*Sqrt[2]*a^(19/4)*b^(5/4)) - (231*d^(3/2)*Log[Sqrt[a]*Sqr
t[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[
2]*a^(19/4)*b^(5/4)) + (231*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(19/4)*b^(5/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{1}{20}\left(b^4d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{\left(3b^3d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{64a}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{\left(11b^2d^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx}{256a^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} + \frac{\left(77bd^2\right)}{}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} + \frac{77}{4096a^4}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} + \frac{77}{4096a^4}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} + \frac{77}{4096a^4}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} + \frac{77}{4096a^4}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} + \frac{77}{4096a^4}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5} + \frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4} + \frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3} + \frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2} + \frac{77}{4096a^4}$$

Mathematica [A] time = 0.178583, size = 298, normalized size = 0.77

$$\frac{d\sqrt{dx}\left(\frac{3080\sqrt[4]{b}}{a^4(a+bx^2)}+\frac{1760\sqrt[4]{b}}{a^3(a+bx^2)^2}+\frac{1280\sqrt[4]{b}}{a^2(a+bx^2)^3}-\frac{1155\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{19/4}\sqrt{x}}+\frac{1155\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{19/4}\sqrt{x}}-\frac{2310\sqrt{2}\tan^{-1}\left(1-\right.}{a^{19/4}\sqrt{x}}\right.}{163840b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d*Sqrt[d*x]*((-16384*b^(1/4))/(a + b*x^2)^5 + (1024*b^(1/4))/(a*(a + b*x^2
)^4) + (1280*b^(1/4))/(a^2*(a + b*x^2)^3) + (1760*b^(1/4))/(a^3*(a + b*x^2)
^2) + (3080*b^(1/4))/(a^4*(a + b*x^2)) - (2310*Sqrt[2]*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(19/4)*Sqrt[x]) + (2310*Sqrt[2]*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(19/4)*Sqrt[x]) - (1155*Sqrt[2]*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(19/4)*Sqrt[x]) +
(1155*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
a^(19/4)*Sqrt[x])))/(163840*b^(5/4))
```

Maple [A] time = 0.064, size = 335, normalized size = 0.9

$$-\frac{231\,d^{11}}{4096\,\left(bd^2x^2+ad^2\right)^5 b}\sqrt{dx}+\frac{331\,d^9}{2560\,\left(bd^2x^2+ad^2\right)^5 a}(dx)^{\frac{5}{2}}+\frac{313\,d^7b}{2048\,\left(bd^2x^2+ad^2\right)^5 a^2}(dx)^{\frac{9}{2}}+\frac{11\,d^5b^2}{128\,\left(bd^2x^2+ad^2\right)^5 a^3}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -231/4096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(1/2)+331/2560*d^9/(b*d^2*x^2+a*
d^2)^5/a*(d*x)^(5/2)+313/2048*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(9/2)+11/
128*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(13/2)+77/4096*d^3/(b*d^2*x^2+a*d
^2)^5/a^4*b^3*(d*x)^(17/2)+231/32768*d/a^5/b*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*
x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+231/16384*d/a^5/b*(a*d^2/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+231/16384*d/a^5/b*(a*d^
2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45077, size = 1100, normalized size = 2.83

$$\frac{4620\left(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b\right)\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{14}b^4d\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{3}{4}}-\sqrt{a^{10}b^2\sqrt{-\frac{d^6}{a^{19}b^5}}+\cdots}}{d^6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(4620*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x
^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^14
*b^4*d*(-d^6/(a^19*b^5))^(3/4) - sqrt(a^10*b^2*sqrt(-d^6/(a^19*b^5)) + d^3*
x)*a^14*b^4*(-d^6/(a^19*b^5))^(3/4))/d^6) + 1155*(a^4*b^6*x^10 + 5*a^5*b^5*
x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*
b^5))^(1/4)*log(231*a^5*b*(-d^6/(a^19*b^5))^(1/4) + 231*sqrt(d*x)*d) - 1155
*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^
2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1/4)*log(-231*a^5*b*(-d^6/(a^19*b^5))^(1/
4) + 231*sqrt(d*x)*d) + 4*(385*b^4*d*x^8 + 1760*a*b^3*d*x^6 + 3130*a^2*b^2*
d*x^4 + 2648*a^3*b*d*x^2 - 1155*a^4*d)*sqrt(d*x))/(a^4*b^6*x^10 + 5*a^5*b^5
*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(a+bx^2\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**6, x)
```

Giac [A] time = 1.1737, size = 460, normalized size = 1.18

$$\frac{231\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^5b^2}+\frac{231\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^5b^2}+\frac{231\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 231/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2) + 231/16384*sqrt(2)*(a*b^3
*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^5*b^2) + 231/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x +
 sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2) - 231/32768*s
qrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sq
rt(a*d^2/b))/(a^5*b^2) + 1/20480*(385*sqrt(d*x)*b^4*d^11*x^8 + 1760*sqrt(d*
x)*a*b^3*d^11*x^6 + 3130*sqrt(d*x)*a^2*b^2*d^11*x^4 + 2648*sqrt(d*x)*a^3*b*
d^11*x^2 - 1155*sqrt(d*x)*a^4*d^11)/((b*d^2*x^2 + a*d^2)^5*a^4*b)
```

3.723 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=387

$$\frac{663\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\tan^{-1}\left(1-\right.}{8192\sqrt{2}a^{21/4}b^{3}}$$

```
[Out] (d*x)^(3/2)/(10*a*d*(a + b*x^2)^5) + (17*(d*x)^(3/2))/(160*a^2*d*(a + b*x^2
)^4) + (221*(d*x)^(3/2))/(1920*a^3*d*(a + b*x^2)^3) + (663*(d*x)^(3/2))/(51
20*a^4*d*(a + b*x^2)^2) + (663*(d*x)^(3/2))/(4096*a^5*d*(a + b*x^2)) - (663
*Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*S
qrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(21/4)*b^(3/4)) - (663*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(21/4)*b
^(3/4))
```

Rubi [A] time = 0.490223, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{663\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\tan^{-1}\left(1-\right.}{8192\sqrt{2}a^{21/4}b^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d*x)^(3/2)/(10*a*d*(a + b*x^2)^5) + (17*(d*x)^(3/2))/(160*a^2*d*(a + b*x^2
)^4) + (221*(d*x)^(3/2))/(1920*a^3*d*(a + b*x^2)^3) + (663*(d*x)^(3/2))/(51
20*a^4*d*(a + b*x^2)^2) + (663*(d*x)^(3/2))/(4096*a^5*d*(a + b*x^2)) - (663
*Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*S
qrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(21/4)*b^(3/4)) - (663*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
```

```
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(21/4)*b
^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{\left(17b^5\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{\left(221b^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{320a^2}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{\left(663b^3\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{1280a^3}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2} + \frac{(663\ldots}{}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2} + \frac{}{409\ldots}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2} + \frac{}{409\ldots}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2} + \frac{}{409\ldots}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2} + \frac{}{409\ldots}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2} + \frac{}{409\ldots}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2} + \frac{}{409\ldots}$$

Mathematica [C] time = 0.0062739, size = 32, normalized size = 0.08

$$\frac{2x\sqrt{dx}\,{}_2F_1\left(\frac{3}{4},6;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*Sqrt[d*x]*Hypergeometric2F1[3/4, 6, 7/4, -((b*x^2)/a)])/(3*a^6)
```

Maple [A] time = 0.067, size = 336, normalized size = 0.9

$$\frac{7529\,d^9}{12288\left(bd^2x^2+ad^2\right)^5 a}(dx)^{\frac{3}{2}}+\frac{527\,d^7b}{384\left(bd^2x^2+ad^2\right)^5 a^2}(dx)^{\frac{7}{2}}+\frac{9061\,d^5b^2}{6144\left(bd^2x^2+ad^2\right)^5 a^3}(dx)^{\frac{11}{2}}+\frac{1989\,d^3b^3}{2560\left(bd^2x^2+ad^2\right)^5 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 7529/12288*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(3/2)+527/384*d^7/(b*d^2*x^2+a*d
^2)^5/a^2*b*(d*x)^(7/2)+9061/6144*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(11
/2)+1989/2560*d^3/(b*d^2*x^2+a*d^2)^5/a^4*b^3*(d*x)^(15/2)+663/4096*d/(b*d^
2*x^2+a*d^2)^5/a^5*b^4*(d*x)^(19/2)+663/32768*d/a^5/b/(a*d^2/b)^(1/4)*2^(1/
2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+663/16384*d/a^5/b/(a*d^2/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+663/16384*d/a^
5/b/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47173, size = 1199, normalized size = 3.1

$$-\frac{39780 \left(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10}\right)\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{1}{4}} \arctan\left(-\frac{291434247\,\sqrt{dx}a^5bd\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{1}{4}}-\sqrt{-84}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(39780*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^
2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*arctan(-1/291434247*(29
1434247*sqrt(d*x)*a^5*b*d*(-d^2/(a^21*b^3))^(1/4) - sqrt(-84933920324457009
*a^11*b*d^2*sqrt(-d^2/(a^21*b^3)) + 84933920324457009*d^3*x)*a^5*b*(-d^2/(a
^21*b^3))^(1/4))/d^2) - 9945*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6
 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*log(2914342
47*a^16*b^2*(-d^2/(a^21*b^3))^(3/4) + 291434247*sqrt(d*x)*d) + 9945*(a^5*b^
5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^
10)*(-d^2/(a^21*b^3))^(1/4)*log(-291434247*a^16*b^2*(-d^2/(a^21*b^3))^(3/4)
 + 291434247*sqrt(d*x)*d) - 4*(9945*b^4*x^9 + 47736*a*b^3*x^7 + 90610*a^2*b
^2*x^5 + 84320*a^3*b*x^3 + 37645*a^4*x)*sqrt(d*x))/(a^5*b^5*x^10 + 5*a^6*b^
4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2259, size = 468, normalized size = 1.21

$$\frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6b^3d}+\frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6b^3d}-\frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(a\right.}{327}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 663/16384*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b^3*d) + 663/16384*sqrt(2)*(a*b^3
*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^6*b^3*d) - 663/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^3*d) + 663/32768*s
qrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(a^6*b^3*d) + 1/61440*(9945*sqrt(d*x)*b^4*d^10*x^9 + 47736*sqrt(
d*x)*a*b^3*d^10*x^7 + 90610*sqrt(d*x)*a^2*b^2*d^10*x^5 + 84320*sqrt(d*x)*a^
3*b*d^10*x^3 + 37645*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^5)
```

3.724 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=387

$$\frac{1463\sqrt{dx}}{4096a^5d\left(a+bx^2\right)}+\frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2}+\frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3}+\frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4}-\frac{4389\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{\ldots}\right.}{16384\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}\ldots}$$

```
[Out] Sqrt[d*x]/(10*a*d*(a + b*x^2)^5) + (19*Sqrt[d*x])/(160*a^2*d*(a + b*x^2)^4)
 + (19*Sqrt[d*x])/(128*a^3*d*(a + b*x^2)^3) + (209*Sqrt[d*x])/(1024*a^4*d*(
a + b*x^2)^2) + (1463*Sqrt[d*x])/(4096*a^5*d*(a + b*x^2)) - (4389*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(23/4)*b^
(1/4)*Sqrt[d]) + (4389*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(8192*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d]) - (4389*Log[Sqrt[a]*Sqrt[d]
+ Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^
(23/4)*b^(1/4)*Sqrt[d]) + (4389*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d])
```

Rubi [A] time = 0.495858, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{1463\sqrt{dx}}{4096a^5d\left(a+bx^2\right)}+\frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2}+\frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3}+\frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4}-\frac{4389\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{\ldots}\right.}{16384\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] Sqrt[d*x]/(10*a*d*(a + b*x^2)^5) + (19*Sqrt[d*x])/(160*a^2*d*(a + b*x^2)^4)
 + (19*Sqrt[d*x])/(128*a^3*d*(a + b*x^2)^3) + (209*Sqrt[d*x])/(1024*a^4*d*(
a + b*x^2)^2) + (1463*Sqrt[d*x])/(4096*a^5*d*(a + b*x^2)) - (4389*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(23/4)*b^
(1/4)*Sqrt[d]) + (4389*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(8192*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d]) - (4389*Log[Sqrt[a]*Sqrt[d]
+ Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^
(23/4)*b^(1/4)*Sqrt[d]) + (4389*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{\left(19b^5\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{\left(57b^4\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{64a^2}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{\left(209b^3\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}}{256a^3}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2} +$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2} +$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2} +$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2} +$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2} +$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2} +$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{dx}}{1024a^4d\left(a+bx^2\right)^2} +$$

Mathematica [A] time = 0.162617, size = 295, normalized size = 0.76

$$\frac{\sqrt{x}\left(\frac{16384a^{19/4}\sqrt{x}}{(a+bx^2)^5}+\frac{19456a^{15/4}\sqrt{x}}{(a+bx^2)^4}+\frac{24320a^{11/4}\sqrt{x}}{(a+bx^2)^3}+\frac{33440a^{7/4}\sqrt{x}}{(a+bx^2)^2}+\frac{58520a^{3/4}\sqrt{x}}{a+bx^2}-\frac{21945\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{21945\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{\ldots}\right.}{\ldots}\right.}{163840a^{23/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (Sqrt[x]*((16384*a^(19/4)*Sqrt[x])/(a + b*x^2)^5 + (19456*a^(15/4)*Sqrt[x])
/(a + b*x^2)^4 + (24320*a^(11/4)*Sqrt[x])/(a + b*x^2)^3 + (33440*a^(7/4)*Sq
rt[x])/(a + b*x^2)^2 + (58520*a^(3/4)*Sqrt[x])/(a + b*x^2) - (43890*Sqrt[2]
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (43890*Sqrt[2]*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (21945*Sqrt[2]*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (21945*Sqr
t[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/
(163840*a^(23/4)*Sqrt[d*x])
```

Maple [A] time = 0.069, size = 333, normalized size = 0.9

$$\frac{3803\,d^9}{4096\left(bd^2x^2+ad^2\right)^5a}\sqrt{dx}+\frac{6289\,d^7b}{2560\left(bd^2x^2+ad^2\right)^5a^2}(dx)^{\frac{5}{2}}+\frac{5947\,d^5b^2}{2048\left(bd^2x^2+ad^2\right)^5a^3}(dx)^{\frac{9}{2}}+\frac{209\,d^3b^3}{128\left(bd^2x^2+ad^2\right)^5a^4}(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x)
```

```
[Out] 3803/4096*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(1/2)+6289/2560*d^7/(b*d^2*x^2+a*
d^2)^5/a^2*b*(d*x)^(5/2)+5947/2048*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(9
/2)+209/128*d^3/(b*d^2*x^2+a*d^2)^5/a^4*b^3*(d*x)^(13/2)+1463/4096*d/(b*d^2
*x^2+a*d^2)^5/a^5*b^4*(d*x)^(17/2)+4389/32768/d/a^6*(a*d^2/b)^(1/4)*2^(1/2)
*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+4389/16384/d/a^6*(a*d^2/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+4389/16384/d/a^6*
(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47251, size = 1104, normalized size = 2.85

$$\frac{87780\left(a^5b^5dx^{10}+5\,a^6b^4dx^8+10\,a^7b^3dx^6+10\,a^8b^2dx^4+5\,a^9bdx^2+a^{10}d\right)\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^{12}d^2\sqrt{-\frac{1}{a^{23}bd^2}}+dxa^{17}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/81920*(87780*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^
8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1/(a^23*b*d^2))^(1/4)*arctan(sqrt(a
^12*d^2*sqrt(-1/(a^23*b*d^2)) + d*x)*a^17*b*d*(-1/(a^23*b*d^2))^(3/4) - sqr
t(d*x)*a^17*b*d*(-1/(a^23*b*d^2))^(3/4)) + 21945*(a^5*b^5*d*x^10 + 5*a^6*b^
4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1
/(a^23*b*d^2))^(1/4)*log(a^6*d*(-1/(a^23*b*d^2))^(1/4) + sqrt(d*x)) - 21945
*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 +
5*a^9*b*d*x^2 + a^10*d)*(-1/(a^23*b*d^2))^(1/4)*log(-a^6*d*(-1/(a^23*b*d^2)
)^(1/4) + sqrt(d*x)) + 4*(7315*b^4*x^8 + 33440*a*b^3*x^6 + 59470*a^2*b^2*x^
4 + 50312*a^3*b*x^2 + 19015*a^4)*sqrt(d*x))/(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x
^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27412, size = 467, normalized size = 1.21

$$\frac{4389\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6bd}+\frac{4389\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6bd}+\frac{4389\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4389/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d) + 4389/16384*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^6*b*d) + 4389/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b*d) - 4389/32768*sqr
t(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^6*b*d) + 1/20480*(7315*sqrt(d*x)*b^4*d^9*x^8 + 33440*sqrt(d*x)*
a*b^3*d^9*x^6 + 59470*sqrt(d*x)*a^2*b^2*d^9*x^4 + 50312*sqrt(d*x)*a^3*b*d^9
*x^2 + 19015*sqrt(d*x)*a^4*d^9)/((b*d^2*x^2 + a*d^2)^5*a^5)
```

3.725 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=404

$$-\frac{13923\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\tan^{-}}{8192\sqrt{2}}$$

```
[Out] -13923/(4096*a^6*d*Sqrt[d*x]) + 1/(10*a*d*Sqrt[d*x]*(a + b*x^2)^5) + 21/(16
0*a^2*d*Sqrt[d*x]*(a + b*x^2)^4) + 119/(640*a^3*d*Sqrt[d*x]*(a + b*x^2)^3)
+ 1547/(5120*a^4*d*Sqrt[d*x]*(a + b*x^2)^2) + 13923/(20480*a^5*d*Sqrt[d*x]*
(a + b*x^2)) + (13923*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/2)) - (13923*b^(1/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/
2)) - (13923*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(25/4)*d^(3/2)) + (13923*b^(1/4)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(16384*Sqrt[2]*a^(25/4)*d^(3/2))
```

Rubi [A] time = 0.529383, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{13923\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\tan^{-}}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```


```
[Out] -13923/(4096*a^6*d*Sqrt[d*x]) + 1/(10*a*d*Sqrt[d*x]*(a + b*x^2)^5) + 21/(16
0*a^2*d*Sqrt[d*x]*(a + b*x^2)^4) + 119/(640*a^3*d*Sqrt[d*x]*(a + b*x^2)^3)
+ 1547/(5120*a^4*d*Sqrt[d*x]*(a + b*x^2)^2) + 13923/(20480*a^5*d*Sqrt[d*x]*
(a + b*x^2)) + (13923*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/2)) - (13923*b^(1/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/
2)) - (13923*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(25/4)*d^(3/2)) + (13923*b^(1/4)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
```

```
)/(16384*Sqrt[2]*a^(25/4)*d^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{\left(21b^5\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{\left(357b^4\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^4}\,dx}{320a^2}$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a+bx^2\right)^3} + \frac{(1547b \cdots}{\cdots}$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a+bx^2\right)^3} + \frac{\cdots}{5120a^4 \cdots}$$

$$= \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a+bx^2\right)^3} + \frac{\cdots}{5120a^4 \cdots}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a \cdots\right.}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a \cdots\right.}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a \cdots\right.}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a \cdots\right.}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a \cdots\right.}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\left(a \cdots\right.}$$

Mathematica [C] time = 0.0133965, size = 30, normalized size = 0.07

$$-\frac{2x\,{}_2F_1\left(-\frac{1}{4},6;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^6(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-1/4, 6, 3/4, -((b*x^2)/a)])/(a^6*(d*x)^(3/2))
```

Maple [A] time = 0.076, size = 349, normalized size = 0.9

$$-2\frac{1}{a^6d\sqrt{dx}}-\frac{11743\,d^7b}{4096\,a^2\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{3}{2}}-\frac{1129\,d^5b^2}{128\,a^3\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{7}{2}}-\frac{22467\,d^3b^3}{2048\,a^4\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{11}{2}}-\frac{1616}{2560\,a^5\left(bd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -2/a^6/d/(d*x)^(1/2)-11743/4096*d^7*b/a^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(3/2)-1
129/128*d^5*b^2/a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(7/2)-22467/2048*d^3*b^3/a^4/
(b*d^2*x^2+a*d^2)^5*(d*x)^(11/2)-16169/2560*d*b^4/a^5/(b*d^2*x^2+a*d^2)^5*(
d*x)^(15/2)-5731/4096/d*b^5/a^6/(b*d^2*x^2+a*d^2)^5*(d*x)^(19/2)-13923/3276
8/d/a^6/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
)-13923/16384/d/a^6/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*
(d*x)^(1/2)+1)-13923/16384/d/a^6/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75432, size = 1411, normalized size = 3.49

$$\frac{278460\left(a^{6}b^{5}d^{2}x^{11}+5\,a^{7}b^{4}d^{2}x^{9}+10\,a^{8}b^{3}d^{2}x^{7}+10\,a^{9}b^{2}d^{2}x^{5}+5\,a^{10}bd^{2}x^{3}+a^{11}d^{2}x\right)\left(-\frac{b}{a^{25}d^{6}}\right)^{\frac{1}{4}}\arctan\left(-\frac{2698972561467\,\sqrt{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(278460*(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7
+ 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)
*arctan(-1/2698972561467*(2698972561467*sqrt(d*x)*a^6*b*d*(-b/(a^25*d^6))^(
1/4) - sqrt(-7284452887551739093192089*a^13*b*d^4*sqrt(-b/(a^25*d^6)) + 728
4452887551739093192089*b^2*d*x)*a^6*d*(-b/(a^25*d^6))^(1/4))/b) - 69615*(a^
6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^
5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)*log(2698972561467*
a^19*d^5*(-b/(a^25*d^6))^(3/4) + 2698972561467*sqrt(d*x)*b) + 69615*(a^6*b^
5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^5 +
5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)*log(-2698972561467*a^1
9*d^5*(-b/(a^25*d^6))^(3/4) + 2698972561467*sqrt(d*x)*b) - 4*(69615*b^5*x^1
0 + 334152*a*b^4*x^8 + 634270*a^2*b^3*x^6 + 590240*a^3*b^2*x^4 + 263515*a^4
*b*x^2 + 40960*a^5)*sqrt(d*x))/(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a
^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2959, size = 493, normalized size = 1.22

$$-\frac{\frac{327680}{\sqrt{dx}a^6} + \frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^7b^2d^2} + \frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^7b^2d^2} - \frac{69615\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\cdots\right.\right.}{a^7b^2d^2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/163840*(327680/(sqrt(d*x)*a^6) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*
b^2*d^2) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b^2*d^2) - 69615*sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^7*b^2*d^2) + 69615*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*b^2*d^2) + 8*(28655*sqrt(d*x)*b
^5*d^9*x^9 + 129352*sqrt(d*x)*a*b^4*d^9*x^7 + 224670*sqrt(d*x)*a^2*b^3*d^9*
x^5 + 180640*sqrt(d*x)*a^3*b^2*d^9*x^3 + 58715*sqrt(d*x)*a^4*b*d^9*x)/((b*d
^2*x^2 + a*d^2)^5*a^6))/d
```

3.726 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=404

$$\frac{33649b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{27/4}d^{5/2}}+\frac{33649b^{3/4}\tan}{8192}$$

```
[Out] -33649/(12288*a^6*d*(d*x)^(3/2)) + 1/(10*a*d*(d*x)^(3/2)*(a + b*x^2)^5) + 2
3/(160*a^2*d*(d*x)^(3/2)*(a + b*x^2)^4) + 437/(1920*a^3*d*(d*x)^(3/2)*(a +
b*x^2)^3) + 437/(1024*a^4*d*(d*x)^(3/2)*(a + b*x^2)^2) + 4807/(4096*a^5*d*(
d*x)^(3/2)*(a + b*x^2)) + (33649*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])]])/(8192*Sqrt[2]*a^(27/4)*d^(5/2)) - (33649*b^(3/4)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]])/(8192*Sqrt[2]*a^(
27/4)*d^(5/2)) + (33649*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(5/2)) - (3364
9*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(5/2))
```

Rubi [A] time = 0.509242, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{33649b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{27/4}d^{5/2}}+\frac{33649b^{3/4}\tan}{8192}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```


```
[Out] -33649/(12288*a^6*d*(d*x)^(3/2)) + 1/(10*a*d*(d*x)^(3/2)*(a + b*x^2)^5) + 2
3/(160*a^2*d*(d*x)^(3/2)*(a + b*x^2)^4) + 437/(1920*a^3*d*(d*x)^(3/2)*(a +
b*x^2)^3) + 437/(1024*a^4*d*(d*x)^(3/2)*(a + b*x^2)^2) + 4807/(4096*a^5*d*(
d*x)^(3/2)*(a + b*x^2)) + (33649*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])]])/(8192*Sqrt[2]*a^(27/4)*d^(5/2)) - (33649*b^(3/4)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]])/(8192*Sqrt[2]*a^(
27/4)*d^(5/2)) + (33649*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(5/2)) - (3364
9*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)
```

```
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{\left(23b^5\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{\left(437b^4\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^4}\,dx}{320a^2}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\left(a+bx^2\right)^3} + \cdots$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\left(a+bx^2\right)^3} + \cdots$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\left(a+bx^2\right)^3} + \cdots$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{\cdots}{1920a^3d(\cdots}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{\cdots}{1920a^3d(\cdots}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{\cdots}{1920a^3d(\cdots}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{\cdots}{1920a^3d(\cdots}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{\cdots}{1920a^3d(\cdots}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{\cdots}{1920a^3d(\cdots}$$

Mathematica [C] time = 0.0151481, size = 32, normalized size = 0.08

$$-\frac{2x\,{}_2F_1\left(-\frac{3}{4},6;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^6(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-2*x*Hypergeometric2F1[-3/4, 6, 1/4, -((b*x^2)/a)])/(3*a^6*(d*x)^(5/2))
```

Maple [A] time = 0.074, size = 352, normalized size = 0.9

$$-\frac{2}{3\,a^6 d}(dx)^{-\frac{3}{2}}-\frac{15503\,d^7 b}{4096\,a^2\left(bd^2x^2+ad^2\right)^5}\sqrt{dx}-\frac{31149\,d^5b^2}{2560\,a^3\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{5}{2}}-\frac{95821\,d^3b^3}{6144\,a^4\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{9}{2}}-\frac{3}{384\,a^5\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -2/3/a^6/d/(d*x)^(3/2)-15503/4096*d^7/a^2*b/(b*d^2*x^2+a*d^2)^5*(d*x)^(1/2)
-31149/2560*d^5/a^3*b^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(5/2)-95821/6144*d^3/a^4*
b^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(9/2)-3527/384*d/a^5*b^4/(b*d^2*x^2+a*d^2)^5*
(d*x)^(13/2)-25457/12288/d/a^6*b^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(17/2)-33649/3
2768/d^3/a^7*b*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))-33649/16384/d^3/a^7*b*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^
2/b)^(1/4)*(d*x)^(1/2)+1)-33649/16384/d^3/a^7*b*(a*d^2/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74076, size = 1320, normalized size = 3.27

$$-\frac{2018940\left(a^{6}b^{5}d^{3}x^{12}+5\,a^{7}b^{4}d^{3}x^{10}+10\,a^{8}b^{3}d^{3}x^{8}+10\,a^{9}b^{2}d^{3}x^{6}+5\,a^{10}bd^{3}x^{4}+a^{11}d^{3}x^{2}\right)\left(-\frac{b^{3}}{a^{27}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{20}bd^{7}\left(-\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(2018940*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*
x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^1
0))^(1/4)*arctan(-(sqrt(d*x)*a^20*b*d^7*(-b^3/(a^27*d^10))^(3/4) - sqrt(a^1
4*d^6*sqrt(-b^3/(a^27*d^10)) + b^2*d*x)*a^20*d^7*(-b^3/(a^27*d^10))^(3/4))/
b^3) + 504735*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*x^8 +
 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^10))^(
1/4)*log(33649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d*x)*b) - 5047
35*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*x^8 + 10*a^9*b^2
*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^10))^(1/4)*log(-3
3649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d*x)*b) + 4*(168245*b^5*
x^10 + 769120*a*b^4*x^8 + 1367810*a^2*b^3*x^6 + 1157176*a^3*b^2*x^4 + 43734
5*a^4*b*x^2 + 40960*a^5)*sqrt(d*x))/(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10
+ 10*a^8*b^3*d^3*x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32536, size = 481, normalized size = 1.19

$$-\frac{33649\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}-\frac{33649\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}-\frac{33649\sqrt{2}\left(ab^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -33649/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/16384*sqrt(2)*(a*
b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x
+ sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*d^3) + 33649/3276
8*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a^7*d^3) - 2/3/(sqrt(d*x)*a^6*d^2*x) - 1/61440*(127285*sqrt(
d*x)*b^5*d^8*x^8 + 564320*sqrt(d*x)*a*b^4*d^8*x^6 + 958210*sqrt(d*x)*a^2*b^
3*d^8*x^4 + 747576*sqrt(d*x)*a^3*b^2*d^8*x^2 + 232545*sqrt(d*x)*a^4*b*d^8)/
((b*d^2*x^2 + a*d^2)^5*a^6*d)
```

3.727 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=422

$$\frac{69615b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\tan^{-}}{8192\sqrt{2}}$$

```
[Out] -13923/(4096*a^6*d*(d*x)^(5/2)) + (69615*b)/(4096*a^7*d^3*Sqrt[d*x]) + 1/(1
0*a*d*(d*x)^(5/2)*(a + b*x^2)^5) + 5/(32*a^2*d*(d*x)^(5/2)*(a + b*x^2)^4) +
 35/(128*a^3*d*(d*x)^(5/2)*(a + b*x^2)^3) + 595/(1024*a^4*d*(d*x)^(5/2)*(a
+ b*x^2)^2) + 7735/(4096*a^5*d*(d*x)^(5/2)*(a + b*x^2)) - (69615*b^(5/4)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(2
9/4)*d^(7/2)) + (69615*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(29/4)*d^(7/2)) + (69615*b^(5/4)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*S
qrt[2]*a^(29/4)*d^(7/2)) - (69615*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqr
t[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(29/4)*d^(7/2
))
```

Rubi [A] time = 0.554436, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{69615b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\tan^{-}}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -13923/(4096*a^6*d*(d*x)^(5/2)) + (69615*b)/(4096*a^7*d^3*Sqrt[d*x]) + 1/(1
0*a*d*(d*x)^(5/2)*(a + b*x^2)^5) + 5/(32*a^2*d*(d*x)^(5/2)*(a + b*x^2)^4) +
 35/(128*a^3*d*(d*x)^(5/2)*(a + b*x^2)^3) + 595/(1024*a^4*d*(d*x)^(5/2)*(a
+ b*x^2)^2) + 7735/(4096*a^5*d*(d*x)^(5/2)*(a + b*x^2)) - (69615*b^(5/4)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(2
9/4)*d^(7/2)) + (69615*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(29/4)*d^(7/2)) + (69615*b^(5/4)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*S
```

```
qrt[2]*a^(29/4)*d^(7/2)) - (69615*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqr
t[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(29/4)*d^(7/2
))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{\left(5b^5\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^5}\,dx}{4a}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{\left(105b^4\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^4}\,dx}{64a^2}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}\left(a+bx^2\right)^3} + \cdots$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}\left(a+bx^2\right)^3} + \cdots$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}\left(a+bx^2\right)^3} + \cdots$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \cdots$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+b\cdots\right.}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+b\cdots\right.}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+b\cdots\right.}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+b\cdots\right.}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+b\cdots\right.}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+b\cdots\right.}$$

Mathematica [C] time = 0.0139302, size = 37, normalized size = 0.09

$$-\frac{2\sqrt{dx}\,{}_2F_1\left(-\frac{5}{4},6;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^6d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (-2*Sqrt[d*x]*Hypergeometric2F1[-5/4, 6, -1/4, -((b*x^2)/a)])/(5*a^6*d^4*x^
3)
```

Maple [A] time = 0.077, size = 368, normalized size = 0.9

$$-\frac{2}{5\,a^6d}(dx)^{-\frac{5}{2}}+12\,\frac{b}{a^7d^3\sqrt{dx}}+\frac{34139\,d^5b^2}{4096\,a^3\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{3}{2}}+\frac{3597\,d^3b^3}{128\,a^4\left(bd^2x^2+ad^2\right)^5}(dx)^{\frac{7}{2}}+\frac{75471\,b^4d}{2048\,a^5\left(bd^2x^2+ad^2\right)^5}(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -2/5/a^6/d/(d*x)^(5/2)+12*b/a^7/d^3/(d*x)^(1/2)+34139/4096*d^5*b^2/a^3/(b*d
^2*x^2+a*d^2)^5*(d*x)^(3/2)+3597/128*d^3*b^3/a^4/(b*d^2*x^2+a*d^2)^5*(d*x)^
(7/2)+75471/2048*d*b^4/a^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(11/2)+56269/2560/d*b^
5/a^6/(b*d^2*x^2+a*d^2)^5*(d*x)^(15/2)+20463/4096/d^3*b^6/a^7/(b*d^2*x^2+a*
d^2)^5*(d*x)^(19/2)+69615/32768/d^3*b/a^7/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+69615/16384/d^3*b/a^7/(a*d^2/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+69615/16384/d^3*b/a^7/
(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71876, size = 1557, normalized size = 3.69

$$-\frac{1392300\left(a^{7}b^{5}d^{4}x^{13}+5\,a^{8}b^{4}d^{4}x^{11}+10\,a^{9}b^{3}d^{4}x^{9}+10\,a^{10}b^{2}d^{4}x^{7}+5\,a^{11}bd^{4}x^{5}+a^{12}d^{4}x^{3}\right)\left(-\frac{b^{5}}{a^{29}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{33737157\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(1392300*(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x
^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^4*x^3)*(-b^5/(a^29*d^1
4))^(1/4)*arctan(-1/337371570183375*(337371570183375*sqrt(d*x)*a^7*b^4*d^3*
(-b^5/(a^29*d^14))^(1/4) - sqrt(-113819576367995923331126390625*a^15*b^5*d^
8*sqrt(-b^5/(a^29*d^14)) + 113819576367995923331126390625*b^8*d*x)*a^7*d^3*
(-b^5/(a^29*d^14))^(1/4))/b^5) - 348075*(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x
^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^
4*x^3)*(-b^5/(a^29*d^14))^(1/4)*log(337371570183375*a^22*d^11*(-b^5/(a^29*d
^14))^(3/4) + 337371570183375*sqrt(d*x)*b^4) + 348075*(a^7*b^5*d^4*x^13 + 5
*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4
*x^5 + a^12*d^4*x^3)*(-b^5/(a^29*d^14))^(1/4)*log(-337371570183375*a^22*d^1
1*(-b^5/(a^29*d^14))^(3/4) + 337371570183375*sqrt(d*x)*b^4) - 4*(348075*b^6
*x^12 + 1670760*a*b^5*x^10 + 3171350*a^2*b^4*x^8 + 2951200*a^3*b^3*x^6 + 13
17575*a^4*b^2*x^4 + 204800*a^5*b*x^2 - 8192*a^6)*sqrt(d*x))/(a^7*b^5*d^4*x^
13 + 5*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11
*b*d^4*x^5 + a^12*d^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35482, size = 489, normalized size = 1.16

$$\frac{69615\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^8bd^5}+\frac{69615\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^8bd^5}-\frac{69615\sqrt{2}\left(ab^3d^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 69615/16384*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^8*b*d^5) + 69615/16384*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^8*b*d^5) - 69615/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d
*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^8*b*d^5) + 69615
/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/(a^8*b*d^5) + 1/20480*(348075*b^6*d^12*x^12 + 1670760*a*
b^5*d^12*x^10 + 3171350*a^2*b^4*d^12*x^8 + 2951200*a^3*b^3*d^12*x^6 + 13175
75*a^4*b^2*d^12*x^4 + 204800*a^5*b*d^12*x^2 - 8192*a^6*d^12)/((sqrt(d*x)*b*
d^2*x^2 + sqrt(d*x)*a*d^2)^5*a^7*d^3)
```

3.728 $\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

```
[Out] (2*a*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (2*b*
(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2))
```

Rubi [A] time = 0.0297022, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (2*b*
(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(ab+b^2x^2\right)dx}{ab+b^2x^2} \\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^{5/2}+\frac{b^2(dx)^{9/2}}{d^2}\right)dx}{ab+b^2x^2} \\ &= \frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0207229, size = 44, normalized size = 0.47

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(11a+7bx^2\right)}{77\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(11*a + 7*b*x^2))/(77*(a + b*x^2))
```

Maple [A] time = 0.044, size = 39, normalized size = 0.4

$$\frac{2\left(7\,bx^2+11\,a\right)x}{77\,bx^2+77\,a}(dx)^{\frac{5}{2}}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 2/77*x*(7*b*x^2+11*a)*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 1.01641, size = 30, normalized size = 0.32

$$\frac{2}{77}\left(7\,bd^{\frac{5}{2}}x^3+11\,ad^{\frac{5}{2}}x\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/77*(7*b*d^(5/2)*x^3 + 11*a*d^(5/2)*x)*x^(5/2)
```

Fricas [A] time = 1.26214, size = 61, normalized size = 0.66

$$\frac{2}{77}\left(7\,bd^2x^5+11\,ad^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b*d^2*x^5 + 11*a*d^2*x^3)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16491, size = 61, normalized size = 0.66

$$\frac{2}{11}\sqrt{dx}bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}ad^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/11*sqrt(d*x)*b*d^2*x^5*sgn(b*x^2 + a) + 2/7*sqrt(d*x)*a*d^2*x^3*sgn(b*x^2
 + a)
```

3.729 $\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}$$

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (2*b*
(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a + b*x^2))
```

Rubi [A] time = 0.0288275, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (2*b*
(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{3/2}\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^{3/2}+\frac{b^2(dx)^{7/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.013489, size = 44, normalized size = 0.47

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(9a+5bx^2\right)}{45\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(9*a + 5*b*x^2))/(45*(a + b*x^2))
```

Maple [A] time = 0.046, size = 39, normalized size = 0.4

$$\frac{2\left(5\,bx^2+9\,a\right)x}{45\,bx^2+45\,a}(dx)^{\frac{3}{2}}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 2/45*x*(5*b*x^2+9*a)*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 1.00629, size = 30, normalized size = 0.32

$$\frac{2}{45}\left(5\,bd^{\frac{3}{2}}x^3+9\,ad^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(5*b*d^(3/2)*x^3 + 9*a*d^(3/2)*x)*x^(3/2)
```

Fricas [A] time = 1.32534, size = 54, normalized size = 0.58

$$\frac{2}{45}\left(5\,bdx^4+9\,adx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b*d*x^4 + 9*a*d*x^2)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2685, size = 55, normalized size = 0.59

$$\frac{2}{9}\sqrt{dx}bdx^4\text{sgn}\left(bx^2+a\right)+\frac{2}{5}\sqrt{dx}adx^2\text{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(d*x)*b*d*x^4*sgn(b*x^2 + a) + 2/5*sqrt(d*x)*a*d*x^2*sgn(b*x^2 + a)
```

3.730 $\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}$$

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (2*b*
(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2))
```

Rubi [A] time = 0.0290742, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (2*b*
(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab\sqrt{dx}+\frac{b^2(dx)^{5/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} + \frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0132072, size = 44, normalized size = 0.47

$$\frac{2\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}\left(7ax+3bx^3\right)}{21\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(7*a*x + 3*b*x^3))/(21*(a + b*x^2))
```

Maple [A] time = 0.043, size = 39, normalized size = 0.4

$$\frac{2\left(3\,bx^2+7\,a\right)x}{21\,bx^2+21\,a}\sqrt{dx}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 2/21*x*(3*b*x^2+7*a)*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 1.03298, size = 30, normalized size = 0.32

$$\frac{2}{21}\left(3\,b\sqrt{d}x^3+7\,a\sqrt{d}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(3*b*sqrt(d)*x^3 + 7*a*sqrt(d)*x)*sqrt(x)
```

Fricas [A] time = 1.25723, size = 46, normalized size = 0.49

$$\frac{2}{21}\left(3\,bx^3+7\,ax\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b*x^3 + 7*a*x)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19223, size = 59, normalized size = 0.63

$$\frac{2\left(3\,\sqrt{dx}bdx^3\mathrm{sgn}\left(bx^2+a\right)+7\,\sqrt{dx}adx\mathrm{sgn}\left(bx^2+a\right)\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*(3*sqrt(d*x)*b*d*x^3*sgn(b*x^2 + a) + 7*sqrt(d*x)*a*d*x*sgn(b*x^2 + a)
)/d
```

3.731 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=91

$$\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}$$

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (2*b*(d*x
)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*x^2))
```

Rubi [A] time = 0.0283024, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (2*b*(d*x
)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{\sqrt{dx}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{\sqrt{dx}}+\frac{b^2(dx)^{3/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)} + \frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0125833, size = 43, normalized size = 0.47

$$\frac{2\sqrt{\left(a+bx^2\right)^2}\left(5ax+bx^3\right)}{5\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]

[Out] (2*Sqrt[(a + b*x^2)^2]*(5*a*x + b*x^3))/(5*Sqrt[d*x]*(a + b*x^2))
```

Maple [A] time = 0.042, size = 38, normalized size = 0.4

$$\frac{2\left(bx^2+5a\right)x}{5bx^2+5a}\sqrt{\left(bx^2+a\right)^2}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x)

[Out] 2/5*x*(b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(1/2)
```

Maxima [A] time = 1.04161, size = 32, normalized size = 0.35

$$\frac{2\left(b\sqrt{d}x^3+5a\sqrt{d}x\right)}{5d\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*sqrt(d)*x^3 + 5*a*sqrt(d)*x)/(d*sqrt(x))
```

Fricas [A] time = 1.30979, size = 42, normalized size = 0.46

$$\frac{2\left(bx^2 + 5a\right)\sqrt{dx}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b*x^2 + 5*a)*sqrt(d*x)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a + bx^2\right)^2}}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/sqrt(d*x), x)
```

Giac [A] time = 1.21156, size = 54, normalized size = 0.59

$$\frac{2\left(\sqrt{dx}bx^2\mathrm{sgn}\left(bx^2 + a\right) + 5\sqrt{dx}a\mathrm{sgn}\left(bx^2 + a\right)\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(sqrt(d*x)*b*x^2*sgn(b*x^2 + a) + 5*sqrt(d*x)*a*sgn(b*x^2 + a))/d
```

3.732 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}$$

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (2*b*(d*
x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2))
```

Rubi [A] time = 0.0279233, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (2*b*(d*
x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{3/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{3/2}}+\frac{b^2\sqrt{dx}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}+\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0143756, size = 43, normalized size = 0.47

$$\frac{2x\left(bx^2-3a\right)\sqrt{\left(a+bx^2\right)^2}}{3(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-3*a + b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(3/2)*(a + b*x^2))
```

Maple [A] time = 0.044, size = 39, normalized size = 0.4

$$-\frac{2\left(-bx^2+3a\right)x}{3bx^2+3a}\sqrt{\left(bx^2+a\right)^2}\,(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x)
```

```
[Out] -2/3*x*(-b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(3/2)
```

Maxima [A] time = 1.01541, size = 32, normalized size = 0.35

$$\frac{2\left(b\sqrt{d}x^3-3a\sqrt{d}x\right)}{3d^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*sqrt(d)*x^3 - 3*a*sqrt(d)*x)/(d^2*x^(3/2))
```

Fricas [A] time = 1.2116, size = 50, normalized size = 0.55

$$\frac{2\left(bx^2-3\,a\right)\sqrt{dx}}{3\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x^2 - 3*a)*sqrt(d*x)/(d^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25953, size = 55, normalized size = 0.6

$$\frac{2\left(\frac{\sqrt{dx}bx\mathrm{sgn}\left(bx^2+a\right)}{d}-\frac{3\,a\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(d*x)*b*x*sgn(b*x^2 + a)/d - 3*a*sgn(b*x^2 + a)/sqrt(d*x))/d
```

3.733 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (2*b
*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2))
```

Rubi [A] time = 0.0280979, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$\frac{2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (2*b
*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{5/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{5/2}}+\frac{b^2}{d^2\sqrt{dx}}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0163486, size = 42, normalized size = 0.46

$$-\frac{2x\left(a-3bx^2\right)\sqrt{\left(a+bx^2\right)^2}}{3(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(a - 3*b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(5/2)*(a + b*x^2))
```

Maple [A] time = 0.045, size = 37, normalized size = 0.4

$$-\frac{2\left(-3\,bx^2+a\right)x}{3\,bx^2+3\,a}\sqrt{\left(bx^2+a\right)^2}\,(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x)
```

```
[Out] -2/3*x*(-3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(5/2)
```

Maxima [A] time = 1.0045, size = 34, normalized size = 0.37

$$\frac{2\left(3\,b\sqrt{d}x^3-a\sqrt{d}x\right)}{3\,d^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*b*sqrt(d)*x^3 - a*sqrt(d)*x)/(d^3*x^(5/2))
```

Fricas [A] time = 1.27042, size = 53, normalized size = 0.58

$$\frac{2\left(3\,bx^2-a\right)\sqrt{dx}}{3\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b*x^2 - a)*sqrt(d*x)/(d^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16802, size = 57, normalized size = 0.63

$$\frac{2\left(3\,\sqrt{dx}b\mathrm{sgn}\left(bx^2+a\right)-\frac{ad\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}x}\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(d*x)*b*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a)/(sqrt(d*x)*x))/d^3
```

3.734 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}$$

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (2*b
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2))
```

Rubi [A] time = 0.0287815, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 14}

$$-\frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (2*b
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{7/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{7/2}}+\frac{b^2}{d^2(dx)^{3/2}}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)} - \frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0160878, size = 42, normalized size = 0.46

$$-\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(a+5bx^2\right)}{5(dx)^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

```
[Out] (-2*x*Sqrt[(a + b*x^2)^2]*(a + 5*b*x^2))/(5*(d*x)^(7/2)*(a + b*x^2))
```

Maple [A] time = 0.044, size = 37, normalized size = 0.4

$$-\frac{2\left(5\,bx^2+a\right)x}{5\,bx^2+5\,a}\sqrt{\left(bx^2+a\right)^2}\,(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x)
```

```
[Out] -2/5*x*(5*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(7/2)
```

Maxima [A] time = 1.00958, size = 32, normalized size = 0.35

$$-\frac{2\left(5\,b\sqrt{d}x^3+a\sqrt{d}x\right)}{5\,d^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*b*sqrt(d)*x^3 + a*sqrt(d)*x)/(d^4*x^(7/2))
```

Fricas [A] time = 1.30679, size = 54, normalized size = 0.59

$$-\frac{2\left(5\,bx^2+a\right)\sqrt{dx}}{5\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(5*b*x^2 + a)*sqrt(d*x)/(d^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27959, size = 59, normalized size = 0.65

$$-\frac{2\left(5\,bd^3x^2\mathrm{sgn}\left(bx^2+a\right)+ad^3\mathrm{sgn}\left(bx^2+a\right)\right)}{5\,\sqrt{dx}d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*b*d^3*x^2*sgn(b*x^2 + a) + a*d^3*sgn(b*x^2 + a))/(sqrt(d*x)*d^6*x^2
)
```

3.735 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=195

$$\frac{2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{7/2}\sqrt{a^2+\cdots}}{7d\left(a+\cdots\right.}$$

```
[Out] (2*a^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (6*
a^2*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2)) +
(2*a*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^5*(a + b*x^2))
+ (2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a + b*x^2))
```

Rubi [A] time = 0.0594125, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{7/2}\sqrt{a^2+\cdots}}{7d\left(a+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (6*
a^2*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2)) +
(2*a*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^5*(a + b*x^2))
+ (2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(ab+b^2x^2\right)^3\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3(dx)^{5/2}+\frac{3a^2b^4(dx)^{9/2}}{d^2}+\frac{3ab^5(dx)^{13/2}}{d^4}+\frac{b^6(dx)^{17/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{2a^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2ab^2(dx)^{15/2}\ldots}{5d\ldots}$$

Mathematica [A] time = 0.0308913, size = 66, normalized size = 0.34

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(1995a^2bx^2+1045a^3+1463ab^2x^4+385b^3x^6\right)}{7315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b^
2*x^4 + 385*b^3*x^6))/(7315*(a + b*x^2))
```

Maple [A] time = 0.167, size = 61, normalized size = 0.3

$$\frac{2\,x\left(385\,b^3x^6+1463\,ax^4b^2+1995\,a^2bx^2+1045\,a^3\right)}{7315\left(bx^2+a\right)^3}(dx)^{\frac{5}{2}}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 2/7315*x*(385*b^3*x^6+1463*a*b^2*x^4+1995*a^2*b*x^2+1045*a^3)*(d*x)^(5/2)*(
(b*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 1.0142, size = 112, normalized size = 0.57

$$\frac{2}{285}\left(15\,b^3d^{\frac{5}{2}}x^3+19\,ab^2d^{\frac{5}{2}}x\right)x^{\frac{13}{2}}+\frac{4}{165}\left(11\,ab^2d^{\frac{5}{2}}x^3+15\,a^2bd^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+\frac{2}{77}\left(7\,a^2bd^{\frac{5}{2}}x^3+11\,a^3d^{\frac{5}{2}}x\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/285*(15*b^3*d^(5/2)*x^3 + 19*a*b^2*d^(5/2)*x)*x^(13/2) + 4/165*(11*a*b^2*
d^(5/2)*x^3 + 15*a^2*b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^2*b*d^(5/2)*x^3 + 11*
a^3*d^(5/2)*x)*x^(5/2)
```

Fricas [A] time = 1.24644, size = 131, normalized size = 0.67

$$\frac{2}{7315}\left(385\,b^3d^2x^9+1463\,ab^2d^2x^7+1995\,a^2bd^2x^5+1045\,a^3d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^3*d^2*x^9 + 1463*a*b^2*d^2*x^7 + 1995*a^2*b*d^2*x^5 + 1045*a^
3*d^2*x^3)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27354, size = 134, normalized size = 0.69

$$\frac{2}{19}\sqrt{dx}b^3d^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{5}\sqrt{dx}ab^2d^2x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{11}\sqrt{dx}a^2bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}a^3d^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*sqrt(d*x)*b^3*d^2*x^9*sgn(b*x^2 + a) + 2/5*sqrt(d*x)*a*b^2*d^2*x^7*sgn
(b*x^2 + a) + 6/11*sqrt(d*x)*a^2*b*d^2*x^5*sgn(b*x^2 + a) + 2/7*sqrt(d*x)*a
^3*d^2*x^3*sgn(b*x^2 + a)
```

3.736 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=195

$$\frac{2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{5/2}\sqrt{a^2+\cdots}}{5d\left(a+\cdots\right.}$$

```
[Out] (2*a^3*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (2*
a^2*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2)) + (6
*a*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^5*(a + b*x^2)) +
 (2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a + b*x^2))
```

Rubi [A] time = 0.0583482, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{5/2}\sqrt{a^2+\cdots}}{5d\left(a+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (2*
a^2*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2)) + (6
*a*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^5*(a + b*x^2)) +
 (2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{3/2}\left(ab+b^2x^2\right)^3\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^3b^3(dx)^{3/2}+\frac{3a^2b^4(dx)^{7/2}}{d^2}+\frac{3ab^5(dx)^{11/2}}{d^4}+\frac{b^6(dx)^{15/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{2a^3(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{13/2}}{13d^5}$$

Mathematica [A] time = 0.0237474, size = 66, normalized size = 0.34

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(1105a^2bx^2+663a^3+765ab^2x^4+195b^3x^6\right)}{3315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2*
x^4 + 195*b^3*x^6))/(3315*(a + b*x^2))
```

Maple [A] time = 0.165, size = 61, normalized size = 0.3

$$\frac{2\,x\left(195\,b^3x^6+765\,ax^4b^2+1105\,a^2bx^2+663\,a^3\right)}{3315\,\left(bx^2+a\right)^3}(dx)^{\frac{3}{2}}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 2/3315*x*(195*b^3*x^6+765*a*b^2*x^4+1105*a^2*b*x^2+663*a^3)*(d*x)^(3/2)*((b
*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.9979, size = 112, normalized size = 0.57

$$\frac{2}{221}\left(13\,b^3d^{\frac{3}{2}}x^3+17\,ab^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+\frac{4}{117}\left(9\,ab^2d^{\frac{3}{2}}x^3+13\,a^2bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}}+\frac{2}{45}\left(5\,a^2bd^{\frac{3}{2}}x^3+9\,a^3d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/221*(13*b^3*d^(3/2)*x^3 + 17*a*b^2*d^(3/2)*x)*x^(11/2) + 4/117*(9*a*b^2*d
^(3/2)*x^3 + 13*a^2*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^2*b*d^(3/2)*x^3 + 9*a^
3*d^(3/2)*x)*x^(3/2)
```

Fricas [A] time = 1.20209, size = 117, normalized size = 0.6

$$\frac{2}{3315}\left(195\,b^3dx^8+765\,ab^2dx^6+1105\,a^2bdx^4+663\,a^3dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*b^3*d*x^8 + 765*a*b^2*d*x^6 + 1105*a^2*b*d*x^4 + 663*a^3*d*x^2)
*sqrt(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.20944, size = 123, normalized size = 0.63

$$\frac{2}{17}\sqrt{dx}b^3dx^8\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{13}\sqrt{dx}ab^2dx^6\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\sqrt{dx}a^2bdx^4\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{5}\sqrt{dx}a^3dx^2\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/17*sqrt(d*x)*b^3*d*x^8*sgn(b*x^2 + a) + 6/13*sqrt(d*x)*a*b^2*d*x^6*sgn(b*
x^2 + a) + 2/3*sqrt(d*x)*a^2*b*d*x^4*sgn(b*x^2 + a) + 2/5*sqrt(d*x)*a^3*d*x
^2*sgn(b*x^2 + a)
```

3.737 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=195

$$\frac{2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{3/2}\sqrt{a^2+\cdots}}{3d\left(a+\cdots\right.}$$

```
[Out] (2*a^3*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (6*
a^2*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2)) + (6
*a*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^5*(a + b*x^2)) +
 (2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^7*(a + b*x^2))
```

Rubi [A] time = 0.0547496, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2a^3(dx)^{3/2}\sqrt{a^2+\cdots}}{3d\left(a+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (6*
a^2*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2)) + (6
*a*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^5*(a + b*x^2)) +
 (2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^7*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(ab+b^2x^2\right)^3\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3\sqrt{dx}+\frac{3a^2b^4(dx)^{5/2}}{d^2}+\frac{3ab^5(dx)^{9/2}}{d^4}+\frac{b^6(dx)^{13/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{11/2}\sqrt{a}}{11d^5\left(\right.}$$

Mathematica [A] time = 0.0207061, size = 66, normalized size = 0.34

$$\frac{2\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}\left(495a^2bx^3+385a^3x+315ab^2x^5+77b^3x^7\right)}{1155\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(385*a^3*x + 495*a^2*b*x^3 + 315*a*b^2*x^5
 + 77*b^3*x^7))/(1155*(a + b*x^2))
```

Maple [A] time = 0.166, size = 61, normalized size = 0.3

$$\frac{2\,x\left(77\,b^3x^6+315\,ax^4b^2+495\,a^2bx^2+385\,a^3\right)}{1155\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x)
```

```
[Out] 2/1155*x*(77*b^3*x^6+315*a*b^2*x^4+495*a^2*b*x^2+385*a^3)*((b*x^2+a)^2)^(3/
2)*(d*x)^(1/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.998312, size = 112, normalized size = 0.57

$$\frac{2}{165}\left(11\,b^3\sqrt{d}x^3+15\,ab^2\sqrt{d}x\right)x^{\frac{9}{2}}+\frac{4}{77}\left(7\,ab^2\sqrt{d}x^3+11\,a^2b\sqrt{d}x\right)x^{\frac{5}{2}}+\frac{2}{21}\left(3\,a^2b\sqrt{d}x^3+7\,a^3\sqrt{d}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/165*(11*b^3*sqrt(d)*x^3 + 15*a*b^2*sqrt(d)*x)*x^(9/2) + 4/77*(7*a*b^2*sqr
t(d)*x^3 + 11*a^2*b*sqrt(d)*x)*x^(5/2) + 2/21*(3*a^2*b*sqrt(d)*x^3 + 7*a^3*
sqrt(d)*x)*sqrt(x)
```

Fricas [A] time = 1.26053, size = 101, normalized size = 0.52

$$\frac{2}{1155}\left(77\,b^3x^7+315\,ab^2x^5+495\,a^2bx^3+385\,a^3x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*sqrt(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.2965, size = 127, normalized size = 0.65

$$\frac{2\left(77\sqrt{dx}b^3dx^7\mathrm{sgn}\left(bx^2+a\right)+315\sqrt{dx}ab^2dx^5\mathrm{sgn}\left(bx^2+a\right)+495\sqrt{dx}a^2bdx^3\mathrm{sgn}\left(bx^2+a\right)+385\sqrt{dx}a^3dx\mathrm{sgn}\left(bx^2+a\right)\right.}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/1155*(77*sqrt(d*x)*b^3*d*x^7*sgn(b*x^2 + a) + 315*sqrt(d*x)*a*b^2*d*x^5*s
gn(b*x^2 + a) + 495*sqrt(d*x)*a^2*b*d*x^3*sgn(b*x^2 + a) + 385*sqrt(d*x)*a^
3*d*x*sgn(b*x^2 + a))/d
```

3.738 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=193

$$\frac{2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a^3\sqrt{dx}\sqrt{a^2+2ab}}{d\left(a+bx\right.}$$

```
[Out] (2*a^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (6*a^2*
b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*x^2)) + (2*a*b
^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2)) + (2*b^
3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2))
```

Rubi [A] time = 0.0544002, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a^3\sqrt{dx}\sqrt{a^2+2ab}}{d\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```



```
[Out] (2*a^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (6*a^2*
b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*x^2)) + (2*a*b
^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2)) + (2*b^
3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{\sqrt{dx}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{\sqrt{dx}}+\frac{3a^2b^4(dx)^{3/2}}{d^2}+\frac{3ab^5(dx)^{7/2}}{d^4}+\frac{b^6(dx)^{11/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{2a^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2ab}}{3d^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0216541, size = 66, normalized size = 0.34

$$\frac{2\sqrt{\left(a+bx^2\right)^2}\left(117a^2bx^3+195a^3x+65ab^2x^5+15b^3x^7\right)}{195\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[(a + b*x^2)^2]*(195*a^3*x + 117*a^2*b*x^3 + 65*a*b^2*x^5 + 15*b^3*x
^7))/(195*Sqrt[d*x]*(a + b*x^2))
```

Maple [A] time = 0.164, size = 61, normalized size = 0.3

$$\frac{2\left(15\,b^3x^6+65\,ax^4b^2+117\,a^2bx^2+195\,a^3\right)x}{195\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] 2/195*x*(15*b^3*x^6+65*a*b^2*x^4+117*a^2*b*x^2+195*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3/(d*x)^(1/2)
```

Maxima [A] time = 1.00415, size = 117, normalized size = 0.61

$$\frac{2\left(5\left(9\,b^3\sqrt{d}x^3+13\,ab^2\sqrt{d}x\right)x^{\frac{7}{2}}+26\left(5\,ab^2\sqrt{d}x^3+9\,a^2b\sqrt{d}x\right)x^{\frac{3}{2}}+\frac{117\left(a^2b\sqrt{d}x^3+5\,a^3\sqrt{d}x\right)}{\sqrt{x}}\right)}{585\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/585*(5*(9*b^3*sqrt(d)*x^3 + 13*a*b^2*sqrt(d)*x)*x^(7/2) + 26*(5*a*b^2*sqr
t(d)*x^3 + 9*a^2*b*sqrt(d)*x)*x^(3/2) + 117*(a^2*b*sqrt(d)*x^3 + 5*a^3*sqrt
(d)*x)/sqrt(x))/d
```

Fricas [A] time = 1.30149, size = 99, normalized size = 0.51

$$\frac{2\left(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3\right)\sqrt{dx}}{195\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(d*x)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/sqrt(d*x), x)
```

Giac [A] time = 1.23547, size = 120, normalized size = 0.62

$$\frac{2\left(15\sqrt{dx}b^3x^6\mathrm{sgn}\left(bx^2+a\right)+65\sqrt{dx}ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+117\sqrt{dx}a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+195\sqrt{dx}a^3\mathrm{sgn}\left(bx^2+a\right)\right)}{195\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/195*(15*sqrt(d*x)*b^3*x^6*sgn(b*x^2 + a) + 65*sqrt(d*x)*a*b^2*x^4*sgn(b*x
^2 + a) + 117*sqrt(d*x)*a^2*b*x^2*sgn(b*x^2 + a) + 195*sqrt(d*x)*a^3*sgn(b*
x^2 + a))/d
```

3.739 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=191

$$\frac{2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+}}{d\sqrt{dx}\left(a+bx\right.}$$

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (2*a^2
*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (6*a*b^
2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^5*(a + b*x^2)) + (2*b^3
*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^2))
```

Rubi [A] time = 0.0576898, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+}}{d\sqrt{dx}\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```



```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (2*a^2
*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (6*a*b^
2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^5*(a + b*x^2)) + (2*b^3
*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{3/2}}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{(dx)^{3/2}}\,dx}{b^2\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{3/2}}+\frac{3a^2b^4\sqrt{dx}}{d^2}+\frac{3ab^5(dx)^{5/2}}{d^4}+\frac{b^6(dx)^{9/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}\\ &= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2}}{7d^5\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0254333, size = 66, normalized size = 0.35

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(77a^2bx^2-77a^3+33ab^2x^4+7b^3x^6\right)}{77(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-77*a^3 + 77*a^2*b*x^2 + 33*a*b^2*x^4 + 7*b^3*x^6
))/(77*(d*x)^(3/2)*(a + b*x^2))
```

Maple [A] time = 0.176, size = 61, normalized size = 0.3

$$-\frac{2\left(-7\,b^3x^6-33\,ax^4b^2-77\,a^2bx^2+77\,a^3\right)x}{77\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x)
```

```
[Out] -2/77*x*(-7*b^3*x^6-33*a*b^2*x^4-77*a^2*b*x^2+77*a^3)*((b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3/(d*x)^(3/2)
```

Maxima [A] time = 1.01135, size = 117, normalized size = 0.61

$$\frac{2\left(3\left(7\,b^3\sqrt{d}x^3+11\,ab^2\sqrt{d}x\right)x^{\frac{5}{2}}+22\left(3\,ab^2\sqrt{d}x^3+7\,a^2b\sqrt{d}x\right)\sqrt{x}+\frac{77\left(a^2b\sqrt{d}x^3-3\,a^3\sqrt{d}x\right)}{x^{\frac{3}{2}}}\right)}{231\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/231*(3*(7*b^3*sqrt(d)*x^3 + 11*a*b^2*sqrt(d)*x)*x^(5/2) + 22*(3*a*b^2*sqr
t(d)*x^3 + 7*a^2*b*sqrt(d)*x)*sqrt(x) + 77*(a^2*b*sqrt(d)*x^3 - 3*a^3*sqrt(
d)*x)/x^(3/2))/d^2
```

Fricas [A] time = 1.28315, size = 101, normalized size = 0.53

$$\frac{2\left(7\,b^3x^6+33\,ab^2x^4+77\,a^2bx^2-77\,a^3\right)\sqrt{dx}}{77\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)*sqrt(d*x)/(d^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(3/2), x)
```

Giac [A] time = 1.3335, size = 138, normalized size = 0.72

$$-\frac{2\left(\frac{77\,a^3\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}-\frac{7\,\sqrt{dx}b^3d^{65}x^5\mathrm{sgn}\left(bx^2+a\right)+33\,\sqrt{dx}ab^2d^{65}x^3\mathrm{sgn}\left(bx^2+a\right)+77\,\sqrt{dx}a^2bd^{65}x\mathrm{sgn}\left(bx^2+a\right)}{d^{66}}\right)}{77\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/77*(77*a^3*sgn(b*x^2 + a)/sqrt(d*x) - (7*sqrt(d*x)*b^3*d^65*x^5*sgn(b*x^
2 + a) + 33*sqrt(d*x)*a*b^2*d^65*x^3*sgn(b*x^2 + a) + 77*sqrt(d*x)*a^2*b*d^
65*x*sgn(b*x^2 + a))/d^66)/d
```

3.740 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=193

$$\frac{2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (6
*a^2*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (6*a*
b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^5*(a + b*x^2)) + (2*b
^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2))
```

Rubi [A] time = 0.0547985, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}+\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```


```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (6
*a^2*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (6*a*
b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^5*(a + b*x^2)) + (2*b
^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{5/2}}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{(dx)^{5/2}}\,dx}{b^2\left(ab+b^2x^2\right)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{5/2}}+\frac{3a^2b^4}{d^2\sqrt{dx}}+\frac{3ab^5(dx)^{3/2}}{d^4}+\frac{b^6(dx)^{7/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}\\ &= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+}}{5d^5\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A] time = 0.0269476, size = 66, normalized size = 0.34

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(135a^2bx^2-15a^3+27ab^2x^4+5b^3x^6\right)}{45(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-15*a^3 + 135*a^2*b*x^2 + 27*a*b^2*x^4 + 5*b^3*x^
6))/(45*(d*x)^(5/2)*(a + b*x^2))
```

Maple [A] time = 0.17, size = 61, normalized size = 0.3

$$-\frac{2\left(-5\,b^3x^6-27\,ax^4b^2-135\,a^2bx^2+15\,a^3\right)x}{45\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x)
```

```
[Out] -2/45*x*(-5*b^3*x^6-27*a*b^2*x^4-135*a^2*b*x^2+15*a^3)*((b*x^2+a)^2)^(3/2)/
(b*x^2+a)^3/(d*x)^(5/2)
```

Maxima [A] time = 1.02564, size = 116, normalized size = 0.6

$$\frac{2\left(\left(5\,b^3\sqrt{d}x^3 + 9\,ab^2\sqrt{d}x\right)x^{\frac{3}{2}} + \frac{18\left(ab^2\sqrt{d}x^3 + 5\,a^2b\sqrt{d}x\right)}{\sqrt{x}} + \frac{15\left(3\,a^2b\sqrt{d}x^3 - a^3\sqrt{d}x\right)}{x^{\frac{5}{2}}}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45*((5*b^3*sqrt(d)*x^3 + 9*a*b^2*sqrt(d)*x)*x^(3/2) + 18*(a*b^2*sqrt(d)*x
^3 + 5*a^2*b*sqrt(d)*x)/sqrt(x) + 15*(3*a^2*b*sqrt(d)*x^3 - a^3*sqrt(d)*x)/
x^(5/2))/d^3
```

Fricas [A] time = 1.49424, size = 105, normalized size = 0.54

$$\frac{2\left(5\,b^3x^6 + 27\,ab^2x^4 + 135\,a^2bx^2 - 15\,a^3\right)\sqrt{dx}}{45\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)*sqrt(d*x)/(d^3*x^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(5/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(5/2), x)
```

Giac [A] time = 1.28162, size = 142, normalized size = 0.74

$$-\frac{2\left(\frac{15\,a^3d\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}x}-\frac{5\,\sqrt{dx}b^3d^{36}x^4\mathrm{sgn}\left(bx^2+a\right)+27\,\sqrt{dx}ab^2d^{36}x^2\mathrm{sgn}\left(bx^2+a\right)+135\,\sqrt{dx}a^2bd^{36}\mathrm{sgn}\left(bx^2+a\right)}{d^{36}}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/45*(15*a^3*d*sgn(b*x^2 + a)/(sqrt(d*x)*x) - (5*sqrt(d*x)*b^3*d^36*x^4*sg
n(b*x^2 + a) + 27*sqrt(d*x)*a*b^2*d^36*x^2*sgn(b*x^2 + a) + 135*sqrt(d*x)*a
^2*b*d^36*sgn(b*x^2 + a))/d^36)/d^3
```

3.741 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=191

$$\frac{2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7(a+bx^2)}+\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5(a+bx^2)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}(a+bx^2)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}(a+bx^2)}$$

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (6
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2)) + (2*a*
b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(a + b*x^2)) + (2*b^3
*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2))
```

Rubi [A] time = 0.0552309, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7(a+bx^2)}+\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5(a+bx^2)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}(a+bx^2)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (6
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2)) + (2*a*
b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(a + b*x^2)) + (2*b^3
*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{(dx)^{7/2}}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{7/2}}+\frac{3a^2b^4}{d^2(dx)^{3/2}}+\frac{3ab^5\sqrt{dx}}{d^4}+\frac{b^6(dx)^{5/2}}{d^6}\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}+\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0260261, size = 66, normalized size = 0.35

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(-105a^2bx^2-7a^3+35ab^2x^4+5b^3x^6\right)}{35(dx)^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-7*a^3 - 105*a^2*b*x^2 + 35*a*b^2*x^4 + 5*b^3*x^6
))/(35*(d*x)^(7/2)*(a + b*x^2))
```

Maple [A] time = 0.17, size = 61, normalized size = 0.3

$$-\frac{2\left(-5\,b^3x^6-35\,ax^4b^2+105\,a^2bx^2+7\,a^3\right)x}{35\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x)
```

```
[Out] -2/35*x*(-5*b^3*x^6-35*a*b^2*x^4+105*a^2*b*x^2+7*a^3)*((b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3/(d*x)^(7/2)
```

Maxima [A] time = 1.00227, size = 116, normalized size = 0.61

$$\frac{2\left(5\left(3\,b^{3}\sqrt{d}x^{3}+7\,ab^{2}\sqrt{d}x\right)\sqrt{x}+\frac{70\left(ab^{2}\sqrt{d}x^{3}-3\,a^{2}b\sqrt{d}x\right)}{x^{\frac{3}{2}}}-\frac{21\left(5\,a^{2}b\sqrt{d}x^{3}+a^{3}\sqrt{d}x\right)}{x^{\frac{7}{2}}}\right)}{105\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/105*(5*(3*b^3*sqrt(d)*x^3 + 7*a*b^2*sqrt(d)*x)*sqrt(x) + 70*(a*b^2*sqrt(d
)*x^3 - 3*a^2*b*sqrt(d)*x)/x^(3/2) - 21*(5*a^2*b*sqrt(d)*x^3 + a^3*sqrt(d)*
x)/x^(7/2))/d^4
```

Fricas [A] time = 1.4689, size = 104, normalized size = 0.54

$$\frac{2\left(5\,b^{3}x^{6}+35\,ab^{2}x^{4}-105\,a^{2}bx^{2}-7\,a^{3}\right)\sqrt{dx}}{35\,d^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)*sqrt(d*x)/(d^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(7/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(7/2), x)
```

Giac [A] time = 1.2064, size = 144, normalized size = 0.75

$$-\frac{2\left(\frac{7\left(15\,a^{2}bd^{3}x^{2}\mathrm{sgn}\left(bx^{2}+a\right)+a^{3}d^{3}\mathrm{sgn}\left(bx^{2}+a\right)\right)}{\sqrt{dx}d^{2}x^{2}}-\frac{5\left(\sqrt{dx}b^{3}d^{21}x^{3}\mathrm{sgn}\left(bx^{2}+a\right)+7\,\sqrt{dx}ab^{2}d^{21}x\mathrm{sgn}\left(bx^{2}+a\right)\right)}{d^{21}}\right)}{35\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/35*(7*(15*a^2*b*d^3*x^2*sgn(b*x^2 + a) + a^3*d^3*sgn(b*x^2 + a))/(sqrt(d
*x)*d^2*x^2) - 5*(sqrt(d*x)*b^3*d^21*x^3*sgn(b*x^2 + a) + 7*sqrt(d*x)*a*b^2
*d^21*x*sgn(b*x^2 + a))/d^21)/d^4
```

3.742 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{27/2}\sqrt{a^2+2abx^2+b^2x^4}}{27d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{15/2}}{3d}$$

```
[Out] (2*a^5*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2)) +
 (4*a^3*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2
)) + (20*a^2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a +
 b*x^2)) + (10*a*b^4*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^9*
(a + b*x^2)) + (2*b^5*(d*x)^(27/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(27*d^1
1*(a + b*x^2))
```

Rubi [A] time = 0.0823243, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{27/2}\sqrt{a^2+2abx^2+b^2x^4}}{27d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{15/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a^5*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2)) +
 (4*a^3*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2
)) + (20*a^2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a +
 b*x^2)) + (10*a*b^4*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^9*
(a + b*x^2)) + (2*b^5*(d*x)^(27/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(27*d^1
1*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5b^5(dx)^{5/2}+\frac{5a^4b^6(dx)^{9/2}}{d^2}+\frac{10a^3b^7(dx)^{13/2}}{d^4}+\frac{10a^2b^8(dx)^{17/2}}{d^6}+\frac{5a\cdots}{\cdots}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$=\frac{2a^5(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{15\cdots}}{3\cdots}$$

Mathematica [A] time = 0.042513, size = 88, normalized size = 0.3

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(478170a^2b^3x^6+605682a^3b^2x^4+412965a^4bx^2+129789a^5+197505ab^4x^8+33649b^5x^{10}\right)}{908523\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(129789*a^5 + 412965*a^4*b*x^2 + 60568
2*a^3*b^2*x^4 + 478170*a^2*b^3*x^6 + 197505*a*b^4*x^8 + 33649*b^5*x^10))/(9
08523*(a + b*x^2))
```

Maple [A] time = 0.174, size = 83, normalized size = 0.3

$$\frac{2\,x\left(33649\,b^5x^{10}+197505\,ab^4x^8+478170\,a^2b^3x^6+605682\,b^2a^3x^4+412965\,a^4bx^2+129789\,a^5\right)}{908523\left(bx^2+a\right)^5}(dx)^{\frac{5}{2}}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 2/908523*x*(33649*b^5*x^10+197505*a*b^4*x^8+478170*a^2*b^3*x^6+605682*a^3*b
^2*x^4+412965*a^4*b*x^2+129789*a^5)*(d*x)^(5/2)*((b*x^2+a)^2)^(5/2)/(b*x^2+
a)^5
```

Maxima [A] time = 1.01273, size = 198, normalized size = 0.67

$$\frac{2}{621}\left(23\,b^5d^{\frac{5}{2}}x^3+27\,ab^4d^{\frac{5}{2}}x\right)x^{\frac{21}{2}}+\frac{8}{437}\left(19\,ab^4d^{\frac{5}{2}}x^3+23\,a^2b^3d^{\frac{5}{2}}x\right)x^{\frac{17}{2}}+\frac{4}{95}\left(15\,a^2b^3d^{\frac{5}{2}}x^3+19\,a^3b^2d^{\frac{5}{2}}x\right)x^{\frac{13}{2}}+\frac{8}{165}\left(11\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/621*(23*b^5*d^(5/2)*x^3 + 27*a*b^4*d^(5/2)*x)*x^(21/2) + 8/437*(19*a*b^4*
d^(5/2)*x^3 + 23*a^2*b^3*d^(5/2)*x)*x^(17/2) + 4/95*(15*a^2*b^3*d^(5/2)*x^3
 + 19*a^3*b^2*d^(5/2)*x)*x^(13/2) + 8/165*(11*a^3*b^2*d^(5/2)*x^3 + 15*a^4*
b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^4*b*d^(5/2)*x^3 + 11*a^5*d^(5/2)*x)*x^(5/2
)
```

Fricas [A] time = 1.47708, size = 215, normalized size = 0.72

$$\frac{2}{908523}\left(33649\,b^5d^2x^{13}+197505\,ab^4d^2x^{11}+478170\,a^2b^3d^2x^9+605682\,a^3b^2d^2x^7+412965\,a^4bd^2x^5+129789\,a^5d^2x^3\right)\sqrt{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/908523*(33649*b^5*d^2*x^13 + 197505*a*b^4*d^2*x^11 + 478170*a^2*b^3*d^2*x
^9 + 605682*a^3*b^2*d^2*x^7 + 412965*a^4*b*d^2*x^5 + 129789*a^5*d^2*x^3)*sq
rt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24999, size = 207, normalized size = 0.7

$$\frac{2}{27}\sqrt{dx}b^5d^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{23}\sqrt{dx}ab^4d^2x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{20}{19}\sqrt{dx}a^2b^3d^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{4}{3}\sqrt{dx}a^3b^2d^2x^7\mathrm{sgn}\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(d*x)*b^5*d^2*x^13*sgn(b*x^2 + a) + 10/23*sqrt(d*x)*a*b^4*d^2*x^11
*sgn(b*x^2 + a) + 20/19*sqrt(d*x)*a^2*b^3*d^2*x^9*sgn(b*x^2 + a) + 4/3*sqrt
(d*x)*a^3*b^2*d^2*x^7*sgn(b*x^2 + a) + 10/11*sqrt(d*x)*a^4*b*d^2*x^5*sgn(b*
x^2 + a) + 2/7*sqrt(d*x)*a^5*d^2*x^3*sgn(b*x^2 + a)
```

3.743 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{25/2}\sqrt{a^2+2abx^2+b^2x^4}}{25d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{13}}{13}$$

```
[Out] (2*a^5*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a + b*x^2)) + (
20*a^3*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^5*(a + b*x^2
)) + (20*a^2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a +
 b*x^2)) + (10*a*b^4*(d*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^9*
(a + b*x^2)) + (2*b^5*(d*x)^(25/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(25*d^1
1*(a + b*x^2))
```

Rubi [A] time = 0.0772811, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{25/2}\sqrt{a^2+2abx^2+b^2x^4}}{25d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (2*a^5*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a + b*x^2)) + (
20*a^3*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^5*(a + b*x^2
)) + (20*a^2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a +
 b*x^2)) + (10*a*b^4*(d*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^9*
(a + b*x^2)) + (2*b^5*(d*x)^(25/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(25*d^1
1*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{3/2}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5(dx)^{3/2}+\frac{5a^4b^6(dx)^{7/2}}{d^2}+\frac{10a^3b^7(dx)^{11/2}}{d^4}+\frac{10a^2b^8(dx)^{15/2}}{d^6}+\frac{5a}{}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$=\frac{2a^5(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{1}}{13}$$

Mathematica [A] time = 0.0338699, size = 88, normalized size = 0.3

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(204750a^2b^3x^6+267750a^3b^2x^4+193375a^4bx^2+69615a^5+82875ab^4x^8+13923b^5x^{10}\right)}{348075\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(69615*a^5 + 193375*a^4*b*x^2 + 267750
*a^3*b^2*x^4 + 204750*a^2*b^3*x^6 + 82875*a*b^4*x^8 + 13923*b^5*x^10))/(348
075*(a + b*x^2))
```

Maple [A] time = 0.174, size = 83, normalized size = 0.3

$$\frac{2\,x\left(13923\,b^5x^{10}+82875\,ab^4x^8+204750\,a^2b^3x^6+267750\,b^2a^3x^4+193375\,a^4bx^2+69615\,a^5\right)}{348075\,\left(bx^2+a\right)^5}(dx)^{\frac{3}{2}}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 2/348075*x*(13923*b^5*x^10+82875*a*b^4*x^8+204750*a^2*b^3*x^6+267750*a^3*b^
2*x^4+193375*a^4*b*x^2+69615*a^5)*(d*x)^(3/2)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)
^5
```

Maxima [A] time = 1.01767, size = 198, normalized size = 0.67

$$\frac{2}{525}\left(21\,b^5d^{\frac{3}{2}}x^3+25\,ab^4d^{\frac{3}{2}}x\right)x^{\frac{19}{2}}+\frac{8}{357}\left(17\,ab^4d^{\frac{3}{2}}x^3+21\,a^2b^3d^{\frac{3}{2}}x\right)x^{\frac{15}{2}}+\frac{12}{221}\left(13\,a^2b^3d^{\frac{3}{2}}x^3+17\,a^3b^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+\frac{8}{117}\left(9\,a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/525*(21*b^5*d^(3/2)*x^3 + 25*a*b^4*d^(3/2)*x)*x^(19/2) + 8/357*(17*a*b^4*
d^(3/2)*x^3 + 21*a^2*b^3*d^(3/2)*x)*x^(15/2) + 12/221*(13*a^2*b^3*d^(3/2)*x
^3 + 17*a^3*b^2*d^(3/2)*x)*x^(11/2) + 8/117*(9*a^3*b^2*d^(3/2)*x^3 + 13*a^4
*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^4*b*d^(3/2)*x^3 + 9*a^5*d^(3/2)*x)*x^(3/2
)
```

Fricas [A] time = 1.49071, size = 196, normalized size = 0.66

$$\frac{2}{348075}\left(13923\,b^5dx^{12}+82875\,ab^4dx^{10}+204750\,a^2b^3dx^8+267750\,a^3b^2dx^6+193375\,a^4bdx^4+69615\,a^5dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^5*d*x^12 + 82875*a*b^4*d*x^10 + 204750*a^2*b^3*d*x^8 + 26
7750*a^3*b^2*d*x^6 + 193375*a^4*b*d*x^4 + 69615*a^5*d*x^2)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27631, size = 190, normalized size = 0.64

$$\frac{2}{25}\sqrt{dx}b^5dx^{12}\operatorname{sgn}\left(bx^2+a\right)+\frac{10}{21}\sqrt{dx}ab^4dx^{10}\operatorname{sgn}\left(bx^2+a\right)+\frac{20}{17}\sqrt{dx}a^2b^3dx^8\operatorname{sgn}\left(bx^2+a\right)+\frac{20}{13}\sqrt{dx}a^3b^2dx^6\operatorname{sgn}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/25*sqrt(d*x)*b^5*d*x^12*sgn(b*x^2 + a) + 10/21*sqrt(d*x)*a*b^4*d*x^10*sgn
(b*x^2 + a) + 20/17*sqrt(d*x)*a^2*b^3*d*x^8*sgn(b*x^2 + a) + 20/13*sqrt(d*x
)*a^3*b^2*d*x^6*sgn(b*x^2 + a) + 10/9*sqrt(d*x)*a^4*b*d*x^4*sgn(b*x^2 + a)
+ 2/5*sqrt(d*x)*a^5*d*x^2*sgn(b*x^2 + a)
```

3.744 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^9\left(a+bx^2\right)}+\frac{4a^2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{11/2}}{11d^5}$$

```
[Out] (2*a^5*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2)) + (
20*a^3*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^5*(a + b*x^2
)) + (4*a^2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^7*(a + b
*x^2)) + (10*a*b^4*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^9*(a
 + b*x^2)) + (2*b^5*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^11*
(a + b*x^2))
```

Rubi [A] time = 0.0812924, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^9\left(a+bx^2\right)}+\frac{4a^2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (2*a^5*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2)) + (
20*a^3*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^5*(a + b*x^2
)) + (4*a^2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^7*(a + b
*x^2)) + (10*a*b^4*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^9*(a
 + b*x^2)) + (2*b^5*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^11*
(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(ab+b^2x^2\right)^5 dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5b^5\sqrt{dx}+\frac{5a^4b^6(dx)^{5/2}}{d^2}+\frac{10a^3b^7(dx)^{9/2}}{d^4}+\frac{10a^2b^8(dx)^{13/2}}{d^6}+\frac{5ab^9(dx)}{d^8}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{2a^5(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{11/2}}{11d}$$

Mathematica [A] time = 0.0307575, size = 88, normalized size = 0.3

$$\frac{2\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}\left(67298a^2b^3x^7+91770a^3b^2x^5+72105a^4bx^3+33649a^5x+26565ab^4x^9+4389b^5x^{11}\right)}{100947\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(33649*a^5*x + 72105*a^4*b*x^3 + 91770*a^3
*b^2*x^5 + 67298*a^2*b^3*x^7 + 26565*a*b^4*x^9 + 4389*b^5*x^11))/(100947*(a
 + b*x^2))
```

Maple [A] time = 0.167, size = 83, normalized size = 0.3

$$\frac{2\,x\left(4389\,b^5x^{10}+26565\,ab^4x^8+67298\,a^2b^3x^6+91770\,b^2a^3x^4+72105\,a^4bx^2+33649\,a^5\right)}{100947\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x)
```

```
[Out] 2/100947*x*(4389*b^5*x^10+26565*a*b^4*x^8+67298*a^2*b^3*x^6+91770*a^3*b^2*x
^4+72105*a^4*b*x^2+33649*a^5)*((b*x^2+a)^2)^(5/2)*(d*x)^(1/2)/(b*x^2+a)^5
```

Maxima [A] time = 0.985182, size = 198, normalized size = 0.67

$$\frac{2}{437}\left(19\,b^5\sqrt{d}x^3 + 23\,ab^4\sqrt{d}x\right)x^{\frac{17}{2}} + \frac{8}{285}\left(15\,ab^4\sqrt{d}x^3 + 19\,a^2b^3\sqrt{d}x\right)x^{\frac{13}{2}} + \frac{4}{55}\left(11\,a^2b^3\sqrt{d}x^3 + 15\,a^3b^2\sqrt{d}x\right)x^{\frac{9}{2}} + \frac{8}{77}\left(7\,a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/437*(19*b^5*sqrt(d)*x^3 + 23*a*b^4*sqrt(d)*x)*x^(17/2) + 8/285*(15*a*b^4*
sqrt(d)*x^3 + 19*a^2*b^3*sqrt(d)*x)*x^(13/2) + 4/55*(11*a^2*b^3*sqrt(d)*x^3
 + 15*a^3*b^2*sqrt(d)*x)*x^(9/2) + 8/77*(7*a^3*b^2*sqrt(d)*x^3 + 11*a^4*b*s
qrt(d)*x)*x^(5/2) + 2/21*(3*a^4*b*sqrt(d)*x^3 + 7*a^5*sqrt(d)*x)*sqrt(x)
```

Fricas [A] time = 1.53238, size = 170, normalized size = 0.57

$$\frac{2}{100947}\left(4389\,b^5x^{11} + 26565\,ab^4x^9 + 67298\,a^2b^3x^7 + 91770\,a^3b^2x^5 + 72105\,a^4bx^3 + 33649\,a^5x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/100947*(4389*b^5*x^11 + 26565*a*b^4*x^9 + 67298*a^2*b^3*x^7 + 91770*a^3*b
^2*x^5 + 72105*a^4*b*x^3 + 33649*a^5*x)*sqrt(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.23254, size = 194, normalized size = 0.65

$$\frac{2\left(4389\sqrt{dx}b^5dx^{11}\mathrm{sgn}\left(bx^2+a\right)+26565\sqrt{dx}ab^4dx^9\mathrm{sgn}\left(bx^2+a\right)+67298\sqrt{dx}a^2b^3dx^7\mathrm{sgn}\left(bx^2+a\right)+91770\sqrt{dx}a^3b\right.}{100947\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/100947*(4389*sqrt(d*x)*b^5*d*x^11*sgn(b*x^2 + a) + 26565*sqrt(d*x)*a*b^4*
d*x^9*sgn(b*x^2 + a) + 67298*sqrt(d*x)*a^2*b^3*d*x^7*sgn(b*x^2 + a) + 91770
*sqrt(d*x)*a^3*b^2*d*x^5*sgn(b*x^2 + a) + 72105*sqrt(d*x)*a^4*b*d*x^3*sgn(b
*x^2 + a) + 33649*sqrt(d*x)*a^5*d*x*sgn(b*x^2 + a))/d
```

3.745 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=293

$$\frac{2b^5(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{9/}}{9d}$$

```
[Out] (2*a^5*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (2*a^4*
b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (20*a^3*
b^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^5*(a + b*x^2)) + (20*
a^2*b^3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2))
+ (10*a*b^4*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^9*(a + b*x^
2)) + (2*b^5*(d*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^11*(a + b*
x^2))
```

Rubi [A] time = 0.078523, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{9/}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```


```
[Out] (2*a^5*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (2*a^4*
b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (20*a^3*
b^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^5*(a + b*x^2)) + (20*
a^2*b^3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2))
+ (10*a*b^4*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^9*(a + b*x^
2)) + (2*b^5*(d*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^11*(a + b*
x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{\sqrt{dx}}\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{\sqrt{dx}}+\frac{5a^4b^6(dx)^{3/2}}{d^2}+\frac{10a^3b^7(dx)^{7/2}}{d^4}+\frac{10a^2b^8(dx)^{11/2}}{d^6}+\frac{5ab^9(dx)^{15/2}}{d^8}+\frac{b^{10}(d}{d}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{2a^5\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}+\frac{2a^4b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{9/2}\sqrt{a^2+2}}{9d^5\left(a+bx^2\right.}$$

Mathematica [A] time = 0.0308095, size = 88, normalized size = 0.3

$$\frac{2\sqrt{\left(a+bx^2\right)^2}\left(10710a^2b^3x^7+15470a^3b^2x^5+13923a^4bx^3+13923a^5x+4095ab^4x^9+663b^5x^{11}\right)}{13923\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[(a + b*x^2)^2]*(13923*a^5*x + 13923*a^4*b*x^3 + 15470*a^3*b^2*x^5 +
 10710*a^2*b^3*x^7 + 4095*a*b^4*x^9 + 663*b^5*x^11))/(13923*Sqrt[d*x]*(a +
b*x^2))
```

Maple [A] time = 0.166, size = 83, normalized size = 0.3

$$\frac{2\left(663\,b^5x^{10}+4095\,ab^4x^8+10710\,a^2b^3x^6+15470\,b^2a^3x^4+13923\,a^4bx^2+13923\,a^5\right)x}{13923\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] 2/13923*x*(663*b^5*x^10+4095*a*b^4*x^8+10710*a^2*b^3*x^6+15470*a^3*b^2*x^4+
13923*a^4*b*x^2+13923*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(1/2)
```

Maxima [A] time = 1.00069, size = 204, normalized size = 0.7

$$\frac{2\left(195\left(17\,b^5\sqrt{d}x^3+21\,ab^4\sqrt{d}x\right)x^{\frac{15}{2}}+1260\left(13\,ab^4\sqrt{d}x^3+17\,a^2b^3\sqrt{d}x\right)x^{\frac{11}{2}}+3570\left(9\,a^2b^3\sqrt{d}x^3+13\,a^3b^2\sqrt{d}x\right)x^{\frac{7}{2}}+6188\right.}{69615\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/69615*(195*(17*b^5*sqrt(d)*x^3 + 21*a*b^4*sqrt(d)*x)*x^(15/2) + 1260*(13*
a*b^4*sqrt(d)*x^3 + 17*a^2*b^3*sqrt(d)*x)*x^(11/2) + 3570*(9*a^2*b^3*sqrt(d
)*x^3 + 13*a^3*b^2*sqrt(d)*x)*x^(7/2) + 6188*(5*a^3*b^2*sqrt(d)*x^3 + 9*a^4
*b*sqrt(d)*x)*x^(3/2) + 13923*(a^4*b*sqrt(d)*x^3 + 5*a^5*sqrt(d)*x)/sqrt(x)
)/d
```

Fricas [A] time = 1.49227, size = 166, normalized size = 0.57

$$\frac{2\left(663\,b^5x^{10}+4095\,ab^4x^8+10710\,a^2b^3x^6+15470\,a^3b^2x^4+13923\,a^4bx^2+13923\,a^5\right)\sqrt{dx}}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^5*x^10 + 4095*a*b^4*x^8 + 10710*a^2*b^3*x^6 + 15470*a^3*b^2*
x^4 + 13923*a^4*b*x^2 + 13923*a^5)*sqrt(d*x)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/sqrt(d*x), x)
```

Giac [A] time = 1.27013, size = 185, normalized size = 0.63

$$\frac{2\left(663\sqrt{dx}b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+4095\sqrt{dx}ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+10710\sqrt{dx}a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+15470\sqrt{dx}a^3b^2x^4\mathrm{sgn}\right.}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/13923*(663*sqrt(d*x)*b^5*x^10*sgn(b*x^2 + a) + 4095*sqrt(d*x)*a*b^4*x^8*s
gn(b*x^2 + a) + 10710*sqrt(d*x)*a^2*b^3*x^6*sgn(b*x^2 + a) + 15470*sqrt(d*x
)*a^3*b^2*x^4*sgn(b*x^2 + a) + 13923*sqrt(d*x)*a^4*b*x^2*sgn(b*x^2 + a) + 1
3923*sqrt(d*x)*a^5*sgn(b*x^2 + a))/d
```

3.746 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=295

$$\frac{2b^5(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^{11}(a+bx^2)}+\frac{2ab^4(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^9(a+bx^2)}+\frac{20a^2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7(a+bx^2)}+\frac{20a^3b^2(dx)^{7/2}}{7d^5}$$

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (10*a^
4*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2)) + (20*
a^3*b^2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^5*(a + b*x^2)) +
(20*a^2*b^3*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^
2)) + (2*a*b^4*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^9*(a + b*
x^2)) + (2*b^5*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^11*(a +
b*x^2))
```

Rubi [A] time = 0.079362, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^{11}(a+bx^2)}+\frac{2ab^4(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^9(a+bx^2)}+\frac{20a^2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7(a+bx^2)}+\frac{20a^3b^2(dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```


```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (10*a^
4*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2)) + (20*
a^3*b^2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^5*(a + b*x^2)) +
(20*a^2*b^3*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^
2)) + (2*a*b^4*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^9*(a + b*
x^2)) + (2*b^5*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^11*(a +
b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{(dx)^{3/2}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{3/2}}+\frac{5a^4b^6\sqrt{dx}}{d^2}+\frac{10a^3b^7(dx)^{5/2}}{d^4}+\frac{10a^2b^8(dx)^{9/2}}{d^6}+\frac{5ab^9(dx)^{13/2}}{d^8}+\frac{b^{10}(dx}{d^{10}}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}+\frac{10a^4b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{7/2}\sqrt{a^2+2ab}}{7d^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0326072, size = 88, normalized size = 0.3

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(3990a^2b^3x^6+6270a^3b^2x^4+7315a^4bx^2-4389a^5+1463ab^4x^8+231b^5x^{10}\right)}{4389(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-4389*a^5 + 7315*a^4*b*x^2 + 6270*a^3*b^2*x^4 + 3
990*a^2*b^3*x^6 + 1463*a*b^4*x^8 + 231*b^5*x^10))/(4389*(d*x)^(3/2)*(a + b*
x^2))
```

Maple [A] time = 0.167, size = 83, normalized size = 0.3

$$-\frac{2\left(-231\,b^5x^{10}-1463\,ab^4x^8-3990\,a^2b^3x^6-6270\,b^2a^3x^4-7315\,a^4bx^2+4389\,a^5\right)x}{4389\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x)
```

```
[Out] -2/4389*x*(-231*b^5*x^10-1463*a*b^4*x^8-3990*a^2*b^3*x^6-6270*a^3*b^2*x^4-7
315*a^4*b*x^2+4389*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(3/2)
```

Maxima [A] time = 1.04978, size = 204, normalized size = 0.69

$$\frac{2\left(77\left(15\,b^5\sqrt{d}x^3+19\,ab^4\sqrt{d}x\right)x^{\frac{13}{2}}+532\left(11\,ab^4\sqrt{d}x^3+15\,a^2b^3\sqrt{d}x\right)x^{\frac{9}{2}}+1710\left(7\,a^2b^3\sqrt{d}x^3+11\,a^3b^2\sqrt{d}x\right)x^{\frac{5}{2}}+4180\left(3\right.}{21945\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21945*(77*(15*b^5*sqrt(d)*x^3 + 19*a*b^4*sqrt(d)*x)*x^(13/2) + 532*(11*a*
b^4*sqrt(d)*x^3 + 15*a^2*b^3*sqrt(d)*x)*x^(9/2) + 1710*(7*a^2*b^3*sqrt(d)*x
^3 + 11*a^3*b^2*sqrt(d)*x)*x^(5/2) + 4180*(3*a^3*b^2*sqrt(d)*x^3 + 7*a^4*b*
sqrt(d)*x)*sqrt(x) + 7315*(a^4*b*sqrt(d)*x^3 - 3*a^5*sqrt(d)*x)/x^(3/2))/d^
2
```

Fricas [A] time = 1.52886, size = 167, normalized size = 0.57

$$\frac{2\left(231\,b^5x^{10}+1463\,ab^4x^8+3990\,a^2b^3x^6+6270\,a^3b^2x^4+7315\,a^4bx^2-4389\,a^5\right)\sqrt{dx}}{4389\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/4389*(231*b^5*x^10 + 1463*a*b^4*x^8 + 3990*a^2*b^3*x^6 + 6270*a^3*b^2*x^4
 + 7315*a^4*b*x^2 - 4389*a^5)*sqrt(d*x)/(d^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(3/2), x)
```

Giac [A] time = 1.2793, size = 211, normalized size = 0.72

$$-\frac{2\left(\frac{4389\,a^5\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}-\frac{231\,\sqrt{dx}b^5d^{189}x^9\mathrm{sgn}\left(bx^2+a\right)+1463\,\sqrt{dx}ab^4d^{189}x^7\mathrm{sgn}\left(bx^2+a\right)+3990\,\sqrt{dx}a^2b^3d^{189}x^5\mathrm{sgn}\left(bx^2+a\right)+6270\,\sqrt{dx}a^3b^2d^{189}x^3\mathrm{sgn}\left(bx^2+\right.}{d^{190}}\right.}{4389\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/4389*(4389*a^5*sgn(b*x^2 + a)/sqrt(d*x) - (231*sqrt(d*x)*b^5*d^189*x^9*s
gn(b*x^2 + a) + 1463*sqrt(d*x)*a*b^4*d^189*x^7*sgn(b*x^2 + a) + 3990*sqrt(d
*x)*a^2*b^3*d^189*x^5*sgn(b*x^2 + a) + 6270*sqrt(d*x)*a^3*b^2*d^189*x^3*sgn
(b*x^2 + a) + 7315*sqrt(d*x)*a^4*b*d^189*x*sgn(b*x^2 + a))/d^190)/d
```

3.747 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=293

$$\frac{2b^5(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{5/2}\sqrt{}}{d^5\left(\right.}$$

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (1
0*a^4*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (4*a
^3*b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(a + b*x^2)) + (20
*a^2*b^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2)) +
 (10*a*b^4*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^9*(a + b*x^2
)) + (2*b^5*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^11*(a + b*x
^2))
```

Rubi [A] time = 0.0819537, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{5/2}\sqrt{}}{d^5\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```



```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (1
0*a^4*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (4*a
^3*b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(a + b*x^2)) + (20
*a^2*b^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2)) +
 (10*a*b^4*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^9*(a + b*x^2
)) + (2*b^5*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^11*(a + b*x
^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{(dx)^{5/2}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{5/2}}+\frac{5a^4b^6}{d^2\sqrt{dx}}+\frac{10a^3b^7(dx)^{3/2}}{d^4}+\frac{10a^2b^8(dx)^{7/2}}{d^6}+\frac{5ab^9(dx)^{11/2}}{d^8}+\frac{b^{10}(dx)^{15/}}{d^{10}}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{10a^4b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{5/2}\sqrt{a^2+2abx^2}}{d^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0378307, size = 88, normalized size = 0.3

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(2210a^2b^3x^6+3978a^3b^2x^4+9945a^4bx^2-663a^5+765ab^4x^8+117b^5x^{10}\right)}{1989(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-663*a^5 + 9945*a^4*b*x^2 + 3978*a^3*b^2*x^4 + 22
10*a^2*b^3*x^6 + 765*a*b^4*x^8 + 117*b^5*x^10))/(1989*(d*x)^(5/2)*(a + b*x^
2))
```

Maple [A] time = 0.178, size = 83, normalized size = 0.3

$$-\frac{2\left(-117\,b^5x^{10}-765\,ab^4x^8-2210\,a^2b^3x^6-3978\,b^2a^3x^4-9945\,a^4bx^2+663\,a^5\right)x}{1989\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x)
```

```
[Out] -2/1989*x*(-117*b^5*x^10-765*a*b^4*x^8-2210*a^2*b^3*x^6-3978*a^3*b^2*x^4-99
45*a^4*b*x^2+663*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(5/2)
```

Maxima [A] time = 1.02199, size = 204, normalized size = 0.7

$$\frac{2\left(45\left(13\,b^5\sqrt{d}x^3+17\,ab^4\sqrt{d}x\right)x^{\frac{11}{2}}+340\left(9\,ab^4\sqrt{d}x^3+13\,a^2b^3\sqrt{d}x\right)x^{\frac{7}{2}}+1326\left(5\,a^2b^3\sqrt{d}x^3+9\,a^3b^2\sqrt{d}x\right)x^{\frac{3}{2}}+\frac{7956\left(a^3b^2\sqrt{\ldots}\right.}{\ldots}\right.}{9945\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9945*(45*(13*b^5*sqrt(d)*x^3 + 17*a*b^4*sqrt(d)*x)*x^(11/2) + 340*(9*a*b^
4*sqrt(d)*x^3 + 13*a^2*b^3*sqrt(d)*x)*x^(7/2) + 1326*(5*a^2*b^3*sqrt(d)*x^3
 + 9*a^3*b^2*sqrt(d)*x)*x^(3/2) + 7956*(a^3*b^2*sqrt(d)*x^3 + 5*a^4*b*sqrt(
d)*x)/sqrt(x) + 3315*(3*a^4*b*sqrt(d)*x^3 - a^5*sqrt(d)*x)/x^(5/2))/d^3
```

Fricas [A] time = 1.47365, size = 167, normalized size = 0.57

$$\frac{2\left(117\,b^5x^{10}+765\,ab^4x^8+2210\,a^2b^3x^6+3978\,a^3b^2x^4+9945\,a^4bx^2-663\,a^5\right)\sqrt{dx}}{1989\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*b^5*x^10 + 765*a*b^4*x^8 + 2210*a^2*b^3*x^6 + 3978*a^3*b^2*x^4
+ 9945*a^4*b*x^2 - 663*a^5)*sqrt(d*x)/(d^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(5/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(5/2), x)
```

Giac [A] time = 1.14683, size = 215, normalized size = 0.73

$$-\frac{2\left(\frac{663\,a^5 d\,\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}\,x}-\frac{117\,\sqrt{dx}b^5d^{136}x^8\mathrm{sgn}\left(bx^2+a\right)+765\,\sqrt{dx}ab^4d^{136}x^6\mathrm{sgn}\left(bx^2+a\right)+2210\,\sqrt{dx}a^2b^3d^{136}x^4\mathrm{sgn}\left(bx^2+a\right)+3978\,\sqrt{dx}a^3b^2d^{136}x^2\mathrm{sgn}\left(bx^2+a\right.}{d^{136}}\right.}{1989\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/1989*(663*a^5*d*sgn(b*x^2 + a)/(sqrt(d*x)*x) - (117*sqrt(d*x)*b^5*d^136*
x^8*sgn(b*x^2 + a) + 765*sqrt(d*x)*a*b^4*d^136*x^6*sgn(b*x^2 + a) + 2210*sq
rt(d*x)*a^2*b^3*d^136*x^4*sgn(b*x^2 + a) + 3978*sqrt(d*x)*a^3*b^2*d^136*x^2
*sgn(b*x^2 + a) + 9945*sqrt(d*x)*a^4*b*d^136*sgn(b*x^2 + a))/d^136)/d^3
```

3.748 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=295

$$\frac{2b^5(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{3/2}}{3d^5}$$

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (1
0*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2)) + (20*
a^3*b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2)) +
(20*a^2*b^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2)
) + (10*a*b^4*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^9*(a + b*
x^2)) + (2*b^5*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^11*(a +
b*x^2))
```

Rubi [A] time = 0.0792314, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1112, 270}

$$\frac{2b^5(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{3/2}}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (1
0*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2)) + (20*
a^3*b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2)) +
(20*a^2*b^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2)
) + (10*a*b^4*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^9*(a + b*
x^2)) + (2*b^5*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^11*(a +
b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{(dx)^{7/2}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{7/2}}+\frac{5a^4b^6}{d^2(dx)^{3/2}}+\frac{10a^3b^7\sqrt{dx}}{d^4}+\frac{10a^2b^8(dx)^{5/2}}{d^6}+\frac{5ab^9(dx)^{9/2}}{d^8}+\frac{b^{10}(dx)^{13/2}}{d^{10}}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}-\frac{10a^4b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{3/2}\sqrt{a^2+2abx^2+b}}{3d^5\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0355773, size = 88, normalized size = 0.3

$$\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(1650a^2b^3x^6+3850a^3b^2x^4-5775a^4bx^2-231a^5+525ab^4x^8+77b^5x^{10}\right)}{1155(dx)^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-231*a^5 - 5775*a^4*b*x^2 + 3850*a^3*b^2*x^4 + 16
50*a^2*b^3*x^6 + 525*a*b^4*x^8 + 77*b^5*x^10))/(1155*(d*x)^(7/2)*(a + b*x^2
))
```

Maple [A] time = 0.171, size = 83, normalized size = 0.3

$$-\frac{2\left(-77\,b^5x^{10}-525\,ab^4x^8-1650\,a^2b^3x^6-3850\,b^2a^3x^4+5775\,a^4bx^2+231\,a^5\right)x}{1155\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x)
```

```
[Out] -2/1155*x*(-77*b^5*x^10-525*a*b^4*x^8-1650*a^2*b^3*x^6-3850*a^3*b^2*x^4+577
5*a^4*b*x^2+231*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(7/2)
```

Maxima [A] time = 1.01593, size = 203, normalized size = 0.69

$$\frac{2\left(7\left(11\,b^5\sqrt{d}x^3+15\,ab^4\sqrt{d}x\right)x^{\frac{9}{2}}+60\left(7\,ab^4\sqrt{d}x^3+11\,a^2b^3\sqrt{d}x\right)x^{\frac{5}{2}}+330\left(3\,a^2b^3\sqrt{d}x^3+7\,a^3b^2\sqrt{d}x\right)\sqrt{x}+\frac{1540\left(a^3b^2\sqrt{d}x^3-\right.}{x^{\frac{3}{2}}}\right.}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(7*(11*b^5*sqrt(d)*x^3 + 15*a*b^4*sqrt(d)*x)*x^(9/2) + 60*(7*a*b^4*s
qrt(d)*x^3 + 11*a^2*b^3*sqrt(d)*x)*x^(5/2) + 330*(3*a^2*b^3*sqrt(d)*x^3 + 7
*a^3*b^2*sqrt(d)*x)*sqrt(x) + 1540*(a^3*b^2*sqrt(d)*x^3 - 3*a^4*b*sqrt(d)*x
)/x^(3/2) - 231*(5*a^4*b*sqrt(d)*x^3 + a^5*sqrt(d)*x)/x^(7/2))/d^4
```

Fricas [A] time = 1.49153, size = 166, normalized size = 0.56

$$\frac{2\left(77\,b^5x^{10}+525\,ab^4x^8+1650\,a^2b^3x^6+3850\,a^3b^2x^4-5775\,a^4bx^2-231\,a^5\right)\sqrt{dx}}{1155\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^5*x^10 + 525*a*b^4*x^8 + 1650*a^2*b^3*x^6 + 3850*a^3*b^2*x^4 -
 5775*a^4*b*x^2 - 231*a^5)*sqrt(d*x)/(d^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26529, size = 219, normalized size = 0.74

$$-\frac{2\left(\frac{231\,\left(25\,a^4bd^3x^2\mathrm{sgn}\left(bx^2+a\right)+a^5d^3\mathrm{sgn}\left(bx^2+a\right)\right)}{\sqrt{dx}d^2x^2}-\frac{77\,\sqrt{dx}b^5d^{105}x^7\mathrm{sgn}\left(bx^2+a\right)+525\,\sqrt{dx}ab^4d^{105}x^5\mathrm{sgn}\left(bx^2+a\right)+1650\,\sqrt{dx}a^2b^3d^{105}x^3\mathrm{sgn}\left(bx^2+a\right)+3850\cdots}{d^{105}}\right.}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/1155*(231*(25*a^4*b*d^3*x^2*sgn(b*x^2 + a) + a^5*d^3*sgn(b*x^2 + a))/(sq
rt(d*x)*d^2*x^2) - (77*sqrt(d*x)*b^5*d^105*x^7*sgn(b*x^2 + a) + 525*sqrt(d*
x)*a*b^4*d^105*x^5*sgn(b*x^2 + a) + 1650*sqrt(d*x)*a^2*b^3*d^105*x^3*sgn(b*
x^2 + a) + 3850*sqrt(d*x)*a^3*b^2*d^105*x*sgn(b*x^2 + a))/d^105)/d^4
```

3.749 $\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=457

$$-\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}b^{9/4}\sqrt{a^2+2ab}}$$

```
[Out] (-2*a*d^3*Sqrt[d*x]*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2
*d*(d*x)^(5/2)*(a + b*x^2))/(5*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4
)*d^(7/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqr
t[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4)*d^(7/2)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.324991, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}b^{9/4}\sqrt{a^2+2ab}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```


```
[Out] (-2*a*d^3*Sqrt[d*x]*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2
*d*(d*x)^(5/2)*(a + b*x^2))/(5*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4
)*d^(7/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqr
t[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4)*d^(7/2)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1
```

```
/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a^2d^4\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(2a^2d^3\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\right.}{b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a^{3/2}d^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}}{ab+\frac{b^2x^4}{d^2}}\right.}{b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a^{5/4}d^{7/2}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{2}}{-\frac{\sqrt{a}d}{\sqrt{b}}-}\right.}{2\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\right.}{2\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+}}$$

$$= -\frac{2ad^3\sqrt{dx}\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}\left(a+bx^2\right)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{5/4}d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.091201, size = 238, normalized size = 0.52

$$\frac{d^3\sqrt{dx}\left(a+bx^2\right)\left(-5\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-10\sqrt{2}a^{5/4}\tan\right.}{20b^{9/4}\sqrt{x}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (d^3*Sqrt[d*x]*(a + b*x^2)*(-40*a*b^(1/4)*Sqrt[x] + 8*b^(5/4)*x^(5/2) - 10*
Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 10*Sqrt[2]*
```

```
a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 5*Sqrt[2]*a^(5/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*a^(5/
4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(20*b^(9/4)
*Sqrt[x]*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.227, size = 239, normalized size = 0.5

$$\frac{(bx^2+a)\,d}{20\,b^2}\left(5\,ad^2\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+10\,ad^2\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/20*(b*x^2+a)*d*(5*a*d^2*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2)))+10*a*d^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+10*a*d^2*(a*d^2/b)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+8*(d*x)^(5/2)*b
-40*a*d^2*(d*x)^(1/2))/((b*x^2+a)^2)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{7}{2}}}{\sqrt{\left(bx^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(7/2)/sqrt((b*x^2 + a)^2), x)
```

Fricas [A] time = 1.59594, size = 498, normalized size = 1.09

$$\frac{20\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\arctan\left(-\frac{\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{dx}ab^7d^3-\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{a^2d^7x+\sqrt{-\frac{a^5d^{14}}{b^9}}b^4}b^7}{a^5d^{14}}\right)+5\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\log\left(\sqrt{dx}ad^3+\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)-}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*(-a^5*d^14/b^9)^(1/4)*b^2*arctan(-((-a^5*d^14/b^9)^(3/4)*sqrt(d*x)
*a*b^7*d^3 - (-a^5*d^14/b^9)^(3/4)*sqrt(a^2*d^7*x + sqrt(-a^5*d^14/b^9)*b^4
)*b^7)/(a^5*d^14)) + 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 + (-a^
5*d^14/b^9)^(1/4)*b^2) - 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 -
(-a^5*d^14/b^9)^(1/4)*b^2) + 4*(b*d^3*x^2 - 5*a*d^3)*sqrt(d*x))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25957, size = 374, normalized size = 0.82

$$\frac{1}{20}d^2\left(\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}+\frac{5\sqrt{2}\left(ab^3d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*d^2*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 + 10*sqrt(2)*(a*b^3*d^2)^(1
/4)*a*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/b^3 + 5*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*log(d*x + sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*log
(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 8*(sqrt(d*x
)*b^4*d^6*x^2 - 5*sqrt(d*x)*a*b^3*d^6)/(b^5*d^5))*sgn(b*x^2 + a)
```

3.750 $\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=412

$$-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (2*d*(d*x)^(3/2)*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3
/4)*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2
)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/4)*d^(5/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.286151, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```


```
[Out] (2*d*(d*x)^(3/2)*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3
/4)*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2
)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/4)*d^(5/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2ad\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ad\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad\left(ab+b^2x^2\right)\right.}{b^{3/2}}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a^{3/4}d^{5/2}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{2}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a^{3/}\right.}{}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^{3/4}d^{5/}}{}$$

$$= \frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\right.}{\sqrt{2}b^{7/4}\sqrt{a^2+2abx}}$$

Mathematica [A] time = 0.0538894, size = 110, normalized size = 0.27

$$\frac{(dx)^{5/2}\left(a+bx^2\right)\left(3(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)-3(-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+2b^{3/4}x^{3/2}\right)}{3b^{7/4}x^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(5/2)*(a + b*x^2)*(2*b^(3/4)*x^(3/2) + 3*(-a)^(3/4)*ArcTan[(b^(1/4)*
Sqrt[x])/(-a)^(1/4)] - 3*(-a)^(3/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)]))
/(3*b^(7/4)*x^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.223, size = 221, normalized size = 0.5

$$\frac{\left(bx^2+a\right)d}{12\,b^2}\left(8\,(dx)^{3/2}\,b\sqrt[4]{\frac{ad^2}{b}}-3\,ad^2\sqrt{2}\ln\left(-\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)-6\,ad^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/12*(b*x^2+a)*d*(8*(d*x)^(3/2)*b*(a*d^2/b)^(1/4)-3*a*d^2*2^(1/2)*ln(-((a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))-6*a*d^2*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-6*a*d^2*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)))/((b*x^2+a)^2)^(1/2)/b^2/(a*d^2/
b)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^{\frac{5}{2}}}{\sqrt{\left(bx^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(5/2)/sqrt((b*x^2 + a)^2), x)
```

Fricas [A] time = 1.63825, size = 486, normalized size = 1.18

$$\frac{4\sqrt{dx}d^2x+12\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}\sqrt{dx}a^2b^2d^7-\sqrt{a^4d^{15}x-\sqrt{-\frac{a^3d^{10}}{b^7}}a^3b^3d^{10}}\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b^2}{a^3d^{10}}\right)-3\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}a^2d^7\right.}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(d*x)*d^2*x + 12*(-a^3*d^10/b^7)^(1/4)*b*arctan(-((-a^3*d^10/b^7
)^(1/4)*sqrt(d*x)*a^2*b^2*d^7 - sqrt(a^4*d^15*x - sqrt(-a^3*d^10/b^7)*a^3*b
^3*d^10)*(-a^3*d^10/b^7)^(1/4)*b^2)/(a^3*d^10)) - 3*(-a^3*d^10/b^7)^(1/4)*b
*log(sqrt(d*x)*a^2*d^7 + (-a^3*d^10/b^7)^(3/4)*b^5) + 3*(-a^3*d^10/b^7)^(1/
4)*b*log(sqrt(d*x)*a^2*d^7 - (-a^3*d^10/b^7)^(3/4)*b^5))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29573, size = 325, normalized size = 0.79

$$\frac{1}{12}\left(\frac{8\sqrt{dx}dx}{b} - \frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4} - \frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4} + 3\sqrt{2}\left(ab^3d^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(8*sqrt(d*x)*d*x/b - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^4 - 6*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/b^4 + 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/
b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^4 - 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d
```

```
*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^4)*d*sgn(b*x^2 +
a)
```

3.751 $\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=410

$$\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\cdots$$

```
[Out] (2*d*Sqrt[d*x]*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(1/4)*
d^(3/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3/2)*(a
 + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[
2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(1/4)*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3/2)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.27544, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*d*Sqrt[d*x]*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(1/4)*
d^(3/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3/2)*(a
 + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[
2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(1/4)*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3/2)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ad^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2ad\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\sqrt{a}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\sqrt{a}\left(ab+b^2x^2\right.\right.}{b}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt[4]{a}d^{3/2}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt[4]{\right.}{}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{a}d^{3/2}}{}$$

$$= \frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\right.}{\sqrt{2}b^{5/4}\sqrt{a^2+2abx^{2}}}$$

Mathematica [A] time = 0.0732777, size = 221, normalized size = 0.54

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\right.\right.}{4b^{5/4}x^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(8*b^(1/4)*Sqrt[x] + 2*Sqrt[2]*a^(1/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)] + Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
```

```
*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(4*b^(5/4)*x^(3/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.227, size = 214, normalized size = 0.5

$$-\frac{\left(bx^2+a\right)d}{4b}\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+2\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\arctan\left(\left(\sqrt{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)*d*((a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2
/b)^(1/2)))+2*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))+2*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-8*(d*x)^(1/2))/((b*x^2+a)^2)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^{\frac{3}{2}}}{\sqrt{\left(bx^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)/sqrt((b*x^2 + a)^2), x)
```

Fricas [A] time = 1.58735, size = 390, normalized size = 0.95

$$-\frac{4\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{ad^6}{b^5}\right)^{\frac{3}{4}}\sqrt{dx}b^4d-\sqrt{d^3x+\sqrt{-\frac{ad^6}{b^5}}b^2}\left(-\frac{ad^6}{b^5}\right)^{\frac{3}{4}}b^4}{ad^6}\right)+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}d+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\right)-\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\sqrt{dxd}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(-a*d^6/b^5)^(1/4)*b*arctan(-((-a*d^6/b^5)^(3/4)*sqrt(d*x)*b^4*d -
sqrt(d^3*x + sqrt(-a*d^6/b^5)*b^2)*(-a*d^6/b^5)^(3/4)*b^4)/(a*d^6)) + (-a*d
^6/b^5)^(1/4)*b*log(sqrt(d*x)*d + (-a*d^6/b^5)^(1/4)*b) - (-a*d^6/b^5)^(1/4
)*b*log(sqrt(d*x)*d - (-a*d^6/b^5)^(1/4)*b) - 4*sqrt(d*x)*d)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27476, size = 327, normalized size = 0.8

$$-\frac{1}{4}\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^2 + 2*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/
b^2 + sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*
```

```
x) + sqrt(a*d^2/b))/b^2 - sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x - sqrt(2)*(a*
d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^2 - 8*sqrt(d*x)*d/b)*sgn(b*x^2 +
a)
```

3.752 $\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=368

$$\frac{\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a\ \cdots\right.}{\sqrt{2}\sqrt[4]{\cdots}}$$

```
[Out] -((Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (Sqrt[d
]*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (Sqrt[d]*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[
d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi [A] time = 0.246074, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1112, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a\ \cdots\right.}{\sqrt{2}\sqrt[4]{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```


```
[Out] -((Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (Sqrt[d
]*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (Sqrt[d]*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[
d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{\sqrt{b}d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}d+\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{\sqrt{b}d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(\sqrt{d}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt{d}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\cdots}{-\frac{\sqrt{a}\cdots}{\sqrt{b}}\cdots}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}\sqrt{a^2+2\cdots}}$$

$$= \frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\cdots\right.}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2\cdots}}$$

$$= -\frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{d}\left(a+bx^2\right)\cdots}{2\sqrt{\cdots}}$$

Mathematica [A] time = 0.0452284, size = 85, normalized size = 0.23

$$\frac{\sqrt{dx}\left(a+bx^2\right)\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+\tanh^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)\right)}{\sqrt[4]{-a}b^{3/4}\sqrt{x}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[d*x]*(a + b*x^2)*(ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + ArcTanh[(a*b
^(1/4)*Sqrt[x])/(-a)^(5/4)]))/((-a)^(1/4)*b^(3/4)*Sqrt[x]*Sqrt[(a + b*x^2)^
2])
```

Maple [A] time = 0.225, size = 183, normalized size = 0.5

$$\frac{d\left(bx^2+a\right)\sqrt{2}}{4b}\left(\ln\left(-\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+2\arctan\left(\left(\sqrt{2}\sqrt{dx}+\sqrt[4]{\frac{ad^2}{b}}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)*d/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln(-((a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(
1/4))/(a*d^2/b)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\sqrt{\left(bx^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)/sqrt((b*x^2 + a)^2), x)
```

Fricas [A] time = 1.62866, size = 390, normalized size = 1.06

$$-2\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}bd\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}-\sqrt{-abd^2\sqrt{-\frac{d^2}{ab^3}}+d^3x}b\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}}{d^2}\right)+\frac{1}{2}\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}d\right)-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-d^2/(a*b^3))^(1/4)*arctan(-(sqrt(d*x)*b*d*(-d^2/(a*b^3))^(1/4) - sqrt(
-a*b*d^2*sqrt(-d^2/(a*b^3)) + d^3*x)*b*(-d^2/(a*b^3))^(1/4))/d^2) + 1/2*(-d
^2/(a*b^3))^(1/4)*log(a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d) - 1/2*(-d^2
/(a*b^3))^(1/4)*log(-a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d)
```

Sympy [A] time = 152.039, size = 41, normalized size = 0.11

$$2d\,\mathrm{RootSum}\left(256t^4ab^3d^2+1,\left(t\mapsto t\log\left(64t^3ab^2d^2+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] 2*d*RootSum(256*_t**4*a*b**3*d**2 + 1, Lambda(_t, _t*log(64*_t**3*a*b**2*d*
*2 + sqrt(d*x))))
```

Giac [A] time = 1.28289, size = 339, normalized size = 0.92

$$\frac{1}{4}\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3d}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3d}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\right.}{ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*d) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))
/(a*b^3*d) - sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a*b^3*d) + sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3*d))*sgn(b*x^2 + a)
```

3.753 $\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=368

$$-\frac{\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\text{ta}}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}}$$

```
[Out] -(((a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + ((a + b
*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a
^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi [A] time = 0.237599, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1112, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\text{ta}}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(((a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + ((a + b
*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a
^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{\sqrt{a}d^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\sqrt{a}d+\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{\right.}{\sqrt{a}d^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}\right.}{2\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+}$$

$$= -\frac{\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2ab}$$

$$= -\frac{\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a+bx^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a+bx^2\right)\text{lo}}{2\sqrt{2}}$$

Mathematica [A] time = 0.0492538, size = 178, normalized size = 0.48

$$-\frac{\sqrt{x}\left(a+bx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\tan^{-1}\left(\right.\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(Sqrt[x]*(a + b*x^2)*(2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x]))/(2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d*x]*Sqrt[(a + b*x^2)^2
])
```

Maple [A] time = 0.227, size = 182, normalized size = 0.5

$$\frac{(bx^2+a)\sqrt{2}}{4\,ad}\sqrt[4]{\frac{ad^2}{b}}\left(\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+2\arctan\left(\left(\sqrt{2}\sqrt{dx}+\sqrt[4]{\frac{ad^2}{b}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/d*(a*d^2/b)^(1/4)/a*2^(1/2)*(ln((d*x+(a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1
/4))/(a*d^2/b)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx^2+a)^2}\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)^2)*sqrt(d*x)), x)
```

Fricas [A] time = 1.63144, size = 400, normalized size = 1.09

$$2\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2d^2\sqrt{-\frac{1}{a^3bd^2}}+dx}a^2bd\left(-\frac{1}{a^3bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}a^2bd\left(-\frac{1}{a^3bd^2}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(a^3*b*d^2))^(1/4)*arctan(sqrt(a^2*d^2*sqrt(-1/(a^3*b*d^2)) + d*x)*a^
2*b*d*(-1/(a^3*b*d^2))^(3/4) - sqrt(d*x)*a^2*b*d*(-1/(a^3*b*d^2))^(3/4)) +
1/2*(-1/(a^3*b*d^2))^(1/4)*log(a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x)) - 1/
2*(-1/(a^3*b*d^2))^(1/4)*log(-a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29153, size = 339, normalized size = 0.92

$$\frac{1}{4}\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{abd}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{abd}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b*d) + 2*sqrt(2)*(a*b^3*d^2)^(1/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(
a*b*d) + sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a*b*d) - sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b*d))*sgn(b*x^2 + a)
```

3.754 $\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=412

$$-\frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\left(a\cdots\right.}{\sqrt{2}\cdots}$$

```
[Out] (-2*(a + b*x^2))/(a*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sq
rt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(5
/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*S
qrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.278618, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\left(a\cdots\right.}{\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(a*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sq
rt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(5
/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*S
qrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2b\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{ad^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt{b}\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{ad^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(\sqrt{}}{}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{2}a^{5/4}b^{3/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(}{}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{b}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{b}\left(a+bx^2\right)\tan}{\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2}}$$

Mathematica [C] time = 0.0127298, size = 50, normalized size = 0.12

$$-\frac{2x\left(a+bx^2\right){}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{bx^2}{a}\right)}{a(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*x*(a + b*x^2)*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^2)/a)])/(a*(d*x)^(
3/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.227, size = 224, normalized size = 0.5

$$-\frac{bx^2+a}{4\,ad}\left(\sqrt{2}\sqrt{dx}\ln\left(-\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+2\sqrt{2}\sqrt{dx}\arctan\left(\left(\sqrt{2}\sqrt{dx}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)/d*(2^(1/2)*(d*x)^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))+2*2^(1/2)*(d*x)^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))+2*2^(1/2)*(d*x)^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))+8*(a*d^2/b)^(1/4))/((b*x^2+a)^2)^(1/2)/a/(a*d^2/
b)^(1/4)/(d*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(bx^2+a\right)^2}\,(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(3/2)), x)
```

Fricas [A] time = 1.61482, size = 464, normalized size = 1.13

$$\frac{4\,ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}abd\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}-\sqrt{-a^3bd^4\sqrt{-\frac{b}{a^5d^6}}+b^2dxad\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}}}{b}\right)-ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\log\left(a^4d^5\left(-\frac{b}{a^5d^6}\right)^{\frac{3}{4}}+\sqrt{dx}b\right)+a}{2\,ad^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*d^2*x*(-b/(a^5*d^6))^(1/4)*arctan(-(sqrt(d*x)*a*b*d*(-b/(a^5*d^6))
^(1/4) - sqrt(-a^3*b*d^4*sqrt(-b/(a^5*d^6)) + b^2*d*x)*a*d*(-b/(a^5*d^6))^(
1/4))/b) - a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(a^4*d^5*(-b/(a^5*d^6))^(3/4) +
sqrt(d*x)*b) + a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(-a^4*d^5*(-b/(a^5*d^6))^(3/
4) + sqrt(d*x)*b) - 4*sqrt(d*x))/(a*d^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16243, size = 356, normalized size = 0.86

$$-\frac{\frac{8}{\sqrt{dx}a} + \frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2} + \frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2} - \frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^2d^2} +}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(8/(sqrt(d*x)*a) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^2) + 2*sqrt
(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt
(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^2) - sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x +
 sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*d^2) + sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^2*b^2*d^2))*sgn(b*x^2 + a)/d
```

3.755 $\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=414

$$\frac{b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}\left(a\right.}{\sqrt{2}a}$$

```
[Out] (-2*(a + b*x^2))/(3*a*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(
3/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])
/(Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)*(a +
b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*
a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/4)*(a + b*x^2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)*(a +
 b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.276258, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}\left(a\right.}{\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```


```
[Out] (-2*(a + b*x^2))/(3*a*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(
3/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])
/(Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)*(a +
b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*
a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/4)*(a + b*x^2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)*(a +
 b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(2b\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{ad^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}d-\sqrt{b}x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{a^{3/2}d^4\sqrt{a^2+2abx^2+b^2x^4}} - \cdots$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(ab+b^2x^2\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{ad}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} \cdots$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\cdots\right.}{2\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} \cdots$$

$$= -\frac{2\left(a+bx^2\right)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b^{3/4}\left(a+bx^2\cdots\right.}{\sqrt{2}a^{7/4}d^{5/2}\cdots}$$

Mathematica [C] time = 0.0140528, size = 52, normalized size = 0.13

$$-\frac{2x\left(a+bx^2\right)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*x*(a + b*x^2)*Hypergeometric2F1[-3/4, 1, 1/4, -((b*x^2)/a)])/(3*a*(d*x)
^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.227, size = 239, normalized size = 0.6

$$-\frac{bx^2+a}{12\,d^3a^2}\left(3\,b\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\,(dx)^{3/2}\ln\left(\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+6\,b\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\,(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/12*(b*x^2+a)/d^3*(3*b*(a*d^2/b)^(1/4)*2^(1/2)*(d*x)^(3/2)*ln((d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+6*b*(a*d^2/b)^(1/4)*2^(1/2)*(d*x)^(3/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+6*b*(a*d^2/b)^(1/
4)*2^(1/2)*(d*x)^(3/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))+8*a*d^2)/((b*x^2+a)^2)^(1/2)/a^2/(d*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(bx^2+a\right)^2}\,(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(5/2)), x)
```

Fricas [A] time = 1.65694, size = 525, normalized size = 1.27

$$-\frac{12\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^5bd^7\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^4d^6\sqrt{-\frac{b^3}{a^7d^{10}}}+b^2dxa^5d^7\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{3}{4}}}}{b^3}\right)+3\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\log\left(a^2d^3\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\right.}{6\,ad^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*d^3*x^2*(-b^3/(a^7*d^10))^(1/4)*arctan(-(sqrt(d*x)*a^5*b*d^7*(-b
^3/(a^7*d^10))^(3/4) - sqrt(a^4*d^6*sqrt(-b^3/(a^7*d^10)) + b^2*d*x)*a^5*d^
7*(-b^3/(a^7*d^10))^(3/4))/b^3) + 3*a*d^3*x^2*(-b^3/(a^7*d^10))^(1/4)*log(a
^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) - 3*a*d^3*x^2*(-b^3/(a^7*d^10
))^(1/4)*log(-a^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) + 4*sqrt(d*x))
/(a*d^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27133, size = 346, normalized size = 0.84

$$-\frac{1}{12}\left(\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*d^3) + 6*sqrt(2)*(a*b^3*d^2)^(1/4)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4
))/(a^2*d^3) + 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4
)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) - 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*
x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) + 8/(sqrt(
d*x)*a*d^2*x))*sgn(b*x^2 + a)
```

3.756 $\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=459

$$\frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\right.}{2\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2}$$

```
[Out] (-2*(a + b*x^2))/(5*a*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*b
*(a + b*x^2))/(a^2*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(5/4
)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x
^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(
9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x^2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*
Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(5/4)*(a + b*
x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(2*Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.326733, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\right.}{2\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(5*a*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*b
*(a + b*x^2))/(a^2*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(5/4
)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x
^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(
9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x^2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*
Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(5/4)*(a + b*
x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
```

```
[d*x]])/(2*Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int}{a^2d^4\sqrt{a^2+2abx}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(2b^2\left(ab+b^2x^2\right)\right)}{a^2d^5\sqrt{}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b^{3/2}\left(ab+b^2x^2\right)\right)}{a^2d^5\sqrt{}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt[4]{b}\left(ab+b^2x^2\right)\right)}{2\sqrt{2}a}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{5/4}\left(a+bx^2\right)\log}{2\sqrt{2}a^{9/}}$$

$$= -\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b^{5/4}\left(a+bx^2\right)\tan}{\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2}}$$

Mathematica [C] time = 0.0139664, size = 52, normalized size = 0.11

$$-\frac{2x\left(a+bx^2\right){}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a(dx)^{7/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*x*(a + b*x^2)*Hypergeometric2F1[-5/4, 1, -1/4, -((b*x^2)/a)])/(5*a*(d*x
```

```
)^(7/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.236, size = 251, normalized size = 0.6

$$\frac{bx^2+a}{20\,d^3a^2}\left(5\,b\sqrt{2}\,(dx)^{5/2}\ln\left(-\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+10\,b\sqrt{2}\,(dx)^{5/2}\arctan\left(\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/20*(b*x^2+a)/d^3*(5*b*2^(1/2)*(d*x)^(5/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2
)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2)))+10*b*2^(1/2)*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))+10*b*2^(1/2)*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(
1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+40*b*x^2*d^2*(a*d^2/b)^(1/4)-8*a*d^2
*(a*d^2/b)^(1/4))/((b*x^2+a)^2)^(1/2)/a^2/(d*x)^(5/2)/(a*d^2/b)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\left(bx^2+a\right)^2}\,(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(7/2)), x)
```

Fricas [A] time = 1.69109, size = 574, normalized size = 1.25

$$-\frac{20\,a^2d^4x^3\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^2b^4d^3\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}-\sqrt{-a^5b^5d^8\sqrt{-\frac{b^5}{a^9d^{14}}}+b^8dxa^2d^3\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}}}{b^5}\right)-5\,a^2d^4x^3\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}\log\left(a^7d^{11}\left(-\right.\right.}{10\,a^2d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*d^4*x^3*(-b^5/(a^9*d^14))^(1/4)*arctan(-(sqrt(d*x)*a^2*b^4*d^
3*(-b^5/(a^9*d^14))^(1/4) - sqrt(-a^5*b^5*d^8*sqrt(-b^5/(a^9*d^14)) + b^8*d
*x)*a^2*d^3*(-b^5/(a^9*d^14))^(1/4))/b^5) - 5*a^2*d^4*x^3*(-b^5/(a^9*d^14))
^(1/4)*log(a^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)*b^4) + 5*a^2*d^4*x^
3*(-b^5/(a^9*d^14))^(1/4)*log(-a^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)
*b^4) - 4*(5*b*x^2 - a)*sqrt(d*x))/(a^2*d^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2879, size = 383, normalized size = 0.83

$$\frac{1}{20}\left(\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3bd^5}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3bd^5}-\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*(10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d^5) + 10*sqrt(2)*(a*b^3*d^2)^(3
```

```
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^3*b*d^5) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d^5) + 5*sqrt(2)*(a*b^3*d^2)^(3/4)
*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d^5) +
 8*(5*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^2*d^5*x^2))*sgn(b*x^2 + a)
```

3.757 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=551

$$-\frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117d^5(dx)^{5/2}\left(a+bx^2\right)}{80b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}\right.}{64\sqrt{2}b^{17/4}\sqrt{a^2+2ab\cdots}}$$

```
[Out] (-13*d^3*(d*x)^(9/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(
13/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a*d^7*Sqrt[
d*x]*(a + b*x^2))/(16*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*d^5*(d*x)
^(5/2)*(a + b*x^2))/(80*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)
*d^(15/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*a^(5/4)*
d^(15/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)*d
^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (117*a^(5/4)*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.39872, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117d^5(dx)^{5/2}\left(a+bx^2\right)}{80b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}\right.}{64\sqrt{2}b^{17/4}\sqrt{a^2+2ab\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```


```
[Out] (-13*d^3*(d*x)^(9/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(
13/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a*d^7*Sqrt[
d*x]*(a + b*x^2))/(16*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*d^5*(d*x)
^(5/2)*(a + b*x^2))/(80*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)
*d^(15/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*a^(5/4)*
```

```
d^(15/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)*d
^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (117*a^(5/4)*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(117d^4\left(ab+b^2x^2\right)\right)\int}{32b^2\sqrt{a^2+2abx^2}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{117d^5(dx)^{5/2}\left(a+bx^2\right.}{80b^3\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2}}$$

Mathematica [A] time = 0.170515, size = 498, normalized size = 0.9

$$\frac{d^7\sqrt{dx}\left(-8424a^2b^{5/4}x^{5/2}-585\sqrt{2}a^{5/4}b^2x^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+585\sqrt{2}a^{5/4}b^2x^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (d^7*Sqrt[d*x]*(-4680*a^3*b^(1/4)*Sqrt[x] - 8424*a^2*b^(5/4)*x^(5/2) - 3328
*a*b^(9/4)*x^(9/2) + 256*b^(13/4)*x^(13/2) - 1170*Sqrt[2]*a^(5/4)*(a + b*x^
2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 1170*Sqrt[2]*a^(5/4)*(
a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 585*Sqrt[2]*a^
(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 1170*Sq
rt[2]*a^(9/4)*b*x^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x] - 585*Sqrt[2]*a^(5/4)*b^2*x^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x] + 585*Sqrt[2]*a^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x] + 1170*Sqrt[2]*a^(9/4)*b*x^2*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 585*Sqrt[2]*a^(5/4)*b^2*x^4*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(640*b^(17/4)*Sqrt[x]
*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [B] time = 0.229, size = 737, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/640*(585*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))*x^4*a*b^2*d^2+1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a*b^2*d^2+1170*(a*d^2/b)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a*b^2*
d^2+256*(d*x)^(5/2)*x^4*b^3+1170*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^2*b*d^2+2340*(a*d^2/b)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^2*b*d^2+2340
*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^
```

```
2/b)^(1/4))*x^2*a^2*b*d^2+512*(d*x)^(5/2)*x^2*a*b^2-3840*(d*x)^(1/2)*x^4*a*
b^2*d^2+585*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2)))*a^3*d^2+1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*d^2+1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*d^2-744*(d*x)^(5/
2)*a^2*b-7680*(d*x)^(1/2)*x^2*a^2*b*d^2-4680*(d*x)^(1/2)*a^3*d^2)*d^5*(b*x^
2+a)/b^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72113, size = 774, normalized size = 1.4

$$2340\left(-\frac{a^{5}d^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)\arctan\left(-\frac{\left(-\frac{a^{5}d^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}ab^{13}d^{7}-\left(-\frac{a^{5}d^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{a^{2}d^{15}x+\sqrt{-\frac{a^{5}d^{30}}{b^{17}}}b^{8}}b^{13}}{a^{5}d^{30}}\right)+585\left(-\frac{a^{5}d^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^{6}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/320*(2340*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*arctan
(-((-a^5*d^30/b^17)^(3/4)*sqrt(d*x)*a*b^13*d^7 - (-a^5*d^30/b^17)^(3/4)*sqr
t(a^2*d^15*x + sqrt(-a^5*d^30/b^17)*b^8)*b^13)/(a^5*d^30)) + 585*(-a^5*d^30
/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*log(117*sqrt(d*x)*a*d^7 + 11
7*(-a^5*d^30/b^17)^(1/4)*b^4) - 585*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b
^5*x^2 + a^2*b^4)*log(117*sqrt(d*x)*a*d^7 - 117*(-a^5*d^30/b^17)^(1/4)*b^4)
 + 4*(32*b^3*d^7*x^6 - 416*a*b^2*d^7*x^4 - 1053*a^2*b*d^7*x^2 - 585*a^3*d^7
)*sqrt(d*x))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37969, size = 571, normalized size = 1.04

$$\frac{1}{640}d^6\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{585\sqrt{2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/640*d^6*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sgn(b*d^4*x^2 + a*d^4))
 + 1170*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sgn(b*d^4*x^2 + a*d^4)) + 585*
sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(b^5*sgn(b*d^4*x^2 + a*d^4)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4
)*a*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^5*sgn
(b*d^4*x^2 + a*d^4)) - 40*(25*sqrt(d*x)*a^2*b*d^5*x^2 + 21*sqrt(d*x)*a^3*d^
5)/((b*d^2*x^2 + a*d^2)^2*b^4*sgn(b*d^4*x^2 + a*d^4)) + 256*(sqrt(d*x)*b^12
*d^6*x^2 - 15*sqrt(d*x)*a*b^11*d^6)/(b^15*d^5*sgn(b*d^4*x^2 + a*d^4)))
```

3.758 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=504

$$\frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+$$

```
[Out] (-11*d^3*(d*x)^(7/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(
11/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^5*(d*x)^(3
/2)*(a + b*x^2))/(48*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*a^(3/4)*d^(
13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(32*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13
/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(32*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13/2
)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7
7*a^(3/4)*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rubi [A] time = 0.368843, antiderivative size = 504, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```


```
[Out] (-11*d^3*(d*x)^(7/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(
11/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^5*(d*x)^(3
/2)*(a + b*x^2))/(48*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*a^(3/4)*d^(
13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(32*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13
/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(32*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13/2
```

```
)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7
7*a^(3/4)*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(77d^4\left(ab+b^2x^2\right)\right)\int \frac{(d}{ab-}}{32b^2\sqrt{a^2+2abx^2+}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x}}$$

Mathematica [C] time = 0.0352888, size = 88, normalized size = 0.17

$$-\frac{2d^5(dx)^{3/2}\left(-77a^2+77\left(a+bx^2\right)^2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)-55abx^2-5b^2x^4\right)}{15b^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-2*d^5*(d*x)^(3/2)*(-77*a^2 - 55*a*b*x^2 - 5*b^2*x^4 + 77*(a + b*x^2)^2*Hy
pergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)]))/(15*b^3*(a + b*x^2)*Sqrt[(a +
b*x^2)^2])
```

Maple [B] time = 0.236, size = 679, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/384*(256*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^4*b^3*d^2-231*2^(1/2)*ln(-((a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a*b^2*d^4-462*2^(1/2)*arctan((2^(1/2)
*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a*b^2*d^4-462*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a*b^2*d^4+4
56*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a*b^2+512*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^2*a*
b^2*d^2-462*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^2*b
*d^4-924*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*x^2*a^2*b*d^4-924*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/
(a*d^2/b)^(1/4))*x^2*a^2*b*d^4+616*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^2*b*d^2-23
1*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*
x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*d^4-462*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*d^4-462*
2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*d
^4)*d^3*(b*x^2+a)/(a*d^2/b)^(1/4)/b^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66543, size = 783, normalized size = 1.55

$$\frac{924\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)\arctan\left(-\frac{\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}\sqrt{dx}a^2b^4d^{19}-\sqrt{a^4d^{39}x-\sqrt{-\frac{a^3d^{26}}{b^{15}}}a^3b^7d^{26}}\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}b^4}{a^3d^{26}}\right)-231\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}\left(b^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/192*(924*(-a^3*d^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*arctan(
-((-a^3*d^26/b^15)^(1/4)*sqrt(d*x)*a^2*b^4*d^19 - sqrt(a^4*d^39*x - sqrt(-a
^3*d^26/b^15)*a^3*b^7*d^26)*(-a^3*d^26/b^15)^(1/4)*b^4)/(a^3*d^26)) - 231*(
-a^3*d^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt(d*x
)*a^2*d^19 + 456533*(-a^3*d^26/b^15)^(3/4)*b^11) + 231*(-a^3*d^26/b^15)^(1/
4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt(d*x)*a^2*d^19 - 456533
*(-a^3*d^26/b^15)^(3/4)*b^11) + 4*(32*b^2*d^6*x^5 + 121*a*b*d^6*x^3 + 77*a^
2*d^6*x)*sqrt(d*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31281, size = 524, normalized size = 1.04

$$\frac{1}{384}d^5\left(\frac{256\sqrt{dx}dx}{b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{b^6\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*d^5*(256*sqrt(d*x)*d*x/(b^3*sgn(b*d^4*x^2 + a*d^4)) - 462*sqrt(2)*(a*
b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(b^6*sgn(b*d^4*x^2 + a*d^4)) - 462*sqrt(2)*(a*b^3*d^2)^(3/4
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/
4))/(b^6*sgn(b*d^4*x^2 + a*d^4)) + 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x +
sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*sgn(b*d^4*x^2 + a*d
^4)) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt
(d*x) + sqrt(a*d^2/b))/(b^6*sgn(b*d^4*x^2 + a*d^4)) + 24*(19*sqrt(d*x)*a*b*
d^5*x^3 + 15*sqrt(d*x)*a^2*d^5*x)/((b*d^2*x^2 + a*d^2)^2*b^3*sgn(b*d^4*x^2
+ a*d^4)))
```

3.759 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=504

$$\frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}d^{11/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-$$

```
[Out] (-9*d^3*(d*x)^(5/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(9
/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^5*Sqrt[d*x]*
(a + b*x^2))/(16*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/2
)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(3
2*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^(1/4)*d^(11/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*
Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/2)*(a
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^
(1/4)*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])
```

Rubi [A] time = 0.371432, antiderivative size = 504, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}d^{11/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-9*d^3*(d*x)^(5/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(9
/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^5*Sqrt[d*x]*
(a + b*x^2))/(16*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/2
)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(3
2*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^(1/4)*d^(11/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*
Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/2)*(a
```

```
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^
(1/4)*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(45d^4\left(ab+b^2x^2\right)\right)\int \frac{}{a}}{32b^2\sqrt{a^2+2abx^2+}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2}}$$

Mathematica [A] time = 0.168992, size = 484, normalized size = 0.96

$$\frac{15a^2(dx)^{11/2}\left(a+bx^2\right)}{4b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}} - \frac{15a(dx)^{11/2}\left(a+bx^2\right)^2}{16b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}} + \frac{6a(dx)^{11/2}\left(a+bx^2\right)}{b^2x^3\left(\left(a+bx^2\right)^2\right)^{3/2}} + \frac{45\sqrt[4]{a}(dx)^{11/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{64\sqrt{2}b^{13/4}x^{11/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (15*a^2*(d*x)^(11/2)*(a + b*x^2))/(4*b^3*x^5*((a + b*x^2)^2)^(3/2)) + (6*a*
(d*x)^(11/2)*(a + b*x^2))/(b^2*x^3*((a + b*x^2)^2)^(3/2)) + (2*(d*x)^(11/2)
*(a + b*x^2))/(b*x*((a + b*x^2)^2)^(3/2)) - (15*a*(d*x)^(11/2)*(a + b*x^2)^
2)/(16*b^3*x^5*((a + b*x^2)^2)^(3/2)) + (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2
)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*b^(13/4)*x^(
11/2)*((a + b*x^2)^2)^(3/2)) - (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*b^(13/4)*x^(11/2)*((a
 + b*x^2)^2)^(3/2)) + (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*b^(13/4)*x^(11/2)
*((a + b*x^2)^2)^(3/2)) - (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*b^(13/4)*x^(1
1/2)*((a + b*x^2)^2)^(3/2))
```

Maple [B] time = 0.235, size = 696, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128*(45*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))*x^4*b^2*d^2+90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2+90*(a*d^2/b)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2+90*(a*
d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a*b*d
^2+180*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*x^2*a*b*d^2+180*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^2-256*(d*x)^(1/2)*x^
4*b^2*d^2+45*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*a^2*d^2+90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2+90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2-136*(d*x)^(5/2)*
a*b-512*(d*x)^(1/2)*x^2*a*b*d^2-360*(d*x)^(1/2)*a^2*d^2)*d^3*(b*x^2+a)/b^3/
```

```
((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66702, size = 693, normalized size = 1.38

$$-\frac{180\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)\arctan\left(-\frac{\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{ad^{22}}{b^{13}}}b^6}\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{3}{4}}b^{10}}{ad^{22}}\right)+45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(180*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*arctan(-(
(-a*d^22/b^13)^(3/4)*sqrt(d*x)*b^10*d^5 - sqrt(d^11*x + sqrt(-a*d^22/b^13)*
b^6)*(-a*d^22/b^13)^(3/4)*b^10)/(a*d^22)) + 45*(-a*d^22/b^13)^(1/4)*(b^5*x^
4 + 2*a*b^4*x^2 + a^2*b^3)*log(45*sqrt(d*x)*d^5 + 45*(-a*d^22/b^13)^(1/4)*b
^3) - 45*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(45*sqrt
(d*x)*d^5 - 45*(-a*d^22/b^13)^(1/4)*b^3) - 4*(32*b^2*d^5*x^4 + 81*a*b*d^5*x
^2 + 45*a^2*d^5)*sqrt(d*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39015, size = 527, normalized size = 1.05

$$-\frac{1}{128}\,d^4\left(\frac{90\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{90\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{45\,\sqrt{2}\left(ab^3d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d^4*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*sgn(b*d^4*x^2 + a*d^4)) +
90*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
 - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*
(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^
2/b))/(b^4*sgn(b*d^4*x^2 + a*d^4)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x
 - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*sgn(b*d^4*x^2 +
a*d^4)) - 256*sqrt(d*x)*d/(b^3*sgn(b*d^4*x^2 + a*d^4)) - 8*(17*sqrt(d*x)*a*
b*d^5*x^2 + 13*sqrt(d*x)*a^2*d^5)/((b*d^2*x^2 + a*d^2)^2*b^3*sgn(b*d^4*x^2
+ a*d^4)))
```

3.760 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} \, dx$

Optimal. Leaf size=458

$$-\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{64\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+\cdots}}$$

```
[Out] (-7*d^3*(d*x)^(3/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(7
/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*d^(9/2)*(a + b
*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2
]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*d^(9
/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])
```

Rubi [A] time = 0.333422, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{64\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```


```
[Out] (-7*d^3*(d*x)^(3/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(7
/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*d^(9/2)*(a + b
*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2
]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*d^(9
```

```
/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^4\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{\cdots}}{ab+\cdots}}{32b^2\sqrt{a^2+2abx^2+\cdots}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^3\left(ab+b^2x^2\right)\right)\text{Subs}\cdots}{16b^2\sqrt{a^2+2\cdots}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(21d^3\left(ab+b^2x^2\right)\right)\text{Subs}\cdots}{32b^{5/2}\sqrt{a^2+\cdots}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^{9/2}\left(ab+b^2x^2\right)\right)\text{Sub}\cdots}{64\sqrt{2}\sqrt[4]{a}b^{15\cdots}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{21d^{9/2}\left(a+bx^2\right)\log\left(\sqrt{a}\cdots\right.}{64\sqrt{2}\sqrt[4]{a}b^{11/4}\cdots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21d^{9/2}\left(a+bx^2\right)\tan^{-1}\left(1\cdots\right.}{32\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2a\cdots}}$$

Mathematica [C] time = 0.0334198, size = 84, normalized size = 0.18

$$\frac{2d^3(dx)^{3/2}\left(7\left(a+bx^2\right)^2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(7a+5bx^2\right)\right)}{5ab^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*d^3*(d*x)^(3/2)*(-(a*(7*a + 5*b*x^2)) + 7*(a + b*x^2)^2*Hypergeometric2F
1[3/4, 3, 7/4, -((b*x^2)/a)]))/(5*a*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [B] time = 0.238, size = 612, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128*(-21*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*b^2*d
^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))
*x^4*b^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4))*x^4*b^2*d^4+88*(a*d^2/b)^(1/4)*(d*x)^(7/2)*b^2-42*2^(1/2)*ln(-((
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a*b*d^4-84*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^4-84*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^4+56*
(a*d^2/b)^(1/4)*(d*x)^(3/2)*a*b*d^2-21*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a*d^2/b)^(1/2)))*a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^
(1/4))/(a*d^2/b)^(1/4))*a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^4)*d*(b*x^2+a)/(a*d^2/b)^(1/4)/b^3/((b*
x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64341, size = 707, normalized size = 1.54

$$-\frac{84\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac14}\arctan\left(-\frac{\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac14}\sqrt{dx}b^3d^{13}-\sqrt{d^{27}x-\sqrt{-\frac{d^{18}}{ab^{11}}}ab^5d^{18}}\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac14}b^3}{d^{18}}\right)-21\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(84*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)*arctan(-
((-d^18/(a*b^11))^(1/4)*sqrt(d*x)*b^3*d^13 - sqrt(d^27*x - sqrt(-d^18/(a*b^
11))*a*b^5*d^18)*(-d^18/(a*b^11))^(1/4)*b^3)/d^18) - 21*(b^4*x^4 + 2*a*b^3*
x^2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)*log(9261*sqrt(d*x)*d^13 + 9261*(-d^18
/(a*b^11))^(3/4)*a*b^8) + 21*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^
11))^(1/4)*log(9261*sqrt(d*x)*d^13 - 9261*(-d^18/(a*b^11))^(3/4)*a*b^8) + 4
*(11*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d*x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33204, size = 497, normalized size = 1.09

$$-\frac{1}{128}d^3\left(\frac{8\left(11\sqrt{dx}bd^5x^3+7\sqrt{dx}ad^5x\right)}{\left(bd^2x^2+ad^2\right)^2b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{ab^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac34}\arctan\left(-\cdots\right)}{ab^5\mathrm{sgn}\left(bd^4x^2\cdots\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d^3*(8*(11*sqrt(d*x)*b*d^5*x^3 + 7*sqrt(d*x)*a*d^5*x)/((b*d^2*x^2 +
a*d^2)^2*b^2*sgn(b*d^4*x^2 + a*d^4)) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5
*sgn(b*d^4*x^2 + a*d^4)) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*d^4*
x^2 + a*d^4)) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1
/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*sgn(b*d^4*x^2 + a*d^4)) - 21*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a*b^5*sgn(b*d^4*x^2 + a*d^4)))
```

3.761 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=458

$$-\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d\right.}}{64\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2a}}$$

```
[Out] (-5*d^3*Sqrt[d*x])/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(5/2
))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^
2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^(7/2)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(3/4
)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*
Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^(7/2)*(a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.322187, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d\right.}}{64\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*d^3*Sqrt[d*x])/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(5/2
))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^
2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^(7/2)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(3/4
)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*
Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^(7/2)*(a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
```

```
rt[d*x]])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^4\left(ab+b^2x^2\right)\right)\int \frac{}{\sqrt{a}}}{32b^2\sqrt{a^2+2abx^2}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^3\left(ab+b^2x^2\right)\right)\mathrm{Sub}}{16b^2\sqrt{a^2+}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^2\left(ab+b^2x^2\right)\right)\mathrm{Sub}}{32\sqrt{a}b^2\sqrt{a^2}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(5d^{7/2}\left(ab+b^2x^2\right)\right)\mathrm{Su}}{64\sqrt{2}a^{3/4}b}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{a}\right.}{64\sqrt{2}a^{3/4}b^{9/}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(\right.}{32\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2}}$$

Mathematica [A] time = 0.150843, size = 447, normalized size = 0.98

$$-\frac{5(dx)^{7/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}x^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}} + \frac{5(dx)^{7/2}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}x^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}} - \frac{5(dx)^{7/2}\left(\right.}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*a*(d*x)^(7/2)*(a + b*x^2))/(12*b^2*x^3*((a + b*x^2)^2)^(3/2)) - (2*(d*x
)^(7/2)*(a + b*x^2))/(3*b*x*((a + b*x^2)^2)^(3/2)) + (5*(d*x)^(7/2)*(a + b*
x^2)^2)/(48*b^2*x^3*((a + b*x^2)^2)^(3/2)) - (5*(d*x)^(7/2)*(a + b*x^2)^3*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)*x
^(7/2)*((a + b*x^2)^2)^(3/2)) + (5*(d*x)^(7/2)*(a + b*x^2)^3*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)*x^(7/2)*((a +
b*x^2)^2)^(3/2)) - (5*(d*x)^(7/2)*(a + b*x^2)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*x^(7/2)*((a +
b*x^2)^2)^(3/2)) + (5*(d*x)^(7/2)*(a + b*x^2)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*x^(7/2)*((a +
b*x^2)^2)^(3/2))
```

Maple [B] time = 0.238, size = 666, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(5*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
))*x^4*b^2*d^2+10*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2+10*(a*d^2/b)^(1/4)*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2+10*(a*d^
2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a*b*d^2
+20*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a
*d^2/b)^(1/4))*x^2*a*b*d^2+20*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)
^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^2+5*(a*d^2/b)^(1/4)*2^(1
/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^2+10*(a*d^2/b)^(1/4)
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*
d^2+10*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*a^2*d^2-72*(d*x)^(5/2)*a*b-40*(d*x)^(1/2)*a^2*d^2)*d*(b*x
^2+a)/a/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67061, size = 695, normalized size = 1.52

$$\frac{20\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{3}{4}}\sqrt{dx}a^2b^7d^3-\sqrt{d^7x+\sqrt{-\frac{d^{14}}{a^3b^9}}a^2b^4}\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{3}{4}}a^2b^7}{d^{14}}\right)+5\left(b^4x^4+2\,ab^3x^2+a^2b^2\right.}{64\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*arctan(-
((-d^14/(a^3*b^9))^(3/4)*sqrt(d*x)*a^2*b^7*d^3 - sqrt(d^7*x + sqrt(-d^14/(a
^3*b^9))*a^2*b^4)*(-d^14/(a^3*b^9))^(3/4)*a^2*b^7)/d^14) + 5*(b^4*x^4 + 2*a
*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*log(5*sqrt(d*x)*d^3 + 5*(-d^14/
(a^3*b^9))^(1/4)*a*b^2) - 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b
^9))^(1/4)*log(5*sqrt(d*x)*d^3 - 5*(-d^14/(a^3*b^9))^(1/4)*a*b^2) - 4*(9*b*
d^3*x^2 + 5*a*d^3)*sqrt(d*x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(7/2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.38142, size = 501, normalized size = 1.09

$$\frac{1}{128}d^2\left(\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d^2*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*sgn(b*d^4*x^2 + a*d^4)) +
 10*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*sgn(b*d^4*x^2 + a*d^4)) + 5*sqrt(2
)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*
d^2/b))/(a*b^3*sgn(b*d^4*x^2 + a*d^4)) - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(
d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3*sgn(b*d^4*x
^2 + a*d^4)) - 8*(9*sqrt(d*x)*b*d^5*x^2 + 5*sqrt(d*x)*a*d^5)/((b*d^2*x^2 +
a*d^2)^2*b^2*sgn(b*d^4*x^2 + a*d^4)))
```

3.762 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=459

$$\frac{3d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5}}{32}$$

```
[Out] (3*d*(d*x)^(3/2))/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(3/2)
)/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(5/2)*(a + b*x^2
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^
(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(5/4)
*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*S
qrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(5/2)*(a + b
*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d*x]])/(64*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.333142, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5}}{32}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```


```
[Out] (3*d*(d*x)^(3/2))/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(3/2)
)/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(5/2)*(a + b*x^2
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^
(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(5/4)
*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*S
qrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(5/2)*(a + b
*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
```

```
t[d*x]])/(64*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{ab+b^2 \ldots}}{32ab\sqrt{a^2+2abx^2+b^2 \ldots}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d\left(ab+b^2x^2\right)\right)\text{Subst}\left(\ldots\right.}{16ab\sqrt{a^2+2ab \ldots}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(3d\left(ab+b^2x^2\right)\right)\text{Subst}\left(\ldots\right.}{32ab^{3/2}\sqrt{a^2+2 \ldots}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^{5/2}\left(ab+b^2x^2\right)\right)\text{Subst} \ldots}{64\sqrt{2}a^{5/4}b^{11/4} \ldots}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{a}\sqrt{\ldots}\right.}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{\ldots}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1- \ldots\right.}{32\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2ab \ldots}}$$

Mathematica [C] time = 0.027012, size = 73, normalized size = 0.16

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right)^2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\right)}{5a^2b\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a^2 + (a + b*x^2)^2*Hypergeometric2F1[3/4, 3, 7/4, -((b*
x^2)/a)]))/(5*a^2*b*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [B] time = 0.24, size = 617, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(3*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/
2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*b^2*d^4+
6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4
*b^2*d^4+6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*x^4*b^2*d^4+24*(a*d^2/b)^(1/4)*(d*x)^(7/2)*b^2+6*2^(1/2)*ln(-((a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a*b*d^4+12*2^(1/2)*arctan((2^(1/2)*(d
*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^4+12*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^4-8*(a*d^2/
b)^(1/4)*(d*x)^(3/2)*a*b*d^2+3*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))*a^2*d^4+6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a
*d^2/b)^(1/4))*a^2*d^4+6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*a^2*d^4)/d*(b*x^2+a)/(a*d^2/b)^(1/4)/b^2/a/((b*x^2+a)^2
)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82999, size = 745, normalized size = 1.62

$$-\frac{12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{27\,\sqrt{dx}ab^2d^7\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}-\sqrt{-729\,a^3b^3d^{10}\sqrt{-\frac{d^{10}}{a^5b^7}}+729\,d^{15}xab^2}\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}}{27\,d^{10}}\right)-3\left(ab^3x^4+2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*arcta
n(-1/27*(27*sqrt(d*x)*a*b^2*d^7*(-d^10/(a^5*b^7))^(1/4) - sqrt(-729*a^3*b^3
*d^10*sqrt(-d^10/(a^5*b^7)) + 729*d^15*x)*a*b^2*(-d^10/(a^5*b^7))^(1/4))/d^
10) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(27*
a^4*b^5*(-d^10/(a^5*b^7))^(3/4) + 27*sqrt(d*x)*d^7) + 3*(a*b^3*x^4 + 2*a^2*
b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(-27*a^4*b^5*(-d^10/(a^5*b^7))^
(3/4) + 27*sqrt(d*x)*d^7) - 4*(3*b*d^2*x^3 - a*d^2*x)*sqrt(d*x))/(a*b^3*x^4
 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^{\frac{5}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(5/2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.38369, size = 498, normalized size = 1.08

$$\frac{1}{128}d\left(\frac{8\left(3\sqrt{dx}bd^5x^3-\sqrt{dx}ad^5x\right)}{\left(bd^2x^2+ad^2\right)^2ab\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\cdots\right.}{\cdots}\right)}{a^2b^4\mathrm{sgn}\left(bd^4x^2+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d*(8*(3*sqrt(d*x)*b*d^5*x^3 - sqrt(d*x)*a*d^5*x)/((b*d^2*x^2 + a*d^2)
^2*a*b*sgn(b*d^4*x^2 + a*d^4)) + 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqr
t(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(
b*d^4*x^2 + a*d^4)) + 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*d^4*x^2
+ a*d^4)) - 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*s
qrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)) + 3*sqrt(2)*(a*b
^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)))
```

3.763 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=459

$$-\frac{3d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^3}{32}$$

```
[Out] (d*Sqrt[d*x])/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[d*x])/(4*b
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*
b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/2)*(a + b*x^2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*b^(5/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]
*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(64*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.326663, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^3}{32}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```


```
[Out] (d*Sqrt[d*x])/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[d*x])/(4*b
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*
b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/2)*(a + b*x^2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*b^(5/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]
*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
```

```
])/(64*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(d^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d^2\left(ab+b^2x^2\right)\right)\int \frac{}{\sqrt{dx}}}{32ab\sqrt{a^2+2abx^2}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3d\left(ab+b^2x^2\right)\right)\text{Subst}}{16ab\sqrt{a^2+2}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3\left(ab+b^2x^2\right)\right)\text{Subst}}{32a^{3/2}b\sqrt{a^2+}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(3d^{3/2}\left(ab+b^2x^2\right)\right)\text{Sub}}{64\sqrt{2}a^{7/4}b^{9/}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{a}\right.}{64\sqrt{2}a^{7/4}b^{5/4}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(1\right.}{32\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2a}}$$

Mathematica [A] time = 0.216866, size = 272, normalized size = 0.59

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(8a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-32a^{7/4}\sqrt[4]{b}\sqrt{x}-3\sqrt{2}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+3\sqrt{2}\left(a+bx\right.\right.}{128a^{7/4}b^{5/4}x^{3/2}\left(\left(a\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(-32*a^(7/4)*b^(1/4)*Sqrt[x] + 8*a^(3/4)*b^(1/4)*S
qrt[x]*(a + b*x^2) - 6*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)] + 6*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)] - 3*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x] + 3*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(128*a^(7/4)*b^(5/4)*x^(3/2)*((a + b*x^2
)^2)^(3/2))
```

Maple [B] time = 0.237, size = 668, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(3*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
))*x^4*b^2*d^2+6*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2+6*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2+6*(a*d^2/b
)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
)/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a*b*d^2+12
*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*x^2*a*b*d^2+12*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^2+3*(a*d^2/b)^(1/4)*2^(1/2)
*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^2+6*(a*d^2/b)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2+
6*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d
^2/b)^(1/4))*a^2*d^2+8*(d*x)^(5/2)*a*b-24*(d*x)^(1/2)*a^2*d^2)/d*(b*x^2+a)/
b/a^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92385, size = 679, normalized size = 1.48

$$\frac{12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^5b^4d\left(-\frac{d^6}{a^7b^5}\right)^{\frac{3}{4}}-\sqrt{a^4b^2\sqrt{-\frac{d^6}{a^7b^5}}+d^3x}a^5b^4\left(-\frac{d^6}{a^7b^5}\right)^{\frac{3}{4}}}{d^6}\right)+3\left(ab^3x^4+2\,a^2b^2x^2+a^3\right.}{64\left(ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*arctan(
-(sqrt(d*x)*a^5*b^4*d*(-d^6/(a^7*b^5))^(3/4) - sqrt(a^4*b^2*sqrt(-d^6/(a^7*
b^5)) + d^3*x)*a^5*b^4*(-d^6/(a^7*b^5))^(3/4))/d^6) + 3*(a*b^3*x^4 + 2*a^2*
b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*log(3*a^2*b*(-d^6/(a^7*b^5))^(1/4)
+ 3*sqrt(d*x)*d) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(
1/4)*log(-3*a^2*b*(-d^6/(a^7*b^5))^(1/4) + 3*sqrt(d*x)*d) + 4*(b*d*x^2 - 3*
a*d)*sqrt(d*x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.39991, size = 497, normalized size = 1.08

$$\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\log\left(dx\right.}{128\,a^2b^2\mathrm{sgn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*sgn(b*d^4*x^2 + a*d^4)) + 3/64*s
qrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*sgn(b*d^4*x^2 + a*d^4)) + 3/128*sqrt(
2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^2*b^2*sgn(b*d^4*x^2 + a*d^4)) - 3/128*sqrt(2)*(a*b^3*d^2)^(1/4)
*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*sg
n(b*d^4*x^2 + a*d^4)) + 1/16*(sqrt(d*x)*b*d^5*x^2 - 3*sqrt(d*x)*a*d^5)/((b*
d^2*x^2 + a*d^2)^2*a*b*sgn(b*d^4*x^2 + a*d^4))
```

3.764 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=460

$$\frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\right.}{64\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (5*(d*x)^(3/2))/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x)^(3/2)/(4
*a*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*Sqrt[d]*(a + b*x^2)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9
/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9/4)*b
^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqr
t[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*Sqrt[d]*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(64*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.332915, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\right.}{64\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (5*(d*x)^(3/2))/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x)^(3/2)/(4
*a*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*Sqrt[d]*(a + b*x^2)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9
/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9/4)*b
^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqr
t[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*Sqrt[d]*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
```

```
d*x]])/(64*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{ab+b^2 \cdots}}{32a^2\sqrt{a^2+2abx^2+b \cdots}}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\left(ab+b^2x^2\right)\right)\text{Subst} \cdots}{16a^2d\sqrt{a^2+2 \cdots}}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(5\left(ab+b^2x^2\right)\right)\text{Subst} \cdots}{32a^2\sqrt{b}d\sqrt{a^2 \cdots}}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\sqrt{d}\left(ab+b^2x^2\right)\right)\text{Sub} \cdots}{64\sqrt{2}a^{9/4}b \cdots}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{a} \cdots\right.}{64\sqrt{2}a^{9/4}b^{3/4} \cdots}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1 \cdots\right.}{32\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2a \cdots}}$$

Mathematica [C] time = 0.0146492, size = 54, normalized size = 0.12

$$\frac{2x\sqrt{dx}\left(a+bx^2\right)^3 \,_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^3\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*(a + b*x^2)^3*Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)])/
(3*a^3*((a + b*x^2)^2)^(3/2))
```

Maple [B] time = 0.225, size = 617, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(5*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/
2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*b^2*d^2+
10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^
4*b^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)
^(1/4))*x^4*b^2*d^2+40*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^2*b^2+10*2^(1/2)*ln(-(
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a*b*d^2+20*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^2+20*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^2+72
*(d*x)^(3/2)*a*b*(a*d^2/b)^(1/4)+5*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2)))*a^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*a^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2)/d*(b*x^2+a)/(a*d^2/b)^(1/4)/b/a^2/((b*x^
2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93447, size = 714, normalized size = 1.55

$$-\frac{20\left(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\right)\left(-\frac{d^{2}}{a^{9}b^{3}}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\sqrt{dx}a^{2}bd\left(-\frac{d^{2}}{a^{9}b^{3}}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^{5}bd^{2}\sqrt{-\frac{d^{2}}{a^{9}b^{3}}}+15625\,d^{3}x}a^{2}b\left(-\frac{d^{2}}{a^{9}b^{3}}\right)^{\frac{1}{4}}}{125\,d^{2}}\right)-5\left(a^{2}b^{2}x^{4}+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(20*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*arctan(-
1/125*(125*sqrt(d*x)*a^2*b*d*(-d^2/(a^9*b^3))^(1/4) - sqrt(-15625*a^5*b*d^2
*sqrt(-d^2/(a^9*b^3)) + 15625*d^3*x)*a^2*b*(-d^2/(a^9*b^3))^(1/4))/d^2) - 5
*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*log(125*a^7*b^2*(
-d^2/(a^9*b^3))^(3/4) + 125*sqrt(d*x)*d) + 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a
^4)*(-d^2/(a^9*b^3))^(1/4)*log(-125*a^7*b^2*(-d^2/(a^9*b^3))^(3/4) + 125*sq
rt(d*x)*d) - 4*(5*b*x^3 + 9*a*x)*sqrt(d*x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^
4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A] time = 1.34084, size = 506, normalized size = 1.1

$$\frac{5\sqrt{dx}bd^{4}x^{3}+9\sqrt{dx}ad^{4}x}{16\left(bd^{2}x^{2}+ad^{2}\right)^{2}a^{2}\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{5\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{3}b^{3}d\,\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{5\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right.\right.}{2\left(\frac{a}{}\right.}\right.}{64\,a^{3}b^{3}d\,\mathrm{sgn}\left(bd^{4}x^{2}+ad^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(5*sqrt(d*x)*b*d^4*x^3 + 9*sqrt(d*x)*a*d^4*x)/((b*d^2*x^2 + a*d^2)^2*a
^2*sgn(b*d^4*x^2 + a*d^4)) + 5/64*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*d*sgn
(b*d^4*x^2 + a*d^4)) + 5/64*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*d*sgn(b*d^
4*x^2 + a*d^4)) - 5/128*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/
b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*d*sgn(b*d^4*x^2 + a*d^4)) + 5/
128*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(a^3*b^3*d*sgn(b*d^4*x^2 + a*d^4))
```

3.765 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=460

$$\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+$$

```
[Out] (7*Sqrt[d*x])/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d*x]/(4*a*d
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4
)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*(a + b*x^2)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*S
qrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^
(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(64*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.335974, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (7*Sqrt[d*x])/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d*x]/(4*a*d
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4
)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*(a + b*x^2)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*S
qrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^
(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
```

```
)/(64*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7b\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^2}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21\left(ab+b^2x^2\right)\right)}{32a^2\sqrt{a^2+2}}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21\left(ab+b^2x^2\right)\right)}{16a^2d}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21\left(ab+b^2x^2\right)\right)}{32a^{5/2}d}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21\left(ab+b^2x^2\right)\right)}{64a^5}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21\left(a+bx^2\right)\log}{64\sqrt{2}a^{11/}}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21\left(a+bx^2\right)\mathrm{ta}}{32\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}}$$

Mathematica [A] time = 0.193036, size = 272, normalized size = 0.59

$$\frac{\sqrt{x}\left(a+bx^2\right)\left(56a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+32a^{7/4}\sqrt[4]{b}\sqrt{x}-21\sqrt{2}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+21\sqrt{2}\left(a+bx\right.\right.}{128a^{11/4}\sqrt[4]{b}\sqrt{dx}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[x]*(a + b*x^2)*(32*a^(7/4)*b^(1/4)*Sqrt[x] + 56*a^(3/4)*b^(1/4)*Sqrt[
x]*(a + b*x^2) - 42*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)] + 42*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] - 21*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x] + 21*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(128*a^(11/4)*b^(1/4)*Sqrt[d*x]*((a + b*
x^2)^2)^(3/2))
```

Maple [B] time = 0.233, size = 638, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] 1/128*(21*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
)))*x^4*b^2+42*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2+42*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2
)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2+42*(a*d^2/b)^(1/4)*
2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a*b+84*(a*d^2/b)^(
1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
x^2*a*b+84*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1
/4))/(a*d^2/b)^(1/4))*x^2*a*b+21*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2)))*a^2+42*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2+42*(a*d^2/b)^(1/4)*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2+56*(d*x)
^(1/2)*x^2*a*b+88*(d*x)^(1/2)*a^2)/d*(b*x^2+a)/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99167, size = 682, normalized size = 1.48

$$84\left(a^2b^2dx^4+2\,a^3bdx^2+a^4d\right)\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^6d^2\sqrt{-\frac{1}{a^{11}bd^2}}+dx}a^8bd\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}a^8bd\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{3}{4}}\right)+21\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(84*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*ar
ctan(sqrt(a^6*d^2*sqrt(-1/(a^11*b*d^2)) + d*x)*a^8*b*d*(-1/(a^11*b*d^2))^(3
/4) - sqrt(d*x)*a^8*b*d*(-1/(a^11*b*d^2))^(3/4)) + 21*(a^2*b^2*d*x^4 + 2*a^
3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*log(a^3*d*(-1/(a^11*b*d^2))^(1/4
) + sqrt(d*x)) - 21*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2
))^(1/4)*log(-a^3*d*(-1/(a^11*b*d^2))^(1/4) + sqrt(d*x)) + 4*(7*b*x^2 + 11*
a)*sqrt(d*x))/(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A] time = 1.2722, size = 505, normalized size = 1.1

$$\frac{7\sqrt{dx}bd^3x^2+11\sqrt{dx}ad^3}{16\left(bd^2x^2+ad^2\right)^2a^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64a^3bd\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\right.\right.}{2}\right.}{64a^3bd\mathrm{sgn}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(7*sqrt(d*x)*b*d^3*x^2 + 11*sqrt(d*x)*a*d^3)/((b*d^2*x^2 + a*d^2)^2*a^
2*sgn(b*d^4*x^2 + a*d^4)) + 21/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sgn(b
*d^4*x^2 + a*d^4)) + 21/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sgn(b*d^4*x
^2 + a*d^4)) + 21/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d*sgn(b*d^4*x^2 + a*d^4)) - 21/128
*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sq
rt(a*d^2/b))/(a^3*b*d*sgn(b*d^4*x^2 + a*d^4))
```

3.766 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=506

$$-\frac{45\sqrt[4]{b}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\cdots$$

```
[Out] 9/(16*a^2*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*Sqrt[d*x]
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*(a + b*x^2))/(16*a^3*d*
Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*b^(1/4)*(a + b*x^2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d
^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d^(3
/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[
2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*b^(1/4)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(64*Sqrt[2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.384298, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{45\sqrt[4]{b}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 9/(16*a^2*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*Sqrt[d*x]
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*(a + b*x^2))/(16*a^3*d*
Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*b^(1/4)*(a + b*x^2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d
^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d^(3
/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[
```

```
2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*b^(1/4)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(64*Sqrt[2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^2}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(45\left(ab \cdots\right.\right.}{3 \cdots}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{16a^3d\sqrt{\cdots}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{16a^3d\sqrt{\cdots}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{16a^3d\sqrt{\cdots}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{16a^3d\sqrt{\cdots}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{16a^3d\sqrt{\cdots}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{16a^3d\sqrt{\cdots}}$$

Mathematica [C] time = 0.0151114, size = 52, normalized size = 0.1

$$-\frac{2x\left(a+bx^2\right)^3\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^3(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^3*Hypergeometric2F1[-1/4, 3, 3/4, -((b*x^2)/a)])/(a^3*(d*
x)^(3/2)*((a + b*x^2)^2)^(3/2))
```

Maple [A] time = 0.237, size = 645, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128/d*(45*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(1
/2)*x^4*b^2+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*(d*x)^(1/2)*x^4*b^2+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(1/2)*x^4*b^2+360*(a*d^2/b)^(1/4)*x^4*b^2
+90*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(
d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(1/2)*x^2*a
*b+180*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*(d*x)^(1/2)*x^2*a*b+180*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*(d*x)^(1/2)*x^2*a*b+648*(a*d^2/b)^(1/4)*x^2*a*b+45*2^(
1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(1/2)*a^2+90*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(1/2
)*a^2+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*(d*x)^(1/2)*a^2+256*(a*d^2/b)^(1/4)*a^2)*(b*x^2+a)/(a*d^2/b)^(1/4)/(d*x
)^(1/2)/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.77116, size = 837, normalized size = 1.65

$$\frac{180\left(a^3b^2d^2x^5+2\,a^4bd^2x^3+a^5d^2x\right)\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\,\sqrt{dx}a^3bd\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,a^7bd^4\sqrt{-\frac{b}{a^{13}d^6}}+8303765625\,b^2dx}a^3d\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}}{91125\,b}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(180*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(-b/(a^13*d^6))^(
1/4)*arctan(-1/91125*(91125*sqrt(d*x)*a^3*b*d*(-b/(a^13*d^6))^(1/4) - sqrt(
-8303765625*a^7*b*d^4*sqrt(-b/(a^13*d^6)) + 8303765625*b^2*d*x)*a^3*d*(-b/(
a^13*d^6))^(1/4))/b) - 45*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(
-b/(a^13*d^6))^(1/4)*log(91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(
d*x)*b) + 45*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(-b/(a^13*d^6)
)^(1/4)*log(-91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(d*x)*b) - 4*
(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2)*sqrt(d*x))/(a^3*b^2*d^2*x^5 + 2*a^4*b*d^
2*x^3 + a^5*d^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A] time = 1.40361, size = 554, normalized size = 1.09

$$-\frac{\frac{256}{\sqrt{dx}a^{3}\operatorname{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{8\left(13\sqrt{dx}b^{2}d^{3}x^{3}+17\sqrt{dx}abd^{3}x\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{2}a^{3}\operatorname{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{90\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4}b^{2}d^{2}\operatorname{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{90\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\cdots\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4}b^{2}d^{2}\operatorname{sgn}\left(bd^{4}x^{2}+ad^{4}\right)}\cdots}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*(256/(sqrt(d*x)*a^3*sgn(b*d^4*x^2 + a*d^4)) + 8*(13*sqrt(d*x)*b^2*d^
3*x^3 + 17*sqrt(d*x)*a*b*d^3*x)/((b*d^2*x^2 + a*d^2)^2*a^3*sgn(b*d^4*x^2 +
a*d^4)) + 90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)
) + 90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) - 4
5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a^4*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b^3*d^2)
^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^
2*d^2*sgn(b*d^4*x^2 + a*d^4)))/d
```

3.767 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{77b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77\ldots}{3\ldots}$$

```
[Out] 11/(16*a^2*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*(d*x)^
(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*(a + b*x^2))/(48*a
^3*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(
15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(15/
4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(6
4*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(64*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.376197, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77\ldots}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 11/(16*a^2*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*(d*x)^
(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*(a + b*x^2))/(48*a
^3*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(
15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(15/
4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(6
```

```
4*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(64*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^2}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} -$$

Mathematica [C] time = 0.0152462, size = 54, normalized size = 0.11

$$-\frac{2x\left(a+bx^2\right)^3 {}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^3(dx)^{5/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^3*Hypergeometric2F1[-3/4, 3, 1/4, -((b*x^2)/a)])/(3*a^3*(
d*x)^(5/2)*((a + b*x^2)^2)^(3/2))
```

Maple [B] time = 0.236, size = 707, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/384/d^3*(231*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))*(d*x)^(3/2)*x^4*b^3+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*x^4*b^3+462*(a*d^2
/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1
/4))*(d*x)^(3/2)*x^4*b^3+462*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a*d^2/b)^(1/2)))*(d*x)^(3/2)*x^2*a*b^2+924*(a*d^2/b)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*x^2
*a*b^2+924*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1
/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*x^2*a*b^2+231*(a*d^2/b)^(1/4)*2^(1/2)*ln(
(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(3/2)*a^2*b+462*(a*d^2/b)^(
1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
(d*x)^(3/2)*a^2*b+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(
a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*a^2*b+616*a*d^2*b^2*x^4+968*x^
2*a^2*b*d^2+256*a^3*d^2)*(b*x^2+a)/(d*x)^(3/2)/a^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70463, size = 826, normalized size = 1.63

$$-\frac{924\left(a^3b^2d^3x^6+2\,a^4bd^3x^4+a^5d^3x^2\right)\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{11}bd^7\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^8d^6\sqrt{-\frac{b^3}{a^{15}d^{10}}}+b^2dx}\,a^{11}d^7\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+231\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/192*(924*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d
^10))^(1/4)*arctan(-(sqrt(d*x)*a^11*b*d^7*(-b^3/(a^15*d^10))^(3/4) - sqrt(a
^8*d^6*sqrt(-b^3/(a^15*d^10)) + b^2*d*x)*a^11*d^7*(-b^3/(a^15*d^10))^(3/4))
/b^3) + 231*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d
^10))^(1/4)*log(77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) - 231
*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d^10))^(1/4)
*log(-77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) + 4*(77*b^2*x^4
 + 121*a*b*x^2 + 32*a^2)*sqrt(d*x))/(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^
5*d^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A] time = 1.2741, size = 541, normalized size = 1.07

$$-\frac{15\sqrt{dx}b^2d^2x^2+19\sqrt{dx}abd^2}{16\left(bd^2x^2+ad^2\right)^2a^3d\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4d^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\right.}{}\right)}{64\,a^4d^3\mathrm{sgn}\left(bd^4x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(15*sqrt(d*x)*b^2*d^2*x^2 + 19*sqrt(d*x)*a*b*d^2)/((b*d^2*x^2 + a*d^2
)^2*a^3*d*sgn(b*d^4*x^2 + a*d^4)) - 77/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d
^3*sgn(b*d^4*x^2 + a*d^4)) - 77/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d^3*sgn
(b*d^4*x^2 + a*d^4)) - 77/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(
a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*d^3*sgn(b*d^4*x^2 + a*d^4))
+ 77/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a^4*d^3*sgn(b*d^4*x^2 + a*d^4)) - 2/3/(sqrt(d*x)*a^3*
d^2*x*sgn(b*d^4*x^2 + a*d^4))
```

3.768 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=553

$$\frac{117b\left(a+bx^2\right)}{16a^4d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{\right.}{64\sqrt{2}a^{17/4}d^{7/}$$

```
[Out] 13/(16*a^2*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*(d*x)^
(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*(a + b*x^2))/(80*
a^3*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b*(a + b*x^2))/(1
6*a^4*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^(5/4)*(a + b*
x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]
*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^
(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])
```

Rubi [A] time = 0.432307, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117b\left(a+bx^2\right)}{16a^4d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{\right.}{64\sqrt{2}a^{17/4}d^{7/}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 13/(16*a^2*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*(d*x)^
(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*(a + b*x^2))/(80*
a^3*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b*(a + b*x^2))/(1
6*a^4*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^(5/4)*(a + b*
x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]
*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^
```

```
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^
(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^2}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(1\cdots}{\cdots}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{80\cdots}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{80\cdots}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{80\cdots}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{80\cdots}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{80\cdots}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{80\cdots}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{80\cdots}$$

Mathematica [C] time = 0.0160301, size = 54, normalized size = 0.1

$$-\frac{2x\left(a+bx^2\right)^3 {}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^3(dx)^{7/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^3*Hypergeometric2F1[-5/4, 3, -1/4, -((b*x^2)/a)])/(5*a^3*
(d*x)^(7/2)*((a + b*x^2)^2)^(3/2))
```

Maple [A] time = 0.247, size = 687, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/640/d^3*(585*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/
b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^
(5/2)*x^4*b^3+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*(d*x)^(5/2)*x^4*b^3+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*x^4*b^3+4680*(a*d^2/b)^(1/4)*
x^6*b^3*d^2+1170*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^
2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x
)^(5/2)*x^2*a*b^2+2340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*(d*x)^(5/2)*x^2*a*b^2+2340*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*x^2*a*b^2+8424*(a*d^2/b
)^(1/4)*x^4*a*b^2*d^2+585*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-
d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))*(d*x)^(5/2)*a^2*b+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(
1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*a^2*b+1170*2^(1/2)*arctan((2^(1/2)*(d*x)
^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*a^2*b+3328*(a*d^2/b)^(
1/4)*x^2*a^2*b*d^2-256*(a*d^2/b)^(1/4)*a^3*d^2)*(b*x^2+a)/(a*d^2/b)^(1/4)/(
d*x)^(5/2)/a^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69873, size = 961, normalized size = 1.74

$$-\frac{2340\left(a^4b^2d^4x^7+2\,a^5bd^4x^5+a^6d^4x^3\right)\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613\,\sqrt{dx}a^4b^4d^3\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-2565164201769\,a^9b^5d^8\sqrt{-\frac{b^5}{a^{17}d^{14}}}+25651\ldots}}{1601613\,b^5}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/320*(2340*(a^4*b^2*d^4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*
d^14))^(1/4)*arctan(-1/1601613*(1601613*sqrt(d*x)*a^4*b^4*d^3*(-b^5/(a^17*d
^14))^(1/4) - sqrt(-2565164201769*a^9*b^5*d^8*sqrt(-b^5/(a^17*d^14)) + 2565
164201769*b^8*d*x)*a^4*d^3*(-b^5/(a^17*d^14))^(1/4))/b^5) - 585*(a^4*b^2*d^
4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*d^14))^(1/4)*log(1601613
*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) + 1601613*sqrt(d*x)*b^4) + 585*(a^4*b^2
*d^4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*d^14))^(1/4)*log(-160
1613*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) + 1601613*sqrt(d*x)*b^4) - 4*(585*b
^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^3)*sqrt(d*x))/(a^4*b^2*d^4*x
^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{7}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(7/2)*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A] time = 1.33656, size = 583, normalized size = 1.05

$$\frac{21\sqrt{dx}b^3d^3x^3 + 25\sqrt{dx}ab^2d^3x}{16\left(bd^2x^2 + ad^2\right)^2a^4d^3\operatorname{sgn}\left(bd^4x^2 + ad^4\right)} + \frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}} + 2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5bd^5\operatorname{sgn}\left(bd^4x^2 + ad^4\right)} + \frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\cdots\right.}{64\,a^5bd^5\operatorname{sgn}\left(bd^4x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(21*sqrt(d*x)*b^3*d^3*x^3 + 25*sqrt(d*x)*a*b^2*d^3*x)/((b*d^2*x^2 + a*
d^2)^2*a^4*d^3*sgn(b*d^4*x^2 + a*d^4)) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/
(a^5*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^
5*b*d^5*sgn(b*d^4*x^2 + a*d^4)) - 117/128*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x
 + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d^5*sgn(b*d^4*
x^2 + a*d^4)) + 117/128*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/
b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 2/
5*(15*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^4*d^5*x^2*sgn(b*d^4*x^2 + a*d^4))
```

3.769 $\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=647

$$-\frac{13923ad^{11}\sqrt{dx}\left(a+bx^2\right)}{1024b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923d^9(dx)^{5/2}\left(a+bx^2\right)}{5120b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{119d^5(dx)^{13/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2}}$$

```
[Out] (-1547*d^7*(d*x)^(9/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*
x)^(21/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7*d^3*(d*
x)^(17/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (119*d^
5*(d*x)^(13/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13
923*a*d^11*Sqrt[d*x]*(a + b*x^2))/(1024*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (13923*d^9*(d*x)^(5/2)*(a + b*x^2))/(5120*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(25/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(4096*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.508811, antiderivative size = 647, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{13923ad^{11}\sqrt{dx}\left(a+bx^2\right)}{1024b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923d^9(dx)^{5/2}\left(a+bx^2\right)}{5120b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{119d^5(dx)^{13/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-1547*d^7*(d*x)^(9/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*
x)^(21/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7*d^3*(d*
x)^(17/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (119*d^
```

```
5*(d*x)^(13/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13
923*a*d^11*Sqrt[d*x]*(a + b*x^2))/(1024*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (13923*d^9*(d*x)^(5/2)*(a + b*x^2))/(5120*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(25/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(4096*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{23/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(119d^4\left(ab\right.\right.}{64\sqrt{}}$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{256b^3\left(a+\right.}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

Mathematica [A] time = 0.318006, size = 401, normalized size = 0.62

$$\frac{(dx)^{23/2}\left(a+bx^2\right)\left(-21446656a^2b^{13/4}x^{13/2}-39829504a^3b^{9/4}x^{9/2}-32587776a^4b^{5/4}x^{5/2}+2042040a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(23/2)*(a + b*x^2)*(-10183680*a^5*b^(1/4)*Sqrt[x] - 32587776*a^4*b^(
5/4)*x^(5/2) - 39829504*a^3*b^(9/4)*x^(9/2) - 21446656*a^2*b^(13/4)*x^(13/2
) - 3784704*a*b^(17/4)*x^(17/2) + 180224*b^(21/4)*x^(21/2) + 848640*a^4*b^(
1/4)*Sqrt[x]*(a + b*x^2) + 1166880*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 2042
040*a^2*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - 1531530*Sqrt[2]*a^(5/4)*(a + b*x^2)
^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 1531530*Sqrt[2]*a^(5/4)*
(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 765765*Sqrt[2
]*a^(5/4)*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x] + 765765*Sqrt[2]*a^(5/4)*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(450560*b^(25/4)*x^(23/2)*((a + b*x^2)^2)^
(5/2))
```

Maple [B] time = 0.236, size = 1287, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/40960*(139230*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))*a^5*d^6+139230*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^5*d^6-1638400*(d*x)^
(1/2)*x^2*a^4*b*d^6+65536*(d*x)^(5/2)*x^6*a*b^4*d^4-409600*(d*x)^(1/2)*x^8*
a*b^4*d^6+98304*(d*x)^(5/2)*x^4*a^2*b^3*d^4-1638400*(d*x)^(1/2)*x^6*a^2*b^3
*d^6+65536*(d*x)^(5/2)*x^2*a^3*b^2*d^4-2457600*(d*x)^(1/2)*x^4*a^3*b^2*d^6+
69615*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*
a^5*d^6-223960*(d*x)^(13/2)*a^2*b^3+16384*(d*x)^(5/2)*x^8*b^5*d^4-565800*(d
*x)^(9/2)*a^3*b^2*d^2-477896*(d*x)^(5/2)*a^4*b*d^4-556920*(d*x)^(1/2)*a^5*d
^6+69615*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
```

```
)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
))*x^8*a*b^4*d^6+556920*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a^2*b^3*d^6+417690*(a*d^2/b)^(1/4)*2
^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^3*b^2*d^6+835380*
(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4))*x^4*a^3*b^2*d^6+835380*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^3*b^2*d^6+278460*(a*d^2/
b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^4*b*d^6
+556920*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4)
)/(a*d^2/b)^(1/4))*x^2*a^4*b*d^6+556920*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^4*b*d^6+139230*(a*
d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)
^(1/4))*x^8*a*b^4*d^6+139230*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*a*b^4*d^6+278460*(a*d^2/b)^(1/4
)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x
-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a^2*b^3*d^6+5569
20*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*x^6*a^2*b^3*d^6)*d^5*(b*x^2+a)/b^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72312, size = 1067, normalized size = 1.65

$$\frac{278460 \left(-\frac{a^5 d^{46}}{b^{25}}\right)^{\frac{1}{4}} \left(b^{10} x^8 + 4\, a b^9 x^6 + 6\, a^2 b^8 x^4 + 4\, a^3 b^7 x^2 + a^4 b^6\right) \arctan\left(-\frac{\left(-\frac{a^5 d^{46}}{b^{25}}\right)^{\frac{3}{4}} \sqrt{dx}\, a b^{19} d^{11} - \left(-\frac{a^5 d^{46}}{b^{25}}\right)^{\frac{3}{4}} \sqrt{a^2 d^{23} x + \sqrt{-\frac{a^5 d^{46}}{b^{25}}}\, b}}{a^5 d^{46}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(278460*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*
x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*arctan(-((-a^5*d^46/b^25)^(3/4)*sqrt(d*x)*a*
b^19*d^11 - (-a^5*d^46/b^25)^(3/4)*sqrt(a^2*d^23*x + sqrt(-a^5*d^46/b^25)*b
^12)*b^19)/(a^5*d^46)) + 69615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x
^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x)*a*d^11 +
13923*(-a^5*d^46/b^25)^(1/4)*b^6) - 69615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8
+ 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x
)*a*d^11 - 13923*(-a^5*d^46/b^25)^(1/4)*b^6) + 4*(2048*b^5*d^11*x^10 - 4300
8*a*b^4*d^11*x^8 - 220507*a^2*b^3*d^11*x^6 - 369733*a^3*b^2*d^11*x^4 - 2645
37*a^4*b*d^11*x^2 - 69615*a^5*d^11)*sqrt(d*x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*
a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36281, size = 622, normalized size = 0.96

$$\frac{1}{40960}d^{10}\left(\frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/40960*d^10*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sqrt(2)*(sqrt
(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*sgn(b*d^4*x^2 + a*
d^4)) + 139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*sgn(b*d^4*x^2 + a*d^4))
 + 69615*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(b^7*sgn(b*d^4*x^2 + a*d^4)) - 69615*sqrt(2)*(a*b^
3*d^2)^(1/4)*a*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(b^7*sgn(b*d^4*x^2 + a*d^4)) - 40*(5599*sqrt(d*x)*a^2*b^3*d^9*x^6 + 1414
5*sqrt(d*x)*a^3*b^2*d^9*x^4 + 12357*sqrt(d*x)*a^4*b*d^9*x^2 + 3683*sqrt(d*x
)*a^5*d^9)/((b*d^2*x^2 + a*d^2)^4*b^6*sgn(b*d^4*x^2 + a*d^4)) + 16384*(sqrt
(d*x)*b^20*d^6*x^2 - 25*sqrt(d*x)*a*b^19*d^6)/(b^25*d^5*sgn(b*d^4*x^2 + a*d
^4)))
```

3.770 $\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=600

$$\frac{7315d^9(dx)^{3/2}\left(a+bx^2\right)}{3072b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{95d^5(dx)^{11/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{19d^3(dx)^{1}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2-}}$$

```
[Out] (-1045*d^7*(d*x)^(7/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*
x)^(19/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (19*d^3*(d
*x)^(15/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (95*d^
5*(d*x)^(11/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (73
15*d^9*(d*x)^(3/2)*(a + b*x^2))/(3072*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x
])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(23/4)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) + (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt
[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.468336, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{7315d^9(dx)^{3/2}\left(a+bx^2\right)}{3072b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{95d^5(dx)^{11/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{19d^3(dx)^{1}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (-1045*d^7*(d*x)^(7/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*
x)^(19/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (19*d^3*(d
*x)^(15/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (95*d^
5*(d*x)^(11/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (73
```

```
15*d^9*(d*x)^(3/2)*(a + b*x^2))/(3072*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x
])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(23/4)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) + (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt
[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{21/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(19b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(95d^4\left(ab\right.\right.}{64\sqrt{}}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{256b^3\left(a+\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

Mathematica [C] time = 0.0501262, size = 110, normalized size = 0.18

$$-\frac{2d^9(dx)^{3/2}\left(-2223a^2b^2x^4-2717a^3bx^2-1463a^4-741ab^3x^6+1463\left(a+bx^2\right)^4\,{}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-39b^4x^8\right)}{117b^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-2*d^9*(d*x)^(3/2)*(-1463*a^4 - 2717*a^3*b*x^2 - 2223*a^2*b^2*x^4 - 741*a*
b^3*x^6 - 39*b^4*x^8 + 1463*(a + b*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -(
(b*x^2)/a)]))/(117*b^5*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [B] time = 0.246, size = 1171, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(16384*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^8*b^5*d^6-21945*2^(1/2)*ln(-((
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^8*a*b^4*d^8-43890*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*a*b^4*d^8-43890*
2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*a
*b^4*d^8+70200*(a*d^2/b)^(1/4)*(d*x)^(15/2)*a*b^4+65536*(a*d^2/b)^(1/4)*(d*
x)^(3/2)*x^6*a*b^4*d^6-87780*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2)))*x^6*a^2*b^3*d^8-175560*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*x^6*a^2*b^3*d^8-175560*2^(1/2)*arctan((2^(1/2)*(d
*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a^2*b^3*d^8+168456*(a*d^2/b
)^(1/4)*(d*x)^(11/2)*a^2*b^3*d^2+98304*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^4*a^2*
b^3*d^6-131670*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/
b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^
3*b^2*d^8-263340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*x^4*a^3*b^2*d^8-263340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^3*b^2*d^8+143464*(a*d^2/b)^(1/4)*(d*x)^
(7/2)*a^3*b^2*d^4+65536*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^2*a^3*b^2*d^6-87780*2
^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(
```

```
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^4*b*d^8-175560*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^
4*b*d^8-175560*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*x^2*a^4*b*d^8+58520*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^4*b*d^6-21945*2
^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^5*d^8-43890*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^5*d^8-43890
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^5*
d^8)*d^3*(b*x^2+a)/(a*d^2/b)^(1/4)/b^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68991, size = 1080, normalized size = 1.8

$$\frac{87780\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)\arctan\left(-\frac{\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}\sqrt{dx}a^2b^6d^{31}-\sqrt{a^4d^{63}x-\sqrt{-\frac{a^3d^{42}}{b^{23}}}a^3b^{11}d^{42}}\left(-\frac{a^3d^{4}}{b^{23}}\right.}{a^3d^{42}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/12288*(87780*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^
4 + 4*a^3*b^6*x^2 + a^4*b^5)*arctan(-((-a^3*d^42/b^23)^(1/4)*sqrt(d*x)*a^2*
b^6*d^31 - sqrt(a^4*d^63*x - sqrt(-a^3*d^42/b^23)*a^3*b^11*d^42)*(-a^3*d^42
/b^23)^(1/4)*b^6)/(a^3*d^42)) - 21945*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a
*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(391419980875*sqrt(d
*x)*a^2*d^31 + 391419980875*(-a^3*d^42/b^23)^(3/4)*b^17) + 21945*(-a^3*d^42
/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b
```

```
^5)*log(391419980875*sqrt(d*x)*a^2*d^31 - 391419980875*(-a^3*d^42/b^23)^(3/
4)*b^17) + 4*(2048*b^4*d^10*x^9 + 16967*a*b^3*d^10*x^7 + 33345*a^2*b^2*d^10
*x^5 + 26125*a^3*b*d^10*x^3 + 7315*a^4*d^10*x)*sqrt(d*x))/(b^9*x^8 + 4*a*b^
8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47986, size = 575, normalized size = 0.96

$$\frac{1}{24576}d^9\left(\frac{16384\sqrt{dx}dx}{b^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\ldots\right.\right.}{2\ldots}\right)}{b^8\mathrm{sgn}\left(bd^4x^2+ad^4\right)}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^9*(16384*sqrt(d*x)*d*x/(b^5*sgn(b*d^4*x^2 + a*d^4)) - 43890*sqrt(
2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d
*x))/(a*d^2/b)^(1/4))/(b^8*sgn(b*d^4*x^2 + a*d^4)) - 43890*sqrt(2)*(a*b^3*d
^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^
2/b)^(1/4))/(b^8*sgn(b*d^4*x^2 + a*d^4)) + 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*
log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*sgn(b*d^4
*x^2 + a*d^4)) - 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*sgn(b*d^4*x^2 + a*d^4)) + 8*(8775*s
```

```
qrt(d*x)*a*b^3*d^9*x^7 + 21057*sqrt(d*x)*a^2*b^2*d^9*x^5 + 17933*sqrt(d*x)*
a^3*b*d^9*x^3 + 5267*sqrt(d*x)*a^4*d^9*x)/((b*d^2*x^2 + a*d^2)^4*b^5*sgn(b*
d^4*x^2 + a*d^4)))
```

3.771 $\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=600

$$\frac{3315d^9\sqrt{dx}\left(a+bx^2\right)}{1024b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{221d^5(dx)^{9/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{17d^3(dx)^{1}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2-}}$$

```
[Out] (-663*d^7*(d*x)^(5/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(17/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (17*d^3*(d*
x)^(13/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (221*d^
5*(d*x)^(9/2))/(768*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (331
5*d^9*Sqrt[d*x]*(a + b*x^2))/(1024*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a
^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(21/4)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]
*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.463117, antiderivative size = 600, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{3315d^9\sqrt{dx}\left(a+bx^2\right)}{1024b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{221d^5(dx)^{9/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{17d^3(dx)^{1}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (-663*d^7*(d*x)^(5/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(17/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (17*d^3*(d*
x)^(13/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (221*d^
5*(d*x)^(9/2))/(768*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (331
```

```
5*d^9*Sqrt[d*x]*(a + b*x^2))/(1024*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a
^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(21/4)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]
*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(17b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(221d^4\left(a\right.\right.}{192}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{768b^3\left(a+\right.}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

Mathematica [A] time = 0.29358, size = 384, normalized size = 0.64

$$\frac{(dx)^{19/2}\left(a+bx^2\right)\left(39829504a^2b^{9/4}x^{9/2}+32587776a^3b^{5/4}x^{5/2}-1166880a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2-848640a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(19/2)*(a + b*x^2)*(10183680*a^4*b^(1/4)*Sqrt[x] + 32587776*a^3*b^(5
/4)*x^(5/2) + 39829504*a^2*b^(9/4)*x^(9/2) + 21446656*a*b^(13/4)*x^(13/2) +
 3784704*b^(17/4)*x^(17/2) - 848640*a^3*b^(1/4)*Sqrt[x]*(a + b*x^2) - 11668
80*a^2*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 - 2042040*a*b^(1/4)*Sqrt[x]*(a + b*x^2
)^3 + 1531530*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)] - 1531530*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)] + 765765*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 765765*Sqrt[2]*a^(1/4)*(
a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(
1892352*b^(21/4)*x^(19/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.24, size = 1202, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576*(9945*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))*x^8*b^4*d^6+19890*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+19890*(a*d^2/b)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*
d^6+39780*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
)))*x^6*a*b^3*d^6+79560*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+79560*(a*d^2/b)^(1/4)*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3
*d^6-49152*(d*x)^(1/2)*x^8*b^4*d^6+59670*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*
```

```
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^6+119340*(a*d^2/b)^(1/4)*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^
2*b^2*d^6+119340*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6-55400*(d*x)^(13/2)*a*b^3-196608
*(d*x)^(1/2)*x^6*a*b^3*d^6+39780*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^3*b*d^6+79560*(a*d^2/b)^(1/4)*2^(1/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6+795
60*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*x^2*a^3*b*d^6-127640*(d*x)^(9/2)*a^2*b^2*d^2-294912*(d*x)^(1/
2)*x^4*a^2*b^2*d^6+9945*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a*d^2/b)^(1/2)))*a^4*d^6+19890*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6+19890*(a*d^2/b)^(1/4)*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6
-105720*(d*x)^(5/2)*a^3*b*d^4-196608*(d*x)^(1/2)*x^2*a^3*b*d^6-79560*(d*x)^
(1/2)*a^4*d^6)*d^3*(b*x^2+a)/b^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69105, size = 964, normalized size = 1.61

$$-\frac{39780 \left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}} \left(b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5\right) \arctan\left(-\frac{\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{3}{4}} \sqrt{dx}b^{16}d^9 - \sqrt{d^{19}x + \sqrt{-\frac{ad^{38}}{b^{21}}}b^{10}}\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{3}{4}}b^{16}}{ad^{38}}\right) +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(39780*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4
 + 4*a^3*b^6*x^2 + a^4*b^5)*arctan(-((-a*d^38/b^21)^(3/4)*sqrt(d*x)*b^16*d^
9 - sqrt(d^19*x + sqrt(-a*d^38/b^21)*b^10)*(-a*d^38/b^21)^(3/4)*b^16)/(a*d^
38)) + 9945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4
*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*d^9 + 3315*(-a*d^38/b^21)^(1/4)*
b^5) - 9945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4
*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*d^9 - 3315*(-a*d^38/b^21)^(1/4)*
b^5) - 4*(6144*b^4*d^9*x^8 + 31501*a*b^3*d^9*x^6 + 52819*a^2*b^2*d^9*x^4 +
37791*a^3*b*d^9*x^2 + 9945*a^4*d^9)*sqrt(d*x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a
^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38807, size = 578, normalized size = 0.96

$$-\frac{1}{24576}d^8\left(\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{99\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*d^8*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sgn(b*d^4*x^2 + a*d^4
```

```
)) + 19890*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/
b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sgn(b*d^4*x^2 + a*d^4)) + 994
5*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(b^6*sgn(b*d^4*x^2 + a*d^4)) - 9945*sqrt(2)*(a*b^3*d^2)^(1/
4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*sgn(
b*d^4*x^2 + a*d^4)) - 49152*sqrt(d*x)*d/(b^5*sgn(b*d^4*x^2 + a*d^4)) - 8*(6
925*sqrt(d*x)*a*b^3*d^9*x^6 + 15955*sqrt(d*x)*a^2*b^2*d^9*x^4 + 13215*sqrt(
d*x)*a^3*b*d^9*x^2 + 3801*sqrt(d*x)*a^4*d^9)/((b*d^2*x^2 + a*d^2)^4*b^5*sgn
(b*d^4*x^2 + a*d^4)))
```

3.772 $\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=554

$$-\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{55d^5(dx)^{7/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}(\ldots}{\ldots}$$

```
[Out] (-385*d^7*(d*x)^(3/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(15/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^3*(d*x
)^(11/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (55*d^5*
(d*x)^(7/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*
d^(17/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (115
5*d^(17/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1
155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (1155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(1
9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.419476, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{55d^5(dx)^{7/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (-385*d^7*(d*x)^(3/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(15/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^3*(d*x
)^(11/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (55*d^5*
(d*x)^(7/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*
d^(17/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
```

```
])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (115
5*d^(17/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1
155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (1155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(1
9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(15b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(55d^4\left(ab\right.\right.}{64\sqrt{}}$$

$$= -\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{256b^3\left(a+\right.}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

Mathematica [C] time = 0.0463936, size = 106, normalized size = 0.19

$$\frac{2d^7(dx)^{3/2}\left(77\left(a+bx^2\right)^4\,{}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a\left(143a^2bx^2+77a^3+117ab^2x^4+39b^3x^6\right)\right)}{39ab^4\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d^7*(d*x)^(3/2)*(-(a*(77*a^3 + 143*a^2*b*x^2 + 117*a*b^2*x^4 + 39*b^3*x^
6)) + 77*(a + b*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -((b*x^2)/a)]))/(39*a
*b^4*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [B] time = 0.234, size = 1046, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/8192*(-1155*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/
b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^8*b^
4*d^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*x^8*b^4*d^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*x^8*b^4*d^8+7144*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4-4620*2^
(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3*d^8-9240*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3
*d^8-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1
/4))*x^6*a*b^3*d^8+14040*(a*d^2/b)^(1/4)*(d*x)^(11/2)*a*b^3*d^2-6930*2^(1/2
)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^8-13860*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^
2*d^8-13860*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^
(1/4))*x^4*a^2*b^2*d^8+11000*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4-4620*2
^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^3*b*d^8-9240*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*
b*d^8-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(
```

```
1/4))*x^2*a^3*b*d^8+3080*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6-1155*2^(1/2)
*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8-2310*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8-2310*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)*d*(
b*x^2+a)/(a*d^2/b)^(1/4)/b^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70759, size = 992, normalized size = 1.79

$$-\frac{4620\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\sqrt{dx}b^5d^{25}-\sqrt{d^{51}x-\sqrt{-\frac{d^{34}}{ab^{19}}}ab^9d^{34}}\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}b^5}{d^{34}}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4096*(4620*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*
b^4)*(-d^34/(a*b^19))^(1/4)*arctan(-((-d^34/(a*b^19))^(1/4)*sqrt(d*x)*b^5*d
^25 - sqrt(d^51*x - sqrt(-d^34/(a*b^19))*a*b^9*d^34)*(-d^34/(a*b^19))^(1/4)
*b^5)/d^34) - 1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 +
 a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*x)*d^25 + 1540798875
*(-d^34/(a*b^19))^(3/4)*a*b^14) + 1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x
^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*
x)*d^25 - 1540798875*(-d^34/(a*b^19))^(3/4)*a*b^14) + 4*(893*b^3*d^8*x^7 +
1755*a*b^2*d^8*x^5 + 1375*a^2*b*d^8*x^3 + 385*a^3*d^8*x)*sqrt(d*x))/(b^8*x^
8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43693, size = 548, normalized size = 0.99

$$\frac{1}{8192}d^7\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^7\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^7\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{1155\sqrt{2}\left(ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^7*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*sgn(b*d^4*x^2 + a*d^4))
+ 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*sgn(b*d^4*x^2 + a*d^4)) - 1155*sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a*b^7*sgn(b*d^4*x^2 + a*d^4)) + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*l
og(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^7*sgn(b*d^
4*x^2 + a*d^4)) - 8*(893*sqrt(d*x)*b^3*d^9*x^7 + 1755*sqrt(d*x)*a*b^2*d^9*x
^5 + 1375*sqrt(d*x)*a^2*b*d^9*x^3 + 385*sqrt(d*x)*a^3*d^9*x)/((b*d^2*x^2 +
a*d^2)^4*b^4*sgn(b*d^4*x^2 + a*d^4)))
```

3.773 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}} dx$

Optimal. Leaf size=554

$$-\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{39d^5(dx)^{5/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^{15/2}(\cdots}{\cdots}$$

```
[Out] (-195*d^7*Sqrt[d*x])/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(13/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13*d^3*(d*x)
^(9/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (39*d^5*(d
*x)^(5/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d^(
15/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*d^
(15/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d
^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]) + (195*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[
d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.422205, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{39d^5(dx)^{5/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^{15/2}(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```



```
[Out] (-195*d^7*Sqrt[d*x])/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(13/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13*d^3*(d*x)
^(9/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (39*d^5*(d
*x)^(5/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d^(
15/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
```

```
)/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*d^
(15/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d
^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]) + (195*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[
d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(39d^4\left(ab+\right.\right.}{64\sqrt{a^2}}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{256b^3\left(a+\right.}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

Mathematica [A] time = 0.282188, size = 366, normalized size = 0.66

$$\frac{(dx)^{15/2}\left(a+bx^2\right)\left(-1916928a^2b^{5/4}x^{5/2}+49920a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-\frac{45045\sqrt{2}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{bx}\right)}{a^{3/4}}+\frac{45045\sqrt{2}\left(a+b\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(15/2)*(a + b*x^2)*(-599040*a^3*b^(1/4)*Sqrt[x] - 1916928*a^2*b^(5/4
)*x^(5/2) - 2342912*a*b^(9/4)*x^(9/2) - 1261568*b^(13/4)*x^(13/2) + 49920*a
^2*b^(1/4)*Sqrt[x]*(a + b*x^2) + 68640*a*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 12
0120*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - (90090*Sqrt[2]*(a + b*x^2)^4*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (90090*Sqrt[2]*(a + b*x^2)^
4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (45045*Sqrt[2]*(
a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^
(3/4) + (45045*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/a^(3/4)))/(1892352*b^(17/4)*x^(15/2)*((a + b*x^2)^2)^
(5/2))
```

Maple [B] time = 0.245, size = 1134, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(585*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*x^8*b^4*d^6+1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+1170*(a*d^2/b)^(1/4)*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+
2340*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x
^6*a*b^3*d^6+4680*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+4680*(a*d^2/b)^(1/4)*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+35
```

```
10*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4
*a^2*b^2*d^6+7020*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6+7020*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^
6-14824*(d*x)^(13/2)*a*b^3+2340*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^3*b*d^6+4680*(a*d^2/b)^(1/4)*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6+4680*
(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4))*x^2*a^3*b*d^6-24856*(d*x)^(9/2)*a^2*b^2*d^2+585*(a*d^2/b)^(1/4)*
2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^6+1170*(a*d^2/b)
^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*a^4*d^6+1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-17784*(d*x)^(5/2)*a^3*b*d^4-4680*(d*x)^(1
/2)*a^4*d^6)*d*(b*x^2+a)/a/b^4/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.698, size = 975, normalized size = 1.76

$$\frac{2340\left(b^{8}x^{8}+4\,ab^{7}x^{6}+6\,a^{2}b^{6}x^{4}+4\,a^{3}b^{5}x^{2}+a^{4}b^{4}\right)\left(-\frac{d^{30}}{a^{3}b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{30}}{a^{3}b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}a^{2}b^{13}d^{7}-\sqrt{d^{15}x+\sqrt{-\frac{d^{30}}{a^{3}b^{17}}}a^{2}b^{8}}\left(-\frac{d^{30}}{a^{3}b^{17}}\right)^{\frac{3}{4}}a^{2}b^{13}}{d^{30}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(2340*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*
b^4)*(-d^30/(a^3*b^17))^(1/4)*arctan(-((-d^30/(a^3*b^17))^(3/4)*sqrt(d*x)*a
^2*b^13*d^7 - sqrt(d^15*x + sqrt(-d^30/(a^3*b^17))*a^2*b^8)*(-d^30/(a^3*b^1
7))^(3/4)*a^2*b^13)/d^30) + 585*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*
a^3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d^7 + 195
*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 585*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x
^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d^
7 - 195*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 4*(1853*b^3*d^7*x^6 + 3107*a*b^2*
d^7*x^4 + 2223*a^2*b*d^7*x^2 + 585*a^3*d^7)*sqrt(d*x))/(b^8*x^8 + 4*a*b^7*x
^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4427, size = 552, normalized size = 1.

$$\frac{1}{24576}d^6\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{585\sqrt{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^6*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*d^4*x^2 + a*d^4
```

```
)) + 1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*d^4*x^2 + a*d^4)) + 58
5*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(a*b^5*sgn(b*d^4*x^2 + a*d^4)) - 585*sqrt(2)*(a*b^3*d^2)^(1
/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*s
gn(b*d^4*x^2 + a*d^4)) - 8*(1853*sqrt(d*x)*b^3*d^9*x^6 + 3107*sqrt(d*x)*a*b
^2*d^9*x^4 + 2223*sqrt(d*x)*a^2*b*d^9*x^2 + 585*sqrt(d*x)*a^3*d^9)/((b*d^2*
x^2 + a*d^2)^4*b^4*sgn(b*d^4*x^2 + a*d^4)))
```

3.774 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^5(dx)^{3/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13/2}\left(a\right.}{}$$

```
[Out] (77*d^5*(d*x)^(3/2))/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(11/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (11*d^3*(d*
x)^(7/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^5*
(d*x)^(3/2))/(768*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^
(13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(13/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (77*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.425292, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^5(dx)^{3/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13/2}\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```



```
[Out] (77*d^5*(d*x)^(3/2))/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(11/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (11*d^3*(d*
x)^(7/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^5*
(d*x)^(3/2))/(768*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^
(13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
```

```
])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(13/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (77*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(77d^4\left(ab+\right.\right.}{192\sqrt{a}}$$

$$= -\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{768b^3\left(a+\right.}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)}{96b^2\left(a+bx^2\right)^2\sqrt{a^2}}$$

Mathematica [C] time = 0.0400176, size = 97, normalized size = 0.17

$$\frac{2d^5(dx)^{3/2}\left(77\left(a+bx^2\right)^4 {}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a^2\left(77a^2+143abx^2+117b^2x^4\right)\right)}{585a^2b^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d^5*(d*x)^(3/2)*(-(a^2*(77*a^2 + 143*a*b*x^2 + 117*b^2*x^4)) + 77*(a + b
*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -((b*x^2)/a)]))/(585*a^2*b^3*(a + b*
x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [B] time = 0.231, size = 1051, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(231*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^8*b^4*
d^8+462*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*x^8*b^4*d^8+462*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*x^8*b^4*d^8+1848*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+924*2^(1/2)
*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3*d^8+1848*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^8+
1848*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
x^6*a*b^3*d^8-2808*(a*d^2/b)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+1386*2^(1/2)*ln(-
((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^8+2772*2^(1/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^8+2
772*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x
^4*a^2*b^2*d^8-2200*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+924*2^(1/2)*ln(
-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^3*b*d^8+1848*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^8+1848
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*
```

```
a^3*b*d^8-616*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+231*2^(1/2)*ln(-((a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8+462*2^(1/2)*arctan((2^(1/2)*(d*x
)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8+462*2^(1/2)*arctan((2^(1/
2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)/d*(b*x^2+a)/(a*d^
2/b)^(1/4)/b^4/a/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7136, size = 1021, normalized size = 1.83

$$-\frac{924\left(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}\sqrt{dx}ab^4d^{19}-\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{a^5b^{15}}}a^3b^7d^{26}}\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}a}{d^{26}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(924*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 +
a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*arctan(-((-d^26/(a^5*b^15))^(1/4)*sqrt(d*
x)*a*b^4*d^19 - sqrt(d^39*x - sqrt(-d^26/(a^5*b^15))*a^3*b^7*d^26)*(-d^26/(
a^5*b^15))^(1/4)*a*b^4)/d^26) - 231*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*
x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*log(456533*sqrt(d*x
)*d^19 + 456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) + 231*(a*b^7*x^8 + 4*a^2
*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4
)*log(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) - 4
*(231*b^3*d^6*x^7 - 351*a*b^2*d^6*x^5 - 275*a^2*b*d^6*x^3 - 77*a^3*d^6*x)*s
qrt(d*x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*
```

```
b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32311, size = 552, normalized size = 0.99

$$\frac{1}{24576}d^5\left(\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^6\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^6\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{231\sqrt{2}\left(ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^5*(462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*sgn(b*d^4*x^2 + a*d^4)
) + 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*sgn(b*d^4*x^2 + a*d^4)) - 231*
sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqr
t(a*d^2/b))/(a^2*b^6*sgn(b*d^4*x^2 + a*d^4)) + 231*sqrt(2)*(a*b^3*d^2)^(3/4
)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^6*sgn
(b*d^4*x^2 + a*d^4)) + 8*(231*sqrt(d*x)*b^3*d^9*x^7 - 351*sqrt(d*x)*a*b^2*d
^9*x^5 - 275*sqrt(d*x)*a^2*b*d^9*x^3 - 77*sqrt(d*x)*a^3*d^9*x)/((b*d^2*x^2
+ a*d^2)^4*a*b^3*sgn(b*d^4*x^2 + a*d^4)))
```

3.775 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15d^5\sqrt{dx}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11/2}\left(a+\right.}{4}$$

```
[Out] (15*d^5*Sqrt[d*x])/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(9/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^3*(d*x)^(
5/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (15*d^5*Sqrt
[d*x])/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(11/2)
*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + (45*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.430248, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15d^5\sqrt{dx}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11/2}\left(a+\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```



```
[Out] (15*d^5*Sqrt[d*x])/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(9/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^3*(d*x)^(
5/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (15*d^5*Sqrt
[d*x])/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
```

```
48*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(11/2)
*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + (45*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(15d^4\left(ab\right.\right.}{64\sqrt{}}$$

$$= -\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{256b^3\left(a+\right.}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx}{32b^2\left(a+bx^2\right)^2\sqrt{a}}$$

Mathematica [A] time = 0.305875, size = 352, normalized size = 0.63

$$\frac{d(dx)^{9/2}\left(a+bx^2\right)\left(-\frac{3465\sqrt{2}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}+\frac{3465\sqrt{2}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}-\frac{6930\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(1-\frac{\sqrt{2}}{\ldots}\right.}{a^{7/4}}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (d*(d*x)^(9/2)*(a + b*x^2)*(-46080*a^2*b^(1/4)*Sqrt[x] - 147456*a*b^(5/4)*x
^(5/2) - 180224*b^(9/4)*x^(9/2) + 3840*a*b^(1/4)*Sqrt[x]*(a + b*x^2) + 5280
*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + (9240*b^(1/4)*Sqrt[x]*(a + b*x^2)^3)/a - (
6930*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a
^(7/4) + (6930*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/a^(7/4) - (3465*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3465*Sqrt[2]*(a + b*x^2)^4*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4)))/(630784*b^
(13/4)*x^(9/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.232, size = 1136, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/8192*(45*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))*x^8*b^4*d^6+90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+90*(a*d^2/b)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+180*(a
*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^
3*d^6+360*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+360*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+270*(a*d^2/
b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
```

```
))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d
^6+540*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6+540*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/
2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6+120*(d*x)^
(13/2)*a*b^3+180*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/
b)^(1/2)))*x^2*a^3*b*d^6+360*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6+360*(a*d^2/b)^(1/4)*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^
3*b*d^6-1912*(d*x)^(9/2)*a^2*b^2*d^2+45*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^6+90*(a*d^2/b)^(1/4)*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6+90*(a*d^2/b
)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*a^4*d^6-1368*(d*x)^(5/2)*a^3*b*d^4-360*(d*x)^(1/2)*a^4*d^6)/d*(b*x^2+a)/
b^3/a^2/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73429, size = 984, normalized size = 1.77

$$\frac{180\left(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}a^5b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{d^{22}}{a^7b^{13}}}a^4b^6\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{3}{4}}a^5b}}{d^{22}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4096*(180*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^
5*b^3)*(-d^22/(a^7*b^13))^(1/4)*arctan(-((-d^22/(a^7*b^13))^(3/4)*sqrt(d*x)
*a^5*b^10*d^5 - sqrt(d^11*x + sqrt(-d^22/(a^7*b^13))*a^4*b^6)*(-d^22/(a^7*b
^13))^(3/4)*a^5*b^10)/d^22) + 45*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4
 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(d*x)*d^5 +
 45*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) - 45*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a
^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(
d*x)*d^5 - 45*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) + 4*(15*b^3*d^5*x^6 - 239*a
*b^2*d^5*x^4 - 171*a^2*b*d^5*x^2 - 45*a^3*d^5)*sqrt(d*x))/(a*b^7*x^8 + 4*a^
2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45915, size = 556, normalized size = 1.

$$\frac{1}{8192}d^4\left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{45\sqrt{2}\left(ab^3d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^4*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)
```

```
) + 90*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)) + 45*
sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a^2*b^4*sgn(b*d^4*x^2 + a*d^4)) - 45*sqrt(2)*(a*b^3*d^2)^(1/
4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*
sgn(b*d^4*x^2 + a*d^4)) + 8*(15*sqrt(d*x)*b^3*d^9*x^6 - 239*sqrt(d*x)*a*b^2
*d^9*x^4 - 171*sqrt(d*x)*a^2*b*d^9*x^2 - 45*sqrt(d*x)*a^3*d^9)/((b*d^2*x^2
+ a*d^2)^4*a*b^3*sgn(b*d^4*x^2 + a*d^4)))
```

3.776 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=560

$$\frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7d^3(dx)^{3/2}}{256ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{9/2}\left(a\right.}{}$$

```
[Out] (35*d^3*(d*x)^(3/2))/(1024*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d
*x)^(7/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7*d^3*(d*
x)^(3/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7*d^3*(
d*x)^(3/2))/(256*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d
^(9/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^
(9/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(
9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) - (35*d^(9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.423949, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7d^3(dx)^{3/2}}{256ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{9/2}\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (35*d^3*(d*x)^(3/2))/(1024*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d
*x)^(7/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7*d^3*(d*
x)^(3/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7*d^3*(
d*x)^(3/2))/(256*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d
^(9/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
```

```
])/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^
(9/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(
9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) - (35*d^(9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7d^4\left(ab+\right.\right.}{64\sqrt{a}}$$

$$= -\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{256ab^2\left(a\right.}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(d}{96b^2\left(a+bx^2\right)^2\sqrt{a}}$$

Mathematica [C] time = 0.0367708, size = 86, normalized size = 0.15

$$\frac{2d^3(dx)^{3/2}\left(7\left(a+bx^2\right)^4\ {}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a^3\left(7a+13bx^2\right)\right)}{117a^3b^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d^3*(d*x)^(3/2)*(-(a^3*(7*a + 13*b*x^2)) + 7*(a + b*x^2)^4*Hypergeometri
c2F1[3/4, 5, 7/4, -((b*x^2)/a)]))/(117*a^3*b^2*(a + b*x^2)^3*Sqrt[(a + b*x^
2)^2])
```

Maple [B] time = 0.234, size = 1051, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(105*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^8*b^4*
d^8+210*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*x^8*b^4*d^8+210*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*x^8*b^4*d^8+840*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+420*2^(1/2)*
ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3*d^8+840*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^8+84
0*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6
*a*b^3*d^8+3192*(a*d^2/b)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+630*2^(1/2)*ln(-((a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^8+1260*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^8+1260*
2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a
^2*b^2*d^8-1000*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+420*2^(1/2)*ln(-((a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^3*b*d^8+840*2^(1/2)*arctan((2^
(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^8+840*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*
```

```
d^8-280*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+105*2^(1/2)*ln(-((a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8+210*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2
)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8+210*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)/d^3*(b*x^2+a)/(a*d^2/b)
^(1/4)/b^3/a^2/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72935, size = 1080, normalized size = 1.93

$$-\frac{420\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{42875\,\sqrt{dx}a^2b^3d^{13}\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}-\sqrt{-1838265625\,a^5b^5d^{18}\sqrt{\cdots}}}{42875\,d^{18}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")

[Out] -1/12288*(420*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2
+ a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*arctan(-1/42875*(42875*sqrt(d*x)*a^2*b^
3*d^13*(-d^18/(a^9*b^11))^(1/4) - sqrt(-1838265625*a^5*b^5*d^18*sqrt(-d^18/
(a^9*b^11)) + 1838265625*d^27*x)*a^2*b^3*(-d^18/(a^9*b^11))^(1/4))/d^18) -
105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
*(-d^18/(a^9*b^11))^(1/4)*log(42875*a^7*b^8*(-d^18/(a^9*b^11))^(3/4) + 4287
5*sqrt(d*x)*d^13) + 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^
5*b^3*x^2 + a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*log(-42875*a^7*b^8*(-d^18/(a^
9*b^11))^(3/4) + 42875*sqrt(d*x)*d^13) - 4*(105*b^3*d^4*x^7 + 399*a*b^2*d^4
*x^5 - 125*a^2*b*d^4*x^3 - 35*a^3*d^4*x)*sqrt(d*x))/(a^2*b^6*x^8 + 4*a^3*b^
```

```
5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45089, size = 552, normalized size = 0.99

$$\frac{1}{24576}d^3\left(\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{105\sqrt{2}\left(ab^3d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^3*(210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*sgn(b*d^4*x^2 + a*d^4)
) + 210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*sgn(b*d^4*x^2 + a*d^4)) - 105*
sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqr
t(a*d^2/b))/(a^3*b^5*sgn(b*d^4*x^2 + a*d^4)) + 105*sqrt(2)*(a*b^3*d^2)^(3/4
)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^5*sgn
(b*d^4*x^2 + a*d^4)) + 8*(105*sqrt(d*x)*b^3*d^9*x^7 + 399*sqrt(d*x)*a*b^2*d
^9*x^5 - 125*sqrt(d*x)*a^2*b*d^9*x^3 - 35*sqrt(d*x)*a^3*d^9*x)/((b*d^2*x^2
+ a*d^2)^4*a^2*b^2*sgn(b*d^4*x^2 + a*d^4)))
```

3.777 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}} \, dx$

Optimal. Leaf size=560

$$\frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^3\sqrt{dx}}{768ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{7/2}(\cdots}{\cdots}$$

```
[Out] (35*d^3*Sqrt[d*x])/(3072*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(5/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^3*Sqrt[
d*x])/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^3*Sqrt[
d*x])/(768*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(7/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(204
8*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (35*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.438324, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^3\sqrt{dx}}{768ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{7/2}(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (35*d^3*Sqrt[d*x])/(3072*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(5/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^3*Sqrt[
d*x])/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^3*Sqrt[
d*x])/(768*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
```

```
48*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(7/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(204
8*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (35*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^4\left(ab+\cdots\right.\right.}{192\sqrt{\cdots}}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{768ab^2\left(a+\cdots\right.}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{\cdots}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2\cdots}}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{\cdots}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2\cdots}}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{\cdots}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2\cdots}}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{\cdots}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2\cdots}}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{\cdots}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2\cdots}}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{\cdots}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2\cdots}}$$

Mathematica [A] time = 0.314182, size = 341, normalized size = 0.61

$$\frac{(dx)^{7/2}\left(a+bx^2\right)\left(-49152a^{11/4}b^{5/4}x^{5/2}+3080a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+1760a^{7/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+1280a^{11/4}\sqrt[4]{b}\sqrt{x}\left(a+b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(7/2)*(a + b*x^2)*(-15360*a^(15/4)*b^(1/4)*Sqrt[x] - 49152*a^(11/4)*
b^(5/4)*x^(5/2) + 1280*a^(11/4)*b^(1/4)*Sqrt[x]*(a + b*x^2) + 1760*a^(7/4)*
b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 3080*a^(3/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^3
- 2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
- 1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x] + 1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x]))/(270336*a^(11/4)*b^(9/4)*x^(7/2)*((a + b*x^2)^2
)^(5/2))
```

Maple [B] time = 0.24, size = 1136, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(105*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*x^8*b^4*d^6+210*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+210*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+42
0*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^
2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*
a*b^3*d^6+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+630*(a*
d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b
^2*d^6+1260*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(
```

```
1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6+1260*(a*d^2/b)^(1/4)*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6+280*
(d*x)^(13/2)*a*b^3+420*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2)))*x^2*a^3*b*d^6+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*
(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6+840*(a*d^2/b)^(
1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
x^2*a^3*b*d^6+1000*(d*x)^(9/2)*a^2*b^2*d^2+105*(a*d^2/b)^(1/4)*2^(1/2)*ln((
d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^6+210*(a*d^2/b)^(1/4)*2^(1/2
)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6+210
*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*a^4*d^6-3192*(d*x)^(5/2)*a^3*b*d^4-840*(d*x)^(1/2)*a^4*d^6)/d^3
*(b*x^2+a)/b^2/a^3/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73798, size = 998, normalized size = 1.78

$$\frac{420\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^8b^7d^3\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{3}{4}}-\sqrt{a^6b^4\sqrt{-\frac{d^{14}}{a^{11}b^9}}+d^7x}\,a^8b^7\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{3}{4}}}{d^{14}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(420*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 +
 a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*arctan(-(sqrt(d*x)*a^8*b^7*d^3*(-d^14/(a
```

```
^11*b^9))^(3/4) - sqrt(a^6*b^4*sqrt(-d^14/(a^11*b^9)) + d^7*x)*a^8*b^7*(-d^
14/(a^11*b^9))^(3/4))/d^14) + 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*
x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(35*a^3*b^2*(-d^
14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d^3) - 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6
 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(-3
5*a^3*b^2*(-d^14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d^3) + 4*(35*b^3*d^3*x^6
+ 125*a*b^2*d^3*x^4 - 399*a^2*b*d^3*x^2 - 105*a^3*d^3)*sqrt(d*x))/(a^2*b^6*
x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{7}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(7/2)/((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.43302, size = 556, normalized size = 0.99

$$\frac{1}{24576}d^2\left(\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{105\sqrt{2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^2*(210*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^
```

```
4)) + 210*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)) +
105*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a^3*b^3*sgn(b*d^4*x^2 + a*d^4)) - 105*sqrt(2)*(a*b^3*d^2
)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3
*b^3*sgn(b*d^4*x^2 + a*d^4)) + 8*(35*sqrt(d*x)*b^3*d^9*x^6 + 125*sqrt(d*x)*
a*b^2*d^9*x^4 - 399*sqrt(d*x)*a^2*b*d^9*x^2 - 105*sqrt(d*x)*a^3*d^9)/((b*d^
2*x^2 + a*d^2)^4*a^2*b^2*sgn(b*d^4*x^2 + a*d^4)))
```

3.778 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{45d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-$$

```
[Out] (45*d*(d*x)^(3/2))/(1024*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(3/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(d*x)^(3/2)
)/(32*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (9*d*(d*x)^(3/2)
)/(256*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(5/2)*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a +
 b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sq
rt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (45*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rubi [A] time = 0.453825, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{45d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (45*d*(d*x)^(3/2))/(1024*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(3/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(d*x)^(3/2)
)/(32*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (9*d*(d*x)^(3/2)
)/(256*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(5/2)*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
```

```
qrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a +
 b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sq
rt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (45*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9bd^2\left(ab+\right.\right.}{64a\sqrt{a}}$$

$$= -\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{256a^2b\left(a+\right.}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/}}{32ab\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/}}{32ab\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/}}{32ab\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/}}{32ab\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/}}{32ab\left(a+bx^2\right)^2\sqrt{a^2}}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/}}{32ab\left(a+bx^2\right)^2\sqrt{a^2}}$$

Mathematica [C] time = 0.0321789, size = 73, normalized size = 0.13

$$\frac{2d(dx)^{3/2}\left(\left(a+bx^2\right)^4\,_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)-a^4\right)}{13a^4b\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*d*(d*x)^(3/2)*(-a^4 + (a + b*x^2)^4*Hypergeometric2F1[3/4, 5, 7/4, -((b*
x^2)/a)]))/(13*a^4*b*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [B] time = 0.237, size = 1051, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/8192*(90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*x^8*b^4*d^8+45*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(
a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*
x^8*b^4*d^8+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*x^8*b^4*d^8+360*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^8+360*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+180
*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x
+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3*d^8+360*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b
^3*d^8+540*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*x^4*a^2*b^2*d^8+1368*(a*d^2/b)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+270*2^(1/
2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^8+540*2^(1/2
)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2
*d^8+360*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*x^2*a^3*b*d^8+1912*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+180*2^(1/2)*
ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^3*b*d^8+360*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^8+90
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*
```

```
d^8-120*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+45*2^(1/2)*ln(-((a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)/d^5*(b*x^2+a)/(a*d^2/b)^(1/4)/b^
2/a^3/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70797, size = 1062, normalized size = 1.91

$$-\frac{180\left(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b\right)\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\,\sqrt{dx}a^3b^2d^7\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,a^7b^3d^{10}\sqrt{-\frac{d^{1}}{a^{13}}}}}{91125\,d^{10}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4096*(180*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 +
 a^7*b)*(-d^10/(a^13*b^7))^(1/4)*arctan(-1/91125*(91125*sqrt(d*x)*a^3*b^2*d
^7*(-d^10/(a^13*b^7))^(1/4) - sqrt(-8303765625*a^7*b^3*d^10*sqrt(-d^10/(a^1
3*b^7)) + 8303765625*d^15*x)*a^3*b^2*(-d^10/(a^13*b^7))^(1/4))/d^10) - 45*(
a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^10
/(a^13*b^7))^(1/4)*log(91125*a^10*b^5*(-d^10/(a^13*b^7))^(3/4) + 91125*sqrt
(d*x)*d^7) + 45*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^
2 + a^7*b)*(-d^10/(a^13*b^7))^(1/4)*log(-91125*a^10*b^5*(-d^10/(a^13*b^7))^
(3/4) + 91125*sqrt(d*x)*d^7) - 4*(45*b^3*d^2*x^7 + 171*a*b^2*d^2*x^5 + 239*
a^2*b*d^2*x^3 - 15*a^3*d^2*x)*sqrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a
^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(5/2)/((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.3794, size = 549, normalized size = 0.99

$$\frac{1}{8192}d\left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d*(90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*sgn(b*d^4*x^2 + a*d^4)) +
90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*sgn(b*d^4*x^2 + a*d^4)) - 45*sqrt(2
)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^
2/b))/(a^4*b^4*sgn(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d
*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^4*sgn(b*d^4*
x^2 + a*d^4)) + 8*(45*sqrt(d*x)*b^3*d^9*x^7 + 171*sqrt(d*x)*a*b^2*d^9*x^5 +
 239*sqrt(d*x)*a^2*b*d^9*x^3 - 15*sqrt(d*x)*a^3*d^9*x)/((b*d^2*x^2 + a*d^2)
^4*a^3*b*sgn(b*d^4*x^2 + a*d^4)))
```

3.779 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$-\frac{77d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-$$

```
[Out] (77*d*Sqrt[d*x])/(3072*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[d*x
])/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*Sqrt[d*x])/(96*
a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (11*d*Sqrt[d*x])/(768*
a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*
a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(3/2)*(a + b*x^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a
^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rubi [A] time = 0.426147, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{77d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (77*d*Sqrt[d*x])/(3072*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[d*x
])/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*Sqrt[d*x])/(96*
a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (11*d*Sqrt[d*x])/(768*
a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*
```

a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(3/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2d^2\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11bd^2\left(a\right.\right.}{192}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{768a^2b\left(a\right.}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{}}{96ab\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{}}{96ab\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{}}{96ab\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{}}{96ab\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{}}{96ab\left(a+bx^2\right)^2\sqrt{a}}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{}}{96ab\left(a+bx^2\right)^2\sqrt{a}}$$

Mathematica [A] time = 0.293068, size = 324, normalized size = 0.58

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(616a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+352a^{7/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+256a^{11/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-3072a^{15/4}\sqrt[4]{b}\sqrt{x}-231\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(-3072*a^(15/4)*b^(1/4)*Sqrt[x] + 256*a^(11/4)*b^(
1/4)*Sqrt[x]*(a + b*x^2) + 352*a^(7/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^2 + 616*
a^(3/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - 462*Sqrt[2]*(a + b*x^2)^4*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 462*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 231*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 231*Sqrt[2]*(a + b*x^2)^4
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(24576*a^(15/
4)*b^(5/4)*x^(3/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.237, size = 1136, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(231*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*x^8*b^4*d^6+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+462*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+92
4*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^
2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*
a*b^3*d^6+1848*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+1848*(a*d^2/b)^(1/4)*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+1386*
(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/
b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^
2*b^2*d^6+2772*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6+2772*(a*d^2/b)^(1/4)*2^(1/2)*arct
```

```
an((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6+6
16*(d*x)^(13/2)*a*b^3+924*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2)))*x^2*a^3*b*d^6+1848*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6+1848*(a*d^2
/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1
/4))*x^2*a^3*b*d^6+2200*(d*x)^(9/2)*a^2*b^2*d^2+231*(a*d^2/b)^(1/4)*2^(1/2)
*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^6+462*(a*d^2/b)^(1/4)*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^
6+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/
(a*d^2/b)^(1/4))*a^4*d^6+2808*(d*x)^(5/2)*a^3*b*d^4-1848*(d*x)^(1/2)*a^4*d^
6)/d^5*(b*x^2+a)/b/a^4/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66781, size = 953, normalized size = 1.71

$$\frac{924\left(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b\right)\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{11}b^4d\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{3}{4}}-\sqrt{a^8b^2\sqrt{-\frac{d^6}{a^{15}b^5}}+d^3x}a^{11}b^4\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{3}{4}}}{d^6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(924*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 +
 a^7*b)*(-d^6/(a^15*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^11*b^4*d*(-d^6/(a^15*b
^5))^(3/4) - sqrt(a^8*b^2*sqrt(-d^6/(a^15*b^5)) + d^3*x)*a^11*b^4*(-d^6/(a^
```

```
15*b^5))^(3/4))/d^6) + 231*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4
*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(77*a^4*b*(-d^6/(a^15*b^5)
)^(1/4) + 77*sqrt(d*x)*d) - 231*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^
4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(-77*a^4*b*(-d^6/(a^1
5*b^5))^(1/4) + 77*sqrt(d*x)*d) + 4*(77*b^3*d*x^6 + 275*a*b^2*d*x^4 + 351*a
^2*b*d*x^2 - 231*a^3*d)*sqrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3
*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(3/2)/((a + b*x**2)**2)**(5/2), x)
```

Giac [A] time = 1.42389, size = 549, normalized size = 0.99

$$\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^4b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^4b^2\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\log\left(\right.}{8192\,a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 77/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sgn(b*d^4*x^2 + a*d^4)) + 77/
4096*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sgn(b*d^4*x^2 + a*d^4)) + 77/81
92*sqrt(2)*(a*b^3*d^2)^(1/4)*d*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/(a^4*b^2*sgn(b*d^4*x^2 + a*d^4)) - 77/8192*sqrt(2)*(a*b^3*
```

```
d^2)^(1/4)*d*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^4*b^2*sgn(b*d^4*x^2 + a*d^4)) + 1/3072*(77*sqrt(d*x)*b^3*d^9*x^6 + 275*sq
rt(d*x)*a*b^2*d^9*x^4 + 351*sqrt(d*x)*a^2*b*d^9*x^2 - 231*sqrt(d*x)*a^3*d^9
)/((b*d^2*x^2 + a*d^2)^4*a^3*b*sgn(b*d^4*x^2 + a*d^4))
```

3.780 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=556

$$\frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{39(dx)^{3/2}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{8ad\left(a+\right.}$$

```
[Out] (195*(d*x)^(3/2))/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x)^(3/2
)/(8*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13*(d*x)^(3/2))/
(96*a^2*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (39*(d*x)^(3/2))
/(256*a^3*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*Sqrt[d]*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a
+ b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a
+ b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (195*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.433782, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{39(dx)^{3/2}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{8ad\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (195*(d*x)^(3/2))/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x)^(3/2
)/(8*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13*(d*x)^(3/2))/
(96*a^2*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (39*(d*x)^(3/2))
/(256*a^3*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*Sqrt[d]*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
```

```
qrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a
+ b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a
+ b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (195*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b^3\left(ab+b^2x^2\right)\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(39b^2\left(a\ldots\right.\right.}{64a^2\ldots}$$

$$= \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\ldots}{256a^3d\left(\ldots\right.}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(\ldots}{96a^2d\left(a+bx^2\right)^2\ldots}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(\ldots}{96a^2d\left(a+bx^2\right)^2\ldots}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(\ldots}{96a^2d\left(a+bx^2\right)^2\ldots}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(\ldots}{96a^2d\left(a+bx^2\right)^2\ldots}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(\ldots}{96a^2d\left(a+bx^2\right)^2\ldots}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(\ldots}{96a^2d\left(a+bx^2\right)^2\ldots}$$

Mathematica [C] time = 0.0128927, size = 54, normalized size = 0.1

$$\frac{2x\sqrt{dx}\left(a+bx^2\right)^5\,{}_2F_1\left(\frac{3}{4},5;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^5\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*(a + b*x^2)^5*Hypergeometric2F1[3/4, 5, 7/4, -((b*x^2)/a)])/
(3*a^5*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.234, size = 1051, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(585*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^8*b^4*
d^8+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*x^8*b^4*d^8+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(
a*d^2/b)^(1/4))*x^8*b^4*d^8+4680*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+2340*2^(1
/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3*d^8+4680*2^(1/2
)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d
^8+4680*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*x^6*a*b^3*d^8+17784*(a*d^2/b)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+3510*2^(1/2)*
ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^8+7020*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d
^8+7020*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*x^4*a^2*b^2*d^8+24856*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+2340*2^(1/
2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^3*b*d^8+4680*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^
8+4680*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*x^2*a^3*b*d^8+14824*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+585*2^(1/2)*ln(
```

```
-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8+1170*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8+1170*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)/d^7*(b*
x^2+a)/(a*d^2/b)^(1/4)/b/a^4/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69494, size = 1027, normalized size = 1.85

$$-\frac{2340\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{7414875\,\sqrt{dx}a^4bd\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}-\sqrt{-54980371265625\,a^9bd^2\sqrt{-}}}{7414875\,d^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(2340*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 +
 a^8)*(-d^2/(a^17*b^3))^(1/4)*arctan(-1/7414875*(7414875*sqrt(d*x)*a^4*b*d*
(-d^2/(a^17*b^3))^(1/4) - sqrt(-54980371265625*a^9*b*d^2*sqrt(-d^2/(a^17*b^
3)) + 54980371265625*d^3*x)*a^4*b*(-d^2/(a^17*b^3))^(1/4))/d^2) - 585*(a^4*
b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)*(-d^2/(a^17*b^
3))^(1/4)*log(7414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4) + 7414875*sqrt(d*x)*
d) + 585*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)*
(-d^2/(a^17*b^3))^(1/4)*log(-7414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4) + 741
4875*sqrt(d*x)*d) - 4*(585*b^3*x^7 + 2223*a*b^2*x^5 + 3107*a^2*b*x^3 + 1853
*a^3*x)*sqrt(d*x))/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x
^2 + a^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39095, size = 558, normalized size = 1.

$$\frac{195\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5b^3d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{195\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5b^3d\,\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{195\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(d\right.}{8192\,a^5b^3ds}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 195/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*d*sgn(b*d^4*x^2 + a*d^4)) + 19
5/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*d*sgn(b*d^4*x^2 + a*d^4)) - 195
/8192*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a^5*b^3*d*sgn(b*d^4*x^2 + a*d^4)) + 195/8192*sqrt(2)*(a*
b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^5*b^3*d*sgn(b*d^4*x^2 + a*d^4)) + 1/3072*(585*sqrt(d*x)*b^3*d^8*x^7 + 2
223*sqrt(d*x)*a*b^2*d^8*x^5 + 3107*sqrt(d*x)*a^2*b*d^8*x^3 + 1853*sqrt(d*x)
*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*a^4*sgn(b*d^4*x^2 + a*d^4))
```

3.781 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=556

$$\frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{55\sqrt{dx}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\quad}{8ad\left(a\right.}$$

```
[Out] (385*Sqrt[d*x])/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d*x]/(8
*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d*x])/(32*a^2
*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (55*Sqrt[d*x])/(256*a^3
*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a + b*x^2)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)*
b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*(a + b*x^2)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)
*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4
096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (11
55*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rubi [A] time = 0.429213, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1112, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{55\sqrt{dx}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\quad}{8ad\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (385*Sqrt[d*x])/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d*x]/(8
*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d*x])/(32*a^2
*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (55*Sqrt[d*x])/(256*a^3
*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a + b*x^2)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)*
b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*(a + b*x^2)*ArcTan
```

```
[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)
*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4
096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (11
55*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(15b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{\sqrt{dx}\left(ab+b^2x^2\right)^4}\,dx}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(55b^2 \cdots}{\cdots}$$

$$= \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{256a^3 \cdots}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32a^2d\left(a+bx^2 \cdots}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32a^2d\left(a+bx^2 \cdots}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32a^2d\left(a+bx^2 \cdots}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32a^2d\left(a+bx^2 \cdots}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32a^2d\left(a+bx^2 \cdots}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\cdots}{32a^2d\left(a+bx^2 \cdots}$$

Mathematica [A] time = 0.137275, size = 319, normalized size = 0.57

$$\frac{\sqrt{x}\left(a+bx^2\right)\left(3080a^{3/4}\sqrt{x}\left(a+bx^2\right)^3+1760a^{7/4}\sqrt{x}\left(a+bx^2\right)^2+1280a^{11/4}\sqrt{x}\left(a+bx^2\right)+1024a^{15/4}\sqrt{x}-\frac{1155\sqrt{2}\left(a+bx^2\right)^4 1}{\cdots}\right.}{8192a^{19/4}\sqrt{a}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (Sqrt[x]*(a + b*x^2)*(1024*a^(15/4)*Sqrt[x] + 1280*a^(11/4)*Sqrt[x]*(a + b*
x^2) + 1760*a^(7/4)*Sqrt[x]*(a + b*x^2)^2 + 3080*a^(3/4)*Sqrt[x]*(a + b*x^2
)^3 - (2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/b^(1/4) + (2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/b^(1/4) - (1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (1155*Sqrt[2]*(a + b*x^2)^
4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/(81
92*a^(19/4)*Sqrt[d*x]*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.227, size = 1133, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] 1/8192*(1155*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*x^8*b^4*d^6+2310*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+2310*(a*d^2/b)^(1/4)*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+
4620*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x
^6*a*b^3*d^6+9240*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+9240*(a*d^2/b)^(1/4)*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+69
30*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4
```

```
*a^2*b^2*d^6+13860*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6+13860*(a*d^2/b)^(1/4)*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*
d^6+3080*(d*x)^(13/2)*a*b^3+4620*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^3*b*d^6+9240*(a*d^2/b)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6+9240
*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*x^2*a^3*b*d^6+11000*(d*x)^(9/2)*a^2*b^2*d^2+1155*(a*d^2/b)^(1/4
)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x
-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^6+2310*(a*d^2/
b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*a^4*d^6+2310*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6+14040*(d*x)^(5/2)*a^3*b*d^4+7144*(d*x)^
(1/2)*a^4*d^6)/d^7*(b*x^2+a)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70475, size = 946, normalized size = 1.7

$$\frac{4620\left(a^4b^4dx^8+4\,a^5b^3dx^6+6\,a^6b^2dx^4+4\,a^7bdx^2+a^8d\right)\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^{10}d^2\sqrt{-\frac{1}{a^{19}bd^2}}+dxa^{14}bd\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{3}{4}}-\sqrt{\vphantom{x}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/4096*(4620*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4 + 4*a^7*b*d
*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*arctan(sqrt(a^10*d^2*sqrt(-1/(a^19*b*
```

```
d^2)) + d*x)*a^14*b*d*(-1/(a^19*b*d^2))^(3/4) - sqrt(d*x)*a^14*b*d*(-1/(a^1
9*b*d^2))^(3/4)) + 1155*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4
+ 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(a^5*d*(-1/(a^19*b*d^2)
)^(1/4) + sqrt(d*x)) - 1155*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*
x^4 + 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(-a^5*d*(-1/(a^19*b
*d^2))^(1/4) + sqrt(d*x)) + 4*(385*b^3*x^6 + 1375*a*b^2*x^4 + 1755*a^2*b*x^
2 + 893*a^3)*sqrt(d*x))/(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4
+ 4*a^7*b*d*x^2 + a^8*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(5/2)), x)
```

Giac [A] time = 1.41937, size = 556, normalized size = 1.

$$\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\,\mathrm{l}}{8192\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1155/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) + 115
5/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) + 1155/
8192*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
```

```
+ sqrt(a*d^2/b))/(a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) - 1155/8192*sqrt(2)*(a*b^
3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^5*b*d*sgn(b*d^4*x^2 + a*d^4)) + 1/1024*(385*sqrt(d*x)*b^3*d^7*x^6 + 1375*
sqrt(d*x)*a*b^2*d^7*x^4 + 1755*sqrt(d*x)*a^2*b*d^7*x^2 + 893*sqrt(d*x)*a^3*
d^7)/((b*d^2*x^2 + a*d^2)^4*a^4*sgn(b*d^4*x^2 + a*d^4))
```

3.782 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=602

$$-\frac{3315\sqrt[4]{b}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right.}{4096\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 663/(1024*a^4*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*Sqrt[
d*x]*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 17/(96*a^2*d*Sqrt[d*x
]*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 221/(768*a^3*d*Sqrt[d*x]
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*(a + b*x^2))/(1024*a^
5*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*b^(1/4)*(a + b*x^2)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^
(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b*x^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a
^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (331
5*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])
```

Rubi [A] time = 0.485422, antiderivative size = 602, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3315\sqrt[4]{b}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right.}{4096\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```



```
[Out] 663/(1024*a^4*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*Sqrt[
d*x]*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 17/(96*a^2*d*Sqrt[d*x
]*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 221/(768*a^3*d*Sqrt[d*x]
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*(a + b*x^2))/(1024*a^
5*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*b^(1/4)*(a + b*x^2)*
```

```
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^
(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b*x^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a
^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (331
5*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(17b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^4}\,dx}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{17}{96a^2d\sqrt{dx}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{17}{96a^2d\sqrt{dx}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a}$$

Mathematica [C] time = 0.0148081, size = 52, normalized size = 0.09

$$-\frac{2x\left(a+bx^2\right)^5\,{}_2F_1\left(-\frac{1}{4},5;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^5(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^5*Hypergeometric2F1[-1/4, 5, 3/4, -((b*x^2)/a)])/(a^5*(d*
x)^(3/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.24, size = 1081, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576/d*(9945*(d*x)^(1/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2)))*x^8*b^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4+39780*(d*x)^(1/2)
*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x
+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3+79560*(d*x
)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*x^6*a*b^3+79560*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3+79560*(a*d^2/b)^(1/4)*x^8*b^4+59670*(d*
x)^(1/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/
2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2+
119340*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*x^4*a^2*b^2+119340*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2+302328*(a*d^2/b)^(1/4)*
x^6*a*b^3+39780*(d*x)^(1/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*x^2*a^3*b+79560*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b+79560*(d*x)^(1/2)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b+422552*(a*d^
```

```
2/b)^(1/4)*x^4*a^2*b^2+9945*(d*x)^(1/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2)))*a^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2
)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4+252008*(a*d^2/b)^(
1/4)*x^2*a^3*b+49152*(a*d^2/b)^(1/4)*a^4)*(b*x^2+a)/(a*d^2/b)^(1/4)/(d*x)^(
1/2)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84966, size = 1211, normalized size = 2.01

$$\frac{39780 \left(a^5 b^4 d^2 x^9 + 4\, a^6 b^3 d^2 x^7 + 6\, a^7 b^2 d^2 x^5 + 4\, a^8 b d^2 x^3 + a^9 d^2 x\right) \left(-\frac{b}{a^{21} d^6}\right)^{\frac{1}{4}} \arctan\left(-\frac{36429280875 \sqrt{dx} a^5 b d \left(-\frac{b}{a^{21} d^6}\right)^{\frac{1}{4}} - \sqrt{-132709}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/12288*(39780*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4
*a^8*b*d^2*x^3 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*arctan(-1/36429280875*(36
429280875*sqrt(d*x)*a^5*b*d*(-b/(a^21*d^6))^(1/4) - sqrt(-13270925050696407
65625*a^11*b*d^4*sqrt(-b/(a^21*d^6)) + 1327092505069640765625*b^2*d*x)*a^5*
d*(-b/(a^21*d^6))^(1/4))/b) - 9945*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6
*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(3
6429280875*a^16*d^5*(-b/(a^21*d^6))^(3/4) + 36429280875*sqrt(d*x)*b) + 9945
*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3
```

```
 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(-36429280875*a^16*d^5*(-b/(a^21*d^6
))^(3/4) + 36429280875*sqrt(d*x)*b) - 4*(9945*b^4*x^8 + 37791*a*b^3*x^6 + 5
2819*a^2*b^2*x^4 + 31501*a^3*b*x^2 + 6144*a^4)*sqrt(d*x))/(a^5*b^4*d^2*x^9
+ 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(5/2)), x)
```

Giac [A] time = 1.37066, size = 605, normalized size = 1.

$$-\frac{\frac{49152}{\sqrt{dx}a^5\operatorname{sgn}\left(bd^4x^2+ad^4\right)}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^2d^2\operatorname{sgn}\left(bd^4x^2+ad^4\right)}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^2d^2\operatorname{sgn}\left(bd^4x^2+ad^4\right)}-\frac{9945\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\,\mathrm{l}}{a^6b^2d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*(49152/(sqrt(d*x)*a^5*sgn(b*d^4*x^2 + a*d^4)) + 19890*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a
*d^2/b)^(1/4))/(a^6*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) + 19890*sqrt(2)*(a*b^3*
d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/(a^6*b^2*d^2*sgn(b*d^4*x^2 + a*d^4)) - 9945*sqrt(2)*(a*b^3*d^2
)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b
^2*d^2*sgn(b*d^4*x^2 + a*d^4)) + 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^2*d^2*sgn(b*d^4*x^
2 + a*d^4)) + 8*(3801*sqrt(d*x)*b^4*d^7*x^7 + 13215*sqrt(d*x)*a*b^3*d^7*x^5
```

```
 + 15955*sqrt(d*x)*a^2*b^2*d^7*x^3 + 6925*sqrt(d*x)*a^3*b*d^7*x)/((b*d^2*x^
2 + a*d^2)^4*a^5*sgn(b*d^4*x^2 + a*d^4)))/d
```

3.783 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=602

$$\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1045/(1024*a^4*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*(d
*x)^(3/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 19/(96*a^2*d*(d*
x)^(3/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 95/(256*a^3*d*(d*
x)^(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*(a + b*x^2))/
(3072*a^5*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*(a
 + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*
Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*b^(3/4)*(
a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048
*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*
(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (7315*b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.483648, antiderivative size = 602, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1045/(1024*a^4*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*(d
*x)^(3/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 19/(96*a^2*d*(d*
x)^(3/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 95/(256*a^3*d*(d*
x)^(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*(a + b*x^2))/
(3072*a^5*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*(a
```

```
 + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*
Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*b^(3/4)*(
a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048
*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*
(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (7315*b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(19b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^4}\,d}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{19}{96a^2d(dx)^{3/2}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{19}{96a^2d(dx)^{3/2}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

Mathematica [C] time = 0.0248275, size = 54, normalized size = 0.09

$$-\frac{2x\left(a+bx^2\right)^5{}_2F_1\left(-\frac{3}{4},5;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^5(dx)^{5/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^5*Hypergeometric2F1[-3/4, 5, 1/4, -((b*x^2)/a)])/(3*a^5*(
d*x)^(5/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.243, size = 1183, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576/d^3*(21945*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2)))*x^8*b^5+43890*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^5+4389
0*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))*x^8*b^5+87780*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^(1/2)*
ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^4+175560*(d*x)^(3/2)*(
a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*x^6*a*b^4+175560*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^4+131670*(d*x)^(3
/2)*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^
4*a^2*b^3+263340*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^3+263340*(d*x)^(3/2)*(a*d
^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^
(1/4))*x^4*a^2*b^3+58520*x^8*a*b^4*d^2+87780*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^
(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^3*b^2+175560*(d*x)
^(3/2)*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
```

```
/(a*d^2/b)^(1/4))*x^2*a^3*b^2+175560*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b^2+209
000*x^6*a^2*b^3*d^2+21945*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*b+43890*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*b+43
890*(d*x)^(3/2)*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))*a^4*b+266760*x^4*a^3*b^2*d^2+135736*x^2*a^4*b*d^
2+16384*a^5*d^2)*(b*x^2+a)/(d*x)^(3/2)/a^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88879, size = 1133, normalized size = 1.88

$$-\frac{87780\left(a^5b^4d^3x^{10}+4\,a^6b^3d^3x^8+6\,a^7b^2d^3x^6+4\,a^8bd^3x^4+a^9d^3x^2\right)\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{17}bd^7\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^{12}d^6\sqrt{-\frac{b^3}{a^{23}d^{10}}}}}{b^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/12288*(87780*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 +
 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*arctan(-(sqrt(d*x)
*a^17*b*d^7*(-b^3/(a^23*d^10))^(3/4) - sqrt(a^12*d^6*sqrt(-b^3/(a^23*d^10))
 + b^2*d*x)*a^17*d^7*(-b^3/(a^23*d^10))^(3/4))/b^3) + 21945*(a^5*b^4*d^3*x^
10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)
```

```
*(-b^3/(a^23*d^10))^(1/4)*log(7315*a^6*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*
sqrt(d*x)*b) - 21945*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*
x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*log(-7315*a^6
*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*sqrt(d*x)*b) + 4*(7315*b^4*x^8 + 26125
*a*b^3*x^6 + 33345*a^2*b^2*x^4 + 16967*a^3*b*x^2 + 2048*a^4)*sqrt(d*x))/(a^
5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 +
a^9*d^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(5/2)), x)
```

Giac [A] time = 1.48317, size = 593, normalized size = 0.99

$$-\frac{7315\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^6d^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{7315\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^6d^3\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{7315\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -7315/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) - 73
15/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) - 7315
/8192*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) + 7315/8192*sqrt(2)*(a*b
```

```
^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(a^6*d^3*sgn(b*d^4*x^2 + a*d^4)) - 2/3/(sqrt(d*x)*a^5*d^2*x*sgn(b*d^4*x^2 +
 a*d^4)) - 1/3072*(5267*sqrt(d*x)*b^4*d^6*x^6 + 17933*sqrt(d*x)*a*b^3*d^6*x
^4 + 21057*sqrt(d*x)*a^2*b^2*d^6*x^2 + 8775*sqrt(d*x)*a^3*b*d^6)/((b*d^2*x^
2 + a*d^2)^4*a^5*d*sgn(b*d^4*x^2 + a*d^4))
```

3.784 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=649

$$\frac{13923b\left(a+bx^2\right)}{1024a^6d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923b^{5/4}\left(a+bx^2\right)}{4096\sqrt{2}}$$

```
[Out] 1547/(1024*a^4*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*(d
*x)^(5/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 7/(32*a^2*d*(d*x
)^(5/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 119/(256*a^3*d*(d*
x)^(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*(a + b*x^2))
/(5120*a^5*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*b*(a + b
*x^2))/(1024*a^6*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*b^
(5/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*
b^(5/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1392
3*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (13923*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sq
rt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.534428, antiderivative size = 649, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1112, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13923b\left(a+bx^2\right)}{1024a^6d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923b^{5/4}\left(a+bx^2\right)}{4096\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```


```
[Out] 1547/(1024*a^4*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*(d
*x)^(5/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 7/(32*a^2*d*(d*x
)^(5/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 119/(256*a^3*d*(d*
x)^(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*(a + b*x^2))
```

```
/(5120*a^5*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*b*(a + b
*x^2))/(1024*a^6*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*b^
(5/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*
b^(5/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1392
3*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (13923*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sq
rt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
```

```
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^4}\,d}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

Mathematica [C] time = 0.0168006, size = 54, normalized size = 0.08

$$-\frac{2x\left(a+bx^2\right)^5 {}_2F_1\left(-\frac{5}{4},5;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^5(dx)^{7/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*x*(a + b*x^2)^5*Hypergeometric2F1[-5/4, 5, -1/4, -((b*x^2)/a)])/(5*a^5*
(d*x)^(7/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.251, size = 1129, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/40960/d^3*(69615*(d*x)^(5/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x
-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
)*2^(1/2)*x^8*b^5+139230*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^
(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*x^8*b^5+139230*(d*x)^(5/2)*arctan((2^(1/2)*
(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*x^8*b^5+278460*(d*x)^
(5/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*2^(1/2)*x^6*a*b^4+55692
0*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))
*2^(1/2)*x^6*a*b^4+556920*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*x^6*a*b^4+556920*(a*d^2/b)^(1/4)*x^10*b^5*
d^2+417690*(d*x)^(5/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/
b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*2^(1/2
)*x^4*a^2*b^3+835380*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*2^(1/2)*x^4*a^2*b^3+835380*(d*x)^(5/2)*arctan((2^(1/2)*
(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*x^4*a^2*b^3+2116296*(
a*d^2/b)^(1/4)*x^8*a*b^4*d^2+278460*(d*x)^(5/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2)))*2^(1/2)*x^2*a^3*b^2+556920*(d*x)^(5/2)*arctan((2^(1/2)*(
d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*x^2*a^3*b^2+556920*(d*
x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1
```

```
/2)*x^2*a^3*b^2+2957864*(a*d^2/b)^(1/4)*x^6*a^2*b^3*d^2+69615*(d*x)^(5/2)*l
n(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*2^(1/2)*a^4*b+139230*(d*x)^(5/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*a^
4*b+139230*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*2^(1/2)*a^4*b+1764056*(a*d^2/b)^(1/4)*x^4*a^3*b^2*d^2+344064*(a*d
^2/b)^(1/4)*x^2*a^4*b*d^2-16384*(a*d^2/b)^(1/4)*a^5*d^2)*(b*x^2+a)/(a*d^2/b
)^(1/4)/(d*x)^(5/2)/a^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84727, size = 1350, normalized size = 2.08

$$-\frac{278460\left(a^{6}b^{4}d^{4}x^{11}+4\,a^{7}b^{3}d^{4}x^{9}+6\,a^{8}b^{2}d^{4}x^{7}+4\,a^{9}bd^{4}x^{5}+a^{10}d^{4}x^{3}\right)\left(-\frac{b^{5}}{a^{25}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{2698972561467\,\sqrt{dx}a^{6}b^{4}d^{3}\left(-\frac{b^{5}}{a^{25}d^{14}}\right)}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/20480*(278460*(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7
+ 4*a^9*b*d^4*x^5 + a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*arctan(-1/269897
2561467*(2698972561467*sqrt(d*x)*a^6*b^4*d^3*(-b^5/(a^25*d^14))^(1/4) - sqr
t(-7284452887551739093192089*a^13*b^5*d^8*sqrt(-b^5/(a^25*d^14)) + 72844528
87551739093192089*b^8*d*x)*a^6*d^3*(-b^5/(a^25*d^14))^(1/4))/b^5) - 69615*(
a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*d^4*x^5
```

```
+ a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*log(2698972561467*a^19*d^11*(-b^5/
(a^25*d^14))^(3/4) + 2698972561467*sqrt(d*x)*b^4) + 69615*(a^6*b^4*d^4*x^11
 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*d^4*x^5 + a^10*d^4*x^3)*
(-b^5/(a^25*d^14))^(1/4)*log(-2698972561467*a^19*d^11*(-b^5/(a^25*d^14))^(3
/4) + 2698972561467*sqrt(d*x)*b^4) - 4*(69615*b^5*x^10 + 264537*a*b^4*x^8 +
 369733*a^2*b^3*x^6 + 220507*a^3*b^2*x^4 + 43008*a^4*b*x^2 - 2048*a^5)*sqrt
(d*x))/(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*
d^4*x^5 + a^10*d^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44379, size = 635, normalized size = 0.98

$$\frac{13923\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^7bd^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}+\frac{13923\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^7bd^5\mathrm{sgn}\left(bd^4x^2+ad^4\right)}-\frac{13923\sqrt{2}\left(ab^3d^2\right.}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 13923/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) +
13923/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) -
 13923/8192*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqr
t(d*x) + sqrt(a*d^2/b))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 13923/8192*sqr
t(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
```

```
*d^2/b))/(a^7*b*d^5*sgn(b*d^4*x^2 + a*d^4)) + 1/1024*(3683*sqrt(d*x)*b^5*d^
7*x^7 + 12357*sqrt(d*x)*a*b^4*d^7*x^5 + 14145*sqrt(d*x)*a^2*b^3*d^7*x^3 + 5
599*sqrt(d*x)*a^3*b^2*d^7*x)/((b*d^2*x^2 + a*d^2)^4*a^6*d^3*sgn(b*d^4*x^2 +
 a*d^4)) + 2/5*(25*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^6*d^5*x^2*sgn(b*d^4*x^2
+ a*d^4))
```

3.785 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=150

$$\frac{15a^4b^2(dx)^{m+5}}{d^5(m+5)}+\frac{20a^3b^3(dx)^{m+7}}{d^7(m+7)}+\frac{15a^2b^4(dx)^{m+9}}{d^9(m+9)}+\frac{6a^5b(dx)^{m+3}}{d^3(m+3)}+\frac{a^6(dx)^{m+1}}{d(m+1)}+\frac{6ab^5(dx)^{m+11}}{d^{11}(m+11)}+\frac{b^6(dx)^{m+13}}{d^{13}(m+13)}$$

```
[Out] (a^6*(d*x)^(1 + m))/(d*(1 + m)) + (6*a^5*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
15*a^4*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (20*a^3*b^3*(d*x)^(7 + m))/(d^7*(
7 + m)) + (15*a^2*b^4*(d*x)^(9 + m))/(d^9*(9 + m)) + (6*a*b^5*(d*x)^(11 + m
))/(d^11*(11 + m)) + (b^6*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rubi [A] time = 0.120609, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 270}

$$\frac{15a^4b^2(dx)^{m+5}}{d^5(m+5)}+\frac{20a^3b^3(dx)^{m+7}}{d^7(m+7)}+\frac{15a^2b^4(dx)^{m+9}}{d^9(m+9)}+\frac{6a^5b(dx)^{m+3}}{d^3(m+3)}+\frac{a^6(dx)^{m+1}}{d(m+1)}+\frac{6ab^5(dx)^{m+11}}{d^{11}(m+11)}+\frac{b^6(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*(d*x)^(1 + m))/(d*(1 + m)) + (6*a^5*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
15*a^4*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (20*a^3*b^3*(d*x)^(7 + m))/(d^7*(
7 + m)) + (15*a^2*b^4*(d*x)^(9 + m))/(d^9*(9 + m)) + (6*a*b^5*(d*x)^(11 + m
))/(d^11*(11 + m)) + (b^6*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 dx = \frac{\int (dx)^m \left(ab+b^2x^2\right)^6 dx}{b^6}$$
$$= \frac{\int \left(a^6b^6(dx)^m + \frac{6a^5b^7(dx)^{2+m}}{d^2} + \frac{15a^4b^8(dx)^{4+m}}{d^4} + \frac{20a^3b^9(dx)^{6+m}}{d^6} + \frac{15a^2b^{10}(dx)^{8+m}}{d^8} + \frac{6ab^{11}(dx)^{10+m}}{d^{10}}\right.}{b^6}$$
$$= \frac{a^6(dx)^{1+m}}{d(1+m)} + \frac{6a^5b(dx)^{3+m}}{d^3(3+m)} + \frac{15a^4b^2(dx)^{5+m}}{d^5(5+m)} + \frac{20a^3b^3(dx)^{7+m}}{d^7(7+m)} + \frac{15a^2b^4(dx)^{9+m}}{d^9(9+m)} + \frac{6a}{d}$$

Mathematica [A] time = 0.0760334, size = 105, normalized size = 0.7

$$x(dx)^m\left(\frac{15a^2b^4x^8}{m+9}+\frac{20a^3b^3x^6}{m+7}+\frac{15a^4b^2x^4}{m+5}+\frac{6a^5bx^2}{m+3}+\frac{a^6}{m+1}+\frac{6ab^5x^{10}}{m+11}+\frac{b^6x^{12}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x*(d*x)^m*(a^6/(1 + m) + (6*a^5*b*x^2)/(3 + m) + (15*a^4*b^2*x^4)/(5 + m) +
 (20*a^3*b^3*x^6)/(7 + m) + (15*a^2*b^4*x^8)/(9 + m) + (6*a*b^5*x^10)/(11 +
 m) + (b^6*x^12)/(13 + m))
```

Maple [B] time = 0.051, size = 602, normalized size = 4.

$$\frac{(dx)^m \left(b^6m^6x^{12} + 36\,b^6m^5x^{12} + 6\,ab^5m^6x^{10} + 505\,b^6m^4x^{12} + 228\,ab^5m^5x^{10} + 3480\,b^6m^3x^{12} + 15\,a^2b^4m^6x^8 + 3330\,ab^5m^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (d*x)^m*(b^6*m^6*x^12+36*b^6*m^5*x^12+6*a*b^5*m^6*x^10+505*b^6*m^4*x^12+228
*a*b^5*m^5*x^10+3480*b^6*m^3*x^12+15*a^2*b^4*m^6*x^8+3330*a*b^5*m^4*x^10+12
139*b^6*m^2*x^12+600*a^2*b^4*m^5*x^8+23640*a*b^5*m^3*x^10+19524*b^6*m*x^12+
20*a^3*b^3*m^6*x^6+9195*a^2*b^4*m^4*x^8+84234*a*b^5*m^2*x^10+10395*b^6*x^12
+840*a^3*b^3*m^5*x^6+67920*a^2*b^4*m^3*x^8+137412*a*b^5*m*x^10+15*a^4*b^2*m
^6*x^4+13580*a^3*b^3*m^4*x^6+249405*a^2*b^4*m^2*x^8+73710*a*b^5*x^10+660*a^
4*b^2*m^5*x^4+105840*a^3*b^3*m^3*x^6+415320*a^2*b^4*m*x^8+6*a^5*b*m^6*x^2+1
1295*a^4*b^2*m^4*x^4+406700*a^3*b^3*m^2*x^6+225225*a^2*b^4*x^8+276*a^5*b*m^
```

```
5*x^2+94200*a^4*b^2*m^3*x^4+699720*a^3*b^3*m*x^6+a^6*m^6+5010*a^5*b*m^4*x^2
+389685*a^4*b^2*m^2*x^4+386100*a^3*b^3*x^6+48*a^6*m^5+45240*a^5*b*m^3*x^2+7
11540*a^4*b^2*m*x^4+925*a^6*m^4+208554*a^5*b*m^2*x^2+405405*a^4*b^2*x^4+912
0*a^6*m^3+438324*a^5*b*m*x^2+48259*a^6*m^2+270270*a^5*b*x^2+129072*a^6*m+13
5135*a^6)*x/(13+m)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.61831, size = 1239, normalized size = 8.26

$$\frac{\left(\left(b^6m^6 + 36\,b^6m^5 + 505\,b^6m^4 + 3480\,b^6m^3 + 12139\,b^6m^2 + 19524\,b^6m + 10395\,b^6\right)x^{13} + 6\left(ab^5m^6 + 38\,ab^5m^5 + 555\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] ((b^6*m^6 + 36*b^6*m^5 + 505*b^6*m^4 + 3480*b^6*m^3 + 12139*b^6*m^2 + 19524
*b^6*m + 10395*b^6)*x^13 + 6*(a*b^5*m^6 + 38*a*b^5*m^5 + 555*a*b^5*m^4 + 39
40*a*b^5*m^3 + 14039*a*b^5*m^2 + 22902*a*b^5*m + 12285*a*b^5)*x^11 + 15*(a^
2*b^4*m^6 + 40*a^2*b^4*m^5 + 613*a^2*b^4*m^4 + 4528*a^2*b^4*m^3 + 16627*a^2
*b^4*m^2 + 27688*a^2*b^4*m + 15015*a^2*b^4)*x^9 + 20*(a^3*b^3*m^6 + 42*a^3*
b^3*m^5 + 679*a^3*b^3*m^4 + 5292*a^3*b^3*m^3 + 20335*a^3*b^3*m^2 + 34986*a^
3*b^3*m + 19305*a^3*b^3)*x^7 + 15*(a^4*b^2*m^6 + 44*a^4*b^2*m^5 + 753*a^4*b
^2*m^4 + 6280*a^4*b^2*m^3 + 25979*a^4*b^2*m^2 + 47436*a^4*b^2*m + 27027*a^4
*b^2)*x^5 + 6*(a^5*b*m^6 + 46*a^5*b*m^5 + 835*a^5*b*m^4 + 7540*a^5*b*m^3 +
34759*a^5*b*m^2 + 73054*a^5*b*m + 45045*a^5*b)*x^3 + (a^6*m^6 + 48*a^6*m^5
+ 925*a^6*m^4 + 9120*a^6*m^3 + 48259*a^6*m^2 + 129072*a^6*m + 135135*a^6)*x
)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 26
4207*m + 135135)
```

Sympy [A] time = 6.4638, size = 3188, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Piecewise(((-a**6/(12*x**12) - 3*a**5*b/(5*x**10) - 15*a**4*b**2/(8*x**8) -
 10*a**3*b**3/(3*x**6) - 15*a**2*b**4/(4*x**4) - 3*a*b**5/x**2 + b**6*log(x
))/d**13, Eq(m, -13)), ((-a**6/(10*x**10) - 3*a**5*b/(4*x**8) - 5*a**4*b**2
/(2*x**6) - 5*a**3*b**3/x**4 - 15*a**2*b**4/(2*x**2) + 6*a*b**5*log(x) + b*
*6*x**2/2)/d**11, Eq(m, -11)), ((-a**6/(8*x**8) - a**5*b/x**6 - 15*a**4*b**
2/(4*x**4) - 10*a**3*b**3/x**2 + 15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6
*x**4/4)/d**9, Eq(m, -9)), ((-a**6/(6*x**6) - 3*a**5*b/(2*x**4) - 15*a**4*b
**2/(2*x**2) + 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2
+ b**6*x**6/6)/d**7, Eq(m, -7)), ((-a**6/(4*x**4) - 3*a**5*b/x**2 + 15*a**4
*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x**6 + b**6
*x**8/8)/d**5, Eq(m, -5)), ((-a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**
2*x**2/2 + 5*a**3*b**3*x**4 + 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x
**10/10)/d**3, Eq(m, -3)), ((a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**
4/4 + 10*a**3*b**3*x**6/3 + 15*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x
**12/12)/d, Eq(m, -1)), (a**6*d**m*m**6*x*x**m/(m**7 + 49*m**6 + 973*m**5 +
 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48*a**6*d**m*
m**5*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m
**2 + 264207*m + 135135) + 925*a**6*d**m*m**4*x*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9120*a*
*6*d**m*m**3*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 48259*a**6*d**m*m**2*x*x**m/(m**7 + 49*m
**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
 + 129072*a**6*d**m*m*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 5737
9*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**6*d**m*x*x**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 6*a**5*b*d**m*m**6*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 276*a**5*b*d**m*m**5*x*
*3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
+ 264207*m + 135135) + 5010*a**5*b*d**m*m**4*x**3*x**m/(m**7 + 49*m**6 + 97
3*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 45240
*a**5*b*d**m*m**3*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379
*m**3 + 177331*m**2 + 264207*m + 135135) + 208554*a**5*b*d**m*m**2*x**3*x**
m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 2642
07*m + 135135) + 438324*a**5*b*d**m*m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5
+ 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 270270*a**5*
```

b*d**m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 15*a**4*b**2*d**m*m**6*x**5*x**m/(m**7 + 49
*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 660*a**4*b**2*d**m*m**5*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m
4 + 57379*m3 + 177331*m**2 + 264207*m + 135135) + 11295*a**4*b**2*d**m*
m**4*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 94200*a**4*b**2*d**m*m**3*x**5*x**m/(m**7 + 4
9*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1351
35) + 389685*a**4*b**2*d**m*m**2*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 100
45*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 711540*a**4*b**2*
d**m*m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 405405*a**4*b**2*d**m*x**5*x**m/(m**7 + 49*
m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + 20*a**3*b**3*d**m*m**6*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 840*a**3*b**3*d**m*m**5
*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m*
*2 + 264207*m + 135135) + 13580*a**3*b**3*d**m*m**4*x**7*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 105840*a**3*b**3*d**m*m**3*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m
4 + 57379*m3 + 177331*m**2 + 264207*m + 135135) + 406700*a**3*b**3*d**m
*m**2*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 1773
31*m**2 + 264207*m + 135135) + 699720*a**3*b**3*d**m*m*x**7*x**m/(m**7 + 49
*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 386100*a**3*b**3*d**m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 15*a**2*b**4*d**m*m**6*
x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**
2 + 264207*m + 135135) + 600*a**2*b**4*d**m*m**5*x**9*x**m/(m**7 + 49*m**6
+ 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9
195*a**2*b**4*d**m*m**4*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 +
 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 67920*a**2*b**4*d**m*m**3*
x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**
2 + 264207*m + 135135) + 249405*a**2*b**4*d**m*m**2*x**9*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 415320*a**2*b**4*d**m*m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 225225*a**2*b**4*d**m*x*
*9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
+ 264207*m + 135135) + 6*a*b**5*d**m*m**6*x**11*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 228*a*b
5*dm*m**5*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m*
*3 + 177331*m**2 + 264207*m + 135135) + 3330*a*b**5*d**m*m**4*x**11*x**m/(m
7 + 49*m6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
 + 135135) + 23640*a*b**5*d**m*m**3*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 +
 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 84234*a*b**5*
d**m*m**2*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
 177331*m**2 + 264207*m + 135135) + 137412*a*b**5*d**m*m*x**11*x**m/(m**7 +

```
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 73710*a*b**5*d**m*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + b**6*d**m*m**6*x**13*x*
*m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264
207*m + 135135) + 36*b**6*d**m*m**5*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 +
 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 505*b**6*d**m
*m**4*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 3480*b**6*d**m*m**3*x**13*x**m/(m**7 + 49*m
**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
 + 12139*b**6*d**m*m**2*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 19524*b**6*d**m*m*x**13*x
**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 26
4207*m + 135135) + 10395*b**6*d**m*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 +
10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135), True))
```

Giac [B] time = 1.26087, size = 1143, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^6*m^6*x^13 + 36*(d*x)^m*b^6*m^5*x^13 + 6*(d*x)^m*a*b^5*m^6*x^11
+ 505*(d*x)^m*b^6*m^4*x^13 + 228*(d*x)^m*a*b^5*m^5*x^11 + 3480*(d*x)^m*b^6*
m^3*x^13 + 15*(d*x)^m*a^2*b^4*m^6*x^9 + 3330*(d*x)^m*a*b^5*m^4*x^11 + 12139
*(d*x)^m*b^6*m^2*x^13 + 600*(d*x)^m*a^2*b^4*m^5*x^9 + 23640*(d*x)^m*a*b^5*m
^3*x^11 + 19524*(d*x)^m*b^6*m*x^13 + 20*(d*x)^m*a^3*b^3*m^6*x^7 + 9195*(d*x
)^m*a^2*b^4*m^4*x^9 + 84234*(d*x)^m*a*b^5*m^2*x^11 + 10395*(d*x)^m*b^6*x^13
 + 840*(d*x)^m*a^3*b^3*m^5*x^7 + 67920*(d*x)^m*a^2*b^4*m^3*x^9 + 137412*(d*
x)^m*a*b^5*m*x^11 + 15*(d*x)^m*a^4*b^2*m^6*x^5 + 13580*(d*x)^m*a^3*b^3*m^4*
x^7 + 249405*(d*x)^m*a^2*b^4*m^2*x^9 + 73710*(d*x)^m*a*b^5*x^11 + 660*(d*x)
^m*a^4*b^2*m^5*x^5 + 105840*(d*x)^m*a^3*b^3*m^3*x^7 + 415320*(d*x)^m*a^2*b^
4*m*x^9 + 6*(d*x)^m*a^5*b*m^6*x^3 + 11295*(d*x)^m*a^4*b^2*m^4*x^5 + 406700*
(d*x)^m*a^3*b^3*m^2*x^7 + 225225*(d*x)^m*a^2*b^4*x^9 + 276*(d*x)^m*a^5*b*m^
5*x^3 + 94200*(d*x)^m*a^4*b^2*m^3*x^5 + 699720*(d*x)^m*a^3*b^3*m*x^7 + (d*x
)^m*a^6*m^6*x + 5010*(d*x)^m*a^5*b*m^4*x^3 + 389685*(d*x)^m*a^4*b^2*m^2*x^5
 + 386100*(d*x)^m*a^3*b^3*x^7 + 48*(d*x)^m*a^6*m^5*x + 45240*(d*x)^m*a^5*b*
m^3*x^3 + 711540*(d*x)^m*a^4*b^2*m*x^5 + 925*(d*x)^m*a^6*m^4*x + 208554*(d*
x)^m*a^5*b*m^2*x^3 + 405405*(d*x)^m*a^4*b^2*x^5 + 9120*(d*x)^m*a^6*m^3*x +
438324*(d*x)^m*a^5*b*m*x^3 + 48259*(d*x)^m*a^6*m^2*x + 270270*(d*x)^m*a^5*b
*x^3 + 129072*(d*x)^m*a^6*m*x + 135135*(d*x)^m*a^6*x)/(m^7 + 49*m^6 + 973*m
```

```
^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

3.786 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2 dx$

Optimal. Leaf size=104

$$\frac{6a^2b^2(dx)^{m+5}}{d^5(m+5)}+\frac{4a^3b(dx)^{m+3}}{d^3(m+3)}+\frac{a^4(dx)^{m+1}}{d(m+1)}+\frac{4ab^3(dx)^{m+7}}{d^7(m+7)}+\frac{b^4(dx)^{m+9}}{d^9(m+9)}$$

```
[Out] (a^4*(d*x)^(1 + m))/(d*(1 + m)) + (4*a^3*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
6*a^2*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (4*a*b^3*(d*x)^(7 + m))/(d^7*(7 +
m)) + (b^4*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rubi [A] time = 0.074949, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 270}

$$\frac{6a^2b^2(dx)^{m+5}}{d^5(m+5)}+\frac{4a^3b(dx)^{m+3}}{d^3(m+3)}+\frac{a^4(dx)^{m+1}}{d(m+1)}+\frac{4ab^3(dx)^{m+7}}{d^7(m+7)}+\frac{b^4(dx)^{m+9}}{d^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*(d*x)^(1 + m))/(d*(1 + m)) + (4*a^3*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
6*a^2*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (4*a*b^3*(d*x)^(7 + m))/(d^7*(7 +
m)) + (b^4*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2 dx = \frac{\int (dx)^m \left(ab+b^2x^2\right)^4 dx}{b^4}$$
$$= \frac{\int \left(a^4b^4(dx)^m + \frac{4a^3b^5(dx)^{2+m}}{d^2} + \frac{6a^2b^6(dx)^{4+m}}{d^4} + \frac{4ab^7(dx)^{6+m}}{d^6} + \frac{b^8(dx)^{8+m}}{d^8}\right) dx}{b^4}$$
$$= \frac{a^4(dx)^{1+m}}{d(1+m)} + \frac{4a^3b(dx)^{3+m}}{d^3(3+m)} + \frac{6a^2b^2(dx)^{5+m}}{d^5(5+m)} + \frac{4ab^3(dx)^{7+m}}{d^7(7+m)} + \frac{b^4(dx)^{9+m}}{d^9(9+m)}$$

Mathematica [A] time = 0.0349877, size = 73, normalized size = 0.7

$$x(dx)^m \left(\frac{6a^2b^2x^4}{m+5} + \frac{4a^3bx^2}{m+3} + \frac{a^4}{m+1} + \frac{4ab^3x^6}{m+7} + \frac{b^4x^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x*(d*x)^m*(a^4/(1 + m) + (4*a^3*b*x^2)/(3 + m) + (6*a^2*b^2*x^4)/(5 + m) +
(4*a*b^3*x^6)/(7 + m) + (b^4*x^8)/(9 + m))
```

Maple [B] time = 0.051, size = 292, normalized size = 2.8

$$\frac{(dx)^m \left(b^4m^4x^8 + 16\,b^4m^3x^8 + 4\,ab^3m^4x^6 + 86\,b^4m^2x^8 + 72\,ab^3m^3x^6 + 176\,b^4mx^8 + 6\,a^2b^2m^4x^4 + 416\,ab^3m^2x^6 + 105\,b^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (d*x)^m*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^8+72*a*b^3
*m^3*x^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*x^6+105*b^4*x^8+120*
a^2*b^2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x^2+780*a^2*b^2*m^2*x^4+540*a*b
^3*x^6+88*a^3*b*m^3*x^2+1800*a^2*b^2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+1134*a
^2*b^2*x^4+24*a^4*m^3+1832*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*a^4*m
+945*a^4)*x/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.6283, size = 576, normalized size = 5.54

$$\frac{\left(\left(b^4m^4 + 16\,b^4m^3 + 86\,b^4m^2 + 176\,b^4m + 105\,b^4\right)x^9 + 4\left(ab^3m^4 + 18\,ab^3m^3 + 104\,ab^3m^2 + 222\,ab^3m + 135\,ab^3\right)x^7 + 6\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] ((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 + 4*(a*b^3*m
^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a*b^3)*x^7 + 6*(a^2*b
^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 300*a^2*b^2*m + 189*a^2*b^2)*x^
5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 164*a^3*b*m^2 + 458*a^3*b*m + 315*a^3*b)*
x^3 + (a^4*m^4 + 24*a^4*m^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*(d*x)^m
/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [A] time = 3.08984, size = 1321, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Piecewise(((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*a
*b**3/x**2 + b**4*log(x))/d**9, Eq(m, -9)), ((-a**4/(6*x**6) - a**3*b/x**4
- 3*a**2*b**2/x**2 + 4*a*b**3*log(x) + b**4*x**2/2)/d**7, Eq(m, -7)), ((-a*
*4/(4*x**4) - 2*a**3*b/x**2 + 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**
4/4)/d**5, Eq(m, -5)), ((-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**
```

```
2 + a*b**3*x**4 + b**4*x**6/6)/d**3, Eq(m, -3)), ((a**4*log(x) + 2*a**3*b*x
**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*x**8/8)/d, Eq(m, -1)), (a
**4*d**m*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 24*a**4*d**m*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 206*a**4*d**m*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 744*a**4*d**m*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 945*a**4*d**m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 4*a**3*b*d**m*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**
3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*d**m*m**3*x**3*x**m/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*d**m*m**2*x**3*x**m/(m
**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1832*a**3*b*d**m*m*x*
*3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1260*a**3*b
*d**m*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 6*a
**2*b**2*d**m*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 120*a**2*b**2*d**m*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 95
0*m**2 + 1689*m + 945) + 780*a**2*b**2*d**m*m**2*x**5*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 1800*a**2*b**2*d**m*m*x**5*x**m/(m*
*5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1134*a**2*b**2*d**m*x*
*5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a*b**3*d*
*m*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 7
2*a*b**3*d**m*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 416*a*b**3*d**m*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 888*a*b**3*d**m*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**
3 + 950*m**2 + 1689*m + 945) + 540*a*b**3*d**m*x**7*x**m/(m**5 + 25*m**4 +
230*m**3 + 950*m**2 + 1689*m + 945) + b**4*d**m*m**4*x**9*x**m/(m**5 + 25*m
**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*b**4*d**m*m**3*x**9*x**m/(m*
*5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**4*d**m*m**2*x**9
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*b**4*d**m
*m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*b*
*4*d**m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), Tr
ue))
```

Giac [B] time = 1.28462, size = 560, normalized size = 5.38

$$(dx)^m b^4m^4x^9 + 16\,(dx)^m b^4m^3x^9 + 4\,(dx)^m ab^3m^4x^7 + 86\,(dx)^m b^4m^2x^9 + 72\,(dx)^m ab^3m^3x^7 + 176\,(dx)^m b^4mx^9 + 6\,(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^4*m^4*x^9 + 16*(d*x)^m*b^4*m^3*x^9 + 4*(d*x)^m*a*b^3*m^4*x^7 + 8
6*(d*x)^m*b^4*m^2*x^9 + 72*(d*x)^m*a*b^3*m^3*x^7 + 176*(d*x)^m*b^4*m*x^9 +
```

```
6*(d*x)^m*a^2*b^2*m^4*x^5 + 416*(d*x)^m*a*b^3*m^2*x^7 + 105*(d*x)^m*b^4*x^9
 + 120*(d*x)^m*a^2*b^2*m^3*x^5 + 888*(d*x)^m*a*b^3*m*x^7 + 4*(d*x)^m*a^3*b*
m^4*x^3 + 780*(d*x)^m*a^2*b^2*m^2*x^5 + 540*(d*x)^m*a*b^3*x^7 + 88*(d*x)^m*
a^3*b*m^3*x^3 + 1800*(d*x)^m*a^2*b^2*m*x^5 + (d*x)^m*a^4*m^4*x + 656*(d*x)^
m*a^3*b*m^2*x^3 + 1134*(d*x)^m*a^2*b^2*x^5 + 24*(d*x)^m*a^4*m^3*x + 1832*(d
*x)^m*a^3*b*m*x^3 + 206*(d*x)^m*a^4*m^2*x + 1260*(d*x)^m*a^3*b*x^3 + 744*(d
*x)^m*a^4*m*x + 945*(d*x)^m*a^4*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689
*m + 945)
```

3.787 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=58

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{b^2(dx)^{m+5}}{d^5(m+5)}$$

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (b^
2*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rubi [A] time = 0.0232997, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {14}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{b^2(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (b^
2*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) \, dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{2+m}}{d^2}+\frac{b^2(dx)^{4+m}}{d^4}\right) \, dx\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{b^2(dx)^{5+m}}{d^5(5+m)}\end{aligned}$$

Mathematica [A] time = 0.0294711, size = 41, normalized size = 0.71

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{2abx^2}{m+3}+\frac{b^2x^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

Maple [A] time = 0.049, size = 94, normalized size = 1.6

$$\frac{(dx)^m \left(b^2m^2x^4+4\,b^2mx^4+2\,abm^2x^2+3\,b^2x^4+12\,abmx^2+a^2m^2+10\,abx^2+8\,ma^2+15\,a^2\right)x}{(5+m)(3+m)(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] (d*x)^m*(b^2*m^2*x^4+4*b^2*m*x^4+2*a*b*m^2*x^2+3*b^2*x^4+12*a*b*m*x^2+a^2*m
^2+10*a*b*x^2+8*a^2*m+15*a^2)*x/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57241, size = 186, normalized size = 3.21

$$\frac{\left(\left(b^2m^2+4\,b^2m+3\,b^2\right)x^5+2\left(abm^2+6\,abm+5\,ab\right)x^3+\left(a^2m^2+8\,a^2m+15\,a^2\right)x\right)(dx)^m}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^2
*m^2 + 8*a^2*m + 15*a^2)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [A] time = 0.894467, size = 345, normalized size = 5.95

$$\begin{cases} \frac{-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)}{d^5} \\ \frac{-\frac{a^2}{2x^2} + 2ab \log(x) + \frac{b^2x^2}{2}}{d^3} \\ \frac{a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}}{d} \\ \frac{a^2d^m m^2 x x^m}{m^3+9m^2+23m+15} + \frac{8a^2d^m m x x^m}{m^3+9m^2+23m+15} + \frac{15a^2d^m x x^m}{m^3+9m^2+23m+15} + \frac{2abd^m m^2 x^3 x^m}{m^3+9m^2+23m+15} + \frac{12abd^m m x^3 x^m}{m^3+9m^2+23m+15} + \frac{10abd^m x^3 x^m}{m^3+9m^2+23m+15} + \frac{b^2d^m m^2 x^5 x^m}{m^3+9m^2+23m+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Piecewise(((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x))/d**5, Eq(m, -5)), ((-a
**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2)/d**3, Eq(m, -3)), ((a**2*log(x)
+ a*b*x**2 + b**2*x**4/4)/d, Eq(m, -1)), (a**2*d**m*m**2*x*x**m/(m**3 + 9*m
**2 + 23*m + 15) + 8*a**2*d**m*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a*
*2*d**m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 2*a*b*d**m*m**2*x**3*x**m/(m**
3 + 9*m**2 + 23*m + 15) + 12*a*b*d**m*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 1
5) + 10*a*b*d**m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + b**2*d**m*m**2*x**
5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*d**m*m*x**5*x**m/(m**3 + 9*m**2
 + 23*m + 15) + 3*b**2*d**m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15), True))
```

Giac [B] time = 1.28529, size = 182, normalized size = 3.14

$$\frac{(dx)^m b^2m^2x^5 + 4 (dx)^m b^2mx^5 + 2 (dx)^m abm^2x^3 + 3 (dx)^m b^2x^5 + 12 (dx)^m abmx^3 + (dx)^m a^2m^2x + 10 (dx)^m abx^3 + 8}{m^3 + 9 m^2 + 23 m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^2*m^2*x^5 + 4*(d*x)^m*b^2*m*x^5 + 2*(d*x)^m*a*b*m^2*x^3 + 3*(d*x
)^m*b^2*x^5 + 12*(d*x)^m*a*b*m*x^3 + (d*x)^m*a^2*m^2*x + 10*(d*x)^m*a*b*x^3
 + 8*(d*x)^m*a^2*m*x + 15*(d*x)^m*a^2*x)/(m^3 + 9*m^2 + 23*m + 15)
```

3.788 $\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=44

$$\frac{(dx)^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^2*d*(1 + m))
```

Rubi [A] time = 0.0176527, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 364}

$$\frac{(dx)^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^2*d*(1 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^m}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{(dx)^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^2d(1+m)}$$

Mathematica [A] time = 0.0081655, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(d*x)^m*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^2*(1 + m))
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b^2x^4+2\,abx^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b^2x^4+2\,abx^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b^2x^4 + 2\,abx^2 + a^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.789 $\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=44

$$\frac{(dx)^{m+1}{}_2F_1\left(4,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^4d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[4, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^4*d*(1 + m))
```

Rubi [A] time = 0.0169372, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 364}

$$\frac{(dx)^{m+1}{}_2F_1\left(4,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^4d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[4, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^4*d*(1 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{(dx)^m}{(ab+b^2x^2)^4}\,dx$$
$$= \frac{(dx)^{1+m}\,{}_2F_1\left(4,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^4d(1+m)}$$

Mathematica [A] time = 0.0079946, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(4,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^4(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*Hypergeometric2F1[4, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^4*(1 + m))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4+2\,abx^2+a^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4+2\,abx^2+a^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^4x^8 + 4\,ab^3x^6 + 6\,a^2b^2x^4 + 4\,a^3bx^2 + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + bx^2)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2, x)
```

3.790 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=44

$$\frac{(dx)^{m+1}\,{}_2F_1\left(6,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^6d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[6, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^6*d*(1 + m))
```

Rubi [A] time = 0.0174174, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {28, 364}

$$\frac{(dx)^{m+1}\,{}_2F_1\left(6,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^6d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[6, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^6*d*(1 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^m}{\left(ab+b^2x^2\right)^6}\,dx$$
$$= \frac{(dx)^{1+m}\,{}_2F_1\left(6,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^6d(1+m)}$$

Mathematica [A] time = 0.0090146, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(6,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^6(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (x*(d*x)^m*Hypergeometric2F1[6, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^6*(1 + m))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^6x^{12} + 6\,ab^5x^{10} + 15\,a^2b^4x^8 + 20\,a^3b^3x^6 + 15\,a^4b^2x^4 + 6\,a^5bx^2 + a^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^6*x^12 + 6*a*b^5*x^10 + 15*a^2*b^4*x^8 + 20*a^3*b^3*x^6
 + 15*a^4*b^2*x^4 + 6*a^5*b*x^2 + a^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3, x)
```

3.791 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=313

$$\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{5ab^4\sqrt{a}}{d}$$

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2))
 + (5*a^4*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a
+ b*x^2)) + (10*a^3*b^2*(d*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5
*(5 + m)*(a + b*x^2)) + (10*a^2*b^3*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(d^7*(7 + m)*(a + b*x^2)) + (5*a*b^4*(d*x)^(9 + m)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(d^9*(9 + m)*(a + b*x^2)) + (b^5*(d*x)^(11 + m)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(d^11*(11 + m)*(a + b*x^2))
```

Rubi [A] time = 0.120312, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 270}

$$\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{5ab^4\sqrt{a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2))
 + (5*a^4*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a
+ b*x^2)) + (10*a^3*b^2*(d*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5
*(5 + m)*(a + b*x^2)) + (10*a^2*b^3*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(d^7*(7 + m)*(a + b*x^2)) + (5*a*b^4*(d*x)^(9 + m)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(d^9*(9 + m)*(a + b*x^2)) + (b^5*(d*x)^(11 + m)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(d^11*(11 + m)*(a + b*x^2))
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m \left(ab+b^2x^2\right)^5 dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5b^5(dx)^m + \frac{5a^4b^6(dx)^{2+m}}{d^2} + \frac{10a^3b^7(dx)^{4+m}}{d^4} + \frac{10a^2b^8(dx)^{6+m}}{d^6} + \frac{5ab^9(\ldots}{\ldots}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)} + \frac{5a^4b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)} + \frac{10a^3b^2(dx)^{5+m}\ldots}{d^5(5+\ldots}$$

Mathematica [A] time = 0.0900514, size = 111, normalized size = 0.35

$$\frac{x\left(\left(a+bx^2\right)^2\right)^{5/2}(dx)^m\left(\frac{10a^2b^3x^6}{m+7}+\frac{10a^3b^2x^4}{m+5}+\frac{5a^4bx^2}{m+3}+\frac{a^5}{m+1}+\frac{5ab^4x^8}{m+9}+\frac{b^5x^{10}}{m+11}\right)}{\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^2)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^2)/(3 + m) + (10
*a^3*b^2*x^4)/(5 + m) + (10*a^2*b^3*x^6)/(7 + m) + (5*a*b^4*x^8)/(9 + m) +
(b^5*x^10)/(11 + m)))/(a + b*x^2)^5
```

Maple [A] time = 0.166, size = 453, normalized size = 1.5

$$\frac{\left(b^5m^5x^{10}+25\,b^5m^4x^{10}+5\,ab^4m^5x^8+230\,b^5m^3x^{10}+135\,ab^4m^4x^8+950\,b^5m^2x^{10}+10\,a^2b^3m^5x^6+1310\,ab^4m^3x^8+1689\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] x*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+135*a*b^4*
m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*m^3*x^8+1689*b^5*m*x
^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+945*b^5*x^10+10*a^3*b^2*m^5*x^4+
3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*x^8+310*a^3*b^2*m^4*x^4+13660*a^2*b^3*m^
2*x^6+5775*a*b^4*x^8+5*a^4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b^3*m*x
^6+165*a^4*b*m^4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+2030*a
^4*b*m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+20790*a^3*b
^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*b*x^2+9129*a^5*
m+10395*a^5)*(d*x)^m*((b*x^2+a)^2)^(5/2)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+
m)/(b*x^2+a)^5
```

Maxima [A] time = 0.994014, size = 328, normalized size = 1.05

$$\frac{\left(\left(m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945\right)b^5 d^m x^{11} + 5\left(m^5 + 27\,m^4 + 262\,m^3 + 1122\,m^2 + 2041\,m + 1155\right)ab^4 d^m \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*d^m*x^11 + 5*(m^5 +
27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*d^m*x^9 + 10*(m^5 + 29*m
^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*d^m*x^7 + 10*(m^5 + 31*m^4
 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*d^m*x^5 + 5*(m^5 + 33*m^4 +
406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*d^m*x^3 + (m^5 + 35*m^4 + 470*m^3
 + 3010*m^2 + 9129*m + 10395)*a^5*d^m*x)*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480
*m^3 + 12139*m^2 + 19524*m + 10395)
```

Fricas [A] time = 1.62986, size = 873, normalized size = 2.79

$$\frac{\left(\left(b^5 m^5 + 25\,b^5 m^4 + 230\,b^5 m^3 + 950\,b^5 m^2 + 1689\,b^5 m + 945\,b^5\right)x^{11} + 5\left(ab^4 m^5 + 27\,ab^4 m^4 + 262\,ab^4 m^3 + 1122\,ab^4 m^2 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m + 945*b^5)*
x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 1122*a*b^4*m^2 + 2041*
a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 + 29*a^2*b^3*m^4 + 302*a^2*b^3*
m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b^2*m
^5 + 31*a^3*b^2*m^4 + 350*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^2*m +
 2079*a^3*b^2)*x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 2262*a^4
*b*m^2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 470*a^5*m
^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*(d*x)^m/(m^6 + 36*m^5 + 505*
m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**(5/2), x)
```

Giac [B] time = 1.29992, size = 1215, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^5*m^5*x^11*sgn(b*x^2 + a) + 25*(d*x)^m*b^5*m^4*x^11*sgn(b*x^2 +
a) + 5*(d*x)^m*a*b^4*m^5*x^9*sgn(b*x^2 + a) + 230*(d*x)^m*b^5*m^3*x^11*sgn(
b*x^2 + a) + 135*(d*x)^m*a*b^4*m^4*x^9*sgn(b*x^2 + a) + 950*(d*x)^m*b^5*m^2
*x^11*sgn(b*x^2 + a) + 10*(d*x)^m*a^2*b^3*m^5*x^7*sgn(b*x^2 + a) + 1310*(d*
x)^m*a*b^4*m^3*x^9*sgn(b*x^2 + a) + 1689*(d*x)^m*b^5*m*x^11*sgn(b*x^2 + a)
+ 290*(d*x)^m*a^2*b^3*m^4*x^7*sgn(b*x^2 + a) + 5610*(d*x)^m*a*b^4*m^2*x^9*s
gn(b*x^2 + a) + 945*(d*x)^m*b^5*x^11*sgn(b*x^2 + a) + 10*(d*x)^m*a^3*b^2*m^
5*x^5*sgn(b*x^2 + a) + 3020*(d*x)^m*a^2*b^3*m^3*x^7*sgn(b*x^2 + a) + 10205*
(d*x)^m*a*b^4*m*x^9*sgn(b*x^2 + a) + 310*(d*x)^m*a^3*b^2*m^4*x^5*sgn(b*x^2
+ a) + 13660*(d*x)^m*a^2*b^3*m^2*x^7*sgn(b*x^2 + a) + 5775*(d*x)^m*a*b^4*x^
```

```
9*sgn(b*x^2 + a) + 5*(d*x)^m*a^4*b*m^5*x^3*sgn(b*x^2 + a) + 3500*(d*x)^m*a^
3*b^2*m^3*x^5*sgn(b*x^2 + a) + 25770*(d*x)^m*a^2*b^3*m*x^7*sgn(b*x^2 + a) +
 165*(d*x)^m*a^4*b*m^4*x^3*sgn(b*x^2 + a) + 17300*(d*x)^m*a^3*b^2*m^2*x^5*s
gn(b*x^2 + a) + 14850*(d*x)^m*a^2*b^3*x^7*sgn(b*x^2 + a) + (d*x)^m*a^5*m^5*
x*sgn(b*x^2 + a) + 2030*(d*x)^m*a^4*b*m^3*x^3*sgn(b*x^2 + a) + 34890*(d*x)^
m*a^3*b^2*m*x^5*sgn(b*x^2 + a) + 35*(d*x)^m*a^5*m^4*x*sgn(b*x^2 + a) + 1131
0*(d*x)^m*a^4*b*m^2*x^3*sgn(b*x^2 + a) + 20790*(d*x)^m*a^3*b^2*x^5*sgn(b*x^
2 + a) + 470*(d*x)^m*a^5*m^3*x*sgn(b*x^2 + a) + 26765*(d*x)^m*a^4*b*m*x^3*s
gn(b*x^2 + a) + 3010*(d*x)^m*a^5*m^2*x*sgn(b*x^2 + a) + 17325*(d*x)^m*a^4*b
*x^3*sgn(b*x^2 + a) + 9129*(d*x)^m*a^5*m*x*sgn(b*x^2 + a) + 10395*(d*x)^m*a
^5*x*sgn(b*x^2 + a))/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524
*m + 10395)
```

3.792 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=205

$$\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{a^3\sqrt{a^2+2abx^2-}}{d(m+1)\left(\right.}$$

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2))
 + (3*a^2*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a
+ b*x^2)) + (3*a*b^2*(d*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(5
 + m)*(a + b*x^2)) + (b^3*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d
^7*(7 + m)*(a + b*x^2))
```

Rubi [A] time = 0.0758189, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 270}

$$\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{a^3\sqrt{a^2+2abx^2-}}{d(m+1)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```



```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2))
 + (3*a^2*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a
+ b*x^2)) + (3*a*b^2*(d*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(5
 + m)*(a + b*x^2)) + (b^3*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d
^7*(7 + m)*(a + b*x^2))
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3(dx)^m+\frac{3a^2b^4(dx)^{2+m}}{d^2}+\frac{3ab^5(dx)^{4+m}}{d^4}+\frac{b^6(dx)^{6+m}}{d^6}\right) dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)}+\frac{3a^2b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)}+\frac{3ab^2(dx)^{5+m}}{d^5(5+}$$

Mathematica [A] time = 0.0693267, size = 131, normalized size = 0.64

$$\frac{x\sqrt{\left(a+bx^2\right)^2}(dx)^m \left(3a^2b\left(m^3+13m^2+47m+35\right)x^2+a^3\left(m^3+15m^2+71m+105\right)+3ab^2\left(m^3+11m^2+31m+21\right)}{(m+1)(m+3)(m+5)(m+7)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^2)^2]*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2*b*(
35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4 + b^
3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a + b*x
^2))
```

Maple [A] time = 0.169, size = 199, normalized size = 1.

$$\frac{\left(b^3m^3x^6+9\,b^3m^2x^6+3\,ab^2m^3x^4+23\,b^3mx^6+33\,ab^2m^2x^4+15\,b^3x^6+3\,a^2bm^3x^2+93\,ab^2mx^4+39\,a^2bm^2x^2+63\,ax^4\right.}{(7+m)\,(5+m)\,(3+m)\,(1+m)\left(bx^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] x*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^2*x^4+
15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x^4+a^3
*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)*(d*x)^m*((b
*x^2+a)^2)^(3/2)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^3
```

Maxima [A] time = 0.984213, size = 161, normalized size = 0.79

$$\frac{\left(\left(m^3+9\,m^2+23\,m+15\right)b^3d^mx^7+3\left(m^3+11\,m^2+31\,m+21\right)ab^2d^mx^5+3\left(m^3+13\,m^2+47\,m+35\right)a^2bd^mx^3+\left(m^3+\right.\right.}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*d^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^2
*d^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*d^m*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*a^3*d^m*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Fricas [A] time = 1.55453, size = 352, normalized size = 1.72

$$\frac{\left(\left(b^3m^3+9\,b^3m^2+23\,b^3m+15\,b^3\right)x^7+3\left(ab^2m^3+11\,ab^2m^2+31\,ab^2m+21\,ab^2\right)x^5+3\left(a^2bm^3+13\,a^2bm^2+47\,a^2bm+\right.\right.}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 11*a*b^2*m^
2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*b*m^2 + 47*a^2*b*m +
 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a^3*m + 105*a^3)*x)*(d*x)^m/(m^
4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**(3/2), x)
```

Giac [B] time = 1.27073, size = 518, normalized size = 2.53

$$\frac{(dx)^m b^3m^3x^7\operatorname{sgn}\left(bx^2+a\right)+9\,(dx)^m b^3m^2x^7\operatorname{sgn}\left(bx^2+a\right)+3\,(dx)^m ab^2m^3x^5\operatorname{sgn}\left(bx^2+a\right)+23\,(dx)^m b^3mx^7\operatorname{sgn}\left(bx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^3*m^3*x^7*sgn(b*x^2 + a) + 9*(d*x)^m*b^3*m^2*x^7*sgn(b*x^2 + a)
+ 3*(d*x)^m*a*b^2*m^3*x^5*sgn(b*x^2 + a) + 23*(d*x)^m*b^3*m*x^7*sgn(b*x^2 +
 a) + 33*(d*x)^m*a*b^2*m^2*x^5*sgn(b*x^2 + a) + 15*(d*x)^m*b^3*x^7*sgn(b*x^
2 + a) + 3*(d*x)^m*a^2*b*m^3*x^3*sgn(b*x^2 + a) + 93*(d*x)^m*a*b^2*m*x^5*sg
n(b*x^2 + a) + 39*(d*x)^m*a^2*b*m^2*x^3*sgn(b*x^2 + a) + 63*(d*x)^m*a*b^2*x
^5*sgn(b*x^2 + a) + (d*x)^m*a^3*m^3*x*sgn(b*x^2 + a) + 141*(d*x)^m*a^2*b*m*
x^3*sgn(b*x^2 + a) + 15*(d*x)^m*a^3*m^2*x*sgn(b*x^2 + a) + 105*(d*x)^m*a^2*
b*x^3*sgn(b*x^2 + a) + 71*(d*x)^m*a^3*m*x*sgn(b*x^2 + a) + 105*(d*x)^m*a^3*
x*sgn(b*x^2 + a))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.793 $\int (dx)^m \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=97

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+1}}{d(m+1)\left(a+bx^2\right)}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2)) +
 (b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a + b*x^2)
)
```

Rubi [A] time = 0.0337345, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 14}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+1}}{d(m+1)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```


```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2)) +
 (b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a + b*x^2)
)
```


Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int (dx)^m\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m\left(ab+b^2x^2\right)dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^m+\frac{b^2(dx)^{2+m}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)} + \frac{b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0247809, size = 53, normalized size = 0.55

$$\frac{x\sqrt{\left(a+bx^2\right)^2}(dx)^m\left(a(m+3)+b(m+1)x^2\right)}{(m+1)(m+3)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^2)^2]*(a*(3 + m) + b*(1 + m)*x^2))/((1 + m)*(3 + m
)*(a + b*x^2))
```

Maple [A] time = 0.166, size = 56, normalized size = 0.6

$$\frac{\left(bmx^2+bx^2+am+3\,a\right)x\,(dx)^m}{(3+m)\,(1+m)\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] x*(b*m*x^2+b*x^2+a*m+3*a)*(d*x)^m*((b*x^2+a)^2)^(1/2)/(3+m)/(1+m)/(b*x^2+a)
```

Maxima [A] time = 0.971085, size = 47, normalized size = 0.48

$$\frac{\left(bd^m(m+1)x^3+ad^m(m+3)x\right)x^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*d^m*(m + 1)*x^3 + a*d^m*(m + 3)*x)*x^m/(m^2 + 4*m + 3)
```

Fricas [A] time = 1.54057, size = 77, normalized size = 0.79

$$\frac{\left((bm+b)x^3+(am+3\,a)x\right)(dx)^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^3 + (a*m + 3*a)*x)*(d*x)^m/(m^2 + 4*m + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m\sqrt{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**2)**2), x)
```

Giac [A] time = 1.28775, size = 112, normalized size = 1.15

$$\frac{(dx)^m\,bmx^3\mathrm{sgn}\left(bx^2+a\right)+(dx)^m\,bx^3\mathrm{sgn}\left(bx^2+a\right)+(dx)^m\,amx\mathrm{sgn}\left(bx^2+a\right)+3\,(dx)^m\,ax\mathrm{sgn}\left(bx^2+a\right)}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b*m*x^3*sgn(b*x^2 + a) + (d*x)^m*b*x^3*sgn(b*x^2 + a) + (d*x)^m*a*
m*x*sgn(b*x^2 + a) + 3*(d*x)^m*a*x*sgn(b*x^2 + a))/(m^2 + 4*m + 3)
```

3.794 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{(a+bx^2)(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0308932, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 364}

$$\frac{(a+bx^2)(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(dx)^m}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(dx)^{1+m}\left(a+bx^2\right)\, {}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0194491, size = 62, normalized size = 0.85

$$\frac{x\left(a+bx^2\right)(dx)^m\, {}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*
x^2)/a)])/(a*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F] time = 0.266, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{b^2x^4 + 2\,abx^2 + a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{\left(a + bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt((a + b*x**2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^4 + 2\,abx^2 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.795 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)(dx)^{m+1}\,_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0339412, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 364}

$$\frac{\left(a+bx^2\right)(dx)^{m+1}\,_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^m}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}} = \frac{(dx)^{1+m}\left(a+bx^2\right){}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0233468, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^2\right)(dx)^m{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int (dx)^m\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^4 + 2\,abx^2 + a^2}\,(dx)^m}{b^4x^8 + 4\,ab^3x^6 + 6\,a^2b^2x^4 + 4\,a^3bx^2 + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m/(b^4*x^8 + 4*a*b^3*x^6 + 6
*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")

[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

3.796 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0327895, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1112, 364}

$$\frac{\left(a+bx^2\right)(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1112

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{(dx)^m}{\left(ab+b^2x^2\right)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(dx)^{1+m}\left(a+bx^2\right)\,{}_2F_1\left(5,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0189352, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^2\right)(dx)^m\,{}_2F_1\left(5,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^5(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F] time = 0.232, size = 0, normalized size = 0.

$$\int (dx)^m\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^4 + 2\,abx^2 + a^2}\,(dx)^m}{b^6x^{12} + 6\,ab^5x^{10} + 15\,a^2b^4x^8 + 20\,a^3b^3x^6 + 15\,a^4b^2x^4 + 6\,a^5bx^2 + a^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m/(b^6*x^12 + 6*a*b^5*x^10 +
 15*a^2*b^4*x^8 + 20*a^3*b^3*x^6 + 15*a^4*b^2*x^4 + 6*a^5*b*x^2 + a^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**2)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")

[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2), x)
```

3.797 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=74

$$\frac{\left(a+bx^2\right)(dx)^{m+1}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(1,\frac{1}{2}(m+4p+3);\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ad(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[
1, (3 + m + 4*p)/2, (3 + m)/2, -((b*x^2)/a)])/(a*d*(1 + m))
```

Rubi [A] time = 0.0263096, antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1113, 364}

$$\frac{(dx)^{m+1}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{m+1}{2},-2p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[(1 + m)/2, -
2*p, (3 + m)/2, -((b*x^2)/a)])/(d*(1 + m)*(1 + (b*x^2)/a)^(2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p \,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int (dx)^m\left(1+\frac{bx^2}{a}\right)^{2p} dx$$
$$= \frac{(dx)^{1+m}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{1+m}{2},-2p;\frac{3+m}{2};-\frac{bx^2}{a}\right)}{d(1+m)}$$

Mathematica [A] time = 0.0192336, size = 66, normalized size = 0.89

$$\frac{x(dx)^m\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p} {}_2F_1\left(\frac{m+1}{2},-2p;\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x*(d*x)^m*((a + b*x^2)^2)^p*Hypergeometric2F1[(1 + m)/2, -2*p, 1 + (1 + m)
/2, -((b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int (dx)^m\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^4 + 2\,abx^2 + a^2\right)^p (dx)^m\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(\left(a + bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

3.798 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=174

$$\frac{\left(a+bx^2\right)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+2)}-\frac{3a\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+3)}+\frac{3a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+1)}-\frac{a^3\left(a+b\right.}{}$$

```
[Out] -(a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(1 + 2*p)) + (3*a^2
*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(1 + p)) - (3*a*(a + b
*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(3 + 2*p)) + ((a + b*x^2)^4*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(2 + p))
```

Rubi [A] time = 0.108686, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 43}

$$\frac{\left(a+bx^2\right)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+2)}-\frac{3a\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+3)}+\frac{3a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+1)}-\frac{a^3\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```


```
[Out] -(a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(1 + 2*p)) + (3*a^2
*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(1 + p)) - (3*a*(a + b
*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(3 + 2*p)) + ((a + b*x^2)^4*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(2 + p))
```


Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7 \left(a^2+2abx^2+b^2x^4\right)^p dx &= \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^7\left(1+\frac{bx^2}{a}\right)^{2p} dx\\ &= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^3\left(1+\frac{bx}{a}\right)^{2p} dx, x, x^2\right)\\ &= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^3\left(1+\frac{bx}{a}\right)^{2p}}{b^3}+\frac{3a^3\left(1+\frac{bx}{a}\right)^{1+2p}}{b^3}\right.\right.\\ &= -\frac{a^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(1+2p)}+\frac{3a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(1+p)}-\frac{3a\left(a+b\right.}{}\end{aligned}$$

Mathematica [A] time = 0.0578408, size = 110, normalized size = 0.63

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\left(3a^2b(2p+1)x^2-3a^3-3ab^2\left(2p^2+3p+1\right)x^4+b^3\left(4p^3+12p^2+11p+3\right)x^6\right)}{4b^4(p+1)(p+2)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^2 - 3*a*b^2*(1
 + 3*p + 2*p^2)*x^4 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^6))/(4*b^4*(1 + p)*
(2 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A] time = 0.05, size = 150, normalized size = 0.9

$$-\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p\left(-4\,b^3p^3x^6-12\,b^3p^2x^6-11\,b^3px^6+6\,ab^2p^2x^4-3\,b^3x^6+9\,ab^2px^4+3\,ax^4b^2-6\,a^2bpx^2-3\,a^2bx\right.}{4\,b^4\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-4*b^3*p^3*x^6-12*b^3*p^2*x^6-11*b^3*p*x^6+
6*a*b^2*p^2*x^4-3*b^3*x^6+9*a*b^2*p*x^4+3*a*b^2*x^4-6*a^2*b*p*x^2-3*a^2*b*x
^2+3*a^3)*(b*x^2+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

Maxima [A] time = 0.978187, size = 155, normalized size = 0.89

$$\frac{\left(\left(4\,p^3+12\,p^2+11\,p+3\right)b^4x^8+2\left(2\,p^3+3\,p^2+p\right)ab^3x^6-3\left(2\,p^2+p\right)a^2b^2x^4+6\,a^3bpx^2-3\,a^4\right)\left(bx^2+a\right)^{2\,p}}{4\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6
- 3*(2*p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((4*
p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

Fricas [A] time = 1.58023, size = 335, normalized size = 1.93

$$\frac{\left(\left(4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4\right)x^8+6\,a^3bpx^2+2\left(2\,ab^3p^3+3\,ab^3p^2+ab^3p\right)x^6-3\left(2\,a^2b^2p^2+a^2b^2p\right)x^4-3\,a^4\right)\left(b^2x^4\right.}{4\left(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^8 + 6*a^3*b*p*x^2 + 2*(2
*a*b^3*p^3 + 3*a*b^3*p^2 + a*b^3*p)*x^6 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*x^4
 - 3*a^4)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^4*p^
2 + 25*b^4*p + 6*b^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.24871, size = 506, normalized size = 2.91

$$\frac{4\left(b^2x^4+2\,abx^2+a^2\right)^p b^4p^3x^8+12\left(b^2x^4+2\,abx^2+a^2\right)^p b^4p^2x^8+4\left(b^2x^4+2\,abx^2+a^2\right)^p ab^3p^3x^6+11\left(b^2x^4+2\,abx^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^3*x^8 + 12*(b^2*x^4 + 2*a*b*x^2
+ a^2)^p*b^4*p^2*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^3*x^6 + 11*(
b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p*x^8 + 6*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*
b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 2*(b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a*b^3*p*x^6 - 6*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4
- 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 + 6*(b^2*x^4 + 2*a*b*x^2 +
a^2)^p*a^3*b*p*x^2 - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4)/(4*b^4*p^4 + 20*b
^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

3.799 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=130

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(p+1)}+\frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+1)}$$

```
[Out] (a^2*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + 2*p)) - (a*(a +
 b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + p)) + ((a + b*x^2)^3*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(3 + 2*p))
```

Rubi [A] time = 0.081633, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 43}

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(p+1)}+\frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + 2*p)) - (a*(a +
 b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + p)) + ((a + b*x^2)^3*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(3 + 2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^5\left(1+\frac{bx^2}{a}\right)^{2p}\,dx$$

$$=\frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(1+\frac{bx}{a}\right)^{2p}}{b^2}-\frac{2a^2\left(1+\frac{bx}{a}\right)^{1+2p}}{b^2}+\right.\right.$$

$$=\frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(1+2p)}-\frac{a\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(1+p)}+\frac{\left(a+bx^2\right)^3\left(a\right.}{2b}$$

Mathematica [A] time = 0.0325615, size = 77, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\left(a^2-ab(2p+1)x^2+b^2\left(2p^2+3p+1\right)x^4\right)}{2b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(a^2 - a*b*(1 + 2*p)*x^2 + b^2*(1 + 3*p + 2*
p^2)*x^4))/(2*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A] time = 0.046, size = 96, normalized size = 0.7

$$\frac{\left(2\,b^2p^2x^4+3\,b^2px^4+b^2x^4-2\,abpx^2-abx^2+a^2\right)\left(bx^2+a\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{2\,b^3\left(4\,p^3+12\,p^2+11\,p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/2*(b*x^2+a)*(2*b^2*p^2*x^4+3*b^2*p*x^4+b^2*x^4-2*a*b*p*x^2-a*b*x^2+a^2)*(
b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

Maxima [A] time = 0.98319, size = 107, normalized size = 0.82

$$\frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^6+\left(2\,p^2+p\right)ab^2x^4-2\,a^2bpx^2+a^3\right)\left(bx^2+a\right)^{2\,p}}{2\left(4\,p^3+12\,p^2+11\,p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a^
3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Fricas [A] time = 1.57287, size = 223, normalized size = 1.72

$$\frac{\left(\left(2\,b^3p^2+3\,b^3p+b^3\right)x^6-2\,a^2bpx^2+\left(2\,ab^2p^2+ab^2p\right)x^4+a^3\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{2\left(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((2*b^3*p^2 + 3*b^3*p + b^3)*x^6 - 2*a^2*b*p*x^2 + (2*a*b^2*p^2 + a*b^2
*p)*x^4 + a^3)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b
^3*p + 3*b^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.21699, size = 317, normalized size = 2.44

$$\frac{2\left(b^2x^4+2\,abx^2+a^2\right)^pb^3p^2x^6+3\left(b^2x^4+2\,abx^2+a^2\right)^pb^3px^6+2\left(b^2x^4+2\,abx^2+a^2\right)^pab^2p^2x^4+\left(b^2x^4+2\,abx^2+a^2\right)^p}{2\left(4\,b^3p^3+12\,b^3p^2+11\,b^3p+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2 +
 a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p^2*x^4 + (b^2*x^
4 + 2*a*b*x^2 + a^2)^p*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4
- 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)
^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

3.800 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=84

$$\frac{\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+1)}$$

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(1 + 2*p)) + ((a + b*
x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p))
```

Rubi [A] time = 0.0566805, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 43}

$$\frac{\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(1 + 2*p)) + ((a + b*
x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^3\left(1+\frac{bx^2}{a}\right)^{2p}\,dx\\ &= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^2\right)\\ &= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1+\frac{bx}{a}\right)^{1+2p}}{b}\right)dx\right.\\ &= -\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(1+2p)}+\frac{\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)}\end{aligned}$$

Mathematica [A] time = 0.021166, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\left(b(2p+1)x^2-a\right)}{4b^2(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(-a + b*(1 + 2*p)*x^2))/(4*b^2*(1 + p)*(1 +
2*p))
```

Maple [A] time = 0.048, size = 60, normalized size = 0.7

$$-\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p\left(-2\,x^2pb-bx^2+a\right)\left(bx^2+a\right)}{4\,b^2\left(2\,p^2+3\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-b*x^2+a)*(b*x^2+a)/b^2/(2*p^2+3*
p+1)
```

Maxima [A] time = 0.974834, size = 73, normalized size = 0.87

$$\frac{\left(b^2(2p+1)x^4 + 2abpx^2 - a^2\right)\left(bx^2+a\right)^{2p}}{4\left(2p^2+3p+1\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p
 + 1)*b^2)
```

Fricas [A] time = 1.61221, size = 142, normalized size = 1.69

$$\frac{\left(2abpx^2 + \left(2b^2p + b^2\right)x^4 - a^2\right)\left(b^2x^4 + 2abx^2 + a^2\right)^p}{4\left(2b^2p^2 + 3b^2p + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*p*x^2 + (2*b^2*p + b^2)*x^4 - a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p
/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.32464, size = 178, normalized size = 2.12

$$\frac{2\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}b^{2}px^{4}+\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}b^{2}x^{4}+2\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}abpx^{2}-\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}a^{2}}{4\left(2\,b^{2}p^{2}+3\,b^{2}p+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^2*p*x^4 + (b^2*x^4 + 2*a*b*x^2 + a^2
)^p*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b*p*x^2 - (b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

3.801 $\int x\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=41

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(2p+1)}$$

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b*(1 + 2*p))
```

Rubi [A] time = 0.0249834, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1107, 609}

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b*(1 + 2*p))
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)\\ &= \frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0042079, size = 29, normalized size = 0.71

$$\frac{(a+bx^2)\left((a+bx^2)^2\right)^p}{4bp+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p)/(2*b + 4*b*p)
```

Maple [A] time = 0.044, size = 40, normalized size = 1.

$$\frac{(bx^2+a)\left(b^2x^4+2\,abx^2+a^2\right)^p}{2\,b\,(1+2\,p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b/(1+2*p)
```

Maxima [A] time = 0.993721, size = 41, normalized size = 1.

$$\frac{(bx^2+a)(bx^2+a)^{2\,p}}{2\,b(2\,p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)/(b*(2*p + 1))
```

Fricas [A] time = 1.60884, size = 80, normalized size = 1.95

$$\frac{\left(bx^{2}+a\right)\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}}{2\left(2\,bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b*p + b)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.18376, size = 78, normalized size = 1.9

$$\frac{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}bx^{2}+\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{p}a}{2\left(2\,bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b^2*x^4 + 2*a*b*x^2 + a^2)^p*b*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a)
/(2*b*p + b)
```

3.802 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x}\,dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a(2p+1)}$$

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1, 1 + 2*p, 2
*(1 + p), 1 + (b*x^2)/a])/(2*a*(1 + 2*p))
```

Rubi [A] time = 0.036719, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 65}

$$-\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1, 1 + 2*p, 2
*(1 + p), 1 + (b*x^2)/a])/(2*a*(1 + 2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{x}\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x}\,dx$$
$$=\frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx,x,x^2\right)$$
$$=-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(1,1+2p;2(1+p);1+\frac{bx^2}{a}\right)}{2a(1+2p)}$$

Mathematica [A] time = 0.0103386, size = 54, normalized size = 0.86

$$-\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\,{}_2F_1\left(1,2p+1;2p+2;\frac{bx^2}{a}+1\right)}{2a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x,x]
```

```
[Out] -((a + b*x^2)*((a + b*x^2)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p, 1 +
(b*x^2)/a])/(2*a*(1 + 2*p))
```

Maple [F] time = 0.202, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

3.803 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3} dx$

Optimal. Leaf size=64

$$\frac{b(a+bx^2)(a^2+2abx^2+b^2x^4)^p \,_2F_1\left(2,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a^2(2p+1)}$$

```
[Out] (b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 + (b*x^2)/a])/(2*a^2*(1 + 2*p))
```

Rubi [A] time = 0.03992, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1113, 266, 65}

$$\frac{b(a+bx^2)(a^2+2abx^2+b^2x^4)^p \,_2F_1\left(2,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 + (b*x^2)/a])/(2*a^2*(1 + 2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{x^3}\,dx &= \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x^3}\,dx\\ &= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^2}\,dx,x,x^2\right)\\ &= \frac{b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(2,1+2p;2(1+p);1+\frac{bx^2}{a}\right)}{2a^2(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.0116051, size = 55, normalized size = 0.86

$$\frac{b\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\,{}_2F_1\left(2,2p+1;2p+2;\frac{bx^2}{a}+1\right)}{2a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)*((a + b*x^2)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1 +
 (b*x^2)/a])/(2*a^2*(1 + 2*p))
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((a + bx^2)^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

3.804 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{1}{5}x^5\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/2, -2*p, 7/2, -((b*x
^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0190635, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1113, 364}

$$\frac{1}{5}x^5\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/2, -2*p, 7/2, -((b*x
^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^4\left(1+\frac{bx^2}{a}\right)^{2p}\,dx$$
$$=\frac{1}{5}x^5\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] time = 0.0068837, size = 51, normalized size = 0.85

$$\frac{1}{5}x^5\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^5*((a + b*x^2)^2)^p*Hypergeometric2F1[5/2, -2*p, 7/2, -((b*x^2)/a)])/(5*
(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.213, size = 0, normalized size = 0.

$$\int x^4\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b^2x^4+2\,abx^2+a^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^4 + 2\,abx^2 + a^2\right)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(\left(a + bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

3.805 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{1}{3}x^3\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/2, -2*p, 5/2, -((b*x
^2)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0181595, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1113, 364}

$$\frac{1}{3}x^3\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/2, -2*p, 5/2, -((b*x
^2)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p\right) \int x^2 \left(1+\frac{bx^2}{a}\right)^{2p} \, dx$$
$$= \frac{1}{3} x^3 \left(1+\frac{bx^2}{a}\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] time = 0.0060333, size = 51, normalized size = 0.85

$$\frac{1}{3} x^3 \left(\left(a+bx^2\right)^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-2p} {}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^3*((a + b*x^2)^2)^p*Hypergeometric2F1[3/2, -2*p, 5/2, -((b*x^2)/a)])/(3*
(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int x^2 \left(b^2x^4+2\,abx^2+a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2x^4 + 2\,abx^2 + a^2\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\left(a + bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b^2x^4 + 2\,abx^2 + a^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

3.806 $\int \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=55

$$x\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/2, -2*p, 3/2, -((b*x^2
)/a)])/(1 + (b*x^2)/a)^(2*p)
```

Rubi [A] time = 0.0118, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1089, 245}

$$x\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/2, -2*p, 3/2, -((b*x^2
)/a)])/(1 + (b*x^2)/a)^(2*p)
```

Rule 1089

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart
[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(2*FracPart[p]), Int
[(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \left(1+\frac{bx^2}{a}\right)^{2p}\,dx$$
$$=x\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A] time = 0.0064605, size = 46, normalized size = 0.84

$$x\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x*((a + b*x^2)^2)^p*Hypergeometric2F1[1/2, -2*p, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(2*p)
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^4 + 2\,abx^2 + a^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

3.807 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{x^2} dx$

Optimal. Leaf size=58

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

```
[Out] -(((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, -((b*x^
2)/a)])/(x*(1 + (b*x^2)/a)^(2*p)))
```

Rubi [A] time = 0.0184011, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1113, 364}

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^2,x]
```

```
[Out] -(((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, -((b*x^
2)/a)])/(x*(1 + (b*x^2)/a)^(2*p)))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x^2}\,dx$$
$$= -\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Mathematica [A] time = 0.00608, size = 49, normalized size = 0.84

$$-\frac{\left((a+bx^2)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^2,x]
```

```
[Out] -((((a + b*x^2)^2)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(2*p)))
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

3.808 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

```
[Out] -((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, -((b*x^
2)/a)])/(3*x^3*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0179137, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1113, 364}

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^4,x]
```

```
[Out] -((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, -((b*x^
2)/a)])/(3*x^3*(1 + (b*x^2)/a)^(2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{x^4}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x^4}\,dx$$
$$= -\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Mathematica [A] time = 0.0062628, size = 51, normalized size = 0.85

$$-\frac{\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^4,x]
```

```
[Out] -(((a + b*x^2)^2)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, -((b*x^2)/a)])/(3*x
^3*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**4,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

3.809 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=67

$$\frac{2(dx)^{5/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)}{5d}$$

```
[Out] (2*(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/4, -2*p, 9
/4, -((b*x^2)/a)])/(5*d*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0211025, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113, 364}

$$\frac{2(dx)^{5/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/4, -2*p, 9
/4, -((b*x^2)/a)])/(5*d*(1 + (b*x^2)/a)^(2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int (dx)^{3/2}\left(1+\frac{bx^2}{a}\right)^{2p}\,dx$$
$$= \frac{2(dx)^{5/2}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)}{5d}$$

Mathematica [A] time = 0.0088855, size = 56, normalized size = 0.84

$$\frac{2}{5}x(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*x*(d*x)^(3/2)*((a + b*x^2)^2)^p*Hypergeometric2F1[5/4, -2*p, 9/4, -((b*x
^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx}\left(b^2x^4 + 2\,abx^2 + a^2\right)^p dx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*d*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \left(\left(a + bx^2\right)^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral((d*x)**(3/2)*((a + b*x**2)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \left(b^2x^4 + 2\,abx^2 + a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

3.810 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{2(dx)^{3/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}$$

```
[Out] (2*(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/4, -2*p, 7
/4, -((b*x^2)/a)])/(3*d*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0204767, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113, 364}

$$\frac{2(dx)^{3/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/4, -2*p, 7
/4, -((b*x^2)/a)])/(3*d*(1 + (b*x^2)/a)^(2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \sqrt{dx}\left(1+\frac{bx^2}{a}\right)^{2p}\,dx$$
$$=\frac{2(dx)^{3/2}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}$$

Mathematica [A] time = 0.0085003, size = 56, normalized size = 0.84

$$\frac{2}{3}x\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*x*Sqrt[d*x]*((a + b*x^2)^2)^p*Hypergeometric2F1[3/4, -2*p, 7/4, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx}\left(b^2x^4 + 2\,abx^2 + a^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(\left(a + bx^2\right)^2\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(b^2x^4 + 2\,abx^2 + a^2\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

3.811 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}} \, dx$

Optimal. Leaf size=65

$$\frac{2\sqrt{dx}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

```
[Out] (2*Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/4, -2*p, 5/4
, -((b*x^2)/a)])/(d*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0206416, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113, 364}

$$\frac{2\sqrt{dx}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/Sqrt[d*x],x]
```


```
[Out] (2*Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/4, -2*p, 5/4
, -((b*x^2)/a)])/(d*(1 + (b*x^2)/a)^(2*p))
```


Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{\sqrt{dx}}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{\sqrt{dx}}\,dx$$
$$= \frac{2\sqrt{dx}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

Mathematica [A] time = 0.0088893, size = 54, normalized size = 0.83

$$\frac{2x\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/Sqrt[d*x],x]
```

```
[Out] (2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[1/4, -2*p, 5/4, -((b*x^2)/a)])/(Sq
rt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p\frac{1}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(1/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

3.812 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/4, -2*p, 3/4, -((b*x
^2)/a)])/(d*Sqrt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0221593, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113, 364}

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(3/2),x]
```

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/4, -2*p, 3/4, -((b*x
^2)/a)])/(d*Sqrt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{(dx)^{3/2}}\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{(dx)^{3/2}}\,dx$$
$$=-\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

Mathematica [A] time = 0.0088025, size = 54, normalized size = 0.83

$$-\frac{2x\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(3/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[-1/4, -2*p, 3/4, -((b*x^2)/a)])/(
(d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p (dx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/(d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2), x)
```

3.813 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^p}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/4, -2*p, 1/4, -((b*x
^2)/a)])/(3*d*(d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.021957, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113, 364}

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(5/2),x]
```

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/4, -2*p, 1/4, -((b*x
^2)/a)])/(3*d*(d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Rule 1113

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/(1 + (2*c*x^2)/b)^(
2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/b)^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{5/2}}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{(dx)^{5/2}}\,dx$$
$$= -\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

Mathematica [A] time = 0.0090595, size = 56, normalized size = 0.84

$$-\frac{2x\left((a+bx^2)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(5/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[-3/4, -2*p, 1/4, -((b*x^2)/a)])/(
3*(d*x)^(5/2)*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p(dx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p}{d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2), x)
```

3.814 $\int x^2 \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rubi [A] time = 0.0070952, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2+cx^4\right) \, dx &= \int \left(ax^2+bx^4+cx^6\right) \, dx\\ &= \frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0022796, size = 25, normalized size = 1.

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Maple [A] time = 0.039, size = 20, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Maxima [A] time = 0.975144, size = 26, normalized size = 1.04

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A] time = 1.29524, size = 47, normalized size = 1.88

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c + 1/5*x^5*b + 1/3*x^3*a
```

Sympy [A] time = 0.062342, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

Giac [A] time = 1.12102, size = 26, normalized size = 1.04

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

3.815 $\int x\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi [A] time = 0.0071979, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)\,dx&=\int \left(ax+bx^3+cx^5\right)\,dx\\&=\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.0012317, size = 25, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Maple [A] time = 0.041, size = 20, normalized size = 0.8

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Maxima [A] time = 0.964554, size = 26, normalized size = 1.04

$$\frac{1}{6}cx^6 + \frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Fricas [A] time = 1.27351, size = 47, normalized size = 1.88

$$\frac{1}{6}x^6c + \frac{1}{4}x^4b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c + 1/4*x^4*b + 1/2*x^2*a
```

Sympy [A] time = 0.062213, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a),x)
```

```
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

Giac [A] time = 1.22588, size = 26, normalized size = 1.04

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

3.816 $\int \left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=20

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi [A] time = 0.0034928, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^2 + c*x^4,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)\,dx=ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Mathematica [A] time = 0.0000322, size = 20, normalized size = 1.

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^2 + c*x^4,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Maple [A] time = 0.04, size = 17, normalized size = 0.9

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^2+a,x)
```

```
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Maxima [A] time = 0.970358, size = 22, normalized size = 1.1

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Fricas [A] time = 1.24892, size = 39, normalized size = 1.95

$$\frac{1}{5}x^5c + \frac{1}{3}x^3b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c + 1/3*x^3*b + x*a
```

Sympy [A] time = 0.058773, size = 15, normalized size = 0.75

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**2+a,x)
```

```
[Out] a*x + b*x**3/3 + c*x**5/5
```

Giac [A] time = 1.20711, size = 22, normalized size = 1.1

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

3.817 $\int \frac{a+bx^2+cx^4}{x}\,dx$

Optimal. Leaf size=21

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rubi [A] time = 0.005449, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x}\,dx &= \int \left(\frac{a}{x}+bx+cx^3\right)dx\\ &= \frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.0015812, size = 21, normalized size = 1.

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Maple [A] time = 0.043, size = 18, normalized size = 0.9

$$\frac{bx^2}{2}+\frac{cx^4}{4}+a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x,x)
```

```
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Maxima [A] time = 0.995727, size = 27, normalized size = 1.29

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

Fricas [A] time = 1.48111, size = 46, normalized size = 2.19

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Sympy [A] time = 0.096976, size = 17, normalized size = 0.81

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*log(x) + b*x**2/2 + c*x**4/4
```

Giac [A] time = 1.32177, size = 27, normalized size = 1.29

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2 + \frac{1}{2}a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

3.818 $\int \frac{a+bx^2+cx^4}{x^2} dx$

Optimal. Leaf size=18

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rubi [A] time = 0.0066606, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^2,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{x^2} dx &= \int \left(b + \frac{a}{x^2} + cx^2\right) dx \\ &= -\frac{a}{x} + bx + \frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.001743, size = 18, normalized size = 1.

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^2,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Maple [A] time = 0.045, size = 17, normalized size = 0.9

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2,x)
```

```
[Out] -a/x+b*x+1/3*c*x^3
```

Maxima [A] time = 0.996086, size = 22, normalized size = 1.22

$$\frac{1}{3}cx^3+bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

Fricas [A] time = 1.42734, size = 42, normalized size = 2.33

$$\frac{cx^4+3\,bx^2-3\,a}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

Sympy [A] time = 0.268721, size = 12, normalized size = 0.67

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a/x + b*x + c*x**3/3
```

Giac [A] time = 1.28605, size = 22, normalized size = 1.22

$$\frac{1}{3}\,cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

3.819 $\int \frac{a+bx^2+cx^4}{x^3} dx$

Optimal. Leaf size=21

$$-\frac{a}{2x^2} + b\log(x) + \frac{cx^2}{2}$$

```
[Out] -a/(2*x^2) + (c*x^2)/2 + b*Log[x]
```

Rubi [A] time = 0.0069357, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{2x^2} + b\log(x) + \frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^3,x]
```

```
[Out] -a/(2*x^2) + (c*x^2)/2 + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{x^3} dx &= \int \left(\frac{a}{x^3} + \frac{b}{x} + cx\right) dx \\ &= -\frac{a}{2x^2} + \frac{cx^2}{2} + b\log(x) \end{aligned}$$

Mathematica [A] time = 0.0019273, size = 21, normalized size = 1.

$$-\frac{a}{2x^2} + b\log(x) + \frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^3,x]
```

```
[Out] -a/(2*x^2) + (c*x^2)/2 + b*Log[x]
```

Maple [A] time = 0.046, size = 18, normalized size = 0.9

$$-\frac{a}{2x^2}+\frac{cx^2}{2}+b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^3,x)
```

```
[Out] -1/2/x^2*a+1/2*c*x^2+b*ln(x)
```

Maxima [A] time = 0.989109, size = 27, normalized size = 1.29

$$\frac{1}{2}cx^2+\frac{1}{2}b\log\left(x^2\right)-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*a/x^2
```

Fricas [A] time = 1.45976, size = 51, normalized size = 2.43

$$\frac{cx^4+2bx^2\log(x)-a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^4 + 2*b*x^2*log(x) - a)/x^2
```

Sympy [A] time = 0.292389, size = 17, normalized size = 0.81

$$-\frac{a}{2x^2} + b\log(x) + \frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x) + c*x**2/2
```

Giac [A] time = 1.27056, size = 35, normalized size = 1.67

$$\frac{1}{2}cx^2 + \frac{1}{2}b\log\left(x^2\right) - \frac{bx^2 + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

3.820 $\int \frac{a+bx^2+cx^4}{x^4} dx$

Optimal. Leaf size=18

$$-\frac{a}{3x^3}-\frac{b}{x}+cx$$

```
[Out] -a/(3*x^3) - b/x + c*x
```

Rubi [A] time = 0.0066929, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{3x^3}-\frac{b}{x}+cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -a/(3*x^3) - b/x + c*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^4}\,dx &= \int \left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)dx\\ &= -\frac{a}{3x^3}-\frac{b}{x}+cx\end{aligned}$$

Mathematica [A] time = 0.0040091, size = 18, normalized size = 1.

$$-\frac{a}{3x^3}-\frac{b}{x}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -a/(3*x^3) - b/x + c*x
```

Maple [A] time = 0.046, size = 17, normalized size = 0.9

$$-\frac{a}{3x^3}-\frac{b}{x}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4,x)
```

```
[Out] -1/3*a/x^3-b/x+c*x
```

Maxima [A] time = 0.959433, size = 23, normalized size = 1.28

$$cx-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
```

```
[Out] c*x - 1/3*(3*b*x^2 + a)/x^3
```

Fricas [A] time = 1.4283, size = 45, normalized size = 2.5

$$\frac{3\,cx^4-3\,bx^2-a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c*x^4 - 3*b*x^2 - a)/x^3
```

Sympy [A] time = 0.303946, size = 15, normalized size = 0.83

$$cx - \frac{a + 3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] c*x - (a + 3*b*x**2)/(3*x**3)
```

Giac [A] time = 1.22534, size = 23, normalized size = 1.28

$$cx - \frac{3\,bx^2 + a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
```

```
[Out] c*x - 1/3*(3*b*x^2 + a)/x^3
```

3.821 $\int \frac{a+bx^2+cx^4}{x^5} dx$

Optimal. Leaf size=21

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

```
[Out] -a/(4*x^4) - b/(2*x^2) + c*Log[x]
```

Rubi [A] time = 0.0069292, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(2*x^2) + c*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{x^5} \, dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^3}+\frac{c}{x}\right) dx \\ &= -\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x) \end{aligned}$$

Mathematica [A] time = 0.0028227, size = 21, normalized size = 1.

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(2*x^2) + c*Log[x]
```

Maple [A] time = 0.047, size = 18, normalized size = 0.9

$$-\frac{a}{4\,x^4}-\frac{b}{2\,x^2}+c\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^5,x)
```

```
[Out] -1/4*a/x^4-1/2/x^2*b+c*ln(x)
```

Maxima [A] time = 0.957028, size = 28, normalized size = 1.33

$$\frac{1}{2}\,c\log\left(x^2\right)-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2) - 1/4*(2*b*x^2 + a)/x^4
```

Fricas [A] time = 1.51292, size = 54, normalized size = 2.57

$$\frac{4\,cx^4\log\left(x\right)-2\,bx^2-a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*c*x^4*log(x) - 2*b*x^2 - a)/x^4
```

Sympy [A] time = 0.40308, size = 17, normalized size = 0.81

$$c\log(x) - \frac{a + 2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] c*log(x) - (a + 2*b*x**2)/(4*x**4)
```

Giac [A] time = 1.30952, size = 36, normalized size = 1.71

$$\frac{1}{2}c\log\left(x^2\right) - \frac{3cx^4 + 2bx^2 + a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2) - 1/4*(3*c*x^4 + 2*b*x^2 + a)/x^4
```

3.822 $\int \frac{a+bx^2+cx^4}{x^6}\,dx$

Optimal. Leaf size=23

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

```
[Out] -a/(5*x^5) - b/(3*x^3) - c/x
```

Rubi [A] time = 0.0068175, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/(3*x^3) - c/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^6}\,dx &= \int \left(\frac{a}{x^6}+\frac{b}{x^4}+\frac{c}{x^2}\right)dx\\ &= -\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}\end{aligned}$$

Mathematica [A] time = 0.0023745, size = 23, normalized size = 1.

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/(3*x^3) - c/x
```

Maple [A] time = 0.048, size = 20, normalized size = 0.9

$$-\frac{a}{5\,x^{5}}-\frac{b}{3\,x^{3}}-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6,x)
```

```
[Out] -1/5*a/x^5-1/3*b/x^3-c/x
```

Maxima [A] time = 0.950352, size = 28, normalized size = 1.22

$$-\frac{15\,cx^{4}+5\,bx^{2}+3\,a}{15\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

Fricas [A] time = 1.39925, size = 51, normalized size = 2.22

$$-\frac{15\,cx^{4}+5\,bx^{2}+3\,a}{15\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

Sympy [A] time = 0.434125, size = 22, normalized size = 0.96

$$-\frac{3a + 5bx^2 + 15cx^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] -(3*a + 5*b*x**2 + 15*c*x**4)/(15*x**5)
```

Giac [A] time = 1.24413, size = 28, normalized size = 1.22

$$-\frac{15\,cx^4 + 5\,bx^2 + 3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

3.823 $\int \frac{a+bx^2+cx^4}{x^7} dx$

Optimal. Leaf size=25

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

```
[Out] -a/(6*x^6) - b/(4*x^4) - c/(2*x^2)
```

Rubi [A] time = 0.0069301, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -a/(6*x^6) - b/(4*x^4) - c/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^7} dx &= \int \left(\frac{a}{x^7}+\frac{b}{x^5}+\frac{c}{x^3}\right) dx\\ &= -\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0023377, size = 25, normalized size = 1.

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -a/(6*x^6) - b/(4*x^4) - c/(2*x^2)
```

Maple [A] time = 0.046, size = 20, normalized size = 0.8

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^7,x)
```

```
[Out] -1/6*a/x^6-1/4*b/x^4-1/2*c/x^2
```

Maxima [A] time = 0.95056, size = 28, normalized size = 1.12

$$-\frac{6\,cx^4+3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

Fricas [A] time = 1.39779, size = 50, normalized size = 2.

$$-\frac{6\,cx^4+3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

Sympy [A] time = 0.486558, size = 22, normalized size = 0.88

$$-\frac{2a + 3bx^2 + 6cx^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] -(2*a + 3*b*x**2 + 6*c*x**4)/(12*x**6)
```

Giac [A] time = 1.24525, size = 28, normalized size = 1.12

$$-\frac{6\,cx^4 + 3\,bx^2 + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

3.824 $\int \frac{a+bx^2+cx^4}{x^8} dx$

Optimal. Leaf size=25

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

```
[Out] -a/(7*x^7) - b/(5*x^5) - c/(3*x^3)
```

Rubi [A] time = 0.0066146, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -a/(7*x^7) - b/(5*x^5) - c/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^8}\,dx &= \int \left(\frac{a}{x^8}+\frac{b}{x^6}+\frac{c}{x^4}\right)dx\\ &= -\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0024899, size = 25, normalized size = 1.

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -a/(7*x^7) - b/(5*x^5) - c/(3*x^3)
```

Maple [A] time = 0.049, size = 20, normalized size = 0.8

$$-\frac{a}{7\,x^{7}}-\frac{b}{5\,x^{5}}-\frac{c}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^8,x)
```

```
[Out] -1/7*a/x^7-1/5*b/x^5-1/3*c/x^3
```

Maxima [A] time = 0.935626, size = 28, normalized size = 1.12

$$-\frac{35\,cx^{4}+21\,bx^{2}+15\,a}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

Fricas [A] time = 1.4354, size = 55, normalized size = 2.2

$$-\frac{35\,cx^{4}+21\,bx^{2}+15\,a}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

Sympy [A] time = 0.487416, size = 22, normalized size = 0.88

$$-\frac{15a + 21bx^2 + 35cx^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8,x)
```

```
[Out] -(15*a + 21*b*x**2 + 35*c*x**4)/(105*x**7)
```

Giac [A] time = 1.20766, size = 28, normalized size = 1.12

$$-\frac{35\,cx^4 + 21\,bx^2 + 15\,a}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

3.825 $\int x^2 \left(a+bx^2+cx^4\right)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Rubi [A] time = 0.0301173, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2+cx^4\right)^2 dx &= \int \left(a^2x^2+2abx^4+\left(b^2+2ac\right)x^6+2bcx^8+c^2x^{10}\right) dx\\ &= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.0079476, size = 54, normalized size = 1.

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Maple [A] time = 0.044, size = 45, normalized size = 0.8

$$\frac{a^2x^3}{3}+\frac{2\,abx^5}{5}+\frac{\left(2\,ac+b^2\right)x^7}{7}+\frac{2\,bcx^9}{9}+\frac{c^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Maxima [A] time = 0.956985, size = 59, normalized size = 1.09

$$\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\left(b^2+2\,ac\right)x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5 + 1/3*a^2
*x^3
```

Fricas [A] time = 1.28016, size = 115, normalized size = 2.13

$$\frac{1}{11}x^{11}c^2+\frac{2}{9}x^9cb+\frac{1}{7}x^7b^2+\frac{2}{7}x^7ca+\frac{2}{5}x^5ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2 + 2/9*x^9*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 2/5*x^5*b*a + 1/3
*x^3*a^2
```

Sympy [A] time = 0.073569, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7 +
 b**2/7)
```

Giac [A] time = 1.28443, size = 62, normalized size = 1.15

$$\frac{1}{11}\,c^2x^{11} + \frac{2}{9}\,bcx^9 + \frac{1}{7}\,b^2x^7 + \frac{2}{7}\,acx^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/3
*a^2*x^3
```

3.826 $\int x \left(a + bx^2 + cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2} + \frac{1}{6}x^6 \left(2ac + b^2\right) + \frac{1}{2}abx^4 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10
)/10
```

Rubi [A] time = 0.0383747, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 611}

$$\frac{a^2x^2}{2} + \frac{1}{6}x^6 \left(2ac + b^2\right) + \frac{1}{2}abx^4 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10
)/10
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\int x\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{2}\,\text{Subst}\left(\int\left(a+bx+cx^2\right)^2\,dx,x,x^2\right)$$
$$=\frac{1}{2}\,\text{Subst}\left(\int\left(a^2+2abx+b^2\left(1+\frac{2ac}{b^2}\right)x^2+2bcx^3+c^2x^4\right)dx,x,x^2\right)$$
$$=\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Mathematica [A] time = 0.0078386, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2\left(30a^2+10x^4\left(2ac+b^2\right)+30abx^2+15bcx^6+6c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8))
/60
```

Maple [A] time = 0.041, size = 45, normalized size = 0.8

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{\left(2\,ac+b^2\right)x^6}{6}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Maxima [A] time = 0.953357, size = 59, normalized size = 1.09

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\left(b^2+2\,ac\right)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

Fricas [A] time = 1.279, size = 115, normalized size = 2.13

$$\frac{1}{10}x^{10}c^2 + \frac{1}{4}x^8cb + \frac{1}{6}x^6b^2 + \frac{1}{3}x^6ca + \frac{1}{2}x^4ba + \frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^2 + 1/4*x^8*c*b + 1/6*x^6*b^2 + 1/3*x^6*c*a + 1/2*x^4*b*a + 1/2
*x^2*a^2
```

Sympy [A] time = 0.077345, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{bcx^8}{4} + \frac{c^2x^{10}}{10} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2/
6)
```

Giac [A] time = 1.35185, size = 62, normalized size = 1.15

$$\frac{1}{10}c^2x^{10} + \frac{1}{4}bcx^8 + \frac{1}{6}b^2x^6 + \frac{1}{3}acx^6 + \frac{1}{2}abx^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/2
*a^2*x^2
```

3.827 $\int \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=49

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rubi [A] time = 0.0207458, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1090}

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2+2abx^2+b^2\left(1+\frac{2ac}{b^2}\right)x^4+2bcx^6+c^2x^8\right) dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0046814, size = 49, normalized size = 1.

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A] time = 0.041, size = 42, normalized size = 0.9

$$a^{2}x+\frac{2\,abx^{3}}{3}+\frac{\left(2\,ac+b^{2}\right)x^{5}}{5}+\frac{2\,bcx^{7}}{7}+\frac{c^{2}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A] time = 0.955626, size = 61, normalized size = 1.24

$$\frac{1}{9}\,c^{2}x^{9}+\frac{2}{7}\,bcx^{7}+\frac{1}{5}\,b^{2}x^{5}+a^{2}x+\frac{2}{15}\left(3\,cx^{5}+5\,bx^{3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)*
a
```

Fricas [A] time = 1.26708, size = 104, normalized size = 2.12

$$\frac{1}{9}x^{9}c^{2}+\frac{2}{7}x^{7}cb+\frac{1}{5}x^{5}b^{2}+\frac{2}{5}x^{5}ca+\frac{2}{3}x^{3}ba+xa^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2 + 2/5*x^5*c*a + 2/3*x^3*b*a + x*a^2
```

Sympy [A] time = 0.081184, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

Giac [A] time = 1.19867, size = 58, normalized size = 1.18

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.828 $\int \frac{(a+bx^2+cx^4)^2}{x} dx$

Optimal. Leaf size=47

$$a^2\log(x) + \frac{1}{4}x^4\left(2ac+b^2\right) + abx^2 + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

```
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2*Log[x]
```

Rubi [A] time = 0.0413996, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$a^2\log(x) + \frac{1}{4}x^4\left(2ac+b^2\right) + abx^2 + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x,x]
```

```
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2*Log[x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+\left(b^2+2ac\right)x+2bcx^2+c^2x^3\right)dx, x, x^2\right)$$
$$= abx^2+\frac{1}{4}\left(b^2+2ac\right)x^4+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}+a^2\log(x)$$

Mathematica [A] time = 0.0113947, size = 47, normalized size = 1.

$$a^2\log(x)+\frac{1}{4}x^4\left(2ac+b^2\right)+abx^2+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x,x]
```

```
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2*Log[x]
```

Maple [A] time = 0.045, size = 44, normalized size = 0.9

$$\frac{c^2x^8}{8}+\frac{bcx^6}{3}+\frac{x^4ac}{2}+\frac{b^2x^4}{4}+abx^2+a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x,x)
```

```
[Out] 1/8*c^2*x^8+1/3*b*c*x^6+1/2*x^4*a*c+1/4*b^2*x^4+a*b*x^2+a^2*ln(x)
```

Maxima [A] time = 0.965898, size = 59, normalized size = 1.26

$$\frac{1}{8}c^2x^8+\frac{1}{3}bcx^6+\frac{1}{4}\left(b^2+2ac\right)x^4+abx^2+\frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + 1/2*a^2*log(x
^2)
```

Fricas [A] time = 1.46607, size = 100, normalized size = 2.13

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}\left(b^2 + 2ac\right)x^4 + abx^2 + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A] time = 0.315114, size = 42, normalized size = 0.89

$$a^2\log(x) + abx^2 + \frac{bcx^6}{3} + \frac{c^2x^8}{8} + x^4\left(\frac{ac}{2} + \frac{b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b*c*x**6/3 + c**2*x**8/8 + x**4*(a*c/2 + b**2/4)
```

Giac [A] time = 1.12332, size = 62, normalized size = 1.32

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}b^2x^4 + \frac{1}{2}acx^4 + abx^2 + \frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4 + 1/2*a*c*x^4 + a*b*x^2 + 1/2*a^2*l
og(x^2)
```

3.829 $\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rubi [A] time = 0.021085, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^2}\,dx &= \int \left(2ab+\frac{a^2}{x^2}+\left(b^2+2ac\right)x^2+2bcx^4+c^2x^6\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{1}{3}\left(b^2+2ac\right)x^3+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0169373, size = 48, normalized size = 1.

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Maple [A] time = 0.046, size = 45, normalized size = 0.9

$$\frac{c^2x^7}{7} + \frac{2\,bcx^5}{5} + \frac{2\,acx^3}{3} + \frac{b^2x^3}{3} + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^2,x)
```

```
[Out] 1/7*c^2*x^7+2/5*b*c*x^5+2/3*a*c*x^3+1/3*b^2*x^3+2*a*b*x-a^2/x
```

Maxima [A] time = 0.951097, size = 57, normalized size = 1.19

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\left(b^2 + 2\,ac\right)x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*(b^2 + 2*a*c)*x^3 + 2*a*b*x - a^2/x
```

Fricas [A] time = 1.42244, size = 111, normalized size = 2.31

$$\frac{15\,c^2x^8 + 42\,bcx^6 + 35\left(b^2 + 2\,ac\right)x^4 + 210\,abx^2 - 105\,a^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c^2*x^8 + 42*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 210*a*b*x^2 - 105*a
^2)/x
```

Sympy [A] time = 0.32274, size = 44, normalized size = 0.92

$$-\frac{a^2}{x} + 2abx + \frac{2bcx^5}{5} + \frac{c^2x^7}{7} + x^3\left(\frac{2ac}{3} + \frac{b^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + 2*b*c*x**5/5 + c**2*x**7/7 + x**3*(2*a*c/3 + b**2/3)
```

Giac [A] time = 1.13804, size = 59, normalized size = 1.23

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3 + \frac{2}{3}\,acx^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + 2*a*b*x - a^2/x
```

3.830 $\int \frac{(a+bx^2+cx^4)^2}{x^3} dx$

Optimal. Leaf size=51

$$-\frac{a^2}{2x^2}+\frac{1}{2}x^2\left(2ac+b^2\right)+2ab\log(x)+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}$$

```
[Out] -a^2/(2*x^2) + ((b^2 + 2*a*c)*x^2)/2 + (b*c*x^4)/2 + (c^2*x^6)/6 + 2*a*b*Lo
g[x]
```

Rubi [A] time = 0.0408121, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{2x^2}+\frac{1}{2}x^2\left(2ac+b^2\right)+2ab\log(x)+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + ((b^2 + 2*a*c)*x^2)/2 + (b*c*x^4)/2 + (c^2*x^6)/6 + 2*a*b*Lo
g[x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(b^2\left(1+\frac{2ac}{b^2}\right)+\frac{a^2}{x^2}+\frac{2ab}{x}+2bcx+c^2x^2\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{2x^2}+\frac{1}{2}\left(b^2+2ac\right)x^2+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}+2ab\log(x)$$

Mathematica [A] time = 0.0144078, size = 46, normalized size = 0.9

$$\frac{1}{6}\left(-\frac{3a^2}{x^2}+3x^2\left(2ac+b^2\right)+12ab\log(x)+3bcx^4+c^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] ((-3*a^2)/x^2 + 3*(b^2 + 2*a*c)*x^2 + 3*b*c*x^4 + c^2*x^6 + 12*a*b*Log[x])/
6
```

Maple [A] time = 0.048, size = 45, normalized size = 0.9

$$\frac{c^2x^6}{6}+\frac{bcx^4}{2}+cax^2+\frac{b^2x^2}{2}+2\,ab\ln(x)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^3,x)
```

```
[Out] 1/6*c^2*x^6+1/2*b*c*x^4+c*a*x^2+1/2*b^2*x^2+2*a*b*ln(x)-1/2/x^2*a^2
```

Maxima [A] time = 0.955824, size = 59, normalized size = 1.16

$$\frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\left(b^2+2\,ac\right)x^2+ab\log\left(x^2\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*(b^2 + 2*a*c)*x^2 + a*b*log(x^2) - 1/2*a^2/
x^2
```

Fricas [A] time = 1.43173, size = 109, normalized size = 2.14

$$\frac{c^2x^8 + 3\,bcx^6 + 3\left(b^2 + 2\,ac\right)x^4 + 12\,abx^2\log\left(x\right) - 3\,a^2}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(c^2*x^8 + 3*b*c*x^6 + 3*(b^2 + 2*a*c)*x^4 + 12*a*b*x^2*log(x) - 3*a^2)
/x^2
```

Sympy [A] time = 0.345443, size = 44, normalized size = 0.86

$$-\frac{a^2}{2x^2} + 2ab\log\left(x\right) + \frac{bcx^4}{2} + \frac{c^2x^6}{6} + x^2\left(ac + \frac{b^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b*c*x**4/2 + c**2*x**6/6 + x**2*(a*c + b**2
/2)
```

Giac [A] time = 1.14875, size = 72, normalized size = 1.41

$$\frac{1}{6}\,c^2x^6 + \frac{1}{2}\,bcx^4 + \frac{1}{2}\,b^2x^2 + acx^2 + ab\log\left(x^2\right) - \frac{2\,abx^2 + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2 + a*c*x^2 + a*b*log(x^2) - 1/2*(2*a
*b*x^2 + a^2)/x^2
```

3.831 $\int \frac{(a+bx^2+cx^4)^2}{x^4} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rubi [A] time = 0.023734, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^4}\,dx &= \int \left(b^2\left(1+\frac{2ac}{b^2}\right)+\frac{a^2}{x^4}+\frac{2ab}{x^2}+2bcx^2+c^2x^4\right)dx\\ &= -\frac{a^2}{3x^3}-\frac{2ab}{x}+\left(b^2+2ac\right)x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.0180374, size = 47, normalized size = 1.

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Maple [A] time = 0.047, size = 42, normalized size = 0.9

$$\frac{c^2x^5}{5} + \frac{2\,bcx^3}{3} + 2\,acx + b^2x - \frac{a^2}{3\,x^3} - 2\,\frac{ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^4,x)
```

```
[Out] 1/5*c^2*x^5+2/3*b*c*x^3+2*a*c*x+b^2*x-1/3*a^2/x^3-2*a*b/x
```

Maxima [A] time = 0.966724, size = 57, normalized size = 1.21

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + \left(b^2 + 2\,ac\right)x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + (b^2 + 2*a*c)*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Fricas [A] time = 1.41883, size = 107, normalized size = 2.28

$$\frac{3\,c^2x^8 + 10\,bcx^6 + 15\left(b^2 + 2\,ac\right)x^4 - 30\,abx^2 - 5\,a^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*c^2*x^8 + 10*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 - 30*a*b*x^2 - 5*a^2)/x
^3
```

Sympy [A] time = 0.341833, size = 44, normalized size = 0.94

$$\frac{2bcx^3}{3} + \frac{c^2x^5}{5} + x\left(2ac + b^2\right) - \frac{a^2 + 6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] 2*b*c*x**3/3 + c**2*x**5/5 + x*(2*a*c + b**2) - (a**2 + 6*a*b*x**2)/(3*x**3
)
```

Giac [A] time = 1.09942, size = 57, normalized size = 1.21

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x + 2\,acx - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x + 2*a*c*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

3.832 $\int \frac{(a+bx^2+cx^4)^2}{x^5}\,dx$

Optimal. Leaf size=45

$$-\frac{a^2}{4x^4}+\log(x)\left(2ac+b^2\right)-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}$$

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b*c*x^2 + (c^2*x^4)/4 + (b^2 + 2*a*c)*Log[x]
```

Rubi [A] time = 0.0375199, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{4x^4}+\log(x)\left(2ac+b^2\right)-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b*c*x^2 + (c^2*x^4)/4 + (b^2 + 2*a*c)*Log[x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^5}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(2bc+\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2+2ac}{x}+c^2x\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{4x^4}-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}+\left(b^2+2ac\right)\log(x)$$

Mathematica [A] time = 0.0189848, size = 41, normalized size = 0.91

$$\log(x)\left(2ac+b^2\right)+\frac{\left(cx^4-a\right)\left(a+4bx^2+cx^4\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] ((-a + c*x^4)*(a + 4*b*x^2 + c*x^4))/(4*x^4) + (b^2 + 2*a*c)*Log[x]
```

Maple [A] time = 0.047, size = 43, normalized size = 1.

$$\frac{c^2x^4}{4}+bcx^2+2\,\ln(x)\,ac+b^2\ln(x)-\frac{a^2}{4\,x^4}-\frac{ab}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^5,x)
```

```
[Out] 1/4*c^2*x^4+b*c*x^2+2*ln(x)*a*c+b^2*ln(x)-1/4*a^2/x^4-1/x^2*a*b
```

Maxima [A] time = 0.960798, size = 61, normalized size = 1.36

$$\frac{1}{4}\,c^2x^4+bcx^2+\frac{1}{2}\left(b^2+2\,ac\right)\log\left(x^2\right)-\frac{4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/
x^4
```

Fricas [A] time = 1.46365, size = 105, normalized size = 2.33

$$\frac{c^2x^8 + 4\,bcx^6 + 4\left(b^2 + 2\,ac\right)x^4 \log\left(x\right) - 4\,abx^2 - a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*x^8 + 4*b*c*x^6 + 4*(b^2 + 2*a*c)*x^4*log(x) - 4*a*b*x^2 - a^2)/x^
4
```

Sympy [A] time = 0.53367, size = 42, normalized size = 0.93

$$bcx^2 + \frac{c^2x^4}{4} + \left(2ac + b^2\right)\log\left(x\right) - \frac{a^2 + 4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] b*c*x**2 + c**2*x**4/4 + (2*a*c + b**2)*log(x) - (a**2 + 4*a*b*x**2)/(4*x**
4)
```

Giac [A] time = 1.12364, size = 81, normalized size = 1.8

$$\frac{1}{4}\,c^2x^4 + bcx^2 + \frac{1}{2}\left(b^2 + 2\,ac\right)\log\left(x^2\right) - \frac{3\,b^2x^4 + 6\,acx^4 + 4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(3*b^2*x^4 + 6*a*c
*x^4 + 4*a*b*x^2 + a^2)/x^4
```

3.833 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^6}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - (b^2 + 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3
```

Rubi [A] time = 0.0230556, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - (b^2 + 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^6}\,dx &= \int \left(2bc+\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2+2ac}{x^2}+c^2x^2\right)dx\\ &= -\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2+2ac}{x}+2bcx+\frac{c^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0209539, size = 49, normalized size = 1.02

$$-\frac{a^2}{5x^5}+\frac{-2ac-b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) + (-b^2 - 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3
```

Maple [A] time = 0.047, size = 43, normalized size = 0.9

$$\frac{c^2x^3}{3}+2\,bcx-\frac{2\,ab}{3\,x^3}-\frac{a^2}{5\,x^5}-\frac{2\,ac+b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^6,x)
```

```
[Out] 1/3*c^2*x^3+2*b*c*x-2/3*a*b/x^3-1/5*a^2/x^5-(2*a*c+b^2)/x
```

Maxima [A] time = 0.978461, size = 61, normalized size = 1.27

$$\frac{1}{3}\,c^2x^3+2\,bcx-\frac{15\left(b^2+2\,ac\right)x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*(b^2 + 2*a*c)*x^4 + 10*a*b*x^2 + 3*a^2)/x^
5
```

Fricas [A] time = 1.37401, size = 107, normalized size = 2.23

$$\frac{5\,c^2x^8+30\,bcx^6-15\left(b^2+2\,ac\right)x^4-10\,abx^2-3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(5*c^2*x^8 + 30*b*c*x^6 - 15*(b^2 + 2*a*c)*x^4 - 10*a*b*x^2 - 3*a^2)/x
^5
```

Sympy [A] time = 0.596332, size = 46, normalized size = 0.96

$$2bcx + \frac{c^2x^3}{3} - \frac{3a^2 + 10abx^2 + x^4\left(30ac + 15b^2\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] 2*b*c*x + c**2*x**3/3 - (3*a**2 + 10*a*b*x**2 + x**4*(30*a*c + 15*b**2))/(1
5*x**5)
```

Giac [A] time = 1.13228, size = 63, normalized size = 1.31

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{15\,b^2x^4 + 30\,acx^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*b^2*x^4 + 30*a*c*x^4 + 10*a*b*x^2 + 3*a^2)
/x^5
```

3.834 $\int \frac{(a+bx^2+cx^4)^2}{x^7} dx$

Optimal. Leaf size=51

$$-\frac{a^2}{6x^6}-\frac{2ac+b^2}{2x^2}-\frac{ab}{2x^4}+2bc\log(x)+\frac{c^2x^2}{2}$$

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - (b^2 + 2*a*c)/(2*x^2) + (c^2*x^2)/2 + 2*b*c*
Log[x]
```

Rubi [A] time = 0.036362, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{6x^6}-\frac{2ac+b^2}{2x^2}-\frac{ab}{2x^4}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - (b^2 + 2*a*c)/(2*x^2) + (c^2*x^2)/2 + 2*b*c*
Log[x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(c^2+\frac{a^2}{x^4}+\frac{2ab}{x^3}+\frac{b^2+2ac}{x^2}+\frac{2bc}{x}\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2+2ac}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)$$

Mathematica [A] time = 0.0174762, size = 50, normalized size = 0.98

$$-\frac{a^2+3abx^2+6acx^4+3b^2x^4-12bcx^6\log(x)-3c^2x^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -(a^2 + 3*a*b*x^2 + 3*b^2*x^4 + 6*a*c*x^4 - 3*c^2*x^8 - 12*b*c*x^6*Log[x])/
(6*x^6)
```

Maple [A] time = 0.046, size = 46, normalized size = 0.9

$$\frac{c^2x^2}{2}+2\,bc\ln(x)-\frac{ab}{2\,x^4}-\frac{ac}{x^2}-\frac{b^2}{2\,x^2}-\frac{a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^7,x)
```

```
[Out] 1/2*c^2*x^2+2*b*c*ln(x)-1/2*a*b/x^4-1/x^2*a*c-1/2*b^2/x^2-1/6*a^2/x^6
```

Maxima [A] time = 0.971878, size = 61, normalized size = 1.2

$$\frac{1}{2}\,c^2x^2+bc\log\left(x^2\right)-\frac{3\left(b^2+2\,ac\right)x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(3*(b^2 + 2*a*c)*x^4 + 3*a*b*x^2 + a^2)/x^
6
```

Fricas [A] time = 1.43859, size = 109, normalized size = 2.14

$$\frac{3\,c^2x^8 + 12\,bcx^6\log\left(x\right) - 3\left(b^2 + 2\,ac\right)x^4 - 3\,abx^2 - a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(3*c^2*x^8 + 12*b*c*x^6*log(x) - 3*(b^2 + 2*a*c)*x^4 - 3*a*b*x^2 - a^2)
/x^6
```

Sympy [A] time = 0.908223, size = 46, normalized size = 0.9

$$2bc\log\left(x\right) + \frac{c^2x^2}{2} - \frac{a^2 + 3abx^2 + x^4\left(6ac + 3b^2\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*b*c*log(x) + c**2*x**2/2 - (a**2 + 3*a*b*x**2 + x**4*(6*a*c + 3*b**2))/(6
*x**6)
```

Giac [A] time = 1.11117, size = 73, normalized size = 1.43

$$\frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{11\,bcx^6 + 3\,b^2x^4 + 6\,acx^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(11*b*c*x^6 + 3*b^2*x^4 + 6*a*c*x^4 + 3*a*
b*x^2 + a^2)/x^6
```

3.835 $\int \frac{(a+bx^2+cx^4)^2}{x^8} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{7x^7}-\frac{2ac+b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - (b^2 + 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*x
```

Rubi [A] time = 0.0240573, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{7x^7}-\frac{2ac+b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - (b^2 + 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*x
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{x^8}\,dx &= \int \left(c^2+\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2+2ac}{x^4}+\frac{2bc}{x^2}\right)dx\\ &= -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2+2ac}{3x^3}-\frac{2bc}{x}+c^2x\end{aligned}$$

Mathematica [A] time = 0.0232191, size = 49, normalized size = 1.04

$$-\frac{a^2}{7x^7}+\frac{-2ac-b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) + (-b^2 - 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*x
```

Maple [A] time = 0.048, size = 42, normalized size = 0.9

$$c^2x - \frac{2\,ac + b^2}{3\,x^3} - \frac{2\,ab}{5\,x^5} - \frac{a^2}{7\,x^7} - 2\,\frac{bc}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^8,x)
```

```
[Out] c^2*x-1/3*(2*a*c+b^2)/x^3-2/5*a*b/x^5-1/7*a^2/x^7-2*b*c/x
```

Maxima [A] time = 0.983932, size = 59, normalized size = 1.26

$$c^2x - \frac{210\,bcx^6 + 35\,\left(b^2 + 2\,ac\right)x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="maxima")
```

```
[Out] c^2*x - 1/105*(210*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 42*a*b*x^2 + 15*a^2)/x^
7
```

Fricas [A] time = 1.45975, size = 113, normalized size = 2.4

$$\frac{105\,c^2x^8 - 210\,bcx^6 - 35\,\left(b^2 + 2\,ac\right)x^4 - 42\,abx^2 - 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*c^2*x^8 - 210*b*c*x^6 - 35*(b^2 + 2*a*c)*x^4 - 42*a*b*x^2 - 15*a
^2)/x^7
```

Sympy [A] time = 0.873508, size = 44, normalized size = 0.94

$$c^2x - \frac{15a^2 + 42abx^2 + 210bcx^6 + x^4\left(70ac + 35b^2\right)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**8,x)
```

```
[Out] c**2*x - (15*a**2 + 42*a*b*x**2 + 210*b*c*x**6 + x**4*(70*a*c + 35*b**2))/(
105*x**7)
```

Giac [A] time = 1.14199, size = 62, normalized size = 1.32

$$c^2x - \frac{210\,bcx^6 + 35\,b^2x^4 + 70\,acx^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="giac")
```

```
[Out] c^2*x - 1/105*(210*b*c*x^6 + 35*b^2*x^4 + 70*a*c*x^4 + 42*a*b*x^2 + 15*a^2)
/x^7
```

3.836 $\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2}{8x^8}-\frac{2ac+b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - (b^2 + 2*a*c)/(4*x^4) - (b*c)/x^2 + c^2*Log[
x]
```

Rubi [A] time = 0.0350476, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{8x^8}-\frac{2ac+b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - (b^2 + 2*a*c)/(4*x^4) - (b*c)/x^2 + c^2*Log[
x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^5}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2+2ac}{x^3}+\frac{2bc}{x^2}+\frac{c^2}{x}\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2+2ac}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Mathematica [A] time = 0.0265462, size = 50, normalized size = 1.04

$$-\frac{a^2}{8x^8}+\frac{-2ac-b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) + (-b^2 - 2*a*c)/(4*x^4) - (b*c)/x^2 + c^2*Log
[x]
```

Maple [A] time = 0.049, size = 45, normalized size = 0.9

$$c^2\ln(x)-\frac{ac}{2x^4}-\frac{b^2}{4x^4}-\frac{bc}{x^2}-\frac{a^2}{8x^8}-\frac{ab}{3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^9,x)
```

```
[Out] c^2*ln(x)-1/2/x^4*a*c-1/4*b^2/x^4-b*c/x^2-1/8*a^2/x^8-1/3*a*b/x^6
```

Maxima [A] time = 0.967694, size = 65, normalized size = 1.35

$$\frac{1}{2}c^2\log\left(x^2\right)-\frac{24\,bcx^6+6\left(b^2+2\,ac\right)x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2) - 1/24*(24*b*c*x^6 + 6*(b^2 + 2*a*c)*x^4 + 8*a*b*x^2 + 3*a
^2)/x^8
```

Fricas [A] time = 1.4253, size = 115, normalized size = 2.4

$$\frac{24\,c^2x^8\log\left(x\right) - 24\,bcx^6 - 6\left(b^2 + 2\,ac\right)x^4 - 8\,abx^2 - 3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(24*c^2*x^8*log(x) - 24*b*c*x^6 - 6*(b^2 + 2*a*c)*x^4 - 8*a*b*x^2 - 3*
a^2)/x^8
```

Sympy [A] time = 1.46377, size = 46, normalized size = 0.96

$$c^2\log\left(x\right) - \frac{3a^2 + 8abx^2 + 24bcx^6 + x^4\left(12ac + 6b^2\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**9,x)
```

```
[Out] c**2*log(x) - (3*a**2 + 8*a*b*x**2 + 24*b*c*x**6 + x**4*(12*a*c + 6*b**2))/
(24*x**8)
```

Giac [A] time = 1.13102, size = 78, normalized size = 1.62

$$\frac{1}{2}\,c^2\log\left(x^2\right) - \frac{25\,c^2x^8 + 24\,bcx^6 + 6\,b^2x^4 + 12\,acx^4 + 8\,abx^2 + 3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2) - 1/24*(25*c^2*x^8 + 24*b*c*x^6 + 6*b^2*x^4 + 12*a*c*x^4 +
 8*a*b*x^2 + 3*a^2)/x^8
```

3.837 $\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=52

$$-\frac{a^2}{9x^9}-\frac{2ac+b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

```
[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - (b^2 + 2*a*c)/(5*x^5) - (2*b*c)/(3*x^3) -
c^2/x
```

Rubi [A] time = 0.0246766, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{9x^9}-\frac{2ac+b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - (b^2 + 2*a*c)/(5*x^5) - (2*b*c)/(3*x^3) -
c^2/x
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^8}+\frac{b^2+2ac}{x^6}+\frac{2bc}{x^4}+\frac{c^2}{x^2}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2+2ac}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}\end{aligned}$$

Mathematica [A] time = 0.0196876, size = 50, normalized size = 0.96

$$-\frac{35a^2 + 90abx^2 + 126acx^4 + 63b^2x^4 + 210bcx^6 + 315c^2x^8}{315x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -(35*a^2 + 90*a*b*x^2 + 63*b^2*x^4 + 126*a*c*x^4 + 210*b*c*x^6 + 315*c^2*x^
8)/(315*x^9)
```

Maple [A] time = 0.047, size = 45, normalized size = 0.9

$$-\frac{2\,bc}{3\,x^3} - \frac{2\,ac + b^2}{5\,x^5} - \frac{2\,ab}{7\,x^7} - \frac{c^2}{x} - \frac{a^2}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^10,x)
```

```
[Out] -2/3*b*c/x^3-1/5*(2*a*c+b^2)/x^5-2/7*a*b/x^7-c^2/x-1/9*a^2/x^9
```

Maxima [A] time = 0.964756, size = 62, normalized size = 1.19

$$-\frac{315\,c^2x^8 + 210\,bcx^6 + 63\,(b^2 + 2\,ac)x^4 + 90\,abx^2 + 35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b*x^2 + 35*
a^2)/x^9
```

Fricas [A] time = 1.40219, size = 115, normalized size = 2.21

$$-\frac{315\,c^2x^8 + 210\,bcx^6 + 63\,(b^2 + 2\,ac)x^4 + 90\,abx^2 + 35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b*x^2 + 35*
a^2)/x^9
```

Sympy [A] time = 1.58361, size = 49, normalized size = 0.94

$$-\frac{35a^2 + 90abx^2 + 210bcx^6 + 315c^2x^8 + x^4\left(126ac + 63b^2\right)}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**10,x)
```

```
[Out] -(35*a**2 + 90*a*b*x**2 + 210*b*c*x**6 + 315*c**2*x**8 + x**4*(126*a*c + 63
*b**2))/(315*x**9)
```

Giac [A] time = 1.11946, size = 65, normalized size = 1.25

$$-\frac{315\,c^2x^8 + 210\,bcx^6 + 63\,b^2x^4 + 126\,acx^4 + 90\,abx^2 + 35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*b^2*x^4 + 126*a*c*x^4 + 90*a*b*x^2 +
 35*a^2)/x^9
```

3.838 $\int \frac{(a+bx^2+cx^4)^2}{x^{11}} dx$

Optimal. Leaf size=54

$$-\frac{a^2}{10x^{10}}-\frac{2ac+b^2}{6x^6}-\frac{ab}{4x^8}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

```
[Out] -a^2/(10*x^10) - (a*b)/(4*x^8) - (b^2 + 2*a*c)/(6*x^6) - (b*c)/(2*x^4) - c^
2/(2*x^2)
```

Rubi [A] time = 0.0374664, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{10x^{10}}-\frac{2ac+b^2}{6x^6}-\frac{ab}{4x^8}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -a^2/(10*x^10) - (a*b)/(4*x^8) - (b^2 + 2*a*c)/(6*x^6) - (b*c)/(2*x^4) - c^
2/(2*x^2)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2+2ac}{x^4}+\frac{2bc}{x^3}+\frac{c^2}{x^2}\right) dx, x, x^2\right)$$
$$= -\frac{a^2}{10x^{10}}-\frac{ab}{4x^8}-\frac{b^2+2ac}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Mathematica [A] time = 0.015071, size = 53, normalized size = 0.98

$$-\frac{6a^2+5a\left(3bx^2+4cx^4\right)+10x^4\left(b^2+3bcx^2+3c^2x^4\right)}{60x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -(6*a^2 + 5*a*(3*b*x^2 + 4*c*x^4) + 10*x^4*(b^2 + 3*b*c*x^2 + 3*c^2*x^4))/(
60*x^10)
```

Maple [A] time = 0.048, size = 45, normalized size = 0.8

$$-\frac{bc}{2\,x^4}-\frac{c^2}{2\,x^2}-\frac{ab}{4\,x^8}-\frac{2\,ac+b^2}{6\,x^6}-\frac{a^2}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^11,x)
```

```
[Out] -1/2*b*c/x^4-1/2*c^2/x^2-1/4*a*b/x^8-1/6*(2*a*c+b^2)/x^6-1/10*a^2/x^10
```

Maxima [A] time = 0.957288, size = 62, normalized size = 1.15

$$-\frac{30\,c^2x^8+30\,bcx^6+10\left(b^2+2\,ac\right)x^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^2 + 6*a^2)
/x^10
```

Fricas [A] time = 1.4316, size = 111, normalized size = 2.06

$$-\frac{30\,c^2x^8 + 30\,bcx^6 + 10\,\left(b^2 + 2\,ac\right)x^4 + 15\,abx^2 + 6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^2 + 6*a^2)
/x^10
```

Sympy [A] time = 2.1357, size = 49, normalized size = 0.91

$$-\frac{6a^2 + 15abx^2 + 30bcx^6 + 30c^2x^8 + x^4\left(20ac + 10b^2\right)}{60x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**11,x)
```

```
[Out] -(6*a**2 + 15*a*b*x**2 + 30*b*c*x**6 + 30*c**2*x**8 + x**4*(20*a*c + 10*b**
2))/(60*x**10)
```

Giac [A] time = 1.15229, size = 65, normalized size = 1.2

$$-\frac{30\,c^2x^8 + 30\,bcx^6 + 10\,b^2x^4 + 20\,acx^4 + 15\,abx^2 + 6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*b^2*x^4 + 20*a*c*x^4 + 15*a*b*x^2 + 6*a
^2)/x^10
```

3.839 $\int \frac{(a+bx^2+cx^4)^2}{x^{12}} \, dx$

Optimal. Leaf size=54

$$-\frac{a^2}{11x^{11}}-\frac{2ac+b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

```
[Out] -a^2/(11*x^11) - (2*a*b)/(9*x^9) - (b^2 + 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5)
- c^2/(3*x^3)
```

Rubi [A] time = 0.0248372, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{a^2}{11x^{11}}-\frac{2ac+b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -a^2/(11*x^11) - (2*a*b)/(9*x^9) - (b^2 + 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5)
- c^2/(3*x^3)
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2+cx^4)^2}{x^{12}} \, dx &= \int \left(\frac{a^2}{x^{12}}+\frac{2ab}{x^{10}}+\frac{b^2+2ac}{x^8}+\frac{2bc}{x^6}+\frac{c^2}{x^4}\right) dx\\
&= -\frac{a^2}{11x^{11}}-\frac{2ab}{9x^9}-\frac{b^2+2ac}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.024197, size = 56, normalized size = 1.04

$$-\frac{a^2}{11x^{11}}+\frac{-2ac-b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -a^2/(11*x^11) - (2*a*b)/(9*x^9) + (-b^2 - 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5)
 - c^2/(3*x^3)
```

Maple [A] time = 0.048, size = 45, normalized size = 0.8

$$-\frac{c^2}{3\,x^3}-\frac{2\,bc}{5\,x^5}-\frac{a^2}{11\,x^{11}}-\frac{2\,ac+b^2}{7\,x^7}-\frac{2\,ab}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^12,x)
```

```
[Out] -1/3*c^2/x^3-2/5*b*c/x^5-1/11*a^2/x^11-1/7*(2*a*c+b^2)/x^7-2/9*a*b/x^9
```

Maxima [A] time = 0.960844, size = 62, normalized size = 1.15

$$-\frac{1155\,c^2x^8+1386\,bcx^6+495\,\left(b^2+2\,ac\right)x^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2
+ 315*a^2)/x^11
```

Fricas [A] time = 1.56134, size = 124, normalized size = 2.3

$$-\frac{1155\,c^2x^8+1386\,bcx^6+495\,\left(b^2+2\,ac\right)x^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2
+ 315*a^2)/x^11
```

Sympy [A] time = 2.04834, size = 49, normalized size = 0.91

$$-\frac{315a^2 + 770abx^2 + 1386bcx^6 + 1155c^2x^8 + x^4\left(990ac + 495b^2\right)}{3465x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**12,x)
```

```
[Out] -(315*a**2 + 770*a*b*x**2 + 1386*b*c*x**6 + 1155*c**2*x**8 + x**4*(990*a*c
+ 495*b**2))/(3465*x**11)
```

Giac [A] time = 1.11809, size = 65, normalized size = 1.2

$$-\frac{1155\,c^2x^8 + 1386\,bcx^6 + 495\,b^2x^4 + 990\,acx^4 + 770\,abx^2 + 315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*b^2*x^4 + 990*a*c*x^4 + 770*a*b*
x^2 + 315*a^2)/x^11
```

3.840 $\int \frac{(a+bx^2+cx^4)^2}{x^{13}} dx$

Optimal. Leaf size=54

$$-\frac{a^2}{12x^{12}}-\frac{2ac+b^2}{8x^8}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

```
[Out] -a^2/(12*x^12) - (a*b)/(5*x^10) - (b^2 + 2*a*c)/(8*x^8) - (b*c)/(3*x^6) - c
^2/(4*x^4)
```

Rubi [A] time = 0.0353243, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$-\frac{a^2}{12x^{12}}-\frac{2ac+b^2}{8x^8}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] -a^2/(12*x^12) - (a*b)/(5*x^10) - (b^2 + 2*a*c)/(8*x^8) - (b*c)/(3*x^6) - c
^2/(4*x^4)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^7}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2+2ac}{x^5}+\frac{2bc}{x^4}+\frac{c^2}{x^3}\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{12x^{12}}-\frac{ab}{5x^{10}}-\frac{b^2+2ac}{8x^8}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

Mathematica [A] time = 0.0153282, size = 50, normalized size = 0.93

$$-\frac{10a^2+24abx^2+30acx^4+15b^2x^4+40bcx^6+30c^2x^8}{120x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] -(10*a^2 + 24*a*b*x^2 + 15*b^2*x^4 + 30*a*c*x^4 + 40*b*c*x^6 + 30*c^2*x^8)/
(120*x^12)
```

Maple [A] time = 0.048, size = 45, normalized size = 0.8

$$-\frac{ab}{5\,x^{10}}-\frac{c^2}{4\,x^4}-\frac{2\,ac+b^2}{8\,x^8}-\frac{bc}{3\,x^6}-\frac{a^2}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^13,x)
```

```
[Out] -1/5*a*b/x^10-1/4*c^2/x^4-1/8*(2*a*c+b^2)/x^8-1/3*b*c/x^6-1/12*a^2/x^12
```

Maxima [A] time = 0.956137, size = 62, normalized size = 1.15

$$-\frac{30\,c^2x^8+40\,bcx^6+15\,(b^2+2\,ac)x^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="maxima")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x^2 + 10*a^
2)/x^12
```

Fricas [A] time = 1.58857, size = 113, normalized size = 2.09

$$-\frac{30\,c^2x^8 + 40\,bcx^6 + 15\left(b^2 + 2\,ac\right)x^4 + 24\,abx^2 + 10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x^2 + 10*a^
2)/x^12
```

Sympy [A] time = 2.80806, size = 49, normalized size = 0.91

$$-\frac{10a^2 + 24abx^2 + 40bcx^6 + 30c^2x^8 + x^4\left(30ac + 15b^2\right)}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**13,x)
```

```
[Out] -(10*a**2 + 24*a*b*x**2 + 40*b*c*x**6 + 30*c**2*x**8 + x**4*(30*a*c + 15*b*
*2))/(120*x**12)
```

Giac [A] time = 1.1095, size = 65, normalized size = 1.2

$$-\frac{30\,c^2x^8 + 40\,bcx^6 + 15\,b^2x^4 + 30\,acx^4 + 24\,abx^2 + 10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*b^2*x^4 + 30*a*c*x^4 + 24*a*b*x^2 + 10
*a^2)/x^12
```

3.841 $\int x^2 \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=89

$$\frac{3}{5}a^2bx^5+\frac{a^3x^3}{3}+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)*
x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```

Rubi [A] time = 0.0594182, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^3}{3}+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)*
x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2+cx^4\right)^3\,dx&=\int\left(a^3x^2+3a^2bx^4+3a\left(b^2+ac\right)x^6+b\left(b^2+6ac\right)x^8+3c\left(b^2+ac\right)x^{10}+3bc^2x^{12}+c^3x^{14}\right)d\\&=\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}a\left(b^2+ac\right)x^7+\frac{1}{9}b\left(b^2+6ac\right)x^9+\frac{3}{11}c\left(b^2+ac\right)x^{11}+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.0131496, size = 89, normalized size = 1.

$$\frac{3}{5}a^2bx^5+\frac{a^3x^3}{3}+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)*
x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```

Maple [A] time = 0.041, size = 111, normalized size = 1.3

$$\frac{c^3x^{15}}{15}+\frac{3\,bc^2x^{13}}{13}+\frac{\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)x^{11}}{11}+\frac{\left(4\,abc+b\left(2\,ac+b^2\right)\right)x^9}{9}+\frac{\left(a\left(2\,ac+b^2\right)+2\,b^2a+a^2c\right)x^7}{7}+\frac{3\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/15*c^3*x^15+3/13*b*c^2*x^13+1/11*(a*c^2+2*b^2*c+c*(2*a*c+b^2))*x^11+1/9*(
4*a*b*c+b*(2*a*c+b^2))*x^9+1/7*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)*x^7+3/5*a^2*b*
x^5+1/3*a^3*x^3
```

Maxima [A] time = 0.952867, size = 109, normalized size = 1.22

$$\frac{1}{15}\,c^3x^{15}+\frac{3}{13}\,bc^2x^{13}+\frac{3}{11}\left(b^2c+ac^2\right)x^{11}+\frac{1}{9}\left(b^3+6\,abc\right)x^9+\frac{3}{5}\,a^2bx^5+\frac{3}{7}\left(ab^2+a^2c\right)x^7+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*(b^2*c + a*c^2)*x^11 + 1/9*(b^3 + 6*
a*b*c)*x^9 + 3/5*a^2*b*x^5 + 3/7*(a*b^2 + a^2*c)*x^7 + 1/3*a^3*x^3
```

Fricas [A] time = 1.52763, size = 217, normalized size = 2.44

$$\frac{1}{15}x^{15}c^3+\frac{3}{13}x^{13}c^2b+\frac{3}{11}x^{11}cb^2+\frac{3}{11}x^{11}c^2a+\frac{1}{9}x^9b^3+\frac{2}{3}x^9cba+\frac{3}{7}x^7b^2a+\frac{3}{7}x^7ca^2+\frac{3}{5}x^5ba^2+\frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*c^3 + 3/13*x^13*c^2*b + 3/11*x^11*c*b^2 + 3/11*x^11*c^2*a + 1/9*x
^9*b^3 + 2/3*x^9*c*b*a + 3/7*x^7*b^2*a + 3/7*x^7*c*a^2 + 3/5*x^5*b*a^2 + 1/
3*x^3*a^3
```

Sympy [A] time = 0.088396, size = 97, normalized size = 1.09

$$\frac{a^3x^3}{3} + \frac{3a^2bx^5}{5} + \frac{3bc^2x^{13}}{13} + \frac{c^3x^{15}}{15} + x^{11}\left(\frac{3ac^2}{11} + \frac{3b^2c}{11}\right) + x^9\left(\frac{2abc}{3} + \frac{b^3}{9}\right) + x^7\left(\frac{3a^2c}{7} + \frac{3ab^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*b*c**2*x**13/13 + c**3*x**15/15 + x**11*(
3*a*c**2/11 + 3*b**2*c/11) + x**9*(2*a*b*c/3 + b**3/9) + x**7*(3*a**2*c/7 +
 3*a*b**2/7)
```

Giac [A] time = 1.11659, size = 117, normalized size = 1.31

$$\frac{1}{15}c^3x^{15} + \frac{3}{13}bc^2x^{13} + \frac{3}{11}b^2cx^{11} + \frac{3}{11}ac^2x^{11} + \frac{1}{9}b^3x^9 + \frac{2}{3}abcx^9 + \frac{3}{7}ab^2x^7 + \frac{3}{7}a^2cx^7 + \frac{3}{5}a^2bx^5 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*b^2*c*x^11 + 3/11*a*c^2*x^11 + 1/9*b
^3*x^9 + 2/3*a*b*c*x^9 + 3/7*a*b^2*x^7 + 3/7*a^2*c*x^7 + 3/5*a^2*b*x^5 + 1/
3*a^3*x^3
```

3.842 $\int x\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=89

$$\frac{3}{4}a^2bx^4+\frac{a^3x^2}{2}+\frac{3}{10}cx^{10}\left(ac+b^2\right)+\frac{1}{8}bx^8\left(6ac+b^2\right)+\frac{1}{2}ax^6\left(ac+b^2\right)+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^4)/4 + (a*(b^2 + a*c)*x^6)/2 + (b*(b^2 + 6*a*c)*x^
8)/8 + (3*c*(b^2 + a*c)*x^10)/10 + (b*c^2*x^12)/4 + (c^3*x^14)/14
```

Rubi [A] time = 0.0830906, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 611}

$$\frac{3}{4}a^2bx^4+\frac{a^3x^2}{2}+\frac{3}{10}cx^{10}\left(ac+b^2\right)+\frac{1}{8}bx^8\left(6ac+b^2\right)+\frac{1}{2}ax^6\left(ac+b^2\right)+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^4)/4 + (a*(b^2 + a*c)*x^6)/2 + (b*(b^2 + 6*a*c)*x^
8)/8 + (3*c*(b^2 + a*c)*x^10)/10 + (b*c^2*x^12)/4 + (c^3*x^14)/14
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\int x\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{2}\operatorname{Subst}\left(\int\left(a+bx+cx^2\right)^3\,dx,x,x^2\right)$$
$$=\frac{1}{2}\operatorname{Subst}\left(\int\left(a^3+3a^2bx+3ab^2\left(1+\frac{ac}{b^2}\right)x^2+b^3\left(1+\frac{6ac}{b^2}\right)x^3+3b^2c\left(1+\frac{ac}{b^2}\right)x^4+3bc^2x^5+\right.\right.$$
$$=\frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{1}{2}a\left(b^2+ac\right)x^6+\frac{1}{8}b\left(b^2+6ac\right)x^8+\frac{3}{10}c\left(b^2+ac\right)x^{10}+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

Mathematica [A] time = 0.0163152, size = 79, normalized size = 0.89

$$\frac{1}{280}x^2\left(210a^2bx^2+140a^3+84cx^8\left(ac+b^2\right)+35bx^6\left(6ac+b^2\right)+140ax^4\left(ac+b^2\right)+70bc^2x^{10}+20c^3x^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(140*a^3 + 210*a^2*b*x^2 + 140*a*(b^2 + a*c)*x^4 + 35*b*(b^2 + 6*a*c)*
x^6 + 84*c*(b^2 + a*c)*x^8 + 70*b*c^2*x^10 + 20*c^3*x^12))/280
```

Maple [A] time = 0.043, size = 111, normalized size = 1.3

$$\frac{c^3x^{14}}{14}+\frac{bc^2x^{12}}{4}+\frac{\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)x^{10}}{10}+\frac{\left(4\,abc+b\left(2\,ac+b^2\right)\right)x^8}{8}+\frac{\left(a\left(2\,ac+b^2\right)+2\,b^2a+a^2c\right)x^6}{6}+\frac{3\,a^2b}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/14*c^3*x^14+1/4*b*c^2*x^12+1/10*(a*c^2+2*b^2*c+c*(2*a*c+b^2))*x^10+1/8*(4
*a*b*c+b*(2*a*c+b^2))*x^8+1/6*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)*x^6+3/4*a^2*b*x
^4+1/2*x^2*a^3
```

Maxima [A] time = 0.980865, size = 109, normalized size = 1.22

$$\frac{1}{14}\,c^3x^{14}+\frac{1}{4}\,bc^2x^{12}+\frac{3}{10}\,\left(b^2c+ac^2\right)x^{10}+\frac{1}{8}\,\left(b^3+6\,abc\right)x^8+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,\left(ab^2+a^2c\right)x^6+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*(b^2*c + a*c^2)*x^10 + 1/8*(b^3 + 6*a
*b*c)*x^8 + 3/4*a^2*b*x^4 + 1/2*(a*b^2 + a^2*c)*x^6 + 1/2*a^3*x^2
```

Fricas [A] time = 1.49572, size = 216, normalized size = 2.43

$$\frac{1}{14}x^{14}c^3 + \frac{1}{4}x^{12}c^2b + \frac{3}{10}x^{10}cb^2 + \frac{3}{10}x^{10}c^2a + \frac{1}{8}x^8b^3 + \frac{3}{4}x^8cba + \frac{1}{2}x^6b^2a + \frac{1}{2}x^6ca^2 + \frac{3}{4}x^4ba^2 + \frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*c^3 + 1/4*x^12*c^2*b + 3/10*x^10*c*b^2 + 3/10*x^10*c^2*a + 1/8*x^
8*b^3 + 3/4*x^8*c*b*a + 1/2*x^6*b^2*a + 1/2*x^6*c*a^2 + 3/4*x^4*b*a^2 + 1/2
*x^2*a^3
```

Sympy [A] time = 0.08057, size = 92, normalized size = 1.03

$$\frac{a^3x^2}{2} + \frac{3a^2bx^4}{4} + \frac{bc^2x^{12}}{4} + \frac{c^3x^{14}}{14} + x^{10}\left(\frac{3ac^2}{10} + \frac{3b^2c}{10}\right) + x^8\left(\frac{3abc}{4} + \frac{b^3}{8}\right) + x^6\left(\frac{a^2c}{2} + \frac{ab^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**2/2 + 3*a**2*b*x**4/4 + b*c**2*x**12/4 + c**3*x**14/14 + x**10*(3*a
*c**2/10 + 3*b**2*c/10) + x**8*(3*a*b*c/4 + b**3/8) + x**6*(a**2*c/2 + a*b*
*2/2)
```

Giac [A] time = 1.12805, size = 117, normalized size = 1.31

$$\frac{1}{14}c^3x^{14} + \frac{1}{4}bc^2x^{12} + \frac{3}{10}b^2cx^{10} + \frac{3}{10}ac^2x^{10} + \frac{1}{8}b^3x^8 + \frac{3}{4}abcx^8 + \frac{1}{2}ab^2x^6 + \frac{1}{2}a^2cx^6 + \frac{3}{4}a^2bx^4 + \frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*b^2*c*x^10 + 3/10*a*c^2*x^10 + 1/8*b^
3*x^8 + 3/4*a*b*c*x^8 + 1/2*a*b^2*x^6 + 1/2*a^2*c*x^6 + 3/4*a^2*b*x^4 + 1/2
*a^3*x^2
```

3.843 $\int \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=81

$$a^2bx^3+a^3x+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c*
(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Rubi [A] time = 0.0447497, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1090}

$$a^2bx^3+a^3x+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c*
(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2+cx^4\right)^3 \, dx &= \int \left(a^3+3a^2bx^2+3ab^2\left(1+\frac{ac}{b^2}\right)x^4+b^3\left(1+\frac{6ac}{b^2}\right)x^6+3b^2c\left(1+\frac{ac}{b^2}\right)x^8+3bc^2x^{10}+c^3x^{12}\right) d\\ &= a^3x+a^2bx^3+\frac{3}{5}a\left(b^2+ac\right)x^5+\frac{1}{7}b\left(b^2+6ac\right)x^7+\frac{1}{3}c\left(b^2+ac\right)x^9+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.0097713, size = 81, normalized size = 1.

$$a^2bx^3+a^3x+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c*
(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Maple [A] time = 0.043, size = 107, normalized size = 1.3

$$\frac{c^3x^{13}}{13}+\frac{3\,bc^2x^{11}}{11}+\frac{\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)x^9}{9}+\frac{\left(4\,abc+b\left(2\,ac+b^2\right)\right)x^7}{7}+\frac{\left(a\left(2\,ac+b^2\right)+2\,b^2a+a^2c\right)x^5}{5}+a^2bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/13*c^3*x^13+3/11*b*c^2*x^11+1/9*(a*c^2+2*b^2*c+c*(2*a*c+b^2))*x^9+1/7*(4*
a*b*c+b*(2*a*c+b^2))*x^7+1/5*(a*(2*a*c+b^2)+2*b^2*a+a^2*c)*x^5+a^2*b*x^3+x*
a^3
```

Maxima [A] time = 0.963325, size = 115, normalized size = 1.42

$$\frac{1}{13}\,c^3x^{13}+\frac{3}{11}\,bc^2x^{11}+\frac{1}{3}\,b^2cx^9+\frac{1}{7}\,b^3x^7+a^3x+\frac{1}{5}\left(3\,cx^5+5\,bx^3\right)a^2+\frac{1}{105}\left(35\,c^2x^9+90\,bcx^7+63\,b^2x^5\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/7*b^3*x^7 + a^3*x + 1/5
*(3*c*x^5 + 5*b*x^3)*a^2 + 1/105*(35*c^2*x^9 + 90*b*c*x^7 + 63*b^2*x^5)*a
```

Fricas [A] time = 1.54389, size = 198, normalized size = 2.44

$$\frac{1}{13}x^{13}c^3+\frac{3}{11}x^{11}c^2b+\frac{1}{3}x^9cb^2+\frac{1}{3}x^9c^2a+\frac{1}{7}x^7b^3+\frac{6}{7}x^7cba+\frac{3}{5}x^5b^2a+\frac{3}{5}x^5ca^2+x^3ba^2+xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*c^3 + 3/11*x^11*c^2*b + 1/3*x^9*c*b^2 + 1/3*x^9*c^2*a + 1/7*x^7*b
^3 + 6/7*x^7*c*b*a + 3/5*x^5*b^2*a + 3/5*x^5*c*a^2 + x^3*b*a^2 + x*a^3
```

Sympy [A] time = 0.080683, size = 87, normalized size = 1.07

$$a^3x + a^2bx^3 + \frac{3bc^2x^{11}}{11} + \frac{c^3x^{13}}{13} + x^9\left(\frac{ac^2}{3} + \frac{b^2c}{3}\right) + x^7\left(\frac{6abc}{7} + \frac{b^3}{7}\right) + x^5\left(\frac{3a^2c}{5} + \frac{3ab^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*b*x**3 + 3*b*c**2*x**11/11 + c**3*x**13/13 + x**9*(a*c**2/3 +
 b**2*c/3) + x**7*(6*a*b*c/7 + b**3/7) + x**5*(3*a**2*c/5 + 3*a*b**2/5)
```

Giac [A] time = 1.11074, size = 112, normalized size = 1.38

$$\frac{1}{13}c^3x^{13} + \frac{3}{11}bc^2x^{11} + \frac{1}{3}b^2cx^9 + \frac{1}{3}ac^2x^9 + \frac{1}{7}b^3x^7 + \frac{6}{7}abcx^7 + \frac{3}{5}ab^2x^5 + \frac{3}{5}a^2cx^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/3*a*c^2*x^9 + 1/7*b^3*x
^7 + 6/7*a*b*c*x^7 + 3/5*a*b^2*x^5 + 3/5*a^2*c*x^5 + a^2*b*x^3 + a^3*x
```

3.844 $\int \frac{(a+bx^2+cx^4)^3}{x} \, dx$

Optimal. Leaf size=85

$$\frac{3}{2}a^2bx^2 + a^3\log(x) + \frac{3}{8}cx^8\left(ac+b^2\right) + \frac{1}{6}bx^6\left(6ac+b^2\right) + \frac{3}{4}ax^4\left(ac+b^2\right) + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12}$$

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)/6 + (3*c*
(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12 + a^3*Log[x]
```

Rubi [A] time = 0.0742707, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$\frac{3}{2}a^2bx^2 + a^3\log(x) + \frac{3}{8}cx^8\left(ac+b^2\right) + \frac{1}{6}bx^6\left(6ac+b^2\right) + \frac{3}{4}ax^4\left(ac+b^2\right) + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)/6 + (3*c*
(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12 + a^3*Log[x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^3}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3a\left(b^2+ac\right)x+b\left(b^2+6ac\right)x^2+3c\left(b^2+ac\right)x^3+3bc^2x^4+c^3x^5\right)\right.$$
$$= \frac{3}{2}a^2bx^2+\frac{3}{4}a\left(b^2+ac\right)x^4+\frac{1}{6}b\left(b^2+6ac\right)x^6+\frac{3}{8}c\left(b^2+ac\right)x^8+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}+a^3\log(x$$

Mathematica [A] time = 0.0211895, size = 85, normalized size = 1.

$$\frac{3}{2}a^2bx^2+a^3\log(x)+\frac{3}{8}cx^8\left(ac+b^2\right)+\frac{1}{6}bx^6\left(6ac+b^2\right)+\frac{3}{4}ax^4\left(ac+b^2\right)+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)/6 + (3*c*
(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12 + a^3*Log[x]
```

Maple [A] time = 0.045, size = 85, normalized size = 1.

$$\frac{c^3x^{12}}{12}+\frac{3\,bc^2x^{10}}{10}+\frac{3\,x^8ac^2}{8}+\frac{3\,x^8b^2c}{8}+x^6abc+\frac{b^3x^6}{6}+\frac{3\,a^2cx^4}{4}+\frac{3\,ax^4b^2}{4}+\frac{3\,a^2bx^2}{2}+a^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x,x)
```

```
[Out] 1/12*c^3*x^12+3/10*b*c^2*x^10+3/8*x^8*a*c^2+3/8*x^8*b^2*c+x^6*a*b*c+1/6*b^3
*x^6+3/4*a^2*c*x^4+3/4*a*x^4*b^2+3/2*a^2*b*x^2+a^3*ln(x)
```

Maxima [A] time = 0.978995, size = 111, normalized size = 1.31

$$\frac{1}{12}\,c^3x^{12}+\frac{3}{10}\,bc^2x^{10}+\frac{3}{8}\left(b^2c+ac^2\right)x^8+\frac{1}{6}\left(b^3+6\,abc\right)x^6+\frac{3}{2}\,a^2bx^2+\frac{3}{4}\left(ab^2+a^2c\right)x^4+\frac{1}{2}\,a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(b^3 + 6*a*
b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + 1/2*a^3*log(x^2)
```

Fricas [A] time = 1.75492, size = 189, normalized size = 2.22

$$\frac{1}{12}c^3x^{12} + \frac{3}{10}bc^2x^{10} + \frac{3}{8}\left(b^2c + ac^2\right)x^8 + \frac{1}{6}\left(b^3 + 6\,abc\right)x^6 + \frac{3}{2}a^2bx^2 + \frac{3}{4}\left(ab^2 + a^2c\right)x^4 + a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(b^3 + 6*a*
b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + a^3*log(x)
```

Sympy [A] time = 0.367357, size = 92, normalized size = 1.08

$$a^3\log\left(x\right) + \frac{3a^2bx^2}{2} + \frac{3bc^2x^{10}}{10} + \frac{c^3x^{12}}{12} + x^8\left(\frac{3ac^2}{8} + \frac{3b^2c}{8}\right) + x^6\left(abc + \frac{b^3}{6}\right) + x^4\left(\frac{3a^2c}{4} + \frac{3ab^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] a**3*log(x) + 3*a**2*b*x**2/2 + 3*b*c**2*x**10/10 + c**3*x**12/12 + x**8*(3
*a*c**2/8 + 3*b**2*c/8) + x**6*(a*b*c + b**3/6) + x**4*(3*a**2*c/4 + 3*a*b*
*2/4)
```

Giac [A] time = 1.14849, size = 117, normalized size = 1.38

$$\frac{1}{12}c^3x^{12} + \frac{3}{10}bc^2x^{10} + \frac{3}{8}b^2cx^8 + \frac{3}{8}ac^2x^8 + \frac{1}{6}b^3x^6 + abcx^6 + \frac{3}{4}ab^2x^4 + \frac{3}{4}a^2cx^4 + \frac{3}{2}a^2bx^2 + \frac{1}{2}a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*b^2*c*x^8 + 3/8*a*c^2*x^8 + 1/6*b^3*x
^6 + a*b*c*x^6 + 3/4*a*b^2*x^4 + 3/4*a^2*c*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*lo
g(x^2)
```

3.845 $\int \frac{(a+bx^2+cx^4)^3}{x^2}\,dx$

Optimal. Leaf size=80

$$3a^2bx-\frac{a^3}{x}+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*(
b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rubi [A] time = 0.039367, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$3a^2bx-\frac{a^3}{x}+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*(
b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^2}\,dx &= \int \left(3a^2b+\frac{a^3}{x^2}+3a\left(b^2+ac\right)x^2+b\left(b^2+6ac\right)x^4+3c\left(b^2+ac\right)x^6+3bc^2x^8+c^3x^{10}\right)dx\\ &= -\frac{a^3}{x}+3a^2bx+a\left(b^2+ac\right)x^3+\frac{1}{5}b\left(b^2+6ac\right)x^5+\frac{3}{7}c\left(b^2+ac\right)x^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.0231187, size = 80, normalized size = 1.

$$3a^2bx-\frac{a^3}{x}+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*(
b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Maple [A] time = 0.046, size = 84, normalized size = 1.1

$$\frac{c^3x^{11}}{11}+\frac{bc^2x^9}{3}+\frac{3\,x^7ac^2}{7}+\frac{3\,b^2cx^7}{7}+\frac{6\,x^5abc}{5}+\frac{b^3x^5}{5}+x^3a^2c+ax^3b^2+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^2,x)
```

```
[Out] 1/11*c^3*x^11+1/3*b*c^2*x^9+3/7*x^7*a*c^2+3/7*b^2*c*x^7+6/5*x^5*a*b*c+1/5*b
^3*x^5+x^3*a^2*c+a*x^3*b^2+3*a^2*b*x-a^3/x
```

Maxima [A] time = 0.955623, size = 105, normalized size = 1.31

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\left(b^2c+ac^2\right)x^7+\frac{1}{5}\left(b^3+6\,abc\right)x^5+3\,a^2bx+\left(ab^2+a^2c\right)x^3-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*(b^2*c + a*c^2)*x^7 + 1/5*(b^3 + 6*a*b*
c)*x^5 + 3*a^2*b*x + (a*b^2 + a^2*c)*x^3 - a^3/x
```

Fricas [A] time = 1.46594, size = 201, normalized size = 2.51

$$\frac{105\,c^3x^{12}+385\,bc^2x^{10}+495\,\left(b^2c+ac^2\right)x^8+231\,\left(b^3+6\,abc\right)x^6+3465\,a^2bx^2+1155\,\left(ab^2+a^2c\right)x^4-1155\,a^3}{1155\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/1155*(105*c^3*x^12 + 385*b*c^2*x^10 + 495*(b^2*c + a*c^2)*x^8 + 231*(b^3
+ 6*a*b*c)*x^6 + 3465*a^2*b*x^2 + 1155*(a*b^2 + a^2*c)*x^4 - 1155*a^3)/x
```

Sympy [A] time = 0.359609, size = 82, normalized size = 1.02

$$-\frac{a^3}{x}+3a^2bx+\frac{bc^2x^9}{3}+\frac{c^3x^{11}}{11}+x^7\left(\frac{3ac^2}{7}+\frac{3b^2c}{7}\right)+x^5\left(\frac{6abc}{5}+\frac{b^3}{5}\right)+x^3\left(a^2c+ab^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*x + b*c**2*x**9/3 + c**3*x**11/11 + x**7*(3*a*c**2/7 + 3
*b**2*c/7) + x**5*(6*a*b*c/5 + b**3/5) + x**3*(a**2*c + a*b**2)
```

Giac [A] time = 1.16089, size = 112, normalized size = 1.4

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{3}{7}\,ac^2x^7+\frac{1}{5}\,b^3x^5+\frac{6}{5}\,abcx^5+ab^2x^3+a^2cx^3+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 3/7*a*c^2*x^7 + 1/5*b^3*x^5
 + 6/5*a*b*c*x^5 + a*b^2*x^3 + a^2*c*x^3 + 3*a^2*b*x - a^3/x
```

3.846 $\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx$

Optimal. Leaf size=86

$$3a^2b\log(x)-\frac{a^3}{2x^2}+\frac{1}{2}cx^6\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+\frac{3}{2}ax^2\left(ac+b^2\right)+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

```
[Out] -a^3/(2*x^2) + (3*a*(b^2 + a*c)*x^2)/2 + (b*(b^2 + 6*a*c)*x^4)/4 + (c*(b^2
+ a*c)*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10 + 3*a^2*b*Log[x]
```

Rubi [A] time = 0.0778557, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1114, 698}

$$3a^2b\log(x)-\frac{a^3}{2x^2}+\frac{1}{2}cx^6\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+\frac{3}{2}ax^2\left(ac+b^2\right)+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*(b^2 + a*c)*x^2)/2 + (b*(b^2 + 6*a*c)*x^4)/4 + (c*(b^2
+ a*c)*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10 + 3*a^2*b*Log[x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^3}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(3a\left(b^2+ac\right)+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b\left(b^2+6ac\right)x+3c\left(b^2+ac\right)x^2+3bc^2x^3+c^3x^4\right)dx,\right.$$
$$= -\frac{a^3}{2x^2}+\frac{3}{2}a\left(b^2+ac\right)x^2+\frac{1}{4}b\left(b^2+6ac\right)x^4+\frac{1}{2}c\left(b^2+ac\right)x^6+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}+3a^2b\log(x)$$

Mathematica [A] time = 0.0334795, size = 78, normalized size = 0.91

$$\frac{1}{40}\left(120a^2b\log(x)-\frac{20a^3}{x^2}+20cx^6\left(ac+b^2\right)+10bx^4\left(6ac+b^2\right)+60ax^2\left(ac+b^2\right)+15bc^2x^8+4c^3x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] ((-20*a^3)/x^2 + 60*a*(b^2 + a*c)*x^2 + 10*b*(b^2 + 6*a*c)*x^4 + 20*c*(b^2
+ a*c)*x^6 + 15*b*c^2*x^8 + 4*c^3*x^10 + 120*a^2*b*Log[x])/40
```

Maple [A] time = 0.047, size = 87, normalized size = 1.

$$\frac{c^3x^{10}}{10}+\frac{3\,bc^2x^8}{8}+\frac{x^6ac^2}{2}+\frac{x^6b^2c}{2}+\frac{3\,x^4abc}{2}+\frac{b^3x^4}{4}+\frac{3\,x^2a^2c}{2}+\frac{3\,ab^2x^2}{2}+3\,a^2b\ln(x)-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^3,x)
```

```
[Out] 1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*x^6*a*c^2+1/2*x^6*b^2*c+3/2*x^4*a*b*c+1/4*b
^3*x^4+3/2*x^2*a^2*c+3/2*a*b^2*x^2+3*a^2*b*ln(x)-1/2*a^3/x^2
```

Maxima [A] time = 0.96496, size = 111, normalized size = 1.29

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\left(b^2c+ac^2\right)x^6+\frac{1}{4}\left(b^3+6\,abc\right)x^4+\frac{3}{2}\,a^2b\log\left(x^2\right)+\frac{3}{2}\left(ab^2+a^2c\right)x^2-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*(b^2*c + a*c^2)*x^6 + 1/4*(b^3 + 6*a*b*
c)*x^4 + 3/2*a^2*b*log(x^2) + 3/2*(a*b^2 + a^2*c)*x^2 - 1/2*a^3/x^2
```

Fricas [A] time = 1.40602, size = 197, normalized size = 2.29

$$\frac{4\,c^3x^{12} + 15\,bc^2x^{10} + 20\left(b^2c + ac^2\right)x^8 + 10\left(b^3 + 6\,abc\right)x^6 + 120\,a^2bx^2\log\left(x\right) + 60\left(ab^2 + a^2c\right)x^4 - 20\,a^3}{40\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/40*(4*c^3*x^12 + 15*b*c^2*x^10 + 20*(b^2*c + a*c^2)*x^8 + 10*(b^3 + 6*a*b
*c)*x^6 + 120*a^2*b*x^2*log(x) + 60*(a*b^2 + a^2*c)*x^4 - 20*a^3)/x^2
```

Sympy [A] time = 0.393836, size = 92, normalized size = 1.07

$$-\frac{a^3}{2x^2} + 3a^2b\log\left(x\right) + \frac{3bc^2x^8}{8} + \frac{c^3x^{10}}{10} + x^6\left(\frac{ac^2}{2} + \frac{b^2c}{2}\right) + x^4\left(\frac{3abc}{2} + \frac{b^3}{4}\right) + x^2\left(\frac{3a^2c}{2} + \frac{3ab^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*b*c**2*x**8/8 + c**3*x**10/10 + x**6*(
a*c**2/2 + b**2*c/2) + x**4*(3*a*b*c/2 + b**3/4) + x**2*(3*a**2*c/2 + 3*a*b
**2/2)
```

Giac [A] time = 1.12736, size = 132, normalized size = 1.53

$$\frac{1}{10}\,c^3x^{10} + \frac{3}{8}\,bc^2x^8 + \frac{1}{2}\,b^2cx^6 + \frac{1}{2}\,ac^2x^6 + \frac{1}{4}\,b^3x^4 + \frac{3}{2}\,abcx^4 + \frac{3}{2}\,ab^2x^2 + \frac{3}{2}\,a^2cx^2 + \frac{3}{2}\,a^2b\log\left(x^2\right) - \frac{3\,a^2bx^2 + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/2*a*c^2*x^6 + 1/4*b^3*x^4
 + 3/2*a*b*c*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*c*x^2 + 3/2*a^2*b*log(x^2) - 1/2
*(3*a^2*b*x^2 + a^3)/x^2
```

3.847 $\int \frac{(a+bx^2+cx^4)^3}{x^4}\,dx$

Optimal. Leaf size=83

$$-\frac{3a^2b}{x}-\frac{a^3}{3x^3}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
(3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rubi [A] time = 0.0406804, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1108}

$$-\frac{3a^2b}{x}-\frac{a^3}{3x^3}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
(3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^4}\,dx &= \int \left(3a\left(b^2+ac\right)+\frac{a^3}{x^4}+\frac{3a^2b}{x^2}+b\left(b^2+6ac\right)x^2+3c\left(b^2+ac\right)x^4+3bc^2x^6+c^3x^8\right)dx\\ &= -\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3a\left(b^2+ac\right)x+\frac{1}{3}b\left(b^2+6ac\right)x^3+\frac{3}{5}c\left(b^2+ac\right)x^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0247444, size = 83, normalized size = 1.

$$-\frac{3a^2b}{x}-\frac{a^3}{3x^3}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
(3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Maple [A] time = 0.049, size = 84, normalized size = 1.

$$\frac{c^3x^9}{9}+\frac{3\,bc^2x^7}{7}+\frac{3\,x^5ac^2}{5}+\frac{3\,b^2cx^5}{5}+2\,x^3abc+\frac{b^3x^3}{3}+3\,a^2cx+3\,b^2ax-\frac{a^3}{3\,x^3}-3\,\frac{ba^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^4,x)
```

```
[Out] 1/9*c^3*x^9+3/7*b*c^2*x^7+3/5*x^5*a*c^2+3/5*b^2*c*x^5+2*x^3*a*b*c+1/3*b^3*x
^3+3*a^2*c*x+3*b^2*a*x-1/3*a^3/x^3-3*a^2*b/x
```

Maxima [A] time = 0.970253, size = 108, normalized size = 1.3

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\left(b^2c+ac^2\right)x^5+\frac{1}{3}\left(b^3+6\,abc\right)x^3+3\left(ab^2+a^2c\right)x-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*(b^2*c + a*c^2)*x^5 + 1/3*(b^3 + 6*a*b*c)
*x^3 + 3*(a*b^2 + a^2*c)*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

Fricas [A] time = 1.39957, size = 197, normalized size = 2.37

$$\frac{35\,c^3x^{12}+135\,bc^2x^{10}+189\,\left(b^2c+ac^2\right)x^8+105\,\left(b^3+6\,abc\right)x^6-945\,a^2bx^2+945\,\left(ab^2+a^2c\right)x^4-105\,a^3}{315\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/315*(35*c^3*x^12 + 135*b*c^2*x^10 + 189*(b^2*c + a*c^2)*x^8 + 105*(b^3 +
6*a*b*c)*x^6 - 945*a^2*b*x^2 + 945*(a*b^2 + a^2*c)*x^4 - 105*a^3)/x^3
```

Sympy [A] time = 0.405694, size = 88, normalized size = 1.06

$$\frac{3bc^2x^7}{7}+\frac{c^3x^9}{9}+x^5\left(\frac{3ac^2}{5}+\frac{3b^2c}{5}\right)+x^3\left(2abc+\frac{b^3}{3}\right)+x\left(3a^2c+3ab^2\right)-\frac{a^3+9a^2bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**4,x)
```

```
[Out] 3*b*c**2*x**7/7 + c**3*x**9/9 + x**5*(3*a*c**2/5 + 3*b**2*c/5) + x**3*(2*a*
b*c + b**3/3) + x*(3*a**2*c + 3*a*b**2) - (a**3 + 9*a**2*b*x**2)/(3*x**3)
```

Giac [A] time = 1.12945, size = 113, normalized size = 1.36

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{3}{5}\,ac^2x^5+\frac{1}{3}\,b^3x^3+2\,abcx^3+3\,ab^2x+3\,a^2cx-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 3/5*a*c^2*x^5 + 1/3*b^3*x^3 +
 2*a*b*c*x^3 + 3*a*b^2*x + 3*a^2*c*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

3.848 $\int \frac{x^7}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=100

$$\frac{(b^2-ac)\log(a+bx^2+cx^4)}{4c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rubi [A] time = 0.117027, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1114, 701, 634, 618, 206, 628}

$$\frac{(b^2-ac)\log(a+bx^2+cx^4)}{4c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4),x]
```


```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7}{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+(b^2-ac)x}{c^2(a+bx+cx^2)}\right)dx, x, x^2\right)$$
$$= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\operatorname{Subst}\left(\int \frac{ab+(b^2-ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2}$$
$$= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}-\frac{(b(b^2-3ac))\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3}+\frac{(b^2-ac)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3}$$
$$= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{(b^2-ac)\log(a+bx^2+cx^4)}{4c^3}+\frac{(b(b^2-3ac))\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^3}$$
$$= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx^2+cx^4)}{4c^3}$$

Mathematica [A] time = 0.0902564, size = 93, normalized size = 0.93

$$\frac{(b^2-ac)\log(a+bx^2+cx^4)-\frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx^2(cx^2-2b)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A] time = 0.174, size = 142, normalized size = 1.4

$$\frac{x^4}{4c}-\frac{bx^2}{2c^2}-\frac{\ln(cx^4+bx^2+a)a}{4c^2}+\frac{\ln(cx^4+bx^2+a)b^2}{4c^3}+\frac{3ab}{2c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{b^3}{2c^3}\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*x^4/c-1/2*b*x^2/c^2-1/4/c^2*ln(c*x^4+b*x^2+a)*a+1/4/c^3*ln(c*x^4+b*x^2+
a)*b^2+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b-
1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58299, size = 675, normalized size = 6.75

$$\left[\frac{\left(b^2c^2 - 4\,ac^3\right)x^4 - 2\left(b^3c - 4\,abc^2\right)x^2 - \left(b^3 - 3\,abc\right)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right) + \left(b^4 - 5\,ab^2c + \right.}{4\left(b^2c^3 - 4\,ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c)
*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)
*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2
*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2
)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] time = 2.08908, size = 391, normalized size = 3.91

$$-\frac{bx^2}{2c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)} - \frac{ac-b^2}{4c^3}\right)\log\left(x^2 + \frac{2a^2c - ab^2 + 8ac^3\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)} - \frac{ac-b^2}{4c^3}\right) - 2b^2c^2\left(-\frac{b\sqrt{-4ac}}{4c^3(}\right.}{3abc - b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c -
b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(-
b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)
/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*
a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c +
 b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(
x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*sqrt(-4*a
*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)))
/(3*a*b*c - b**3)) + x**4/(4*c)
```

Giac [A] time = 1.20849, size = 124, normalized size = 1.24

$$\frac{cx^4 - 2\,bx^2}{4\,c^2} + \frac{\left(b^2 - ac\right)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1/
2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*c^3)
```

3.849 $\int \frac{x^5}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=81

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 0.0799677, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1114, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2c}+\frac{\operatorname{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx,x,x^2\right)}{2c}\\
&= \frac{x^2}{2c}-\frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}+\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}\\
&= \frac{x^2}{2c}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}
$$

Mathematica [A] time = 0.0449408, size = 78, normalized size = 0.96

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+bx^2+cx^4\right)+2cx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] time = 0.171, size = 111, normalized size = 1.4

$$\frac{x^2}{2\,c}-\frac{b\ln\left(cx^4+bx^2+a\right)}{4\,c^2}-\frac{a}{c}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{2\,c^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x^2/c-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53332, size = 556, normalized size = 6.86

$$\left[\frac{2\left(b^2c-4\,ac^2\right)x^2-\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(b^3-4\,abc\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c^2-4\,ac^3\right)},\frac{2\left(b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b
*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 1.70457, size = 316, normalized size = 3.9

$$\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)+2b^2c\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{4c^{\cdot}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*
log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2) + sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (-a*b
- 8*a*c**2*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
 - b**2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

Giac [A] time = 1.18008, size = 101, normalized size = 1.25

$$\frac{x^2}{2\,c} - \frac{b\log\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/4*b*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.850 $\int \frac{x^3}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=63

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(4*c)
```

Rubi [A] time = 0.0549137, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(4*c)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}\\
&= \frac{\log\left(a+bx^2+cx^4\right)}{4c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^2+cx^4\right)}{4c}
\end{aligned}$$

Mathematica [A] time = 0.0232091, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^2+cx^4\right) - \frac{2b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^2 + c*x^4])/(4*c)
```

Maple [A] time = 0.162, size = 60, normalized size = 1.

$$\frac{\ln\left(cx^4+bx^2+a\right)}{4c}-\frac{b}{2c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54093, size = 443, normalized size = 7.03

$$\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)+\left(b^2-4ac\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c-4ac^2\right)},\frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{(2cx^2+b)\sqrt{}}{b^2-}\right.}{4\left(b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*x^2
```

```
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] time = 0.874535, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-b*
sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c +
b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/
(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

Giac [A] time = 1.15594, size = 80, normalized size = 1.27

$$-\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c}+\frac{\log\left(cx^4+bx^2+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
4*log(c*x^4 + b*x^2 + a)/c
```

3.851 $\int \frac{x}{a+bx^2+cx^4} dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.033514, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= -\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.0081687, size = 39, normalized size = 1.08

$$\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.162, size = 36, normalized size = 1.

$$\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44921, size = 290, normalized size = 8.06

$$\left[\frac{\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] time = 0.476949, size = 131, normalized size = 3.64

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2
```

Giac [A] time = 1.15805, size = 47, normalized size = 1.31

$$\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.852 $\int \frac{1}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rubi [A] time = 0.0699097, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$
$$= \frac{\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a} + \frac{\mathrm{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}$$
$$= \frac{\log(x)}{a} - \frac{\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}$$
$$= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a}$$

Mathematica [A] time = 0.0709346, size = 113, normalized size = 1.64

$$\frac{-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4\log(x)\sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.169, size = 66, normalized size = 1.

$$\frac{\ln(x)}{a} - \frac{\ln\left(cx^4+bx^2+a\right)}{4\,a} - \frac{b}{2\,a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a),x)
```

```
[Out] ln(x)/a-1/4*ln(c*x^4+b*x^2+a)/a-1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59068, size = 510, normalized size = 7.39

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^4+bx^2+a\right)+4\left(b^2-4\,ac\right)\log\left(x\right)}{4\left(ab^2-4\,a^2c\right)}, \frac{2\sqrt{-b^2+4\,ac}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] time = 3.19689, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) +
 (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(b*sqr
t(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) + l
og(x)/a
```

Giac [A] time = 1.16852, size = 92, normalized size = 1.33

$$-\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a
```

3.853 $\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=89

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a^2
)
```

Rubi [A] time = 0.130687, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a^2
)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2ax^2} + \frac{\operatorname{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2} + \frac{\operatorname{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{\operatorname{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}$$
$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} + \frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}$$
$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log\left(a+bx^2+cx^4\right)}{4a^2} - \frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^2}$$
$$= -\frac{1}{2ax^2} - \frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}$$

Mathematica [A] time = 0.124095, size = 135, normalized size = 1.52

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}} - \frac{2a}{x^2} - 4b\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A] time = 0.17, size = 119, normalized size = 1.3

$$-\frac{1}{2\,ax^2} - \frac{b\ln(x)}{a^2} + \frac{b\ln\left(cx^4+bx^2+a\right)}{4\,a^2} - \frac{c}{a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}} + \frac{b^2}{2\,a^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58336, size = 664, normalized size = 7.46

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(b^3-4\,abc\right)x^2\log\left(cx^4+bx^2+a\right)+4\left(b^3-4\,abc\right)x^2}{4\left(a^2b^2-4\,a^3c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [B] time = 7.79971, size = 345, normalized size = 3.88

$$\left(\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-8a^3c\left(\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)+2a^2b^2\left(\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)+3abc-b^3}{2ac^2-b^2c}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] (b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*l
og(x**2 + (-8*a**3*c*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a*
*2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
- b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) + (
b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*lo
g(x**2 + (-8*a**3*c*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**
2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c -
 b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) - 1/
(2*a*x**2) - b*log(x)/a**2
```

Giac [A] time = 1.17467, size = 127, normalized size = 1.43

$$\frac{b\log\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*x
^2 - a)/(a^2*x^2)
```

3.854 $\int \frac{1}{x^5(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=114

$$-\frac{(b^2-ac)\log(a+bx^2+cx^4)}{4a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2
 - a*c)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi [A] time = 0.196236, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-ac)\log(a+bx^2+cx^4)}{4a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2
 - a*c)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$
$$= -\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{\left(b^2-ac\right)\log(x)}{a^3}+\frac{\text{Subst}\left(\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^3}$$
$$= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^3}-\frac{\left(b^2-ac\right)\text{S}}{}$$
$$= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int\right.}{2}$$
$$= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx^2+\right.}{4a^3}$$

Mathematica [A] time = 0.222882, size = 188, normalized size = 1.65

$$\frac{-\frac{a^2}{x^4}-\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2\sqrt{b^2-4ac}+ac\sqrt{b^2-4ac}-3abc+b^3\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+4\log(x)\left(b^2-ac\right.}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(a^2/x^4) + (2*a*b)/x^2 + 4*(b^2 - a*c)*Log[x] - ((b^3 - 3*a*b*c + b^2*Sq
rt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/Sqrt[b^2 - 4*a*c] + ((b^3 - 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a
^3)
```

Maple [A] time = 0.171, size = 159, normalized size = 1.4

$$-\frac{1}{4\,ax^4}-\frac{c\ln(x)}{a^2}+\frac{b^2\ln(x)}{a^3}+\frac{b}{2\,a^2x^2}+\frac{c\ln\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{\ln\left(cx^4+bx^2+a\right)b^2}{4\,a^3}+\frac{3\,bc}{2\,a^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/a/x^4-c*ln(x)/a^2+b^2*ln(x)/a^3+1/2*b/a^2/x^2+1/4/a^2*c*ln(c*x^4+b*x^2
+a)-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2+3/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*b*c-1/2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7616, size = 819, normalized size = 7.18

$$\Bigg[-\frac{\left(b^3-3\,abc\right)\sqrt{b^2-4\,ac}x^4\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(b^4-5\,ab^2c+4\,a^2c^2\right)x^4\log\left(cx^4+bx^2+a\right)-4\,(b}{4\left(a^3b^2-4\,a^4c\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b
^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 -
 5*a*b^2*c + 4*a^2*c^2)*x^4*log(c*x^4 + b*x^2 + a) - 4*(b^4 - 5*a*b^2*c + 4
*a^2*c^2)*x^4*log(x) + a^2*b^2 - 4*a^3*c - 2*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3
*b^2 - 4*a^4*c)*x^4), 1/4*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^4*arctan(
-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^4 - 5*a*b^2*c + 4*a^2
*c^2)*x^4*log(c*x^4 + b*x^2 + a) + 4*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^4*log(
x) - a^2*b^2 + 4*a^3*c + 2*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2 - 4*a^4*c)*x^
4)]
```

Sympy [B] time = 9.74828, size = 423, normalized size = 3.71

$$\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4a^3\left(4ac-b^2\right)}+\frac{ac-b^2}{4a^3}\right)\log\left(x^2+\frac{8a^4c\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4a^3(4ac-b^2)}+\frac{ac-b^2}{4a^3}\right)-2a^3b^2\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4a^3(4ac-b^2)}+\frac{ac-b^2}{4a^3}\right)-2}{3abc^2-b^3c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b**
2)/(4*a**3))*log(x**2 + (8*a**4*c*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4
*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a**3)) - 2*a**3*b**2*(-b*sqrt(-4*a*
c + b**2)*(3*a*c - b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a**3)) -
 2*a**2*c**2 + 4*a*b**2*c - b**4)/(3*a*b*c**2 - b**3*c)) + (b*sqrt(-4*a*c +
 b**2)*(3*a*c - b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a**3))*log(
x**2 + (8*a**4*c*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*a**3*(4*a*c - b**
2)) + (a*c - b**2)/(4*a**3)) - 2*a**3*b**2*(b*sqrt(-4*a*c + b**2)*(3*a*c -
b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a**3)) - 2*a**2*c**2 + 4*a*
b**2*c - b**4)/(3*a*b*c**2 - b**3*c)) + (-a + 2*b*x**2)/(4*a**2*x**4) - (a*
c - b**2)*log(x)/a**3
```

Giac [A] time = 1.17367, size = 170, normalized size = 1.49

$$-\frac{\left(b^2-ac\right)\log\left(cx^4+bx^2+a\right)}{4\,a^3}+\frac{\left(b^2-ac\right)\log\left(x^2\right)}{2\,a^3}-\frac{\left(b^3-3\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^3}-\frac{3\,b^2x^4-3\,acx^4-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2 - a*c)*log(x^2)/a^3
- 1/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
 4*a*c)*a^3) - 1/4*(3*b^2*x^4 - 3*a*c*x^4 - 2*a*b*x^2 + a^2)/(a^3*x^4)
```

3.855 $\int \frac{x^6}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=203

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.670122, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1122, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{a+bx^2+cx^4}\,dx &= \frac{x^3}{3c}-\frac{\int \frac{x^2(3a+3bx^2)}{a+bx^2+cx^4}\,dx}{3c}\\
&= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\int \frac{3ab+3(b^2-ac)x^2}{a+bx^2+cx^4}\,dx}{3c^2}\\
&= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}\\
&= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.153356, size = 250, normalized size = 1.23

$$\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 3
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.187, size = 467, normalized size = 2.3

$$\frac{x^3}{3\,c}-\frac{bx}{c^2}+\frac{\sqrt{2}a}{2\,c}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{\sqrt{2}b^2}{2\,c^2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*x^3/c-b*x/c^2+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x
*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c^2*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a+1/2/c^2*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1
```

```
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^3 - 3\,bx}{3\,c^2} - \frac{-\int \frac{(b^2-ac)x^2+ab}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x^4 + b*x^2
 + a), x)/c^2
```

Fricas [B] time = 1.72572, size = 3217, normalized size = 15.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b
^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c
^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c
^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(
1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
```

```
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sq
rt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^
2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*
a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

Sympy [A] time = 2.45874, size = 194, normalized size = 0.96

$$-\frac{bx}{c^2} + \operatorname{RootSum}\left(t^4\left(256a^2c^7 - 128ab^2c^6 + 16b^4c^5\right) + t^2\left(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7\right) + a^5, \left(t \mapsto t\log\left(x + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
+ _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a**
5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8*_t
**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c -
 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

Giac [C] time = 2.61815, size = 5466, normalized size = 26.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 2*((a*c^3)^(1/4)*a*b*c^4*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*
c^3)^(1/4)*a*b*c^4*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3
)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
) - ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*c
os(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)
*a*c^3)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*arctan(-((a/c)^(1
/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*
sin(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*b
*c^5*abs(c) - (b^2 - 4*a*c)*c^6) + 2*((a*c^3)^(1/4)*a*b*c^4*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*c^4*sin(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/
4)*a*c^3)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/
4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2
```

*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*b*c^5*abs(c) - (b^2 - 4*a*c)*c^6) - ((a*c^3)^(1/4)*a*b*c^4*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*c^4*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))

```
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*log(-2*x*(a/c)^(1/4)*cos
(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(
b^2 - 4*a*c)*b*c^5*abs(c) - (b^2 - 4*a*c)*c^6) - ((a*c^3)^(1/4)*a*b*c^4*cos
(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*c^4*cos(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4
)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*
cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4
)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*
((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
 + 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(
(a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
- 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*
c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3 + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3)*log(-2*x*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*b*c^5*ab
s(c) - (b^2 - 4*a*c)*c^6) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

3.856 $\int \frac{x^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.268775, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4),x]
```


```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+bx^2+cx^4}\,dx &= \frac{x}{c}-\frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c}\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.108353, size = 202, normalized size = 1.13

$$-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
```

```
[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.186, size = 343, normalized size = 1.9

$$\frac{x}{c}+\frac{\sqrt{2}b}{2c}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\sqrt{2}a\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] x/c+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*a-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-1/2/c*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [B] time = 1.61186, size = 2168, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A] time = 1.7736, size = 129, normalized size = 0.72

$$\mathrm{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8t^3b^3c^3-4\ldots}{a^2c-\ldots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

Giac [C] time = 2.389, size = 4593, normalized size = 25.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -2*(3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(3/4)
*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*b*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^
3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2 - 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3
/4)*b*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*c^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*c^2*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1
/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*b*c^2*abs(c) - (b^2
*c - 4*a*c^2)*c^2) - 2*(3*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
)))) - (a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9
*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*p
```

```
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))) + 9*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3 + (a*c^3)^(3/4)*b*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
+ (a*c^3)^(1/4)*a*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3
)^(1/4)*a*c^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^
(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4
)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)
*b*c^2*abs(c) - (b^2*c - 4*a*c^2)*c^2) + ((a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3
)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2 - 9*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (a*c^3)^(3/4)*b*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
```

```
bs(c))))) - (a*c^3)^(1/4)*a*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*b*c^2*abs(c) - (b^2*c - 4*a*c^2)*c^
2) + ((a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*
(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*
c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*(a*c^3)^(3/4)*b*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^2 - (a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 +
 3*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*
c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*c^2*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi +
 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a
*c)*b*c^2*abs(c) - (b^2*c - 4*a*c^2)*c^2) + x/c
```

3.857 $\int \frac{x^2}{a+bx^2+cx^4} dx$

Optimal. Leaf size=150

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.10949, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+
$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.0833702, size = 165, normalized size = 1.1

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.179, size = 208, normalized size = 1.4

$$-\frac{\sqrt{2}}{2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{\sqrt{2}b}{2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1
/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 1.50843, size = 1206, normalized size = 8.04

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{-\cdots}}{\sqrt{b^2c^2-4\,\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/
2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqr
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2
*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4
*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(
b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*s
qrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a
```

```
*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A] time = 0.849726, size = 75, normalized size = 0.5

$$\operatorname{RootSum}\left(t^4\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(-16abc+4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c-2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a*
b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 2*
_t*b + x)))
```

Giac [C] time = 2.30385, size = 5350, normalized size = 35.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(a
*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (
a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2
 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
```

```
abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)
^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2 - 4*(a
*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^3)*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) + 1/2*(3*((a*c^3)^(3/4)*b^
2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3
)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
 (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c
^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*co
s(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/
4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^
2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3
)^(3/4)*sqrt(b^2 - 4*a*c)*b)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^3)*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
```

```
/(a*b^2*c^3 - 4*a^2*c^4) - 1/4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
(a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(
b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^
2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4
)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3
)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*
cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
(a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(
b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*log(-2*x*(a/c)^(1/4)
*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a
*b^2*c^3 - 4*a^2*c^4) - 1/4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*
c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2
 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*s
qrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(
```

```
3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*
a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cos
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*
c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2
 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*log(-2*x*(a/c)^(1/4)*co
s(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^
2*c^3 - 4*a^2*c^4)
```

3.858 $\int \frac{1}{a+bx^2+cx^4} dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.0862244, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1093, 205}

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.0755575, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A] time = 0.177, size = 116, normalized size = 0.8

$$-c\sqrt{2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}} - c\sqrt{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 1.53411, size = 1323, normalized size = 8.82

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx+\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
```

```
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A] time = 0.972327, size = 87, normalized size = 0.58

$$\text{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Giac [C] time = 1.38361, size = 1365, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 -
 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*s
qrt(a*c)*b/(a*abs(c))))))/(a*b^2*c - 4*a^2*c^2) + 1/2*(((a*c^3)^(1/4)*b^2 -
 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a
*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
)))*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))
```

```
/(a*b^2*c - 4*a^2*c^2) - 1/4*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a
*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
)) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*
cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*
b^2*c - 4*a^2*c^2) - 1/4*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3
)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) -
 ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)
*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(
1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*
c - 4*a^2*c^2)
```

3.859 $\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=174

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.221775, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx &= -\frac{1}{ax}+\frac{\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx}{a}\\
&= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}\\
&= -\frac{1}{ax}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] time = 0.384809, size = 191, normalized size = 1.1

$$
-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{x}}{2a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]
```

```
]))/(2*a)
```

Maple [A] time = 0.192, size = 232, normalized size = 1.3

$$\frac{c\sqrt{2}}{2a}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{c\sqrt{2}b}{2a}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b-1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b-1/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

Fricas [B] time = 1.66304, size = 2279, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2
*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a
^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))
)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 -
 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5
 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^
3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
- 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*s
qrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqr
t(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*
c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x)
```

Sympy [A] time = 1.96385, size = 148, normalized size = 0.85

$$\text{RootSum}\left(t^4\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2+48t^3a^4b^2}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t**
3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c**2
+ 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

Giac [C] time = 2.42736, size = 4581, normalized size = 26.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -2*(3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(3/4)
*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*a*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^
3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2 - 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3
/4)*a*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*b*
c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*c*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1
/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a) - (b^2
*c - 4*a*c^2)*a^2) - 2*(3*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
)))) - (a*c^3)^(3/4)*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9
*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
```

```
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*a*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))) + 9*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3 + (a*c^3)^(3/4)*a*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
+ (a*c^3)^(1/4)*a*b*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3
)^(1/4)*a*b*c*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^
(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4
)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)
*a*b*c*abs(a) - (b^2*c - 4*a*c^2)*a^2) + ((a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3
)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2 - 9*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (a*c^3)^(3/4)*a*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*b*c*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) - (a*c^3)^(1/4)*a*b*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
```

```
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a) - (b^2*c - 4*a*c^2)*a^
2) + ((a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*
(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*
c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*(a*c^3)^(3/4)*a*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^2 - (a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 +
 3*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*
b*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*c*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi +
 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a
*c)*a*b*c*abs(a) - (b^2*c - 4*a*c^2)*a^2) - 1/(a*x)
```

3.860 $\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=196

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (Sqrt[c]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.422824, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1123, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (Sqrt[c]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^2+cx^4\right)}\,dx &= -\frac{1}{3ax^3}+\frac{\int \frac{-3b-3cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{3a}\\
&= -\frac{1}{3ax^3}+\frac{b}{a^2x}-\frac{\int \frac{-3\left(b^2-ac\right)-3bcx^2}{a+bx^2+cx^4}\,dx}{3a^2}\\
&= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2}+\frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2}\\
&= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.136853, size = 216, normalized size = 1.1

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{x^3}+\frac{6b}{x}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^3 + (6*b)/x + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-b^2 + 2*a*c + b*Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

Maple [B] time = 0.191, size = 368, normalized size = 1.9

$$-\frac{1}{3\,ax^3}+\frac{b}{a^2x}-\frac{c\sqrt{2}b}{2\,a^2}\text{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{c^2\sqrt{2}}{a}\text{Artanh}\left(cx\sqrt{2}\frac{}{\sqrt{(-b+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3/a/x^3+b/a^2/x-1/2/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/a*c^2/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b^2+1/2/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((b*c*x^2 + b^2 - a*c)/(c*x^4 + b*x^2 + a), x)/a^2 + 1/3*(3*b*x^2
- a)/(a^2*x^3)
```

Fricas [B] time = 1.70473, size = 3343, normalized size = 17.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 -
 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c^4
+ a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3
 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*sqrt(-
(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^
2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 +
 (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3
*a*b^2*c^4 + a^2*c^5)*x - sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*
a^3*b^2*c^3 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*
c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c
)))/(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(
b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 + (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^
2 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*
```

```
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2
 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*
c))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x - sqrt(1/2)*(b^8 - 8*a*b^6*c
+ 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 + (a^5*b^5 - 7*a^6*b^3*c + 12
*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2
- 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*b*x^2 + 2*a)/(a^2*x^3)
```

Sympy [A] time = 2.72619, size = 211, normalized size = 1.08

$$\operatorname{RootSum}\left(t^4\left(256a^7c^2-128a^6b^2c+16a^5b^4\right)+t^2\left(-80a^3bc^3+100a^2b^3c^2-36ab^5c+4b^7\right)+c^5,\left(t\mapsto t\log\left(x+\frac{-96t^3a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**7*c**2 - 128*a**6*b**2*c + 16*a**5*b**4) + _t**2*(-80
*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + c**5, Lambda(_t
, _t*log(x + (-96*_t**3*a**7*b*c**2 + 56*_t**3*a**6*b**3*c - 8*_t**3*a**5*b
**5 - 4*_t*a**4*c**4 + 32*_t*a**3*b**2*c**3 - 40*_t*a**2*b**4*c**2 + 16*_t*
a*b**6*c - 2*_t*b**8)/(a**2*c**5 - 3*a*b**2*c**4 + b**4*c**3)))) + (-a + 3*
b*x**2)/(3*a**2*x**3)
```

Giac [C] time = 2.42314, size = 6892, normalized size = 35.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2
- 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^3 -
4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cosh(1/2*imag_
```

```
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)
*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^
3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b
*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(
3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c +
 (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
 + ((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*
a*c)*b^2)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)
*b^4*c - 5*(a*c^3)^(1/4)*a*b^2*c^2 + 4*(a*c^3)^(1/4)*a^2*c^3 + ((a*c^3)^(1/
4)*b^3*c - (a*c^3)^(1/4)*a*b*c^2)*sqrt(b^2 - 4*a*c))*cosh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^4*c - 5*(a*c^3)^(1/4)*a*b^2*c^2 +
 4*(a*c^3)^(1/4)*a^2*c^3 + ((a*c^3)^(1/4)*b^3*c - (a*c^3)^(1/4)*a*b*c^2)*sq
rt(b^2 - 4*a*c))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/
c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(
1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a^3*b^2*c^2 -
4*a^4*c^3) + 1/2*(3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3
/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)
^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^3 - 4*
(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(1/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
)) + 3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2
```

- 4*a*c)*b^2)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^4*c - 5*(a*c^3)^(1/4)*a*b^2*c^2 + 4*(a*c^3)^(1/4)*a^2*c^3 + ((a*c^3)^(1/4)*b^3*c - (a*c^3)^(1/4)*a*b*c^2)*sqrt(b^2 - 4*a*c))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^4*c - 5*(a*c^3)^(1/4)*a*b^2*c^2 + 4*(a*c^3)^(1/4)*a^2*c^3 + ((a*c^3)^(1/4)*b^3*c - (a*c^3)^(1/4)*a*b*c^2)*sqrt(b^2 - 4*a*c))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a^3*b^2*c^2 - 4*a^4*c^3) - 1/4*(((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*

c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^3 -
 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4
)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(5/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^3 + ((a*c^3)^(1/4)*b^4*c - 5*(a*c^3)^(1/4)*a*b^2*c^2 + 4*(a*c^3)^(1
/4)*a^2*c^3 + ((a*c^3)^(1/4)*b^3*c - (a*c^3)^(1/4)*a*b*c^2)*sqrt(b^2 - 4*a*
c))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^4*c - 5
*(a*c^3)^(1/4)*a*b^2*c^2 + 4*(a*c^3)^(1/4)*a^2*c^3 + ((a*c^3)^(1/4)*b^3*c -
 (a*c^3)^(1/4)*a*b*c^2)*sqrt(b^2 - 4*a*c))*cos(5/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))) + x^2 + sqrt(a/c))/(a^3*b^2*c^2 - 4*a^4*c^3) - 1/4*(((a*c^3)^(
3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos
(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)*b^3 - 4*(a
*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(1/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3
)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(
3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos
(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3
/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)
b^3 - 4(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b^2)*cos(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2 - ((a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sq
rt(b^2 - 4*a*c)*b^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(
(a*c^3)^(3/4)*b^3 - 4*(a*c^3)^(3/4)*a*b*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)

```
*b^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^4*c - 5*(a*c
^3)^(1/4)*a*b^2*c^2 + 4*(a*c^3)^(1/4)*a^2*c^3 + ((a*c^3)^(1/4)*b^3*c - (a*c
^3)^(1/4)*a*b*c^2)*sqrt(b^2 - 4*a*c))*cos(1/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c))))) - ((a*c^3)^(1/4)*b^4*c - 5*(a*c^3)^(1/4)*a*b^2*c^2 + 4*(a*c^3)^(1/4
)*a^2*c^3 + ((a*c^3)^(1/4)*b^3*c - (a*c^3)^(1/4)*a*b*c^2)*sqrt(b^2 - 4*a*c)
)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/
4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a^3*b^2*
c^2 - 4*a^4*c^3) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

3.861 $\int \frac{x^7}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=132

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 + c*x^4]/(4*c^2)
```

Rubi [A] time = 0.168111, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 738, 773, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 + c*x^4]/(4*c^2)
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
```

```
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$
$$= \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx,x,x^2\right)}{2\left(b^2-4ac\right)}$$
$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-ab+\left(-b^2+4ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c\left(b^2-4ac\right)}$$
$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2} - \frac{\left(b\left(b^2-6ac\right)\right)\text{S}}{4}$$
$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\int\right.}{2c^2\left(b\right.}$$
$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}} + \frac{\log\left(a+bx^2\right.}{4c^2}$$

Mathematica [A] time = 0.182405, size = 121, normalized size = 0.92

$$\frac{\frac{2\left(-2a^2c+ab\left(b-3cx^2\right)+b^3x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2
 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] time = 0.18, size = 222, normalized size = 1.7

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(\frac{b\left(3\,ac-b^2\right)x^2}{c^2\left(4\,ac-b^2\right)} + \frac{a\left(2\,ac-b^2\right)}{c^2\left(4\,ac-b^2\right)}\right) + \frac{\ln\left(cx^4+bx^2+a\right)a}{c\left(4\,ac-b^2\right)} - \frac{\ln\left(cx^4+bx^2+a\right)b^2}{4\,c^2\left(4\,ac-b^2\right)} - 3\,\frac{ab}{c\left(4\,ac-b^2\right)^{3/2}}\,\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^
4+b*x^2+a)+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a-1/4/c^2/(4*a*c-b^2)*ln(c*x^4
+b*x^2+a)*b^2-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a
*b+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59107, size = 1412, normalized size = 10.7

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\left(b^5-7\,ab^3c+12\,a^2bc^2\right)x^2+\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\sqrt{b^2-4\,}}{4\left(ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c
^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*
x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2
+ b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a
^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3
*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (
```

```
b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 +
 b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

Sympy [B] time = 3.86407, size = 745, normalized size = 5.64

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)\log\left(x^2+\frac{-32a^2c^3\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)+8a^2c+16ab^2c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(
-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c
**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(b*sqrt(-(4*a*c
 - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(b*sqrt(-(4*a*c
 - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x**2*(3*a
*b*c - b**3))/(8*a**2*c**3 - 2*a*b**2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3)
+ x**2*(8*a*b*c**3 - 2*b**3*c**2))
```

Giac [A] time = 23.448, size = 205, normalized size = 1.55

$$-\frac{\left(b^{3}-6\,abc\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(b^{2}c^{2}-4\,ac^{3}\right)\sqrt{-b^{2}+4\,ac}}-\frac{b^{2}cx^{4}-4\,ac^{2}x^{4}-b^{3}x^{2}+2\,abcx^{2}-ab^{2}}{4\left(cx^{4}+bx^{2}+a\right)\left(b^{2}c^{2}-4\,ac^{3}\right)}+\frac{\log\left(cx^{4}+bx^{2}+a\right)}{4\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4
*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*a*
b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x^4
+ b*x^2 + a)/c^2
```

3.862 $\int \frac{x^5}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=78

$$\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0656601, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 722, 618, 206}

$$\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
```

```
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{a\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac}\\
&= \frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\
&= \frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0894118, size = 93, normalized size = 1.19

$$\frac{a\left(b-2cx^2\right)+b^2x^2}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.174, size = 104, normalized size = 1.3

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(-\frac{\left(2\,ac-b^2\right)x^2}{c\left(4\,ac-b^2\right)}+\frac{ab}{c\left(4\,ac-b^2\right)}\right)+2\,\frac{a}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2*a/
(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.76937, size = 864, normalized size = 11.08

$$\left[-\frac{ab^3-4\,a^2bc+\left(b^4-6\,ab^2c+8\,a^2c^2\right)x^2+2\left(ac^2x^4+abcx^2+a^2c\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\left(ab^4c-8\,a^2b^2c^2+16\,a^3c^3+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^4+\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^2\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^4
 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c - 8
*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a
*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*c - 8*a
^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

Sympy [B] time = 1.98307, size = 282, normalized size = 3.62

$$-a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+ab}{2ac}\right)+a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*c
- b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a
**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)*
*3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) - (-a*b + x**2*(2*a
*c - b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**
2*(8*a*b*c**2 - 2*b**3*c))
```

Giac [A] time = 20.65, size = 130, normalized size = 1.67

$$-\frac{2\,a\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{b^2x^2-2\,acx^2+ab}{2\left(cx^4+bx^2+a\right)\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c
^2))
```

3.863 $\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=75

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0612293, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 638, 618, 206}

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}\\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0638087, size = 79, normalized size = 1.05

$$
\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x^
2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.171, size = 77, normalized size = 1.

$$\frac{-bx^2-2a}{(8ac-2b^2)(cx^4+bx^2+a)} - b\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)(4ac-b^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.74573, size = 778, normalized size = 10.37

$$\left[\frac{2ab^2-8a^2c+(b^3-4abc)x^2-(bcx^4+b^2x^2+ab)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{2\left(ab^4-8a^2b^2c+16a^3c^2+(b^4c-8ab^2c^2+16a^2c^3)x^4+(b^5-8ab^3c+16a^2bc^2)x^2\right)}, \frac{2ab^2-8a^2c+(}{2(ab^4-8a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*
x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*x^2 +
 a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
```

```
a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] time = 1.74307, size = 267, normalized size = 3.56

$$\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)}{2}-\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^3c\cdots}{\cdots}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b*
*2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a*
*2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3
) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 - (2*a + b*x**2)/(8*
a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

Giac [A] time = 21.0379, size = 111, normalized size = 1.48

$$\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{bx^2+2\,a}{2\left(cx^4+bx^2+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

3.864 $\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0579318, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1107, 614, 618, 206}

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)\\
&= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{c\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac}\\
&= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(2c)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\
&= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0780499, size = 79, normalized size = 1.07

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2cx^2}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c))
```

Maple [A] time = 0.172, size = 75, normalized size = 1.

$$\frac{2\,cx^2+b}{\left(8\,ac-2\,b^2\right)\left(cx^4+bx^2+a\right)}+2\,\frac{c}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82307, size = 783, normalized size = 10.58

$$\left[-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2+2\left(c^2x^4+bcx^2+ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\left(ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^4+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^2\right)},-\frac{b^3-4\,abc+}{2\left(ab^4-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*c
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 + b*c*x
```

```
^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] time = 1.74412, size = 267, normalized size = 3.61

$$-c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc}{2c^2}\right)+c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(-1/(4*a*c
- b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*
a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x**2)/
(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)
)
```

Giac [A] time = 20.6235, size = 111, normalized size = 1.5

$$-\frac{2\,c\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,cx^2+b}{2\left(cx^4+bx^2+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

3.865 $\int \frac{1}{x(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=122

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}-\frac{\log(a+bx^2+cx^4)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)
```

Rubi [A] time = 0.197928, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 740, 800, 634, 618, 206, 628}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}-\frac{\log(a+bx^2+cx^4)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
```

```
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\operatorname{Subst}\left(\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\operatorname{Subst}\left(\int \left(\frac{-b^2+4ac}{ax} + \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right) dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\operatorname{Subst}\left(\int \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} - \frac{\left(b\left(b^2-6ac\right)\right)\operatorname{Subs}}{4a^2\left(\right.}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(b\left(b^2-6ac\right)\right)\operatorname{Subst}\left(\int \frac{}{b^2-}\right.}{2a^2\left(b^2-\right.}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2+cx\right.}{4a^2}$$

Mathematica [A] time = 0.311731, size = 207, normalized size = 1.7

$$\frac{\frac{2a\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} +}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log[
x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log
[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c -
b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [B] time = 0.184, size = 253, normalized size = 2.1

$$\frac{\ln(x)}{a^2} - \frac{bcx^2}{2a\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)} + \frac{c}{\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)} - \frac{b^2}{2a\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)} - \frac{c\ln\left(cx^4+bx^2+\right.}{a\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] ln(x)/a^2-1/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x^2+1/(c*x^4+b*x^2+a)/(4*a*
c-b^2)*c-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2-1/a/(4*a*c-b^2)*c*ln(c*x^4+b
*x^2+a)+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2-3/a/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.47028, size = 1728, normalized size = 14.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 +
 ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (
```

```
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^
2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^
4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a*b^4 - 12
*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b
*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*
a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
]
```

Sympy [B] time = 45.2521, size = 772, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2))*log(x**2 + (-32*a**4*c**2*(
-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) + 16*a**3*b**2*c*(-b*sqrt(-(
4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 2*a**2*b**4*(-b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) - 1/(4*a**2)) - 8*a**2*c**2 + 7*a*b**2*c - b**4)/(6*a*b*c**2 - b*
*3*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 -
 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2))*log(x**2 + (-32*a**
4*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) + 16*a**3*b**2*c*(b*s
qrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 2*a**2*b**4*(b*sqrt(-(4*a*c -
b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - 1/(4*a**2)) - 8*a**2*c**2 + 7*a*b**2*c - b**4)/(6*a*b*c**2
 - b**3*c)) - (-2*a*c + b**2 + b*c*x**2)/(8*a**3*c - 2*a**2*b**2 + x**4*(8*
a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + log(x)/a**2
```

Giac [A] time = 21.4641, size = 224, normalized size = 1.84

$$-\frac{\left(b^{3}-6\,abc\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(a^{2}b^{2}-4\,a^{3}c\right)\sqrt{-b^{2}+4\,ac}}+\frac{b^{2}cx^{4}-4\,ac^{2}x^{4}+b^{3}x^{2}-2\,abcx^{2}+3\,ab^{2}-8\,a^{2}c}{4\,\left(cx^{4}+bx^{2}+a\right)\left(a^{2}b^{2}-4\,a^{3}c\right)}-\frac{\log\left(cx^{4}+bx^{2}+a\right)}{4\,a^{2}}+\frac{\log\left(x^{2}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4
*a^3*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*a*
b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) - 1/
4*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

3.866 $\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=162

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}+\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+bc}{2ax^2\left(b^2-4ac\right)\left(a+b\right.}$$

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^
2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x]
)/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)
```

Rubi [A] time = 0.25055, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}+\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+bc}{2ax^2\left(b^2-4ac\right)\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^
2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x]
)/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
```

```
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2} - \frac{2b\left(-b^2+4ac\right)}{a^2x} + \frac{2\left(-b^4+5ab^2c-3a^2c^2-bc(b\right.}{a^2\left(a+bx+cx^2\right)}\right.\right.}{2a\left(b^2-4ac\right)}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} - \frac{\text{Subst}\left(\int \frac{-b^4+5ab^2c-3a^2c^2}{a+bx+}\right.}{a^3\left(b^2-\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,\right.}{2a^3}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx^2+cx^4\right)}{2a^3} -$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4}}\right.}{a^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.26558, size = 248, normalized size = 1.53

$$\frac{\frac{\left(6a^2c^2+b^3\sqrt{b^2-4ac}-6ab^2c-4abc\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-6a^2c^2+b^3\sqrt{b^2-4ac}+6ab^2c-4abc\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} - \frac{a(-3ab}{(b^2-}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 -
4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
b^2 - 4*a*c)^(3/2))/(2*a^3)
```

Maple [B] time = 0.184, size = 352, normalized size = 2.2

$$-\frac{1}{2\,a^{2}x^{2}}-2\,\frac{b\ln(x)}{a^{3}}-\frac{c^{2}x^{2}}{a\left(cx^{4}+bx^{2}+a\right)\left(4\,ac-b^{2}\right)}+\frac{b^{2}cx^{2}}{2\,a^{2}\left(cx^{4}+bx^{2}+a\right)\left(4\,ac-b^{2}\right)}-\frac{3\,bc}{2\,a\left(cx^{4}+bx^{2}+a\right)\left(4\,ac-b^{2}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2/x^2-2*b*ln(x)/a^3-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2+1/2/a^2/
(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*c
+1/2/a^2/(c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)+2/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2
+a)*b-1/2/a^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3-6/a/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2+6/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*c*b^2-1/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.58553, size = 2103, normalized size = 12.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12
*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6*a
```

```
*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 -
6*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)
*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4
*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 +
 (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^
2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*
b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2
+ a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*
b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b
*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [B] time = 78.9888, size = 906, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (b/(2*a**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2
*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x**2 +
(-16*a**5*c**2*(b/(2*a**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b*
*2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
))) + 8*a**4*b**2*c*(b/(2*a**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6
*a*b**2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6))) - a**3*b**4*(b/(2*a**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
6*a*b**2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6))) + 11*a**2*b*c**2 - 7*a*b**3*c + b**5)/(6*a**2*c**3 - 6*a*b**2*c**
2 + b**4*c)) + (b/(2*a**3) + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b*
*2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
)))*log(x**2 + (-16*a**5*c**2*(b/(2*a**3) + sqrt(-(4*a*c - b**2)**3)*(6*a**
2*c**2 - 6*a*b**2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6))) + 8*a**4*b**2*c*(b/(2*a**3) + sqrt(-(4*a*c - b**2)**3)*(
```

```
6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6))) - a**3*b**4*(b/(2*a**3) + sqrt(-(4*a*c - b**2)**3)*
(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6))) + 11*a**2*b*c**2 - 7*a*b**3*c + b**5)/(6*a**2*c**3
 - 6*a*b**2*c**2 + b**4*c)) - (4*a**2*c - a*b**2 + x**4*(6*a*c**2 - 2*b**2*
c) + x**2*(7*a*b*c - 2*b**3))/(x**6*(8*a**3*c**2 - 2*a**2*b**2*c) + x**4*(8
*a**3*b*c - 2*a**2*b**3) + x**2*(8*a**4*c - 2*a**3*b**2)) - 2*b*log(x)/a**3
```

Giac [A] time = 20.868, size = 246, normalized size = 1.52

$$\frac{\left(b^4 - 6\,ab^2c + 6\,a^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^2 - 4\,a^4c\right)\sqrt{-b^2+4\,ac}} - \frac{2\,b^2cx^4 - 6\,ac^2x^4 + 2\,b^3x^2 - 7\,abcx^2 + ab^2 - 4\,a^2c}{2\left(cx^6 + bx^4 + ax^2\right)\left(a^2b^2 - 4\,a^3c\right)} + \frac{b\log\left(cx^4 + bx^2 + a\right)}{2\,a^3} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2*b
^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 -
 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3
```

3.867 $\int \frac{x^8}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=331

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(3b^2\right.}{2c^2\left(b\right.}$$

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) +
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*
b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.844144, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1279, 1166, 205}

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(3b^2\right.}{2c^2\left(b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2 + c*x^4)^2,x]
```



```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) +
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*
b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^4\left(10a+3bx^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{x^2\left(9ab+3\left(3b^2-10ac\right)x^2\right)}{a+bx^2+cx^4}\,dx}{6c\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{3a\left(3b^2-10ac\right)+3b\left(3b^2-13ac\right)x^2}{a+bx^2+cx^4}\,dx}{6c^2\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+20a^2\ldots}{\sqrt{b^2-4ac}}\right)\ldots}{4c^2\left(b^2-\ldots\right.}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+20a^2\ldots}{\sqrt{b^2-4ac}}\right)\ldots}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\ldots}$$

Mathematica [A] time = 0.685977, size = 327, normalized size = 0.99

$$\frac{-\frac{\sqrt{2}\left(-20a^2c^2+3b^3\sqrt{b^2-4ac}+19ab^2c-13abc\sqrt{b^2-4ac}-3b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(20a^2c^2+3b^3\sqrt{b^2-4ac}-19ab^2c-13abc\sqrt{b^2-4ac}+3b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\ldots}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^2
+ 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[b^2 -
 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c
^(5/2))
```

Maple [B] time = 0.205, size = 844, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x/c^2+3/2/c/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^3*a-1/2/c^2/(c*x^4+b*x^2+a)*b^3
/(4*a*c-b^2)*x^3+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x-1/2/c^2/(c*x^4+b*x^2
+a)*a/(4*a*c-b^2)*x*b^2+13/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b-3/4/c^
2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*a^2-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*a*b^2+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^4-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+3/4/c^2/(4*a*c-b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2-19/4
/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2+3/4/c^2/(4*a*c-b
^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^3-3\,abc\right)x^3+\left(ab^2-2\,a^2c\right)x}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^4+\left(b^3c^2-4\,abc^3\right)x^2\right)}+\frac{-\int\frac{3\,ab^2-10\,a^2c+\left(3\,b^3-13\,abc\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(b^2c^2-4\,ac^3\right)}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b
^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*b^
2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4
```

```
*a*c^3) + x/c^2
```

Fricas [B] time = 2.44139, size = 6311, normalized size = 19.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/2)*(a*b^2*
c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt
(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*
b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^
2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64
*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5
*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^
3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 +
336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^
6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385
*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b
^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*s
qrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051
*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*
a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818
*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6
 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c +
385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^1
3)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(
a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2
```

```
)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 -
 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 25
00*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8
818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b
^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2
)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*
x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 -
 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2
- 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 -
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10
 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^
5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(3*
a*b^2 - 10*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^
3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [A] time = 7.15399, size = 450, normalized size = 1.36

$$\frac{x^3\left(3abc-b^3\right)+x\left(2a^2c-ab^2\right)}{8a^2c^3-2ab^2c^2+x^4\left(8ac^4-2b^2c^3\right)+x^2\left(8abc^3-2b^3c^2\right)}+\text{RootSum}\left(t^4\left(1048576a^6c^{11}-1572864a^5b^2c^{10}+983040a^4b^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (x**3*(3*a*b*c - b**3) + x*(2*a**2*c - a*b**2))/(8*a**2*c**3 - 2*a*b**2*c**
```

```
2 + x**4*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b**3*c**2)) + Root
Sum(_t**4*(1048576*a**6*c**11 - 1572864*a**5*b**2*c**10 + 983040*a**4*b**4*
c**9 - 327680*a**3*b**6*c**8 + 61440*a**2*b**8*c**7 - 6144*a*b**10*c**6 + 2
56*b**12*c**5) + _t**2*(430080*a**6*b*c**6 - 716800*a**5*b**3*c**5 + 483840
*a**4*b**5*c**4 - 170496*a**3*b**7*c**3 + 33232*a**2*b**9*c**2 - 3408*a*b**
11*c + 144*b**13) + 10000*a**7*c**2 - 4200*a**6*b**2*c + 441*a**5*b**4, Lam
bda(_t, _t*log(x + (65536*_t**3*a**4*b*c**9 - 61440*_t**3*a**3*b**3*c**8 +
21504*_t**3*a**2*b**5*c**7 - 3328*_t**3*a*b**7*c**6 + 192*_t**3*b**9*c**5 -
 8000*_t*a**5*c**5 + 36160*_t*a**4*b**2*c**4 - 32476*_t*a**3*b**4*c**3 + 11
592*_t*a**2*b**6*c**2 - 1836*_t*a*b**8*c + 108*_t*b**10)/(2500*a**5*c**3 -
5625*a**4*b**2*c**2 + 1971*a**3*b**4*c - 189*a**2*b**6)))) + x/c**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.868 $\int \frac{x^6}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=271

$$\frac{\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-$$

```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.571902, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1279, 1166, 205}

$$\frac{\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
```

```
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^2\left(6a+bx^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{ab+\left(b^2-6ac\right)x^2}{a+bx^2+cx^4}\,dx}{2c\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c\left(b^2-4ac\right)} + \ldots$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \ldots$$

Mathematica [A] time = 0.539951, size = 282, normalized size = 1.04

$$\frac{-\frac{2\sqrt{c}x\left(a\left(b-2cx^2\right)+b^2x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+8abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}-8abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
(4*c^(3/2))
```

Maple [B] time = 0.201, size = 602, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2*a*b/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a
)-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b^2+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b-1/4/(
4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3+3/2/(4*a*c-b^2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*a-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+2/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^{2}-2\,ac\right)x^{3}+abx}{2\left(\left(b^{2}c^{2}-4\,ac^{3}\right)x^{4}+ab^{2}c-4\,a^{2}c^{2}+\left(b^{3}c-4\,abc^{2}\right)x^{2}\right)}-\frac{-\int\frac{\left(b^{2}-6\,ac\right)x^{2}+ab}{cx^{4}+bx^{2}+a}\,dx}{2\left(b^{2}c-4\,ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2
*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2 - 6*a*c)*x^2 + a*b)/
(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [B] time = 1.89699, size = 4806, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

[Out] -1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x - 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x - 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*

```
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6))))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a
^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

Sympy [A] time = 4.92884, size = 379, normalized size = 1.4

$$-\frac{-abx + x^3\left(2ac - b^2\right)}{8a^2c^2 - 2ab^2c + x^4\left(8ac^3 - 2b^2c^2\right) + x^2\left(8abc^2 - 2b^3c\right)} + \text{RootSum}\left(t^4\left(1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a^4b^4c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(-a*b*x + x**3*(2*a*c - b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3
- 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + RootSum(_t**4*(1048576*a**
6*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4*b**4*c**7 - 327680*a**3*b**6*
c**6 + 61440*a**2*b**8*c**5 - 6144*a*b**10*c**4 + 256*b**12*c**3) + _t**2*(
-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**
2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**5*c**2 - 360*a**4*b**2*c +
 25*a**3*b**4, Lambda(_t, _t*log(x + (49152*_t**3*a**4*c**7 - 40960*_t**3*a
**3*b**2*c**6 + 12288*_t**3*a**2*b**4*c**5 - 1536*_t**3*a*b**6*c**4 + 64*_t
**3*b**8*c**3 - 1728*_t*a**3*b*c**3 + 656*_t*a**2*b**3*c**2 - 88*_t*a*b**5*
c + 4*_t*b**7)/(324*a**3*c**2 - 81*a**2*b**2*c + 5*a*b**4))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.869 $\int \frac{x^4}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=237

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 +
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rubi [A] time = 0.410998, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1120, 1166, 205}

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 +
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
```

```
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\left(b^2-4ac\right)^{3/2}} + \frac{\left(b^2+4ac+b\sqrt{}\right.}{}$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2+4ac+b\sqrt{}\right.}{2\sqrt{2}\sqrt{c}\left(b^2\right.}$$

Mathematica [A] time = 0.423217, size = 235, normalized size = 0.99

$$\frac{1}{4}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}\right.}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/4
```

Maple [B] time = 0.194, size = 452, normalized size = 1.9

$$\frac{1}{cx^4+bx^2+a}\left(-\frac{bx^3}{8\,ac-2\,b^2}-\frac{ax}{4\,ac-b^2}\right)+\frac{\sqrt{2}b}{16\,ac-4\,b^2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b-c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-1/4/(4*a*c
-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*b-c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c
)
```

Fricas [B] time = 1.67601, size = 3584, normalized size = 15.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*
x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b
*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b
^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*
c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2
+ 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^
2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a
```

```
*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3
+ 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [A] time = 3.57616, size = 294, normalized size = 1.24

$$-\frac{2ax+bx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}+\operatorname{RootSum}\left(t^4\left(1048576a^6c^7-1572864a^5b^2c^6+983040a^4b^4c^5-32\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(2*a*x + b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*
(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**2*
c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c**3
 - 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3
*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a
**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2*c**2
 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.870 $\int \frac{x^2}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=221

$$-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(2*b -
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
+ Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.259512, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1119, 1166, 205}

$$-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(2*b -
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
+ Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
```

```
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx &= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}\\ &= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(c\left(1+\frac{2b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(b^2-4ac\right)}+\frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)\right)\int \cdots}{2\left(b^2-4\cdots\right.}\\ &= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(2b+\sqrt{b^2-4a\cdots}\right.}{\sqrt{2}\left(b^2-4ac\right)^{3/\cdots}}\end{aligned}$$

Mathematica [A] time = 0.442591, size = 222, normalized size = 1.

$$\frac{-bx-2cx^3}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b +
 Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
 + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A] time = 0.232, size = 342, normalized size = 1.6

$$\frac{x}{8\,ac-2\,b^2}\left(x^2+\frac{b}{2\,c}-\frac{1}{2\,c}\sqrt{-4\,ac+b^2}\right)^{-1}+\frac{c\sqrt{2}b}{4\,ac-b^2}\text{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))+c/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-1/2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))+1/2/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+c/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*
c)
```

Fricas [B] time = 1.7401, size = 3623, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*
c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6
*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3
*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
+ 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256
*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a
^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2
- 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2
)*x - 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c
+ 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^
2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + 2*b*x)/((b^2*
```

```
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [A] time = 3.74574, size = 298, normalized size = 1.35

$$\frac{bx+2cx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}+\text{RootSum}\left(t^4\left(1048576a^7c^6-1572864a^6b^2c^5+983040a^5b^4c^4-32\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (b*x + 2*c*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(
8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c**6 - 1572864*a**6*b**2*c
**5 + 983040*a**5*b**4*c**4 - 327680*a**4*b**6*c**3 + 61440*a**3*b**8*c**2
- 6144*a**2*b**10*c + 256*a*b**12) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3*
b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**2*c**3 + 24*a*b**2*c**2
+ 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4 - 8192*_t**3*a**4*
b**2*c**3 + 512*_t**3*a**2*b**6*c - 64*_t**3*a*b**8 - 128*_t*a**2*b*c**2 -
16*_t*a*b**3*c - 4*_t*b**5)/(4*a*c**2 + 3*b**2*c))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.871 $\int \frac{1}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=252

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\cdots}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4\cdots}}}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.513073, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1092, 1166, 205}

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\cdots}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}} + \frac{\left(c\left(b^2-\right.\right.}{}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2-\right.}{2\sqrt{}}$$

Mathematica [A] time = 0.447863, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.22, size = 733, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*b+c/(-4*a*c+b^2
)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))-1/4/(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*b^2+1/4*c/(
4*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*b-c/(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/4/
(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*b
^2-1/4*c/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c
-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [B] time = 2.02123, size = 4918, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6
- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 67
2*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 4
48*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*
x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
```

```
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [A] time = 4.91032, size = 394, normalized size = 1.56

$$-\frac{bcx^3 + x\left(-2ac + b^2\right)}{8a^3c - 2a^2b^2 + x^4\left(8a^2c^2 - 2ab^2c\right) + x^2\left(8a^2bc - 2ab^3\right)} + \operatorname{RootSum}\left(t^4\left(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(b*c*x**3 + x*(-2*a*c + b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2
 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a**
9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*
c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**2*(
-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**
2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 +
 25*b**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672*_t**3
*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c + 64*_
t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_t*a**2*b
**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*c**3 + 5*
b**4*c**2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.872 $\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{3b^2-10ac}{2a^2x\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16\ldots\right.}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\ldots}$$

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b
^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 -
10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.44276, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2x\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16\ldots\right.}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b
^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 -
10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3b^2+10ac-3bcx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-b\left(3b^2-13ac\right)-c\left(3b^2-10ac\right)x^2}{a+bx^2+cx^4}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\right)\int}{4a^2\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\text{ta}}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2}}}$$

Mathematica [A] time = 0.631005, size = 302, normalized size = 0.98

$$\frac{-\frac{2x\left(-3abc-2ac^2x^2+b^2cx^2+b^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\text{t}}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4
*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2
]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [B] time = 0.209, size = 712, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a^2/x-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*x^4+b*x^2+a)*c/
(4*a*c-b^2)*x^3*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x*c+1/2/a^2/(c*x^4+
b*x^2+a)*b^3/(4*a*c-b^2)*x+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-3/4/
a^2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b^3-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+3/4/a^2*c/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(3\,b^2c-10\,ac^2\right)x^4+2\,ab^2-8\,a^2c+\left(3\,b^3-11\,abc\right)x^2}{2\left(\left(a^2b^2c-4\,a^3c^2\right)x^5+\left(a^2b^3-4\,a^3bc\right)x^3+\left(a^3b^2-4\,a^4c\right)x\right)}+\frac{-\int\frac{3\,b^3-13\,abc+\left(3\,b^2c-10\,ac^2\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(a^2b^2-4\,a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^2
)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a
^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/(c*
x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Fricas [B] time = 2.42947, size = 6460, normalized size = 20.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c
)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3
+ (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
+ 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^
5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*
a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*
b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^
5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7
*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-
(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sq
rt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 144
08*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*
c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/
2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*
a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b
^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*
c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7
*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^10
 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1
280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
```

```
c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 -
4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a
^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 -
1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sqrt(1/2)*(27*b^1
1 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
- 5200*a^5*b*c^5 + (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*
b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3
*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2
)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [A] time = 7.70261, size = 481, normalized size = 1.56

$$\mathrm{RootSum}\left(t^4\left(1048576a^{11}c^6 - 1572864a^{10}b^2c^5 + 983040a^9b^4c^4 - 327680a^8b^6c^3 + 61440a^7b^8c^2 - 6144a^6b^{10}c + 256a^5b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] RootSum(_t**4*(1048576*a**11*c**6 - 1572864*a**10*b**2*c**5 + 983040*a**9*b
**4*c**4 - 327680*a**8*b**6*c**3 + 61440*a**7*b**8*c**2 - 6144*a**6*b**10*c
 + 256*a**5*b**12) + _t**2*(430080*a**6*b*c**6 - 716800*a**5*b**3*c**5 + 48
3840*a**4*b**5*c**4 - 170496*a**3*b**7*c**3 + 33232*a**2*b**9*c**2 - 3408*a
*b**11*c + 144*b**13) + 10000*a**2*c**7 - 4200*a*b**2*c**6 + 441*b**4*c**5,
 Lambda(_t, _t*log(x + (-81920*_t**3*a**10*c**5 + 139264*_t**3*a**9*b**2*c*
*4 - 86016*_t**3*a**8*b**4*c**3 + 25088*_t**3*a**7*b**6*c**2 - 3520*_t**3*a
**6*b**8*c + 192*_t**3*a**5*b**10 - 27200*_t*a**5*b*c**5 + 60176*_t*a**4*b*
*3*c**4 - 42448*_t*a**3*b**5*c**3 + 13320*_t*a**2*b**7*c**2 - 1944*_t*a*b**
9*c + 108*_t*b**11)/(2500*a**3*c**6 - 5625*a**2*b**2*c**5 + 1971*a*b**4*c**
```

```
4 - 189*b**6*c**3)))) - (8*a**2*c - 2*a*b**2 + x**4*(10*a*c**2 - 3*b**2*c)
+ x**2*(11*a*b*c - 3*b**3))/(x**5*(8*a**3*c**2 - 2*a**2*b**2*c) + x**3*(8*a
**3*b*c - 2*a**2*b**3) + x*(8*a**4*c - 2*a**3*b**2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.873 $\int \frac{x^{11}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=209

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx^2\left(b^2-7ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^4\left(bx^2\left(b^2-10ac\right)+a\right.}{4c\left(b^2-4ac\right)^2\left(a+b\right.}$$

```
[Out] -(b*(b^2 - 7*a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^8*(2*a + b*x^2))/(4*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^4*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*
c)*x^2))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c +
 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)
^(5/2)) + Log[a + b*x^2 + c*x^4]/(4*c^3)
```

Rubi [A] time = 0.40082, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1114, 738, 818, 773, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx^2\left(b^2-7ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^4\left(bx^2\left(b^2-10ac\right)+a\right.}{4c\left(b^2-4ac\right)^2\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*(b^2 - 7*a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^8*(2*a + b*x^2))/(4*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^4*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*
c)*x^2))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c +
 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)
^(5/2)) + Log[a + b*x^2 + c*x^4]/(4*c^3)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^5}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^3(8a+bx)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x\left(2a\left(b^2-16ac\right)+\right.}{a+bx\cdots}\right.}{4c\left(b^2-\cdots\right.}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\text{Su}\cdots}{\cdots}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Su}\cdots}{\cdots}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{lo}\cdots}{\cdots}$$

$$= -\frac{b\left(b^2-7ac\right)x^2}{2c^2\left(b^2-4ac\right)^2} + \frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^4\left(a\left(b^2-16ac\right)+b\left(b^2-10ac\right)x^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{b\left(\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.341251, size = 244, normalized size = 1.17

$$\frac{\frac{-39a^2b^2c^2+50a^2bc^3x^2+32a^3c^3-30ab^3c^2x^2+11ab^4c+4b^5cx^2-b^6}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{a^2bc(5cx^2-4b)+2a^3c^2+ab^3(b-5cx^2)+b^5x^2}{(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{2bc(30a^2c^2-10ab^2c+b^4)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}}+c}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x^2 - 30*a*b^3*
c^2*x^2 + 50*a^2*b*c^3*x^2)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (2*a^3*
c^2 + b^5*x^2 + a*b^3*(b - 5*c*x^2) + a^2*b*c*(-4*b + 5*c*x^2))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[
(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + b*x^2 +
 c*x^4])/(4*c^4)
```

Maple [B] time = 0.184, size = 547, normalized size = 2.6

$$\frac{1}{2\left(cx^4+bx^2+a\right)^2}\left(\frac{b\left(25a^2c^2-15acb^2+2b^4\right)x^6}{c^2\left(16a^2c^2-8acb^2+b^4\right)}+\frac{\left(32a^3c^3+11a^2b^2c^2-19ab^4c+3b^6\right)x^4}{2c^3\left(16a^2c^2-8acb^2+b^4\right)}+\frac{ab\left(31a^2c^2-22acb^2+3\right.}{c^3\left(16a^2c^2-8acb^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1
/2*(32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^4+b*a*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^
2+3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+
b*x^2+a)^2+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a^2-2/c^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a*b^2+1/4/c^3/(16*a^2*c^2-8*a*b^2*c
+b^4)*ln(c*x^4+b*x^2+a)*b^4-15/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b+5/c^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^3-1/2/c^3/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)
^(1/2))*b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88714, size = 3467, normalized size = 16.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
- 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^6 + (3*b^8 - 31*a*b^6*c
+ 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(3*a*b^7 - 34*a^2*
b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x^2 + ((b^5*c^2 - 10*a*b^3*c^3 + 3
0*a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*
b^4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*
b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*
c))/(c*x^4 + b*x^2 + a)) + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*
c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b
^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*
a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3
*c^5 - 64*a^4*b*c^6)*x^2), 1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2
- 96*a^5*c^3 + 2*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*
x^6 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*
x^4 + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x^2 + 2*
((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*
a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*
c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b
```

```
^2*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b
^2 - 4*a*c)) + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8
+ a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4
*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3
*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 - 12*a^3
*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b
^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*
a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6
- 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^
4*b*c^6)*x^2)]
```

Sympy [B] time = 17.0604, size = 1520, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(4*c**3))*log(x**2 + (-128*a**3*c**5*(-b*sqr
t(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**
5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20
*a*b**8*c - b**10)) + 1/(4*c**3)) + 32*a**3*c**2 + 96*a**2*b**2*c**4*(-b*sq
rt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a*
*5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 2
0*a*b**8*c - b**10)) + 1/(4*c**3)) - 9*a**2*b**2*c - 24*a*b**4*c**3*(-b*sqr
t(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**
5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20
*a*b**8*c - b**10)) + 1/(4*c**3)) + a*b**4 + 2*b**6*c**2*(-b*sqrt(-(4*a*c -
 b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5 - 12
80*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
 b**10)) + 1/(4*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) + 1/(4*c**3))*log(x**2 + (-128*a**3*c**5*(b*sqrt(-(4*a*c -
b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5 - 128
0*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
b**10)) + 1/(4*c**3)) + 32*a**3*c**2 + 96*a**2*b**2*c**4*(b*sqrt(-(4*a*c -
b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5 - 128
```

```
0*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
b**10)) + 1/(4*c**3)) - 9*a**2*b**2*c - 24*a*b**4*c**3*(b*sqrt(-(4*a*c - b*
*2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5 - 1280*
a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b*
*10)) + 1/(4*c**3)) + a*b**4 + 2*b**6*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(30*
a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c*
*4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(4
*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (24*a**4*c**2 - 21*a**3*b
**2*c + 3*a**2*b**4 + x**6*(50*a**2*b*c**3 - 30*a*b**3*c**2 + 4*b**5*c) + x
**4*(32*a**3*c**3 + 11*a**2*b**2*c**2 - 19*a*b**4*c + 3*b**6) + x**2*(62*a*
*3*b*c**2 - 44*a**2*b**3*c + 6*a*b**5))/(64*a**4*c**5 - 32*a**3*b**2*c**4 +
 4*a**2*b**4*c**3 + x**8*(64*a**2*c**7 - 32*a*b**2*c**6 + 4*b**4*c**5) + x*
*6*(128*a**2*b*c**6 - 64*a*b**3*c**5 + 8*b**5*c**4) + x**4*(128*a**3*c**6 -
 24*a*b**4*c**4 + 4*b**6*c**3) + x**2*(128*a**3*b*c**5 - 64*a**2*b**3*c**4
+ 8*a*b**5*c**3))
```

Giac [A] time = 27.3785, size = 413, normalized size = 1.98

$$-\frac{\left(b^5-10\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)\sqrt{-b^2+4\,ac}}-\frac{3\,b^4c^2x^8-24\,ab^2c^3x^8+48\,a^2c^4x^8-2\,b^5cx^6+12\,ab^3c^2x^6-4\,a^2bc^3x^6-}{8\left(b^4c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a
*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) - 1/8*(3*b^4
*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8 - 2*b^5*c*x^6 + 12*a*b^3*c^2*x
^6 - 4*a^2*b*c^3*x^6 - 3*b^6*x^4 + 20*a*b^4*c*x^4 - 22*a^2*b^2*c^2*x^4 + 32
*a^3*c^3*x^4 - 6*a*b^5*x^2 + 40*a^2*b^3*c*x^2 - 28*a^3*b*c^2*x^2 - 3*a^2*b^
4 + 18*a^3*b^2*c)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)
^2) + 1/4*log(c*x^4 + b*x^2 + a)/c^3
```

3.874 $\int \frac{x^9}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=121

$$-\frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

```
[Out] (x^6*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*a*x^2*(2*a
 + b*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*a^2*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.111931, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 722, 618, 206}

$$-\frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] (x^6*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*a*x^2*(2*a
 + b*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*a^2*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
```

```
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$
$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$
$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x\right.}{\left(b^2-4ac\right)^2}$$
$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x\right.}{\left(b^2-4ac\right)^2}$$
$$= \frac{x^6\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3ax^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.175756, size = 194, normalized size = 1.6

$$\frac{1}{4}\left(\frac{a^2c\left(2cx^2-3b\right)+ab^2\left(b-4cx^2\right)+b^4x^2}{c^3\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2} + \frac{22a^2bc^2-20a^2c^3x^2+16ab^2c^2x^2-8ab^3c-2b^4cx^2+b^5}{c^3\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{24a^2\tan^{-1}\left(\frac{b+}{\sqrt{4}}\right.}{\left(4ac-b^2\right)^{5}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x^2 + 16*a*b^2*c^2*x^2 - 20*a^2*
c^3*x^2)/(c^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^4*x^2 + a*b^2*(b -
4*c*x^2) + a^2*c*(-3*b + 2*c*x^2))/(c^3*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^
2) + (24*a^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
)/4
```

Maple [B] time = 0.179, size = 267, normalized size = 2.2

$$\frac{1}{2\left(cx^4+bx^2+a\right)^2}\left(-\frac{\left(10\,a^2c^2-8\,acb^2+b^4\right)x^6}{c\left(16\,a^2c^2-8\,acb^2+b^4\right)}+\frac{b\left(2\,a^2c^2+8\,acb^2-b^4\right)x^4}{2\,c^2\left(16\,a^2c^2-8\,acb^2+b^4\right)}-\frac{a\left(6\,a^2c^2-10\,acb^2+b^4\right)x^2}{c^2\left(16\,a^2c^2-8\,acb^2+b^4\right)}+\frac{ba^2(}{2\,c^2\left(16\,a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*b*(
2*a^2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-a*(6*a^2*c^2-10
*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2+1/2*b*a^2*(10*a*c-b^2)/c^2
/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+6*a^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59675, size = 2033, normalized size = 16.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42
*a^2*b^2*c^3 - 40*a^3*c^4)*x^6 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3
*b*c^3)*x^4 + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x^2 -
12*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 + a^4*c^2 + (a^2*b^2*c^
2 + 2*a^3*c^3)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*
a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a^2*b^6*c^2 -
 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 4
8*a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^
5 - 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b
^2*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4
- 64*a^4*b*c^5)*x^2), -1/4*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*
c - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^6 + (b^7 - 12*a*b^5*c + 3
0*a^2*b^3*c^2 + 8*a^3*b*c^3)*x^4 + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2
 - 24*a^4*c^3)*x^2 + 24*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 +
a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 4
8*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*
a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x
^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^
6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2
)]
```

Sympy [B] time = 12.3407, size = 554, normalized size = 4.58

$$-3a^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^5c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^4b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36a^3b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3a^2b^6\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}}{6a^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**5*c**3*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**3*b**
4*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 3
*a**2*b)/(6*a**2*c)) + 3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a*
*5*c**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5) + 36*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a**2*b**6*sqrt(-1/(
4*a*c - b**2)**5) + 3*a**2*b)/(6*a**2*c)) - (-10*a**3*b*c + a**2*b**3 + x**
6*(20*a**2*c**3 - 16*a*b**2*c**2 + 2*b**4*c) + x**4*(-2*a**2*b*c**2 - 8*a*b
**3*c + b**5) + x**2*(12*a**3*c**2 - 20*a**2*b**2*c + 2*a*b**4))/(64*a**4*c
**4 - 32*a**3*b**2*c**3 + 4*a**2*b**4*c**2 + x**8*(64*a**2*c**6 - 32*a*b**2
*c**5 + 4*b**4*c**4) + x**6*(128*a**2*b*c**5 - 64*a*b**3*c**4 + 8*b**5*c**3
) + x**4*(128*a**3*c**5 - 24*a*b**4*c**3 + 4*b**6*c**2) + x**2*(128*a**3*b*
c**4 - 64*a**2*b**3*c**3 + 8*a*b**5*c**2))
```

Giac [A] time = 27.3263, size = 286, normalized size = 2.36

$$\frac{6\,a^{2}\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{-b^{2}+4\,ac}}-\frac{2\,b^{4}cx^{6}-16\,ab^{2}c^{2}x^{6}+20\,a^{2}c^{3}x^{6}+b^{5}x^{4}-8\,ab^{3}cx^{4}-2\,a^{2}bc^{2}x^{4}+2\,ab^{4}x^{2}-20\,a^{2}b^{2}c}{4\left(b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}\right)\left(cx^{4}+bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 6*a^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*b^4*c*x^6 - 16*a*b^2*c^2*x^6 + 20*a^2*c^3*
x^6 + b^5*x^4 - 8*a*b^3*c*x^4 - 2*a^2*b*c^2*x^4 + 2*a*b^4*x^2 - 20*a^2*b^2*
c*x^2 + 12*a^3*c^2*x^2 + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

3.875 $\int \frac{x^7}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=119

$$-\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -(x^6*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*b*x^2*(2*
a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*a*b*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.103785, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 728, 722, 618, 206}

$$-\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^6*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*b*x^2*(2*
a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*a*b*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 728

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
```

```
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{(3ab)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,\right.}{2\left(b^2-4ac\right)^2}$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{(3ab)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,\right.}{\left(b^2-4ac\right)^2}$$

$$= -\frac{x^6\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3bx^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.200809, size = 137, normalized size = 1.15

$$-\frac{a^2\left(b^2+10bcx^2+16c^2x^4\right)+8a^3c+abx^2\left(2b^2+bcx^2+6c^2x^4\right)+b^4x^4}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2} - \frac{3ab\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(8*a^3*c + b^4*x^4 + a*b*x^2*(2*b^2 + b*c*x^2 + 6*c^2*x^4) + a^2*(b^2 + 10
*b*c*x^2 + 16*c^2*x^4))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) - (3*a*
b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

Maple [B] time = 0.175, size = 230, normalized size = 1.9

$$\frac{1}{2\left(cx^4+bx^2+a\right)^2}\left(-3\frac{x^6abc}{16a^2c^2-8acb^2+b^4} - \frac{\left(16a^2c^2+acb^2+b^4\right)x^4}{2c\left(16a^2c^2-8acb^2+b^4\right)} - \frac{\left(5ac+b^2\right)abx^2}{c\left(16a^2c^2-8acb^2+b^4\right)} - \frac{a^2\left(8ac+\right.}{2c\left(16a^2c^2-8\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-3
*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.56206, size = 1875, normalized size = 15.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^6 + a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2
 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^4 + 2*(a*b^5 + a^2*b^3
*c - 20*a^3*b*c^2)*x^2 - 6*(a*b*c^3*x^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2
 + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4
+ 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)))/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^
3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5
*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*
b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 +
 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2), -1/4*(6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^6
 + a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 -
 64*a^3*c^3)*x^4 + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x^2 - 12*(a*b*c^3*x
^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2 + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*
```

```
x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)))/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3
- 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^
4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 4
8*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2)]
```

Sympy [B] time = 10.6245, size = 520, normalized size = 4.37

$$\frac{3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^4bc^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36a^2b^5c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^7\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^2}{6abc}\right)}{2}-\frac{3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**4*b*c**3*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**
5*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*
b**2)/(6*a*b*c))/2 - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**4*
b*c**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**
2)**5) + 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*
c - b**2)**5) + 3*a*b**2)/(6*a*b*c))/2 - (8*a**3*c + a**2*b**2 + 6*a*b*c**2
*x**6 + x**4*(16*a**2*c**2 + a*b**2*c + b**4) + x**2*(10*a**2*b*c + 2*a*b**
3))/(64*a**4*c**3 - 32*a**3*b**2*c**2 + 4*a**2*b**4*c + x**8*(64*a**2*c**5
- 32*a*b**2*c**4 + 4*b**4*c**3) + x**6*(128*a**2*b*c**4 - 64*a*b**3*c**3 +
8*b**5*c**2) + x**4*(128*a**3*c**4 - 24*a*b**4*c**2 + 4*b**6*c) + x**2*(128
*a**3*b*c**3 - 64*a**2*b**3*c**2 + 8*a*b**5*c))
```

Giac [A] time = 28.7942, size = 231, normalized size = 1.94

$$-\frac{3\,ab\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,abc^2x^6+b^4x^4+ab^2cx^4+16\,a^2c^2x^4+2\,ab^3x^2+10\,a^2bcx^2+a^2b^2+8\,a^3c}{4\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)\left(cx^4+bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*a*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*
c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*a*b*c^2*x^6 + b^4*x^4 + a*b^2*c*x^4 + 16*
a^2*c^2*x^4 + 2*a*b^3*x^2 + 10*a^2*b*c*x^2 + a^2*b^2 + 8*a^3*c)/((b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

3.876 $\int \frac{x^5}{\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=130

$$\frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^2\left(2ac+b^2\right)+3ab}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*a*b + (b^2
 + 2*a*c)*x^2)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2 + 2*a*c)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.129447, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 738, 638, 618, 206}

$$\frac{x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^2\left(2ac+b^2\right)+3ab}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(2ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*a*b + (b^2
 + 2*a*c)*x^2)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2 + 2*a*c)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
```

```
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= \frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\text{Subst}\left(\int \frac{2a-2bx}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4(b^2-4ac)}$$

$$= \frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3ab+(b^2+2ac)x^2}{2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(b^2+2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\right.}{2(b^2-4ac)^2}$$

$$= \frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3ab+(b^2+2ac)x^2}{2(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(b^2+2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\right.}{(b^2-4ac)^2}$$

$$= \frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3ab+(b^2+2ac)x^2}{2(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(b^2+2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Mathematica [A] time = 0.137033, size = 145, normalized size = 1.12

$$\frac{1}{4}\left(\frac{(2ac+b^2)(b+2cx^2)}{c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{a(b-2cx^2)+b^2x^2}{c(4ac-b^2)(a+bx^2+cx^4)^2} + \frac{4(2ac+b^2)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2 + 2*a*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b
^2*x^2 + a*(b - 2*c*x^2))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(b^
2 + 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/
4
```

Maple [B] time = 0.18, size = 270, normalized size = 2.1

$$\frac{1}{2(cx^4+bx^2+a)^2}\left(\frac{c(2ac+b^2)x^6}{16a^2c^2-8acb^2+b^4} + \frac{3b(2ac+b^2)x^4}{32a^2c^2-16acb^2+2b^4} - \frac{a(2ac-5b^2)x^2}{16a^2c^2-8acb^2+b^4} + 3\frac{ba^2}{16a^2c^2-8acb^2+b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3/2*b*(2*a*c+b^2)/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^4-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+3*a^
2*b/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+2/(16*a^2*c^2-8*a*b^2*c+b
^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*c+1/(16*a^2*c
^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b
^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.61505, size = 1920, normalized size = 14.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^6 + 6*a^2*b^3 - 24*a^3*b*c + 3*
(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^4 + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2
)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b^4 + 4*a
*b^2*c + 4*a^2*c^2)*x^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sq
rt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 -
 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7
 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), 1/4*(2*(b^4*c - 2*a*
```

```
b^2*c^2 - 8*a^2*c^3)*x^6 + 6*a^2*b^3 - 24*a^3*b*c + 3*(b^5 - 2*a*b^3*c - 8*
a^2*b*c^2)*x^4 + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x^2 - 4*((b^2*c^2 +
 2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x
^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arct
an(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4
)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*
x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

Sympy [B] time = 10.4376, size = 580, normalized size = 4.46

$$-\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)\log\left(x^2+\frac{-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)-12ab^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+2abc+b^6\sqrt{-\frac{1}{\left(4ac-\right.}}}{4ac^2+2b^2c}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 + (-64*a**3*c**3*sqrt(-
1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b*
*2) + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*a
*c**2 + 2*b**2*c))/2 + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 +
 (64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**
2*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c -
b**2)**5)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c
 + b**2) + b**3)/(4*a*c**2 + 2*b**2*c))/2 + (6*a**2*b + x**6*(4*a*c**2 + 2*
b**2*c) + x**4*(6*a*b*c + 3*b**3) + x**2*(-4*a**2*c + 10*a*b**2))/(64*a**4*
c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 +
 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(
128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**
3*c + 8*a*b**5))
```

Giac [A] time = 28.9732, size = 217, normalized size = 1.67

$$\frac{\left(b^2+2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{2\,b^2cx^6+4\,ac^2x^6+3\,b^3x^4+6\,abcx^4+10\,ab^2x^2-4\,a^2cx^2+6\,a^2b}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,ab^2c+16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (b^2 + 2*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*b^2*c*x^6 + 4*a*c^2*x^6 + 3*b^3*x^
4 + 6*a*b*c*x^4 + 10*a*b^2*x^2 - 4*a^2*c*x^2 + 6*a^2*b)/((c*x^4 + b*x^2 + a
)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

3.877 $\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=113

$$\frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] (2*a + b*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*b*(b + 2*c*x^2))
/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*b*c*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0903027, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 638, 614, 618, 206}

$$\frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^3,x]
```



```
[Out] (2*a + b*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*b*(b + 2*c*x^2))
/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*b*c*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2+cx^4\right)^3}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$=\frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{(3b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)}{4\left(b^2-4ac\right)}$$

$$=\frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{(3bc)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x\right.}{2\left(b^2-4ac\right)^2}$$

$$=\frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{(3bc)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b\right.}{\left(b^2-4ac\right)^2}$$

$$=\frac{2a+bx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3b\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.103696, size = 114, normalized size = 1.01

$$\frac{\frac{(b^2-4ac)(2a+bx^2)}{(a+bx^2+cx^4)^2}-\frac{12bc\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{3b(b+2cx^2)}{a+bx^2+cx^4}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(2*a + b*x^2))/(a + b*x^2 + c*x^4)^2 - (3*b*(b + 2*c*x^2))/
(a + b*x^2 + c*x^4) - (12*b*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqr
t[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2)
```

Maple [A] time = 0.184, size = 142, normalized size = 1.3

$$\frac{-bx^2-2a}{\left(16ac-4b^2\right)\left(cx^4+bx^2+a\right)^2}-\frac{3bcx^2}{2\left(4ac-b^2\right)^2\left(cx^4+bx^2+a\right)}-\frac{3b^2}{4\left(4ac-b^2\right)^2\left(cx^4+bx^2+a\right)}-3\frac{bc}{\left(4ac-b^2\right)^{5/2}}\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2-3/2*b/(4*a*c-b^2)^2/(c*x^4+b
*x^2+a)*x^2*c-3/4*b^2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)-3*b/(4*a*c-b^2)^(5/2)*c
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.51806, size = 1719, normalized size = 15.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*x^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(
b^4*c - 4*a*b^2*c^2)*x^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x^2 - 6*(b*c^3*
x^8 + 2*b^2*c^2*x^6 + 2*a*b^2*c*x^2 + (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sq
rt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 -
 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7
 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*(6*(b^3*c^2 - 4
*a*b*c^3)*x^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*
x^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x^2 - 12*(b*c^3*x^8 + 2*b^2*c^2*x^6
+ 2*a*b^2*c*x^2 + (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)*sqrt(-b^2 + 4*a*c)*arc
tan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)
*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

Sympy [B] time = 9.81427, size = 490, normalized size = 4.34

$$\frac{3bc\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^3bc^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36ab^5c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^7c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^2c}{6bc^2}\right)}{2}-\frac{3bc\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*b*c**4*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c
```

```
**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b*
*2*c)/(6*b*c**2))/2 - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**3
*b*c**4*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b*
*2)**5) + 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a
*c - b**2)**5) + 3*b**2*c)/(6*b*c**2))/2 - (8*a**2*c + a*b**2 + 9*b**2*c*x*
*4 + 6*b*c**2*x**6 + x**2*(10*a*b*c + 2*b**3))/(64*a**4*c**2 - 32*a**3*b**2
*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**
6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*
a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

Giac [A] time = 28.591, size = 193, normalized size = 1.71

$$-\frac{3\,bc\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2x^6+9\,b^2cx^4+2\,b^3x^2+10\,abcx^2+ab^2+8\,a^2c}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,ab^2c+16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*
c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*b*c^2*x^6 + 9*b^2*c*x^4 + 2*b^3*x^2 + 10*
a*b*c*x^2 + a*b^2 + 8*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a
^2*c^2))
```

3.878 $\int \frac{x}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=113

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3c(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{b+2cx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[Out] -(b + 2*c*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*c*(b + 2*c*x^2)
)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*c^2*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.0875405, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1107, 614, 618, 206}

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3c(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{b+2cx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b + 2*c*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*c*(b + 2*c*x^2)
)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*c^2*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{(3c)\operatorname{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c^2\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,\right.}{\left(b^2-4ac\right)^2}$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(6c^2\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,\right.}{\left(b^2-4ac\right)^2}$$

$$= -\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.0996406, size = 106, normalized size = 0.94

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{\left(b+2cx^2\right)\left(-2c\left(5a+3cx^4\right)+b^2-6bcx^2\right)}{\left(a+bx^2+cx^4\right)^2}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-(((b + 2*c*x^2)*(b^2 - 6*b*c*x^2 - 2*c*(5*a + 3*c*x^4)))/(a + b*x^2 + c*x
^4)^2) + (24*c^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c])/(4*(b^2 - 4*a*c)^2)
```

Maple [A] time = 0.176, size = 141, normalized size = 1.3

$$\frac{2\,cx^2+b}{\left(16\,ac-4\,b^2\right)\left(cx^4+bx^2+a\right)^2}+3\,\frac{c^2x^2}{\left(4\,ac-b^2\right)^2\left(cx^4+bx^2+a\right)}+\frac{3\,bc}{2\,\left(4\,ac-b^2\right)^2\left(cx^4+bx^2+a\right)}+6\,\frac{c^2}{\left(4\,ac-b^2\right)^{5/2}}\,\mathrm{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3*c^2/(4*a*c-b^2)^2/(c*x^4+b*
x^2+a)*x^2+3/2*c/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b+6*c^2/(4*a*c-b^2)^(5/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.58014, size = 1709, normalized size = 15.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*x^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^
3*c^2 - 4*a*b*c^3)*x^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x^2 + 12*(c^4*x
```

```
^8 + 2*b*c^3*x^6 + 2*a*b*c^2*x^2 + (b^2*c^2 + 2*a*c^3)*x^4 + a^2*c^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), 1/4*(12*(b^2*c^3 - 4*a*
c^4)*x^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*x^4 +
 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x^2 - 24*(c^4*x^8 + 2*b*c^3*x^6 + 2*a*b
*c^2*x^2 + (b^2*c^2 + 2*a*c^3)*x^4 + a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6
+ (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 +
2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

Sympy [B] time = 9.5687, size = 481, normalized size = 4.26

$$-3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^3c^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^2b^2c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36ab^4c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3b^6c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}}{6c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*c**5*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**4*c
**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3
*b*c**2)/(6*c**3)) + 3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**3
*c**5*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2
)**5) + 36*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*
a*c - b**2)**5) + 3*b*c**2)/(6*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**4 +
 12*c**3*x**6 + x**2*(20*a*c**2 + 4*b**2*c))/(64*a**4*c**2 - 32*a**3*b**2*c
 + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*
(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*a*
b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

Giac [A] time = 27.711, size = 194, normalized size = 1.72

$$\frac{6\,c^2\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2+20\,ac^2x^2-b^3+10\,abc}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,ab^2c+16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 + 20
*a*c^2*x^2 - b^3 + 10*a*b*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a
^2*c^2))
```

3.879 $\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=200

$$\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)}{a^3}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*b^4
- 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x^2)/(4*a^2*(b^2 - 4*a*c)^2
*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a
+ b*x^2 + c*x^4]/(4*a^3)
```

Rubi [A] time = 0.29849, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*b^4
- 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x^2)/(4*a^2*(b^2 - 4*a*c)^2
*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a
+ b*x^2 + c*x^4]/(4*a^3)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-4ac\right)-3bcx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{2\left(b^2-\right.}{}\right.}{4a}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int\left(\frac{2(-b}{}\right.\right.}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^3} + \frac{\text{Subs}}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^3} - \frac{\text{Subs}}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^3} - \frac{\log\left(a\right.}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{b\left(b^4-10ab^2c\right.}{2a}$$

<u>Mathematica</u> [A] time = 0.496247, size = 342, normalized size = 1.71

$$\frac{\frac{a\left(16a^2c^2-15ab^2c-14abc^2x^2+2b^3cx^2+2b^4\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2+b^4\sqrt{b^2-4ac}-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^5\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{5/2}} + \frac{\left(-16a^2c^2\sqrt{b^2-4a\right.}{}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(
2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2))/((b^2 - 4*
a*c)^2*(a + b*x^2 + c*x^4)) + 4*Log[x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2
+ b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2
 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + ((b^
5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2
- 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

Maple [B] time = 0.198, size = 822, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] ln(x)/a^3-7/2/a/(c*x^4+b*x^2+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/
a^2/(c*x^4+b*x^2+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+4/(c*x^4+b*x^2
+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-29/4/a/(c*x^4+b*x^2+a)^2*c^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^4*b^2+1/a^2/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2
*c+b^4)*x^4*b^4-1/2/(c*x^4+b*x^2+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^2-
3/a/(c*x^4+b*x^2+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c+1/2/a^2/(c*x^4+b
*x^2+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+6*a/(c*x^4+b*x^2+a)^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*c^2-21/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*
b^2+3/4/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4-4/a/(16*a^2*c^2-
8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a)+2/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(
c*x^4+b*x^2+a)*b^2-1/4/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*b^4
-15/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b*c^2+5/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*c-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.54151, size = 4271, normalized size = 21.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*
a^3*b^2*c^3 - 64*a^4*c^4)*x^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 +
4*a^4*b*c^3)*x^2 + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b^5 -
 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^
6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a
^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^
4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b
^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(x))/(a^5*b^6 -
12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3
+ 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5
*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 3
2*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c
^2 - 64*a^7*b*c^3)*x^2), 1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 -
96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6 + (4*a*b^6*c
 - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*x^4 + 2*(a*b^7 - 10*a^2*b
^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x^2 + 2*((b^5*c^2 - 10*a*b^3*c^3 + 30*
a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^
4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*
c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(-b^2 + 4*a*c
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^
```

```
4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*
log(c*x^4 + b*x^2 + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(x))/(a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c
^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*
a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3
- 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*
c^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 32.726, size = 436, normalized size = 2.18

$$-\frac{\left(b^5-10\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2\right)\sqrt{-b^2+4\,ac}}+\frac{3\,b^4c^2x^8-24\,ab^2c^3x^8+48\,a^2c^4x^8+6\,b^5cx^6-44\,ab^3c^2x^6+68\,a^2bc^3x^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a
*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*b^4
*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8 + 6*b^5*c*x^6 - 44*a*b^3*c^2*x
^6 + 68*a^2*b*c^3*x^6 + 3*b^6*x^4 - 10*a*b^4*c*x^4 - 58*a^2*b^2*c^2*x^4 + 1
28*a^3*c^3*x^4 + 10*a*b^5*x^2 - 72*a^2*b^3*c*x^2 + 92*a^3*b*c^2*x^2 + 9*a^2
*b^4 - 66*a^3*b^2*c + 96*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*
x^4 + b*x^2 + a)^2) - 1/4*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*log(x^2)/a^3
```

3.880 $\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=255

$$\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}}-\frac{3\left(b^2-5ac\right)\left(b^2-2\right.}{2a^3x^2\left(b^2-4ac\right)}$$

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*x^2) + (b^2 - 2*a*c
 + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^2) + (3*b^4 - 20*a*b
^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x^2)/(4*a^2*(b^2 - 4*a*c)^2*x^2*(a
+ b*x^2 + c*x^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*L
og[x])/a^4 + (3*b*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rubi [A] time = 0.391344, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(30a^2b^2c^2-20a^3c^3-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}}-\frac{3\left(b^2-5ac\right)\left(b^2-2\right.}{2a^3x^2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*x^2) + (b^2 - 2*a*c
 + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^2) + (3*b^4 - 20*a*b
^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x^2)/(4*a^2*(b^2 - 4*a*c)^2*x^2*(a
+ b*x^2 + c*x^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*L
og[x])/a^4 + (3*b*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{\mathrm{Subst}\left(\int \frac{-3b^2+10ac-4bcx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} + \frac{\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc\left(b^2-6ac\right)x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} + \frac{\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3b}{4a^2\left(b^2-4ac\right)^2x^2\left(a+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3b}{4a^2\left(b^2-4ac\right)^2x^2\left(a+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3b}{4a^2\left(b^2-4ac\right)^2x^2\left(a+\right.}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2x^2} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3b}{4a^2\left(b^2-4ac\right)^2x^2\left(a+\right.}$$

Mathematica [A] time = 0.614424, size = 402, normalized size = 1.58

$$\frac{\frac{a^2\left(-3abc-2ac^2x^2+b^2cx^2+b^3\right)}{\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2} - \frac{a\left(46a^2bc^2+28a^2c^3x^2-26ab^2c^2x^2-29ab^3c+4b^4cx^2+4b^5\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\left(30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}-20a^3c^3+b^5\sqrt{b^2-4ac}-10ab^4c-8ab^3c\right.}{\left(b^2-4ac\right)^{5/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a)/x^2 + (a^2*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c
*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) - 12*b*Log[x] + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 +
b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2
 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + (3*(
-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8
*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

Maple [B] time = 0.192, size = 1002, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -1/2/a^3/x^2-3*b*ln(x)/a^4-7/a/(c*x^4+b*x^2+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^6+13/2/a^2/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b^2-
1/a^3/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b^4-37/2/a/(c*x^
4+b*x^2+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+55/4/a^2/(c*x^4+b*x^2+a)^
2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-2/a^3/(c*x^4+b*x^2+a)^2*b^5*c/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^4-9/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^2*c^3-7/2/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c^2+6/a^2
/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4*c-1/a^3/(c*x^4+b*x^2+
a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^6-29/2/(c*x^4+b*x^2+a)^2*b/(16*a^2*c^
2-8*a*b^2*c+b^4)*c^2+9/a/(c*x^4+b*x^2+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c
-5/4/a^2/(c*x^4+b*x^2+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)+12/a^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a)*b-6/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c
*ln(c*x^4+b*x^2+a)*b^3+3/4/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)
*b^5-30/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(
4*a*c-b^2)^(1/2))*c^3+45/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*c^2-15/a^3/(16*a^2*c^2-8*a*b^2*c+b
^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*c+3/2/a^4/(
16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^
(1/2))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.09586, size = 4906, normalized size = 19.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c
^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^8 + 3*(4*a*b^7*c - 45*
a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^6 + 2*(3*a*b^8 - 30*a^2*b^
6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^4 + (9*a^2*b^7 - 104
*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x^2 + 3*((b^6*c^2 - 10*a*b^4*
c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^10 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*
b^3*c^3 - 20*a^3*b*c^4)*x^8 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^
2*c^3 - 40*a^4*c^4)*x^6 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4
*b*c^3)*x^4 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x^2)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^
2*b^3*c^4 - 64*a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 -
 64*a^3*b^2*c^4)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3
- 128*a^4*b*c^4)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^
2*c^3)*x^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*
log(c*x^4 + b*x^2 + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*
a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4
)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4
)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^4 + (a
^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(x))/((a^4*b
^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c
- 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b
^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12
*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)*x^2), -1/4*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5
*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 4
0*a^4*c^5)*x^8 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*
b*c^4)*x^6 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 -
```

```
200*a^5*c^4)*x^4 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b
*c^3)*x^2 + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^10
+ 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^8 + (b^8 - 8*a
*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^6 + 2*(a*b^7 - 10*
a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x^4 + (a^2*b^6 - 10*a^3*b^4*c +
30*a^4*b^2*c^2 - 20*a^5*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3
*c^4 - 64*a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a
^3*b^2*c^4)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128
*a^4*b*c^4)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3
)*x^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(c
*x^4 + b*x^2 + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^8
 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^6
 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^4 + (a^2*b^
7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(x))/((a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*
a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c
+ 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*
b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 31.9524, size = 516, normalized size = 2.02

$$\frac{3\left(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^4b^4-8\,a^5b^2c+16\,a^6c^2\right)\sqrt{-b^2+4\,ac}}-\frac{9\,b^5c^2x^8-72\,ab^3c^3x^8+144\,a^2bc^4x^8+18\,b^6cx^6-136\,ab^4c^2x^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x^2 + b)/s
qrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)
) - 1/8*(9*b^5*c^2*x^8 - 72*a*b^3*c^3*x^8 + 144*a^2*b*c^4*x^8 + 18*b^6*c*x^
6 - 136*a*b^4*c^2*x^6 + 236*a^2*b^2*c^3*x^6 + 56*a^3*c^4*x^6 + 9*b^7*x^4 -
38*a*b^5*c*x^4 - 110*a^2*b^3*c^2*x^4 + 436*a^3*b*c^3*x^4 + 26*a*b^6*x^2 - 1
92*a^2*b^4*c*x^2 + 316*a^3*b^2*c^2*x^2 + 72*a^4*c^3*x^2 + 19*a^2*b^5 - 144*
a^3*b^3*c + 260*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(c*x^4 + b
*x^2 + a)^2) + 3/4*b*log(c*x^4 + b*x^2 + a)/a^4 - 3/2*b*log(x^2)/a^4 + 1/2*
(3*b*x^2 - a)/(a^4*x^2)
```

3.881 $\int \frac{x^{10}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=400

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (-3*b*(b^2 - 8*a*c)*x)/(8*c^2*(b^2 - 4*a*c)^2) + ((b^2 - 28*a*c)*x^3)/(8*c*
(b^2 - 4*a*c)^2) + (x^7*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) + (x^5*(12*a*b - (b^2 - 28*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*(b^4 - 9*a*b^2*c + 28*a^2*c^2 - (b^5 - 11*a*b^3*c + 44*a^2*b*c^
2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b
^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.72723, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1120, 1275, 1279, 1166, 205}

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3*b*(b^2 - 8*a*c)*x)/(8*c^2*(b^2 - 4*a*c)^2) + ((b^2 - 28*a*c)*x^3)/(8*c*
(b^2 - 4*a*c)^2) + (x^7*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) + (x^5*(12*a*b - (b^2 - 28*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*(b^4 - 9*a*b^2*c + 28*a^2*c^2 - (b^5 - 11*a*b^3*c + 44*a^2*b*c^
2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b
^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
```

```
*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{x^6\left(14a+bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-28ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^4\left(60ab-3\left(b^2-28ac\right)x^2\right)}{a+bx^2+cx^4}\,dx}{8\left(b^2-4ac\right)^2}$$

$$= \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-28ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\int \frac{x^2\left(-9a\left(b^2-28\ldots\right.\right.}{a+\ldots}}{24c\left(\ldots\right.}$$

$$= -\frac{3b\left(b^2-8ac\right)x}{8c^2\left(b^2-4ac\right)^2} + \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-28ac\right)x\ldots\right.}{8\left(b^2-4ac\right)^2\left(a+bx^2+c\ldots\right.}$$

$$= -\frac{3b\left(b^2-8ac\right)x}{8c^2\left(b^2-4ac\right)^2} + \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-28ac\right)x\ldots\right.}{8\left(b^2-4ac\right)^2\left(a+bx^2+c\ldots\right.}$$

$$= -\frac{3b\left(b^2-8ac\right)x}{8c^2\left(b^2-4ac\right)^2} + \frac{\left(b^2-28ac\right)x^3}{8c\left(b^2-4ac\right)^2} + \frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^5\left(12ab-\left(b^2-28ac\right)x\ldots\right.}{8\left(b^2-4ac\right)^2\left(a+bx^2+c\ldots\right.}$$

Mathematica [A] time = 1.19307, size = 455, normalized size = 1.14

$$\frac{\frac{2x\left(48a^2bc^2-44a^2c^3x^2+37ab^2c^2x^2-17ab^3c-5b^4cx^2+2b^5\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2\sqrt{b^2-4ac}-44a^2bc^2+b^4\sqrt{b^2-4ac}+11ab^3c-9ab^2c\sqrt{b^2-4ac}-b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \ldots}{16c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(2*b^5 - 17*a*b^3*c + 48*a^2*b*c^2 - 5*b^4*c*x^2 + 37*a*b^2*c^2*x^2 -
 44*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*(b^4*x^3 + a*b
^2*x*(b - 4*c*x^2) + a^2*c*x*(-3*b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^5 + 11*a*b^3*c - 44*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2*c^2*Sqrt[b^2 - 4*a*c]
```

```
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^5 - 11*a*b^3*c +
44*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2
*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^3)
```

Maple [B] time = 0.214, size = 1141, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*(44*a^2*c^2-37*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^7+1/8*b*
(4*a^2*c^2+20*a*b^2*c-3*b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2*(
28*a^2*c^2-49*a*b^2*c+6*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/8*b*a^2*(8*a*
c-b^2)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-21/4/(16*a^2*c^2
-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2+27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*a*b^2-3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^4+33/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*a^2*b-33/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*a*b^3+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5+21/4/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*a^2-27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2+3
/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4+33/4/(16*a^2*c^2-
8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*b-33/16/c/(16*a^2*
c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^3+3/16/c^2/(16
*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + (3*b^5 - 20*a*b^3*c - 4*a
^2*b*c^2)*x^5 + (6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c^2)*x^3 + 3*(a^2*b^3 - 8*
a^3*b*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3
*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6
*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^
3*b*c^4)*x^2) + 3/8*integrate((a*b^3 - 8*a^2*b*c + (b^4 - 9*a*b^2*c + 28*a^
2*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

Fricas [B] time = 3.58792, size = 9721, normalized size = 24.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + 2*(3*b^5 - 20*a*b^3*c
- 4*a^2*b*c^2)*x^5 + 2*(6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c^2)*x^3 + 3*sqrt(1
/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6
*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)
*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^
4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 12
80*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 -
1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c
^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 2
0*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a
^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5
*b^2*c^3 + 38416*a^6*c^4)*x + 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*
b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*
a^6*b*c^6 - (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8
 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*
```

c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) - 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4)*x - 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 - (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4)*x + 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 8192

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0*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 -
 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*
c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 -
 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^
5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1
024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 +
 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*
c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*
a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) - 3*sqr
t(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*
c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2
- 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c
^4)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680
*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 +
 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2
 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^
6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5
- 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 102
4*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*
a^5*b^2*c^3 + 38416*a^6*c^4)*x - 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a
^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 250
88*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*
c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a
^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401
*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13
+ 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^
5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a
^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8
- 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^1
0 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14
 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^
4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + 6*(a^2*b^3 - 8*a^3*b*c)*x)/((
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*
a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4
*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)
```

Sympy [B] time = 23.413, size = 804, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -(x**7*(44*a**2*c**3 - 37*a*b**2*c**2 + 5*b**4*c) + x**5*(-4*a**2*b*c**2 -
20*a*b**3*c + 3*b**5) + x**3*(28*a**3*c**2 - 49*a**2*b**2*c + 6*a*b**4) + x
*(-24*a**3*b*c + 3*a**2*b**3))/(128*a**4*c**4 - 64*a**3*b**2*c**3 + 8*a**2*
b**4*c**2 + x**8*(128*a**2*c**6 - 64*a*b**2*c**5 + 8*b**4*c**4) + x**6*(256
*a**2*b*c**5 - 128*a*b**3*c**4 + 16*b**5*c**3) + x**4*(256*a**3*c**5 - 48*a
*b**4*c**3 + 8*b**6*c**2) + x**2*(256*a**3*b*c**4 - 128*a**2*b**3*c**3 + 16
*a*b**5*c**2)) + RootSum(_t**4*(68719476736*a**10*c**15 - 171798691840*a**9
*b**2*c**14 + 193273528320*a**8*b**4*c**13 - 128849018880*a**7*b**6*c**12 +
 56371445760*a**6*b**8*c**11 - 16911433728*a**5*b**10*c**10 + 3523215360*a*
*4*b**12*c**9 - 503316480*a**3*b**14*c**8 + 47185920*a**2*b**16*c**7 - 2621
440*a*b**18*c**6 + 65536*b**20*c**5) + _t**2*(-3963617280*a**9*b*c**9 + 693
6330240*a**8*b**3*c**8 - 5400428544*a**7*b**5*c**7 + 2464874496*a**6*b**7*c
**6 - 730054656*a**5*b**9*c**5 + 146165760*a**4*b**11*c**4 - 19860480*a**3*
b**13*c**3 + 1771776*a**2*b**15*c**2 - 94464*a*b**17*c + 2304*b**19) + 4978
7136*a**9*c**4 - 27433728*a**8*b**2*c**3 + 6446304*a**7*b**4*c**2 - 734832*
a**6*b**6*c + 35721*a**5*b**8, Lambda(_t, _t*log(x + (234881024*_t**3*a**7*
c**12 - 335544320*_t**3*a**6*b**2*c**11 + 203423744*_t**3*a**5*b**4*c**10 -
 68157440*_t**3*a**4*b**6*c**9 + 13762560*_t**3*a**3*b**8*c**8 - 1703936*_t
**3*a**2*b**10*c**7 + 122880*_t**3*a*b**12*c**6 - 4096*_t**3*b**14*c**5 - 8
580096*_t*a**6*b*c**6 + 6582528*_t*a**5*b**3*c**5 - 2387520*_t*a**4*b**5*c*
*4 + 498096*_t*a**3*b**7*c**3 - 62496*_t*a**2*b**9*c**2 + 4464*_t*a*b**11*c
 - 144*_t*b**13)/(1037232*a**6*c**4 - 518616*a**5*b**2*c**3 + 113103*a**4*b
**4*c**2 - 12069*a**3*b**6*c + 567*a**2*b**8))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.882 $\int \frac{x^8}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=348

$$\frac{\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^5 (\ldots}{4(b^2-4ac\ldots}$$

```
[Out] -((b^2 + 20*a*c)*x)/(8*c*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x^2))/(4*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) + (x^3*(12*a*b + (b^2 + 20*a*c)*x^2))/(8*(b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3 - 16*a*b*c - (b^4 - 18*a*b^2*c - 40
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((b^3 - 16*a*b*c + (b^4 - 18*a*b^2*c - 40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b
^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.885895, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1120, 1275, 1279, 1166, 205}

$$\frac{\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^5 (\ldots}{4(b^2-4ac\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b^2 + 20*a*c)*x)/(8*c*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x^2))/(4*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) + (x^3*(12*a*b + (b^2 + 20*a*c)*x^2))/(8*(b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3 - 16*a*b*c - (b^4 - 18*a*b^2*c - 40
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((b^3 - 16*a*b*c + (b^4 - 18*a*b^2*c - 40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b
^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{x^4\left(10a-bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^2\left(36ab+\left(b^2+20ac\right)x^2\right)}{a+bx^2+cx^4}\,dx}{8\left(b^2-4ac\right)^2}$$

$$= -\frac{\left(b^2+20ac\right)x}{8c\left(b^2-4ac\right)^2} + \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\int \frac{a\left(b^2+20ac\right)+}{a+bx}}{8c\left(b^2\right.}$$

$$= -\frac{\left(b^2+20ac\right)x}{8c\left(b^2-4ac\right)^2} + \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(b^3-16abc\right.}{}$$

$$= -\frac{\left(b^2+20ac\right)x}{8c\left(b^2-4ac\right)^2} + \frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^3\left(12ab+\left(b^2+20ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(b^3-16abc\right.}{8\sqrt{2}c}$$

Mathematica [A] time = 0.97939, size = 381, normalized size = 1.09

$$\frac{\frac{2x\left(-36a^2c^2+11ab^2c-16abc^2x^2+b^3cx^2-2b^4\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(40a^2c^2+b^3\sqrt{b^2-4ac}+18ab^2c-16abc\sqrt{b^2-4ac}-b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-40a^2c^2+b^3\sqrt{b^2-4ac}-18\right.}{\left(b^2-4\right.}}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4 + 11*a*b^2*c - 36*a^2*c^2 + b^3*c*x^2 - 16*a*b*c^2*x^2))/((b^
2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (4*(-2*a^2*c*x + b^3*x^3 + a*b*x*(b - 3
*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-b^4 +
18*a*b^2*c + 40*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b^4 - 18*a*b^2*c - 4
0*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

Maple [B] time = 0.211, size = 953, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*b*(16*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-1/8*(36*a^2*c^2+5*a*b^2
*c+b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*a/c*b*(14*a*c+b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^3-1/8*a^2*(20*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c
*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b-1/16
/(16*a^2*c^2-8*a*b^2*c+b^4)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3-5/2/(16*a^2*c^2-8*a
*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2-9/8/(16*a^2*c^2-
8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2+1/16/(16*a^2*
c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4-1/(16*a^2
*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/
c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^3-5/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*a^2-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*a*b^2+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^3c-16\,abc^2\right)x^7-\left(b^4+5\,ab^2c+36\,a^2c^2\right)x^5-2\left(ab^3+14\,a^2bc\right)x^3-\left(a^2b^2+20\,a^3\right.}{8\left(\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)x^8+a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3+2\left(b^5c^2-8\,ab^3c^3+16\,a^2bc^4\right)x^6+\left(b^6c-6\,ab^4c^2+32\,a^3c^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^3*c - 16*a*b*c^2)*x^7 - (b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^5 - 2*(a*b
^3 + 14*a^2*b*c)*x^3 - (a^2*b^2 + 20*a^3*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b
^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2) - 1/8*integrate(-(a*b^2 + 20*a^2
*c + (b^3 - 16*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)
```

Fricas [B] time = 2.56748, size = 8352, normalized size = 24.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^3*c - 16*a*b*c^2)*x^7 - 2*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^5 - 4
*(a*b^3 + 14*a^2*b*c)*x^3 + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 +
16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8
*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2
 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^
4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c
^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4
*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 6
40*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2
*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x + 1/2*sqrt(1/2)*(b^10 - 17*a*
b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*
c^5 - (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 345
60*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*
c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c
^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2
*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 64
0*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 6
25*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 +
1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6
*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - sqrt(1/2)*((b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4
*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2
+ 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(
b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*
c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*
```

sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x - 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 - (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11))) *sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x + 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 + (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x - 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 + (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c

```
^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b
*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*
a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-
(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8
*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)
*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b
^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 -
 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024
*a^5*c^8))) - 2*(a^2*b^2 + 20*a^3*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3
+ 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c -
 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)
```

Sympy [B] time = 16.6963, size = 716, normalized size = 2.06

$$-\frac{x^7\left(16abc^2-b^3c\right)+x^5\left(36a^2c^2+5ab^2c+b^4\right)+x^3\left(28a^2bc+2ab^3\right)+x\left(20a^3\right.}{128a^4c^3-64a^3b^2c^2+8a^2b^4c+x^8\left(128a^2c^5-64ab^2c^4+8b^4c^3\right)+x^6\left(256a^2bc^4-128ab^3c^3+16b^5c^2\right)+x^4\left(256a^3c^4-48\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -(x**7*(16*a*b*c**2 - b**3*c) + x**5*(36*a**2*c**2 + 5*a*b**2*c + b**4) + x
**3*(28*a**2*b*c + 2*a*b**3) + x*(20*a**3*c + a**2*b**2))/(128*a**4*c**3 -
64*a**3*b**2*c**2 + 8*a**2*b**4*c + x**8*(128*a**2*c**5 - 64*a*b**2*c**4 +
8*b**4*c**3) + x**6*(256*a**2*b*c**4 - 128*a*b**3*c**3 + 16*b**5*c**2) + x*
*4*(256*a**3*c**4 - 48*a*b**4*c**2 + 8*b**6*c) + x**2*(256*a**3*b*c**3 - 12
8*a**2*b**3*c**2 + 16*a*b**5*c)) + RootSum(_t**4*(68719476736*a**10*c**13 -
 171798691840*a**9*b**2*c**12 + 193273528320*a**8*b**4*c**11 - 128849018880
*a**7*b**6*c**10 + 56371445760*a**6*b**8*c**9 - 16911433728*a**5*b**10*c**8
 + 3523215360*a**4*b**12*c**7 - 503316480*a**3*b**14*c**6 + 47185920*a**2*b
**16*c**5 - 2621440*a*b**18*c**4 + 65536*b**20*c**3) + _t**2*(-440401920*a*
*8*b*c**8 + 477102080*a**7*b**3*c**7 - 174325760*a**6*b**5*c**6 + 11206656*
a**5*b**7*c**5 + 8929280*a**4*b**9*c**4 - 2600960*a**3*b**11*c**3 + 291840*
a**2*b**13*c**2 - 14080*a*b**15*c + 256*b**17) + 160000*a**7*c**4 + 492800*
a**6*b**2*c**3 + 351456*a**5*b**4*c**2 - 43120*a**4*b**6*c + 1225*a**3*b**8
, Lambda(_t, _t*log(x + (218103808*_t**3*a**6*b*c**9 - 276824064*_t**3*a**5
*b**3*c**8 + 141557760*_t**3*a**4*b**5*c**7 - 36700160*_t**3*a**3*b**7*c**6
 + 4915200*_t**3*a**2*b**9*c**5 - 294912*_t**3*a*b**11*c**4 + 4096*_t**3*b*
*13*c**3 + 256000*_t*a**5*c**5 - 888320*_t*a**4*b**2*c**4 - 57472*_t*a**3*b
**4*c**3 + 13664*_t*a**2*b**6*c**2 - 832*_t*a*b**8*c + 16*_t*b**10)/(10000*
```

```
a**4*c**3 + 15000*a**3*b**2*c**2 - 1491*a**2*b**4*c + 35*a*b**6))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.883 $\int \frac{x^6}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=298

$$\frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(x^2\left(4ac+b^2\right)+4ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\left(-\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{b(12ac}{\sqrt{b^2-}}\right.}{8\sqrt{2}}$$

```
[Out] (x^3*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x*(4*a*b +
 (b^2 + 4*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2 + 4*
a*c - (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (3*(b^2 + 4*a*c + (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(
b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.682967, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1275, 1166, 205}

$$\frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(x^2\left(4ac+b^2\right)+4ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\left(-\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{b(12ac}{\sqrt{b^2-}}\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^3*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x*(4*a*b +
 (b^2 + 4*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2 + 4*
a*c - (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (3*(b^2 + 4*a*c + (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(
b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{x^2\left(6a-3bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4ab+\left(b^2+4ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\int \frac{12ab-3\left(b^2+4ac\right)x^2}{a+bx^2+cx^4}\,dx}{8\left(b^2-4ac\right)^2}$$

$$= \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4ab+\left(b^2+4ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3\left(b^2+4ac-\frac{b\left(b^2+12ac\right)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\frac{b}{2}}}{16\left(b^2-4ac\right)^2}$$

$$= \frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x\left(4ab+\left(b^2+4ac\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\left(b^2+4ac-\frac{b\left(b^2+12ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{}}}$$

Mathematica [A] time = 0.874455, size = 343, normalized size = 1.15

$$\frac{\frac{8abcx+24ac^2x^3+6b^2cx^3+4b^3x}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{4\left(ax\left(b-2cx^2\right)+b^2x^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-12abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2}\right.}{\left(b^2-4ac\right.}}{16c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^3*x + 8*a*b*c*x + 6*b^2*c*x^3 + 24*a*c^2*x^3)/((b^2 - 4*a*c)^2*(a + b
*x^2 + c*x^4)) - (4*(b^2*x^3 + a*x*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^3 - 12*a*b*c + b^2*Sqrt[b^2 - 4*a*c]
 + 4*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sq
rt[c]*(b^3 + 12*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

Maple [B] time = 0.204, size = 753, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (3/8*c*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*b*(16*a*c+5*b^2)/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(4*a*c-19*b^2)*a/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^3+3/2*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-3/4/(16*a^2*c
^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-3/16/(16*a^2*c^2-8*a*b^2*c+b^4)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^2+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*a*b+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^3+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*a+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+9/4/(16*a
^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+3/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^2*c + 4*a*c^2)*x^7 + (5*b^3 + 16*a*b*c)*x^5 + 12*a^2*b*x + (19*a*
b^2 - 4*a^2*c)*x^3)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6
 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2
) + 3/8*integrate(((b^2 + 4*a*c)*x^2 - 4*a*b)/(c*x^4 + b*x^2 + a), x)/(b^4
- 8*a*b^2*c + 16*a^2*c^2)
```

Fricas [B] time = 2.03805, size = 6966, normalized size = 23.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^2*c + 4*a*c^2)*x^7 + 2*(5*b^3 + 16*a*b*c)*x^5 + 24*a^2*b*x + 2*(
19*a*b^2 - 4*a^2*c)*x^3 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c +
 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c +
 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a
*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*
c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 128
0*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 6
40*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*
c + 16*a^2*c^2)*x + 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*
a^3*b*c^3 + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*
b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c
^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b^8*c^2 + 160*
a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*
c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 -
 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4
+ 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (
b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^
5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*
b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^
2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^
4 + 40*a*b^2*c + 16*a^2*c^2)*x - 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b
^3*c^2 - 128*a^3*b*c^3 + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^1
0*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6
 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b
^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^
6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*
a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640
*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) + 3*sqrt(1/2)*((b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6
 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4
+ 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*
a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 12
80*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c
^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8
```

```
*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)
)*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x + 3*sqrt(1/2)*(2*b^7 - 24*a*b^5
*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 - (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b
^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*
c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 128
0*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^
10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5
- 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^
4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*
b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) - 3*sqrt(1/2
)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*
a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*
a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3
*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 +
160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10
*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 -
1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x - 3*sqrt(1/2)*(2*b
^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 - (3*b^12*c - 56*a*b^10*c^
2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^
6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*
b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^
2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280
*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4
 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c
^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)))
)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*
a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)
```

Sympy [B] time = 14.0397, size = 627, normalized size = 2.1

$$\frac{12a^2bx + x^7\left(12ac^2 + 3b^2c\right) + x^5\left(16abc + 5b^3\right) + x^3\left(-4a^2c + 19ab^2\right)}{128a^4c^2 - 64a^3b^2c + 8a^2b^4 + x^8\left(128a^2c^4 - 64ab^2c^3 + 8b^4c^2\right) + x^6\left(256a^2bc^3 - 128ab^3c^2 + 16b^5c\right) + x^4\left(256a^3c^3 - 48ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] (12*a**2*b*x + x**7*(12*a*c**2 + 3*b**2*c) + x**5*(16*a*b*c + 5*b**3) + x**
3*(-4*a**2*c + 19*a*b**2))/(128*a**4*c**2 - 64*a**3*b**2*c + 8*a**2*b**4 +
x**8*(128*a**2*c**4 - 64*a*b**2*c**3 + 8*b**4*c**2) + x**6*(256*a**2*b*c**3
```

```
 - 128*a*b**3*c**2 + 16*b**5*c) + x**4*(256*a**3*c**3 - 48*a*b**4*c + 8*b**
6) + x**2*(256*a**3*b*c**2 - 128*a**2*b**3*c + 16*a*b**5)) + RootSum(_t**4*
(68719476736*a**10*c**11 - 171798691840*a**9*b**2*c**10 + 193273528320*a**8
*b**4*c**9 - 128849018880*a**7*b**6*c**8 + 56371445760*a**6*b**8*c**7 - 169
11433728*a**5*b**10*c**6 + 3523215360*a**4*b**12*c**5 - 503316480*a**3*b**1
4*c**4 + 47185920*a**2*b**16*c**3 - 2621440*a*b**18*c**2 + 65536*b**20*c) +
 _t**2*(-188743680*a**7*b*c**7 + 141557760*a**6*b**3*c**6 - 2359296*a**5*b*
*5*c**5 - 26542080*a**4*b**7*c**4 + 9584640*a**3*b**9*c**3 - 1290240*a**2*b
**11*c**2 + 46080*a*b**13*c + 2304*b**15) + 20736*a**5*c**4 + 103680*a**4*b
**2*c**3 + 142560*a**3*b**4*c**2 + 32400*a**2*b**6*c + 2025*a*b**8, Lambda(
_t, _t*log(x + (33554432*_t**3*a**6*c**7 - 16777216*_t**3*a**5*b**2*c**6 -
10485760*_t**3*a**4*b**4*c**5 + 10485760*_t**3*a**3*b**6*c**4 - 3276800*_t*
*3*a**2*b**8*c**3 + 458752*_t**3*a*b**10*c**2 - 24576*_t**3*b**12*c - 64512
*_t*a**3*b*c**3 - 43776*_t*a**2*b**3*c**2 - 21312*_t*a*b**5*c - 144*_t*b**7
)/(432*a**2*c**2 + 1080*a*b**2*c + 135*b**4))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.884 $\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=289

$$\frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-4ac+7b^2+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-$$

```
[Out] (x*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x*(7*b^2 - 4*a
*c + 12*b*c*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(3*b
^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.705426, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1120, 1178, 1166, 205}

$$\frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-4ac+7b^2+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x*(7*b^2 - 4*a
*c + 12*b*c*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(3*b
^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{2a-5bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\int \frac{3a\left(b^2+4ac\right)-12abcx^2}{a+bx^2+cx^4}\,dx}{8a\left(b^2-4ac\right)^2}$$

$$= \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right.\right.}{8\left(b^2-4ac\right.}$$

$$= \frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4a}\right.}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b}}$$

Mathematica [A] time = 0.717406, size = 285, normalized size = 0.99

$$\frac{1}{8}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{4acx-7b^2x-12bcx^3}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (-7*b^2*x + 4*
a*c*x - 12*b*c*x^3)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt
[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/8
```

Maple [B] time = 0.2, size = 617, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-3/2*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*c*(4*a*c-19*b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^5-1/8*b*(16*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8
*a*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/4/(16*a^2*
c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)
*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x
*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-9/8/(16*a^2*c^2-8*a*b^2*c+b
^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-3/4/(16*a^2*c^2-8*a*b^2*
c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*a-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(12*b*c^2*x^7 + (19*b^2*c - 4*a*c^2)*x^5 + (5*b^3 + 16*a*b*c)*x^3 + 3*
(a*b^2 + 4*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^
6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^
2) - 3/8*integrate((4*b*c*x^2 - b^2 - 4*a*c)/(c*x^4 + b*x^2 + a), x)/(b^4 -
 8*a*b^2*c + 16*a^2*c^2)
```

Fricas [B] time = 2.1491, size = 6978, normalized size = 24.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] -1/16*(24*b*c^2*x^7 + 2*(19*b^2*c - 4*a*c^2)*x^5 + 2*(5*b^3 + 16*a*b*c)*x^3
 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*
a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqr
t(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*
c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20
*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^
7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a
^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x +
 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4 - (a*b^13 -
 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 1433
6*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6
*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*
a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4
*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c +
160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^
10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 -
1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*
b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c
 + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqr
t(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^
2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b
^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 +
16*a^2*c^3)*x - 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*
c^4 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5
*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*
c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*s
qrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^
6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 -
20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*
a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280
*a^5*b^2*c^4 - 1024*a^6*c^5))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*
b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10
 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 10
24*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^
3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*
c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c +
40*a*b^2*c^2 + 16*a^2*c^3)*x + 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2
```

```
*c^3 - 256*a^4*c^4 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7
*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^1
0 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1
024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8
*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/s
qrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*
b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4
*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a
^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*
b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a
^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 -
 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c
 + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*lo
g(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x - 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*
c + 128*a^3*b^2*c^3 - 256*a^4*c^4 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2
 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^
6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*
a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^
10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 -
1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*
c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^
6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) + 6*(a*b^2 + 4
*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^
4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)
```

Sympy [B] time = 14.1283, size = 644, normalized size = 2.23

$$-\frac{12bc^2x^7 + x^5\left(-4ac^2 + 19b^2c\right) + x^3\left(16abc + 5b^3\right) + x\left(12a^2c + 3ab^2\right)}{128a^4c^2 - 64a^3b^2c + 8a^2b^4 + x^8\left(128a^2c^4 - 64ab^2c^3 + 8b^4c^2\right) + x^6\left(256a^2bc^3 - 128ab^3c^2 + 16b^5c\right) + x^4\left(256a^3c^3 - 48ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -(12*b*c**2*x**7 + x**5*(-4*a*c**2 + 19*b**2*c) + x**3*(16*a*b*c + 5*b**3)
+ x*(12*a**2*c + 3*a*b**2))/(128*a**4*c**2 - 64*a**3*b**2*c + 8*a**2*b**4 +
 x**8*(128*a**2*c**4 - 64*a*b**2*c**3 + 8*b**4*c**2) + x**6*(256*a**2*b*c**
3 - 128*a*b**3*c**2 + 16*b**5*c) + x**4*(256*a**3*c**3 - 48*a*b**4*c + 8*b*
*6) + x**2*(256*a**3*b*c**2 - 128*a**2*b**3*c + 16*a*b**5)) + RootSum(_t**4
```

```
*(68719476736*a**11*c**10 - 171798691840*a**10*b**2*c**9 + 193273528320*a**
9*b**4*c**8 - 128849018880*a**8*b**6*c**7 + 56371445760*a**7*b**8*c**6 - 16
911433728*a**6*b**10*c**5 + 3523215360*a**5*b**12*c**4 - 503316480*a**4*b**
14*c**3 + 47185920*a**3*b**16*c**2 - 2621440*a**2*b**18*c + 65536*a*b**20)
+ _t**2*(-188743680*a**7*b*c**7 + 141557760*a**6*b**3*c**6 - 2359296*a**5*b
**5*c**5 - 26542080*a**4*b**7*c**4 + 9584640*a**3*b**9*c**3 - 1290240*a**2*
b**11*c**2 + 46080*a*b**13*c + 2304*b**15) + 20736*a**4*c**5 + 103680*a**3*
b**2*c**4 + 142560*a**2*b**4*c**3 + 32400*a*b**6*c**2 + 2025*b**8*c, Lambda
(_t, _t*log(x + (50331648*_t**3*a**7*b*c**6 - 58720256*_t**3*a**6*b**3*c**5
 + 26214400*_t**3*a**5*b**5*c**4 - 5242880*_t**3*a**4*b**7*c**3 + 327680*_t
**3*a**3*b**9*c**2 + 32768*_t**3*a**2*b**11*c - 4096*_t**3*a*b**13 + 18432*
_t*a**4*c**4 - 78336*_t*a**3*b**2*c**3 - 40320*_t*a**2*b**4*c**2 - 3168*_t*
a*b**6*c - 144*_t*b**8)/(432*a**2*c**3 + 1080*a*b**2*c**2 + 135*b**4*c))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.885 $\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=311

$$-\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(cx^2\left(20ac+b^2\right)+b\left(8ac+b^2\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\cdot$$

```
[Out] -(x*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b*(b^2 + 8
*a*c) + c*(b^2 + 20*a*c)*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
(Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.700862, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1119, 1178, 1166, 205}

$$-\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(cx^2\left(20ac+b^2\right)+b\left(8ac+b^2\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\cdot$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b*(b^2 + 8
*a*c) + c*(b^2 + 20*a*c)*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
(Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\int \frac{b-10cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b\left(b^2+8ac\right)+c\left(b^2+20ac\right)x^2\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b\left(b^2-16ac\right)-c\left(b^2+20ac\right)x^2}{a+bx^2+cx^4}}{8a\left(b^2-4ac\right)^2}$$

$$= -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b\left(b^2+8ac\right)+c\left(b^2+20ac\right)x^2\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(b^2+20ac-\frac{b\left(b^2-52ac\right.}{\sqrt{b^2-4ac}}\right.\right.}{16a\left(b^2\right.}$$

$$= -\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b\left(b^2+8ac\right)+c\left(b^2+20ac\right)x^2\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2+20ac+\frac{b\left(b^2-52a\right.}{\sqrt{b^2-4a}}\right.}{8\sqrt{2}a\left(b^2-4ac\right)^2}$$

Mathematica [A] time = 0.850991, size = 334, normalized size = 1.07

$$\frac{1}{16}\left(\frac{2x\left(8abc+20ac^2x^2+b^2cx^2+b^3\right)}{a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{4x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b\right.}{a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4a}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(b^3 + 8
*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
 + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b
^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^3 + 5
2*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/16
```

Maple [B] time = 0.28, size = 2958, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -9*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^
2*a*x^3*b^2+4*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^
(1/2)+1/2*b/c)^2*a^2*x^3*b+1/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b^6-1/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^
6-1/16*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^7+9*c^3/(-
4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-27*c^3/(-4*a*c+b^2)^
(5/2)/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3+1/16/(-4*a*c+b^2)^(5/2)/(4*
a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a*x^3*b^7+20*c^3/(-4*a*
c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a^2*x^3+1/1
6/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a*x
^3*b^6-3*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)
+1/2*b/c)^2*a*x^3*b^3+20*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*
c+b^2)^(1/2)+1/2*b/c)^2*a^2*x^3+7/8/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1
/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*b^6+3/4/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(
x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*b^5-7/8/(-4*a*c+b^2)^(5/2)/(4*a*c
-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*b^6+3/4/(-4*a*c+b^2)^2/(
4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*b^5+52*c^4/(-4*a*c+
b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+52*c^4/(-4*a*c+b^2)^(5
/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-9*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)
^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b^2-1/16*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^7-27*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b^3+3/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)
+1/2*b/c)^2*x^3*b^4+3/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(
-4*a*c+b^2)^(1/2))^2*x^3*b^4+56*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1
/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*a^3-3/4*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^
2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x^3*b^5+1/16/(-4*a*c+b^2)^2/(
4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a*x^3*b^6-1/16/(-4*a*
c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a*x^3*b
^7+3/4*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2
*b/c)^2*x^3*b^5-21/2*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+
```

```
b^2)^(1/2)+1/2*b/c)^2*x*a*b^4+15/4*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b^5-4*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2
/c*(-4*a*c+b^2)^(1/2))^2*a^2*x^3*b+3*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(
x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a*x^3*b^3-6*c/(-4*a*c+b^2)^2/(4*a*c
-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*a*b^3-20*c^4/(-4*a*c+b^2
)^2/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-3/4*c^2/(-4*a*c+b^2)^2/(4*a*c-b
^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^4-56*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^
2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*a^3+42*c^2/(-4*a*c+b^2)^(5/2)/(4*a*
c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*a^2*b^2-9*c^2/(-4*a*c+b
^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a*x^3*b^2+12*c
^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*
a^2*b-42*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^
2)^(1/2))^2*x*a^2*b^2+21/2*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-
1/2/c*(-4*a*c+b^2)^(1/2))^2*x*a*b^4+15/4*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)
^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*b^5+12*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c
*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*a^2*b-6*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^
2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*a*b^3+20*c^4/(-4*a*c+b^2)^2/(4*a*c-
b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))+3/4*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*c^2 + 20*a*c^3)*x^7 + 2*(b^3*c + 14*a*b*c^2)*x^5 + (b^4 + 5*a*b^2
*c + 36*a^2*c^2)*x^3 - (a*b^3 - 16*a^2*b*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3
+ 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2
*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + 1/8*integrate((b^3 - 16*a*b*c
+ (b^2*c + 20*a*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)
```

Fricas [B] time = 2.70466, size = 8466, normalized size = 27.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*x^7 + 4*(b^3*c + 14*a*b*c^2)*x^5 + 2*(b^4 + 5*
a*b^2*c + 36*a^2*c^2)*x^3 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2
+ 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 -
 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2
 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^
4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c
^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^1
0*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 6
40*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a
*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x + 1/2*sqrt(1/2)*(b^11 - 53*
a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^
5*b*c^5 - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 +
 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*
sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^
6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 -
 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c +
 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt(
(b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2
 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^
4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c
^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 -
 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6
+ (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 +
(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2
*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a
^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^1
1*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 128
0*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2
*b^2*c^4 + 10000*a^3*c^5)*x - 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^
7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 - (a^3*b^14
- 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 +
1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2
*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*
```

c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x + 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x - 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c

```
^4 - 1024*a^8*c^5))) - 2*(a*b^3 - 16*a^2*b*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^
3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a
^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2
*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
```

Sympy [B] time = 15.8848, size = 733, normalized size = 2.36

$$\frac{x^7\left(20ac^3+b^2c^2\right)+x^5\left(28abc^2+2b^3c\right)+x^3\left(36a^2c^2+5ab^2c+b^4\right)+x\left(16a\right.}{128a^5c^2-64a^4b^2c+8a^3b^4+x^8\left(128a^3c^4-64a^2b^2c^3+8ab^4c^2\right)+x^6\left(256a^3bc^3-128a^2b^3c^2+16ab^5c\right)+x^4\left(256a^4c^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] (x**7*(20*a*c**3 + b**2*c**2) + x**5*(28*a*b*c**2 + 2*b**3*c) + x**3*(36*a*
*2*c**2 + 5*a*b**2*c + b**4) + x*(16*a**2*b*c - a*b**3))/(128*a**5*c**2 - 6
4*a**4*b**2*c + 8*a**3*b**4 + x**8*(128*a**3*c**4 - 64*a**2*b**2*c**3 + 8*a
*b**4*c**2) + x**6*(256*a**3*b*c**3 - 128*a**2*b**3*c**2 + 16*a*b**5*c) + x
**4*(256*a**4*c**3 - 48*a**2*b**4*c + 8*a*b**6) + x**2*(256*a**4*b*c**2 - 1
28*a**3*b**3*c + 16*a**2*b**5)) + RootSum(_t**4*(68719476736*a**13*c**10 -
171798691840*a**12*b**2*c**9 + 193273528320*a**11*b**4*c**8 - 128849018880*
a**10*b**6*c**7 + 56371445760*a**9*b**8*c**6 - 16911433728*a**8*b**10*c**5
+ 3523215360*a**7*b**12*c**4 - 503316480*a**6*b**14*c**3 + 47185920*a**5*b*
*16*c**2 - 2621440*a**4*b**18*c + 65536*a**3*b**20) + _t**2*(-440401920*a**
8*b*c**8 + 477102080*a**7*b**3*c**7 - 174325760*a**6*b**5*c**6 + 11206656*a
**5*b**7*c**5 + 8929280*a**4*b**9*c**4 - 2600960*a**3*b**11*c**3 + 291840*a
**2*b**13*c**2 - 14080*a*b**15*c + 256*b**17) + 160000*a**4*c**7 + 492800*a
**3*b**2*c**6 + 351456*a**2*b**4*c**5 - 43120*a*b**6*c**4 + 1225*b**8*c**3,
 Lambda(_t, _t*log(x + (167772160*_t**3*a**10*c**7 - 134217728*_t**3*a**9*b
**2*c**6 + 6291456*_t**3*a**8*b**4*c**5 + 26214400*_t**3*a**7*b**6*c**4 - 1
1141120*_t**3*a**6*b**8*c**3 + 1966080*_t**3*a**5*b**10*c**2 - 155648*_t**3
*a**4*b**12*c + 4096*_t**3*a**3*b**14 - 742400*_t*a**5*b*c**5 - 156928*_t*a
**4*b**3*c**4 - 70336*_t*a**3*b**5*c**3 + 14480*_t*a**2*b**7*c**2 - 848*_t*
a*b**9*c + 16*_t*b**11)/(10000*a**3*c**5 + 15000*a**2*b**2*c**4 - 1491*a*b*
*4*c**3 + 35*b**6*c**2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.886 $\int \frac{1}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=355

$$\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\tan^{-1}\left(\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+}}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*
((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2))/(8*a^2*(b^2 - 4*
a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b
*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (3*Sqrt[c]*(b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt
[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.83832, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1092, 1178, 1166, 205}

$$\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\tan^{-1}\left(\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*
((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2))/(8*a^2*(b^2 - 4*
a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b
*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (3*Sqrt[c]*(b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt
[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{b^2-2ac-4\left(b^2-4ac\right)-5bcx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)+3bc\left(b^2-8ac\right)x^2\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\int \frac{3\left(b^4-9ab^2c\cdots\right.}{\cdots}}{8a\cdots}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)+3bc\left(b^2-8ac\right)x^2\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3c\left(b^4-10\cdots\right.\right.}{\cdots}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)+3bc\left(b^2-8ac\right)x^2\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{c}\left(b^4-1\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.05164, size = 372, normalized size = 1.05

$$\frac{\frac{2x\left(28a^2c^2-25ab^2c-24abc^2x^2+3b^3cx^2+3b^4\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{2}\sqrt{c}\left(56a^2c^2+b^3\sqrt{b^2-4ac}-10ab^2c-8abc\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\sqrt{c}\left(56a^2c^2-b^3\sqrt{b^2-4ac}-1\cdots\right.}{\left(b^2-\cdots\right.}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] ((4*a*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2
*x*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2))/((b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56
*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b^
3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]))/(16*a^2)
```

Maple [B] time = 0.261, size = 3360, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -3*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5+3/16*c/(-4*a*c
+b^2)^2/(4*a*c-b^2)^2/a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x
*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^7-3/16*c/(-4*a*c+b^2)^(5/2)/
(4*a*c-b^2)^2/a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^8+114*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-
b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+114*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b^2-3/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a^2*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^7+27/8*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b^6-3/16*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a^2*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b^8+24*c^4/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-24*c^4
/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+3*c^2/(-4*a*c+b^2)^2/(
4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5+27/8*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c
-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b^6-24*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1
/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a*x^3*b-15*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2
)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b^3+27*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/
c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*a*x*b^2-3*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x
^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a*x^3*b^5-168*c^5/(-4*a*c+b^2)^(5/2)
/(4*a*c-b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+27/8*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)
^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a*x^3*b^6-15*c^2/(-4*a*c+b^2)^(
5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*a*x*b^3-168*c^5
/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-27/8*c/(-4*a*c+
b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a*x^3*b^6
+15*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1
/2))^2*a*x*b^3-3*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+
b^2)^(1/2))^2/a*x^3*b^5+27*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/
2/c*(-4*a*c+b^2)^(1/2))^2*a*x*b^2-57/2*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c
```

```
+b^2)^(1/2))*c)^(1/2))*b^4-57/2*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^4+15*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3
-24*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)
^2*a*x^3*b+20*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^
(1/2)+1/2*b/c)^2*a^2*x*b+66*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c
*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*a*x^3*b^2-15/4*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b
^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*b^5+5/16/(-4*a*c+b^2)^2/(4
*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a*x*b^6+3/16/(-4*a*c+b
^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a^2*x^3*b^7-3/
16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^
2/a^2*x^3*b^8-5/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)
^(1/2)+1/2*b/c)^2/a*x*b^7+3/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/
c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a^2*x^3*b^8+5/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^
2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a*x*b^7+3/16/(-4*a*c+b^2)^2/(
4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a^2*x^3*b^7-44*c^3/(-
4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a^2*x-4
4*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2
*a^2*x-21/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/
2*b/c)^2*x*b^4+15*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^
(1/2)+1/2*b/c)^2*x^3*b^3+15*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1
/2/c*(-4*a*c+b^2)^(1/2))^2*x^3*b^3-21/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2
+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*b^4-72*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c
-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*a^2*x^3-45/2*c^2/(-4*a*c+b
^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x^3*b^4+15
/4*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c
)^2*x*b^5+72*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*
c+b^2)^(1/2))^2*a^2*x^3+45/2*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*
b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x^3*b^4+5/16/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(
x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a*x*b^6-66*c^3/(-4*a*c+b^2)^(5/2)/(
4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a*x^3*b^2-20*c^3/(-4*
a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a^2*x
*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(b^3c^2 - 8\,abc^3\right)x^7 + \left(6\,b^4c - 49\,ab^2c^2 + 28\,a^2c^3\right)x^5 + \left(3\,b^5 - 20\,ab^3c - 4\,a^2bc^2\right)x^3 + \left(5\,ab^4 - 37\right.}{8\left(\left(a^2b^4c^2 - 8\,a^3b^2c^3 + 16\,a^4c^4\right)x^8 + a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2 + 2\left(a^2b^5c - 8\,a^3b^3c^2 + 16\,a^4bc^3\right)x^6 + \left(a^2b^6 - 6\,a^3b^4c + 32\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*x^7 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*x^
5 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 + (5*a*b^4 - 37*a^2*b^2*c + 44*a
^3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^
5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (
a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5
*b*c^2)*x^2) - 3/8*integrate(-(b^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^3*c - 8*a*
b*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [B] time = 3.7484, size = 9839, normalized size = 27.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*x^7 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)
*x^5 + 2*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 - 3*sqrt(1/2)*((a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2
*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c +
32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9
- 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^
10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -
1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3
+ 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^
4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160
*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*
(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*
a^4*c^7)*x + 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a
^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 8
7808*a^7*c^7 - (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*
c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a
^12*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 24
01*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^
3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*
b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160
*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^
8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b
^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c
^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*
```

b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*x - 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7 - (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*x + 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7 + (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 -

```
 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*
c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*
b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5
*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21*a
*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 2
0*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a
^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401
*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
+ 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b
^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*b^
8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^
7)*x - 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8
*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a
^7*c^7 + (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 +
15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*
c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4
*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 12
80*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^
2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b
^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22
*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 -
20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1
024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^
3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 2*(5*a*b^4 - 37*a^2*b^2*c + 44*a^
3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a
^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*
b*c^2)*x^2)
```

Sympy [B] time = 21.0109, size = 818, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -(x**7*(24*a*b*c**3 - 3*b**3*c**2) + x**5*(-28*a**2*c**3 + 49*a*b**2*c**2 -
 6*b**4*c) + x**3*(4*a**2*b*c**2 + 20*a*b**3*c - 3*b**5) + x*(-44*a**3*c**2
 + 37*a**2*b**2*c - 5*a*b**4))/(128*a**6*c**2 - 64*a**5*b**2*c + 8*a**4*b**
4 + x**8*(128*a**4*c**4 - 64*a**3*b**2*c**3 + 8*a**2*b**4*c**2) + x**6*(256
```

```
*a**4*b*c**3 - 128*a**3*b**3*c**2 + 16*a**2*b**5*c) + x**4*(256*a**5*c**3 -
 48*a**3*b**4*c + 8*a**2*b**6) + x**2*(256*a**5*b*c**2 - 128*a**4*b**3*c +
16*a**3*b**5)) + RootSum(_t**4*(68719476736*a**15*c**10 - 171798691840*a**1
4*b**2*c**9 + 193273528320*a**13*b**4*c**8 - 128849018880*a**12*b**6*c**7 +
 56371445760*a**11*b**8*c**6 - 16911433728*a**10*b**10*c**5 + 3523215360*a*
*9*b**12*c**4 - 503316480*a**8*b**14*c**3 + 47185920*a**7*b**16*c**2 - 2621
440*a**6*b**18*c + 65536*a**5*b**20) + _t**2*(-3963617280*a**9*b*c**9 + 693
6330240*a**8*b**3*c**8 - 5400428544*a**7*b**5*c**7 + 2464874496*a**6*b**7*c
**6 - 730054656*a**5*b**9*c**5 + 146165760*a**4*b**11*c**4 - 19860480*a**3*
b**13*c**3 + 1771776*a**2*b**15*c**2 - 94464*a*b**17*c + 2304*b**19) + 4978
7136*a**4*c**9 - 27433728*a**3*b**2*c**8 + 6446304*a**2*b**4*c**7 - 734832*
a*b**6*c**6 + 35721*b**8*c**5, Lambda(_t, _t*log(x + (184549376*_t**3*a**12
*b*c**7 - 276824064*_t**3*a**11*b**3*c**6 + 177209344*_t**3*a**10*b**5*c**5
 - 62914560*_t**3*a**9*b**7*c**4 + 13434880*_t**3*a**8*b**9*c**3 - 1736704*
_t**3*a**7*b**11*c**2 + 126976*_t**3*a**6*b**13*c - 4096*_t**3*a**5*b**15 +
 6322176*_t*a**7*c**7 - 13515264*_t*a**6*b**2*c**6 + 8576640*_t*a**5*b**4*c
**5 - 2831328*_t*a**4*b**6*c**4 + 556416*_t*a**3*b**8*c**3 - 66816*_t*a**2*
b**10*c**2 + 4608*_t*a*b**12*c - 144*_t*b**14)/(1037232*a**4*c**7 - 518616*
a**3*b**2*c**6 + 113103*a**2*b**4*c**5 - 12069*a*b**6*c**4 + 567*b**8*c**3)
)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.887 $\int \frac{1}{x^2(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=425

$$\frac{36a^2c^2+bcx^2\left(5b^2-32ac\right)-35ab^2c+5b^4}{8a^2x\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}-$$

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*
c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)^2) + (5*b^4 - 35*a*b^
2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x^2)/(8*a^2*(b^2 - 4*a*c)^2*x*(a +
b*x^2 + c*x^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 -
47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 47*a*b
^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```

Rubi [A] time = 0.962972, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2+bcx^2\left(5b^2-32ac\right)-35ab^2c+5b^4}{8a^2x\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\sqrt{c}\left(\frac{b\left(124a^2c^2-47ab^2c+5b^4\right)}{\sqrt{b^2-4ac}}+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*
c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)^2) + (5*b^4 - 35*a*b^
2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x^2)/(8*a^2*(b^2 - 4*a*c)^2*x*(a +
b*x^2 + c*x^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 -
47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 47*a*b
```

```
^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx = \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{-5b^2+18ac-7bcx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc\left(5b^2-32ac\right)x^2}{8a^2\left(b^2-4ac\right)^2x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{3\left(5b^2-12ac\right.}{\ldots}}{8\ldots}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc\ldots}{8a^2\left(b^2-4ac\right)^2x\left(a+\ldots\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc\ldots}{8a^2\left(b^2-4ac\right)^2x\left(a+\ldots\right.}$$

$$= -\frac{3\left(5b^2-12ac\right)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc\ldots}{8a^2\left(b^2-4ac\right)^2x\left(a+\ldots\right.}$$

Mathematica [A] time = 1.80367, size = 454, normalized size = 1.07

$$-\frac{\frac{2x\left(84a^2bc^2+52a^2c^3x^2-47ab^2c^2x^2-52ab^3c+7b^4cx^2+7b^5\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{2}\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}+124a^2bc^2+5b^4\sqrt{b^2-4ac}-47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4\ldots}}}\right.}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\ldots}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] -(16/x + (4*a*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)^2) + (2*x*(7*b^5 - 52*a*b^3*c + 84*a^2*b*c^2 + 7*b^4*c*x
^2 - 47*a*b^2*c^2*x^2 + 52*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^
4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b
^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/
```

```
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^5 + 47*a*b^3*c -
 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 6
0*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*a^3)
```

Maple [B] time = 0.222, size = 1567, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -1/a^3/x-9/8/a^2/(c*x^4+b*x^2+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x-13/2/a/
(c*x^4+b*x^2+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-7/8/a^3/(c*x^4+b*x^2+a
)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^6-27/2/(c*x^4+b*x^2+a)^2*b/(16*a^2*c^2
-8*a*b^2*c+b^4)*x*c^2-17/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3
*c^3+93/4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b-141/16/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*b^3+93/4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b+15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^5+15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5-141/16/a^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3-111/16/a^2/(16*a^2*c^2-8*
a*b^2*c+b^4)*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+111/16/a^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*b^4-15/16/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*b^4-7/8/a^3/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7*b^4-
7/4/a^3/(c*x^4+b*x^2+a)^2*c*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+47/8/a^2/(c*
x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7*b^2-17/a/(c*x^4+b*x^2+a)^
2*c^3*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+99/8/a^2/(c*x^4+b*x^2+a)^2*c^2*b^3/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^5-25/8/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*
```

```
c+b^4)*x^3*b^2*c^2+43/8/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
3*b^4*c+33/4/a/(c*x^4+b*x^2+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c+45/4/a/
(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-45/4/a/(16*a^2*c^2-8*a
*b^2*c+b^4)*c^3*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + (30*b^5*c - 227*a*b^3
*c^2 + 392*a^2*b*c^3)*x^6 + 8*a^2*b^4 - 64*a^3*b^2*c + 128*a^4*c^2 + (15*b^
6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*x^4 + (25*a*b^5 - 194*a^2*b^
3*c + 364*a^3*b*c^2)*x^2)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 +
 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c
+ 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*x) - 3/8*integrate((5*b^5 - 42*a*b^3*c + 92*a^
2*b*c^2 + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*x^2)/(c*x^4 + b*x^2 + a), x
)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
```

Fricas [B] time = 5.67095, size = 11826, normalized size = 27.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + 2*(30*b^5*c - 227*a*
b^3*c^2 + 392*a^2*b*c^3)*x^6 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2
*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*x^4 + 2*(25*a*b^5 - 1
94*a^2*b^3*c + 364*a^3*b*c^2)*x^2 + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (
a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6
*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*
```

a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*
a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 +
1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725
*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5
+ 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*
b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 1
60*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log
(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^
4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x + 27/2*sqrt(1/2)*(125*b^17
- 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^
9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1
324800*a^8*b*c^8 - (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960
*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4
*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10
*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a
^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 -
640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 4
95*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 184
80*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^
3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94
725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*
c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^
17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c
+ 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)))
- 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5
*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^
3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 1501
5*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^
8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^
5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3
+ 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20
*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 102
4*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3
+ 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77825*a*b^8
*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810
000*a^5*c^9)*x - 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*
c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 714
6736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 - (5*a^7*b^16 -
152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c
^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 1228
80*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a
^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14
*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2
*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 1

5015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x + 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 + (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77

```
825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*
c^8 - 810000*a^5*c^9)*x - 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*
a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*
c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 + (5*a^
7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a
^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c
^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 -
 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c
^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280
*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^
7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10
 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 -
1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310
*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^
14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b
^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*
a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))))/((a^3*b^4*c^2 - 8*a^4*
b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^
7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 1
6*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)
```

Sympy [B] time = 77.1202, size = 925, normalized size = 2.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] RootSum(_t**4*(68719476736*a**17*c**10 - 171798691840*a**16*b**2*c**9 + 193
273528320*a**15*b**4*c**8 - 128849018880*a**14*b**6*c**7 + 56371445760*a**1
3*b**8*c**6 - 16911433728*a**12*b**10*c**5 + 3523215360*a**11*b**12*c**4 -
503316480*a**10*b**14*c**3 + 47185920*a**9*b**16*c**2 - 2621440*a**8*b**18*
c + 65536*a**7*b**20) + _t**2*(43599790080*a**10*b*c**10 - 119899422720*a**
9*b**3*c**9 + 144424304640*a**8*b**5*c**8 - 101155405824*a**7*b**7*c**7 + 4
5862502400*a**6*b**9*c**6 - 14115778560*a**5*b**11*c**5 + 2994877440*a**4*b
**13*c**4 - 433370880*a**3*b**15*c**3 + 40997376*a**2*b**17*c**2 - 2292480*
a*b**19*c + 57600*b**21) + 1049760000*a**4*c**11 - 872467200*a**3*b**2*c**1
0 + 277507296*a**2*b**4*c**9 - 39988080*a*b**6*c**8 + 2205225*b**8*c**7, La
mbda(_t, _t*log(x + (-503316480*_t**3*a**15*c**8 + 1325400064*_t**3*a**14*b
**2*c**7 - 1402994688*_t**3*a**13*b**4*c**6 + 807403520*_t**3*a**12*b**6*c*
*5 - 281149440*_t**3*a**11*b**8*c**4 + 61276160*_t**3*a**10*b**10*c**3 - 82
```

```
16576*_t**3*a**9*b**12*c**2 + 622592*_t**3*a**8*b**14*c - 20480*_t**3*a**7*
b**16 - 255052800*_t*a**8*b*c**8 + 869670144*_t*a**7*b**3*c**7 - 1044793152
*_t*a**6*b**5*c**6 + 644886000*_t*a**5*b**7*c**5 - 233907696*_t*a**4*b**9*c
**4 + 52233552*_t*a**3*b**11*c**3 - 7107840*_t*a**2*b**13*c**2 + 543600*_t*
a*b**15*c - 18000*_t*b**17)/(21870000*a**5*c**9 - 76545000*a**4*b**2*c**8 +
 52848423*a**3*b**4*c**7 - 15417810*a**2*b**6*c**6 + 2101275*a*b**8*c**5 -
111375*b**10*c**4)))) - (128*a**4*c**2 - 64*a**3*b**2*c + 8*a**2*b**4 + x**
8*(180*a**2*c**4 - 111*a*b**2*c**3 + 15*b**4*c**2) + x**6*(392*a**2*b*c**3
- 227*a*b**3*c**2 + 30*b**5*c) + x**4*(324*a**3*c**3 + 25*a**2*b**2*c**2 -
91*a*b**4*c + 15*b**6) + x**2*(364*a**3*b*c**2 - 194*a**2*b**3*c + 25*a*b**
5))/(x**9*(128*a**5*c**4 - 64*a**4*b**2*c**3 + 8*a**3*b**4*c**2) + x**7*(25
6*a**5*b*c**3 - 128*a**4*b**3*c**2 + 16*a**3*b**5*c) + x**5*(256*a**6*c**3
- 48*a**4*b**4*c + 8*a**3*b**6) + x**3*(256*a**6*b*c**2 - 128*a**5*b**3*c +
 16*a**4*b**5) + x*(128*a**7*c**2 - 64*a**6*b**2*c + 8*a**5*b**4))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.888 $\int \frac{x^5}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=82

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

```
[Out] x^2/(2*c) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) + (b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 0.091598, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1114, 703, 634, 618, 206, 628}

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) + (b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a-bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{a-bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2c}+\frac{\operatorname{Subst}\left(\int \frac{-a+bx}{a-bx+cx^2}\,dx, x, x^2\right)}{2c}\\
&= \frac{x^2}{2c}+\frac{b\operatorname{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx, x, x^2\right)}{4c^2}+\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx, x, x^2\right)}{4c^2}\\
&= \frac{x^2}{2c}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}-\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, -b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c}+\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.0527682, size = 80, normalized size = 0.98

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log\left(a-bx^2+cx^4\right)+2cx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] time = 0.173, size = 116, normalized size = 1.4

$$\frac{x^2}{2c}+\frac{b\ln\left(cx^4-bx^2+a\right)}{4c^2}-\frac{a}{c}\arctan\left((2cx^2-b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{b^2}{2c^2}\arctan\left((2cx^2-b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/2*x^2/c+1/4*b*ln(c*x^4-b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
-b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*
c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56773, size = 556, normalized size = 6.78

$$\left[\frac{2\left(b^2c-4\,ac^2\right)x^2-\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac+\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+\left(b^3-4\,abc\right)\log\left(cx^4-bx^2+a\right)}{4\left(b^2c^2-4\,ac^3\right)}, \frac{2\left(b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 - 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b
*x^2 + a)) + (b^3 - 4*a*b*c)*log(c*x^4 - b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*log(c*x^4 -
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 1.60576, size = 311, normalized size = 3.79

$$\left(\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{ab-8ac^2\left(\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)+2b^2c\left(\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(\frac{b}{4c^2}+\frac{\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4-b*x**2+a),x)
```

```
[Out] (b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*l
og(x**2 + (a*b - 8*a*c**2*(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*
c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (b/(4*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (a*b - 8*a*
c**2*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2
))) + 2*b**2*c*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*
a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

Giac [A] time = 1.34545, size = 105, normalized size = 1.28

$$\frac{x^2}{2\,c} + \frac{b\log\left(cx^4 - bx^2 + a\right)}{4\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^2 - b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c + 1/4*b*log(c*x^4 - b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.889 $\int \frac{x^3}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=64

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a - b*x^2 + c*x^4]/(4*c)
```

Rubi [A] time = 0.0621696, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1114, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a - b*x^2 + c*x^4]/(4*c)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a-bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a-bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4c}+\frac{b\operatorname{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x,x^2\right)}{4c}\\
&= \frac{\log\left(a-bx^2+cx^4\right)}{4c}-\frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+2cx^2\right)}{2c}\\
&= \frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}
\end{aligned}$$

Mathematica [A] time = 0.0232969, size = 65, normalized size = 1.02

$$\frac{\frac{2b\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\log\left(a-bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] ((2*b*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 - b*x^2 + c*x^4])/(4*c)
```

Maple [A] time = 0.158, size = 63, normalized size = 1.

$$\frac{\ln\left(cx^4-bx^2+a\right)}{4c}+\frac{b}{2c}\arctan\left((2cx^2-b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/4*ln(c*x^4-b*x^2+a)/c+1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47675, size = 444, normalized size = 6.94

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^4-bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)},-\frac{2\sqrt{-b^2+4\,ac}b\arctan\left(-\frac{(2\,cx^2-b)}{b^2\text{-}}\right.}{4(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x
^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 -
 b*x^2 + a))/(b^2*c - 4*a*c^2), -1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x
^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^4 - b*x^2
```

```
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] time = 0.842036, size = 223, normalized size = 3.48

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\frac{8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)-2a-2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (8*a*c*(
-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) - 2*a - 2*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (8*a*c*(b*sqrt(-4*a*c + b**
2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) - 2*a - 2*b**2*(b*sqrt(-4*a*c + b**2)/(4
*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

Giac [A] time = 1.36896, size = 84, normalized size = 1.31

$$\frac{b\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c}+\frac{\log\left(cx^4-bx^2+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/4
*log(c*x^4 - b*x^2 + a)/c
```

3.890 $\int \frac{x}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.0430031, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1107, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2 + c*x^4),x]
```

```
[Out] ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{a-bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx, x, x^2\right)$$
$$= -\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, -b+2cx^2\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{-b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.0087044, size = 41, normalized size = 1.17

$$\frac{\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.159, size = 38, normalized size = 1.1

$$\arctan\left((2\,cx^2-b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44559, size = 290, normalized size = 8.29

$$\left[\frac{\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\left(2\,cx^2-b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 - b)*sqrt(b^2 - 4*
a*c))/(c*x^4 - b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] time = 0.471686, size = 131, normalized size = 3.74

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}-b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}-b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4-b*x**2+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) - b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) - b)/(2*c))/2
```

Giac [A] time = 1.40388, size = 50, normalized size = 1.43

$$\frac{\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.891 $\int \frac{1}{x(a-bx^2+cx^4)} \, dx$

Optimal. Leaf size=70

$$\frac{b \tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} - \frac{\log\left(a-bx^2+cx^4\right)}{4a} + \frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a - b*x^2 + c*x^4]/(4*a)
```

Rubi [A] time = 0.0837868, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1114, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} - \frac{\log\left(a-bx^2+cx^4\right)}{4a} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a - b*x^2 + c*x^4]/(4*a)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a-bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(a-bx+cx^2\right)}\,dx,x,x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a} + \frac{\operatorname{Subst}\left(\int \frac{b-cx}{a-bx+cx^2}\,dx,x,x^2\right)}{2a}$$
$$= \frac{\log(x)}{a} - \frac{\operatorname{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4a} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x,x^2\right)}{4a}$$
$$= \frac{\log(x)}{a} - \frac{\log\left(a-bx^2+cx^4\right)}{4a} - \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+2cx^2\right)}{2a}$$
$$= \frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a-bx^2+cx^4\right)}{4a}$$

Mathematica [A] time = 0.0713104, size = 117, normalized size = 1.67

$$\frac{\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}-b+2cx^2\right)-\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}-b+2cx^2\right)+4\log(x)\sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] + (b - Sqrt[b^2 - 4*a*c])*Log[-b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2] - (b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.165, size = 69, normalized size = 1.

$$\frac{\ln(x)}{a} - \frac{\ln\left(cx^4-bx^2+a\right)}{4\,a} + \frac{b}{2\,a}\arctan\left((2\,cx^2-b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4-b*x^2+a),x)
```

```
[Out] ln(x)/a-1/4*ln(c*x^4-b*x^2+a)/a+1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b
)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56106, size = 512, normalized size = 7.31

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^4-bx^2+a\right)+4\left(b^2-4\,ac\right)\log\left(x\right)}{4\left(ab^2-4\,a^2c\right)},-\frac{2\sqrt{-b^2+}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x
^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 -
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), -1/4*(2*sqrt(-b^2
+ 4*a*c)*b*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 -
 4*a*c)*log(c*x^4 - b*x^2 + a) - 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] time = 3.0117, size = 253, normalized size = 3.61

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)-2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)+2ac-b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (8*a**2*
c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(-b*sq
rt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c)) +
(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (8*a**2*c
*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(b*sqrt(
-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c)) + log
(x)/a
```

Giac [A] time = 1.33826, size = 96, normalized size = 1.37

$$\frac{b\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a}-\frac{\log\left(cx^4-bx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/4
*log(c*x^4 - b*x^2 + a)/a + 1/2*log(x^2)/a
```

3.892 $\int \frac{1}{x^3\left(a-bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=89

$$\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) + (b*Log[x])/a^2 - (b*Log[a - b*x^2 + c*x^4])/(4*a^2
)
```

Rubi [A] time = 0.139838, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) + (b*Log[x])/a^2 - (b*Log[a - b*x^2 + c*x^4])/(4*a^2
)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a-bx^2+cx^4\right)}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a-bx+cx^2\right)}\,dx,x,x^2\right)$$
$$=-\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \frac{b-cx}{x\left(a-bx+cx^2\right)}\,dx,x,x^2\right)}{2a}$$
$$=-\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \left(\frac{b}{ax}-\frac{-b^2+ac+bcx}{a\left(a-bx+cx^2\right)}\right)\,dx,x,x^2\right)}{2a}$$
$$=-\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{\text{Subst}\left(\int \frac{-b^2+ac+bcx}{a-bx+cx^2}\,dx,x,x^2\right)}{2a^2}$$
$$=-\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\,\text{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x,x^2\right)}{4a^2}$$
$$=-\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+2cx^2\right)}{2a^2}$$
$$=-\frac{1}{2ax^2}+\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}$$

Mathematica [A] time = 0.142451, size = 139, normalized size = 1.56

$$\frac{\frac{\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}-b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(\sqrt{b^2-4ac}-b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{x^2}+4b\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^2 + 4*b*Log[x] + ((b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*Log[-b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((b^2 - 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A] time = 0.166, size = 123, normalized size = 1.4

$$-\frac{1}{2\,ax^2}+\frac{b\ln(x)}{a^2}-\frac{b\ln\left(cx^4-bx^2+a\right)}{4\,a^2}-\frac{c}{a}\arctan\left((2\,cx^2-b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{2\,a^2}\arctan\left((2\,cx^2-b)\frac{1}{\sqrt{4\,a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/2/a/x^2+b*ln(x)/a^2-1/4*b*ln(c*x^4-b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c
*x^2-b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59929, size = 664, normalized size = 7.46

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac+\left(2\,cx^2-b\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+\left(b^3-4\,abc\right)x^2\log\left(cx^4-bx^2+a\right)-4\left(b^3-4\,abc\right)x^2\,\mathrm{l}}{4\left(a^2b^2-4\,a^3c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 - 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) + (b^3 - 4
*a*b*c)*x^2*log(c*x^4 - b*x^2 + a) - 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4
*a*b*c)*x^2*log(c*x^4 - b*x^2 + a) - 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [B] time = 7.50391, size = 350, normalized size = 3.93

$$\left(-\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-8a^3c\left(-\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+2a^2b^2\left(-\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)-3abc+b}{2ac^2-b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*
log(x**2 + (-8*a**3*c*(-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
a**2*(4*a*c - b**2))) + 2*a**2*b**2*(-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a
*c - b**2)/(4*a**2*(4*a*c - b**2))) - 3*a*b*c + b**3)/(2*a*c**2 - b**2*c))
+ (-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))
)*log(x**2 + (-8*a**3*c*(-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2
*a*c - b**2)/(4*a**2*(4*a*c - b**2))) - 3*a*b*c + b**3)/(2*a*c**2 - b**2*c)
) - 1/(2*a*x**2) + b*log(x)/a**2
```

Giac [A] time = 1.44124, size = 128, normalized size = 1.44

$$-\frac{b\log\left(cx^4-bx^2+a\right)}{4\,a^2}+\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2}-\frac{bx^2+a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*b*log(c*x^4 - b*x^2 + a)/a^2 + 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*
arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(b*
x^2 + a)/(a^2*x^2)
```

3.893 $\int \frac{x^4}{a-bx^2+cx^4} \, dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.364524, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1122, 1166, 208}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a-bx^2+cx^4}\,dx &= \frac{x}{c}-\frac{\int \frac{a-bx^2}{a-bx^2+cx^4}\,dx}{c}\\ &= \frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.131441, size = 208, normalized size = 1.16

$$\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] x/c + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - Sqrt
[b^2 - 4*a*c]]) + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*S
```

```
qrt[-b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.183, size = 343, normalized size = 1.9

$$\frac{x}{c}+\frac{\sqrt{2}\,b}{2\,c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}+\sqrt{2}a\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4-b*x^2+a),x)
```

```
[Out] x/c+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
a-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-1/2/c*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] x/c + integrate((b*x^2 - a)/(c*x^4 - b*x^2 + a), x)/c
```

Fricas [B] time = 1.61687, size = 2157, normalized size = 12.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
- a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c
 + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)
))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a
*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 -
(b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))
)*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt((b^3 - 3*a
*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*
c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a
*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1
/2)*c*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x -
 sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2*c^3
- 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4))) - 2*x)/c
```

Sympy [A] time = 1.68431, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5 - 128ab^2c^4 + 16b^4c^3\right) + t^2\left(-48a^2bc^2 + 28ab^3c - 4b^5\right) + a^3, \left(t \mapsto t\log\left(x + \frac{-32t^3abc^4 + 8t^3b^3c^3 \cdots}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(-48
*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + a**3, Lambda(_t, _t*log(x + (-32*_t*
*3*a*b*c**4 + 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**
4)/(a**2*c - a*b**2)))) + x/c
```

Giac [C] time = 2.62217, size = 4250, normalized size = 23.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] -2*(3*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/2*real_
part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(3/4)*b*cosh(1/2*imag_p
art(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/2*real_part(arccos(1/2*sqr
t(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs
(c)))))^2*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*i
mag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cosh(1/2*
imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a
*abs(c))))) + 9*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1
/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(arc
cos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 -
 3*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(
arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3/4)*b*sin(1/2*real_part(
arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a
*c)*b/(a*abs(c)))))^3 - (a*c^3)^(1/4)*a*c^2*cosh(1/2*imag_part(arccos(1/2*s
qrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)
)))) + (a*c^3)^(1/4)*a*c^2*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(
c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(((a/
c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c)))) + x)/((a/c)^(1/4)*sin(
1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*b*c^2*abs(c) +
 (b^2*c - 4*a*c^2)*c^2) - 2*(3*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^3*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3
)^(3/4)*b*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/2
*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*b*cos(1
/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcc
os(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b
/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*
(a*c^3)^(3/4)*b*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*s
```

```
in(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(
arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*b*cos(1/2*real_part(
arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a
*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*
sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)
*b*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_par
t(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*sq
rt(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c
)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(
3/4)*b*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*im
ag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - (a*c^3)^(1/4)*a*c^2*cosh(1
/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(
1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*c^2*sin(1/2*real_part(arcco
s(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(
a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c)
))) - x)/((a/c)^(1/4)*sin(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b
^2 - 4*a*c)*b*c^2*abs(c) + (b^2*c - 4*a*c^2)*c^2) - ((a*c^3)^(3/4)*b*cos(1/
2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcco
s(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*(a*c^3)^(3/4)*b*cos(1/2*real_part(arc
cos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 -
 3*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_pa
rt(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*b*cos(1/2*real_pa
rt(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3
/4)*b*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*ima
g_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2 - 9*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2
*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*i
mag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (a*c^3)^(3/4)*b*cos(1/2*r
eal_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - (a
*c^3)^(1/4)*a*c^2*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*co
sh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*c^2
*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(
arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(2*x*(a/c)^(1/4)*cos(1/2*arccos(1/
2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*b*c^2*abs(
c) + (b^2*c - 4*a*c^2)*c^2) + ((a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*a
```

```
bs(c)))))^3 - 3*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a
*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1
/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cos
(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(ar
ccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c
)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*sqrt(a*c
)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_par
t(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cos(1/2*real_par
t(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arccos(1/2*sqrt
(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2 - 9*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_p
art(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arccos(1/2*sq
rt(a*c)*b/(a*abs(c)))))^2 - (a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3 + 3*(a*c^3)^(3/4)*b*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*
imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - (a*c^3)^(1/4)*a*c^2*cos(
1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcco
s(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*c^2*cos(1/2*real_part(arc
cos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b
/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(
c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*b*c^2*abs(c) + (b^2*c - 4*a*c^2
)*c^2) + x/c
```

3.894 $\int \frac{x^2}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{b-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] (Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - (Sqrt[b + Sqrt[b^2 - 4*a*
c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.108731, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1130, 208}

$$\frac{\sqrt{b-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - (Sqrt[b + Sqrt[b^2 - 4*a*
c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx = -\left(\frac{1}{2}\left(-1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+}$$

$$= \frac{\sqrt{b-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b+\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.107481, size = 137, normalized size = 0.91

$$\frac{\sqrt{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)-\sqrt{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] (-(Sqrt[-b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b
^2 - 4*a*c]]]) + Sqrt[-b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.17, size = 208, normalized size = 1.4

$$\frac{\sqrt{2}}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{\sqrt{2}b}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{}{\sqrt{-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))-1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-1/2*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))-1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^4 - bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 - b*x^2 + a), x)
```

Fricas [B] time = 1.52462, size = 1196, normalized size = 7.97

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{\frac{b+}{}}}{\sqrt{b^2c^2-4\,ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/sqr
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2
*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*
a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*sq
rt(1/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c
^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c
^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2*sqrt(1
/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))
```

```
*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2
- 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A] time = 0.799969, size = 75, normalized size = 0.5

$$\mathrm{RootSum}\left(t^4\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(16abc-4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c+2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(16*a*b
*c - 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c + 2*_
t*b + x)))
```

Giac [C] time = 2.63881, size = 5098, normalized size = 33.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2
*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/2*real_part(arccos(
1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c -
(a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
- 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a
*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*im
ag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1/2
*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs
(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b
^2 - 4*a*c)*b)*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*si
n(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(a
rccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4
)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c
```

```
)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_
part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c
^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcco
s(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a
*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3
*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)
*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_p
art(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3
)^(3/4)*sqrt(b^2 - 4*a*c)*b)*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*arcta
n(((a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c)))) + x)/((a/c)^(1/4
)*sin(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) + 1
/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4
*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*
imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/2*real_part(arccos(1
/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (
a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 -
 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*
c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*ima
g_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1/2*
sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(
c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^
2 - 4*a*c)*b)*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin
(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(ar
ccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)
*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^
3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arccos
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*
abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*
((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*
b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_pa
rt(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(
a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)
^(3/4)*sqrt(b^2 - 4*a*c)*b)*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs
(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*arctan
(-((a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4
)*sin(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) + 1
/4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a
*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*im
ag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)*b^2 - 4*(
```

a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arc
cos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 -
 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*
c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*ima
g_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arccos(1/2
*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (
a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/
(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin
(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(ar
ccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)
*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)
^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1
/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((
a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)
*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_par
t(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)
^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/
2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs
(c)))))^2*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*log(2*
x*(a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c)
)/(a*b^2*c^3 - 4*a^2*c^4) - 1/4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c -
 (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*
a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*ima
g_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/2*real_part(arccos(1/2*
sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c -
(a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*
sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*
b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real
_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sq
rt(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3
)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(
1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arc
cos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/
4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))
)))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_pa

```
rt(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arccos(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^
3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(
(a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c - (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b
)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(
arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arccos(1/2*sqrt(a
*c)*b/(a*abs(c)))))^3)*log(-2*x*(a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/
(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4)
```

3.895 $\int \frac{1}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTanh[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.0724572, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1093, 208}

$$\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTanh[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-bx^2+cx^4}\,dx = \frac{c\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.0839519, size = 137, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{-\sqrt{b^2-4ac}-b}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]]/
Sqrt[-b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^
2 - 4*a*c]]]/Sqrt[-b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A] time = 0.172, size = 116, normalized size = 0.8

$$-c\sqrt{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}} - c\sqrt{2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4-b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^4 - bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 - b*x^2 + a), x)
```

Fricas [B] time = 1.58474, size = 1312, normalized size = 8.75

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx+\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
- 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(a
^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*
b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 -
 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b
^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt((b - (a*b^2 - 4*a^2
*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2
- 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b - (a*b^2 - 4
*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt((
b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x
```

```
 - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sq
rt((b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)))
```

Sympy [A] time = 0.943064, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(16abc-4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{-32t^3a^2bc+8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(16*a*b
*c - 4*b**3) + c, Lambda(_t, _t*log(x + (-32*_t**3*a**2*b*c + 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Giac [C] time = 1.56458, size = 1296, normalized size = 8.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c - (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*rea
l_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3
)^(1/4)*a*c - (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(1/2*real_part(arccos(1
/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*a
bs(c))))))*arctan(((a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c))))
+ x)/((a/c)^(1/4)*sin(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c -
4*a^2*c^2) + 1/2*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c - (a*c^3)^(1/4)*
sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b
^2 - 4*(a*c^3)^(1/4)*a*c - (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(1/2*real_
part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqr
t(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b
/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c))))
))/(a*b^2*c - 4*a^2*c^2) + 1/4*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c -
(a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b
```

```
/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((
a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c - (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)
*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(
arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(2*x*(a/c)^(1/4)*cos(1/2*arccos(1/
2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*c - 4*a^2*c^2) - 1/4*(
((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c - (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*
b)*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_par
t(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/
4)*a*c - (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/2*real_part(arccos(1/2*sq
rt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)
)))))*log(-2*x*(a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^
2 + sqrt(a/c))/(a*b^2*c - 4*a^2*c^2)
```

3.896 $\int \frac{1}{x^2(a-bx^2+cx^4)}\,dx$

Optimal. Leaf size=172

$$\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) + (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.203356, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1123, 1166, 208}

$$\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a-bx^2+cx^4\right)}\,dx &= -\frac{1}{ax}+\frac{\int \frac{b-cx^2}{a-bx^2+cx^4}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}\\ &= -\frac{1}{ax}+\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] time = 0.396669, size = 199, normalized size = 1.16

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b}}+\frac{2}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -(2/x + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - Sqrt[b^2 - 4*
a*c]]) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b + Sqrt[b^2 - 4*
```

```
a*c]]))/(2*a)
```

Maple [A] time = 0.178, size = 232, normalized size = 1.4

$$-\frac{c\sqrt{2}}{2a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{c\sqrt{2}b}{2a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(x*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b+1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(x*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b-1/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 - b)/(c*x^4 - b*x^2 + a), x)/a - 1/(a*x)
```

Fricas [B] time = 1.63054, size = 2267, normalized size = 13.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c
^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4
*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*
sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^3 - 3*a
*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5 - 5
*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4
*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4
*a^4*c))) + sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2
*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4
- 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^
3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(
b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c - (a^3*
b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*
b^2 - 4*a^4*c))) - 2)/(a*x)
```

Sympy [A] time = 1.89738, size = 148, normalized size = 0.86

$$\operatorname{RootSum}\left(t^4\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\left(-48a^2bc^2+28ab^3c-4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2+48t^3a^4b^2}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(-48
*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t*
*3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 + 10*_t*a**2*b*c**2
 - 10*_t*a*b**3*c + 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

Giac [C] time = 2.70335, size = 4238, normalized size = 24.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 2*(3*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/2*real_p
art(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(3/4)*a*cosh(1/2*imag_pa
rt(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/2*real_part(arccos(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sq
rt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(
c)))))^2*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*im
ag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*a*cosh(1/2*i
mag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*
abs(c))))) + 9*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/
2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(
1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cosh(1/2*imag_part(arcc
os(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(
a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 -
3*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*
sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
rccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3/4)*a*sin(1/2*real_part(a
rccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*
c)*b/(a*abs(c)))))^3 - (a*c^3)^(1/4)*a*b*c*cosh(1/2*imag_part(arccos(1/2*sq
rt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))
))) + (a*c^3)^(1/4)*a*b*c*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c
)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(((a/c
)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c)))) + x)/((a/c)^(1/4)*sin(1
/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a) +
(b^2*c - 4*a*c^2)*a^2) + 2*(3*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*
sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^3*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)
^(3/4)*a*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/2*
real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*a*cos(1/
2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcco
s(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/
(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(
a*c^3)^(3/4)*a*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*si
n(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(a
```

```
rccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*a*cos(1/2*real_part(a
rccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arccos(1/2*sqrt(a*
c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*s
inh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*
a*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part
(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3
/4)*a*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*ima
g_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - (a*c^3)^(1/4)*a*b*c*cosh(1/
2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1
/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*b*c*sin(1/2*real_part(arccos
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a
*abs(c))))))*arctan(-((a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c))
)) - x)/((a/c)^(1/4)*sin(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^
2 - 4*a*c)*a*b*c*abs(a) + (b^2*c - 4*a*c^2)*a^2) + ((a*c^3)^(3/4)*a*cos(1/2
*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arccos
(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*(a*c^3)^(3/4)*a*cos(1/2*real_part(arcc
os(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 -
3*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*
cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_par
t(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*a*cos(1/2*real_par
t(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/
4)*a*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag
_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sq
rt(a*c)*b/(a*abs(c)))))^2 - 9*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*
sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(
c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*im
ag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (a*c^3)^(3/4)*a*cos(1/2*re
al_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(
1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - (a*
c^3)^(1/4)*a*b*c*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cos
h(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*b*c*
cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
rccos(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(2*x*(a/c)^(1/4)*cos(1/2*arccos(1/2
*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a
) + (b^2*c - 4*a*c^2)*a^2) - ((a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*
sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^3 - 3*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*
```

```
abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/
2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cos(
1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arc
cos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)
*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sqrt(a*c)
*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*
sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part
(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*a*cos(1/2*real_part
(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arccos(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2 - 9*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c))
)))*cosh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/2*real_pa
rt(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arccos(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2 - (a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sqr
t(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3 + 3*(a*c^3)^(3/4)*a*cos(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(1/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*i
mag_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - (a*c^3)^(1/4)*a*b*c*cos(1
/2*real_part(arccos(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arccos
(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*b*c*cos(1/2*real_part(arcc
os(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arccos(1/2*sqrt(a*c)*b/
(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/2*arccos(1/2*sqrt(a*c)*b/(a*abs(c
)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a) + (b^2*c - 4*a*c^2)
*a^2) - 1/(a*x)
```

3.897 $\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=69

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{2a}+\frac{x^2}{2a}$$

```
[Out] x^2/(2*a) - ((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[
b]) - Log[a - b + 2*a*x^2 + a*x^4]/(2*a)
```

Rubi [A] time = 0.0840896, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1114, 703, 634, 618, 206, 628}

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{2a}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x^2/(2*a) - ((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[
b]) - Log[a - b + 2*a*x^2 + a*x^4]/(2*a)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{a-b+2ax+ax^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2a}+\frac{\text{Subst}\left(\int \frac{-a+b-2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2a}+\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{2a}-\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)}{a}\\
&= \frac{x^2}{2a}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0379755, size = 62, normalized size = 0.9

$$\frac{x^2 - \log\left(a\left(x^2+1\right)^2 - b\right)}{2a} - \frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[b]) + (x^2
- Log[-b + a*(1 + x^2)^2])/(2*a)
```

Maple [A] time = 0.045, size = 86, normalized size = 1.3

$$\frac{x^2}{2a} - \frac{\ln\left(ax^4 + 2ax^2 + a - b\right)}{2a} - \frac{1}{2}\mathrm{Artanh}\left(\frac{2ax^2 + 2a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{b}{2a}\mathrm{Artanh}\left(\frac{2ax^2 + 2a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] 1/2*x^2/a-1/2*ln(a*x^4+2*a*x^2+a-b)/a-1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+
2*a)/(a*b)^(1/2))-1/2/a/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*
b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49233, size = 374, normalized size = 5.42

$$\left[\frac{2\,abx^2-2\,ab\log\left(ax^4+2\,ax^2+a-b\right)+\sqrt{ab}(a+b)\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{ab}\left(x^2+1\right)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,a^2b},\ \frac{abx^2-ab\log\left(ax^4+2\,ax^2+a-b\right)}{2\,a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x^2 - 2*a*b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(a*b)*(a + b)*lo
g((a*x^4 + 2*a*x^2 - 2*sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^4 + 2*a*x^2 + a -
b)))/(a^2*b), 1/2*(a*b*x^2 - a*b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(-a*b)*
(a + b)*arctan(sqrt(-a*b)/(a*x^2 + a)))/(a^2*b)]
```

Sympy [B] time = 1.08395, size = 138, normalized size = 2.

$$\left(-\frac{1}{2a}-\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{2a}-\frac{\sqrt{a^3b}(a+b)}{4a^3b}\right)+a-b}{a+b}\right)+\left(-\frac{1}{2a}+\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{2a}+\frac{\sqrt{a^3b}(a+b)}{4a^3b}\right.}{a+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (-1/(2*a) - sqrt(a**3*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-1/(2*a) -
 sqrt(a**3*b)*(a + b)/(4*a**3*b)) + a - b)/(a + b)) + (-1/(2*a) + sqrt(a**3
*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-1/(2*a) + sqrt(a**3*b)*(a + b)
/(4*a**3*b)) + a - b)/(a + b)) + x**2/(2*a)
```

Giac [A] time = 3.48536, size = 81, normalized size = 1.17

$$\frac{x^2}{2\,a}+\frac{(a+b)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}a}-\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/2*(a + b)*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2
*log(a*x^4 + 2*a*x^2 + a - b)/a
```

3.898 $\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=56

$$\frac{\log\left(ax^4+2ax^2+a-b\right)}{4a}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a - b + 2*a*
x^2 + a*x^4]/(4*a)
```

Rubi [A] time = 0.0489732, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1114, 634, 618, 206, 628}

$$\frac{\log\left(ax^4+2ax^2+a-b\right)}{4a}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a - b + 2*a*
x^2 + a*x^4]/(4*a)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{a-b+2ax+ax^2}\,dx, x, x^2\right) \\
&= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)\right) + \frac{\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{4a} \\
&= \frac{\log\left(a-b+2ax^2+ax^4\right)}{4a} + \text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right) \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\log\left(a-b+2ax^2+ax^4\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0174984, size = 51, normalized size = 0.91

$$\frac{\log\left(a\left(x^2+1\right)^2-b\right)+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{\sqrt{b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ((2*Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[-b + a*(1 +
 x^2)^2])/(4*a)
```

Maple [A] time = 0.043, size = 49, normalized size = 0.9

$$\frac{\ln\left(ax^4+2\,ax^2+a-b\right)}{4\,a}+\frac{1}{2}\mathrm{Artanh}\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] 1/4*ln(a*x^4+2*a*x^2+a-b)/a+1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49557, size = 312, normalized size = 5.57

$$\left[\frac{b\log\left(ax^4+2\,ax^2+a-b\right)+\sqrt{ab}\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{ab}\left(x^2+1\right)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,ab},\frac{b\log\left(ax^4+2\,ax^2+a-b\right)-2\,\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{ax^2+a}\right)}{4\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(a*b)*log((a*x^4 + 2*a*x^2 + 2*s
qrt(a*b)*(x^2 + 1) + a + b)/(a*x^4 + 2*a*x^2 + a - b)))/(a*b), 1/4*(b*log(a
*x^4 + 2*a*x^2 + a - b) - 2*sqrt(-a*b)*arctan(sqrt(-a*b)/(a*x^2 + a)))/(a*b
)]
```

Sympy [B] time = 0.488198, size = 110, normalized size = 1.96

$$\left(\frac{1}{4a} - \frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2 + \frac{4ab\left(\frac{1}{4a} - \frac{\sqrt{a^3b}}{4a^2b}\right) + a - b}{a}\right) + \left(\frac{1}{4a} + \frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2 + \frac{4ab\left(\frac{1}{4a} + \frac{\sqrt{a^3b}}{4a^2b}\right) + a - b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (1/(4*a) - sqrt(a**3*b)/(4*a**2*b))*log(x**2 + (4*a*b*(1/(4*a) - sqrt(a**3*
b)/(4*a**2*b)) + a - b)/a) + (1/(4*a) + sqrt(a**3*b)/(4*a**2*b))*log(x**2 +
 (4*a*b*(1/(4*a) + sqrt(a**3*b)/(4*a**2*b)) + a - b)/a)
```

Giac [A] time = 3.55729, size = 62, normalized size = 1.11

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}} + \frac{\log\left(ax^4 + 2\,ax^2 + a - b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b) + 1/4*log(a*x^4 + 2*a*x^2 +
a - b)/a
```

3.899 $\int \frac{x}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] -ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.028482, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a-b+2ax^2+ax^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)\\ &= -\,\text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0078592, size = 31, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.043, size = 26, normalized size = 0.8

$$-\frac{1}{2}\text{Artanh}\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58671, size = 215, normalized size = 6.94

$$\left[\frac{\sqrt{ab}\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{ab}\left(x^2+1\right)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,ab}, \frac{\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{ax^2+a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^4
 + 2*a*x^2 + a - b))/(a*b), 1/2*sqrt(-a*b)*arctan(sqrt(-a*b)/(a*x^2 + a))/(
a*b)]
```

Sympy [A] time = 0.286649, size = 53, normalized size = 1.71

$$\frac{\sqrt{\frac{1}{ab}}\log\left(-b\sqrt{\frac{1}{ab}}+x^2+1\right)}{4}-\frac{\sqrt{\frac{1}{ab}}\log\left(b\sqrt{\frac{1}{ab}}+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] sqrt(1/(a*b))*log(-b*sqrt(1/(a*b)) + x**2 + 1)/4 - sqrt(1/(a*b))*log(b*sqrt
(1/(a*b)) + x**2 + 1)/4
```

Giac [A] time = 4.48347, size = 31, normalized size = 1.

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b)
```

3.900 $\int \frac{1}{x(a-b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=77

$$-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a-b)}+\frac{\log(x)}{a-b}$$

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*(a - b)*Sqrt[b]) + Log[x]
/(a - b) - Log[a - b + 2*a*x^2 + a*x^4]/(4*(a - b))
```

Rubi [A] time = 0.0717382, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1114, 705, 29, 634, 618, 206, 628}

$$-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a-b)}+\frac{\log(x)}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*(a - b)*Sqrt[b]) + Log[x]
/(a - b) - Log[a - b + 2*a*x^2 + a*x^4]/(4*(a - b))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a-b+2ax^2+ax^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(a-b+2ax+ax^2\right)}\,dx,x,x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2(a-b)} + \frac{\text{Subst}\left(\int \frac{-2a-ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2(a-b)}$$
$$= \frac{\log(x)}{a-b} - \frac{\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{4(a-b)} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2(a-b)}$$
$$= \frac{\log(x)}{a-b} - \frac{\log\left(a-b+2ax^2+ax^4\right)}{4(a-b)} + \frac{a\,\text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx,x,2a\left(1+x^2\right)\right)}{a-b}$$
$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2(a-b)\sqrt{b}} + \frac{\log(x)}{a-b} - \frac{\log\left(a-b+2ax^2+ax^4\right)}{4(a-b)}$$

Mathematica [A] time = 0.0484948, size = 90, normalized size = 1.17

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{a}\left(x^2+1\right)-\sqrt{b}\right)+\left(\sqrt{b}-\sqrt{a}\right)\log\left(\sqrt{a}\left(x^2+1\right)+\sqrt{b}\right)-4\sqrt{b}\log(x)}{4\sqrt{b}(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (-4*Sqrt[b]*Log[x] + (Sqrt[a] + Sqrt[b])*Log[-Sqrt[b] + Sqrt[a]*(1 + x^2)]
+ (-Sqrt[a] + Sqrt[b])*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)])/(4*Sqrt[b]*(-a + b
))
```

Maple [A] time = 0.049, size = 71, normalized size = 0.9

$$\frac{\ln(x)}{a-b} - \frac{\ln\left(ax^4+2\,ax^2+a-b\right)}{4\,a-4\,b} + \frac{a}{2\,a-2\,b}\text{Artanh}\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] ln(x)/(a-b)-1/4*ln(a*x^4+2*a*x^2+a-b)/(a-b)+1/2*a/(a-b)/(a*b)^(1/2)*arctanh
(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53002, size = 350, normalized size = 4.55

$$\left[-\frac{\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\left(bx^2+b\right)\sqrt{\frac{a}{b}}+a+b}{ax^4+2\,ax^2+a-b}\right)+\log\left(ax^4+2\,ax^2+a-b\right)-4\,\log\left(x\right)}{4\left(a-b\right)},-\frac{2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{-\frac{a}{b}}}{ax^2+a}\right)+\log\left(ax^4+2\,ax^2+\right.}{4\left(a-b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b) + a + b)/(a
*x^4 + 2*a*x^2 + a - b)) + log(a*x^4 + 2*a*x^2 + a - b) - 4*log(x))/(a - b)
, -1/4*(2*sqrt(-a/b)*arctan(b*sqrt(-a/b)/(a*x^2 + a)) + log(a*x^4 + 2*a*x^2
 + a - b) - 4*log(x))/(a - b)]
```

Sympy [B] time = 2.29165, size = 184, normalized size = 2.39

$$\left(-\frac{1}{4\left(a-b\right)}-\frac{\sqrt{ab}}{4b\left(a-b\right)}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+a-4b^2\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+b}{a}\right)+\left(-\frac{1}{4\left(a-b\right)}+\frac{\sqrt{ab}}{4b\left(a-b\right)}\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (-1/(4*(a - b)) - sqrt(a*b)/(4*b*(a - b)))*log(x**2 + (4*a*b*(-1/(4*(a - b)
) - sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b)) - sqrt(a*b)/(4*b*
(a - b))) + b)/a) + (-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a - b)))*log(x**2 + (
4*a*b*(-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b
)) + sqrt(a*b)/(4*b*(a - b))) + b)/a) + log(x)/(a - b)
```

Giac [A] time = 4.18689, size = 96, normalized size = 1.25

$$-\frac{a \arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}(a-b)} - \frac{\log\left(ax^4 + 2\,ax^2 + a - b\right)}{4\,(a-b)} + \frac{\log\left(x^2\right)}{2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*(a - b)) - 1/4*log(a*x^4
+ 2*a*x^2 + a - b)/(a - b) + 1/2*log(x^2)/(a - b)
```

3.901 $\int \frac{1}{x^3(a-b+2ax^2+ax^4)} dx$

Optimal. Leaf size=97

$$-\frac{1}{2x^2(a-b)}+\frac{a\log\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{\sqrt{a}(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a-b)^2}-\frac{2a\log(x)}{(a-b)^2}$$

```
[Out] -1/(2*(a - b)*x^2) - (Sqrt[a]*(a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])
/(2*(a - b)^2*Sqrt[b]) - (2*a*Log[x])/(a - b)^2 + (a*Log[a - b + 2*a*x^2 +
a*x^4])/(2*(a - b)^2)
```

Rubi [A] time = 0.140764, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1114, 709, 800, 634, 618, 206, 628}

$$-\frac{1}{2x^2(a-b)}+\frac{a\log\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{\sqrt{a}(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a-b)^2}-\frac{2a\log(x)}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/(2*(a - b)*x^2) - (Sqrt[a]*(a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])
/(2*(a - b)^2*Sqrt[b]) - (2*a*Log[x])/(a - b)^2 + (a*Log[a - b + 2*a*x^2 +
a*x^4])/(2*(a - b)^2)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

, -1]

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a-b+2ax^2+ax^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(a-b+2ax+ax^2\right)}\,dx,x,x^2\right)$$

$$= -\frac{1}{2(a-b)x^2} + \frac{\operatorname{Subst}\left(\int \frac{-2a-ax}{x\left(a-b+2ax+ax^2\right)}\,dx,x,x^2\right)}{2(a-b)}$$

$$= -\frac{1}{2(a-b)x^2} + \frac{\operatorname{Subst}\left(\int \left(-\frac{2a}{(a-b)x} + \frac{a(3a+b+2ax)}{(a-b)\left(a-b+2ax+ax^2\right)}\right)dx,x,x^2\right)}{2(a-b)}$$

$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\operatorname{Subst}\left(\int \frac{3a+b+2ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2(a-b)^2}$$

$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\operatorname{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2(a-b)^2} + \frac{(a(a+b))\operatorname{Subst}\left(\int \frac{}{a-b+2}\right.}{2(a-b)^2}$$

$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\log\left(a-b+2ax^2+ax^4\right)}{2(a-b)^2} - \frac{(a(a+b))\operatorname{Subst}\left(\int \frac{1}{4ab-x^2}\,dx,x\right.}{(a-b)^2}$$

$$= -\frac{1}{2(a-b)x^2} - \frac{\sqrt{a}(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2(a-b)^2\sqrt{b}} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\log\left(a-b+2ax^2+ax^4\right)}{2(a-b)^2}$$

Mathematica [A] time = 0.0936018, size = 146, normalized size = 1.51

$$\frac{-8a\sqrt{b}x^2\log(x)+\sqrt{a}x^2\left(\sqrt{a}+\sqrt{b}\right)^2\log\left(\sqrt{a}\left(x^2+1\right)-\sqrt{b}\right)-\left(\sqrt{a}-\sqrt{b}\right)\left(\left(ax^2-\sqrt{a}\sqrt{b}x^2\right)\log\left(\sqrt{a}\left(x^2+1\right)+\sqrt{b}\right)+2\left(\right.\right.}{4\sqrt{b}x^2(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (-8*a*Sqrt[b]*x^2*Log[x] + Sqrt[a]*(Sqrt[a] + Sqrt[b])^2*x^2*Log[-Sqrt[b] +
 Sqrt[a]*(1 + x^2)] - (Sqrt[a] - Sqrt[b])*(2*(Sqrt[a]*Sqrt[b] + b) + (a*x^2
 - Sqrt[a]*Sqrt[b]*x^2)*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)]))/(4*(a - b)^2*Sqr
t[b]*x^2)
```

Maple [A] time = 0.049, size = 122, normalized size = 1.3

$$-\frac{1}{(2a-2b)x^2}-2\frac{a\ln(x)}{(a-b)^2}+\frac{a\ln\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{a^2}{2(a-b)^2}\operatorname{Artanh}\left(\frac{2ax^2+2a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{ab}{2(a-b)^2}\operatorname{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a-b)/x^2-2*a*ln(x)/(a-b)^2+1/2*a*ln(a*x^4+2*a*x^2+a-b)/(a-b)^2-1/2*a^
2/(a-b)^2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))-1/2*a/(a-b)^2/
(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57919, size = 487, normalized size = 5.02

$$\left[\frac{(a+b)x^2\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\left(bx^2+b\right)\sqrt{\frac{a}{b}}+a+b}{ax^4+2\,ax^2+a-b}\right)+2\,ax^2\log\left(ax^4+2\,ax^2+a-b\right)-8\,ax^2\log\left(x\right)-2\,a+2\,b}{4\left(a^2-2\,ab+b^2\right)x^2}, \frac{(a+b)x^2\sqrt{-\frac{a}{b}}\ \ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*x^2*sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b)
+ a + b)/(a*x^4 + 2*a*x^2 + a - b)) + 2*a*x^2*log(a*x^4 + 2*a*x^2 + a - b)
- 8*a*x^2*log(x) - 2*a + 2*b)/((a^2 - 2*a*b + b^2)*x^2), 1/2*((a + b)*x^2*s
qrt(-a/b)*arctan(b*sqrt(-a/b)/(a*x^2 + a)) + a*x^2*log(a*x^4 + 2*a*x^2 + a
- b) - 4*a*x^2*log(x) - a + b)/((a^2 - 2*a*b + b^2)*x^2)]
```

Sympy [B] time = 5.24125, size = 372, normalized size = 3.84

$$-\frac{2a\log(x)}{(a-b)^2}+\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{ab}\,(a+b)}{4b\left(a^2-2ab+b^2\right)}\right)\log\left(x^2+\frac{-4a^2b\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{ab}(a+b)}{4b\left(a^2-2ab+b^2\right)}\right)+a^2+8ab^2\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{ab}(a+b)}{4b\left(a^2-2ab+b\right.}\right.}{a^2+ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] -2*a*log(x)/(a - b)**2 + (a/(2*(a - b)**2) - sqrt(a*b)*(a + b)/(4*b*(a**2 -
 2*a*b + b**2)))*log(x**2 + (-4*a**2*b*(a/(2*(a - b)**2) - sqrt(a*b)*(a + b
)/(4*b*(a**2 - 2*a*b + b**2))) + a**2 + 8*a*b**2*(a/(2*(a - b)**2) - sqrt(a
*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2)
 - sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))))/(a**2 + a*b)) + (a/(2*(a
 - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2)))*log(x**2 + (-4*a
**2*b*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))) +
a**2 + 8*a*b**2*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b +
b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 -
 2*a*b + b**2))))/(a**2 + a*b)) - 1/(x**2*(2*a - 2*b))
```

Giac [A] time = 2.32143, size = 170, normalized size = 1.75

$$\frac{a\log\left(ax^4+2\,ax^2+a-b\right)}{2\left(a^2-2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2-2\,ab+b^2}+\frac{\left(a^2+ab\right)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\left(a^2-2\,ab+b^2\right)\sqrt{-ab}}+\frac{2\,ax^2-a+b}{2\left(a^2-2\,ab+b^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a - b)/(a^2 - 2*a*b + b^2) - a*log(x^2)/(a^2 -
2*a*b + b^2) + 1/2*(a^2 + a*b)*arctan((a*x^2 + a)/sqrt(-a*b))/((a^2 - 2*a*b
 + b^2)*sqrt(-a*b)) + 1/2*(2*a*x^2 - a + b)/((a^2 - 2*a*b + b^2)*x^2)
```

3.902 $\int \frac{x^4}{a-b+2ax^2+ax^4} dx$

Optimal. Leaf size=114

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}+\frac{x}{a}$$

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]
])/(2*a^(5/4)*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*a^(5/4)*Sqrt[b])
```

Rubi [A] time = 0.165205, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1122, 1166, 205}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]
])/(2*a^(5/4)*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*a^(5/4)*Sqrt[b])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx &= \frac{x}{a}-\frac{\int \frac{a-b+2ax^2}{a-b+2ax^2+ax^4}\,dx}{a}\\ &= \frac{x}{a}-\frac{1}{2}\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx-\frac{1}{2}\left(2+\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx\\ &= \frac{x}{a}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0870461, size = 144, normalized size = 1.26

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2a\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]])
/(2*a*Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^2*ArcTan[(S
qrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(2*a*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b
])
```

Maple [B] time = 0.17, size = 210, normalized size = 1.8

$$\frac{x}{a}+\operatorname{Artanh}\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}-\frac{a}{2}\operatorname{Artanh}\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}-\frac{b}{2}\operatorname{Artanh}\left(ax\frac{1}{\sqrt{(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] x/a+1/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2))-1/2/
(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2)
)*a-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*
a)^(1/2))*b-1/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a)^(1/2
))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a)
^(1/2))*a-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2
)+a)*a)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 1.60055, size = 1287, normalized size = 11.29

$$\frac{a\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\log\left(-\left(3a^2-2ab-b^2\right)x+\left(a^4b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-3a^2b-ab^2\right)\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\right)-a\sqrt{-\frac{a^2b}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*b)/(a^2*b))
*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b))
- 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3
*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*
b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x - (a^4*b*sqrt((9*a^2 + 6*a*b + b^2
)/(a^5*b)) - 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*
b)) + a + 3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)
) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9*a^2 + 6
*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b
^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + a*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2
)/(a^5*b)) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x - (a^4*b*sqrt((
9*a^2 + 6*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sqrt((9*a^2 +
6*a*b + b^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + 4*x)/a
```

Sympy [A] time = 1.09527, size = 105, normalized size = 0.92

$$\operatorname{RootSum}\left(256t^4a^5b^2 + t^2\left(32a^4b + 96a^3b^2\right) + a^3 - 3a^2b + 3ab^2 - b^3, \left(t \mapsto t\log\left(x + \frac{64t^3a^4b + 4ta^3 + 24ta^2b + 4tab^2}{3a^2 - 2ab - b^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(256*_t**4*a**5*b**2 + _t**2*(32*a**4*b + 96*a**3*b**2) + a**3 - 3*a
**2*b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (64*_t**3*a**4*b + 4*_t*a**3
 + 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 - 2*a*b - b**2)))) + x/a
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.903 $\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] -(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a
^(3/4)*Sqrt[b]) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a]
+ Sqrt[b]]])/(2*a^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.0537356, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1130, 205}

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a
^(3/4)*Sqrt[b]) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a]
+ Sqrt[b]]])/(2*a^(3/4)*Sqrt[b])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx = -\left(\frac{1}{2}\left(-1+\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx\right)+\frac{1}{2}\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx$$
$$= -\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [A] time = 0.104889, size = 128, normalized size = 1.17

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (-(((Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]])/Sqrt
[a - Sqrt[a]*Sqrt[b]]) + ((Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a + S
qrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.133, size = 134, normalized size = 1.2

$$-\frac{1}{2}\mathrm{Artanh}\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}+\frac{a}{2}\mathrm{Artanh}\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}+\frac{1}{2}\arctan\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2))+1/2/(
a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2))
*a+1/2/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a)^(1/2))+1/2/
(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a)^(1/2))
```

```
*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{ax^4 + 2\,ax^2 + a - b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x^4 + 2*a*x^2 + a - b), x)
```

Fricas [B] time = 1.47349, size = 617, normalized size = 5.66

$$\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt(-(a*b*sqrt(1/(a^3
*b)) + 1)/(a*b))*sqrt(1/(a^3*b)) + x) - 1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)
/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*sqrt(1/(a^3*b)) +
 x) - 1/4*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*log(a^2*b*sqrt((a*b*sqrt(1/
(a^3*b)) - 1)/(a*b))*sqrt(1/(a^3*b)) + x) + 1/4*sqrt((a*b*sqrt(1/(a^3*b)) -
 1)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*sqrt(1/(a^3*b))
 + x)
```

Sympy [A] time = 0.373015, size = 44, normalized size = 0.4

$$\mathrm{RootSum}\left(256t^4a^3b^2 + 32t^2a^2b + a - b, \left(t \mapsto t\log\left(-64t^3a^2b - 4ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b + a - b, Lambda(_t, _t*log(-6
4*_t**3*a**2*b - 4*_t*a + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.904 $\int \frac{1}{a-b+2ax^2+ax^4} dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[
b]]*Sqrt[b]) - ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]]/(2*a^(1/4)*Sqrt[
Sqrt[a] + Sqrt[b]]*Sqrt[b])
```

Rubi [A] time = 0.0467918, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1093, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[
b]]*Sqrt[b]) - ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]]/(2*a^(1/4)*Sqrt[
Sqrt[a] + Sqrt[b]]*Sqrt[b])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx = \frac{\sqrt{a}\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\sqrt{b}} - \frac{\sqrt{a}\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\sqrt{b}}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}}$$

Mathematica [A] time = 0.0628166, size = 105, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a - Sqrt[a]*Sqrt[b]]*
Sqrt[b]) - ArcTan[(Sqrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a + Sqrt[a
]*Sqrt[b]]*Sqrt[b])
```

Maple [A] time = 0.139, size = 74, normalized size = 0.7

$$-\frac{a}{2}\text{Artanh}\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}} - \frac{a}{2}\arctan\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^
(1/2))*a-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)
+a)*a)^(1/2))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax^4 + 2\,ax^2 + a - b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] integrate(1/(a*x^4 + 2*a*x^2 + a - b), x)
```

Fricas [B] time = 1.50551, size = 1115, normalized size = 10.23

$$-\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(\left(b-\frac{a^2b-ab^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}\right)\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}+1}{ab-b^2}}+x\right)+\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2))*l
og((b - (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(-((a*b - b^2)
/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2)) + x) + 1/4*sqrt(-((a*b -
 b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2))*log(-(b - (a^2*b -
a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a
^2*b^2 + a*b^3) + 1)/(a*b - b^2)) + x) - 1/4*sqrt(((a*b - b^2)/sqrt(a^3*b -
 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2))*log((b + (a^2*b - a*b^2)/sqrt(a^3*b -
 2*a^2*b^2 + a*b^3))*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) - 1)
/(a*b - b^2)) + x) + 1/4*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3)
- 1)/(a*b - b^2))*log(-(b + (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3)
)*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2)) + x)
```

Sympy [A] time = 0.639959, size = 63, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^2b^2 - 256ab^3\right) + 32t^2ab + 1, \left(t \mapsto t\log\left(-64t^3a^2b + 64t^3ab^2 - 4ta - 4tb + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 - 256*a*b**3) + 32*_t**2*a*b + 1, Lambda(_t, _
t*log(-64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a - 4*_t*b + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.905 $\int \frac{1}{x^2(a-b+2ax^2+ax^4)} \, dx$

Optimal. Leaf size=121

$$-\frac{1}{x(a-b)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}$$

```
[Out] -(1/((a - b)*x)) - (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/(2
*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]) + (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt
[a] + Sqrt[b]]])/(2*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.112851, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1123, 1166, 205}

$$-\frac{1}{x(a-b)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(1/((a - b)*x)) - (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/(2
*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]) + (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt
[a] + Sqrt[b]]])/(2*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a-b+2ax^2+ax^4\right)}\,dx &= -\frac{1}{(a-b)x}-\frac{\int \frac{-2a-ax^2}{a-b+2ax^2+ax^4}\,dx}{-a+b}\\ &= -\frac{1}{(a-b)x}-\frac{a\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}+\frac{a\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}}\\ &= -\frac{1}{(a-b)x}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.15068, size = 143, normalized size = 1.18

$$\frac{\frac{\left(\sqrt{a}\sqrt{b}+a\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}}-\frac{\left(a-\sqrt{a}\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{2}{x}}{2(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (2/x + ((a + Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]
)/(Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt
[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]))/(2*
(-a + b))
```

Maple [B] time = 0.144, size = 180, normalized size = 1.5

$$\frac{a}{2a-2b}\mathrm{Artanh}\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}+\frac{a^{2}}{2a-2b}\mathrm{Artanh}\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}-\frac{a}{2a-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] 1/2*a/(a-b)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2)
)+1/2*a^2/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(
1/2)-a)*a)^(1/2))-1/2*a/(a-b)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^
(1/2)+a)*a)^(1/2))+1/2*a^2/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arct
an(a*x/(((a*b)^(1/2)+a)*a)^(1/2))-1/(a-b)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 1.67398, size = 3290, normalized size = 27.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*((a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sqr
t((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 +
15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log((3*a
^2 + a*b)*x + (6*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*sqrt
((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 1
```

```
5*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*
b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 -
 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 -
b^4))) - (a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)
*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^
4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log(
(3*a^2 + a*b)*x - (6*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*
sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4
 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b
^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4))) + (a - b)*x*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^3 -
b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^
3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*
log((3*a^2 + a*b)*x + (6*a^2*b + 2*a*b^2 + (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b
^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3
*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^
2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a
^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*
a*b^3 - b^4))) - (a - b)*x*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 2
0*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^
4))*log((3*a^2 + a*b)*x - (6*a^2*b + 2*a*b^2 + (a^4*b - 2*a^3*b^2 + 2*a*b^4
 - b^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20
*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^
2*b^2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 +
15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2
+ 3*a*b^3 - b^4))) - 4)/((a - b)*x)
```

Sympy [A] time = 2.26557, size = 134, normalized size = 1.11

$$\operatorname{RootSum}\left(t^4\left(256a^3b^2-768a^2b^3+768ab^4-256b^5\right)+t^2\left(32a^2b+96ab^2\right)+a,\left(t\mapsto t\log\left(x+\frac{64t^3a^4b-128t^3a^3b^2+12\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(_t**4*(256*a**3*b**2 - 768*a**2*b**3 + 768*a*b**4 - 256*b**5) + _t*
*2*(32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (64*_t**3*a**4*b - 12
8*_t**3*a**3*b**2 + 128*_t**3*a*b**4 - 64*_t**3*b**5 + 4*_t*a**3 + 40*_t*a*
*2*b + 20*_t*a*b**2)/(3*a**2 + a*b)))) - 1/(x*(a - b))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.906 $\int \frac{x^5}{a+b+2ax^2+ax^4} \, dx$

Optimal. Leaf size=69

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{2a}+\frac{x^2}{2a}$$

```
[Out] x^2/(2*a) + ((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[b
]) - Log[a + b + 2*a*x^2 + a*x^4]/(2*a)
```

Rubi [A] time = 0.0757646, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 703, 634, 618, 204, 628}

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{2a}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x^2/(2*a) + ((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[b
]) - Log[a + b + 2*a*x^2 + a*x^4]/(2*a)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{a+b+2ax+ax^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2a}+\frac{\mathrm{Subst}\left(\int \frac{-a-b-2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\mathrm{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2a}+\frac{(a-b)\,\mathrm{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{2a}-\frac{(a-b)\,\mathrm{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx,x,2a\left(1+x^2\right)\right)}{a}\\
&= \frac{x^2}{2a}+\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.0347361, size = 62, normalized size = 0.9

$$\frac{\sqrt{a}\left(x^2-\log\left(a\left(x^2+1\right)^2+b\right)\right)+\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Sqrt[a]*(x^2 - Log
[b + a*(1 + x^2)^2]))/(2*a^(3/2))
```

Maple [A] time = 0.046, size = 84, normalized size = 1.2

$$\frac{x^2}{2a}-\frac{\ln\left(ax^4+2ax^2+a+b\right)}{2a}+\frac{1}{2}\arctan\left(\frac{2ax^2+2a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{b}{2a}\arctan\left(\frac{2ax^2+2a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/2*x^2/a-1/2*ln(a*x^4+2*a*x^2+a+b)/a+1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2
*a)/(a*b)^(1/2))-1/2/a/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49749, size = 374, normalized size = 5.42

$$\left[\frac{2\,abx^2 - 2\,ab\log\left(ax^4 + 2\,ax^2 + a + b\right) + \sqrt{-ab}(a-b)\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{-ab}\left(x^2+1\right)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,a^2b}, \frac{abx^2 - ab\log\left(ax^4 + 2\,ax^2 + a + \right.}{2\,a^{\cdot}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x^2 - 2*a*b*log(a*x^4 + 2*a*x^2 + a + b) + sqrt(-a*b)*(a - b)*l
og((a*x^4 + 2*a*x^2 + 2*sqrt(-a*b)*(x^2 + 1) + a - b)/(a*x^4 + 2*a*x^2 + a
+ b)))/(a^2*b), 1/2*(a*b*x^2 - a*b*log(a*x^4 + 2*a*x^2 + a + b) - sqrt(a*b)
*(a - b)*arctan(sqrt(a*b)/(a*x^2 + a)))/(a^2*b)]
```

Sympy [B] time = 1.11192, size = 144, normalized size = 2.09

$$\left(-\frac{1}{2a} - \frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)\log\left(x^2 + \frac{4ab\left(-\frac{1}{2a} - \frac{\sqrt{-a^3b}(a-b)}{4a^3b}\right) + a + b}{a-b}\right) + \left(-\frac{1}{2a} + \frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)\log\left(x^2 + \frac{4ab\left(-\frac{1}{2a} + \frac{\sqrt{-a^3b}}{4a^3}\right.}{a-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (-1/(2*a) - sqrt(-a**3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(-1/(2*a) -
 sqrt(-a**3*b)*(a - b)/(4*a**3*b)) + a + b)/(a - b)) + (-1/(2*a) + sqrt(-a*
*3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(-1/(2*a) + sqrt(-a**3*b)*(a -
b)/(4*a**3*b)) + a + b)/(a - b)) + x**2/(2*a)
```

Giac [A] time = 3.5121, size = 78, normalized size = 1.13

$$\frac{x^2}{2\,a} + \frac{(a-b)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a} - \frac{\log\left(ax^4 + 2\,ax^2 + a + b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/2*(a - b)*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*l
og(a*x^4 + 2*a*x^2 + a + b)/a
```

3.907 $\int \frac{x^3}{a+b+2ax^2+ax^4} dx$

Optimal. Leaf size=54

$$\frac{\log\left(ax^4+2ax^2+a+b\right)}{4a}-\frac{\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a + b + 2*a*
x^2 + a*x^4]/(4*a)
```

Rubi [A] time = 0.0449957, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 634, 618, 204, 628}

$$\frac{\log\left(ax^4+2ax^2+a+b\right)}{4a}-\frac{\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a + b + 2*a*
x^2 + a*x^4]/(4*a)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a+b+2ax+ax^2}\,dx, x, x^2\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx, x, x^2\right)\right)+\frac{\operatorname{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{4a}\\
&= \frac{\log\left(a+b+2ax^2+ax^4\right)}{4a}+\operatorname{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\log\left(a+b+2ax^2+ax^4\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.016579, size = 49, normalized size = 0.91

$$\frac{\log\left(a\left(x^2+1\right)^2+b\right)-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ((-2*Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[b + a*(1 +
x^2)^2])/(4*a)
```

Maple [A] time = 0.045, size = 47, normalized size = 0.9

$$\frac{\ln\left(ax^4+2\,ax^2+a+b\right)}{4\,a}-\frac{1}{2}\arctan\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/4*ln(a*x^4+2*a*x^2+a+b)/a-1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56662, size = 312, normalized size = 5.78

$$\left[\frac{b\log\left(ax^4+2\,ax^2+a+b\right)-\sqrt{-ab}\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{-ab}\left(x^2+1\right)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,ab},\frac{b\log\left(ax^4+2\,ax^2+a+b\right)+2\,\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{ax^2+a}\right)}{4\,ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log(a*x^4 + 2*a*x^2 + a + b) - sqrt(-a*b)*log((a*x^4 + 2*a*x^2 + 2*
sqrt(-a*b)*(x^2 + 1) + a - b)/(a*x^4 + 2*a*x^2 + a + b)))/(a*b), 1/4*(b*log
(a*x^4 + 2*a*x^2 + a + b) + 2*sqrt(a*b)*arctan(sqrt(a*b)/(a*x^2 + a)))/(a*b
)]
```

Sympy [B] time = 0.508829, size = 117, normalized size = 2.17

$$\left(\frac{1}{4a} - \frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2 + \frac{-4ab\left(\frac{1}{4a} - \frac{\sqrt{-a^3b}}{4a^2b}\right) + a + b}{a}\right) + \left(\frac{1}{4a} + \frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2 + \frac{-4ab\left(\frac{1}{4a} + \frac{\sqrt{-a^3b}}{4a^2b}\right) + a + b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (1/(4*a) - sqrt(-a**3*b)/(4*a**2*b))*log(x**2 + (-4*a*b*(1/(4*a) - sqrt(-a*
*3*b)/(4*a**2*b)) + a + b)/a) + (1/(4*a) + sqrt(-a**3*b)/(4*a**2*b))*log(x*
*2 + (-4*a*b*(1/(4*a) + sqrt(-a**3*b)/(4*a**2*b)) + a + b)/a)
```

Giac [A] time = 3.41815, size = 57, normalized size = 1.06

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}} + \frac{\log\left(ax^4 + 2\,ax^2 + a + b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b) + 1/4*log(a*x^4 + 2*a*x^2 + a
+ b)/a
```

3.908 $\int \frac{x}{a+b+2ax^2+ax^4} \, dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0255772, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1107, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.0072065, size = 31, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.042, size = 26, normalized size = 0.8

$$\frac{1}{2}\arctan\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48632, size = 217, normalized size = 7.

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{-ab}(x^2+1)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,ab}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{ax^2+a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(-a*b)*(x^2 + 1) + a - b)/(a*
x^4 + 2*a*x^2 + a + b))/(a*b), -1/2*sqrt(a*b)*arctan(sqrt(a*b)/(a*x^2 + a))
/(a*b)]
```

Sympy [B] time = 0.278275, size = 60, normalized size = 1.94

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}+\frac{\sqrt{-\frac{1}{ab}}\log\left(b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-b*sqrt(-1/(a*b)) + x**2 + 1)/4 + sqrt(-1/(a*b))*log(b*
sqrt(-1/(a*b)) + x**2 + 1)/4
```

Giac [A] time = 3.37943, size = 28, normalized size = 0.9

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b)
```

3.909 $\int \frac{1}{x(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=69

$$-\frac{\log\left(ax^4+2ax^2+a+b\right)}{4(a+b)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)}+\frac{\log(x)}{a+b}$$

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*Sqrt[b]*(a + b)) + Log[x]
/(a + b) - Log[a + b + 2*a*x^2 + a*x^4]/(4*(a + b))
```

Rubi [A] time = 0.067098, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1114, 705, 29, 634, 618, 204, 628}

$$-\frac{\log\left(ax^4+2ax^2+a+b\right)}{4(a+b)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)}+\frac{\log(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*Sqrt[b]*(a + b)) + Log[x]
/(a + b) - Log[a + b + 2*a*x^2 + a*x^4]/(4*(a + b))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+b+2ax^2+ax^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(a+b+2ax+ax^2\right)}\,dx,x,x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2(a+b)}+\frac{\operatorname{Subst}\left(\int \frac{-2a-ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2(a+b)}\\
&= \frac{\log(x)}{a+b}-\frac{\operatorname{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{4(a+b)}-\frac{a\operatorname{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2(a+b)}\\
&= \frac{\log(x)}{a+b}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{4(a+b)}+\frac{a\operatorname{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx,x,2a\left(1+x^2\right)\right)}{a+b}\\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)}+\frac{\log(x)}{a+b}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{4(a+b)}
\end{aligned}
$$

Mathematica [C] time = 0.0514506, size = 105, normalized size = 1.52

$$
\frac{i\left(\sqrt{a}+i\sqrt{b}\right)\log\left(\sqrt{a}\left(x^2+1\right)-i\sqrt{b}\right)+\left(-\sqrt{b}-i\sqrt{a}\right)\log\left(\sqrt{a}\left(x^2+1\right)+i\sqrt{b}\right)+4\sqrt{b}\log(x)}{4\sqrt{b}(a+b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (4*Sqrt[b]*Log[x] + I*(Sqrt[a] + I*Sqrt[b])*Log[(-I)*Sqrt[b] + Sqrt[a]*(1 +
 x^2)] + ((-I)*Sqrt[a] - Sqrt[b])*Log[I*Sqrt[b] + Sqrt[a]*(1 + x^2)])/(4*Sq
rt[b]*(a + b))
```

Maple [A] time = 0.049, size = 63, normalized size = 0.9

$$
\frac{\ln(x)}{a+b}-\frac{\ln\left(ax^4+2\,ax^2+a+b\right)}{4\,a+4\,b}-\frac{a}{2\,a+2\,b}\arctan\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] ln(x)/(a+b)-1/4*ln(a*x^4+2*a*x^2+a+b)/(a+b)-1/2*a/(a+b)/(a*b)^(1/2)*arctan(
1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54784, size = 347, normalized size = 5.03

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\left(bx^2+b\right)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-\log\left(ax^4+2\,ax^2+a+b\right)+4\,\log\left(x\right)}{4\left(a+b\right)}, \frac{2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{ax^2+a}\right)-\log\left(ax^4+2\,ax^2+a\right.}{4\left(a+b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a/b) + a - b)/(
a*x^4 + 2*a*x^2 + a + b)) - log(a*x^4 + 2*a*x^2 + a + b) + 4*log(x))/(a + b
), 1/4*(2*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*x^2 + a)) - log(a*x^4 + 2*a*x^2 +
 a + b) + 4*log(x))/(a + b)]
```

Sympy [B] time = 2.26912, size = 194, normalized size = 2.81

$$\left(-\frac{1}{4\left(a+b\right)}-\frac{\sqrt{-ab}}{4b\left(a+b\right)}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{4(a+b)}-\frac{\sqrt{-ab}}{4b(a+b)}\right)+a-4b^2\left(-\frac{1}{4(a+b)}-\frac{\sqrt{-ab}}{4b(a+b)}\right)-b}{a}\right)+\left(-\frac{1}{4\left(a+b\right)}+\frac{\sqrt{-ab}}{4b\left(a+b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (-1/(4*(a + b)) - sqrt(-a*b)/(4*b*(a + b)))*log(x**2 + (-4*a*b*(-1/(4*(a +
b)) - sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4*(a + b)) - sqrt(-a*b)/(
4*b*(a + b))) - b)/a) + (-1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b)))*log(x**
2 + (-4*a*b*(-1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4
*(a + b)) + sqrt(-a*b)/(4*b*(a + b))) - b)/a) + log(x)/(a + b)
```

Giac [A] time = 3.48728, size = 82, normalized size = 1.19

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}(a+b)}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,(a+b)}+\frac{\log\left(x^2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/4*log(a*x^4 +
2*a*x^2 + a + b)/(a + b) + 1/2*log(x^2)/(a + b)
```

3.910 $\int \frac{1}{x^3(a+b+2ax^2+ax^4)} \, dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2(a+b)}+\frac{a\log\left(ax^4+2ax^2+a+b\right)}{2(a+b)^2}+\frac{\sqrt{a}(a-b)\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}$$

```
[Out] -1/(2*(a + b)*x^2) + (Sqrt[a]*(a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/
(2*Sqrt[b]*(a + b)^2) - (2*a*Log[x])/(a + b)^2 + (a*Log[a + b + 2*a*x^2 + a
*x^4])/(2*(a + b)^2)
```

Rubi [A] time = 0.131711, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1114, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{2x^2(a+b)}+\frac{a\log\left(ax^4+2ax^2+a+b\right)}{2(a+b)^2}+\frac{\sqrt{a}(a-b)\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/(2*(a + b)*x^2) + (Sqrt[a]*(a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/
(2*Sqrt[b]*(a + b)^2) - (2*a*Log[x])/(a + b)^2 + (a*Log[a + b + 2*a*x^2 + a
*x^4])/(2*(a + b)^2)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

, -1]

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b+2ax^2+ax^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+b+2ax+ax^2\right)}\,dx,x,x^2\right)$$

$$= -\frac{1}{2(a+b)x^2} + \frac{\operatorname{Subst}\left(\int \frac{-2a-ax}{x\left(a+b+2ax+ax^2\right)}\,dx,x,x^2\right)}{2(a+b)}$$

$$= -\frac{1}{2(a+b)x^2} + \frac{\operatorname{Subst}\left(\int \left(-\frac{2a}{(a+b)x}+\frac{a(3a-b+2ax)}{(a+b)\left(a+b+2ax+ax^2\right)}\right)dx,x,x^2\right)}{2(a+b)}$$

$$= -\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\operatorname{Subst}\left(\int \frac{3a-b+2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2(a+b)^2}$$

$$= -\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\operatorname{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2(a+b)^2} + \frac{(a(a-b))\operatorname{Subst}\left(\int \frac{}{a+b+}\right.}{2(a+b)^2}$$

$$= -\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\log\left(a+b+2ax^2+ax^4\right)}{2(a+b)^2} - \frac{(a(a-b))\operatorname{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx,\right.}{(a+b)^2}$$

$$= -\frac{1}{2(a+b)x^2} + \frac{\sqrt{a}(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(1+x^2\right)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\log\left(a+b+2ax^2+ax^4\right)}{2(a+b)^2}$$

Mathematica [C] time = 0.0924026, size = 163, normalized size = 1.83

$$\frac{\left(2a^{3/2}\sqrt{b}-ia^2+iab\right)\log\left(\sqrt{a}x^2+\sqrt{a}-i\sqrt{b}\right)}{4\sqrt{a}\sqrt{b}(a+b)^2} + \frac{\left(2a^{3/2}\sqrt{b}+ia^2-iab\right)\log\left(\sqrt{a}x^2+\sqrt{a}+i\sqrt{b}\right)}{4\sqrt{a}\sqrt{b}(a+b)^2} - \frac{1}{2x^2(a+b)} - \frac{2a\log(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/(2*(a + b)*x^2) - (2*a*Log[x])/(a + b)^2 + (((-I)*a^2 + 2*a^(3/2)*Sqrt[b
] + I*a*b)*Log[Sqrt[a] - I*Sqrt[b] + Sqrt[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(a +
b)^2) + ((I*a^2 + 2*a^(3/2)*Sqrt[b] - I*a*b)*Log[Sqrt[a] + I*Sqrt[b] + Sqrt
[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(a + b)^2)
```

Maple [A] time = 0.051, size = 110, normalized size = 1.2

$$-\frac{1}{(2a+2b)x^2} - 2\frac{a\ln(x)}{(a+b)^2} + \frac{a\ln\left(ax^4+2ax^2+a+b\right)}{2(a+b)^2} + \frac{a^2}{2(a+b)^2}\arctan\left(\frac{2ax^2+2a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{ab}{2(a+b)^2}\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] -1/2/(a+b)/x^2-2*a*ln(x)/(a+b)^2+1/2*a*ln(a*x^4+2*a*x^2+a+b)/(a+b)^2+1/2/(a
+b)^2*a^2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))-1/2/(a+b)^2*a/(
a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5574, size = 490, normalized size = 5.51

$$\left[-\frac{(a-b)x^2\sqrt{-\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\left(bx^2+b\right)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-2\,ax^2\log\left(ax^4+2\,ax^2+a+b\right)+8\,ax^2\log\left(x\right)+2\,a+2\,b}{4\left(a^2+2\,ab+b^2\right)x^2}, -\frac{(a-b)x^2 \cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - b)*x^2*sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a/
b) + a - b)/(a*x^4 + 2*a*x^2 + a + b)) - 2*a*x^2*log(a*x^4 + 2*a*x^2 + a +
b) + 8*a*x^2*log(x) + 2*a + 2*b)/((a^2 + 2*a*b + b^2)*x^2), -1/2*((a - b)*x
^2*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*x^2 + a)) - a*x^2*log(a*x^4 + 2*a*x^2 +
a + b) + 4*a*x^2*log(x) + a + b)/((a^2 + 2*a*b + b^2)*x^2)]
```

Sympy [B] time = 5.18822, size = 386, normalized size = 4.34

$$-\frac{2a\log(x)}{(a+b)^2}+\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}\,(a-b)}{4b\left(a^2+2ab+b^2\right)}\right)\log\left(x^2+\frac{4a^2b\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b\left(a^2+2ab+b^2\right)}\right)+a^2+8ab^2\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b\left(a^2+2ab+b^2\right)}\right)}{a^2-ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -2*a*log(x)/(a + b)**2 + (a/(2*(a + b)**2) - sqrt(-a*b)*(a - b)/(4*b*(a**2
+ 2*a*b + b**2)))*log(x**2 + (4*a**2*b*(a/(2*(a + b)**2) - sqrt(-a*b)*(a -
b)/(4*b*(a**2 + 2*a*b + b**2))) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) - sqrt(
-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**
2) - sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))))/(a**2 - a*b)) + (a/(2
*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2)))*log(x**2 + (
4*a**2*b*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))
) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a
*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(
a**2 + 2*a*b + b**2))))/(a**2 - a*b)) - 1/(x**2*(2*a + 2*b))
```

Giac [A] time = 3.40728, size = 169, normalized size = 1.9

$$\frac{a\log\left(ax^4+2\,ax^2+a+b\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2+2\,ab+b^2}+\frac{\left(a^2-ab\right)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{ab}}+\frac{2\,ax^2-a-b}{2\left(a^2+2\,ab+b^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a + b)/(a^2 + 2*a*b + b^2) - a*log(x^2)/(a^2 +
2*a*b + b^2) + 1/2*(a^2 - a*b)*arctan((a*x^2 + a)/sqrt(a*b))/((a^2 + 2*a*b
+ b^2)*sqrt(a*b)) + 1/2*(2*a*x^2 - a - b)/((a^2 + 2*a*b + b^2)*x^2)
```

3.911 $\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=432

$$\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}\right.}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}}}$$

```
[Out] x/a + ((a + b + 2*Sqrt[a]*Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]]
 - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[2]*a^(5/4)*Sqrt
[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - ((a + b + 2*Sqrt[a]*Sqrt[a + b])*Arc
Tan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[
a + b]]])/(2*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ((a
 + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[
a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[-Sq
rt[a] + Sqrt[a + b]]) - ((a + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] +
Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a
^(5/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rubi [A] time = 0.890587, antiderivative size = 432, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1122, 1169, 634, 618, 204, 628}

$$\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}\right.}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((a + b + 2*Sqrt[a]*Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]]
 - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[2]*a^(5/4)*Sqrt
[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - ((a + b + 2*Sqrt[a]*Sqrt[a + b])*Arc
Tan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[
a + b]]])/(2*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ((a
 + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[
a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[-Sq
rt[a] + Sqrt[a + b]]) - ((a + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] +
```

```
Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a
^(5/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx = \frac{x}{a} - \frac{\int \frac{a+b+2ax^2}{a+b+2ax^2+ax^4}\,dx}{a}$$

$$= \frac{x}{a} - \frac{\int \frac{\frac{\sqrt{2}(a+b)\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}-\left(a+b-2\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\int \frac{\frac{\sqrt{2}(a+b)\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+\left(a+b-2\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= \frac{x}{a} + \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+2x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\int \frac{\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a}\cdots}}{\sqrt[4]{a}}\cdots}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\cdots}}{\sqrt[4]{a}}\cdots}}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a}\cdots}}$$

$$= \frac{x}{a} + \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}x+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\left(a+b-2\sqrt{a}\cdots\right.}{\cdots}$$

$$= \frac{x}{a} + \frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} - \frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\cdots}}{\sqrt{\cdots}}\right.}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a}\cdots}}$$

Mathematica [C] time = 0.09335, size = 164, normalized size = 0.38

$$-\frac{i\left(\sqrt{a}-i\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a-i\sqrt{a}\sqrt{b}}}+\frac{i\left(\sqrt{a}+i\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a+i\sqrt{a}\sqrt{b}}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a - ((I/2)*(Sqrt[a] - I*Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*
Sqrt[b]]])/(a*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((I/2)*(Sqrt[a] + I*Sq
rt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/(a*Sqrt[a + I*Sqr
t[a]*Sqrt[b]]*Sqrt[b])
```

Maple [B] time = 0.23, size = 1658, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] x/a+1/8/a/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(a
+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/a^2/b*ln(-x^2*a^(1/2)+x*(2*(a*(
a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(a+b)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b
)^(1/2)-2*a)^(1/2)-1/4/a^(3/2)/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^
(1/2)-(a+b)^(1/2))*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a^(1/2
)/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(2*(a^2+a*
b)^(1/2)-2*a)^(1/2)+1/a/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)
*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)
-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(a+b)^(1/2)-1/4/a/b/(4*a^(1/2)*(a+b)^(1/2)-2
*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1
/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2
)-2*a)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a^2/b/(4*a^(1/2)
*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))
^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*
(a*(a+b))^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2
*a)^(1/2)+1/2/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)
*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)
-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2
)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/2/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(
a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/
(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a
)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/a/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))
^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/
8/a^2/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(a+b)^(
1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/4/a^(3/2)/b*ln(x^2*a^(
1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(a^2+a*b)^(1/2)*(2*(a^2+a
*b)^(1/2)-2*a)^(1/2)+1/4/a^(1/2)/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)
^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/a/(4*a^(1/2)*(a+b)^(1/2
)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^
(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(a+b)^(1/2)+1/4
/a/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2
)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2
*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a
)^(1/2)+1/4/a^2/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arcta
n((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(
```

```
a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*(a^2+a*b)
^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/2/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2
*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/
2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)
-2*a)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/2/a^(1/2)/b/(4*
a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*
(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2
))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [A] time = 1.60866, size = 1303, normalized size = 3.02

$$\frac{a\sqrt{\frac{a^2b\sqrt{-\frac{9\,a^2-6\,ab+b^2}{a^5b}}+a-3\,b}{a^2b}}\log\left(-\left(3\,a^2+2\,ab-b^2\right)x+\left(a^4b\sqrt{-\frac{9\,a^2-6\,ab+b^2}{a^5b}}+3\,a^2b-ab^2\right)\sqrt{\frac{a^2b\sqrt{-\frac{9\,a^2-6\,ab+b^2}{a^5b}}+a-3\,b}{a^2b}}\right)-a\sqrt{\frac{a^2b\sqrt{\cdots}}{\cdots}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3*b)/(a^2*b))
*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b))
 + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a -
3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3
*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x - (a^4*b*sqrt(-(9*a^2 - 6*a*b + b
^2)/(a^5*b)) + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^
5*b)) + a - 3*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^
5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqrt(-(9*a^
2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*
```

```
a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*
a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x - (a^4
*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(-(a^2*b*sqr
t(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + 4*x)/a
```

Sympy [A] time = 1.06445, size = 105, normalized size = 0.24

$$\text{RootSum}\left(256t^4a^5b^2 + t^2\left(-32a^4b + 96a^3b^2\right) + a^3 + 3a^2b + 3ab^2 + b^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^4b + 4ta^3 - 24ta^2b + 4tab}{3a^2 + 2ab - b^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(256*_t**4*a**5*b**2 + _t**2*(-32*a**4*b + 96*a**3*b**2) + a**3 + 3*
a**2*b + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b + 4*_t*a*
*3 - 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 + 2*a*b - b**2)))) + x/a
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.912 $\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=331

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\cdots}{\sqrt{\sqrt{a+b}+\cdots}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}\cdots}}$$

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + S
qrt[a + b]]]/(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ArcTan[(Sqrt
[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]]/
(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt[a + b] - Sqrt[2]
*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*S
qrt[-Sqrt[a] + Sqrt[a + b]]) - Log[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt
[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*Sqrt[-Sqrt[a] + Sqrt
[a + b]])
```

Rubi [A] time = 0.255547, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1129, 634, 618, 204, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\cdots}{\sqrt{\sqrt{a+b}+\cdots}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + S
qrt[a + b]]]/(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ArcTan[(Sqrt
[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]]/
(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt[a + b] - Sqrt[2]
*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*S
qrt[-Sqrt[a] + Sqrt[a + b]]) - Log[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt
[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*Sqrt[-Sqrt[a] + Sqrt
[a + b]])
```

Rule 1129

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = \frac{\int \frac{x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}dx}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\int \frac{x}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}dx}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}dx}{4a} + \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}dx}{4a} + \frac{\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+2x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}dx}{4\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\int \frac{}{\frac{\sqrt{a}}{\sqrt{}}}}{4\sqrt{2}}$$

$$= \frac{\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}x+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\log\left(\sqrt{a+b}+\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}x\right.}{4\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\right.}{4\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+}}$$

Mathematica [C] time = 0.116752, size = 143, normalized size = 0.43

$$\frac{\frac{\left(\sqrt{b}+i\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{b}-i\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (((I*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/Sq
rt[a - I*Sqrt[a]*Sqrt[b]] + (((-I)*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sq
rt[a + I*Sqrt[a]*Sqrt[b]]])/Sqrt[a + I*Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*Sqrt[b]
)
```

Maple [B] time = 0.159, size = 724, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/8/a^(3/2)/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*
(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a^(3/2)/b/(4*a^(1/2)*(a+b
)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2
)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a
+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/a^(
1/2)/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(2*(a^2
+a*b)^(1/2)-2*a)^(1/2)-1/4/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/
2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2
)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(
2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/a^(3/2)/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1
/2)-2*a)^(1/2)+(a+b)^(1/2))*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1
/4/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*
x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))
^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*
b)^(1/2)-2*a)^(1/2)-1/8/a^(1/2)/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^
(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/4/a^(1/2)/b/(4*a^(1/2)*(
a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1
/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*
(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{ax^4 + 2\,ax^2 + a + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x^4 + 2*a*x^2 + a + b), x)
```

Fricas [A] time = 1.52766, size = 633, normalized size = 1.91

$$\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt((a*b*sqrt(-1/(a^3
*b)) + 1)/(a*b))*sqrt(-1/(a^3*b)) + x) - 1/4*sqrt((a*b*sqrt(-1/(a^3*b)) + 1
)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*sqrt(-1/(a^3*b))
 + x) - 1/4*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b))*log(a^2*b*sqrt(-(a*b*sq
rt(-1/(a^3*b)) - 1)/(a*b))*sqrt(-1/(a^3*b)) + x) + 1/4*sqrt(-(a*b*sqrt(-1/(
a^3*b)) - 1)/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b))*sqrt
(-1/(a^3*b)) + x)
```

Sympy [A] time = 0.355501, size = 44, normalized size = 0.13

$$\operatorname{RootSum}\left(256t^4a^3b^2 - 32t^2a^2b + a + b, \left(t \mapsto t\log\left(64t^3a^2b - 4ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b + a + b, Lambda(_t, _t*log(64
*_t**3*a**2*b - 4*_t*a + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.913 $\int \frac{1}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=359

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{\ldots}}}{\sqrt{\sqrt{a+\ldots}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\ldots}}$$

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + S
qrt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) +
ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sq
rt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - L
og[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x
^2]/(4*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt
[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*
Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rubi [A] time = 0.261469, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{\ldots}}}{\sqrt{\sqrt{a+\ldots}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + S
qrt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) +
ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sq
rt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - L
og[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x
^2]/(4*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt
[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*
Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx = \frac{\int \frac{\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}-x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \frac{\int \frac{\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+x}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{a}\sqrt{a+b}} + \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{a}\sqrt{a+b}} - \frac{\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+2x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \cdots$$

$$= -\frac{\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}x+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \frac{\log\left(\sqrt{a+b}+\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}x \cdots\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} \cdots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} - \frac{\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{- \cdots}\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\cdots}} \cdots$$

Mathematica [C] time = 0.0709081, size = 119, normalized size = 0.33

$$\frac{i\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a+i\sqrt{a}\sqrt{b}}} - \frac{i\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a-i\sqrt{a}\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ((-I/2)*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/(Sqrt[a - I*Sqrt[a
]*Sqrt[b]]*Sqrt[b]) + ((I/2)*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]
])/(Sqrt[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b])
```

Maple [B] time = 0.167, size = 913, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] -1/8/(a+b)^(1/2)/a/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^
(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/8/(a+b)^(1/2)/b*ln(-
x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)
-2*a)^(1/2)-1/(a+b)^(1/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/
2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/
2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))+1/4/(a+b)^(1/2)/a/b/(4*a^(1/2)*(a+b)^(1/2)
-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^
(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1
/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4/(a+b)^(1/2
)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2
)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2
*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/
(a+b)^(1/2)/a/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))
*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/8/(a+b)^(1/2)/b*ln(x^2*a^(
1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(
1/2)+1/(a+b)^(1/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arct
an((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*
(a+b))^(1/2)+2*a)^(1/2))-1/4/(a+b)^(1/2)/a/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a
+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4
*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^
(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4/(a+b)^(1/2)/b/(4*a^
(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a
+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))
*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax^4 + 2\,ax^2 + a + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] integrate(1/(a*x^4 + 2*a*x^2 + a + b), x)
```

Fricas [B] time = 1.52051, size = 1195, normalized size = 3.33

$$\frac{1}{4}\sqrt{\frac{\left(ab+b^2\right)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}+1}{ab+b^2}}\log\left(\left(\left(a^2b+ab^2\right)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}+b\right)\sqrt{\frac{\left(ab+b^2\right)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}+1}{ab+b^2}}+x\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)
)*log(((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a*b
 + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/4*s
qrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2))*log
(-((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a*b + b
^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/4*sqrt(
-((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))*log(((
a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a*b + b^2)
*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x) + 1/4*sqrt(-((
a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))*log(-((a^
2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a*b + b^2)*s
qrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x)
```

Sympy [A] time = 0.616945, size = 63, normalized size = 0.18

$$\operatorname{RootSum}\left(t^4\left(256a^2b^2+256ab^3\right)-32t^2ab+1,\left(t\mapsto t\log\left(64t^3a^2b+64t^3ab^2-4ta+4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 + 256*a*b**3) - 32*_t**2*a*b + 1, Lambda(_t, _
t*log(64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a + 4*_t*b + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.914 $\int \frac{1}{x^2(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=433

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\right.}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

```
[Out] -(1/((a + b)*x)) + (a^(1/4)*(2*Sqrt[a] + Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a]
 + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[
2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] + Sqrt[
a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt
[a] + Sqrt[a + b]]])/(2*Sqrt[2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]])
+ (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt
[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*(a + b)^(3/2)*Sqrt[-S
qrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a + b]
 + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2
]*(a + b)^(3/2)*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rubi [A] time = 0.518721, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1123, 1169, 634, 618, 204, 628}

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\right.}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```


```
[Out] -(1/((a + b)*x)) + (a^(1/4)*(2*Sqrt[a] + Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a]
 + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[
2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] + Sqrt[
a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt
[a] + Sqrt[a + b]]])/(2*Sqrt[2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]])
+ (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt
[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*(a + b)^(3/2)*Sqrt[-S
qrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a + b]
```

```
 + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2
]*(a + b)^(3/2)*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b+2ax^2+ax^4\right)}\,dx = -\frac{1}{(a+b)x}+\frac{\int \frac{-2a-ax^2}{a+b+2ax^2+ax^4}\,dx}{a+b}$$

$$= -\frac{1}{(a+b)x}+\frac{\int \frac{-2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}-\left(-2a+\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}+\frac{\int \frac{-2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}+\left(-2a+\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}}{2\sqrt{2}\sqrt[4]{a}(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+}}}$$

$$= -\frac{1}{(a+b)x}+\frac{\left(\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\right)\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+2x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{2}(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}-\frac{\left(\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\right)\int}{4\sqrt{2}(a+b)^{3/2}\sqrt{}}$$

$$= -\frac{1}{(a+b)x}+\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}x+\sqrt{a}x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}-\frac{\sqrt[4]{a}\,(}{}$$

$$= -\frac{1}{(a+b)x}+\frac{\sqrt[4]{a}\left(2\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a}+\sqrt{a+b}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}}{2\sqrt{2}(a+b)^{3/2}\sqrt{}}$$

Mathematica [C] time = 0.141398, size = 174, normalized size = 0.4

$$\frac{1}{x(-a-b)}+\frac{\left(-\sqrt{a}\sqrt{b}+ia\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a-i\sqrt{a}\sqrt{b}}(a+b)}+\frac{\left(-\sqrt{a}\sqrt{b}-ia\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a+i\sqrt{a}\sqrt{b}}(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] 1/((-a - b)*x) + ((I*a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqr
t[a]*Sqrt[b]]])/(2*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a + b)) + (((-I)*a
- Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/(2*Sqrt
[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a + b))
```

Maple [B] time = 0.173, size = 3318, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] -1/4*a^(1/2)/(a+b)^2/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*
arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2
*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/
2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/2*a/(a+b)^(5/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*
(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2
))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-
2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/2*a/(a+b)^(5/2)/
b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+
(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a
)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a
*b)^(1/2)+1/4*a^(1/2)/(a+b)^2/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*
a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+
b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^
2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/8/a^(1/2)/(a+b)^2*ln(x^2*a^(1/2)+
x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*
(a^2+a*b)^(1/2)+1/8*a^(3/2)/(a+b)^2/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2
*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4*a^2/(a+b)^(5/2)/b*
ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1
/2)-2*a)^(1/2)+1/4*a^2/(a+b)^(5/2)/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2
*a)^(1/2)-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+2*a/(a+b)^(5/2)*b/(4*a
^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*
(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2
))-1/8/a^(1/2)/(a+b)^2*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b
)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/8*a^(3/2)/(a+b)^2/
b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(2*(a^2+a*b)
^(1/2)-2*a)^(1/2)-2*a/(a+b)^(5/2)*b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2
)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*
(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))+1/4*a/(a+b)^(5/2)*ln(-x^2*a^(1/2)
+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
+2*a^2/(a+b)^(5/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arct
an((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a
*(a+b))^(1/2)+2*a)^(1/2))-1/8*a^(1/2)/(a+b)^2*ln(-x^2*a^(1/2)+x*(2*(a*(a+b)
)^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8*a^(1/2)/(
a+b)^2*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+
a*b)^(1/2)-2*a)^(1/2)-1/4*a/(a+b)^(5/2)*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)
```

```
-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-2*a^2/(a+b)^(5/2)/(4
*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a
*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/
2))-1/4/(a+b)^(5/2)*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1
/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4/(a+b)^(5/2)*ln(-x^2*
a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a
)^(1/2)*(a^2+a*b)^(1/2)-1/(a+b)/x-1/4/a^(1/2)/(a+b)^2/(4*a^(1/2)*(a+b)^(1/2
)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^
(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1
/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/2*a^2/(a+b)^
(5/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^
(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/
2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+
1/2*a^2/(a+b)^(5/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*a
rctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*
(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2
)-2*a)^(1/2)+1/4/a^(1/2)/(a+b)^2/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2
*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a
+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a
^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4*a^(3/2)/(a+b)^2/b/(4*a^(1/2)*(
a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(
1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a
*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4*a^(3/2)/(a+b)^2/
b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(
2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)
^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/2/(a+
b)^(5/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^
(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/
2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*
(a^2+a*b)^(1/2)-1/2/(a+b)^(5/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*
a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+
b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^
2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4*a^(1/2)/(a+b)^2/(4*a^(1/2)*(a+b
)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2
)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a
+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4*a/(a+b)^(5/2)/b*ln(
x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)
-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4*a/(a+b)^(5/2)/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+
b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(
1/2)-1/8*a^(1/2)/(a+b)^2/b*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-
(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4*a^(1/2)/(a+b
)^2/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)
+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*
a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8*a
^(1/2)/(a+b)^2/b*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2)
```

```
)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/2*a/(a+b)^(5/2)/(4*a^(1/2
)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b)
)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2
*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/2*a/(a+b)^(5/2)
/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2
*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^
(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 1.61757, size = 3301, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*((a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt
(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 +
15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*log(-(3*
a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*sqr
t(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 +
 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3
 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4))) - (a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4
)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*
b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*lo
g(-(3*a^2 - a*b)*x - (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^
5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3
*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2
```

```
 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a
^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4))) + (a + b)*x*sqrt((a^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 2
0*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4))*log(-(3*a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 - (a^4*b + 2*a^3*b^2 - 2*a*b^
4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 +
20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b - (a^3*b + 3*a
^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2
+ 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^
2 + 3*a*b^3 + b^4))) - (a + b)*x*sqrt((a^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3
*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b
^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4))*log(-(3*a^2 - a*b)*x - (6*a^2*b - 2*a*b^2 - (a^4*b + 2*a^3*b^2 -
2*a*b^4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*
b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b - (a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^
5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*
a^2*b^2 + 3*a*b^3 + b^4))) - 4)/((a + b)*x)
```

Sympy [A] time = 2.23106, size = 134, normalized size = 0.31

$$\operatorname{RootSum}\left(t^4\left(256a^3b^2+768a^2b^3+768ab^4+256b^5\right)+t^2\left(-32a^2b+96ab^2\right)+a,\left(t\mapsto t\log\left(x+\frac{-64t^3a^4b-128t^3a^3b^2\cdots}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(_t**4*(256*a**3*b**2 + 768*a**2*b**3 + 768*a*b**4 + 256*b**5) + _t*
*2*(-32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b -
128*_t**3*a**3*b**2 + 128*_t**3*a*b**4 + 64*_t**3*b**5 + 4*_t*a**3 - 40*_t*
a**2*b + 20*_t*a*b**2)/(3*a**2 - a*b)))) - 1/(x*(a + b))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.915 $\int \frac{x}{1+x^2+x^4} \, dx$

Optimal. Leaf size=20

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0198906, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1107, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right)$$
$$= \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.005332, size = 20, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Maple [A] time = 0.042, size = 19, normalized size = 1.

$$\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+x^2+1),x)
```

```
[Out] 1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.46805, size = 24, normalized size = 1.2

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Fricas [A] time = 1.46866, size = 61, normalized size = 3.05

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Sympy [A] time = 0.102124, size = 26, normalized size = 1.3

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+x**2+1),x)
```

```
[Out] sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/3
```

Giac [A] time = 1.36519, size = 24, normalized size = 1.2

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

3.916 $\int \frac{x}{10+2x^2+x^4} \, dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rubi [A] time = 0.0155259, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1107, 618, 204}

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{10+2x+x^2}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx, x, 2\left(1+x^2\right)\right)$$
$$= \frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(1+x^2\right)\right)$$

Mathematica [A] time = 0.0052661, size = 14, normalized size = 1.

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Maple [A] time = 0.043, size = 11, normalized size = 0.8

$$\frac{1}{6}\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+10),x)
```

```
[Out] 1/6*arctan(1/3*x^2+1/3)
```

Maxima [A] time = 1.43092, size = 14, normalized size = 1.

$$\frac{1}{6}\,\arctan\left(\frac{1}{3}\,x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="maxima")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Fricas [A] time = 1.42479, size = 36, normalized size = 2.57

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Sympy [A] time = 0.1016, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**2+10),x)
```

```
[Out] atan(x**2/3 + 1/3)/6
```

Giac [A] time = 1.26396, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="giac")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

3.917 $\int \frac{x^2}{20+9x^2+x^4} dx$

Optimal. Leaf size=23

$$\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Rubi [A] time = 0.0124621, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1130, 203}

$$\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(20 + 9*x^2 + x^4),x]
```

```
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{20+9x^2+x^4}\,dx = -\left(4\int \frac{1}{4+x^2}\,dx\right)+5\int \frac{1}{5+x^2}\,dx$$
$$= -2\tan^{-1}\left(\frac{x}{2}\right)+\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.0129906, size = 23, normalized size = 1.

$$\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(20 + 9*x^2 + x^4),x]
```

```
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Maple [A] time = 0.049, size = 19, normalized size = 0.8

$$-2\arctan(x/2)+\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+9*x^2+20),x)
```

```
[Out] -2*arctan(1/2*x)+arctan(1/5*x*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.44195, size = 24, normalized size = 1.04

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-2\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="maxima")
```

[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)

Fricas [A] time = 1.44118, size = 66, normalized size = 2.87

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - 2\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="fricas")

[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)

Sympy [A] time = 0.133676, size = 20, normalized size = 0.87

$$-2\operatorname{atan}\left(\frac{x}{2}\right) + \sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**4+9*x**2+20),x)

[Out] -2*atan(x/2) + sqrt(5)*atan(sqrt(5)*x/5)

Giac [A] time = 1.23379, size = 24, normalized size = 1.04

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - 2\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="giac")

[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)

3.918 $\int \frac{x^2}{1-x^2+x^4} dx$

Optimal. Leaf size=74

$$\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 + Log[1 - Sqrt[3]*x + x^
2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi [A] time = 0.0502575, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 + Log[1 - Sqrt[3]*x + x^
2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{1-x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1-x^2+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}\\
&= \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)-\frac{1}{2}\text{Subst}\left(\int \right.\\
&= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}
\end{aligned}
$$

Mathematica [C] time = 0.141765, size = 94, normalized size = 1.27

$$\frac{\sqrt{-1-i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+\sqrt{-1+i\sqrt{3}}\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(1 - x^2 + x^4),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2] + Sqrt[-1
 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/(2*Sqrt[6])
```

Maple [A] time = 0.053, size = 57, normalized size = 0.8

$$\frac{\arctan\left(2x-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x+\sqrt{3}\right)}{2}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-x^2+1),x)
```

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))+1/12*ln(1+x^2-x*3^(1/2))*3^
(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^4 - x^2 + 1), x)
```

Fricas [B] time = 1.57218, size = 529, normalized size = 7.15

$$-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sq
rt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/6*sqrt(6)*
sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)
*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - 1/24*sqrt(6)*sqrt(2)*log
(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 1/24*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2)
*x + 2*x^2 + 2)
```

Sympy [A] time = 0.168825, size = 63, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-x**2+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/12
 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^4 - x^2 + 1), x)
```

3.919 $\int \frac{x^2}{2-2x^2+x^4} \, dx$

Optimal. Leaf size=188

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(\right.}$$

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1 + S
qrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/
Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*
Sqrt[2*(1 + Sqrt[2])])
```

Rubi [A] time = 0.177248, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1 + S
qrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/
Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*
Sqrt[2*(1 + Sqrt[2])])
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{2-2x^2+x^4}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx+\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{-\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x-x^2}\,dx}{4\sqrt{2\left(1+\sqrt{2}\right)}}+$$

$$= \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,d\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2}+\sqrt{2(1\right.}{4\sqrt{2(1}}$$

Mathematica [C] time = 0.0320793, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^
(3/2)
```

Maple [B] time = 0.076, size = 308, normalized size = 1.6

$$-\frac{\sqrt{2+2\sqrt{2}}\sqrt{2}\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)}{4\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)+\frac{\sqrt{2+2\sqrt{2}}\ln\left(x^2+\sqrt{2}+x\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-2*x^2+2),x)
```

```
[Out] -1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))+1/4*
2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2
))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^
(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)
-x*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arct
an((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*
ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2
)*arctan((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^4-2x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^4 - 2*x^2 + 2), x)
```

Fricas [A] time = 1.58498, size = 771, normalized size = 4.1

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2^(3/4)*x*sqrt(2*sqrt(2)
 + 4) + 2*x^2 + 2*sqrt(2)) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)
*log(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/4*2^(3/4)*sqrt
(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 1/2*2^(1/4)*sqr
t(2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2
*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2
*sqrt(2))*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)
```

Sympy [A] time = 0.461508, size = 24, normalized size = 0.13

$$\mathrm{RootSum}\left(128t^4 + 16t^2 + 1, \left(t \mapsto t\log\left(64t^3 + 4t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-2*x**2+2),x)
```

```
[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^4 - 2x^2 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^4 - 2*x^2 + 2), x)
```

3.920 $\int x^7\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=171

$$\frac{\left(-32ac+35b^2-42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{480c^3}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tan}{512c^{9/2}}$$

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^4) + (x^
4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + ((35*b^2 - 32*a*c - 42*b*c*x^2)*(a +
b*x^2 + c*x^4)^(3/2))/(480*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(9/2))
```

Rubi [A] time = 0.155271, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 742, 779, 612, 621, 206}

$$\frac{\left(-32ac+35b^2-42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{480c^3}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tan}{512c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```



```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^4) + (x^
4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + ((35*b^2 - 32*a*c - 42*b*c*x^2)*(a +
b*x^2 + c*x^4)^(3/2))/(480*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(9/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
```

```
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int x^3\sqrt{a+bx+cx^2}\,dx,x,x^2\right)$$
$$= \frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c} + \frac{\operatorname{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{10c}$$
$$= \frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c} + \frac{\left(35b^2-32ac-42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{480c^3} - \frac{\left(b\left(7b^2-12ac\right)\right)\operatorname{Subst}\left(\right.}{6}$$
$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4} + \frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c} + \frac{\left(35b^2-32ac-42bcx^2\right)}{480c^3}$$
$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4} + \frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c} + \frac{\left(35b^2-32ac-42bcx^2\right)}{480c^3}$$
$$= -\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4} + \frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c} + \frac{\left(35b^2-32ac-42bcx^2\right)}{480c^3}$$

Mathematica [A] time = 0.149221, size = 164, normalized size = 0.96

$$\frac{-\frac{\left(32ac-35b^2+42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{5\left(12abc-7b^3\right)\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)}{256c^{7/2}} + x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(3/2) - ((-35*b^2 + 32*a*c + 42*b*c*x^2)*(a + b*x^
2 + c*x^4)^(3/2))/(48*c^2) + (5*(-7*b^3 + 12*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^2
)*Sqrt[a + b*x^2 + c*x^4] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*
Sqrt[a + b*x^2 + c*x^4])]))/(256*c^(7/2)))/(10*c)
```

Maple [A] time = 0.175, size = 296, normalized size = 1.7

$$\frac{x^4}{10\,c}\left(cx^4+bx^2+a\right)^{\frac{3}{2}} - \frac{7\,bx^2}{80\,c^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}} + \frac{7\,b^2}{96\,c^3}\left(cx^4+bx^2+a\right)^{\frac{3}{2}} - \frac{7\,b^3x^2}{128\,c^3}\sqrt{cx^4+bx^2+a} - \frac{7\,b^4}{256\,c^4}\sqrt{cx^4+bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/10*x^4*(c*x^4+b*x^2+a)^(3/2)/c-7/80*b/c^2*x^2*(c*x^4+b*x^2+a)^(3/2)+7/96*
b^2/c^3*(c*x^4+b*x^2+a)^(3/2)-7/128*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)*x^2-7/256
*b^4/c^4*(c*x^4+b*x^2+a)^(1/2)-5/64*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c
*x^4+b*x^2+a)^(1/2))*a+7/512*b^5/c^(9/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*
x^2+a)^(1/2))+3/32*b/c^2*a*(c*x^4+b*x^2+a)^(1/2)*x^2+3/64*b^2/c^3*a*(c*x^4+
b*x^2+a)^(1/2)+3/32*b/c^(5/2)*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^
(1/2))-1/15*a/c^2*(c*x^4+b*x^2+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64125, size = 859, normalized size = 5.02

$$\left[\frac{15\left(7b^5 - 40ab^3c + 48a^2bc^2\right)\sqrt{c}\log\left(-8c^2x^4 - 8bcx^2 - b^2 - 4\sqrt{cx^4 + bx^2 + a}\left(2cx^2 + b\right)\sqrt{c} - 4ac\right) + 4\left(384c^5x^8 + 48b\right.}{15360c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8
*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) +
 4*(384*c^5*x^8 + 48*b*c^4*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 -
8*(7*b^2*c^3 - 16*a*c^4)*x^4 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^2)*sqrt(c*x^4
 + b*x^2 + a))/c^5, -1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(-c
)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*
x^2 + a*c)) - 2*(384*c^5*x^8 + 48*b*c^4*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 2
56*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^4 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^
```

```
2)*sqrt(c*x^4 + b*x^2 + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \sqrt{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.38191, size = 248, normalized size = 1.45

$$\frac{1}{3840} \sqrt{cx^4 + bx^2 + a}\left(2\left(4\left(6\left(8\,x^2 + \frac{b}{c}\right)x^2 - \frac{7\,b^2c^5 - 16\,ac^6}{c^7}\right)x^2 + \frac{35\,b^3c^4 - 116\,abc^5}{c^7}\right)x^2 - \frac{105\,b^4c^3 - 460\,ab^2c^4 + 256\,a}{c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^5 - 16
*a*c^6)/c^7)*x^2 + (35*b^3*c^4 - 116*a*b*c^5)/c^7)*x^2 - (105*b^4*c^3 - 460
*a*b^2*c^4 + 256*a^2*c^5)/c^7) - 1/512*(7*b^5*c^3 - 40*a*b^3*c^4 + 48*a^2*b
*c^5)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(1
5/2)
```

3.921 $\int x^5\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=153

$$\frac{(5b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(a+$$

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^3) - (5*b*(a
 + b*x^2 + c*x^4)^(3/2))/(48*c^2) + (x^2*(a + b*x^2 + c*x^4)^(3/2))/(8*c) -
 ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(256*c^(7/2))
```

Rubi [A] time = 0.129359, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 742, 640, 612, 621, 206}

$$\frac{(5b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{7/2}}-\frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(a+$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^3) - (5*b*(a
 + b*x^2 + c*x^4)^(3/2))/(48*c^2) + (x^2*(a + b*x^2 + c*x^4)^(3/2))/(8*c) -
 ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(256*c^(7/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
```

```
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a+bx+cx^2}\,dx,x,x^2\right)$$
$$= \frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c} + \frac{\text{Subst}\left(\int\left(-a-\frac{5bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{8c}$$
$$= -\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c} + \frac{\left(5b^2-4ac\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^2\right.}{32c^2}$$
$$= \frac{\left(5b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^3} - \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c} - \frac{\left(\left(b^2\right.\right.}{}$$
$$= \frac{\left(5b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^3} - \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c} - \frac{\left(\left(b^2\right.\right.}{}$$
$$= \frac{\left(5b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^3} - \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48c^2} + \frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{8c} - \frac{\left(b^2-\right.}{}$$

Mathematica [A] time = 0.0698704, size = 136, normalized size = 0.89

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(b\left(8c^2x^4-52ac\right)+24c^2x^2\left(a+2cx^4\right)-10b^2cx^2+15b^3\right)-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{b+2}{2\sqrt{c}\sqrt{a+}}\right.}{768c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^3 - 10*b^2*c*x^2 + 24*c^2*x^2*(a +
 2*c*x^4) + b*(-52*a*c + 8*c^2*x^4)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(768*c^(7/2))
```

Maple [A] time = 0.167, size = 247, normalized size = 1.6

$$\frac{x^2}{8c}\left(cx^4+bx^2+a\right)^{\frac{3}{2}} - \frac{5b}{48c^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}} + \frac{5b^2x^2}{64c^2}\sqrt{cx^4+bx^2+a} + \frac{5b^3}{128c^3}\sqrt{cx^4+bx^2+a} + \frac{3b^2a}{32}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*x^2*(c*x^4+b*x^2+a)^(3/2)/c-5/48*b*(c*x^4+b*x^2+a)^(3/2)/c^2+5/64*b^2/c
^2*(c*x^4+b*x^2+a)^(1/2)*x^2+5/128*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)+3/32*b^2/c
^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-5/256*b^4/c^(7/2)*
ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/16*a/c*(c*x^4+b*x^2+a)^(1
/2)*x^2-1/32*a/c^2*(c*x^4+b*x^2+a)^(1/2)*b-1/16*a^2/c^(3/2)*ln((1/2*b+c*x^2
)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66946, size = 698, normalized size = 4.56

$$\left[\frac{3\left(5\,b^{4}-24\,ab^{2}c+16\,a^{2}c^{2}\right)\sqrt{c}\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}+4\,\sqrt{cx^{4}+bx^{2}+a}\left(2\,cx^{2}+b\right)\sqrt{c}-4\,ac\right)+4\left(48\,c^{4}x^{6}+8\,bc^{3}\right.}{1536\,c^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c
*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(
48*c^4*x^6 + 8*b*c^3*x^4 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)
*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c
^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2
*x^4 + b*c*x^2 + a*c)) + 2*(48*c^4*x^6 + 8*b*c^3*x^4 + 15*b^3*c - 52*a*b*c^
2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.43727, size = 197, normalized size = 1.29

$$\frac{1}{384}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c^3-12ac^4}{c^5}\right)x^2+\frac{15b^3c^2-52abc^3}{c^5}\right)+\frac{\left(5b^4c^2-24ab^2c^3+16a^2c^4\right)\log\left(\left|-2\right.\right.}{256c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c^3 - 12*a*c
^4)/c^5)*x^2 + (15*b^3*c^2 - 52*a*b*c^3)/c^5) + 1/256*(5*b^4*c^2 - 24*a*b^2
*c^3 + 16*a^2*c^4)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(
c) - b))/c^(11/2)
```

3.922 $\int x^3\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=108

$$\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}-\frac{b(b+2cx^2)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{(a+bx^2+cx^4)^{3/2}}{6c}$$

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c^2) + (a + b*x^2 + c*x^4)^(
3/2)/(6*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/(32*c^(5/2))
```

Rubi [A] time = 0.0807406, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 640, 612, 621, 206}

$$\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}-\frac{b(b+2cx^2)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{(a+bx^2+cx^4)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c^2) + (a + b*x^2 + c*x^4)^(
3/2)/(6*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/(32*c^(5/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} - \frac{b\,\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{32c^2}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} + \frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{16c^2}\\
&= -\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c} + \frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0461451, size = 101, normalized size = 0.94

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(8c\left(a+cx^4\right)-3b^2+2bcx^2\right)+3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{96c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 2*b*c*x^2 + 8*c*(a + c*x^4)) +
 3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]
)])/(96*c^(5/2))
```

Maple [A] time = 0.158, size = 139, normalized size = 1.3

$$\frac{1}{6c}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{bx^2}{8c}\sqrt{cx^4+bx^2+a}-\frac{b^2}{16c^2}\sqrt{cx^4+bx^2+a}-\frac{ab}{8}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}+\frac{b^3}{32}\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c-1/8*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16*b^2/c^2*
(c*x^4+b*x^2+a)^(1/2)-1/8*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))*a+1/32*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72775, size = 552, normalized size = 5.11

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)-4\left(8\,c^3x^4+2\,bc^2x^2-3\,b^2c+8\right.}{192\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqr
t(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^4 + 2*b*c^
2*x^2 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/96*(3*(b^3 - 4*
a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(
c^2*x^4 + b*c*x^2 + a*c)) - 2*(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c + 8*a*c^2)
*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.37998, size = 132, normalized size = 1.22

$$\frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{\left(b^3-4abc\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{32c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) -
1/32*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqr
t(c) - b))/c^(5/2)
```

3.923 $\int x\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=83

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2))
```

Rubi [A] time = 0.0546726, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1107, 612, 621, 206}

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2))
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8c}-\frac{\left(b^2-4ac\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{16c}\\ &= \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8c}-\frac{\left(b^2-4ac\right)\operatorname{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8c}\\ &= \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0228379, size = 83, normalized size = 1.

$$\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2))
```

Maple [A] time = 0.155, size = 101, normalized size = 1.2

$$\frac{2\,cx^2+b}{8\,c}\sqrt{cx^4+bx^2+a}+\frac{a}{4}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}-\frac{b^2}{16}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+1/4/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))*a-1/16/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59443, size = 463, normalized size = 5.58

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2+bc\right)}{32\,c^2},\ -\frac{(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a)*
(2*c^2*x^2 + b*c))/c^2, 1/16*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x
^4 + b*x^2 + a)*(2*c^2*x^2 + b*c))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.33372, size = 103, normalized size = 1.24

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{\left(b^2-4ac\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(-2*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2)
```

3.924 $\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx$

Optimal. Leaf size=109

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

```
[Out] Sqrt[a + b*x^2 + c*x^4]/2 - (Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])/2 + (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(4*Sqrt[c])
```

Rubi [A] time = 0.109071, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 734, 843, 621, 206, 724}

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/2 - (Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])/2 + (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(4*Sqrt[c])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
```

```
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\sqrt{a+bx^2+cx^4} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\sqrt{a+bx^2+cx^4} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) + \frac{1}{4}b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx\right.$$
$$= \frac{1}{2}\sqrt{a+bx^2+cx^4} - a\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right) + \frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x,\right.$$
$$= \frac{1}{2}\sqrt{a+bx^2+cx^4} - \frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

Mathematica [A] time = 0.04063, size = 106, normalized size = 0.97

$$\frac{1}{4}\left(2\sqrt{a+bx^2+cx^4} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) + \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^2 + c*x^4] - 2*Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])] + (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2
 + c*x^4])])/Sqrt[c])/4
```

Maple [A] time = 0.163, size = 91, normalized size = 0.8

$$\frac{1}{2}\sqrt{cx^4+bx^2+a} + \frac{b}{4}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}} + \sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}} - \frac{1}{2}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x,x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)+1/4*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))/c^(1/2)-1/2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82483, size = 1332, normalized size = 12.22

$$\left[\frac{b\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+2\,\sqrt{a}c\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right.}{x^4}\right.}{8\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4
*sqrt(c*x^4 + b*x^2 + a)*c)/c, -1/4*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x
^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*c*log(-
((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(a) + 8*a^2)/x^4) - 2*sqrt(c*x^4 + b*x^2 + a)*c)/c, 1/8*(4*sqrt(-a)*c*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 +
 a^2)) + b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*c)/c, 1/4*(
2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c
*x^4 + a*b*x^2 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c
*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*c
)/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x, x)
```

3.925 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3} \, dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(4*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rubi [A] time = 0.103766, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 732, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(4*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
```

```
|| LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}+\frac{1}{4}b\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)+\frac{1}{2}c\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x\right.\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)+c\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,-\right.\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0472109, size = 112, normalized size = 1.

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(4*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Maple [A] time = 0.162, size = 140, normalized size = 1.3

$$-\frac{1}{2\,ax^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{b}{2\,a}\sqrt{cx^4+bx^2+a}-\frac{b}{4}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{cx^2}{2\,a}\sqrt{cx^4+bx^2+a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(c*x^4+b*x^2+a)^(3/2)+1/2*b/a*(c*x^4+b*x^2+a)^(1/2)-1/4*b/a^(1/2
)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/2*c/a*(c*x^4+b*x^2+
```

```
a)^(1/2)*x^2+1/2*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86422, size = 1408, normalized size = 12.57

$$\left[\frac{2\,a\sqrt{c}x^2\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+\sqrt{a}bx^2\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}}{x^4}\right.}{8\,ax^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x
^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*
x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/
x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2), -1/8*(4*a*sqrt(-c)*x^2*arctan(
1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c
)) - sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x
^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 + b*x^2 + a)*a)/
(a*x^2), 1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*
a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + a*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b
*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 2
*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2), 1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*
x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*a*sq
rt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x
^4 + b*c*x^2 + a*c)) - 2*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^3, x)
```

3.926 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx$

Optimal. Leaf size=88

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

```
[Out] -((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a*x^4) + ((b^2 - 4*a*c)*ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(3/2))
```

Rubi [A] time = 0.0709131, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1114, 720, 724, 206}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a*x^4) + ((b^2 - 4*a*c)*ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(3/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{8ax^4} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16a}\\
&= -\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{8ax^4} + \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8a}\\
&= -\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{8ax^4} + \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0388379, size = 88, normalized size = 1.

$$\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}} - \frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a*x^4) + ((b^2 - 4*a*c)*ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(3/2))
```

Maple [B] time = 0.16, size = 193, normalized size = 2.2

$$-\frac{1}{4\,a x^{4}}\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}+\frac{b}{8\,a^{2} x^{2}}\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}-\frac{b^{2}}{8\,a^{2}}\sqrt{c x^{4}+b x^{2}+a}+\frac{b^{2}}{16}\ln\left(\frac{1}{x^{2}}\left(2\,a+b x^{2}+2\,\sqrt{a}\sqrt{c x^{4}+b x^{2}+a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(c*x^4+b*x^2+a)^(3/2)+1/8*b/a^2/x^2*(c*x^4+b*x^2+a)^(3/2)-1/8*b^
2/a^2*(c*x^4+b*x^2+a)^(1/2)+1/16*b^2/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4
+b*x^2+a)^(1/2))/x^2)-1/8*b/a^2*c*(c*x^4+b*x^2+a)^(1/2)*x^2+1/4*c/a*(c*x^4+
b*x^2+a)^(1/2)-1/4*c/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))
/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67553, size = 502, normalized size = 5.7

$$\left[-\frac{\left(b^{2}-4\,ac\right)\sqrt{a}x^{4}\log\left(-\frac{\left(b^{2}+4\,ac\right)x^{4}+8\,abx^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}\left(bx^{2}+2\,a\right)\sqrt{a}+8\,a^{2}}{x^{4}}\right)+4\,\sqrt{cx^{4}+bx^{2}+a}\left(abx^{2}+2\,a^{2}\right)}{32\,a^{2}x^{4}},-\frac{\left(b^{2}-4\,ac\right)\sqrt{-a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*s
qrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 +
 b*x^2 + a)*(a*b*x^2 + 2*a^2))/(a^2*x^4), -1/16*((b^2 - 4*a*c)*sqrt(-a)*x^4
```

```
*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x
^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2))/(a^2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^5, x)
```

3.927 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx$

Optimal. Leaf size=116

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}+\frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}$$

```
[Out] (b*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a^2*x^4) - (a + b*x^2 + c*x^4
)^(3/2)/(6*a*x^6) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])/(32*a^(5/2))
```

Rubi [A] time = 0.0964375, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 730, 720, 724, 206}

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}+\frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] (b*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a^2*x^4) - (a + b*x^2 + c*x^4
)^(3/2)/(6*a*x^6) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])/(32*a^(5/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
```

```
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{b\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^2\right)}{4a}\\
&= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}+\frac{\left(b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{32a^2}\\
&= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{\left(b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{16a^2}\\
&= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0756663, size = 108, normalized size = 0.93

$$-\frac{\sqrt{a+bx^2+cx^4}\left(8a^2+2ax^2\left(b+4cx^2\right)-3b^2x^4\right)}{48a^2x^6}-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]*(8*a^2 - 3*b^2*x^4 + 2*a*x^2*(b + 4*c*x^2)))/(48*
a^2*x^6) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2
 + c*x^4])])/(32*a^(5/2))
```

Maple [B] time = 0.164, size = 222, normalized size = 1.9

$$-\frac{1}{6\,ax^6}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{b}{8\,a^2x^4}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{b^2}{16\,x^2a^3}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{b^3}{16\,a^3}\sqrt{cx^4+bx^2+a}-\frac{b^3}{32}\ln\left(\frac{1}{x^2}\left(2\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^7,x)
```

```
[Out] -1/6*(c*x^4+b*x^2+a)^(3/2)/a/x^6+1/8*b/a^2/x^4*(c*x^4+b*x^2+a)^(3/2)-1/16*b
^2/a^3/x^2*(c*x^4+b*x^2+a)^(3/2)+1/16*b^3/a^3*(c*x^4+b*x^2+a)^(1/2)-1/32*b^
3/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/16*b^2/a^3*
c*(c*x^4+b*x^2+a)^(1/2)*x^2-1/8*b/a^2*c*(c*x^4+b*x^2+a)^(1/2)+1/8*b/a^(3/2)
*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78987, size = 594, normalized size = 5.12

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{a}x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(2\,a^2bx^2-\left(3\,ab^2-8\,a^2c\right)x^4+8\,a^3\right)\sqrt{cx^4+bx}}{192\,a^3x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
+ 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(2*a^2*
b*x^2 - (3*a*b^2 - 8*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^6)
, 1/96*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(2*a^2*b*x^2 - (3*a*b^
2 - 8*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^7, x)
```

3.928 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4}+\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{7/2}}+\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6}-\ (a$$

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^3*x^4) - (a
 + b*x^2 + c*x^4)^(3/2)/(8*a*x^8) + (5*b*(a + b*x^2 + c*x^4)^(3/2))/(48*a^2
*x^6) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])])/(256*a^(7/2))
```

Rubi [A] time = 0.149705, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 744, 806, 720, 724, 206}

$$-\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4}+\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{7/2}}+\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6}-\ (a$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^3*x^4) - (a
 + b*x^2 + c*x^4)^(3/2)/(8*a*x^8) + (5*b*(a + b*x^2 + c*x^4)^(3/2))/(48*a^2
*x^6) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])])/(256*a^(7/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^2\right)}{8a}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6} + \frac{\left(5b^2-4ac\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^2\right)}{32a^2}$$

$$= -\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6} - \frac{\left(\left(b^2-\right.\right.}{}$$

$$= -\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6} + \frac{\left(\left(b^2-\right.\right.}{}$$

$$= -\frac{\left(5b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8} + \frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{48a^2x^6} + \frac{\left(b^2-\right.}{}$$

Mathematica [A] time = 0.0950067, size = 141, normalized size = 0.88

$$\frac{3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) - \frac{2\sqrt{a}\sqrt{a+bx^2+cx^4}\left(8a^2x^2\left(b+3cx^2\right)+48a^3-2abx^4\left(5b+26cx^2\right)+15b^3x^6\right)}{x^8}}{768a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] ((-2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(48*a^3 + 15*b^3*x^6 + 8*a^2*x^2*(b +
3*c*x^2) - 2*a*b*x^4*(5*b + 26*c*x^2)))/x^8 + 3*(5*b^4 - 24*a*b^2*c + 16*a^
2*c^2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(768*a^(
7/2))
```

Maple [B] time = 0.161, size = 387, normalized size = 2.4

$$-\frac{1}{8\,ax^8}\left(cx^4+bx^2+a\right)^{\frac{3}{2}} + \frac{5\,b}{48\,a^2x^6}\left(cx^4+bx^2+a\right)^{\frac{3}{2}} - \frac{5\,b^2}{64\,a^3x^4}\left(cx^4+bx^2+a\right)^{\frac{3}{2}} + \frac{5\,b^3}{128\,a^4x^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}} - \frac{5\,b^4}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^9,x)
```

```
[Out] -1/8*(c*x^4+b*x^2+a)^(3/2)/a/x^8+5/48*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^6-5/64*
b^2/a^3/x^4*(c*x^4+b*x^2+a)^(3/2)+5/128*b^3/a^4/x^2*(c*x^4+b*x^2+a)^(3/2)-5
/128*b^4/a^4*(c*x^4+b*x^2+a)^(1/2)+5/256*b^4/a^(7/2)*ln((2*a+b*x^2+2*a^(1/2
)*(c*x^4+b*x^2+a)^(1/2))/x^2)-5/128*b^3/a^4*c*(c*x^4+b*x^2+a)^(1/2)*x^2+7/6
4*b^2/a^3*c*(c*x^4+b*x^2+a)^(1/2)-3/32*b^2/a^(5/2)*c*ln((2*a+b*x^2+2*a^(1/2
)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/16*c/a^2/x^4*(c*x^4+b*x^2+a)^(3/2)-1/32*c/a
^3*b/x^2*(c*x^4+b*x^2+a)^(3/2)+1/32*c^2/a^3*b*(c*x^4+b*x^2+a)^(1/2)*x^2-1/1
6*c^2/a^2*(c*x^4+b*x^2+a)^(1/2)+1/16*c^2/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c
*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03444, size = 743, normalized size = 4.61

$$\left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{a}x^8\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\left(\left(15\,ab^3-52\,a^2bc\right)x^6+8\,a^3bx^2-\right.}{1536\,a^4x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^8*log(-((b^2 + 4*a*c
)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2
)/x^4) - 4*((15*a*b^3 - 52*a^2*b*c)*x^6 + 8*a^3*b*x^2 - 2*(5*a^2*b^2 - 12*a
^3*c)*x^4 + 48*a^4)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^8), -1/768*(3*(5*b^4 -
24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^3 - 52*a^2*b*c
```

```
)*x^6 + 8*a^3*b*x^2 - 2*(5*a^2*b^2 - 12*a^3*c)*x^4 + 48*a^4)*sqrt(c*x^4 + b
*x^2 + a))/(a^4*x^8)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**9, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^9, x)
```

3.929 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=199

$$-\frac{(35b^2-32ac)(a+bx^2+cx^4)^{3/2}}{480a^3x^6}+\frac{b(7b^2-12ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{b(7b^2-12ac)(b^2-4ac)\tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a}}\right.}{512a^{9/2}}$$

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a^4*x^4) -
(a + b*x^2 + c*x^4)^(3/2)/(10*a*x^10) + (7*b*(a + b*x^2 + c*x^4)^(3/2))/(80
*a^2*x^8) - ((35*b^2 - 32*a*c)*(a + b*x^2 + c*x^4)^(3/2))/(480*a^3*x^6) - (
b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
b*x^2 + c*x^4])])/(512*a^(9/2))
```

Rubi [A] time = 0.229035, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1114, 744, 834, 806, 720, 724, 206}

$$-\frac{(35b^2-32ac)(a+bx^2+cx^4)^{3/2}}{480a^3x^6}+\frac{b(7b^2-12ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{b(7b^2-12ac)(b^2-4ac)\tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a}}\right.}{512a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```


```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a^4*x^4) -
(a + b*x^2 + c*x^4)^(3/2)/(10*a*x^10) + (7*b*(a + b*x^2 + c*x^4)^(3/2))/(80
*a^2*x^8) - ((35*b^2 - 32*a*c)*(a + b*x^2 + c*x^4)^(3/2))/(480*a^3*x^6) - (
b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
b*x^2 + c*x^4])])/(512*a^(9/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+2cx\right)\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^2\right)}{10a}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{4}\left(35b^2-32ac\right)+\frac{7bcx}{2}\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^2\right)}{40a^2}$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8} - \frac{\left(35b^2-32ac\right)\left(a+bx^2+cx^4\right)^{3/2}}{480a^3x^6} - \frac{\left(b\left(7b^2-12ac\right.\right.}{}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8} - \frac{\left(35b^2\right.}{}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8} - \frac{\left(35b^2\right.}{}$$

$$= \frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8} - \frac{\left(35b^2\right.}{}$$

Mathematica [A] time = 0.121544, size = 173, normalized size = 0.87

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-8a^2\left(7b^2x^4+29bcx^6+32c^2x^8\right)+16a^3\left(3bx^2+8cx^4\right)+384a^4+10ab^2x^6\left(7b+46cx^2\right)-105b^4x^8\right)}{3840a^4x^{10}} - \frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]*(384*a^4 - 105*b^4*x^8 + 10*a*b^2*x^6*(7*b + 46*c
*x^2) + 16*a^3*(3*b*x^2 + 8*c*x^4) - 8*a^2*(7*b^2*x^4 + 29*b*c*x^6 + 32*c^2
```

```
*x^8)))/(3840*a^4*x^10) - (b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a
 + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]])])/(512*a^(9/2))
```

Maple [B] time = 0.165, size = 442, normalized size = 2.2

$$-\frac{1}{10\,a x^{10}}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{7\,b}{80\,a^2x^8}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{7\,b^2}{96\,a^3x^6}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{7\,b^3}{128\,a^4x^4}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{7\,b}{256\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^11,x)
```

```
[Out] -1/10*(c*x^4+b*x^2+a)^(3/2)/a/x^10+7/80*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^8-7/9
6*b^2/a^3/x^6*(c*x^4+b*x^2+a)^(3/2)+7/128*b^3/a^4/x^4*(c*x^4+b*x^2+a)^(3/2)
-7/256*b^4/a^5/x^2*(c*x^4+b*x^2+a)^(3/2)+7/256*b^5/a^5*(c*x^4+b*x^2+a)^(1/2
)-7/512*b^5/a^(9/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+7/2
56*b^4/a^5*c*(c*x^4+b*x^2+a)^(1/2)*x^2-13/128*b^3/a^4*c*(c*x^4+b*x^2+a)^(1/
2)+5/64*b^3/a^(7/2)*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-3
/32*b/a^3*c/x^4*(c*x^4+b*x^2+a)^(3/2)+3/64*b^2/a^4*c/x^2*(c*x^4+b*x^2+a)^(3
/2)-3/64*b^2/a^4*c^2*(c*x^4+b*x^2+a)^(1/2)*x^2+3/32*b/a^3*c^2*(c*x^4+b*x^2+
a)^(1/2)-3/32*b/a^(5/2)*c^2*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/
x^2)+1/15*c/a^2/x^6*(c*x^4+b*x^2+a)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.66864, size = 906, normalized size = 4.55

$$\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{a}x^{10}\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(\left(105\,ab^4-460\,a^2b^2c+256\,a\right.\right.}{15360\,a^5x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^10*log(-((b^2 +
4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) +
8*a^2)/x^4) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^8 - 48*a^4*b*x
^2 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^6 - 384*a^5 + 8*(7*a^3*b^2 - 16*a^4*c)*
x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^10), 1/7680*(15*(7*b^5 - 40*a*b^3*c +
48*a^2*b*c^2)*sqrt(-a)*x^10*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*c + 256
*a^3*c^2)*x^8 - 48*a^4*b*x^2 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^6 - 384*a^5 +
 8*(7*a^3*b^2 - 16*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^10)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**11, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^11, x)
```

3.930 $\int x^4\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=395

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}\left(2b^2-5ac\right)-29abc+8b^3\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{210c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{2x\left(2b^2-\right.}{}$$

```
[Out] (-2*(2*b^2 - 5*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^2) + (b*(8*b^2 - 29*a
*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (x^3
*(b + 5*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) - (a^(1/4)*b*(8*b^2 - 29*a*c
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
105*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^3 - 29*a*b*c + 2*Sqrt
[a]*Sqrt[c]*(2*b^2 - 5*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(210*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.251072, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1116, 1279, 1197, 1103, 1195}

$$-\frac{2x\left(2b^2-5ac\right)\sqrt{a+bx^2+cx^4}}{105c^2}+\frac{bx\left(8b^2-29ac\right)\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}\left(2b^2-5ac\right)-29abc+8b^3\right)\left(\sqrt{a}+\sqrt{c}\right.}{210c^{11/4}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (-2*(2*b^2 - 5*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^2) + (b*(8*b^2 - 29*a
*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (x^3
*(b + 5*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) - (a^(1/4)*b*(8*b^2 - 29*a*c
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
105*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^3 - 29*a*b*c + 2*Sqrt
[a]*Sqrt[c]*(2*b^2 - 5*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(210*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1116

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*(2*b*p + c*(m + 4*p - 1)*x^
2))/(c*(m + 4*p + 1)*(m + 4*p - 1)), x] - Dist[(2*p*d^2)/(c*(m + 4*p + 1)*(
m + 4*p - 1)), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx = \frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c} - \frac{\int \frac{x^2\left(3ab+2\left(2b^2-5ac\right)x^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx}{35c}$$

$$= -\frac{2\left(2b^2-5ac\right)x\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c} + \frac{\int \frac{2a\left(2b^2-5ac\right)+b\left(8b^2-29ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,d}{105c^2}$$

$$= -\frac{2\left(2b^2-5ac\right)x\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c} - \frac{\left(\sqrt{a}b\left(8b^2-29ac\right)\right)\int \frac{1-}{\sqrt{a+}}}{105c^{5/2}}$$

$$= -\frac{2\left(2b^2-5ac\right)x\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{b\left(8b^2-29ac\right)x\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2}}{35c}$$

Mathematica [C] time = 1.55288, size = 538, normalized size = 1.36

$$\frac{-i\left(-20a^2c^2+8b^3\sqrt{b^2-4ac}+37ab^2c-29abc\sqrt{b^2-4ac}-8b^4\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(10*a^2*c - 4*b^3*x^2 - b^2*c*x^4 +
18*b*c^2*x^6 + 15*c^3*x^8 + a*(-4*b^2 + 13*b*c*x^2 + 25*c^2*x^4)) + I*b*(8*
b^2 - 29*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^4 + 37
*a*b^2*c - 20*a^2*c^2 + 8*b^3*Sqrt[b^2 - 4*a*c] - 29*a*b*c*Sqrt[b^2 - 4*a*c
])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*
b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Arc
Sinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c])])/(420*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b
*x^2 + c*x^4])
```

Maple [A] time = 0.273, size = 476, normalized size = 1.2

$$\frac{x^5}{7}\sqrt{cx^4+bx^2+a}+\frac{bx^3}{35\,c}\sqrt{cx^4+bx^2+a}+\frac{x}{3\,c}\left(\frac{2\,a}{7}-\frac{4\,b^2}{35\,c}\right)\sqrt{cx^4+bx^2+a}-\frac{a\sqrt{2}}{12\,c}\left(\frac{2\,a}{7}-\frac{4\,b^2}{35\,c}\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+\right.}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/7*x^5*(c*x^4+b*x^2+a)^(1/2)+1/35*b/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(2/7*a
-4/35*b^2/c)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(2/7*a-4/35*b^2/c)/c*a*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))-1/2*(-3/35*b/c*a-2/3*(2/7*a-4/35*b^2/c)/c*b)*a*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*
(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b
^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4+bx^2+a}x^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

3.931 $\int x^2\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=342

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] (-2*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(3/2)*(Sqrt[a] + Sqrt[c]
*x^2)) + (x*(b + 3*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) + (2*a^(1/4)*(b^2
 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4])/(15*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*S
qrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(30*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.141442, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1116, 1197, 1103, 1195}

$$-\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{a+bx^2+cx^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-2*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(3/2)*(Sqrt[a] + Sqrt[c]
*x^2)) + (x*(b + 3*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) + (2*a^(1/4)*(b^2
 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4])/(15*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*S
qrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(30*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1116

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*(2*b*p + c*(m + 4*p - 1)*x^
2))/(c*(m + 4*p + 1)*(m + 4*p - 1)), x] - Dist[(2*p*d^2)/(c*(m + 4*p + 1)*(
m + 4*p - 1)), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx = \frac{x\left(b+3cx^2\right)\sqrt{a+bx^2+cx^4}}{15c} - \frac{\int \frac{ab+2\left(b^2-3ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15c}$$

$$= \frac{x\left(b+3cx^2\right)\sqrt{a+bx^2+cx^4}}{15c} + \frac{\left(2\sqrt{a}\left(b^2-3ac\right)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^{3/2}} - \frac{\left(\sqrt{a}\left(\sqrt{a}b+\frac{2\left(b^2-3ac\right)}{\sqrt{c}}\right)\right)\int \cdots}{15c \cdots}$$

$$= -\frac{2\left(b^2-3ac\right)x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{x\left(b+3cx^2\right)\sqrt{a+bx^2+cx^4}}{15c} + \frac{2\sqrt[4]{a}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\cdots}{15c\cdots}$$

Mathematica [C] time = 1.24161, size = 479, normalized size = 1.4

$$\frac{i\left(b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}+4abc-b^3\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a + b*x^2 + c*x^4) -
I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*
a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*
c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
])/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.213, size = 417, normalized size = 1.2

$$\frac{x^3}{5}\sqrt{cx^4+bx^2+a}+\frac{bx}{15\,c}\sqrt{cx^4+bx^2+a}-\frac{ab\sqrt{2}}{60\,c}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\text{EllipticF}\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*x^3*(c*x^4+b*x^2+a)^(1/2)+1/15*b/c*x*(c*x^4+b*x^2+a)^(1/2)-1/60*b/c*a*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Elli
pticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2))-1/2*(2/5*a-2/15*b^2/c)*a*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

3.932 $\int \sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=309

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E}{3c^{3/4}\sqrt{a+b}}$$

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/3 + (b*x*Sqrt[a + b*x^2 + c*x^4])/(3*Sqrt[c]*(S
qrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*b*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(
1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.100735, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1091, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\right.}{3c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/3 + (b*x*Sqrt[a + b*x^2 + c*x^4])/(3*Sqrt[c]*(S
qrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*b*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(
1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
```

```
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{a+bx^2+cx^4}\,dx = \frac{1}{3}x\sqrt{a+bx^2+cx^4}+\frac{1}{3}\int \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{1}{3}x\sqrt{a+bx^2+cx^4}+\frac{1}{3}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx-\frac{\left(\sqrt{a}b\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{c}}$$

$$= \frac{1}{3}x\sqrt{a+bx^2+cx^4}+\frac{bx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{3c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.828546, size = 445, normalized size = 1.44

$$\frac{-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)+4cx}{12c\sqrt{\frac{c}{\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*(-b + Sqrt
[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ell
ipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
- 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSi
nh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])])/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2
+ c*x^4])
```

Maple [A] time = 0.206, size = 379, normalized size = 1.2

$$\frac{x}{3}\sqrt{cx^4+bx^2+a}+\frac{a\sqrt{2}}{6}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,a\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/6*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/6*b*a*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*
c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(
```

```
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

3.933 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^2} dx$

Optimal. Leaf size=303

$$\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}-\frac{\sqrt{a+bx^2+c}}{x}$$

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/x) + (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[
a] + Sqrt[c]*x^2) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[
a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.0843806, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1117, 1197, 1103, 1195}

$$\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}-\frac{\sqrt{a+bx^2+cx^4}}{x}+\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/x) + (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[
a] + Sqrt[c]*x^2) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[
a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
```

```
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{x} + \int \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{x} + \left(b+2\sqrt{a}\sqrt{c}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \left(2\sqrt{a}\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{x} + \frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2} - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.816292, size = 435, normalized size = 1.44

$$\frac{-i\sqrt{2}x\sqrt{b^2-4ac}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)-2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(a+\right.}{2x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^2,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*(-b + Sqrt[b^2
- 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellipt
icE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*x*Sqrt[(b - S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.217, size = 381, normalized size = 1.3

$$-\frac{1}{x}\sqrt{cx^4+bx^2+a}+\frac{b\sqrt{2}}{4}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/x+1/4*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-c*a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2
*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

3.934 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^4} dx$

Optimal. Leaf size=341

$$\frac{\sqrt[4]{c}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E}{3a^{3/4}\sqrt{a+b}}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(3*x^3) - (b*Sqrt[a + b*x^2 + c*x^4])/(3*a*x) + (b
*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a*(Sqrt[a] + Sqrt[c]*x^2)) - (b*c^(1
/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
/(3*a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + ((b + 2*Sqrt[a]*Sqrt[c])*c^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(
3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.155129, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1117, 1281, 1197, 1103, 1195}

$$\frac{\sqrt[4]{c}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\right.}{3a^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^4,x]
```


```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(3*x^3) - (b*Sqrt[a + b*x^2 + c*x^4])/(3*a*x) + (b
*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a*(Sqrt[a] + Sqrt[c]*x^2)) - (b*c^(1
/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
/(3*a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + ((b + 2*Sqrt[a]*Sqrt[c])*c^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(
3/4)*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{3x^3}+\frac{1}{3}\int \frac{b+2cx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3x^3}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax}-\frac{\int \frac{-2ac-bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3x^3}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax}-\frac{\left(b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{a}}+\frac{1}{3}\left(\left(\frac{b}{\sqrt{a}}+2\sqrt{c}\right)\sqrt{c}\right)\int \frac{}{\sqrt{a+}}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3x^3}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax}+\frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}}{3a^{3/4}\sqrt{a+}}$$

Mathematica [C] time = 0.895, size = 459, normalized size = 1.35

$$\frac{-ix^3\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)-4}{12ax^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^4,x]
```

```
[Out] (-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2)*(a + b*x^2 + c*x^4) + I*b*(
-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.216, size = 404, normalized size = 1.2

$$-\frac{1}{3\,x^3}\sqrt{cx^4+bx^2+a}-\frac{b}{3\,ax}\sqrt{cx^4+bx^2+a}+\frac{c\sqrt{2}}{6}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\mathrm{EllipticI}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/x^3-1/3*b*(c*x^4+b*x^2+a)^(1/2)/a/x+1/6*c*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF
(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))-1/6*b*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2
*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

3.935 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=397

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+}}{15a^2x}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(5*x^5) - (b*Sqrt[a + b*x^2 + c*x^4])/(15*a*x^3) +
 (2*(b^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*x) - (2*Sqrt[c]*(b^2 - 3
*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*(Sqrt[a] + Sqrt[c]*x^2)) + (2*c^(1
/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(15*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(2*b^2 + Sqr
t[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(30*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.2611, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1117, 1281, 1197, 1103, 1195}

$$\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2x}-\frac{2\sqrt{c}x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\right.}{30a^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^6,x]
```


```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(5*x^5) - (b*Sqrt[a + b*x^2 + c*x^4])/(15*a*x^3) +
 (2*(b^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*x) - (2*Sqrt[c]*(b^2 - 3
*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*(Sqrt[a] + Sqrt[c]*x^2)) + (2*c^(1
/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(15*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(2*b^2 + Sqr
t[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(30*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} + \frac{1}{5}\int \frac{b+2cx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} - \frac{\int \frac{2(b^2-3ac)+bcx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{15a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} + \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2x} + \frac{\int \frac{-abc-2c(b^2-3ac)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15a^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} + \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2x} + \frac{\left(2\sqrt{c}\left(b^2-3ac\right)\right)\int \frac{1-\sqrt{}}{\sqrt{a+bx^2}}}{15a^{3/2}}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} + \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2x} - \frac{2\sqrt{c}\left(b^2-3ac\right)x\sqrt{a+bx^2}}{15a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Mathematica [C] time = 1.34032, size = 530, normalized size = 1.34

$$\frac{ix^5\left(b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}+4abc-b^3\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^6,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(3*a^3 - 2*b^2*x^6*(b + c*x^2) + a^2*(4
*b*x^2 + 9*c*x^4) + a*(-(b^2*x^4) + 7*b*c*x^6 + 6*c^2*x^8)) - I*(b^2 - 3*a*
c)*(-b + Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b
^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] +
4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(3
0*a^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.226, size = 452, normalized size = 1.1

$$-\frac{1}{5x^5}\sqrt{cx^4+bx^2+a}-\frac{b}{15\,ax^3}\sqrt{cx^4+bx^2+a}-\frac{6\,ac-2\,b^2}{15\,a^2x}\sqrt{cx^4+bx^2+a}-\frac{bc\sqrt{2}}{60\,a}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

```
[Out] -1/5*(c*x^4+b*x^2+a)^(1/2)/x^5-1/15*b*(c*x^4+b*x^2+a)^(1/2)/a/x^3-2/15*(3*a
*c-b^2)/a^2*(c*x^4+b*x^2+a)^(1/2)/x-1/60*b*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1
/15*c*(3*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

3.936 $\int x^7 \left(a + bx^2 + cx^4\right)^{3/2} dx$

Optimal. Leaf size=223

$$\frac{\left(-16ac + 21b^2 - 30bcx^2\right)\left(a + bx^2 + cx^4\right)^{5/2}}{560c^3} - \frac{b\left(3b^2 - 4ac\right)\left(b + 2cx^2\right)\left(a + bx^2 + cx^4\right)^{3/2}}{256c^4} + \frac{3b\left(b^2 - 4ac\right)\left(3b^2 - 4ac\right)(b}{2048}$$

```
[Out] (3*b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(
2048*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(25
6*c^4) + (x^4*(a + b*x^2 + c*x^4)^(5/2))/(14*c) + ((21*b^2 - 16*a*c - 30*b*
c*x^2)*(a + b*x^2 + c*x^4)^(5/2))/(560*c^3) - (3*b*(b^2 - 4*a*c)^2*(3*b^2 -
 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4096*c
^(11/2))
```

Rubi [A] time = 0.211753, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 742, 779, 612, 621, 206}

$$\frac{\left(-16ac + 21b^2 - 30bcx^2\right)\left(a + bx^2 + cx^4\right)^{5/2}}{560c^3} - \frac{b\left(3b^2 - 4ac\right)\left(b + 2cx^2\right)\left(a + bx^2 + cx^4\right)^{3/2}}{256c^4} + \frac{3b\left(b^2 - 4ac\right)\left(3b^2 - 4ac\right)(b}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```



```
[Out] (3*b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(
2048*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(25
6*c^4) + (x^4*(a + b*x^2 + c*x^4)^(5/2))/(14*c) + ((21*b^2 - 16*a*c - 30*b*
c*x^2)*(a + b*x^2 + c*x^4)^(5/2))/(560*c^3) - (3*b*(b^2 - 4*a*c)^2*(3*b^2 -
 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4096*c
^(11/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^7\left(a+bx^2+cx^4\right)^{3/2}dx=\frac{1}{2}\text{Subst}\left(\int x^3\left(a+bx+cx^2\right)^{3/2}dx,x,x^2\right)$$

$$=\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c}+\frac{\text{Subst}\left(\int x\left(-2a-\frac{9bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}dx,x,x^2\right)}{14c}$$

$$=\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c}+\frac{\left(21b^2-16ac-30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{560c^3}-\frac{\left(b\left(3b^2-4ac\right)\right)\text{Subs}}{}$$

$$=-\frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{256c^4}+\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c}+\frac{\left(21b^2-16ac-30b\right.}{5}$$

$$=\frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+\right.}{256c^4}$$

$$=\frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+\right.}{256c^4}$$

$$=\frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+\right.}{256c^4}$$

Mathematica [A] time = 0.251913, size = 192, normalized size = 0.86

$$\frac{-\frac{\left(16ac-21b^2+30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{40c^2}+\frac{7\left(4abc-3b^3\right)\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(4c\left(5a+2cx^4\right)-3b^2+8bcx^2\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)}{2048c^{9/2}}+x^4\left(\right.}{14c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(5/2) - ((-21*b^2 + 16*a*c + 30*b*c*x^2)*(a + b*x^
2 + c*x^4)^(5/2))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^2)
*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^
2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(
2048*c^(9/2)))/(14*c)
```

Maple [B] time = 0.174, size = 534, normalized size = 2.4

$$\frac{11\,abx^6}{1120\,c}\sqrt{cx^4+bx^2+a}-\frac{31\,ax^4b^2}{2240\,c^2}\sqrt{cx^4+bx^2+a}+\frac{13\,ab^3x^2}{640\,c^3}\sqrt{cx^4+bx^2+a}-\frac{73\,a^2bx^2}{2240\,c^2}\sqrt{cx^4+bx^2+a}-\frac{9\,ab^4}{256\,c^4}\sqrt{cx^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 11/1120*a*b*x^6/c*(c*x^4+b*x^2+a)^(1/2)-31/2240*a*b^2/c^2*x^4*(c*x^4+b*x^2+
a)^(1/2)+13/640*a*b^3/c^3*x^2*(c*x^4+b*x^2+a)^(1/2)-73/2240*a^2*b/c^2*x^2*(
c*x^4+b*x^2+a)^(1/2)-9/256*a*b^4/c^4*(c*x^4+b*x^2+a)^(1/2)-1/35*a^3/c^2*(c*
x^4+b*x^2+a)^(1/2)+21/1024*a*b^5/c^(9/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*
x^2+a)^(1/2))-15/256*a^2*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))+49/640*a^2*b^2/c^3*(c*x^4+b*x^2+a)^(1/2)+3/64*a^3*b/c^(5/2)*ln((1
/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+4/35*a*x^8*(c*x^4+b*x^2+a)^(1/2)
+9/2048*b^6/c^5*(c*x^4+b*x^2+a)^(1/2)-9/4096*b^7/c^(11/2)*ln((1/2*b+c*x^2)/
c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/70*a^2*x^4/c*(c*x^4+b*x^2+a)^(1/2)+1/560*b
^2*x^8/c*(c*x^4+b*x^2+a)^(1/2)-9/4480*b^3/c^2*x^6*(c*x^4+b*x^2+a)^(1/2)+3/1
280*b^4/c^3*x^4*(c*x^4+b*x^2+a)^(1/2)-3/1024*b^5/c^4*x^2*(c*x^4+b*x^2+a)^(1
/2)+1/14*c*x^12*(c*x^4+b*x^2+a)^(1/2)+5/56*b*x^10*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75869, size = 1265, normalized size = 5.67

$$\left[-\frac{105\left(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)-4\,(\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/286720*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
```

```
)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^12 + 6400*b*c^6*x^10 + 128*(b^2*c^5 + 64
*a*c^6)*x^8 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4
- 16*(9*b^3*c^4 - 44*a*b*c^5)*x^6 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2
*c^5)*x^4 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^2)*sqrt(c*x^
4 + b*x^2 + a))/c^6, 1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 6
4*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt
(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(5120*c^7*x^12 + 6400*b*c^6*x^10 + 128*
(b^2*c^5 + 64*a*c^6)*x^8 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 -
2048*a^3*c^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^6 + 8*(21*b^4*c^3 - 124*a*b^2*
c^4 + 128*a^2*c^5)*x^4 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x
^2)*sqrt(c*x^4 + b*x^2 + a))/c^6]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A] time = 1.36957, size = 374, normalized size = 1.68

$$\frac{1}{71680}\sqrt{cx^4 + bx^2 + a}\left(2\left(4\left(2\left(8\left(10\left(4\,cx^2 + 5\,b\right)x^2 + \frac{b^2c^{10} + 64\,ac^{11}}{c^{11}}\right)x^2 - \frac{9\,b^3c^9 - 44\,abc^{10}}{c^{11}}\right)x^2 + \frac{21\,b^4c^8 - 124\,ab^2c^9}{c^{11}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/71680*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*(4*c*x^2 + 5*b)*x^2 + (b^2*
c^10 + 64*a*c^11)/c^11)*x^2 - (9*b^3*c^9 - 44*a*b*c^10)/c^11)*x^2 + (21*b^4
*c^8 - 124*a*b^2*c^9 + 128*a^2*c^10)/c^11)*x^2 - (105*b^5*c^7 - 728*a*b^3*c
^8 + 1168*a^2*b*c^9)/c^11)*x^2 + (315*b^6*c^6 - 2520*a*b^4*c^7 + 5488*a^2*b
^2*c^8 - 2048*a^3*c^9)/c^11) + 3/4096*(3*b^7*c^6 - 28*a*b^5*c^7 + 80*a^2*b^
3*c^8 - 64*a^3*b*c^9)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sq
rt(c) - b))/c^(23/2)
```

3.937 $\int x^5 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=204

$$\frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384c^3}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4}+\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\text{t}}{2048c}$$

```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(102
4*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(384*c^3
) - (7*b*(a + b*x^2 + c*x^4)^(5/2))/(120*c^2) + (x^2*(a + b*x^2 + c*x^4)^(5
/2))/(12*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(2048*c^(9/2))
```

Rubi [A] time = 0.182342, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 742, 640, 612, 621, 206}

$$\frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384c^3}-\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4}+\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\text{t}}{2048c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(102
4*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(384*c^3
) - (7*b*(a + b*x^2 + c*x^4)^(5/2))/(120*c^2) + (x^2*(a + b*x^2 + c*x^4)^(5
/2))/(12*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(2048*c^(9/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5 \left(a + bx^2 + cx^4\right)^{3/2} dx = \frac{1}{2}\,\text{Subst}\left(\int x^2 \left(a + bx + cx^2\right)^{3/2} dx, x, x^2\right)$$
$$= \frac{x^2 \left(a + bx^2 + cx^4\right)^{5/2}}{12c} + \frac{\text{Subst}\left(\int \left(-a - \frac{7bx}{2}\right)\left(a + bx + cx^2\right)^{3/2} dx, x, x^2\right)}{12c}$$
$$= -\frac{7b \left(a + bx^2 + cx^4\right)^{5/2}}{120c^2} + \frac{x^2 \left(a + bx^2 + cx^4\right)^{5/2}}{12c} + \frac{\left(7b^2 - 4ac\right)\text{Subst}\left(\int \left(a + bx + cx^2\right)^{3/2} dx\right.}{48c^2}$$
$$= \frac{\left(7b^2 - 4ac\right)\left(b + 2cx^2\right)\left(a + bx^2 + cx^4\right)^{3/2}}{384c^3} - \frac{7b \left(a + bx^2 + cx^4\right)^{5/2}}{120c^2} + \frac{x^2 \left(a + bx^2 + cx^4\right)^{5/2}}{12c} -$$
$$= -\frac{\left(b^2 - 4ac\right)\left(7b^2 - 4ac\right)\left(b + 2cx^2\right)\sqrt{a + bx^2 + cx^4}}{1024c^4} + \frac{\left(7b^2 - 4ac\right)\left(b + 2cx^2\right)\left(a + bx^2 + cx^4\right.}{384c^3}$$
$$= -\frac{\left(b^2 - 4ac\right)\left(7b^2 - 4ac\right)\left(b + 2cx^2\right)\sqrt{a + bx^2 + cx^4}}{1024c^4} + \frac{\left(7b^2 - 4ac\right)\left(b + 2cx^2\right)\left(a + bx^2 + cx^4\right.}{384c^3}$$
$$= -\frac{\left(b^2 - 4ac\right)\left(7b^2 - 4ac\right)\left(b + 2cx^2\right)\sqrt{a + bx^2 + cx^4}}{1024c^4} + \frac{\left(7b^2 - 4ac\right)\left(b + 2cx^2\right)\left(a + bx^2 + cx^4\right.}{384c^3}$$

Mathematica [A] time = 0.159113, size = 175, normalized size = 0.86

$$\frac{\frac{\left(7b^2-4ac\right)\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(4c\left(5a+2cx^4\right)-3b^2+8bcx^2\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)}{512c^{7/2}} + x^2\left(a + bx^2 + cx^4\right)^{5/2} - \frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{10c}}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((-7*b*(a + b*x^2 + c*x^4)^(5/2))/(10*c) + x^2*(a + b*x^2 + c*x^4)^(5/2) +
((7*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 +
 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(512*c^(7/2)))/(12*c)
```

Maple [B] time = 0.174, size = 432, normalized size = 2.1

$$\frac{7\,ax^6}{48}\sqrt{cx^4 + bx^2 + a} - \frac{7\,b^5}{1024\,c^4}\sqrt{cx^4 + bx^2 + a} + \frac{7\,b^6}{2048}\ln\left(\left(\frac{b}{2} + cx^2\right)\frac{1}{\sqrt{c}} + \sqrt{cx^4 + bx^2 + a}\right)c^{-\frac{9}{2}} + \frac{13\,bx^8}{120}\sqrt{cx^4 + bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 7/48*a*x^6*(c*x^4+b*x^2+a)^(1/2)-7/1024*b^5/c^4*(c*x^4+b*x^2+a)^(1/2)+7/204
8*b^6/c^(9/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+13/120*b*x^8*
(c*x^4+b*x^2+a)^(1/2)+9/128*a^2*b^2/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4
+b*x^2+a)^(1/2))-27/320*a^2*b/c^2*(c*x^4+b*x^2+a)^(1/2)-1/32*a^3/c^(3/2)*ln
((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/12*c*x^10*(c*x^4+b*x^2+a)^(
1/2)+3/160*a*b*x^4/c*(c*x^4+b*x^2+a)^(1/2)-9/320*a*b^2/c^2*x^2*(c*x^4+b*x^2
+a)^(1/2)+19/384*a*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)-15/512*a*b^4/c^(7/2)*ln((1
/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/32*a^2*x^2/c*(c*x^4+b*x^2+a)^(
1/2)+7/1536*b^4/c^3*x^2*(c*x^4+b*x^2+a)^(1/2)+1/320*b^2*x^6/c*(c*x^4+b*x^2+
a)^(1/2)-7/1920*b^3/c^2*x^4*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73758, size = 1061, normalized size = 5.2

$$\left[-\frac{15\left(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)-4\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*l
og(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^10 + 1664*b*c^5*x^8 + 16*(3*b^2*c^4 + 140*a
```

```
*c^5)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 - 8*(7*b^3*c^3 - 36*
a*b*c^4)*x^4 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^2)*sqrt(c*x^4
 + b*x^2 + a))/c^5, -1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64
*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c
)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(1280*c^6*x^10 + 1664*b*c^5*x^8 + 16*(3*b^
2*c^4 + 140*a*c^5)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 - 8*(7*
b^3*c^3 - 36*a*b*c^4)*x^4 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^
2)*sqrt(c*x^4 + b*x^2 + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A] time = 1.27394, size = 311, normalized size = 1.52

$$\frac{1}{15360} \sqrt{cx^4 + bx^2 + a} \left(2 \left(4 \left(2 \left(8 \left(10 \, cx^2 + 13 \, b\right) x^2 + \frac{3 \, b^2 c^8 + 140 \, ac^9}{c^9}\right) x^2 - \frac{7 \, b^3 c^7 - 36 \, abc^8}{c^9}\right) x^2 + \frac{35 \, b^4 c^6 - 216 \, ab^2 c^7 + 2}{c^9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*c*x^2 + 13*b)*x^2 + (3*b^2*
c^8 + 140*a*c^9)/c^9)*x^2 - (7*b^3*c^7 - 36*a*b*c^8)/c^9)*x^2 + (35*b^4*c^6
 - 216*a*b^2*c^7 + 240*a^2*c^8)/c^9)*x^2 - (105*b^5*c^5 - 760*a*b^3*c^6 + 1
296*a^2*b*c^7)/c^9) - 1/2048*(7*b^6*c^5 - 60*a*b^4*c^6 + 144*a^2*b^2*c^7 -
64*a^3*c^8)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b)
)/c^(19/2)
```

3.938 $\int x^3 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=150

$$\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\ (a$$

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3) - (b*(b
 + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (a + b*x^2 + c*x^4)^(5/2)
/(10*c) - (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*
x^2 + c*x^4])])/(512*c^(7/2))
```

Rubi [A] time = 0.116978, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 640, 612, 621, 206}

$$\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\ (a$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3) - (b*(b
 + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (a + b*x^2 + c*x^4)^(5/2)
/(10*c) - (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*
x^2 + c*x^4])])/(512*c^(7/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+bx^2+cx^4\right)^{3/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}-\frac{b\operatorname{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}+\frac{\left(3b\left(b^2-4ac\right)\right)\operatorname{Subst}\left(\int\sqrt{a+bx}\right.}{64c^2}\\
&= \frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right.}{10c}\\
&= \frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right.}{10c}\\
&= \frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right.}{10c}
\end{aligned}$$

Mathematica [A] time = 0.142122, size = 149, normalized size = 0.99

$$-\frac{3b\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)}{512c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(b*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (a + b*x^2 + c*x^4)
^(5/2)/(10*c) - (3*b*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2
 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4])]))/(512*c^(7/2))
```

Maple [B] time = 0.167, size = 316, normalized size = 2.1

$$-\frac{5\,b^2a}{64\,c^2}\sqrt{cx^4+bx^2+a}+\frac{3\,ab^3}{64}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}-\frac{3\,ba^2}{32}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -5/64*a*b^2/c^2*(c*x^4+b*x^2+a)^(1/2)+3/64*a*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c
^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/32*a^2*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(
c*x^4+b*x^2+a)^(1/2))+1/10*c*x^8*(c*x^4+b*x^2+a)^(1/2)+1/5*a*x^4*(c*x^4+b*x
^2+a)^(1/2)+3/256*b^4/c^3*(c*x^4+b*x^2+a)^(1/2)-3/512*b^5/c^(7/2)*ln((1/2*b
+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/10*a^2/c*(c*x^4+b*x^2+a)^(1/2)+11/
80*b*x^6*(c*x^4+b*x^2+a)^(1/2)+1/160*b^2*x^4/c*(c*x^4+b*x^2+a)^(1/2)-1/128*
b^3/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+7/160*a*b*x^2/c*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.68222, size = 837, normalized size = 5.58

$$\left[\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)+4\left(128c^5x^8+176bc^4\right.}{5120c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c
*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(
128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b
^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2
 + a))/c^4, 1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/2
*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c))
+ 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 +
 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^4 +
b*x^2 + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A] time = 1.22629, size = 232, normalized size = 1.55

$$\frac{1}{1280}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(2\left(8cx^2+11b\right)x^2+\frac{b^2c^3+32ac^4}{c^4}\right)x^2-\frac{5b^3c^2-28abc^3}{c^4}\right)x^2+\frac{15b^4c-100ab^2c^2+128a^2c^3}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*c*x^2 + 11*b)*x^2 + (b^2*c^3 + 3
2*a*c^4)/c^4)*x^2 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^2 + (15*b^4*c - 100*a*b
^2*c^2 + 128*a^2*c^3)/c^4) + 3/512*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*log(abs
(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(7/2)
```

3.939 $\int x\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}$$

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^2) + ((b +
2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*c) + (3*(b^2 - 4*a*c)^2*ArcTanh[(b
+ 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2))
```

Rubi [A] time = 0.0855727, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1107, 612, 621, 206}

$$-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^2) + ((b +
2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*c) + (3*(b^2 - 4*a*c)^2*ArcTanh[(b
+ 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2))
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)^{3/2}dx &= \frac{1}{2}\,\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}dx,x,x^2\right)\\ &= \frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}-\frac{\left(3\left(b^2-4ac\right)\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{32c}\\ &= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3\left(b^2-4ac\right)^2\right)}{}\\ &= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{\left(3\left(b^2-4ac\right)^2\right)}{}\\ &= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{3\left(b^2-4ac\right)^2\text{ta}}{}\end{aligned}$$

Mathematica [A] time = 0.0826465, size = 126, normalized size = 1.02

$$\frac{\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)}{8c^{3/2}}+2\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2) + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(
b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/
(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(8*c^(3/2)))/(32*c)
```

Maple [B] time = 0.178, size = 242, normalized size = 2.

$$\frac{cx^6}{8}\sqrt{cx^4+bx^2+a}+\frac{b^2x^2}{64\,c}\sqrt{cx^4+bx^2+a}+\frac{5\,ab}{32\,c}\sqrt{cx^4+bx^2+a}-\frac{3\,b^2a}{32}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}+\frac{3\,bx}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/8*c*x^6*(c*x^4+b*x^2+a)^(1/2)+1/64*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)+5/32*a
*b/c*(c*x^4+b*x^2+a)^(1/2)-3/32*a*b^2/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))+3/16*b*x^4*(c*x^4+b*x^2+a)^(1/2)+3/16*a^2*ln((1/2*b+c*x^
2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+5/16*a*x^2*(c*x^4+b*x^2+a)^(1/2)-
3/128*b^3/c^2*(c*x^4+b*x^2+a)^(1/2)+3/256*b^4/c^(5/2)*ln((1/2*b+c*x^2)/c^(1
/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66658, size = 684, normalized size = 5.52

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+4\left(16\,c^4x^6+24\,bc^3x^4-\right.}{512\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2
 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(16*c
```

```
^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*
sqrt(c*x^4 + b*x^2 + a))/c^3, -1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b
*c*x^2 + a*c)) - 2*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b
^2*c^2 + 20*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A] time = 1.26562, size = 182, normalized size = 1.47

$$\frac{1}{128}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(2\,cx^2+3\,b\right)x^2+\frac{b^2c^2+20\,ac^3}{c^3}\right)x^2-\frac{3\,b^3c-20\,abc^2}{c^3}\right)-\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(\left|-2\left(\sqrt{c}\right.\right.\right.}{256\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*c*x^2 + 3*b)*x^2 + (b^2*c^2 + 20*a*c
^3)/c^3)*x^2 - (3*b^3*c - 20*a*b*c^2)/c^3) - 3/256*(b^4 - 8*a*b^2*c + 16*a^
2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(
5/2)
```

3.940 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x} dx$

Optimal. Leaf size=155

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}}+\frac{\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c}+\frac{1}{6}($$

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c) + (a + b*x^2 + c
*x^4)^(3/2)/6 - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
c*x^4])])/2 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(32*c^(3/2))
```

Rubi [A] time = 0.184235, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1114, 734, 814, 843, 621, 206, 724}

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}}+\frac{\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c}+\frac{1}{6}($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```


```
[Out] ((b^2 + 8*a*c + 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c) + (a + b*x^2 + c
*x^4)^(3/2)/6 - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
c*x^4])])/2 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(32*c^(3/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} + \frac{\mathrm{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12ac\right)x}{x\sqrt{a+bx+cx^2}}\,dx, x, x\right.}{16c}$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} + \frac{1}{2}a^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,d\right.$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - a^2\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a}{\sqrt{a+}}\right.$$

$$= \frac{\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx}}\right.$$

Mathematica [A] time = 0.140945, size = 143, normalized size = 0.92

$$\frac{1}{96}\left(-48a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) - \frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{c^{3/2}} + \frac{2\sqrt{a+bx^2+cx^4}\left(8c\left(4a+cx^4\right)+\right.}{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((2*Sqrt[a + b*x^2 + c*x^4]*(3*b^2 + 14*b*c*x^2 + 8*c*(4*a + c*x^4)))/c - 4
8*a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] - (3*b
*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])
/c^(3/2))/96
```

Maple [A] time = 0.168, size = 192, normalized size = 1.2

$$\frac{cx^4}{6}\sqrt{cx^4+bx^2+a}+\frac{7\,bx^2}{24}\sqrt{cx^4+bx^2+a}+\frac{b^2}{16\,c}\sqrt{cx^4+bx^2+a}-\frac{b^3}{32}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}+\frac{3\,ab}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/6*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/c*b^2
*(c*x^4+b*x^2+a)^(1/2)-1/32/c^(3/2)*b^3*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))+3/8*a*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2
)+2/3*a*(c*x^4+b*x^2+a)^(1/2)-1/2*a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*
x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.49747, size = 1715, normalized size = 11.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/192*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 3*(b^3 - 12*a*b*c)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*sqrt
(c*x^4 + b*x^2 + a))/c^2, 1/96*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 3
```

```
*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)
*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*
c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2, 1/192*(96*sqrt(-a)*a*c^2*arctan
(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^
2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(
c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^4 + 14*b*c^2
*x^2 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2, 1/96*(48*sqrt(-a)*
a*c^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 + 14
*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x, x)
```

3.941 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=150

$$\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2}+\frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\frac{3}{4}\sqrt{a}b\tanh^{-1}\left(\frac{2a+b}{2\sqrt{a}\sqrt{a+b}}\right.$$

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/8 - (a + b*x^2 + c*x^4)^(3/2)/(
2*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x
^4])])/4 + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2
 + c*x^4])])/(16*Sqrt[c])
```

Rubi [A] time = 0.169245, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1114, 732, 814, 843, 621, 206, 724}

$$\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^2}+\frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\frac{3}{4}\sqrt{a}b\tanh^{-1}\left(\frac{2a+b}{2\sqrt{a}\sqrt{a+b}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/8 - (a + b*x^2 + c*x^4)^(3/2)/(
2*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x
^4])])/4 + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2
 + c*x^4])])/(16*Sqrt[c])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
```

```
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2+cx^4)^{3/2}}{2x^2} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)$$
$$= \frac{3}{8}(3b+2cx^2)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{2x^2} - \frac{3\operatorname{Subst}\left(\int \frac{-4abc-c(b^2+4ac)x}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16c}$$
$$= \frac{3}{8}(3b+2cx^2)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{2x^2} + \frac{1}{4}(3ab)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,$$
$$= \frac{3}{8}(3b+2cx^2)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{2x^2} - \frac{1}{2}(3ab)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a}{\sqrt{a+}}\right.$$
$$= \frac{3}{8}(3b+2cx^2)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{2x^2} - \frac{3}{4}\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) +$$

Mathematica [A] time = 0.11658, size = 134, normalized size = 0.89

$$\frac{1}{16}\left(\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}} + \frac{2\sqrt{a+bx^2+cx^4}(-4a+5bx^2+2cx^4)}{x^2} - 12\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] ((2*Sqrt[a + b*x^2 + c*x^4]*(-4*a + 5*b*x^2 + 2*c*x^4))/x^2 - 12*Sqrt[a]*b*
ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] + (3*(b^2 + 4*a*
c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c])/16
```

Maple [A] time = 0.176, size = 170, normalized size = 1.1

$$\frac{cx^2}{4}\sqrt{cx^4+bx^2+a}+\frac{5\,b}{8}\sqrt{cx^4+bx^2+a}+\frac{3\,b^2}{16}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}+\frac{3\,a}{4}\sqrt{c}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] 1/4*c*x^2*(c*x^4+b*x^2+a)^(1/2)+5/8*b*(c*x^4+b*x^2+a)^(1/2)+3/16*b^2*ln((1/
2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/4*a*c^(1/2)*ln((1/2*b+c
*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/4*a^(1/2)*b*ln((2*a+b*x^2+2*a^(1/2)*
(c*x^4+b*x^2+a)^(1/2))/x^2)-1/2*a/x^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18708, size = 1667, normalized size = 11.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*sqrt(a)*b*c*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^
4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 3*(b^2 + 4*a*c)*sqrt(c
)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c*x^4 + b*x
^2 + a))/(c*x^2), 1/16*(6*sqrt(a)*b*c*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
```

```
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c
*x^4 + b*x^2 + a))/(c*x^2), 1/32*(24*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 3*(b^2 +
4*a*c)*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqr
t(c*x^4 + b*x^2 + a))/(c*x^2), 1/16*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*
x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c
*x^4 + b*x^2 + a))/(c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^3, x)
```

3.942 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=151

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4}-\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2}+\frac{3}{4}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx^2}}\right.$$

```
[Out] (-3*(b - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*x^2) - (a + b*x^2 + c*x^4)^(3
/2)/(4*x^4) - (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*
x^2 + c*x^4])])/(16*Sqrt[a]) + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c
]*Sqrt[a + b*x^2 + c*x^4])])/4
```

Rubi [A] time = 0.163027, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1114, 732, 812, 843, 621, 206, 724}

$$-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4}-\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2}+\frac{3}{4}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```


```
[Out] (-3*(b - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*x^2) - (a + b*x^2 + c*x^4)^(3
/2)/(4*x^4) - (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*
x^2 + c*x^4])])/(16*Sqrt[a]) + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c
]*Sqrt[a + b*x^2 + c*x^4])])/4
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
```

```
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4}+\frac{3}{8}\,\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4}-\frac{3}{16}\,\text{Subst}\left(\int \frac{-b^2-4ac-4bcx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right.$$
$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4}+\frac{1}{4}(3bc)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x\right.$$
$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4}+\frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2}{\sqrt{a+bx}}\right.$$
$$= -\frac{3\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{8x^2}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4}-\frac{3\left(b^2+4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}+\frac{3}{4}$$

Mathematica [A] time = 0.1728, size = 134, normalized size = 0.89

$$\frac{1}{16}\left(-\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}-\frac{2\sqrt{a+bx^2+cx^4}\left(2a+5bx^2-4cx^4\right)}{x^4}+12b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] ((-2*(2*a + 5*b*x^2 - 4*c*x^4)*Sqrt[a + b*x^2 + c*x^4])/x^4 - (3*(b^2 + 4*a
*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a] + 1
2*b*Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/16
```

Maple [A] time = 0.171, size = 174, normalized size = 1.2

$$\frac{c}{2}\sqrt{cx^{4}+bx^{2}+a}+\frac{3b}{4}\sqrt{c}\ln\left(\left(\frac{b}{2}+cx^{2}\right)\frac{1}{\sqrt{c}}+\sqrt{cx^{4}+bx^{2}+a}\right)-\frac{3c}{4}\sqrt{a}\ln\left(\frac{1}{x^{2}}\left(2a+bx^{2}+2\sqrt{a}\sqrt{cx^{4}+bx^{2}+a}\right)\right)-\frac{3}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^5,x)
```

```
[Out] 1/2*c*(c*x^4+b*x^2+a)^(1/2)+3/4*b*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))-3/4*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x
^2)*c-3/16/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*b^2-
1/4*a/x^4*(c*x^4+b*x^2+a)^(1/2)-5/8*b/x^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.43635, size = 1670, normalized size = 11.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(12*a*b*sqrt(c)*x^4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 +
 b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^4*lo
g(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(a) + 8*a^2)/x^4) + 4*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt(c*x^4 + b*x
^2 + a))/(a*x^4), -1/32*(24*a*b*sqrt(-c)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 3*(b^2 + 4*a*c)*sq
rt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(
```

```
b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt
(c*x^4 + b*x^2 + a))/(a*x^4), 1/16*(6*a*b*sqrt(c)*x^4*log(-8*c^2*x^4 - 8*b*
c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 3*
(b^2 + 4*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sq
rt(c*x^4 + b*x^2 + a))/(a*x^4), -1/16*(12*a*b*sqrt(-c)*x^4*arctan(1/2*sqrt(
c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 3*(b
^2 + 4*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*s
qrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt
(c*x^4 + b*x^2 + a))/(a*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**5,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^5, x)
```

3.943 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=163

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}}-\frac{\left(x^2\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^2+cx^4}}{16ax^4}+\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-$$

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a*x^4) - (a + b*
x^2 + c*x^4)^(3/2)/(6*x^6) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqr
t[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rubi [A] time = 0.183787, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1114, 732, 810, 843, 621, 206, 724}

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}}-\frac{\left(x^2\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^2+cx^4}}{16ax^4}+\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a*x^4) - (a + b*
x^2 + c*x^4)^(3/2)/(6*x^6) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqr
t[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
```

```
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx,x,x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + \frac{1}{4}\,\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^2\right)$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b\left(b^2-12ac\right)-8ac^2x}{x\sqrt{a+bx+cx^2}}\,dx,\right.}{16a}$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + \frac{1}{2}c^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + c^2\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,-\right.$$

$$= -\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6} + \frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+}{2\sqrt{a}\sqrt{a+}}\right.}{32a^{3/2}}$$

Mathematica [A] time = 0.203306, size = 149, normalized size = 0.91

$$\frac{1}{96}\left(-\frac{2\sqrt{a+bx^2+cx^4}\left(8a^2+14abx^2+32acx^4+3b^2x^4\right)}{ax^6} + \frac{3b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}} + 48c^{3/2}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] ((-2*Sqrt[a + b*x^2 + c*x^4]*(8*a^2 + 14*a*b*x^2 + 3*b^2*x^4 + 32*a*c*x^4))
/(a*x^6) + (3*b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*
x^2 + c*x^4])])/a^(3/2) + 48*c^(3/2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4])])/96
```

Maple [A] time = 0.177, size = 202, normalized size = 1.2

$$\frac{1}{2}c^{\frac{3}{2}}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)-\frac{3bc}{8}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{a}{6x^6}\sqrt{cx^4+bx^2+a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^7,x)
```

```
[Out] 1/2*c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/8*b*c/a^(1/2)
*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/6*a/x^6*(c*x^4+b*x^2
+a)^(1/2)-7/24*b/x^4*(c*x^4+b*x^2+a)^(1/2)-1/16/a*b^2/x^2*(c*x^4+b*x^2+a)^(
1/2)+1/32/a^(3/2)*b^3*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-2
/3*c/x^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.80551, size = 1804, normalized size = 11.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(48*a^2*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^
6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8
```

```
*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6), -1/192*(96*a^2*sqrt(-c)*c*x^6*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 +
 a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(14*a^
2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x
^6), 1/96*(24*a^2*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(-a
)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) - 2*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8*a^3)*sqrt(
c*x^4 + b*x^2 + a))/(a^2*x^6), -1/96*(48*a^2*sqrt(-c)*c*x^6*arctan(1/2*sqrt
(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 3*(
b^3 - 12*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*
a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^
2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**7,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^7, x)
```

3.944 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=133

$$\frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{5/2}}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8}$$

```
[Out] (3*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^2*x^4) - ((2
*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*a*x^8) - (3*(b^2 - 4*a*c)^2*ArcT
anh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(256*a^(5/2))
```

Rubi [A] time = 0.116517, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1114, 720, 724, 206}

$$\frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{5/2}}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (3*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^2*x^4) - ((2
*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*a*x^8) - (3*(b^2 - 4*a*c)^2*ArcT
anh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(256*a^(5/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx,x,x^2\right)$$

$$= -\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} - \frac{\left(3\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^2\right)}{32a}$$

$$= \frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} + \frac{\left(3\left(b^2-4ac\right)^2\right)\text{Su}}{}$$

$$= \frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} - \frac{\left(3\left(b^2-4ac\right)^2\right)\text{Su}}{}$$

$$= \frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} - \frac{3\left(b^2-4ac\right)^2\tanh}{256}$$

Mathematica [A] time = 0.162051, size = 138, normalized size = 1.04

$$-\frac{\frac{3\left(b^2-4ac\right)\left(x^4\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{a}\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}\right)}{8a^{3/2}x^4}+\frac{2\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{x^8}}{32a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -((2*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/x^8 + (3*(b^2 - 4*a*c)*(-2*Sq
rt[a]*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*x^4*ArcTanh[(2*
a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]))/(8*a^(3/2)*x^4))/(32*a)
```

Maple [B] time = 0.18, size = 260, normalized size = 2.

$$-\frac{b^2}{64\,ax^4}\sqrt{cx^4+bx^2+a}+\frac{3\,b^3}{128\,a^2x^2}\sqrt{cx^4+bx^2+a}-\frac{a}{8\,x^8}\sqrt{cx^4+bx^2+a}-\frac{5\,c}{16\,x^4}\sqrt{cx^4+bx^2+a}+\frac{3\,b^2c}{32}\ln\left(\frac{1}{x^2}\left(2\,a+bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^9,x)
```

```
[Out] -1/64/a*b^2/x^4*(c*x^4+b*x^2+a)^(1/2)+3/128/a^2*b^3/x^2*(c*x^4+b*x^2+a)^(1/
2)-1/8*a/x^8*(c*x^4+b*x^2+a)^(1/2)-5/16*c/x^4*(c*x^4+b*x^2+a)^(1/2)+3/32/a^
(3/2)*b^2*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-5/32/a*b*c/
x^2*(c*x^4+b*x^2+a)^(1/2)-3/16*b/x^6*(c*x^4+b*x^2+a)^(1/2)-3/256/a^(5/2)*b^
4*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-3/16*c^2/a^(1/2)*ln((
2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.39964, size = 726, normalized size = 5.46

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{a}x^8\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(\left(3\,ab^3-20\,a^2bc\right)x^6-24\,a^3bx^2-2\,\left(\right.\right.}{512\,a^3x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^8*log(-((b^2 + 4*a*c)*x^
4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^
4) + 4*((3*a*b^3 - 20*a^2*b*c)*x^6 - 24*a^3*b*x^2 - 2*(a^2*b^2 + 20*a^3*c)*
x^4 - 16*a^4)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^8), 1/256*(3*(b^4 - 8*a*b^2*c
 + 16*a^2*c^2)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^6 - 24*
a^3*b*x^2 - 2*(a^2*b^2 + 20*a^3*c)*x^4 - 16*a^4)*sqrt(c*x^4 + b*x^2 + a))/(
a^3*x^8)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**9,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**9, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^9, x)
```

3.945 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=162

$$-\frac{3b(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{7/2}}+\frac{b(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{32a^2x^8}-\frac{(\ldots}{\ldots}$$

```
[Out] (-3*b*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a^3*x^4) +
(b*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*a^2*x^8) - (a + b*x^2 + c*x
^4)^(5/2)/(10*a*x^10) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[
a]*Sqrt[a + b*x^2 + c*x^4])])/(512*a^(7/2))
```

Rubi [A] time = 0.14011, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 730, 720, 724, 206}

$$-\frac{3b(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{7/2}}+\frac{b(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{32a^2x^8}-\frac{(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] (-3*b*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a^3*x^4) +
(b*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*a^2*x^8) - (a + b*x^2 + c*x
^4)^(5/2)/(10*a*x^10) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[
a]*Sqrt[a + b*x^2 + c*x^4])])/(512*a^(7/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
```

```
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ax^{10}} - \frac{b\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^2\right)}{4a}$$

$$= \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ax^{10}} + \frac{\left(3b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,\right.}{64a^2}$$

$$= -\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx^2+cx^4\right)}{10ax^{10}}$$

$$= -\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx^2+cx^4\right)}{10ax^{10}}$$

$$= -\frac{3b\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32a^2x^8} - \frac{\left(a+bx^2+cx^4\right)}{10ax^{10}}$$

Mathematica [A] time = 0.143446, size = 167, normalized size = 1.03

$$\frac{b\left(16a^{3/2}\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2} - 3x^4\left(b^2-4ac\right)\left(2\sqrt{a}\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4} - x^4\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx}}\right.\right.\right.}{512a^{7/2}x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -(a + b*x^2 + c*x^4)^(5/2)/(10*a*x^10) + (b*(16*a^(3/2)*(2*a + b*x^2)*(a +
b*x^2 + c*x^4)^(3/2) - 3*(b^2 - 4*a*c)*x^4*(2*Sqrt[a]*(2*a + b*x^2)*Sqrt[a
+ b*x^2 + c*x^4] - (b^2 - 4*a*c)*x^4*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])))/(512*a^(7/2)*x^8)
```

Maple [B] time = 0.175, size = 337, normalized size = 2.1

$$-\frac{c}{5\,x^6}\sqrt{cx^4+bx^2+a} + \frac{3\,b^5}{512}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{7}{2}} - \frac{a}{10\,x^{10}}\sqrt{cx^4+bx^2+a} - \frac{11\,b}{80\,x^8}\sqrt{cx^4+bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^11,x)
```

```
[Out] -1/5*c/x^6*(c*x^4+b*x^2+a)^(1/2)+3/512*b^5/a^(7/2)*ln((2*a+b*x^2+2*a^(1/2)*
(c*x^4+b*x^2+a)^(1/2))/x^2)-1/10*a/x^10*(c*x^4+b*x^2+a)^(1/2)-11/80*b/x^8*(
c*x^4+b*x^2+a)^(1/2)-7/160/a*c*b/x^4*(c*x^4+b*x^2+a)^(1/2)+5/64/a^2*c*b^2/x
^2*(c*x^4+b*x^2+a)^(1/2)-3/64/a^(5/2)*c*b^3*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+
b*x^2+a)^(1/2))/x^2)+3/32/a^(3/2)*c^2*b*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^
2+a)^(1/2))/x^2)-1/10/a*c^2/x^2*(c*x^4+b*x^2+a)^(1/2)-1/160*b^2/a/x^6*(c*x^
4+b*x^2+a)^(1/2)+1/128*b^3/a^2/x^4*(c*x^4+b*x^2+a)^(1/2)-3/256*b^4/a^3/x^2*
(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.0542, size = 887, normalized size = 5.48

$$\left[\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{a}x^{10}\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2+4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)-4\left(\left(15ab^4-100a^2b^2c+128a^3c^2\right)\right.}{5120a^4x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^10*log(-((b^2 + 4*a*
c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^
2)/x^4) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^8 + 176*a^4*b*x^2 -
 2*(5*a^2*b^3 - 28*a^3*b*c)*x^6 + 128*a^5 + 8*(a^3*b^2 + 32*a^4*c)*x^4)*sqr
t(c*x^4 + b*x^2 + a))/(a^4*x^10), -1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*sqrt(-a)*x^10*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)
/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x
```

```
^8 + 176*a^4*b*x^2 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^6 + 128*a^5 + 8*(a^3*b^2
+ 32*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^10)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**11,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**11, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^11, x)
```

3.946 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=216

$$-\frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384a^3x^8}+\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{\left(b^2-4ac\right)^2\left(7b^2-4a\right.}{20}$$

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(1024
*a^4*x^4) - ((7*b^2 - 4*a*c)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(384*
a^3*x^8) - (a + b*x^2 + c*x^4)^(5/2)/(12*a*x^12) + (7*b*(a + b*x^2 + c*x^4)
^(5/2))/(120*a^2*x^10) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2048*a^(9/2))
```

Rubi [A] time = 0.217046, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 744, 806, 720, 724, 206}

$$-\frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384a^3x^8}+\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{\left(b^2-4ac\right)^2\left(7b^2-4a\right.}{20}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(1024
*a^4*x^4) - ((7*b^2 - 4*a*c)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(384*
a^3*x^8) - (a + b*x^2 + c*x^4)^(5/2)/(12*a*x^12) + (7*b*(a + b*x^2 + c*x^4)
^(5/2))/(120*a^2*x^10) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2048*a^(9/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2+cx^4)^{5/2}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+cx\right)(a+bx+cx^2)^{3/2}}{x^6}\,dx, x, x^2\right)}{12a}$$

$$= -\frac{(a+bx^2+cx^4)^{5/2}}{12ax^{12}} + \frac{7b(a+bx^2+cx^4)^{5/2}}{120a^2x^{10}} + \frac{(7b^2-4ac)\,\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\,dx, x, x^2\right)}{48a^2}$$

$$= -\frac{(7b^2-4ac)(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{384a^3x^8} - \frac{(a+bx^2+cx^4)^{5/2}}{12ax^{12}} + \frac{7b(a+bx^2+cx^4)^{5/2}}{120a^2x^{10}} - \dots$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{(7b^2-4ac)(2a+bx^2)(a+bx^2+cx^4)^{3/\dots}}{384a^3x^8}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{(7b^2-4ac)(2a+bx^2)(a+bx^2+cx^4)^{3/\dots}}{384a^3x^8}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{(7b^2-4ac)(2a+bx^2)(a+bx^2+cx^4)^{3/\dots}}{384a^3x^8}$$

Mathematica [A] time = 0.210329, size = 206, normalized size = 0.95

$$-\frac{\frac{\left(\frac{7b^2}{2}-2ac\right)\left(16a^{3/2}(2a+bx^2)(a+bx^2+cx^4)^{3/2}-3x^4(b^2-4ac)\left(2\sqrt{a}(2a+bx^2)\sqrt{a+bx^2+cx^4}-x^4(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)\right)\right)}{256a^{7/2}x^8} - \frac{7b(a+bx^2+cx^4)^{5/2}}{10ax^{10}} + \frac{(a+b\dots}{\dots}}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -((a + b*x^2 + c*x^4)^(5/2)/x^12 - (7*b*(a + b*x^2 + c*x^4)^(5/2))/(10*a*x^
10) + (((7*b^2)/2 - 2*a*c)*(16*a^(3/2)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3
/2) - 3*(b^2 - 4*a*c)*x^4*(2*Sqrt[a]*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4]
- (b^2 - 4*a*c)*x^4*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4
])])))/(256*a^(7/2)*x^8))/(12*a)
```

Maple [B] time = 0.184, size = 457, normalized size = 2.1

$$-\frac{b^2}{320\,a x^8}\sqrt{c x^4+b x^2+a}+\frac{7\,b^3}{1920\,a^2 x^6}\sqrt{c x^4+b x^2+a}-\frac{7\,b^4}{1536\,a^3 x^4}\sqrt{c x^4+b x^2+a}+\frac{7\,b^5}{1024\,a^4 x^2}\sqrt{c x^4+b x^2+a}-\frac{7\,b^6}{2048}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^13,x)
```

```
[Out] -1/320/a*b^2/x^8*(c*x^4+b*x^2+a)^(1/2)+7/1920/a^2*b^3/x^6*(c*x^4+b*x^2+a)^(
1/2)-7/1536/a^3*b^4/x^4*(c*x^4+b*x^2+a)^(1/2)+7/1024/a^4*b^5/x^2*(c*x^4+b*x
^2+a)^(1/2)-7/2048/a^(9/2)*b^6*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2
))/x^2)+27/320*b*c^2/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)+9/320*b^2*c/a^2/x^4*(c*x
^4+b*x^2+a)^(1/2)-19/384*b^3*c/a^3/x^2*(c*x^4+b*x^2+a)^(1/2)-3/160*b*c/a/x^
6*(c*x^4+b*x^2+a)^(1/2)-13/120*b/x^10*(c*x^4+b*x^2+a)^(1/2)-7/48*c/x^8*(c*x
^4+b*x^2+a)^(1/2)+15/512*b^4*c/a^(7/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2
+a)^(1/2))/x^2)-9/128*b^2*c^2/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+
a)^(1/2))/x^2)-1/12*a/x^12*(c*x^4+b*x^2+a)^(1/2)-1/32/a*c^2/x^4*(c*x^4+b*x^
2+a)^(1/2)+1/32/a^(3/2)*c^3*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/
x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.90069, size = 1108, normalized size = 5.13

$$\left[-\frac{15\left(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{a}x^{12}\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\left(\left(105\,ab^5-760\,a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*x
^12*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c
^2)*x^10 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^8 - 1664*a^5*b*x^
2 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^6 - 1280*a^6 - 16*(3*a^4*b^2 + 140*a^5*c)*
x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^12), 1/30720*(15*(7*b^6 - 60*a*b^4*c +
 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((105*a*b^5 - 76
0*a^2*b^3*c + 1296*a^3*b*c^2)*x^10 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^
4*c^2)*x^8 - 1664*a^5*b*x^2 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^6 - 1280*a^6 - 1
6*(3*a^4*b^2 + 140*a^5*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^12)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**13,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**13, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^13, x)
```

3.947 $\int x^4 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=495

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(60a^2c^2-51ab^2c+8b^4\right)+8b\left(2b^2-9ac\right)\left(b^2-3ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\right.\right.}{2310c^{15/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] ((8*b^4 - 51*a*b^2*c + 60*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(1155*c^3) -
(8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(1155*c^(7/2)
*(Sqrt[a] + Sqrt[c]*x^2)) - (x^3*(b*(2*b^2 + a*c) + 10*c*(b^2 - 3*a*c)*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(385*c^2) + (x^3*(b + 3*c*x^2)*(a + b*x^2 + c*x^4
)^(3/2))/(33*c) + (8*a^(1/4)*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(1155*c^(15/4)*Sq
rt[a + b*x^2 + c*x^4]) - (a^(1/4)*(8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c) + Sqrt
[a]*Sqrt[c]*(8*b^4 - 51*a*b^2*c + 60*a^2*c^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2310*c^(15/4)*Sqrt[a + b*x^2 +
 c*x^4])
```

Rubi [A] time = 0.440438, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1116, 1273, 1279, 1197, 1103, 1195}

$$\frac{x\left(60a^2c^2-51ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(60a^2c^2-51ab^2c+8b^4\right)+8b\left(2b^2-9ac\right)\left(b^2-3ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right.}{2310c^{15/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((8*b^4 - 51*a*b^2*c + 60*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(1155*c^3) -
(8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(1155*c^(7/2)
*(Sqrt[a] + Sqrt[c]*x^2)) - (x^3*(b*(2*b^2 + a*c) + 10*c*(b^2 - 3*a*c)*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(385*c^2) + (x^3*(b + 3*c*x^2)*(a + b*x^2 + c*x^4
)^(3/2))/(33*c) + (8*a^(1/4)*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(1155*c^(15/4)*Sq
```

```
rt[a + b*x^2 + c*x^4]) - (a^(1/4)*(8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c) + Sqrt
[a]*Sqrt[c]*(8*b^4 - 51*a*b^2*c + 60*a^2*c^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2310*c^(15/4)*Sqrt[a + b*x^2 +
 c*x^4])
```

Rule 1116

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*(2*b*p + c*(m + 4*p - 1)*x^
2))/(c*(m + 4*p + 1)*(m + 4*p - 1)), x] - Dist[(2*p*d^2)/(c*(m + 4*p + 1)*(
m + 4*p - 1)), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1273

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*
p + 3)), x] + Dist[(2*p)/(c*(4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
```

```
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int x^4\left(a+bx^2+cx^4\right)^{3/2}dx=\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}-\frac{\int x^2\left(3ab+6\left(b^2-3ac\right)x^2\right)\sqrt{a+bx^2+cx^4}\,dx}{33c}$$

$$=-\frac{x^3\left(b\left(2b^2+ac\right)+10c\left(b^2-3ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{385c^2}+\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}-$$

$$=\frac{\left(8b^4-51ab^2c+60a^2c^2\right)x\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{x^3\left(b\left(2b^2+ac\right)+10c\left(b^2-3ac\right)x^2\right)\sqrt{a+bx^2}}{385c^2}$$

$$=\frac{\left(8b^4-51ab^2c+60a^2c^2\right)x\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{x^3\left(b\left(2b^2+ac\right)+10c\left(b^2-3ac\right)x^2\right)\sqrt{a+bx^2}}{385c^2}$$

$$=\frac{\left(8b^4-51ab^2c+60a^2c^2\right)x\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{8b\left(2b^2-9ac\right)\left(b^2-3ac\right)x\sqrt{a+bx^2+cx^4}}{1155c^{7/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{x}{}$$

Mathematica [C] time = 2.21416, size = 657, normalized size = 1.33

$$i\left(-159a^2b^2c^2+108a^2bc^2\sqrt{b^2-4ac}+60a^3c^3+8b^5\sqrt{b^2-4ac}+68ab^4c-60ab^3c\sqrt{b^2-4ac}-8b^6\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{}}{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(60*a^3*c^2 + a^2*c*(-51*b^2 + 92*b*
c*x^2 + 255*c^2*x^4) + a*(8*b^4 - 57*b^3*c*x^2 - 14*b^2*c^2*x^4 + 367*b*c^3
*x^6 + 300*c^4*x^8) + x^2*(8*b^5 + 2*b^4*c*x^2 - b^3*c^2*x^4 + 145*b^2*c^3*
x^6 + 245*b*c^4*x^8 + 105*c^5*x^10)) - (4*I)*b*(2*b^4 - 15*a*b^2*c + 27*a^2
*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x]
, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-8*b^6 + 68*a*b^4*c
 - 159*a^2*b^2*c^2 + 60*a^3*c^3 + 8*b^5*Sqrt[b^2 - 4*a*c] - 60*a*b^3*c*Sqrt
[b^2 - 4*a*c] + 108*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2310
*c^4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.219, size = 674, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/11*c*x^9*(c*x^4+b*x^2+a)^(1/2)+4/33*b*x^7*(c*x^4+b*x^2+a)^(1/2)+1/7*(13/1
1*a*c+1/33*b^2)/c*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(38/33*a*b-6/7*(13/11*a*c+1
/33*b^2)/c*b)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(a^2-5/7*(13/11*a*c+1/33*b^2)
/c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*b)/c*x*(c*x^4+b*x^2+a)^
(1/2)-1/12*(a^2-5/7*(13/11*a*c+1/33*b^2)/c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+
1/33*b^2)/c*b)/c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5*(38/33*a*b-6/7
*(13/11*a*c+1/33*b^2)/c*b)/c*a-2/3*(a^2-5/7*(13/11*a*c+1/33*b^2)/c*a-4/5*(3
8/33*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
```

```
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^8 + bx^6 + ax^4\right)\sqrt{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^8 + b*x^6 + a*x^4)*sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

3.948 $\int x^2 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=443

$$\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{630c^{11/4}\sqrt{a+bx^2+cx^4}}+\frac{x}{}$$

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(315*c^(5/2)*
(Sqrt[a] + Sqrt[c]*x^2)) - (x*(b*(4*b^2 - 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(315*c^2) + (x*(3*b + 7*c*x^2)*(a + b*x^2 + c*x^4
)^(3/2))/(63*c) - (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqr
t[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[
a]*b*Sqrt[c]*(b^2 - 6*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.284968, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1116, 1176, 1197, 1103, 1195}

$$\frac{x\left(84a^2c^2-57ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}}{630c^{11/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(315*c^(5/2)*
(Sqrt[a] + Sqrt[c]*x^2)) - (x*(b*(4*b^2 - 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(315*c^2) + (x*(3*b + 7*c*x^2)*(a + b*x^2 + c*x^4
)^(3/2))/(63*c) - (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqr
t[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[
a]*b*Sqrt[c]*(b^2 - 6*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
```

```
b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1116

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*(2*b*p + c*(m + 4*p - 1)*x^
2))/(c*(m + 4*p + 1)*(m + 4*p - 1)), x] - Dist[(2*p*d^2)/(c*(m + 4*p + 1)*(
m + 4*p - 1)), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int x^2\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-\frac{\int\left(ab+2\left(2b^2-7ac\right)x^2\right)\sqrt{a+bx^2+cx^4}\,dx}{21c}$$
$$=-\frac{x\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-$$
$$=-\frac{x\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-$$
$$=\frac{\left(8b^4-57ab^2c+84a^2c^2\right)x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{x\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{a+bx^2}}{315c^2}$$

Mathematica [C] time = 1.92701, size = 602, normalized size = 1.36

$$\frac{-i\left(84a^2c^2\sqrt{b^2-4ac}-132a^2bc^2+8b^4\sqrt{b^2-4ac}+65ab^3c-57ab^2c\sqrt{b^2-4ac}-8b^5\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3*c*x^4 + 53*b^2*c^2
*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*x^2) + a*(-4*b^3 + 2
7*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) + I*(8*b^4 - 57*a*b^2*c + 84*a^
2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^5 + 65*a*b^3*
c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*Sqrt[b^2 - 4*a*c]
+ 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(1260*c^3*Sqrt[c/(b +
```

```
Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.214, size = 545, normalized size = 1.2

$$\frac{cx^7}{9}\sqrt{cx^4+bx^2+a}+\frac{10\,bx^5}{63}\sqrt{cx^4+bx^2+a}+\frac{x^3}{5\,c}\left(\frac{11\,ac}{9}+\frac{b^2}{21}\right)\sqrt{cx^4+bx^2+a}+\frac{x}{3\,c}\left(\frac{76\,ab}{63}-\frac{4\,b}{5\,c}\left(\frac{11\,ac}{9}+\frac{b^2}{21}\right)\right)\sqrt{cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/9*c*x^7*(c*x^4+b*x^2+a)^(1/2)+10/63*b*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(11/9
*a*c+1/21*b^2)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(76/63*a*b-4/5*(11/9*a*c+1/2
1*b^2)/c*b)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(76/63*a*b-4/5*(11/9*a*c+1/21*b^
2)/c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(a^2-3/5*(11/9*a*c+1/21*b^2)/c*a
-2/3*(76/63*a*b-4/5*(11/9*a*c+1/21*b^2)/c*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(El
lipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^4 + ax^2\right)\sqrt{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^4 + a*x^2)*sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

3.949 $\int \left(a+bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=381

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{2bx\left(b^2-\right.}{35c}$$

```
[Out] (-2*b*(b^2 - 8*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(35*c^(3/2)*(Sqrt[a] + Sqrt[
c]*x^2)) + (x*(b^2 + 10*a*c + 3*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) +
(x*(a + b*x^2 + c*x^4)^(3/2))/7 + (2*a^(1/4)*b*(b^2 - 8*a*c)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[
a + b*x^2 + c*x^4]) - (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 -
 8*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(70*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.251015, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1091, 1176, 1197, 1103, 1195}

$$-\frac{2bx\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{70c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (-2*b*(b^2 - 8*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(35*c^(3/2)*(Sqrt[a] + Sqrt[
c]*x^2)) + (x*(b^2 + 10*a*c + 3*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) +
(x*(a + b*x^2 + c*x^4)^(3/2))/7 + (2*a^(1/4)*b*(b^2 - 8*a*c)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[
a + b*x^2 + c*x^4]) - (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 -
 8*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(70*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2} + \frac{3}{7}\int \left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{x\left(b^2+10ac+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{35c} + \frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2} + \frac{\int \frac{-a\left(b^2-20ac\right)-2b\left(b^2-8ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{35c}$$

$$= \frac{x\left(b^2+10ac+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{35c} + \frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2} + \frac{\left(2\sqrt{a}b\left(b^2-8ac\right)\right)\int \frac{1-\frac{\sqrt{c}}{\sqrt{}}}{\sqrt{a+bx^2}}}{35c^{3/2}}$$

$$= -\frac{2b\left(b^2-8ac\right)x\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{x\left(b^2+10ac+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{35c} + \frac{1}{7}x\left(a+bx^2+cx\right.$$

Mathematica [C] time = 1.51035, size = 533, normalized size = 1.4

$$i\left(-20a^2c^2+b^3\sqrt{b^2-4ac}+9ab^2c-8abc\sqrt{b^2-4ac}-b^4\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*c*x^2 + 20
*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6)) - I*b*(b^2
- 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a*b^2*c
 - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[
b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])])/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4
])
```

Maple [A] time = 0.214, size = 471, normalized size = 1.2

$$\frac{cx^5}{7}\sqrt{cx^4+bx^2+a}+\frac{8\,bx^3}{35}\sqrt{cx^4+bx^2+a}+\frac{x}{3\,c}\left(\frac{9\,ac}{7}+\frac{3\,b^2}{35}\right)\sqrt{cx^4+bx^2+a}+\frac{\sqrt{2}}{4}\left(a^2-\frac{a}{3\,c}\left(\frac{9\,ac}{7}+\frac{3\,b^2}{35}\right)\right)\sqrt{4-2\,\frac{(-}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/7*c*x^5*(c*x^4+b*x^2+a)^(1/2)+8/35*b*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(9/7*a
*c+3/35*b^2)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(a^2-1/3*(9/7*a*c+3/35*b^2)/c*a)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(46/35*a*b-2/3*(9/7*a*c+3/35*b^2)/c*b)*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^4+bx^2+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

3.950 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}\right.}{}$$

```
[Out] ((b^2 + 12*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(5*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^
2)) + (x*(7*b + 6*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/5 - (a + b*x^2 + c*x^4)^(
3/2)/x - (a^(1/4)*(b^2 + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(5*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4
)*(b^2 + 8*Sqrt[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(10*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.206346, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1117, 1176, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\cdots}}{5c^{3/4}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^2,x]
```


```
[Out] ((b^2 + 12*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(5*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^
2)) + (x*(7*b + 6*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/5 - (a + b*x^2 + c*x^4)^(
3/2)/x - (a^(1/4)*(b^2 + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(5*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4
)*(b^2 + 8*Sqrt[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(10*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^2}\,dx = -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x} + 3\int \left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{5}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x} + \frac{\int \frac{8abc+c\left(b^2+12ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{1}{5}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x} - \frac{\left(\sqrt{a}\left(b^2+12ac\right)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{5\sqrt{c}} + \frac{(\sqrt{}}{}$$

$$= \frac{\left(b^2+12ac\right)x\sqrt{a+bx^2+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{5}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x} - \frac{\sqrt[4]{a}\left(b^2\right.}{}$$

Mathematica [C] time = 1.26276, size = 505, normalized size = 1.4

$$\frac{-ix\left(b^2\sqrt{b^2-4ac}+12ac\sqrt{b^2-4ac}+4abc-b^3\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-5*a^2 - 3*a*b*x^2 + 2*b^2*x^4 - 4*a*
c*x^4 + 3*b*c*x^6 + c^2*x^8) + I*(b^2 + 12*a*c)*(-b + Sqrt[b^2 - 4*a*c])*x*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c])] - I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 12*a*c*Sq
rt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(20*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.223, size = 430, normalized size = 1.2

$$-\frac{a}{x}\sqrt{cx^4+bx^2+a}+\frac{cx^3}{5}\sqrt{cx^4+bx^2+a}+\frac{2\,bx}{5}\sqrt{cx^4+bx^2+a}+\frac{2\,ab\sqrt{2}}{5}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{\right.}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^2,x)
```

```
[Out] -a*(c*x^4+b*x^2+a)^(1/2)/x+1/5*c*x^3*(c*x^4+b*x^2+a)^(1/2)+2/5*b*x*(c*x^4+b
*x^2+a)^(1/2)+2/5*a*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(12/5*a*c+1/5*b^2)*a*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

3.951 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=353

$$\frac{\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}-\frac{(3b-$$

```
[Out] (8*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*(Sqrt[a] + Sqrt[c]*x^2)) - ((3*b
 - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(3*x) - (a + b*x^2 + c*x^4)^(3/2)/(3*x
^3) - (8*a^(1/4)*b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(3*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 + 8*Sqrt[a]*b
*Sqrt[c] + 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(6*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.152889, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1117, 1271, 1197, 1103, 1195}

$$\frac{\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}-\frac{\left(3b-2cx^2\right)\sqrt{}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^4,x]
```


```
[Out] (8*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*(Sqrt[a] + Sqrt[c]*x^2)) - ((3*b
 - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(3*x) - (a + b*x^2 + c*x^4)^(3/2)/(3*x
^3) - (8*a^(1/4)*b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(3*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 + 8*Sqrt[a]*b
*Sqrt[c] + 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(6*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx = -\frac{(a+bx^2+cx^4)^{3/2}}{3x^3} + \int \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

$$= -\frac{(3b-2cx^2)\sqrt{a+bx^2+cx^4}}{3x} - \frac{(a+bx^2+cx^4)^{3/2}}{3x^3} - \frac{1}{3}\int \frac{-3b^2-4ac-8bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{(3b-2cx^2)\sqrt{a+bx^2+cx^4}}{3x} - \frac{(a+bx^2+cx^4)^{3/2}}{3x^3} - \frac{1}{3}\left(8\sqrt{a}b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx - \frac{1}{3}$$

$$= \frac{8b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{(3b-2cx^2)\sqrt{a+bx^2+cx^4}}{3x} - \frac{(a+bx^2+cx^4)^{3/2}}{3x^3} - \frac{8\sqrt[4]{a}b\sqrt[4]{c}\left(\sqrt{a}+\right.}{}$$

Mathematica [C] time = 0.903069, size = 473, normalized size = 1.34

$$\frac{-ix^3\left(4b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)+2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-a^2 - 5*a*b*x^2 - 4*b^2*x^4 - 3*b*c*x^
6 + c^2*x^8) + (4*I)*b*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] +
4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I
*(-b^2 + 4*a*c + 4*b*Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(6*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.226, size = 428, normalized size = 1.2

$$-\frac{a}{3x^3}\sqrt{cx^4+bx^2+a} - \frac{4b}{3x}\sqrt{cx^4+bx^2+a} + \frac{cx}{3}\sqrt{cx^4+bx^2+a} + \frac{\sqrt{2}}{4}\left(\frac{4ac}{3}+b^2\right)\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^4,x)
```

```
[Out] -1/3*a*(c*x^4+b*x^2+a)^(1/2)/x^3-4/3*b*(c*x^4+b*x^2+a)^(1/2)/x+1/3*c*x*(c*x
^4+b*x^2+a)^(1/2)+1/4*(4/3*a*c+b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-4/3*b*c*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**4,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

3.952 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^6} dx$

Optimal. Leaf size=400

$$\frac{\sqrt[4]{c}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}\right.}{}$$

```
[Out] -((b^2 + 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(5*a*x) + (Sqrt[c]*(b^2 + 12*a*c)
*x*Sqrt[a + b*x^2 + c*x^4])/(5*a*(Sqrt[a] + Sqrt[c]*x^2)) - ((b - 6*c*x^2)*
Sqrt[a + b*x^2 + c*x^4])/(5*x^3) - (a + b*x^2 + c*x^4)^(3/2)/(5*x^5) - (c^(
1/4)*(b^2 + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[
a]*Sqrt[c]))/4])/(5*a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(b^2 + 8*Sq
rt[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(10*a^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.250915, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1117, 1271, 1281, 1197, 1103, 1195}

$$\frac{\sqrt[4]{c}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+}{(\sqrt{}}}}{5a^{3/4}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^6,x]
```


```
[Out] -((b^2 + 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(5*a*x) + (Sqrt[c]*(b^2 + 12*a*c)
*x*Sqrt[a + b*x^2 + c*x^4])/(5*a*(Sqrt[a] + Sqrt[c]*x^2)) - ((b - 6*c*x^2)*
Sqrt[a + b*x^2 + c*x^4])/(5*x^3) - (a + b*x^2 + c*x^4)^(3/2)/(5*x^5) - (c^(
1/4)*(b^2 + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[
a]*Sqrt[c]))/4])/(5*a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(b^2 + 8*Sq
rt[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
```

```
(Sqrt[a]*Sqrt[c]))/4])/(10*a^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^6}\,dx = -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}+\frac{3}{5}\int \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

$$= -\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{5x^3}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}-\frac{1}{5}\int \frac{-b^2-12ac-8bcx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\left(b^2+12ac\right)\sqrt{a+bx^2+cx^4}}{5ax}-\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{5x^3}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}+\frac{\int \frac{8abc+c\left(b^2+\right.}{\sqrt{a+bx^2}}}{5a}$$

$$= -\frac{\left(b^2+12ac\right)\sqrt{a+bx^2+cx^4}}{5ax}-\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{5x^3}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}-\frac{\left(\sqrt{c}\left(b^2+1\right.\right.}{}$$

$$= -\frac{\left(b^2+12ac\right)\sqrt{a+bx^2+cx^4}}{5ax}+\frac{\sqrt{c}\left(b^2+12ac\right)x\sqrt{a+bx^2+cx^4}}{5a\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{5x^3}$$

Mathematica [C] time = 1.3484, size = 527, normalized size = 1.32

$$\frac{-ix^5\left(b^2\sqrt{b^2-4ac}+12ac\sqrt{b^2-4ac}+4abc-b^3\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a^3 + b^2*x^6*(b + c*x^2) + a^2*(3*b*x
^2 + 8*c*x^4) + a*(3*b^2*x^4 + 9*b*c*x^6 + 7*c^2*x^8)) + I*(b^2 + 12*a*c)*(
-b + Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
```

```
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^3 + 4*a*b*c + b^2*S
qrt[b^2 - 4*a*c] + 12*a*c*Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(20*a
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.224, size = 450, normalized size = 1.1

$$-\frac{a}{5x^5}\sqrt{cx^4+bx^2+a}-\frac{2b}{5x^3}\sqrt{cx^4+bx^2+a}-\frac{7ac+b^2}{5ax}\sqrt{cx^4+bx^2+a}+\frac{2bc\sqrt{2}}{5}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^6,x)
```

```
[Out] -1/5*a*(c*x^4+b*x^2+a)^(1/2)/x^5-2/5*b*(c*x^4+b*x^2+a)^(1/2)/x^3-1/5*(7*a*c
+b^2)/a*(c*x^4+b*x^2+a)^(1/2)/x+2/5*b*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(c^2
+1/5*c*(7*a*c+b^2)/a)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**6,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

3.953 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=447

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\left(b^2-\right.}{}$$

```
[Out] -((b^2 - 20*a*c)*Sqrt[a + b*x^2 + c*x^4])/(35*a*x^3) + (2*b*(b^2 - 8*a*c)*S
qrt[a + b*x^2 + c*x^4])/(35*a^2*x) - (2*b*Sqrt[c]*(b^2 - 8*a*c)*x*Sqrt[a +
b*x^2 + c*x^4])/(35*a^2*(Sqrt[a] + Sqrt[c]*x^2)) - (3*(b + 10*c*x^2)*Sqrt[a
 + b*x^2 + c*x^4])/(35*x^5) - (a + b*x^2 + c*x^4)^(3/2)/(7*x^7) + (2*b*c^(1
/4)*(b^2 - 8*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(35*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqrt[a]*Sqr
t[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(70*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.394906, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1117, 1271, 1281, 1197, 1103, 1195}

$$\frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{2b\sqrt{c}x\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right.}{70a^{7/4}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -((b^2 - 20*a*c)*Sqrt[a + b*x^2 + c*x^4])/(35*a*x^3) + (2*b*(b^2 - 8*a*c)*S
qrt[a + b*x^2 + c*x^4])/(35*a^2*x) - (2*b*Sqrt[c]*(b^2 - 8*a*c)*x*Sqrt[a +
b*x^2 + c*x^4])/(35*a^2*(Sqrt[a] + Sqrt[c]*x^2)) - (3*(b + 10*c*x^2)*Sqrt[a
 + b*x^2 + c*x^4])/(35*x^5) - (a + b*x^2 + c*x^4)^(3/2)/(7*x^7) + (2*b*c^(1
/4)*(b^2 - 8*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(35*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqrt[a]*Sqr
t[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
```

```
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(70*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
```

```
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^8}\,dx = -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}+\frac{3}{7}\int \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

$$= -\frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}+\frac{3}{35}\int \frac{b^2-20ac-8bcx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3}-\frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}-\int \frac{2b(b^2\ldots}{\ldots}$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3}+\frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5}\ldots$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3}+\frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5}\ldots$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3}+\frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{2b\sqrt{c}\left(b^2-8ac\right)x\sqrt{a+bx^2\ldots}}{35a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}\ldots$$

Mathematica [C] time = 1.59206, size = 572, normalized size = 1.28

$$\frac{ix^7\left(-20a^2c^2+b^3\sqrt{b^2-4ac}+9ab^2c-8abc\sqrt{b^2-4ac}-b^4\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(5*a^4 - 2*b^3*x^8*(b + c*x^2) + a^3*(1
3*b*x^2 + 20*c*x^4) + a*b*x^6*(-b^2 + 17*b*c*x^2 + 16*c^2*x^4) + 3*a^2*(3*b
^2*x^4 + 13*b*c*x^6 + 5*c^2*x^8)) - I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*
c])*x^7*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b
^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*x^7*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(70*a^2*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^7*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.231, size = 495, normalized size = 1.1

$$-\frac{a}{7x^7}\sqrt{cx^4+bx^2+a}-\frac{8b}{35x^5}\sqrt{cx^4+bx^2+a}-\frac{15ac+b^2}{35ax^3}\sqrt{cx^4+bx^2+a}-\frac{2b\left(8ac-b^2\right)}{35a^2x}\sqrt{cx^4+bx^2+a}+\frac{\sqrt{2}}{4}\left(c^2-\frac{c(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^8,x)
```

```
[Out] -1/7*a*(c*x^4+b*x^2+a)^(1/2)/x^7-8/35*b*(c*x^4+b*x^2+a)^(1/2)/x^5-1/35*(15*
a*c+b^2)/a*(c*x^4+b*x^2+a)^(1/2)/x^3-2/35*b*(8*a*c-b^2)/a^2*(c*x^4+b*x^2+a)
^(1/2)/x+1/4*(c^2-1/35*c*(15*a*c+b^2)/a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/35*b*
c*(8*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elliptic
E(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^
2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**8,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**8, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

3.954 $\int \sqrt{3-2x^2-x^4}\,dx$

Optimal. Leaf size=48

$$\frac{4\text{EllipticF}\left(\sin^{-1}(x),-\frac{1}{3}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{-x^4-2x^2+3}x-\frac{2E\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt[3] + (4*E
llipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rubi [A] time = 0.0425514, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}x+\frac{4F\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 2*x^2 - x^4],x]
```


```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt[3] + (4*E
llipticF[ArcSin[x], -1/3])/Sqrt[3]
```


Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-2x^2-x^4}\,dx &= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{1}{3}\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{2}{3}\int \frac{6-2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2}{3}\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx+8\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.0575083, size = 59, normalized size = 1.23

$$\frac{1}{3}\left(-4i\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right),-3\right)+\sqrt{-x^4-2x^2+3}x-2iE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (4*I
)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

Maple [B] time = 0.048, size = 114, normalized size = 2.4

$$\frac{x}{3}\sqrt{-x^4-2x^2+3}+\frac{2\,\mathrm{EllipticF}\left(x,i/3\sqrt{3}\right)}{3}\sqrt{-x^2+1}\sqrt{3x^2+9}\frac{1}{\sqrt{-x^4-2x^2+3}}+\frac{2\,\mathrm{EllipticF}\left(x,i/3\sqrt{3}\right)-2\,\mathrm{EllipticE}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4-2*x^2+3)^(1/2),x)
```

```
[Out] 1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3
)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4
-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^4-2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 - 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-x^4-2x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 - 2x^2 + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 - 2*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 - 2\,x^2 + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 - 2*x^2 + 3), x)
```

3.955 $\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{\left(-16ac+15b^2-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}$$

```
[Out] (x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2 - 16*a*c - 10*b*c*x^2)*Sqrt[
a + b*x^2 + c*x^4])/(48*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rubi [A] time = 0.114397, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 742, 779, 621, 206}

$$\frac{\left(-16ac+15b^2-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2 - 16*a*c - 10*b*c*x^2)*Sqrt[
a + b*x^2 + c*x^4])/(48*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6c}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2-16ac-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+}}\right.}{32c^3}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2-16ac-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{\left(b\left(5b^2-12ac\right)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,d\right.}{16c^3}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2-16ac-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2}}\right.}{32c^{7/2}}$$

Mathematica [A] time = 0.0605068, size = 104, normalized size = 0.86

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(8c\left(cx^4-2a\right)+15b^2-10bcx^2\right)+\left(36abc-15b^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^2 - 10*b*c*x^2 + 8*c*(-2*a + c*x^4
)) + (-15*b^3 + 36*a*b*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4])])/(96*c^(7/2))
```

Maple [A] time = 0.179, size = 162, normalized size = 1.3

$$\frac{x^4}{6\,c}\sqrt{cx^4+bx^2+a}-\frac{5\,bx^2}{24\,c^2}\sqrt{cx^4+bx^2+a}+\frac{5\,b^2}{16\,c^3}\sqrt{cx^4+bx^2+a}-\frac{5\,b^3}{32}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{7}{2}}+\frac{3\,a}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*x^4*(c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+5/16*b
^2/c^3*(c*x^4+b*x^2+a)^(1/2)-5/32*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)
^(1/2))-1/3*a/c^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68011, size = 567, normalized size = 4.69

$$\left[-\frac{3\left(5\,b^{3}-12\,abc\right)\sqrt{c}\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}\left(2\,cx^{2}+b\right)\sqrt{c}-4\,ac\right)-4\left(8\,c^{3}x^{4}-10\,bc^{2}x^{2}+15\,b^{2}c-\right.}{192\,c^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*
sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^4 - 10*
b*c^2*x^2 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(5*b
^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sq
rt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c
- 16*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{7}}{\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.24593, size = 147, normalized size = 1.21

$$\frac{1}{48}\,\sqrt{cx^{4}+bx^{2}+a}\left(2\,x^{2}\left(\frac{4\,x^{2}}{c}-\frac{5\,b}{c^{2}}\right)+\frac{15\,b^{2}c-16\,ac^{2}}{c^{4}}\right)+\frac{\left(5\,b^{3}c-12\,abc^{2}\right)\log\left(\left|-2\left(\sqrt{c}x^{2}-\sqrt{cx^{4}+bx^{2}+a}\right)\sqrt{c}-b\right|\right)}{32\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c - 5*b/c^2) + (15*b^2*c - 16*a*
c^2)/c^4) + 1/32*(5*b^3*c - 12*a*b*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2)
```

3.956 $\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}$$

```
[Out] (-3*b*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[a + b*x^2 + c*x^4])/(4*c
) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Rubi [A] time = 0.0990005, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 742, 640, 621, 206}

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (-3*b*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[a + b*x^2 + c*x^4])/(4*c
) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\operatorname{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4c}\\
&= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16c^2}\\
&= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\left(3b^2-4ac\right)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8c^2}\\
&= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.032765, size = 88, normalized size = 0.85

$$\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\left(2cx^2-3b\right)\sqrt{a+bx^2+cx^4}}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*(-3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (3*b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(5/2))
```

Maple [A] time = 0.172, size = 116, normalized size = 1.1

$$\frac{x^2}{4\,c}\sqrt{cx^4+bx^2+a}-\frac{3\,b}{8\,c^2}\sqrt{cx^4+bx^2+a}+\frac{3\,b^2}{16}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}-\frac{a}{4}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^2*(c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(c*x^4+b*x^2+a)^(1/2)/c^2+3/16*b^2/c^
(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*a/c^(3/2)*ln((1/2
*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60532, size = 475, normalized size = 4.57

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2-3\,bc\right)}{32\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c
*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c^2*x^2 - 3*b*c))/c^3, -1/16*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt
(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*s
qrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 - 3*b*c))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.2738, size = 111, normalized size = 1.07

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(\frac{2\,x^2}{c}-\frac{3\,b}{c^2}\right)-\frac{\left(3\,b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2/c - 3*b/c^2) - 1/16*(3*b^2 - 4*a*c)*log(
abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2)
```

3.957 $\int \frac{x^3}{\sqrt{a+bx^2+cx^4}} \, dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rubi [A] time = 0.0539312, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1114, 640, 621, 206}

$$\frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4c}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2c}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0140484, size = 68, normalized size = 1.

$$\frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 + c*x^4],x]

[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])])/(4*c^(3/2))
```

Maple [A] time = 0.172, size = 56, normalized size = 0.8

$$\frac{1}{2c}\sqrt{cx^4+bx^2+a} - \frac{b}{4}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}} + \sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)/c-1/4*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62224, size = 383, normalized size = 5.63

$$\left[\frac{b\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+4\,\sqrt{cx^4+bx^2+a}\,c}{8\,c^2},\ \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2}{2\left(c^2x^4+bc\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*c)/c^2, 1/4*(b
*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^
4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*c)/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.19704, size = 82, normalized size = 1.21

$$\frac{b\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3
/2) + 1/2*sqrt(c*x^4 + b*x^2 + a)/c
```

3.958 $\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c])
```

Rubi [A] time = 0.031136, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1107, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\ &= \operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0052868, size = 43, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c])
```

Maple [A] time = 0.165, size = 35, normalized size = 0.8

$$\frac{1}{2}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61098, size = 285, normalized size = 6.63

$$\left[\frac{\log\left(-8\,c^2x^4 - 8\,bcx^2 - b^2 - 4\,\sqrt{cx^4 + bx^2 + a}\left(2\,cx^2 + b\right)\sqrt{c} - 4\,ac\right)}{4\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{-c}}{2\left(c^2x^4+bcx^2+ac\right)}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
+ b)*sqrt(c) - 4*a*c)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.19675, size = 54, normalized size = 1.26

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/sqrt(
c)
```

3.959 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rubi [A] time = 0.0409892, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1114, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.0101086, size = 44, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Maple [A] time = 0.169, size = 39, normalized size = 0.9

$$-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63345, size = 294, normalized size = 6.68

$$\left[\frac{\log\left(-\frac{\left(b^{2}+4\,ac\right)x^{4}+8\,abx^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}\left(bx^{2}+2\,a\right)\sqrt{a}+8\,a^{2}}{x^{4}}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\left(bx^{2}+2\,a\right)\sqrt{-a}}{2\left(acx^{4}+abx^{2}+a^{2}\right)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + bx^{2} + cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x), x)
```

3.960 $\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=72

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rubi [A] time = 0.0611296, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1114, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```


```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4a}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2a}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0232132, size = 72, normalized size = 1.

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4]])])/(4*a^(3/2))
```

Maple [A] time = 0.166, size = 63, normalized size = 0.9

$$-\frac{1}{2\,a{x}^{2}}\sqrt{c{x}^{4}+b{x}^{2}+a}+{\frac{b}{4}\ln \left({\frac{1}{{x}^{2}}} \left(2\,a+b{x}^{2}+2\,\sqrt{a}\sqrt{c{x}^{4}+b{x}^{2}+a} \right) } \right) {a}^{-{\frac{3}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*b/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x
^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70678, size = 423, normalized size = 5.88

$$\left[\frac{\sqrt{a} b x^{2} \log\left(-\frac{{\left(b^{2} + 4 \, a c\right)} x^{4} + 8 \, a b x^{2} + 4 \, \sqrt{c x^{4} + b x^{2} + a} {\left(b x^{2} + 2 \, a\right)} \sqrt{a} + 8 \, a^{2}}{x^{4}}\right) - 4 \, \sqrt{c x^{4} + b x^{2} + a} a}{8 \, a^{2} x^{2}}, -\frac{\sqrt{-a} b x^{2} \arctan\left(\frac{\sqrt{c x^{4} + b x^{2} + a} {\left(b x^{2} + 2 \, a\right)} \sqrt{-a}}{2 \, {\left(a c x^{4} + a b x^{2} + a^{2}\right)}}\right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a)
/(a^2*x^2), -1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*a)/(
a^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^3), x)
```

3.961 $\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x
^2) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*
x^4])])/(16*a^(5/2))
```

Rubi [A] time = 0.1054, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 744, 806, 724, 206}

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x
^2) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*
x^4])])/(16*a^(5/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
```

```
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4} - \frac{\operatorname{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4a}$$
$$= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2} + \frac{(3b^2-4ac)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16a^2}$$
$$= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{(3b^2-4ac)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8a^2}$$
$$= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}$$

Mathematica [A] time = 0.0494393, size = 91, normalized size = 0.84

$$\frac{(4ac-3b^2)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}} + \frac{(3bx^2-2a)\sqrt{a+bx^2+cx^4}}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*a + 3*b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((-3*b^2 + 4*a*c)*
ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Maple [A] time = 0.168, size = 127, normalized size = 1.2

$$-\frac{1}{4\,ax^4}\sqrt{cx^4+bx^2+a} + \frac{3\,b}{8\,a^2x^2}\sqrt{cx^4+bx^2+a} - \frac{3\,b^2}{16}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}} + \frac{c}{4}\ln\left(\frac{1}{x^2}\left(2\,a+bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^2-3/16*b
^2/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/4*c/a^(3/2
```

```
)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73873, size = 512, normalized size = 4.74

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{a}x^4\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}\left(3\,abx^2-2\,a^2\right)}{32\,a^3x^4}, \frac{\left(3\,b^2-4\,ac\right)\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 - 4*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4
*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4
 + b*x^2 + a)*(3*a*b*x^2 - 2*a^2))/(a^3*x^4), 1/16*((3*b^2 - 4*a*c)*sqrt(-a
)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(3*a*b*x^2 - 2*a^2))/(a^3*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^5), x)
```

3.962 $\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[a + b*x^2 + c*x^4])/(24*a^2*
x^4) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^2 + c*x^4])/(48*a^3*x^2) + (b*(5*b^2
 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*
a^(7/2))
```

Rubi [A] time = 0.165968, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 744, 834, 806, 724, 206}

$$-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[a + b*x^2 + c*x^4])/(24*a^2*
x^4) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^2 + c*x^4])/(48*a^3*x^2) + (b*(5*b^2
 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*
a^(7/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} - \frac{\operatorname{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{6a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{4}(15b^2-16ac)+\frac{5bcx}{2}}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{12a^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2} - \frac{(b(5b^2-12ac))\,\mathrm{Su}}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{(b(5b^2-12ac))\,\mathrm{S}}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{b(5b^2-12ac)\tan}{32}$$

Mathematica [A] time = 0.0800423, size = 112, normalized size = 0.77

$$\frac{\sqrt{a+bx^2+cx^4}\left(-8a^2+2a\left(5bx^2+8cx^4\right)-15b^2x^4\right)}{48a^3x^6} + \frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 - 15*b^2*x^4 + 2*a*(5*b*x^2 + 8*c*x^4)))/(
48*a^3*x^6) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
 b*x^2 + c*x^4])])/(32*a^(7/2))
```

Maple [A] time = 0.172, size = 176, normalized size = 1.2

$$-\frac{1}{6\,ax^6}\sqrt{cx^4+bx^2+a} + \frac{5\,b}{24\,a^2x^4}\sqrt{cx^4+bx^2+a} - \frac{5\,b^2}{16\,x^2a^3}\sqrt{cx^4+bx^2+a} + \frac{5\,b^3}{32}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-5/16*
b^2/a^3/x^2*(c*x^4+b*x^2+a)^(1/2)+5/32*b^3/a^(7/2)*ln((2*a+b*x^2+2*a^(1/2)*
(c*x^4+b*x^2+a)^(1/2))/x^2)-3/8*b/a^(5/2)*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+
b*x^2+a)^(1/2))/x^2)+1/3*c/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83151, size = 612, normalized size = 4.22

$$\left[-\frac{3\left(5b^3-12abc\right)\sqrt{a}x^6\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)-4\left(10a^2bx^2-\left(15ab^2-16a^2c\right)x^4-8a^3\right)\sqrt{cx^4}}{192a^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(10*
a^2*b*x^2 - (15*a*b^2 - 16*a^2*c)*x^4 - 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^
4*x^6), -1/96*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(10*a^2*b*x^
2 - (15*a*b^2 - 16*a^2*c)*x^4 - 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^7), x)
```

3.963 $\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=313

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{2\sqrt[4]{a}b\left(\sqrt{a}\right.}{}$$

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (2*b*x*Sqrt[a + b*x^2 + c*x^4])/(3*c^(3
/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (2*a^(1/4)*b*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]
) - (a^(1/4)*(2*b + Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.113461, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1122, 1197, 1103, 1195}

$$-\frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (2*b*x*Sqrt[a + b*x^2 + c*x^4])/(3*c^(3
/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (2*a^(1/4)*b*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]
) - (a^(1/4)*(2*b + Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
```

```
x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{x\sqrt{a+bx^2+cx^4}}{3c} - \frac{\int \frac{a+2bx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c}$$

$$= \frac{x\sqrt{a+bx^2+cx^4}}{3c} + \frac{\left(2\sqrt{a}b\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}} - \frac{\left(\sqrt{a}\left(2b+\sqrt{a}\sqrt{c}\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}}$$

$$= \frac{x\sqrt{a+bx^2+cx^4}}{3c} - \frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{2\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\right.\right.}{3c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.861165, size = 444, normalized size = 1.42

$$\frac{i\left(b\sqrt{b^2-4ac}+ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)+2cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\ldots}{6c^2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*b*(-b + Sqrt
[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ell
ipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c])*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 +
 c*x^4])
```

Maple [A] time = 0.212, size = 388, normalized size = 1.2

$$\frac{x}{3c}\sqrt{cx^4+bx^2+a}-\frac{a\sqrt{2}}{12c}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12*a/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*a
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
```

```
(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

3.964 $\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=267

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3
/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.0689026, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1139, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3
/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}} - \frac{\sqrt{a}\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{a}+\right.}{}$$

Mathematica [C] time = 0.130685, size = 278, normalized size = 1.04

$$\frac{i\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\right.\right.\right.}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE
```

```
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqr
t[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.217, size = 216, normalized size = 0.8

$$-\frac{a\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.965 $\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=114

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.0138488, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1103}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.1, size = 186, normalized size = 1.63

$$-\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.217, size = 144, normalized size = 1.3

$$\frac{\sqrt{2}}{4}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b(b}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(
-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

3.966 $\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=294

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{a^{3/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(
Sqrt[a] + Sqrt[c]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
2*a^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.11646, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1123, 12, 1139, 1103, 1195}

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{a^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(
Sqrt[a] + Sqrt[c]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
2*a^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
```

```
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{ax} + \frac{\int \frac{cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{ax} + \frac{c\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{ax} + \frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}} - \frac{\sqrt{c}\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{ax} + \frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{a^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.489957, size = 298, normalized size = 1.01

$$\frac{-\frac{4\left(a+bx^2+cx^4\right)}{x} + \frac{i\sqrt{2}\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2}}{b-\sqrt{}}\right.\right.}{\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}}{4a\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-4*(a + b*x^2 + c*x^4))/x + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a
+ b*x^2 + c*x^4])
```

Maple [A] time = 0.226, size = 239, normalized size = 0.8

$$-\frac{1}{ax}\sqrt{cx^4+bx^2+a} - \frac{c\sqrt{2}}{2}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/a/x-1/2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}}{cx^6 + bx^4 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c*x^6 + b*x^4 + a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

3.967 $\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=345

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{2b\sqrt{c}x\sqrt{a+}}{3a^2\left(\sqrt{a}\right.}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*x
) - (2*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(Sqrt[a] + Sqrt[c]*x^2))
 + (2*b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(3*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - ((2*b + Sqrt[a]*Sqrt[c])
*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(6*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.156436, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1123, 1281, 1197, 1103, 1195}

$$\frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{2b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*x
) - (2*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(Sqrt[a] + Sqrt[c]*x^2))
 + (2*b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(3*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - ((2*b + Sqrt[a]*Sqrt[c])
*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(6*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{\int \frac{-2b-cx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{\int \frac{ac+2bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x} + \frac{\left(2b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^{3/2}} - \frac{\left(\left(2b+\sqrt{a}\sqrt{c}\right)\sqrt{c}\right)\int \frac{1}{\sqrt{a+bx}}}{3a^{3/2}}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{2b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{2b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{}}}}{3a^{7/4}}$$

Mathematica [C] time = 0.892117, size = 459, normalized size = 1.33

$$\frac{ix^3\left(b\sqrt{b^2-4ac}+ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)-2\sqrt{\frac{}{\sqrt{b}}}}{6a^2x^3\sqrt{\frac{}{\sqrt{b}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a - 2*b*x^2)*(a + b*x^2 + c*x^4) - I*b
*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
 (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + a*c + b*Sqrt[
b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(6*a^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.22, size = 413, normalized size = 1.2

$$-\frac{1}{3\,a x^{3}}\sqrt{c x^{4}+b x^{2}+a}+\frac{2\,b}{3\,a^{2} x}\sqrt{c x^{4}+b x^{2}+a}-\frac{c\sqrt{2}}{12\,a}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^{2}}\right)x^{2}}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^{2}}\right)x^{2}}{a}}\mathrm{Ellip}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x-1/12*c/a
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c x^{4}+b x^{2}+a x^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c x^{4}+b x^{2}+a}}{c x^{8}+b x^{6}+a x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c*x^8 + b*x^6 + a*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

3.968 $\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=124

$$-\frac{\left(16ac+15b^2+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3}-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}$$

```
[Out] -(x^4*Sqrt[a + b*x^2 - c*x^4])/(6*c) - ((15*b^2 + 16*a*c + 10*b*c*x^2)*Sqrt
[a + b*x^2 - c*x^4])/(48*c^3) - (b*(5*b^2 + 12*a*c)*ArcTan[(b - 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(32*c^(7/2))
```

Rubi [A] time = 0.113001, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1114, 742, 779, 621, 204}

$$-\frac{\left(16ac+15b^2+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3}-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(x^4*Sqrt[a + b*x^2 - c*x^4])/(6*c) - ((15*b^2 + 16*a*c + 10*b*c*x^2)*Sqrt
[a + b*x^2 - c*x^4])/(48*c^3) - (b*(5*b^2 + 12*a*c)*ArcTan[(b - 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(32*c^(7/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\left(15b^2+16ac+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3} + \frac{\left(b\left(5b^2+12ac\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\cdots}}\right.}{32c^3}$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\left(15b^2+16ac+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3} + \frac{\left(b\left(5b^2+12ac\right)\right)\text{Subst}\left(\int \frac{1}{-4c-x\cdots}\right.}{16c^3}$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\left(15b^2+16ac+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3} - \frac{b\left(5b^2+12ac\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2\cdots}}\right.}{32c^{7/2}}$$

Mathematica [A] time = 0.075698, size = 107, normalized size = 0.86

$$\frac{-2\sqrt{c}\sqrt{a+bx^2-cx^4}\left(8c\left(2a+cx^4\right)+15b^2+10bcx^2\right)-3b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]*(15*b^2 + 10*b*c*x^2 + 8*c*(2*a + c*x^4
)) - 3*b*(5*b^2 + 12*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 -
c*x^4])])/(96*c^(7/2))
```

Maple [A] time = 0.17, size = 168, normalized size = 1.4

$$-\frac{x^4}{6\,c}\sqrt{-cx^4+bx^2+a}-\frac{5\,bx^2}{24\,c^2}\sqrt{-cx^4+bx^2+a}-\frac{5\,b^2}{16\,c^3}\sqrt{-cx^4+bx^2+a}+\frac{5\,b^3}{32}\arctan\left(\sqrt{c}\left(x^2-\frac{b}{2\,c}\right)\frac{1}{\sqrt{-cx^4+bx^2+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*x^4*(-c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(-c*x^4+b*x^2+a)^(1/2)-5/1
6*b^2/c^3*(-c*x^4+b*x^2+a)^(1/2)+5/32*b^3/c^(7/2)*arctan(c^(1/2)*(x^2-1/2*b
/c)/(-c*x^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*arctan(c^(1/2)*(x^2-1/2*b/c)/(-
c*x^4+b*x^2+a)^(1/2))-1/3*a/c^2*(-c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59295, size = 572, normalized size = 4.61

$$\left[-\frac{3\left(5\,b^{3}+12\,abc\right)\sqrt{-c}\log\left(8\,c^{2}x^{4}-8\,bcx^{2}+b^{2}-4\,\sqrt{-cx^{4}+bx^{2}+a}\left(2\,cx^{2}-b\right)\sqrt{-c}-4\,ac\right)+4\left(8\,c^{3}x^{4}+10\,bc^{2}x^{2}+15\,b\right.}{192\,c^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 + 12*a*b*c)*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*
sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*(8*c^3*x^4 + 1
0*b*c^2*x^2 + 15*b^2*c + 16*a*c^2)*sqrt(-c*x^4 + b*x^2 + a))/c^4, -1/96*(3*
(5*b^3 + 12*a*b*c)*sqrt(c)*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b
)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c)) + 2*(8*c^3*x^4 + 10*b*c^2*x^2 + 15*b^2
*c + 16*a*c^2)*sqrt(-c*x^4 + b*x^2 + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{7}}{\sqrt{a+bx^{2}-cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [A] time = 1.31004, size = 159, normalized size = 1.28

$$-\frac{1}{48}\sqrt{-cx^{4}+bx^{2}+a}\left(2\,x^{2}\left(\frac{4\,x^{2}}{c}+\frac{5\,b}{c^{2}}\right)+\frac{15\,b^{2}c+16\,ac^{2}}{c^{4}}\right)-\frac{\left(5\,b^{3}c+12\,abc^{2}\right)\log\left(\left|2\left(\sqrt{-c}x^{2}-\sqrt{-cx^{4}+bx^{2}+a}\right)\sqrt{-c}+\right.\right.}{32\,\sqrt{-c}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c + 5*b/c^2) + (15*b^2*c + 16*
a*c^2)/c^4) - 1/32*(5*b^3*c + 12*a*b*c^2)*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c
*x^4 + b*x^2 + a))*sqrt(-c) + b))/(sqrt(-c)*c^4)
```

3.969 $\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=107

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}$$

```
[Out] (-3*b*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) - (x^2*Sqrt[a + b*x^2 - c*x^4])/(4*c
) - ((3*b^2 + 4*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4
])])/(16*c^(5/2))
```

Rubi [A] time = 0.0927536, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1114, 742, 640, 621, 204}

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```


```
[Out] (-3*b*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) - (x^2*Sqrt[a + b*x^2 - c*x^4])/(4*c
) - ((3*b^2 + 4*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4
])])/(16*c^(5/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}-\frac{\operatorname{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)}{4c}\\
&= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}+\frac{\left(3b^2+4ac\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx,x,x^2\right)}{16c^2}\\
&= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}+\frac{\left(3b^2+4ac\right)\operatorname{Subst}\left(\int \frac{1}{-4c-x^2}\,dx,x,\frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)}{8c^2}\\
&= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}-\frac{\left(3b^2+4ac\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0425406, size = 89, normalized size = 0.83

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{(3b+2cx^2)\sqrt{a+bx^2-cx^4}}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -((3*b + 2*c*x^2)*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) - ((3*b^2 + 4*a*c)*ArcTa
n[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(16*c^(5/2))
```

Maple [A] time = 0.168, size = 120, normalized size = 1.1

$$-\frac{x^2}{4c}\sqrt{-cx^4+bx^2+a}-\frac{3b}{8c^2}\sqrt{-cx^4+bx^2+a}+\frac{3b^2}{16}\arctan\left(\sqrt{c}\left(x^2-\frac{b}{2c}\right)\frac{1}{\sqrt{-cx^4+bx^2+a}}\right)c^{-\frac{5}{2}}+\frac{a}{4}\arctan\left(\sqrt{c}\left(x^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*x^2*(-c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(-c*x^4+b*x^2+a)^(1/2)/c^2+3/16*b^2
/c^(5/2)*arctan(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2))+1/4*a/c^(3/2)
*arctan(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60464, size = 479, normalized size = 4.48

$$\left[-\frac{\left(3\,b^2+4\,ac\right)\sqrt{-c}\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{-c}-4\,ac\right)+4\,\sqrt{-cx^4+bx^2+a}\left(2\,c^2x^2+\right.}{32\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 + 4*a*c)*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-
c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*sqrt(-c*x^4 + b*x^2
+ a)*(2*c^2*x^2 + 3*b*c))/c^3, -1/16*((3*b^2 + 4*a*c)*sqrt(c)*arctan(1/2*sq
rt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c)) + 2
*sqrt(-c*x^4 + b*x^2 + a)*(2*c^2*x^2 + 3*b*c))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [A] time = 1.28701, size = 123, normalized size = 1.15

$$-\frac{1}{8}\,\sqrt{-cx^4+bx^2+a}\left(\frac{2\,x^2}{c}+\frac{3\,b}{c^2}\right)-\frac{\left(3\,b^2+4\,ac\right)\log\left(\left|2\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{16\,\sqrt{-c}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2/c + 3*b/c^2) - 1/16*(3*b^2 + 4*a*c)*lo
g(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/(sqrt(-c)*
```

```
c^2)
```

3.970 $\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}$$

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(2*c) - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 - c*x^4])])/(4*c^(3/2))
```

Rubi [A] time = 0.0581514, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1114, 640, 621, 204}

$$-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(2*c) - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 - c*x^4])])/(4*c^(3/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2-cx^4}}{2c} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)}{4c}\\
&= -\frac{\sqrt{a+bx^2-cx^4}}{2c} + \frac{b\,\text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx, x, \frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)}{2c}\\
&= -\frac{\sqrt{a+bx^2-cx^4}}{2c} - \frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0160885, size = 70, normalized size = 1.

$$
-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}} - \frac{\sqrt{a+bx^2-cx^4}}{2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(2*c) - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 - c*x^4])])/(4*c^(3/2))
```

Maple [A] time = 0.161, size = 58, normalized size = 0.8

$$
-\frac{1}{2\,c}\sqrt{-cx^4+bx^2+a} + \frac{b}{4}\arctan\left(\sqrt{c}\left(x^2-\frac{b}{2\,c}\right)\frac{1}{\sqrt{-cx^4+bx^2+a}}\right)c^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(-c*x^4+b*x^2+a)^(1/2)/c+1/4*b/c^(3/2)*arctan(c^(1/2)*(x^2-1/2*b/c)/(-
c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56805, size = 390, normalized size = 5.57

$$\left[-\frac{b\sqrt{-c}\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{-c}-4\,ac\right)+4\,\sqrt{-cx^4+bx^2+a}\,c}{8\,c^2},-\frac{b\sqrt{c}\arctan\left(\frac{\sqrt{-c}}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(b*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-c*x^4 + b*x^2 +
 a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*sqrt(-c*x^4 + b*x^2 + a)*c)/c^2, -1
/4*(b*sqrt(c)*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(c)/(c^
2*x^4 - b*c*x^2 - a*c)) + 2*sqrt(-c*x^4 + b*x^2 + a)*c)/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [A] time = 1.24795, size = 95, normalized size = 1.36

$$-\frac{b\log\left(\left|2\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{4\sqrt{-c}c}-\frac{\sqrt{-cx^4+bx^2+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/(
sqrt(-c)*c) - 1/2*sqrt(-c*x^4 + b*x^2 + a)/c
```

3.971 $\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] -ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/(2*Sqrt[c])
```

Rubi [A] time = 0.0382176, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1107, 621, 204}

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/(2*Sqrt[c])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx, x, \frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.0056679, size = 44, normalized size = 1.

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/(2*Sqrt[c])
```

Maple [A] time = 0.158, size = 36, normalized size = 0.8

$$\frac{1}{2}\arctan\left(\sqrt{c}\left(x^2-\frac{b}{2\,c}\right)\frac{1}{\sqrt{-cx^4+bx^2+a}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/c^(1/2)*arctan(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53858, size = 288, normalized size = 6.55

$$\left[-\frac{\sqrt{-c}\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{-c}-4\,ac\right)}{4\,c},-\frac{\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}\left(2\,cx^2-b\right)\sqrt{c}}{2\left(c^2x^4-bcx^2-ac\right)}\right)}{2\,\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-c*x^4 + b*x^2 + a)
*(2*c*x^2 - b)*sqrt(-c) - 4*a*c)/c, -1/2*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a
)*(2*c*x^2 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c))/sqrt(c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [A] time = 1.2244, size = 61, normalized size = 1.39

$$-\frac{\log\left(\left|2\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{2\,\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/sqr
t(-c)
```

3.972 $\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rubi [A] time = 0.042422, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1114, 724, 204}

$$-\frac{\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx, x, \frac{-2a+bx^2}{\sqrt{-a+bx^2+cx^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{-2a+bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.0142757, size = 46, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Maple [A] time = 0.165, size = 45, normalized size = 1.

$$-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(-2\,a+bx^2+2\,\sqrt{-a}\sqrt{cx^4+bx^2-a}\right)\right)\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] -1/2/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62648, size = 294, normalized size = 6.26

$$\left[-\frac{\sqrt{-a}\log\left(\frac{(b^2-4\,ac)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}(bx^2-2\,a)\sqrt{-a}+8\,a^2}{x^4}\right)}{4\,a}, \frac{\arctan\left(\frac{\sqrt{cx^4+bx^2-a}(bx^2-2\,a)\sqrt{a}}{2\,(acx^4+abx^2-a^2)}\right)}{2\,\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a)*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 -
a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4)/a, 1/2*arctan(1/2*sqrt(c*x^4 + b*x^
2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2))/sqrt(a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.33649, size = 59, normalized size = 1.26

$$\frac{\log\left(\left|-2\sqrt{-a}\left(\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}-\frac{\sqrt{-a}}{x^2}\right)+b\right|\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*sqrt(-a)*(sqrt(c + b/x^2 - a/x^4) - sqrt(-a)/x^2) + b))/sqrt
(-a)
```

3.973 $\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) - (b*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqr
t[-a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rubi [A] time = 0.0619014, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1114, 730, 724, 204}

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) - (b*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqr
t[-a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)}{4a}\\ &= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx,x,\frac{-2a+bx^2}{\sqrt{-a+bx^2+cx^4}}\right)}{2a}\\ &= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0229716, size = 76, normalized size = 0.99

$$\frac{b\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}+\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) + (b*ArcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[-a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Maple [A] time = 0.169, size = 74, normalized size = 1.

$$\frac{1}{2\,a x^2}\sqrt{c x^4+b x^2-a}-\frac{b}{4\,a}\ln\left(\frac{1}{x^2}\left(-2\,a+b x^2+2\,\sqrt{-a}\sqrt{c x^4+b x^2-a}\right)\right)\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2-a)^(1/2)/a/x^2-1/4*b/a/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1
/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64411, size = 421, normalized size = 5.47

$$\left[-\frac{\sqrt{-a} b x^2 \log\left(\frac{\left(b^2-4\,ac\right)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}\left(bx^2-2\,a\right)\sqrt{-a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2-a}\,a}{8\,a^2x^2}, \frac{\sqrt{a} b x^2 \arctan\left(\frac{\sqrt{cx^4+bx^2-a}\left(bx^2-2\,a\right)\sqrt{a}}{2\left(acx^4+abx^2-a^2\right)}\right)+}{4\,a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b*x^2*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4*sqrt(c*x^4 + b
*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 - a)*
a)/(a^2*x^2), 1/4*(sqrt(a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x^2
- 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2)) + 2*sqrt(c*x^4 + b*x^2 - a)*a)/(a
^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.33424, size = 93, normalized size = 1.21

$$\frac{b\log\left(\left|-2\sqrt{-a}\left(\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}-\frac{\sqrt{-a}}{x^2}\right)+b\right|\right)}{4\sqrt{-a}a}+\frac{\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*sqrt(-a)*(sqrt(c + b/x^2 - a/x^4) - sqrt(-a)/x^2) + b))/(s
qrt(-a)*a) + 1/2*sqrt(c + b/x^2 - a/x^4)/a
```

3.974 $\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=115

$$-\frac{\left(4ac+3b^2\right)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}$$

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*
x^2) - ((3*b^2 + 4*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c
*x^4])])/(16*a^(5/2))
```

Rubi [A] time = 0.123475, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1114, 744, 806, 724, 204}

$$-\frac{\left(4ac+3b^2\right)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*
x^2) - ((3*b^2 + 4*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c
*x^4])])/(16*a^(5/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
```

```
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)$$
$$= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4} + \frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)}{4a}$$
$$= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2} + \frac{(3b^2+4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)}{16a^2}$$
$$= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2} - \frac{(3b^2+4ac)\,\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx,x,\frac{-2a+bx^2}{\sqrt{-a+bx^2+cx^4}}\right)}{8a^2}$$
$$= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4} + \frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2} - \frac{(3b^2+4ac)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}$$

Mathematica [A] time = 0.0462942, size = 95, normalized size = 0.83

$$\frac{(4ac+3b^2)\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}} + \frac{(2a+3bx^2)\sqrt{-a+bx^2+cx^4}}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((2*a + 3*b*x^2)*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((3*b^2 + 4*a*c)*A
rcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Maple [A] time = 0.17, size = 149, normalized size = 1.3

$$\frac{1}{4\,ax^4}\sqrt{cx^4+bx^2-a} + \frac{3\,b}{8\,a^2x^2}\sqrt{cx^4+bx^2-a} - \frac{3\,b^2}{16\,a^2}\ln\left(\frac{1}{x^2}\left(-2\,a+bx^2+2\,\sqrt{-a}\sqrt{cx^4+bx^2-a}\right)\right)\frac{1}{\sqrt{-a}} - \frac{c}{4\,a}\ln\left(\frac{1}{x^2}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/4*(c*x^4+b*x^2-a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^2-3/16*b^
2/a^2/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)-1/
```

```
4*c/a/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71857, size = 510, normalized size = 4.43

$$\left[-\frac{\left(3\,b^2+4\,ac\right)\sqrt{-a}x^4\log\left(\frac{\left(b^2-4\,ac\right)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}\left(bx^2-2\,a\right)\sqrt{-a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2-a}\left(3\,abx^2+2\,a^2\right)}{32\,a^3x^4}, \frac{\left(3\,b^2+4\,ac\right)\sqrt{a}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")

[Out] [-1/32*((3*b^2 + 4*a*c)*sqrt(-a)*x^4*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4
*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*sqrt(c*x^
4 + b*x^2 - a)*(3*a*b*x^2 + 2*a^2))/(a^3*x^4), 1/16*((3*b^2 + 4*a*c)*sqrt(a
)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a
*b*x^2 - a^2)) + 2*sqrt(c*x^4 + b*x^2 - a)*(3*a*b*x^2 + 2*a^2))/(a^3*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.84369, size = 122, normalized size = 1.06

$$\frac{1}{8}\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}\left(\frac{3b}{a^2}+\frac{2}{ax^2}\right)+\frac{\left(3b^2+4ac\right)\log\left(\left|-2\sqrt{-a}\left(\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}-\frac{\sqrt{-a}}{x^2}\right)+b\right|\right)}{16\sqrt{-a}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c + b/x^2 - a/x^4)*(3*b/a^2 + 2/(a*x^2)) + 1/16*(3*b^2 + 4*a*c)*lo
g(abs(-2*sqrt(-a)*(sqrt(c + b/x^2 - a/x^4) - sqrt(-a)/x^2) + b))/(sqrt(-a)*
a^2)
```

3.975 $\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=154

$$\frac{(16ac+15b^2)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}-\frac{b(12ac+5b^2)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}$$

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[-a + b*x^2 + c*x^4])/(24*a^2
*x^4) + ((15*b^2 + 16*a*c)*Sqrt[-a + b*x^2 + c*x^4])/(48*a^3*x^2) - (b*(5*b
^2 + 12*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(3
2*a^(7/2))
```

Rubi [A] time = 0.171621, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1114, 744, 834, 806, 724, 204}

$$\frac{(16ac+15b^2)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}-\frac{b(12ac+5b^2)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```



```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[-a + b*x^2 + c*x^4])/(24*a^2
*x^4) + ((15*b^2 + 16*a*c)*Sqrt[-a + b*x^2 + c*x^4])/(48*a^3*x^2) - (b*(5*b
^2 + 12*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(3
2*a^(7/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{\text{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)}{6a}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(15b^2+16ac)+\frac{5bcx}{2}}{x^2\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)}{12a^2}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{(15b^2+16ac)\sqrt{-a+bx^2+cx^4}}{48a^3x^2} + \frac{(b(5b^2+12ac)}{}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{(15b^2+16ac)\sqrt{-a+bx^2+cx^4}}{48a^3x^2} - \frac{(b(5b^2+12ac)}{}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{(15b^2+16ac)\sqrt{-a+bx^2+cx^4}}{48a^3x^2} - \frac{b(5b^2+12ac)\,t}{}$$

Mathematica [A] time = 0.0767074, size = 116, normalized size = 0.75

$$\frac{\sqrt{-a+bx^2+cx^4}\left(8a^2+2a\left(5bx^2+8cx^4\right)+15b^2x^4\right)}{48a^3x^6} + \frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{bx^2-2a}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-a + b*x^2 + c*x^4]*(8*a^2 + 15*b^2*x^4 + 2*a*(5*b*x^2 + 8*c*x^4)))/(
48*a^3*x^6) + (b*(5*b^2 + 12*a*c)*ArcTan[(-2*a + b*x^2)/(2*Sqrt[a]*Sqrt[-a
+ b*x^2 + c*x^4])])/(32*a^(7/2))
```

Maple [A] time = 0.17, size = 202, normalized size = 1.3

$$\frac{1}{6\,ax^6}\sqrt{cx^4+bx^2-a} + \frac{5\,b}{24\,a^2x^4}\sqrt{cx^4+bx^2-a} + \frac{5\,b^2}{16\,x^2a^3}\sqrt{cx^4+bx^2-a} - \frac{5\,b^3}{32\,a^3}\ln\left(\frac{1}{x^2}\left(-2\,a+bx^2+2\,\sqrt{-a}\sqrt{cx^4+bx^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/6*(c*x^4+b*x^2-a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^4+5/16*b
^2/a^3/x^2*(c*x^4+b*x^2-a)^(1/2)-5/32*b^3/a^3/(-a)^(1/2)*ln((-2*a+b*x^2+2*(
-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)-3/8*b/a^2*c/(-a)^(1/2)*ln((-2*a+b*x^2
+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)+1/3*c/a^2/x^2*(c*x^4+b*x^2-a)^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87892, size = 609, normalized size = 3.95

$$\left[-\frac{3\left(5\,b^{3}+12\,abc\right)\sqrt{-a}x^{6}\log\left(\frac{\left(b^{2}-4\,ac\right)x^{4}-8\,abx^{2}-4\,\sqrt{cx^{4}+bx^{2}-a}\left(bx^{2}-2\,a\right)\sqrt{-a}+8\,a^{2}}{x^{4}}\right)-4\left(10\,a^{2}bx^{2}+\left(15\,ab^{2}+16\,a^{2}c\right)x^{4}+8\,a^{3}\right)\sqrt{}}{192\,a^{4}x^{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 + 12*a*b*c)*sqrt(-a)*x^6*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*(10
*a^2*b*x^2 + (15*a*b^2 + 16*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 - a))/(a
^4*x^6), 1/96*(3*(5*b^3 + 12*a*b*c)*sqrt(a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x
^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2)) + 2*(10*a^2*b*x^2
+ (15*a*b^2 + 16*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 - a))/(a^4*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.79854, size = 158, normalized size = 1.03

$$\frac{1}{48}\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}\left(\frac{2\left(\frac{5\,b}{a^2}+\frac{4}{ax^2}\right)}{x^2}+\frac{15\,ab^2+16\,a^2c}{a^4}\right)+\frac{\left(5\,ab^3+12\,a^2bc\right)\log\left(\left|-2\,\sqrt{-a}\left(\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}-\frac{\sqrt{-a}}{x^2}\right)+b\right|\right)}{32\,\sqrt{-a}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c + b/x^2 - a/x^4)*(2*(5*b/a^2 + 4/(a*x^2))/x^2 + (15*a*b^2 + 16*
a^2*c)/a^4) + 1/32*(5*a*b^3 + 12*a^2*b*c)*log(abs(-2*sqrt(-a)*(sqrt(c + b/x
^2 - a/x^4) - sqrt(-a)/x^2) + b))/(sqrt(-a)*a^4)
```

3.976 $\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=409

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}} - \cdots$$

```
[Out] -(x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - (b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sq
rt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*
Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^
2 + a*c - b*Sqrt[b^2 + 4*a*c])*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*
Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 +
 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.587549, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1122, 1202, 524, 424, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}} - \frac{b\left(b-\sqrt{4\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2 - c*x^4],x]
```


```
[Out] -(x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - (b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sq
rt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*
Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^
2 + a*c - b*Sqrt[b^2 + 4*a*c])*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*
Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 +
 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{x\sqrt{a+bx^2-cx^4}}{3c} + \frac{\int \frac{a+2bx^2}{\sqrt{a+bx^2-cx^4}}\,dx}{3c}$$

$$= -\frac{x\sqrt{a+bx^2-cx^4}}{3c} + \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{a+2bx^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{3c\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(b\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx \quad (}{3c^2\sqrt{a+bx^2-cx^4}} + -$$

$$= -\frac{x\sqrt{a+bx^2-cx^4}}{3c} - \frac{b\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}E\left(\sin^{-1}\right.}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.773824, size = 459, normalized size = 1.12

$$\frac{i\sqrt{2}\left(b\sqrt{4ac+b^2}-ac-b^2\right)\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)+2cx\sqrt{}}{6c^2\sqrt{-\frac{}{\sqrt{4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x*(-a - b*x^2 + c*x^4) - I*Sqrt[2]*
b*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt
[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*
a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (
b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-b^2 - a*c + b
*Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 +
4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqr
t[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*c^2*Sqrt[-(c/(b + Sqrt[b^2 + 4
*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.281, size = 391, normalized size = 1.

$$-\frac{x}{3c}\sqrt{-cx^4+bx^2+a}+\frac{a\sqrt{2}}{12c}\sqrt{4-2\frac{\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4ac\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/12*a/c*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/c*a*2
^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*
a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-cx^4+bx^2+a}\,x^4}{cx^4-bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)*x^4/(c*x^4 - b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.977 $\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=377

$$\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}-\frac{\left(b-\sqrt{4ac+\right.}{}$$

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b
 - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elliptic
E[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4
]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^
2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c
*x^4])
```

Rubi [A] time = 0.24461, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1140, 493, 424, 419}

$$\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b
 - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elliptic
E[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4
]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^
2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c
*x^4])
```

Rule 1140

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/
(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[x^2/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqr
t[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && NegQ[c/a]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{\sqrt{a+bx^2-cx^4}}$$

$$= \frac{\left(\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{2c\sqrt{a+bx^2-cx^4}} - \frac{\left(\left(b-\sqrt{b^2+4\right.\right.}{}$$

$$= -\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4a}}{b-\sqrt{b^2+4a}}\right.}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.118048, size = 271, normalized size = 0.72

$$-\frac{i\left(\sqrt{4ac+b^2}-b\right)\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|-\frac{b+\sqrt{b^2+4ac}}{\sqrt{b^2+4ac}-b}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\right.\right.\right.}{2\sqrt{2}c\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-I/2)*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c]
)]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]
*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqr
t[b^2 + 4*a*c]))] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*
a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))]))/(Sqrt[2]
*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.224, size = 217, normalized size = 0.6

$$-\frac{a\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b\right.}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2
)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)
^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(
1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-cx^4 + bx^2 + a}\,x^2}{cx^4 - bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)*x^2/(c*x^4 - b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.978 $\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4
*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.0663954, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1104, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4
*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{a+bx^2-cx^4}}$$
$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.0811004, size = 177, normalized size = 1.05

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\right),-\frac{\sqrt{4ac+b^2}+b}{\sqrt{4ac+b^2}-b}\right)}{\sqrt{2}\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*
a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[2]*
Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.21, size = 145, normalized size = 0.9

$$\frac{\sqrt{2}}{4}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\sqrt{\right.}\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*E
llipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a
*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4 + bx^2 + a}}{cx^4 - bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)/(c*x^4 - b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.979 $\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=408

$$-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{\left(b-\sqrt{4a\right.}$$

```
[Out] -(Sqrt[a + b*x^2 - c*x^4]/(a*x)) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b
^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)
/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt
[2]*a*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b +
Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*
c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(
2*Sqrt[2]*a*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.331777, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1123, 12, 1140, 493, 424, 419}

$$-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{\left(b-\sqrt{4ac+b^2}\right)\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 - c*x^4]/(a*x)) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b
^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)
/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt
[2]*a*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b +
Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*
c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(
2*Sqrt[2]*a*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1140

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/
(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[x^2/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqr
t[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && NegQ[c/a]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{\int \frac{cx^2}{\sqrt{a+bx^2-cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{c\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{\left(c\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{a\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{\left(\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2c\cdots}{b+\sqrt{b^2\cdots}}}}\cdots}{2a\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}E\left(\sin^{-1}\cdots\right.}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.432216, size = 283, normalized size = 0.69

$$\frac{\frac{i\left(\sqrt{4ac+b^2}-b\right)\sqrt{\frac{4cx^2}{\sqrt{4ac+b^2}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)\right)}{\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}}-\frac{4a}{x}-4b\cdots}{4a\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-4*a)/x - 4*b*x + 4*c*x^3 + (I*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[2 + (4*c*x^2
)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(El
lipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[
b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-
(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*
a*c])]))/Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))])/(4*a*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.225, size = 241, normalized size = 0.6

$$-\frac{1}{ax}\sqrt{-cx^4+bx^2+a}+\frac{c\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -(-c*x^4+b*x^2+a)^(1/2)/a/x+1/2*c*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*
(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)
*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1
/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(
b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-cx^4+bx^2+a}}{cx^6-bx^4-ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)/(c*x^6 - b*x^4 - a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**2 - c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

3.980 $\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=445

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}-\frac{b\left(\cdots\right.}{\cdots}$$

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2 - c*x^4])/(3*a^2*x
) - (b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^
2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ell
ipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b
^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*a^2*Sqrt[c]*Sqrt[a + b*x^
2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^2 + a*c - b*Sqrt[b^2 + 4*a*c]
)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a
*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*a^2*Sqr
t[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.469404, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1123, 1281, 1202, 524, 424, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}-\frac{b\left(b-\sqrt{4ac}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2 - c*x^4])/(3*a^2*x
) - (b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^
2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ell
ipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b
^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*a^2*Sqrt[c]*Sqrt[a + b*x^
2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^2 + a*c - b*Sqrt[b^2 + 4*a*c]
)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a
*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*a^2*Sqr
```

```
t[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{\int \frac{-2b+cx^2}{x^2\sqrt{a+bx^2-cx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{\int \frac{-ac-2bcx^2}{\sqrt{a+bx^2-cx^4}}\,dx}{3a^2}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{-ac-2bcx^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\ldots}}}}{3a^2\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{\left(b\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \ldots}{3a^2\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{b\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1\ldots}}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+b\ldots}}$$

Mathematica [C] time = 0.743167, size = 472, normalized size = 1.06

$$\frac{i\sqrt{2}x^3\left(b\sqrt{4ac+b^2}-ac-b^2\right)\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)-2\sqrt{-\ldots}}{6a^2x^3\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] (-2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*(a - 2*b*x^2)*(a + b*x^2 - c*x^4) -
I*Sqrt[2]*b*(-b + Sqrt[b^2 + 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b +
 Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4
```

```
*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(
-b^2 - a*c + b*Sqrt[b^2 + 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2
)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sq
rt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*
c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*a^2*Sqrt[-(c
/(b + Sqrt[b^2 + 4*a*c]))]*x^3*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.233, size = 417, normalized size = 0.9

$$-\frac{1}{3\,a x^3}\sqrt{-c x^4+b x^2+a}+\frac{2\,b}{3\,a^2 x}\sqrt{-c x^4+b x^2+a}+\frac{c\sqrt{2}}{12\,a}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\mathrm{Ellip}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(-c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(-c*x^4+b*x^2+a)^(1/2)/a^2/x+1/12*c
/a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*
c+b^2)^(1/2))/a/c)^(1/2))-1/3*b*c/a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/
2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^
(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b
*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-c x^4+b x^2+a x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4+bx^2+a}}{cx^8-bx^6-ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)/(c*x^8 - b*x^6 - a*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**2 - c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

3.981 $\int \frac{x^9}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{\left(b\left(15b^2-52ac\right)-2cx^2\left(5b^2-12ac\right)\right)\sqrt{a+bx^2+cx^4}}{8c^3\left(b^2-4ac\right)}+\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2}}$$

```
[Out] (x^6*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x^4*Sqrt[a
 + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*(5*b^2 -
 12*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^3*(b^2 - 4*a*c)) + (3*(5*b^2 -
4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(7
/2))
```

Rubi [A] time = 0.237195, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 738, 832, 779, 621, 206}

$$-\frac{\left(b\left(15b^2-52ac\right)-2cx^2\left(5b^2-12ac\right)\right)\sqrt{a+bx^2+cx^4}}{8c^3\left(b^2-4ac\right)}+\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^6*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x^4*Sqrt[a
 + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*(5*b^2 -
 12*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^3*(b^2 - 4*a*c)) + (3*(5*b^2 -
4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(7
/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{x^2(6a+3bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\text{Subst}\left(\int \frac{x\left(-6ab-\frac{3}{2}\left(5b^2-12ac\right)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{3c\left(b^2-4ac\right)}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12ac\right)x^2\right)\sqrt{a}}{8c^3\left(b^2-4ac\right)}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12ac\right)x^2\right)\sqrt{a}}{8c^3\left(b^2-4ac\right)}$$

$$= \frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} - \frac{\left(b\left(15b^2-52ac\right)-2c\left(5b^2-12ac\right)x^2\right)\sqrt{a}}{8c^3\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.19284, size = 181, normalized size = 0.95

$$\frac{\frac{2\sqrt{c}\left(4a^2c\left(6cx^2-13b\right)+a\left(-62b^2cx^2+15b^3-20bc^2x^4+8c^3x^6\right)+b^2x^2\left(15b^2+5bcx^2-2c^2x^4\right)\right)}{\sqrt{a+bx^2+cx^4}}-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x^2) + b^2*x^2*(15*b^2 + 5*b*c*x^2 - 2*c^
2*x^4) + a*(15*b^3 - 62*b^2*c*x^2 - 20*b*c^2*x^4 + 8*c^3*x^6)))/Sqrt[a + b*
x^2 + c*x^4] - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(7/2)*(-b^2 + 4*a*c))
```

Maple [B] time = 0.173, size = 354, normalized size = 1.9

$$\frac{x^6}{4c}\frac{1}{\sqrt{cx^4+bx^2+a}} - \frac{5bx^4}{8c^2}\frac{1}{\sqrt{cx^4+bx^2+a}} - \frac{15b^2x^2}{16c^3}\frac{1}{\sqrt{cx^4+bx^2+a}} + \frac{15b^3}{32c^4}\frac{1}{\sqrt{cx^4+bx^2+a}} + \frac{15b^4x^2}{16c^3\left(4ac-b^2\right)\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x^6/c/(c*x^4+b*x^2+a)^(1/2)-5/8*b/c^2*x^4/(c*x^4+b*x^2+a)^(1/2)-15/16*b
^2/c^3*x^2/(c*x^4+b*x^2+a)^(1/2)+15/32*b^3/c^4/(c*x^4+b*x^2+a)^(1/2)+15/16*
b^4/c^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+15/32*b^5/c^4/(4*a*c-b^2)/(c*
x^4+b*x^2+a)^(1/2)+15/16*b^2/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))-13/8*b/c^3*a/(c*x^4+b*x^2+a)^(1/2)-13/4*b^2/c^2*a/(4*a*c-b^2)/(c*
x^4+b*x^2+a)^(1/2)*x^2-13/8*b^3/c^3*a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+3/4
*a/c^2*x^2/(c*x^4+b*x^2+a)^(1/2)-3/4*a/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*
x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.26827, size = 1272, normalized size = 6.69

$$\left[-\frac{3\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2+\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^4+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^2\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2\right.}{32\left(ab^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^4*c - 24*a*b^2*c^2 +
16*a^2*c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2)*sqrt(c)*log(-8*c
^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c)
- 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - 5*(b^
3*c^2 - 4*a*b*c^3)*x^4 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^2)*sqrt(c
```

```
*x^4 + b*x^2 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^4 + (b^3*
c^4 - 4*a*b*c^5)*x^2), -1/16*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b
^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)
*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c
^2*x^4 + b*c*x^2 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^6 - 15*a*b^3*c + 52*a
^2*b*c^2 - 5*(b^3*c^2 - 4*a*b*c^3)*x^4 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*
c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*
c^6)*x^4 + (b^3*c^4 - 4*a*b*c^5)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**9/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [B] time = 1.38098, size = 490, normalized size = 2.58

$$\frac{\left(\left(\frac{2\left(b^4c^3-8ab^2c^4+16a^2c^5\right)x^2}{b^4c^4-8ab^2c^5+16a^2c^6} - \frac{5\left(b^5c^2-8ab^3c^3+16a^2bc^4\right)}{b^4c^4-8ab^2c^5+16a^2c^6}\right)x^2 - \frac{15b^6c-122ab^4c^2+272a^2b^2c^3-96a^3c^4}{b^4c^4-8ab^2c^5+16a^2c^6}\right)x^2 - \frac{15ab^5c-112a^2b^3c^2+208a^3bc^3}{b^4c^4-8ab^2c^5+16a^2c^6}}{8\sqrt{cx^4+bx^2+a}} - \frac{3\left(5b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(((2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2/(b^4*c^4 - 8*a*b^2*c^5 +
16*a^2*c^6) - 5*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)/(b^4*c^4 - 8*a*b^2*c
^5 + 16*a^2*c^6))*x^2 - (15*b^6*c - 122*a*b^4*c^2 + 272*a^2*b^2*c^3 - 96*a^
3*c^4)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x^2 - (15*a*b^5*c - 112*a^2*b^
3*c^2 + 208*a^3*b*c^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))/sqrt(c*x^4 + b
*x^2 + a) - 3/16*(5*b^6*c - 44*a*b^4*c^2 + 112*a^2*b^2*c^3 - 64*a^3*c^4)*lo
g(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/((b^4*c^4 -
8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(c))
```

3.982 $\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\left(-8ac+3b^2-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

```
[Out] (x^4*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 - 8*a
*c - 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(2*c^2*(b^2 - 4*a*c)) - (3*b*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(5/2))
```

Rubi [A] time = 0.111292, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 738, 779, 621, 206}

$$\frac{\left(-8ac+3b^2-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (x^4*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 - 8*a
*c - 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(2*c^2*(b^2 - 4*a*c)) - (3*b*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(5/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
```

```
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(3b^2-8ac-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,d\right.}{4c^2}$$

$$= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(3b^2-8ac-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x,\right.}{2c^2}$$

$$= \frac{x^4\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(3b^2-8ac-2bcx^2\right)\sqrt{a+bx^2+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

<u>Mathematica</u> [A] time = 0.119625, size = 137, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}\left(8a^2c+a\left(-3b^2+10bcx^2+4c^2x^4\right)-b^2x^2\left(3b+cx^2\right)\right)}{\sqrt{a+bx^2+cx^4}} + 3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(8*a^2*c - b^2*x^2*(3*b + c*x^2) + a*(-3*b^2 + 10*b*c*x^2 + 4*c
^2*x^4)))/Sqrt[a + b*x^2 + c*x^4] + 3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(5/2)*(-b^2 + 4*a*c))
```

<u>Maple</u> [B] time = 0.173, size = 264, normalized size = 2.

$$\frac{x^4}{2c}\frac{1}{\sqrt{cx^4+bx^2+a}} + \frac{3bx^2}{4c^2}\frac{1}{\sqrt{cx^4+bx^2+a}} - \frac{3b^2}{8c^3}\frac{1}{\sqrt{cx^4+bx^2+a}} - \frac{3b^3x^2}{4c^2\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}} - \frac{3b^4}{8c^3\left(4ac-b^2\right)}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*x^4/c/(c*x^4+b*x^2+a)^(1/2)+3/4*b/c^2*x^2/(c*x^4+b*x^2+a)^(1/2)-3/8*b^2
/c^3/(c*x^4+b*x^2+a)^(1/2)-3/4*b^3/c^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^
2-3/8*b^4/c^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-3/4*b/c^(5/2)*ln((1/2*b+c*x
^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+a/c^2/(c*x^4+b*x^2+a)^(1/2)+2*a/c*b/(4*a
*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+a/c^2*b^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01745, size = 980, normalized size = 7.31

$$\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^4+ab^3-4\,a^2bc+\left(b^4-4\,ab^2c\right)x^2\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-\right.}{8\left(ab^2c^3-4\,a^2c^4+\left(b^2c^4-4\,ac^5\right)x^4+\left(b^3c^3-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^
2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^4 + 3*a*b^2*c - 8*a^
2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^3 - 4
*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2), 1/4*(3*((b
^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^2)*sqrt(-c)
*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x
^2 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^4 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c
- 10*a*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c
```

```
^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [B] time = 1.37653, size = 359, normalized size = 2.68

$$\frac{\left(\frac{\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{3\,b^5-22\,ab^3c+40\,a^2bc^2}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x^2+\frac{3\,ab^4-20\,a^2b^2c+32\,a^3c^2}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}}{2\,\sqrt{cx^4+bx^2+a}}+\frac{3\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+b\right.\right.\right.}{4\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4) + (3*b^5 - 22*a*b^3*c + 40*a^2*b*c^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4))*x^2 + (3*a*b^4 - 20*a^2*b^2*c + 32*a^3*c^2)/(b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4))/sqrt(c*x^4 + b*x^2 + a) + 3/4*(b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/((b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*sqrt(c))
```

3.983 $\int \frac{x^5}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] (x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*Sqrt[a + b
*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])]/(2*c^(3/2))
```

Rubi [A] time = 0.0922341, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 738, 640, 621, 206}

$$\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*Sqrt[a + b
*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])]/(2*c^(3/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
```

```
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)$$

$$= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{2a+bx}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{b^2-4ac}$$

$$= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2c}$$

$$= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{c}$$

$$= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

Mathematica [A] time = 0.0996468, size = 107, normalized size = 0.93

$$\frac{\frac{2\sqrt{c}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{\sqrt{a+bx^2+cx^4}}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(b^2*x^2 + a*(b - 2*c*x^2)))/Sqrt[a + b*x^2 + c*x^4] - (b^2 - 4
*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2
)*(-b^2 + 4*a*c))
```

Maple [A] time = 0.171, size = 149, normalized size = 1.3

$$-\frac{x^2}{2c}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{b}{4c^2}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{b^2x^2}{2c\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{b^3}{4c^2\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{1}{2}\ln\Bigg(\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/4*b/c^2/(c*x^4+b*x^2+a)^(1/2)+1/2*b^2/c/
(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^4+b*x^2+
a)^(1/2)+1/2/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83386, size = 833, normalized size = 7.24

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)-}{4\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^4+\left(b^3c^2-4\,abc^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(
c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a)*(a*b*c + (b^2*c - 2*a*c^2)*
x^2))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c
^3)*x^2), -1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*
x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^
2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b*c + (b^2*c - 2*a*c
^2)*x^2))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a
*b*c^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A] time = 1.28782, size = 255, normalized size = 2.22

$$-\frac{\frac{\left(b^4-6\,ab^2c+8\,a^2c^2\right)x^2}{b^4c-8\,ab^2c^2+16\,a^2c^3}+\frac{ab^3-4\,a^2bc}{b^4c-8\,ab^2c^2+16\,a^2c^3}}{\sqrt{cx^4+bx^2+a}}-\frac{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{2\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3) + (a
*b^3 - 4*a^2*b*c)/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))/sqrt(c*x^4 + b*x^2 +
a) - 1/2*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))*sqrt(c) - b))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(c))
```

3.984 $\int \frac{x^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2a+bx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.028675, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1114, 636}

$$\frac{2a+bx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{2a+bx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.0955075, size = 36, normalized size = 1.

$$\frac{2a+bx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.046, size = 38, normalized size = 1.1

$$-\frac{bx^2+2\,a}{4\,ac-b^2}\frac{1}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -(b*x^2+2*a)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71841, size = 139, normalized size = 3.86

$$\frac{\sqrt{cx^4 + bx^2 + a}\left(bx^2 + 2a\right)}{\left(b^2c - 4ac^2\right)x^4 + ab^2 - 4a^2c + \left(b^3 - 4abc\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^
2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A] time = 1.28001, size = 59, normalized size = 1.64

$$\frac{\frac{bx^2}{b^2-4ac} + \frac{2a}{b^2-4ac}}{\sqrt{cx^4 + bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (b*x^2/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

3.985 $\int \frac{x}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{b+2cx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] -((b + 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]))
```

Rubi [A] time = 0.0231567, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1107, 613}

$$-\frac{b+2cx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b + 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]))
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)$$
$$= -\frac{b+2cx^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.0212924, size = 37, normalized size = 1.03

$$\frac{b+2cx^2}{\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (b + 2*c*x^2)/((-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.046, size = 36, normalized size = 1.

$$\frac{2\,cx^2+b}{4\,ac-b^2}\frac{1}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] (2*c*x^2+b)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70216, size = 140, normalized size = 3.89

$$-\frac{\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)}{\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a
^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A] time = 1.32656, size = 61, normalized size = 1.69

$$-\frac{\frac{2\,cx^2}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(2*c*x^2/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

3.986 $\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2))
```

Rubi [A] time = 0.0808448, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 740, 12, 724, 206}

$$\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
```

```
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{a}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.107392, size = 89, normalized size = 1.

$$\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2))
```

Maple [A] time = 0.166, size = 99, normalized size = 1.1

$$\frac{1}{2\,a}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{b\left(2\,cx^2+b\right)}{2\,a\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2/a/(c*x^4+b*x^2+a)^(1/2)-1/2*b/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)
^(1/2)-1/2/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94186, size = 837, normalized size = 9.4

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)\sqrt{a}\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(abcx^2+ab^2-\right.}{4\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a^3bc\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(
a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(c*x^4 +
b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4
*a^3*b*c)*x^2), 1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a
)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(c*x^4 +
 b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 -
4*a^3*b*c)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x), x)
```

3.987 $\int \frac{1}{x^3(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2x^2(b^2-4ac)}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}+\frac{-2ac+b^2+bcx^2}{ax^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]) - ((3
*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^2) + (3*b*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2))
```

Rubi [A] time = 0.125908, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 740, 806, 724, 206}

$$-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2x^2(b^2-4ac)}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}+\frac{-2ac+b^2+bcx^2}{ax^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]) - ((3
*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^2) + (3*b*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
```

```
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\right.}{4a^2}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x,\right.}{2a^2}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2} + \frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}$$

Mathematica [A] time = 0.0853652, size = 137, normalized size = 0.99

$$\frac{\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx^2-8c^2x^4\right)+3b^2x^2\left(b+cx^2\right)\right)}{x^2\sqrt{a+bx^2+cx^4}} - 3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2
*x^4)))/(x^2*Sqrt[a + b*x^2 + c*x^4]) - 3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*(-b^2 + 4*a*c))
```

Maple [A] time = 0.17, size = 195, normalized size = 1.4

$$-\frac{1}{2\,ax^2}\frac{1}{\sqrt{cx^4+bx^2+a}} - \frac{3\,b}{4\,a^2}\frac{1}{\sqrt{cx^4+bx^2+a}} + \frac{3\,b^2cx^2}{2\,a^2\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}} + \frac{3\,b^3}{4\,a^2\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}} + \frac{3\,b}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/a/x^2/(c*x^4+b*x^2+a)^(1/2)-3/4*b/a^2/(c*x^4+b*x^2+a)^(1/2)+3/2*b^2/a^
2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2*c+3/4*b^3/a^2/(4*a*c-b^2)/(c*x^4+b*
x^2+a)^(1/2)+3/4*b/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x
^2)-2*c/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17469, size = 1025, normalized size = 7.37

$$\left[\frac{3\left(\left(b^{3}c-4\,abc^{2}\right)x^{6}+\left(b^{4}-4\,ab^{2}c\right)x^{4}+\left(ab^{3}-4\,a^{2}bc\right)x^{2}\right)\sqrt{a}\log\left(-\frac{\left(b^{2}+4\,ac\right)x^{4}+8\,abx^{2}+4\,\sqrt{cx^{4}+bx^{2}+a}\left(bx^{2}+2\,a\right)\sqrt{a}+8\,a^{2}}{x^{4}}\right)-4\left(\left(3\,ab^{2}\right.\right.}{8\left(\left(a^{3}b^{2}c-4\,a^{4}c^{2}\right)x^{6}+\left(a^{3}b^{3}-4\,a^{4}bc\right)x^{4}+\left(a^{4}b^{2}-4\,a^{5}c\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2
 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((3*a*b^2*c - 8*a^2*c^2)*x^4
+ a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5
*c)*x^2), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3
 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^
2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^
3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*
x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^3), x)
```

3.988 $\int \frac{1}{x^5(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{b(15b^2-52ac)\sqrt{a+bx^2+cx^4}}{8a^3x^2(b^2-4ac)}-\frac{(5b^2-12ac)\sqrt{a+bx^2+cx^4}}{4a^2x^4(b^2-4ac)}-\frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}}+\frac{-2ac+b}{ax^4(b^2-4ac)}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^4*Sqrt[a + b*x^2 + c*x^4]) - ((5
*b^2 - 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(15*
b^2 - 52*a*c)*Sqrt[a + b*x^2 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x^2) - (3*(5*b^
2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*
a^(7/2))
```

Rubi [A] time = 0.211298, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1114, 740, 834, 806, 724, 206}

$$\frac{b(15b^2-52ac)\sqrt{a+bx^2+cx^4}}{8a^3x^2(b^2-4ac)}-\frac{(5b^2-12ac)\sqrt{a+bx^2+cx^4}}{4a^2x^4(b^2-4ac)}-\frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}}+\frac{-2ac+b}{ax^4(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^4*Sqrt[a + b*x^2 + c*x^4]) - ((5
*b^2 - 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(15*
b^2 - 52*a*c)*Sqrt[a + b*x^2 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x^2) - (3*(5*b^
2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*
a^(7/2))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+12ac\right)-2bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}b\left(15b^2-52ac\right)-\frac{1}{2}c}{x^2\sqrt{a+bx+}}\right.}{2a^2\left(b^2-4\right.}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2}}{8a^3\left(b^2-4ac\right)x^2}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2}}{8a^3\left(b^2-4ac\right)x^2}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2}}{8a^3\left(b^2-4ac\right)x^2}$$

Mathematica [A] time = 0.130459, size = 179, normalized size = 0.92

$$\frac{\frac{2\sqrt{a}\left(2a^2\left(b^2+10bcx^2-12c^2x^4\right)-8a^3c+abx^2\left(-5b^2+62bcx^2+52c^2x^4\right)-15b^3x^4\left(b+cx^2\right)\right)}{x^4\sqrt{a+bx^2+cx^4}}+3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-8*a^3*c - 15*b^3*x^4*(b + c*x^2) + 2*a^2*(b^2 + 10*b*c*x^2 -
12*c^2*x^4) + a*b*x^2*(-5*b^2 + 62*b*c*x^2 + 52*c^2*x^4)))/(x^4*Sqrt[a + b*
x^2 + c*x^4]) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(7/2)*(-b^2 + 4*a*c))
```

Maple [A] time = 0.173, size = 314, normalized size = 1.6

$$-\frac{1}{4\,a x^4}\frac{1}{\sqrt{c x^4+b x^2+a}}+\frac{5\,b}{8\,a^2 x^2}\frac{1}{\sqrt{c x^4+b x^2+a}}+\frac{15\,b^2}{16\,a^3}\frac{1}{\sqrt{c x^4+b x^2+a}}-\frac{15\,b^3 x^2 c}{8\,a^3\left(4\,ac-b^2\right)}\frac{1}{\sqrt{c x^4+b x^2+a}}-\frac{15\,b^4}{16\,a^3\left(4\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/4/a/x^4/(c*x^4+b*x^2+a)^(1/2)+5/8*b/a^2/x^2/(c*x^4+b*x^2+a)^(1/2)+15/16*
b^2/a^3/(c*x^4+b*x^2+a)^(1/2)-15/8*b^3/a^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2
)*x^2*c-15/16*b^4/a^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-15/16*b^2/a^(7/2)*l
n((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+13/2*b/a^2*c^2/(4*a*c-b^
2)/(c*x^4+b*x^2+a)^(1/2)*x^2+13/4*b^2/a^2*c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/
2)-3/4*c/a^2/(c*x^4+b*x^2+a)^(1/2)+3/4*c/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c
*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.61057, size = 1314, normalized size = 6.74

$$\left[-\frac{3\left(\left(5\,b^4 c-24\,ab^2c^2+16\,a^2c^3\right)x^8+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^6+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x^4\right)\sqrt{a}\log\left(-\frac{\left(b^2+4\,ac\right)x^4}{}\right.}{32\left(\left(a^4b^2c-4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^6 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^4)*sqrt(a)*lo
g(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(a) + 8*a^2)/x^4) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^6 - 2*a^3*b^2 + 8
*a^4*c + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^4 + 5*(a^2*b^3 - 4*a^3*b*
c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^8 + (a^4*b^3 -
4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4), 1/16*(3*((5*b^4*c - 24*a*b^2*c^2
 + 16*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^6 + (5*a*b^4 - 2
4*a^2*b^2*c + 16*a^3*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*
(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^3*c - 52*a^2
*b*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*
x^4 + 5*(a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^4*b^2*c - 4
*a^5*c^2)*x^8 + (a^4*b^3 - 4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^5), x)
```

3.989 $\int \frac{x^6}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=408

$$\frac{\sqrt[4]{a}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+}}{c^{3/2}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{}\right.}$$

```
[Out] (x^3*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x*Sqrt[a +
 b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + (2*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*
x^4])/(c^(3/2)*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*(b^2 - 3
*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
])/(c^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*b^2 + Sqrt
[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*c^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.209713, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1120, 1279, 1197, 1103, 1195}

$$\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{c^{3/2}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^3*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x*Sqrt[a +
 b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + (2*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*
x^4])/(c^(3/2)*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*(b^2 - 3
*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
])/(c^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*b^2 + Sqrt
[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
```

```
rt[a]*Sqrt[c]))/4])/(2*c^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\int \frac{x^2\left(6a+3bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx}{-b^2+4ac}$$

$$= \frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{\int \frac{3ab+6\left(b^2-3ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c\left(b^2-4ac\right)}$$

$$= \frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(\sqrt{a}\left(2b-3\sqrt{a}\sqrt{c}\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\left(b-2\sqrt{a}\sqrt{c}\right)c^{3/2}} - \frac{(2\cdots}{\cdots}$$

$$= \frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{2\left(b^2-3ac\right)x\sqrt{a+bx^2+cx^4}}{c^{3/2}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{2\sqrt[4]{a}\left(b^2-\cdots\right.}{\cdots}$$

Mathematica [C] time = 1.32001, size = 489, normalized size = 1.2

$$\frac{i\left(b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}+4abc-b^3\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\cdots\right.}{2c\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2*x^2 + a*(b - 2*c*x^2)) - I*(b^2
 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2*
c^2*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.248, size = 482, normalized size = 1.2

$$-2\,c\left(1/2\,\frac{\left(2\,ac-b^{2}\right)x^{3}}{c^{2}\left(4\,ac-b^{2}\right)}-1/2\,\frac{abx}{c^{2}\left(4\,ac-b^{2}\right)}\right)\frac{1}{\sqrt{\left(x^{4}+\frac{bx^{2}}{c}+\frac{a}{c}\right)c}}-\frac{ab\sqrt{2}}{4\,c\left(4\,ac-b^{2}\right)}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^{2}}\right)x^{2}}{a}}\sqrt{4+2\,\frac{\left(b\right.}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2/c^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3-1/2*a*b/c^2/(4*a*c-b^2)*x)/((x^4+
b/c*x^2+a/c)*c)^(1/2)-1/4*a*b/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*
(1/c+(2*a*c-b^2)/c/(4*a*c-b^2))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,x^{6}}{c^{2}x^{8}+2\,bcx^{6}+\left(b^{2}+2\,ac\right)x^{4}+2\,abx^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^6/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x
^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**6/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.990 $\int \frac{x^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=342

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (x*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x*Sqrt[a + b
*x^2 + c*x^4])/(Sqrt[c]*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*b
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c
^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*(b - 2*Sqrt[a]*Sqr
t[c])*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.130048, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1120, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{x\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x*Sqrt[a + b
*x^2 + c*x^4])/(Sqrt[c]*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*b
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c
^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*(b - 2*Sqrt[a]*Sqr
t[c])*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\int \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{-b^2+4ac}$$

$$= \frac{x\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{a}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{c}} + \frac{\left(\sqrt{a}b\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}\left(b^2-4ac\right)}$$

$$= \frac{x\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(\right.}{c^{3/4}\left(b^2-4ac\right)\sqrt{a}}$$

Mathematica [C] time = 0.833395, size = 452, normalized size = 1.32

$$\frac{i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)+4cx\sqrt{-}}{4c\left(b^2-4ac\right)\sqrt{\frac{c}{\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(2*a + b*x^2) - I*b*(-b + Sqrt[b^2 -
 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sq
rt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])])/(4*c*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a +
 b*x^2 + c*x^4])
```

Maple [A] time = 0.225, size = 450, normalized size = 1.3

$$-2\,c\left(1/2\,\frac{bx^3}{c\left(4\,ac-b^2\right)}+\frac{ax}{c\left(4\,ac-b^2\right)}\right)\frac{1}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}+\frac{a\sqrt{2}}{8\,ac-2\,b^2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\right.}\right.}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2/(4*a*c-b^2)/c*b*x^3+a/c/(4*a*c-b^2)*x)/((x^4+b/c*x^2+a/c)*c)^(1/2
)+1/2*a/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+
b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\,x^4}{c^2x^8 + 2\,bcx^6 + \left(b^2 + 2\,ac\right)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^4/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x
^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**4/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.991 $\int \frac{x^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=341

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+}}$$

```
[Out] -((x*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (2*Sqrt[c]*x
*Sqrt[a + b*x^2 + c*x^4])/((b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1
/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(b - 2*Sqrt[a]*
Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.125243, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1119, 1197, 1103, 1195}

$$\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (2*Sqrt[c]*x
*Sqrt[a + b*x^2 + c*x^4])/((b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1
/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(b - 2*Sqrt[a]*
Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\int \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{-b^2+4ac}$$

$$= -\frac{x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\left(2\sqrt{a}\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{b^2-4ac} - \frac{\left(b+2\sqrt{a}\sqrt{c}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{-b^2+4ac}$$

$$= -\frac{x\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\ldots}{\left(b^2-4ac\right)\sqrt{a}\ldots}$$

Mathematica [C] time = 0.775495, size = 437, normalized size = 1.28

$$\frac{-i\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)-2x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(b+\ldots\right.}{2\left(b^2-4ac\right)\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 2*c*x^2) + I*(-b + Sqrt[b^2 - 4*
a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2
*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.217, size = 446, normalized size = 1.3

$$-2\,c\left(-\frac{x^3}{4\,ac-b^2}-1/2\,\frac{bx}{c\left(4\,ac-b^2\right)}\right)\frac{1}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}-\frac{b\sqrt{2}}{16\,ac-4\,b^2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\ldots}\right.}{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(-1/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)/c*b*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)
-1/4*b/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2
*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,x^2}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^2/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x
^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.992 $\int \frac{1}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=353

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}+\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*
Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)
) + (b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3
/4)*(b - 2*Sqrt[a]*Sqrt[c])*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.135121, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1092, 1197, 1103, 1195}

$$\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*
Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)
) + (b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3
/4)*(b - 2*Sqrt[a]*Sqrt[c])*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\int \frac{2ac+bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b-2\sqrt{a}\sqrt{c}\right)} + \frac{\left(b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\Big(2}{a^{3/4}\left(b^2-4ac\right)\sqrt{a}}$$

Mathematica [C] time = 0.858165, size = 456, normalized size = 1.29

$$-\frac{-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)-4x\sqrt{}}{4a\left(b^2-4ac\right)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] -(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2) + I*b*(-b +
Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a
*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(
2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*A
rcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/
(b - Sqrt[b^2 - 4*a*c])])/(4*a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.226, size = 481, normalized size = 1.4

$$-2c\left(1/2\,\frac{bx^3}{a\left(4ac-b^2\right)}-1/2\,\frac{\left(2ac-b^2\right)x}{a\left(4ac-b^2\right)c}\right)\frac{1}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}+\frac{\sqrt{2}}{4}\left(a^{-1}-\frac{2ac-b^2}{a\left(4ac-b^2\right)}\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+b/c*
x^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}}{c^2x^8 + 2\,bcx^6 + \left(b^2 + 2\,ac\right)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 +
 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

3.993 $\int \frac{1}{x^2(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=428

$$\frac{\sqrt[4]{c}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+}}{a^2x\left(b^2-4ac\right)}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x*Sqrt[a + b*x^2 + c*x^4]) - (2*(b
^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)*x) + (2*Sqrt[c]*(b^
2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]
*x^2)) - (2*c^(1/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^
4]) + (c^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(7/4)*(b^2 - 4*a*c)*Sq
rt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.216941, antiderivative size = 428, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1121, 1281, 1197, 1103, 1195}

$$-\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2x\left(b^2-4ac\right)}+\frac{2\sqrt{c}x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}}{2a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```



```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x*Sqrt[a + b*x^2 + c*x^4]) - (2*(b
^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)*x) + (2*Sqrt[c]*(b^
2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]
*x^2)) - (2*c^(1/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^
4]) + (c^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(
```

```
1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(7/4)*(b^2 - 4*a*c)*Sq
rt[a + b*x^2 + c*x^4])
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x\sqrt{a+bx^2+cx^4}} - \frac{\int \frac{-2\left(b^2-3ac\right)-bcx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x\sqrt{a+bx^2+cx^4}} - \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)x} + \frac{\int \frac{abc+2c\left(b^2-3ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x\sqrt{a+bx^2+cx^4}} - \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)x} - \frac{\left(2\sqrt{c}\left(b^2-3ac\right)\right)\int \frac{1-\frac{\sqrt{c}x}{\sqrt{a}}}{\sqrt{a+bx^2+}}}{a^{3/2}\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x\sqrt{a+bx^2+cx^4}} - \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)x} + \frac{2\sqrt{c}\left(b^2-3ac\right)x\sqrt{a+bx^2}}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\right.}$$

Mathematica [C] time = 1.32956, size = 515, normalized size = 1.2

$$-\frac{ix\left(b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}+4abc-b^3\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*x^2) + a*(
b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*x*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqr
t[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)*Sqrt[c/(b + Sqr
```

```
t[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.229, size = 536, normalized size = 1.3

$$-2\,c\left(1/2\,\frac{\left(2\,ac-b^{2}\right)x^{3}}{\left(4\,ac-b^{2}\right)a^{2}}+1/2\,\frac{b\left(3\,ac-b^{2}\right)x}{a^{2}\left(4\,ac-b^{2}\right)c}\right)\frac{1}{\sqrt{\left(x^{4}+\frac{bx^{2}}{c}+\frac{a}{c}\right)c}}-\frac{1}{a^{2}x}\sqrt{cx^{4}+bx^{2}+a}+\frac{\sqrt{2}}{4}\left(-\frac{b}{a^{2}}+\frac{b\left(3\,ac-b^{2}\right)}{\left(4\,ac-b^{2}\right)a^{2}}\right)\sqrt{4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2*(2*a*c-b^2)/(4*a*c-b^2)/a^2*x^3+1/2*b*(3*a*c-b^2)/a^2/(4*a*c-b^2)
/c*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)-1/a^2*(c*x^4+b*x^2+a)^(1/2)/x+1/4*(-b/a^2
+b*(3*a*c-b^2)/a^2/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(
4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(c*(2*a*c-b^
2)/(4*a*c-b^2)/a^2+1/a^2*c)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-
2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}}{c^2x^{10}+2\,bcx^8+\left(b^2+2\,ac\right)x^6+2\,abx^4+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c^2*x^10 + 2*b*c*x^8 + (b^2 + 2*a*c)*x^6
+ 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

3.994 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=50

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Rubi [A] time = 0.0677387, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3, 2016, 1588}

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
```

```
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{(2b)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}\\ &= -\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}+\frac{x\sqrt{bx^2+cx^4}}{3c}\end{aligned}$$

Mathematica [A] time = 0.0181659, size = 34, normalized size = 0.68

$$\frac{\left(cx^2-2b\right)\sqrt{x^2\left(b+cx^2\right)}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] ((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

Maple [A] time = 0.046, size = 37, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(-cx^2+2\,b\right)x}{3\,c^2}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-c*x^2+2*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 0.992086, size = 46, normalized size = 0.92

$$\frac{c^2x^4 - bcx^2 - 2\,b^2}{3\,\sqrt{cx^2 + b}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

Fricas [A] time = 1.51448, size = 63, normalized size = 1.26

$$\frac{\sqrt{cx^4 + bx^2}\left(cx^2 - 2\,b\right)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2), x)
```

3.995 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(2*c^(3/2))
```

Rubi [A] time = 0.0836604, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3, 2018, 640, 620, 206}

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(2*c^(3/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{4c}\\ &= \frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c}\\ &= \frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0313152, size = 73, normalized size = 1.26

$$\frac{x\left(\sqrt{c}x\left(b+cx^2\right)-b\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2) - b*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b +
c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.046, size = 64, normalized size = 1.1

$$\frac{x}{2}\sqrt{cx^2+b}\left(x\sqrt{cx^2+b}c^{\frac{3}{2}}-b\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)c\right)\frac{1}{\sqrt{cx^4+bx^2}}c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(3/2)-b*ln(x*c^(1/2)+(c*x^2+b)^(
1/2))*c)/(c*x^4+b*x^2)^(1/2)/c^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47644, size = 265, normalized size = 4.57

$$\left[\frac{b\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}c}{4\,c^2}, \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}c}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/
```

```
(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.19506, size = 80, normalized size = 1.38

$$\frac{b \log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2}\right)\sqrt{c} - b\right|\right)}{4\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^4 + bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/c^(3/2)
+ 1/2*sqrt(c*x^4 + b*x^2)/c
```

3.996 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rubi [A] time = 0.0174382, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3, 1588}

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{\sqrt{bx^2+cx^4}}{cx}$$

Mathematica [A] time = 0.0053655, size = 22, normalized size = 1.

$$\frac{\sqrt{x^2\left(b+cx^2\right)}}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[x^2*(b + c*x^2)]/(c*x)
```

Maple [A] time = 0.044, size = 26, normalized size = 1.2

$$\frac{x\left(cx^2+b\right)}{c}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] (c*x^2+b)/c*x/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 0.99111, size = 18, normalized size = 0.82

$$\frac{\sqrt{cx^2+b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

[Out] sqrt(c*x^2 + b)/c

Fricas [A] time = 1.45743, size = 36, normalized size = 1.64

$$\frac{\sqrt{cx^4+bx^2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")

[Out] sqrt(c*x^4 + b*x^2)/(c*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**4+b*x**2)**(1/2),x)

[Out] Integral(x**2/sqrt(x**2*(b + c*x**2)), x)

Giac [A] time = 1.14299, size = 42, normalized size = 1.91

$$-\frac{2\sqrt{b}}{\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")

[Out] -2*sqrt(b)/((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)

3.997 $\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rubi [A] time = 0.0489377, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3, 2013, 620, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]

[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{x}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= \operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0113331, size = 52, normalized size = 1.68

$$\frac{x\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]*Sqrt[x^2*
(b + c*x^2)])
```

Maple [A] time = 0.043, size = 44, normalized size = 1.4

$$x\sqrt{cx^2+b}\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\frac{1}{\sqrt{cx^4+bx^2}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)*ln(x*c^(1/2)+(c*x^2+b)^(1/2))/c^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55389, size = 174, normalized size = 5.61

$$\left[\frac{\log\left(-2\,cx^2 - b - 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c), -sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A] time = 1.17951, size = 53, normalized size = 1.71

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/sqrt(c)
```

3.998 $\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rubi [A] time = 0.0099777, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3, 2008, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0107241, size = 52, normalized size = 1.73

$$-\frac{x\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] -((x*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[x^2*(b
 + c*x^2)]))
```

Maple [B] time = 0.044, size = 50, normalized size = 1.7

$$-x\sqrt{cx^2+b}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\frac{1}{\sqrt{cx^4+bx^2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1
/2)+b)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2), x)
```

Fricas [A] time = 1.52442, size = 186, normalized size = 6.2

$$\left[\frac{\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)/sqrt(b), sqr
t(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*x**2 + c*x**4), x)
```

Giac [A] time = 1.13706, size = 62, normalized size = 2.07

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(c*x^2 + b)/sqrt(-b)
)/(sqrt(-b)*sgn(x))
```

3.999 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rubi [A] time = 0.0400943, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3, 2014}

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Mathematica [A] time = 0.0070438, size = 23, normalized size = 1.

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

Maple [A] time = 0.043, size = 26, normalized size = 1.1

$$-\frac{cx^2+b}{b}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -(c*x^2+b)/b/(c*x^4+b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45414, size = 41, normalized size = 1.78

$$-\frac{\sqrt{cx^4 + bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.13326, size = 19, normalized size = 0.83

$$-\frac{\sqrt{c + \frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(c + b/x^2)/b
```

3.1000 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(2*b^(3/2))
```

Rubi [A] time = 0.05555, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3, 2025, 2008, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(2*b^(3/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}-\frac{c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0629977, size = 68, normalized size = 1.15

$$\frac{c\sqrt{x^2\left(b+cx^2\right)}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{cx^2}{b}+1}\right)}{2\sqrt{\frac{cx^2}{b}+1}}-\frac{b}{2cx^2}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] (c*Sqrt[x^2*(b + c*x^2)]*(-b/(2*c*x^2) + ArcTanh[Sqrt[1 + (c*x^2)/b]]/(2*Sq
rt[1 + (c*x^2)/b])))/(b^2*x)
```

Maple [A] time = 0.045, size = 73, normalized size = 1.2

$$-\frac{1}{2x}\sqrt{cx^2+b}\left(-c\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^2b+\sqrt{cx^2+b}b^{\frac{3}{2}}\right)\frac{1}{\sqrt{cx^4+bx^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(-c*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2*b+(c*x^2
+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^4+bx^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

Fricas [A] time = 1.52262, size = 306, normalized size = 5.19

$$\left[\frac{\sqrt{b}cx^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}b}{4\,b^2x^3},-\frac{\sqrt{-b}cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}b}{2\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1001 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Rubi [A] time = 0.0837913, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3, 2016, 2014}

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
```

```
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{\sqrt{bx^2+cx^4}}{3bx^4} - \frac{(2c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{3bx^4} + \frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

Mathematica [A] time = 0.0146715, size = 35, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(2cx^2-b\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

Maple [A] time = 0.043, size = 37, normalized size = 0.7

$$-\frac{\left(cx^2+b\right)\left(-2\,cx^2+b\right)}{3\,b^2x^2}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-2*c*x^2+b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50202, size = 66, normalized size = 1.27

$$\frac{\sqrt{cx^4 + bx^2}\left(2\,cx^2 - b\right)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A] time = 1.19894, size = 36, normalized size = 0.69

$$-\frac{\left(c + \frac{b}{x^2}\right)^{\frac{3}{2}} - 3\sqrt{c + \frac{b}{x^2}}\,c}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*((c + b/x^2)^(3/2) - 3*sqrt(c + b/x^2)*c)/b^2
```

3.1002 $\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^3) - (3
*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

Rubi [A] time = 0.0995023, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3, 2025, 2008, 206}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^3) - (3
*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(3c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}+\frac{\left(3c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\left(3c^2\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0119407, size = 44, normalized size = 0.51

$$
-\frac{c^2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{cx^2}{b}+1\right)}{b^3x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -((c^2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (c*x^2)/b])
/(b^3*x))
```

Maple [A] time = 0.047, size = 94, normalized size = 1.1

$$-\frac{1}{8x^3}\sqrt{cx^2+b}\left(3\ln\left(2\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4bc^2-3\sqrt{cx^2+b}b^{3/2}x^2c+2\sqrt{cx^2+b}b^{5/2}\right)\frac{1}{\sqrt{cx^4+bx^2}}b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b)^(1/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^4*b*c^2-3*(c*
x^2+b)^(1/2)*b^(3/2)*x^2*c+2*(c*x^2+b)^(1/2)*b^(5/2))/x^3/(c*x^4+b*x^2)^(1/
2)/b^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

Fricas [A] time = 1.57034, size = 366, normalized size = 4.21

$$\left[\frac{3\sqrt{b}c^2x^5\log\left(-\frac{cx^3+2\,bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\sqrt{cx^4+bx^2}\left(3\,bcx^2-2\,b^2\right)}{16\,b^3x^5},\frac{3\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\left(3\,bc\right.}{8\,b^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5), 1/8*(3*sqrt
(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^
```

```
4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1003 $\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{x\sqrt{a+cx^4}}{3c}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

```
[Out] (x*Sqrt[a + c*x^4])/(3*c) - (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(6*c^(5/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0251027, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4, 321, 220}

$$\frac{x\sqrt{a+cx^4}}{3c}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/(3*c) - (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(6*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{x\sqrt{a+cx^4}}{3c} - \frac{a\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3c}$$

$$= \frac{x\sqrt{a+cx^4}}{3c} - \frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.0218178, size = 62, normalized size = 0.57

$$\frac{x\left(-a\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+a+cx^4\right)}{3c\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x*(a + c*x^4 - a*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c
*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])
```

Maple [C] time = 0.176, size = 91, normalized size = 0.8

$$\frac{x}{3\,c}\sqrt{cx^4+a}-\frac{a}{3\,c}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+a)^(1/2)/c-1/3*a/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(c*x^4 + a), x)
```

Sympy [C] time = 0.717134, size = 37, normalized size = 0.34

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+a)**(1/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + a), x)
```

3.1004 $\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{a+cx^4}}{2c}$$

```
[Out] Sqrt[a + c*x^4]/(2*c)
```

Rubi [A] time = 0.0044898, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4, 261}

$$\frac{\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + c*x^4]/(2*c)
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\int \frac{x^3}{\sqrt{a+cx^4}}\,dx$$
$$=\frac{\sqrt{a+cx^4}}{2c}$$

Mathematica [A] time = 0.003454, size = 18, normalized size = 1.

$$\frac{\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + c*x^4]/(2*c)
```

Maple [A] time = 0.044, size = 15, normalized size = 0.8

$$\frac{1}{2c}\sqrt{cx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+a)^(1/2)/c
```

Maxima [A] time = 0.967832, size = 19, normalized size = 1.06

$$\frac{\sqrt{cx^4+a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^4 + a)/c
```

Fricas [A] time = 1.44087, size = 31, normalized size = 1.72

$$\frac{\sqrt{cx^4 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^4 + a)/c
```

Sympy [A] time = 0.457962, size = 22, normalized size = 1.22

$$\begin{cases} \frac{\sqrt{a+cx^4}}{2c} & \text{for } c \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + c*x**4)/(2*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [A] time = 1.11893, size = 19, normalized size = 1.06

$$\frac{\sqrt{cx^4 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + a)/c
```

3.1005 $\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=210

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{x}{\sqrt{c}\left(\right.}$$

```
[Out] (x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0533998, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4, 305, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```


```
[Out] (x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```


Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{\sqrt{a}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} - \frac{\sqrt{a}\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{\right.}}{}$$

Mathematica [C] time = 0.008359, size = 51, normalized size = 0.24

$$\frac{x^3\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(3
*Sqrt[a + c*x^4])
```

Maple [C] time = 0.171, size = 97, normalized size = 0.5

$$i\sqrt{a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+a)^(1/2),x)
```

```
[Out] I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^
(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(c*x^4 + a), x)
```

Sympy [C] time = 0.67028, size = 37, normalized size = 0.18

$$\frac{x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+a)**(1/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + a), x)
```

3.1006 $\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}} dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Rubi [A] time = 0.0161084, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4, 275, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{x}{\sqrt{a+cx^4}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0058223, size = 30, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Maple [A] time = 0.152, size = 24, normalized size = 0.8

$$\frac{1}{2}\ln\left(x^2\sqrt{c}+\sqrt{cx^4+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5281, size = 163, normalized size = 5.43

$$\left[\frac{\log\left(-2\,cx^4 - 2\,\sqrt{cx^4 + a}\sqrt{c}x^2 - a\right)}{4\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+a}}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a)/sqrt(c), -1/2*sqrt(-
c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + a))/c]
```

Sympy [A] time = 1.03413, size = 20, normalized size = 0.67

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+a)**(1/2),x)
```

```
[Out] asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

Giac [A] time = 1.16057, size = 34, normalized size = 1.13

$$-\frac{\log\left(\left|-\sqrt{c}x^2+\sqrt{cx^4+a}\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/sqrt(c)
```

3.1007 $\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]
)
```

Rubi [A] time = 0.0097947, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {4, 220}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```


```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]
)
```


Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.0347523, size = 74, normalized size = 0.84

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
 -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

Maple [C] time = 0.174, size = 70, normalized size = 0.8

$$\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(c*x^4 + a), x)
```

Sympy [C] time = 0.631345, size = 36, normalized size = 0.41

$$\frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

3.1008 $\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=27

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rubi [A] time = 0.0199715, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{1}{x\sqrt{a+cx^4}}\,dx\\ &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^4}\right)}{2c}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0054562, size = 27, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Maple [A] time = 0.158, size = 29, normalized size = 1.1

$$-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{cx^4+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^4+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49163, size = 161, normalized size = 5.96

$$\left[\frac{\log\left(\frac{cx^4-2\,\sqrt{cx^4+a}\sqrt{a}+2\,a}{x^4}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-a}}{a}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4)/sqrt(a), 1/2*sqrt(-
a)*arctan(sqrt(c*x^4 + a)*sqrt(-a)/a)/a]
```

Sympy [A] time = 1.0753, size = 22, normalized size = 0.81

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(c)*x**2))/(2*sqrt(a))
```

Giac [A] time = 1.15403, size = 31, normalized size = 1.15

$$\frac{\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a)
```

3.1009 $\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=232

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+cx^4}}+\frac{\sqrt{c}x}{a\left(\sqrt{\right.}\right.}$$

```
[Out] -(Sqrt[a + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + c*x^4])/(a*(Sqrt[a] + Sqrt[c
]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a
+ c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqr
t[a + c*x^4])
```

Rubi [A] time = 0.0717008, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4, 325, 305, 220, 1196}

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+cx^4}}+\frac{\sqrt{c}x\sqrt{a+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[a + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + c*x^4])/(a*(Sqrt[a] + Sqrt[c
]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a
+ c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqr
t[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{a+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+cx^4}}{ax} + \frac{c\int \frac{x^2}{\sqrt{a+cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+cx^4}}{ax} + \frac{\sqrt{c}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{a}} - \frac{\sqrt{c}\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{a}}$$

$$= -\frac{\sqrt{a+cx^4}}{ax} + \frac{\sqrt{c}x\sqrt{a+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{a^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.0089387, size = 49, normalized size = 0.21

$$-\frac{\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{cx^4}{a}\right)}{x\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -((Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((c*x^4)/a)])/(x*
Sqrt[a + c*x^4]))
```

Maple [C] time = 0.18, size = 115, normalized size = 0.5

$$-\frac{1}{ax}\sqrt{cx^4+a}+i\sqrt{c}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+a)^(1/2),x)
```

```
[Out] -(c*x^4+a)^(1/2)/a/x+I*c^(1/2)/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(Elli
pticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),
I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+a}}{cx^6+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/(c*x^6 + a*x^2), x)
```

Sympy [C] time = 0.717127, size = 39, normalized size = 0.17

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+a)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*x*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^4+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^2), x)
```

3.1010 $\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

```
[Out] -Sqrt[a + c*x^4]/(2*a*x^2)
```

Rubi [A] time = 0.0046456, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4, 264}

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + c*x^4]/(2*a*x^2)
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{1}{x^3\sqrt{a+cx^4}}\,dx\\ &= -\frac{\sqrt{a+cx^4}}{2ax^2}\end{aligned}$$

Mathematica [A] time = 0.0040815, size = 21, normalized size = 1.

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + c*x^4]/(2*a*x^2)
```

Maple [A] time = 0.045, size = 18, normalized size = 0.9

$$-\frac{1}{2\,ax^2}\sqrt{cx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+a)^(1/2),x)
```

```
[Out] -1/2*(c*x^4+a)^(1/2)/a/x^2
```

Maxima [A] time = 0.944374, size = 23, normalized size = 1.1

$$-\frac{\sqrt{cx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^4 + a)/(a*x^2)
```

Fricas [A] time = 1.4529, size = 41, normalized size = 1.95

$$-\frac{\sqrt{cx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^4 + a)/(a*x^2)
```

Sympy [A] time = 0.616605, size = 20, normalized size = 0.95

$$-\frac{\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(c)*sqrt(a/(c*x**4) + 1)/(2*a)
```

Giac [A] time = 1.1882, size = 19, normalized size = 0.9

$$-\frac{\sqrt{c+\frac{a}{x^4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(c + a/x^4)/a
```

3.1011 $\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=110

$$-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3ax^3}$$

```
[Out] -Sqrt[a + c*x^4]/(3*a*x^3) - (c^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(6*a^(5/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.022057, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4, 325, 220}

$$-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + c*x^4]/(3*a*x^3) - (c^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(6*a^(5/4)*Sqrt[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\int \frac{1}{x^4\sqrt{a+cx^4}}\,dx$$
$$=-\frac{\sqrt{a+cx^4}}{3ax^3}-\frac{c\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3a}$$
$$=-\frac{\sqrt{a+cx^4}}{3ax^3}-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.0090398, size = 51, normalized size = 0.46

$$-\frac{\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{cx^4}{a}\right)}{3x^3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((c*x^4)/a)])/(3*x
^3*Sqrt[a + c*x^4])
```

Maple [C] time = 0.174, size = 93, normalized size = 0.9

$$-\frac{1}{3\,ax^3}\sqrt{cx^4+a}-\frac{c}{3\,a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+a)^(1/2),x)
```

```
[Out] -1/3*(c*x^4+a)^(1/2)/a/x^3-1/3*c/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + a}}{cx^8 + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/(c*x^8 + a*x^4), x)
```

Sympy [C] time = 0.849603, size = 41, normalized size = 0.37

$$\frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+a)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*x**3*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^4), x)
```

3.1012 $\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) + (3*a^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] time = 0.022045, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 321, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) + (3*a^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^4}{\sqrt{a+bx^2}}\,dx \\
&= \frac{x^3\sqrt{a+bx^2}}{4b} - \frac{(3a)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b} \\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2} \\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^2} \\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0240006, size = 62, normalized size = 0.85

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\sqrt{b}x\sqrt{a+bx^2}\left(2bx^2-3a\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a + 2*b*x^2) + 3*a^2*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

Maple [A] time = 0.045, size = 59, normalized size = 0.8

$$\frac{x^3}{4\,b}\sqrt{bx^2+a}-\frac{3\,ax}{8\,b^2}\sqrt{bx^2+a}+\frac{3\,a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^3*(b*x^2+a)^(1/2)/b-3/8*a*x*(b*x^2+a)^(1/2)/b^2+3/8/b^(5/2)*a^2*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6038, size = 300, normalized size = 4.11

$$\left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(2\,b^2x^3-3\,abx\right)\sqrt{bx^2+a}}{16\,b^3},-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2x^3-3\,abx\right)\sqrt{}}{8\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] time = 3.776, size = 95, normalized size = 1.3

$$-\frac{3a^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqrt(1 + b*x*
*2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + x**5/(4*sqrt(a)*sqr
t(1 + b*x**2/a))
```

Giac [A] time = 1.22533, size = 73, normalized size = 1.

$$\frac{1}{8}\sqrt{bx^2+a}x\left(\frac{2x^2}{b}-\frac{3a}{b^2}\right)-\frac{3a^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(5/2)
```

3.1013 $\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^2)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rubi [A] time = 0.0225047, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5, 266, 43}

$$\frac{(a+bx^2)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^3}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, x^2\right)\\ &= -\frac{a\sqrt{a+bx^2}}{b^2}+\frac{\left(a+bx^2\right)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.013652, size = 27, normalized size = 0.75

$$\frac{\left(bx^2-2a\right)\sqrt{a+bx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

Maple [A] time = 0.043, size = 25, normalized size = 0.7

$$-\frac{-bx^2+2\,a}{3\,b^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-b*x^2+2*a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45639, size = 53, normalized size = 1.47

$$\frac{\sqrt{bx^2+a}\left(bx^2-2\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

Sympy [A] time = 0.458261, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne
(b, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [A] time = 1.11574, size = 36, normalized size = 1.

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}-3\,\sqrt{bx^2+a}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x^2 + a)^(3/2) - 3*sqrt(b*x^2 + a)*a)/b^2
```

3.1014 $\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Rubi [A] time = 0.0128613, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 321, 217, 206}

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^2}{\sqrt{a+bx^2}}\,dx\\ &= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\ &= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\ &= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0179933, size = 49, normalized size = 1.

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Maple [A] time = 0.045, size = 39, normalized size = 0.8

$$\frac{x}{2\,b}\sqrt{bx^2+a}-\frac{a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64332, size = 238, normalized size = 4.86

$$\left[\frac{2\sqrt{bx^2+a}\,bx + a\sqrt{b}\log\left(-2\,bx^2 + 2\sqrt{bx^2+a}\sqrt{b}x - a\right)}{4\,b^2}, \frac{\sqrt{bx^2+a}\,bx + a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*x + a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b^2]
```

Sympy [A] time = 2.11753, size = 42, normalized size = 0.86

$$\frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b} - \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)
)
```

Giac [A] time = 1.13947, size = 54, normalized size = 1.1

$$\frac{\sqrt{bx^2+a}\,x}{2\,b}+\frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3
/2)
```

3.1015 $\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}} dx$

Optimal. Leaf size=15

$$\frac{\sqrt{a+bx^2}}{b}$$

```
[Out] Sqrt[a + b*x^2]/b
```

Rubi [A] time = 0.0033684, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {5, 261}

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] Sqrt[a + b*x^2]/b
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}} dx &= \int \frac{x}{\sqrt{a+bx^2}} dx\\ &= \frac{\sqrt{a+bx^2}}{b}\end{aligned}$$

Mathematica [A] time = 0.0020604, size = 15, normalized size = 1.

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] Sqrt[a + b*x^2]/b
```

Maple [A] time = 0.041, size = 14, normalized size = 0.9

$$\frac{1}{b}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)/b
```

Maxima [A] time = 0.9383, size = 18, normalized size = 1.2

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)/b
```

Fricas [A] time = 1.5682, size = 26, normalized size = 1.73

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)/b
```

Sympy [A] time = 0.368195, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

Giac [A] time = 1.12855, size = 18, normalized size = 1.2

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b
```

3.1016 $\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0058784, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {5, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0045132, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.042, size = 21, normalized size = 0.8

$$\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x)
```

```
[Out] ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58478, size = 153, normalized size = 6.12

$$\left[\frac{\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A] time = 1.01721, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Giac [A] time = 1.21282, size = 31, normalized size = 1.24

$$-\frac{\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

3.1017 $\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rubi [A] time = 0.01713, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0052582, size = 25, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Maple [A] time = 0.045, size = 29, normalized size = 1.2

$$-\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54177, size = 154, normalized size = 6.16

$$\left[\frac{\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)}{2\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)/sqrt(a), sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a))/a]
```

Sympy [A] time = 1.04702, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Giac [A] time = 1.19971, size = 30, normalized size = 1.2

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)
```

3.1018 $\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a+bx^2}}{ax}$$

```
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Rubi [A] time = 0.0044502, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {5, 264}

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x^2\sqrt{a+bx^2}}\,dx\\ &= -\frac{\sqrt{a+bx^2}}{ax}\end{aligned}$$

Mathematica [A] time = 0.003643, size = 19, normalized size = 1.

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Maple [A] time = 0.043, size = 18, normalized size = 1.

$$-\frac{1}{ax}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] -(b*x^2+a)^(1/2)/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52835, size = 32, normalized size = 1.68

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)/(a*x)
```

Sympy [A] time = 0.57357, size = 19, normalized size = 1.

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Giac [A] time = 1.18817, size = 41, normalized size = 2.16

$$\frac{2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.1019 $\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2
))
```

Rubi [A] time = 0.0269898, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5, 266, 51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2
))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x^3\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a} \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a} \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0476049, size = 61, normalized size = 1.22

$$
\frac{b\sqrt{a+bx^2}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\sqrt{\frac{bx^2}{a}+1}} - \frac{a}{2bx^2}\right)}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*(-a/(2*b*x^2) + ArcTanh[Sqrt[1 + (b*x^2)/a]]/(2*Sqrt[1 +
 (b*x^2)/a])))/a^2
```

Maple [A] time = 0.046, size = 48, normalized size = 1.

$$-\frac{1}{2\,a x^{2}}\sqrt{b x^{2}+a}+\frac{b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^{2}+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5495, size = 263, normalized size = 5.26

$$\left[\frac{\sqrt{a}b x^{2}\log\left(-\frac{b x^{2}+2\,\sqrt{b x^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)-2\,\sqrt{b x^{2}+a}a}{4\,a^{2}x^{2}},-\frac{\sqrt{-a}b x^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^{2}+a}}\right)+\sqrt{b x^{2}+a}a}{2\,a^{2}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*x^2)]
```

Sympy [A] time = 2.22184, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**
(3/2))
```

Giac [A] time = 1.16995, size = 65, normalized size = 1.3

$$-\frac{1}{2}b\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx^2+a}}{abx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*(arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^2 + a)/(a*
b*x^2))
```

3.1020 $\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

```
[Out] -Sqrt[a + b*x^2]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rubi [A] time = 0.0104717, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5, 271, 264}

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x^4\sqrt{a+bx^2}}\,dx\\
&= -\frac{\sqrt{a+bx^2}}{3ax^3}-\frac{(2b)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}\\
&= -\frac{\sqrt{a+bx^2}}{3ax^3}+\frac{2b\sqrt{a+bx^2}}{3a^2x}
\end{aligned}
$$

Mathematica [A] time = 0.0063714, size = 29, normalized size = 0.66

$$
-\frac{\left(a-2bx^2\right)\sqrt{a+bx^2}}{3a^2x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -((a - 2*b*x^2)*Sqrt[a + b*x^2])/(3*a^2*x^3)
```

Maple [A] time = 0.044, size = 26, normalized size = 0.6

$$
-\frac{-2\,bx^2+a}{3\,a^2x^3}\sqrt{bx^2+a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49855, size = 61, normalized size = 1.39

$$\frac{\left(2\,bx^2-a\right)\sqrt{bx^2+a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

Sympy [A] time = 0.753629, size = 46, normalized size = 1.05

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a**2)
```

Giac [A] time = 1.19944, size = 74, normalized size = 1.68

$$\frac{4\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3
```

3.1021 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$\frac{x^5}{3\sqrt{cx^4}}$$

```
[Out] x^5/(3*Sqrt[c*x^4])
```

Rubi [A] time = 0.0015658, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1, 15, 30}

$$\frac{x^5}{3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^5/(3*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^4}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int x^2\,dx}{\sqrt{cx^4}}$$
$$= \frac{x^5}{3\sqrt{cx^4}}$$

Mathematica [A] time = 0.002233, size = 16, normalized size = 1.

$$\frac{x^5}{3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^5/(3*Sqrt[c*x^4])
```

Maple [A] time = 0.044, size = 13, normalized size = 0.8

$$\frac{x^5}{3}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4)^(1/2),x)
```

```
[Out] 1/3*x^5/(c*x^4)^(1/2)
```

Maxima [A] time = 0.955182, size = 16, normalized size = 1.

$$\frac{x^5}{3\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^5/sqrt(c*x^4)
```

Fricas [A] time = 1.26011, size = 28, normalized size = 1.75

$$\frac{\sqrt{cx^4}x}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4)*x/c
```

Sympy [A] time = 0.532797, size = 15, normalized size = 0.94

$$\frac{x^5}{3\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4)**(1/2),x)
```

```
[Out] x**5/(3*sqrt(c)*sqrt(x**4))
```

Giac [A] time = 1.21064, size = 11, normalized size = 0.69

$$\frac{x^3}{3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*x^3/sqrt(c)
```

3.1022 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$\frac{x^4}{2\sqrt{cx^4}}$$

```
[Out] x^4/(2*Sqrt[c*x^4])
```

Rubi [A] time = 0.0017394, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1, 15, 30}

$$\frac{x^4}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^4/(2*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^3}{\sqrt{cx^4}}\,dx = \frac{x^2\int x\,dx}{\sqrt{cx^4}} = \frac{x^4}{2\sqrt{cx^4}}$$

Mathematica [A] time = 0.0016441, size = 16, normalized size = 1.

$$\frac{x^4}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^4/(2*Sqrt[c*x^4])
```

Maple [A] time = 0.042, size = 13, normalized size = 0.8

$$\frac{x^4}{2}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4)^(1/2),x)
```

```
[Out] 1/2*x^4/(c*x^4)^(1/2)
```

Maxima [A] time = 0.946274, size = 16, normalized size = 1.

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^4)/c
```

Fricas [A] time = 1.22158, size = 26, normalized size = 1.62

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^4)/c
```

Sympy [A] time = 0.465126, size = 15, normalized size = 0.94

$$\frac{x^4}{2\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4)**(1/2),x)
```

```
[Out] x**4/(2*sqrt(c)*sqrt(x**4))
```

Giac [A] time = 1.25238, size = 11, normalized size = 0.69

$$\frac{x^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/sqrt(c)
```

3.1023 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=13

$$\frac{x^3}{\sqrt{cx^4}}$$

```
[Out] x^3/Sqrt[c*x^4]
```

Rubi [A] time = 0.0011957, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1, 15, 8}

$$\frac{x^3}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^3/Sqrt[c*x^4]
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^2}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2\int 1\,dx}{\sqrt{cx^4}}$$
$$= \frac{x^3}{\sqrt{cx^4}}$$

Mathematica [A] time = 0.0011859, size = 13, normalized size = 1.

$$\frac{x^3}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^3/Sqrt[c*x^4]
```

Maple [A] time = 0.042, size = 12, normalized size = 0.9

$$x^3\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4)^(1/2),x)
```

```
[Out] x^3/(c*x^4)^(1/2)
```

Maxima [A] time = 0.960865, size = 15, normalized size = 1.15

$$\frac{x^3}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] x^3/sqrt(c*x^4)
```

Fricas [A] time = 1.21399, size = 26, normalized size = 2.

$$\frac{\sqrt{cx^4}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4)/(c*x)
```

Sympy [A] time = 0.420185, size = 14, normalized size = 1.08

$$\frac{x^3}{\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4)**(1/2),x)
```

```
[Out] x**3/(sqrt(c)*sqrt(x**4))
```

Giac [A] time = 1.13143, size = 7, normalized size = 0.54

$$\frac{x}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] x/sqrt(c)
```

3.1024 $\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=15

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

```
[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

Rubi [A] time = 0.0013116, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1, 15, 29}

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x}\,dx}{\sqrt{cx^4}}$$
$$= \frac{x^2 \log(x)}{\sqrt{cx^4}}$$

Mathematica [A] time = 0.0015753, size = 15, normalized size = 1.

$$\frac{x^2 \log(x)}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

Maple [A] time = 0.046, size = 14, normalized size = 0.9

$$x^2 \ln(x) \frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4)^(1/2),x)
```

```
[Out] x^2*ln(x)/(c*x^4)^(1/2)
```

Maxima [A] time = 0.951511, size = 18, normalized size = 1.2

$$\frac{x^2 \log(x)}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] x^2*log(x)/sqrt(c*x^4)
```

Fricas [A] time = 1.29413, size = 38, normalized size = 2.53

$$\frac{\sqrt{cx^4}\log(x)}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4)*log(x)/(c*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(c*x**4), x)
```

Giac [A] time = 1.17966, size = 9, normalized size = 0.6

$$\frac{\log(|x|)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(x))/sqrt(c)
```

3.1025 $\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{cx^4}}$$

[Out] -(x/Sqrt[c*x^4])

Rubi [A] time = 0.0014219, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1, 15, 30}

$$-\frac{x}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]

[Out] -(x/Sqrt[c*x^4])

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x^2}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{x}{\sqrt{cx^4}}$$

Mathematica [A] time = 0.001179, size = 12, normalized size = 1.

$$-\frac{x}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] -(x/Sqrt[c*x^4])
```

Maple [A] time = 0.041, size = 11, normalized size = 0.9

$$-x\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4)^(1/2),x)
```

```
[Out] -x/(c*x^4)^(1/2)
```

Maxima [A] time = 0.948303, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -x/sqrt(c*x^4)
```

Fricas [A] time = 1.32357, size = 30, normalized size = 2.5

$$-\frac{\sqrt{cx^4}}{cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4)/(c*x^3)
```

Sympy [A] time = 0.383548, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4)**(1/2),x)
```

```
[Out] -x/(sqrt(c)*sqrt(x**4))
```

Giac [A] time = 1.17937, size = 11, normalized size = 0.92

$$-\frac{1}{\sqrt{c}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(c)*x)
```

3.1026 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2\sqrt{cx^4}}$$

```
[Out] -1/(2*Sqrt[c*x^4])
```

Rubi [A] time = 0.0013866, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1, 15, 30}

$$-\frac{1}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(2*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x^3}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{2\sqrt{cx^4}}$$

Mathematica [A] time = 0.0012897, size = 13, normalized size = 1.

$$-\frac{1}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(2*Sqrt[c*x^4])
```

Maple [A] time = 0.042, size = 10, normalized size = 0.8

$$-\frac{1}{2}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4)^(1/2),x)
```

```
[Out] -1/2/(c*x^4)^(1/2)
```

Maxima [A] time = 0.940772, size = 12, normalized size = 0.92

$$-\frac{1}{2\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2/sqrt(c*x^4)
```

Fricas [A] time = 1.27602, size = 35, normalized size = 2.69

$$-\frac{\sqrt{cx^4}}{2\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^4)/(c*x^4)
```

Sympy [A] time = 0.422664, size = 15, normalized size = 1.15

$$-\frac{1}{2\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4)**(1/2),x)
```

```
[Out] -1/(2*sqrt(c)*sqrt(x**4))
```

Giac [A] time = 1.18627, size = 11, normalized size = 0.85

$$-\frac{1}{2\,\sqrt{c}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2/(sqrt(c)*x^2)
```

3.1027 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3x\sqrt{cx^4}}$$

```
[Out] -1/(3*x*Sqrt[c*x^4])
```

Rubi [A] time = 0.0014903, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1, 15, 30}

$$-\frac{1}{3x\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(3*x*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{cx^4}}\,dx$$
$$= \frac{x^2 \int \frac{1}{x^4}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{3x\sqrt{cx^4}}$$

Mathematica [A] time = 0.0015786, size = 16, normalized size = 1.

$$-\frac{1}{3x\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(3*x*Sqrt[c*x^4])
```

Maple [A] time = 0.041, size = 13, normalized size = 0.8

$$-\frac{1}{3\,x}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4)^(1/2),x)
```

```
[Out] -1/3/x/(c*x^4)^(1/2)
```

Maxima [A] time = 0.947221, size = 16, normalized size = 1.

$$-\frac{1}{3\sqrt{cx^4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3/(sqrt(c*x^4)*x)
```

Fricas [A] time = 1.19101, size = 35, normalized size = 2.19

$$-\frac{\sqrt{cx^4}}{3\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(c*x^4)/(c*x^5)
```

Sympy [A] time = 0.482531, size = 17, normalized size = 1.06

$$-\frac{1}{3\sqrt{c}x\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4)**(1/2),x)
```

```
[Out] -1/(3*sqrt(c)*x*sqrt(x**4))
```

Giac [A] time = 1.15646, size = 11, normalized size = 0.69

$$-\frac{1}{3\,\sqrt{c}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3/(sqrt(c)*x^3)
```

3.1028 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

```
[Out] -1/(4*x^2*Sqrt[c*x^4])
```

Rubi [A] time = 0.0016737, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1, 15, 30}

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(4*x^2*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x^3\sqrt{cx^4}}\,dx$$
$$= \frac{x^2\int \frac{1}{x^5}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{4x^2\sqrt{cx^4}}$$

Mathematica [A] time = 0.0030231, size = 17, normalized size = 1.06

$$-\frac{cx^2}{4\left(cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -(c*x^2)/(4*(c*x^4)^(3/2))
```

Maple [A] time = 0.042, size = 13, normalized size = 0.8

$$-\frac{1}{4\,x^2}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4)^(1/2),x)
```

```
[Out] -1/4/x^2/(c*x^4)^(1/2)
```

Maxima [A] time = 0.955371, size = 16, normalized size = 1.

$$-\frac{1}{4\,\sqrt{cx^4}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4/(sqrt(c*x^4)*x^2)
```

Fricas [A] time = 1.25511, size = 35, normalized size = 2.19

$$-\frac{\sqrt{cx^4}}{4\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(c*x^4)/(c*x^6)
```

Sympy [A] time = 0.546345, size = 19, normalized size = 1.19

$$-\frac{1}{4\sqrt{c}x^2\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4)**(1/2),x)
```

```
[Out] -1/(4*sqrt(c)*x**2*sqrt(x**4))
```

Giac [A] time = 1.19921, size = 11, normalized size = 0.69

$$-\frac{1}{4\,\sqrt{c}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4/(sqrt(c)*x^4)
```

3.1029 $\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

```
[Out] -1/(5*x^3*Sqrt[c*x^4])
```

Rubi [A] time = 0.0016077, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1, 15, 30}

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(5*x^3*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{1}{x^4\sqrt{cx^4}}\,dx$$
$$= \frac{x^2\int \frac{1}{x^6}\,dx}{\sqrt{cx^4}}$$
$$= -\frac{1}{5x^3\sqrt{cx^4}}$$

Mathematica [A] time = 0.0018054, size = 15, normalized size = 0.94

$$-\frac{cx}{5\left(cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -(c*x)/(5*(c*x^4)^(3/2))
```

Maple [A] time = 0.041, size = 13, normalized size = 0.8

$$-\frac{1}{5\,x^3}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4)^(1/2),x)
```

```
[Out] -1/5/x^3/(c*x^4)^(1/2)
```

Maxima [A] time = 0.943107, size = 16, normalized size = 1.

$$-\frac{1}{5\,\sqrt{cx^4}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5/(sqrt(c*x^4)*x^3)
```

Fricas [A] time = 1.31814, size = 35, normalized size = 2.19

$$-\frac{\sqrt{cx^4}}{5\,cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(c*x^4)/(c*x^7)
```

Sympy [A] time = 0.623809, size = 19, normalized size = 1.19

$$-\frac{1}{5\sqrt{c}x^3\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4)**(1/2),x)
```

```
[Out] -1/(5*sqrt(c)*x**3*sqrt(x**4))
```

Giac [A] time = 1.12828, size = 11, normalized size = 0.69

$$-\frac{1}{5\,\sqrt{c}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/5/(sqrt(c)*x^5)
```

3.1030 $\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^5}{5\sqrt{a}}$$

```
[Out] x^5/(5*Sqrt[a])
```

Rubi [A] time = 0.0014659, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$\frac{x^5}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^5/(5*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^4}{\sqrt{a}}\,dx$$
$$= \frac{\int x^4\,dx}{\sqrt{a}}$$
$$= \frac{x^5}{5\sqrt{a}}$$

Mathematica [A] time = 0.0006259, size = 12, normalized size = 1.

$$\frac{x^5}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^5/(5*Sqrt[a])
```

Maple [A] time = 0.047, size = 9, normalized size = 0.8

$$\frac{x^5}{5}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/a^(1/2),x)
```

```
[Out] 1/5*x^5/a^(1/2)
```

Maxima [A] time = 0.948889, size = 11, normalized size = 0.92

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^5/sqrt(a)
```

Fricas [A] time = 1.2801, size = 23, normalized size = 1.92

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*x^5/sqrt(a)
```

Sympy [A] time = 0.054869, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/a**(1/2),x)
```

```
[Out] x**5/(5*sqrt(a))
```

Giac [A] time = 1.13269, size = 11, normalized size = 0.92

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*x^5/sqrt(a)
```

3.1031 $\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^4}{4\sqrt{a}}$$

```
[Out] x^4/(4*Sqrt[a])
```

Rubi [A] time = 0.0013625, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$\frac{x^4}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x^4/(4*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^3}{\sqrt{a}}\,dx$$
$$= \frac{\int x^3\,dx}{\sqrt{a}}$$
$$= \frac{x^4}{4\sqrt{a}}$$

Mathematica [A] time = 0.0005364, size = 12, normalized size = 1.

$$\frac{x^4}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^4/(4*Sqrt[a])
```

Maple [A] time = 0.04, size = 9, normalized size = 0.8

$$\frac{x^4}{4}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/a^(1/2),x)
```

```
[Out] 1/4*x^4/a^(1/2)
```

Maxima [A] time = 0.949651, size = 11, normalized size = 0.92

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/sqrt(a)
```

Fricas [A] time = 1.24138, size = 23, normalized size = 1.92

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4/sqrt(a)
```

Sympy [A] time = 0.055062, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/a**(1/2),x)
```

```
[Out] x**4/(4*sqrt(a))
```

Giac [A] time = 1.17581, size = 11, normalized size = 0.92

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*x^4/sqrt(a)
```

3.1032 $\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^3}{3\sqrt{a}}$$

```
[Out] x^3/(3*Sqrt[a])
```

Rubi [A] time = 0.0014377, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$\frac{x^3}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^3/(3*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^2}{\sqrt{a}}\,dx$$
$$= \frac{\int x^2\,dx}{\sqrt{a}}$$
$$= \frac{x^3}{3\sqrt{a}}$$

Mathematica [A] time = 0.0005254, size = 12, normalized size = 1.

$$\frac{x^3}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^3/(3*Sqrt[a])
```

Maple [A] time = 0.041, size = 9, normalized size = 0.8

$$\frac{x^3}{3}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/a^(1/2),x)
```

```
[Out] 1/3*x^3/a^(1/2)
```

Maxima [A] time = 0.953583, size = 11, normalized size = 0.92

$$\frac{x^3}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/sqrt(a)
```

Fricas [A] time = 1.253, size = 23, normalized size = 1.92

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*x^3/sqrt(a)
```

Sympy [A] time = 0.055926, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/a**(1/2),x)
```

```
[Out] x**3/(3*sqrt(a))
```

Giac [A] time = 1.16584, size = 11, normalized size = 0.92

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*x^3/sqrt(a)
```

3.1033 $\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^2}{2\sqrt{a}}$$

```
[Out] x^2/(2*Sqrt[a])
```

Rubi [A] time = 0.0012983, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2, 12, 30}

$$\frac{x^2}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x^2/(2*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x}{\sqrt{a}}\,dx$$
$$= \frac{\int x\,dx}{\sqrt{a}}$$
$$= \frac{x^2}{2\sqrt{a}}$$

Mathematica [A] time = 0.0004321, size = 12, normalized size = 1.

$$\frac{x^2}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^2/(2*Sqrt[a])
```

Maple [A] time = 0.04, size = 9, normalized size = 0.8

$$\frac{x^2}{2}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/a^(1/2),x)
```

```
[Out] 1/2*x^2/a^(1/2)
```

Maxima [A] time = 0.951067, size = 11, normalized size = 0.92

$$\frac{x^2}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/sqrt(a)
```

Fricas [A] time = 1.20608, size = 23, normalized size = 1.92

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2/sqrt(a)
```

Sympy [A] time = 0.054073, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a**(1/2),x)
```

```
[Out] x**2/(2*sqrt(a))
```

Giac [A] time = 1.16745, size = 11, normalized size = 0.92

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/sqrt(a)
```

3.1034 $\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=7

$$\frac{x}{\sqrt{a}}$$

```
[Out] x/Sqrt[a]
```

Rubi [A] time = 0.000861, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2, 8}

$$\frac{x}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x/Sqrt[a]
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}}\,dx\\ &= \frac{x}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0003813, size = 7, normalized size = 1.

$$\frac{x}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x/Sqrt[a]
```

Maple [A] time = 0.039, size = 6, normalized size = 0.9

$$x\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a^(1/2),x)
```

```
[Out] x/a^(1/2)
```

Maxima [A] time = 0.949851, size = 7, normalized size = 1.

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(a)
```

Fricas [A] time = 1.18099, size = 15, normalized size = 2.14

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="fricas")
```

```
[Out] x/sqrt(a)
```

Sympy [A] time = 0.049509, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a**(1/2),x)
```

```
[Out] x/sqrt(a)
```

Giac [A] time = 1.15081, size = 7, normalized size = 1.

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="giac")
```

```
[Out] x/sqrt(a)
```

3.1035 $\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=8

$$\frac{\log(x)}{\sqrt{a}}$$

```
[Out] Log[x]/Sqrt[a]
```

Rubi [A] time = 0.001119, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 29}

$$\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] Log[x]/Sqrt[a]
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a}x}\,dx$$
$$= \frac{\int \frac{1}{x}\,dx}{\sqrt{a}}$$
$$= \frac{\log(x)}{\sqrt{a}}$$

Mathematica [A] time = 0.0004301, size = 8, normalized size = 1.

$$\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] Log[x]/Sqrt[a]
```

Maple [A] time = 0.04, size = 7, normalized size = 0.9

$$\ln(x)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/a^(1/2),x)
```

```
[Out] ln(x)/a^(1/2)
```

Maxima [A] time = 0.944304, size = 8, normalized size = 1.

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="maxima")
```

```
[Out] log(x)/sqrt(a)
```

Fricas [A] time = 1.25804, size = 22, normalized size = 2.75

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="fricas")
```

```
[Out] log(x)/sqrt(a)
```

Sympy [A] time = 0.061173, size = 7, normalized size = 0.88

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a**(1/2),x)
```

```
[Out] log(x)/sqrt(a)
```

Giac [A] time = 1.21322, size = 9, normalized size = 1.12

$$\frac{\log(|x|)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(x))/sqrt(a)
```

3.1036 $\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{\sqrt{a}x}$$

```
[Out] -(1/(Sqrt[a]*x))
```

Rubi [A] time = 0.0014484, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$-\frac{1}{\sqrt{a}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(1/(Sqrt[a]*x))
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a}x^2}\,dx$$
$$= \frac{\int \frac{1}{x^2}\,dx}{\sqrt{a}}$$
$$= -\frac{1}{\sqrt{a}x}$$

Mathematica [A] time = 0.0004757, size = 10, normalized size = 1.

$$-\frac{1}{\sqrt{a}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -(1/(Sqrt[a]*x))
```

Maple [A] time = 0.04, size = 9, normalized size = 0.9

$$-\frac{1}{x}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/a^(1/2),x)

[Out] -1/x/a^(1/2)
```

Maxima [A] time = 0.950732, size = 11, normalized size = 1.1

$$-\frac{1}{\sqrt{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(a)*x)
```

Fricas [A] time = 1.125, size = 22, normalized size = 2.2

$$-\frac{1}{\sqrt{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="fricas")
```

```
[Out] -1/(sqrt(a)*x)
```

Sympy [A] time = 0.061107, size = 8, normalized size = 0.8

$$-\frac{1}{\sqrt{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/a**(1/2),x)
```

```
[Out] -1/(sqrt(a)*x)
```

Giac [A] time = 1.20013, size = 11, normalized size = 1.1

$$-\frac{1}{\sqrt{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(a)*x)
```

3.1037 $\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{2\sqrt{a}x^2}$$

```
[Out] -1/(2*Sqrt[a]*x^2)
```

Rubi [A] time = 0.001611, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$-\frac{1}{2\sqrt{a}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/(2*Sqrt[a]*x^2)
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a}x^3}\,dx$$
$$= \frac{\int \frac{1}{x^3}\,dx}{\sqrt{a}}$$
$$= -\frac{1}{2\sqrt{a}x^2}$$

Mathematica [A] time = 0.0004923, size = 12, normalized size = 1.

$$-\frac{1}{2\sqrt{a}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/(2*Sqrt[a]*x^2)
```

Maple [A] time = 0.04, size = 9, normalized size = 0.8

$$-\frac{1}{2x^2}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/a^(1/2),x)
```

```
[Out] -1/2/x^2/a^(1/2)
```

Maxima [A] time = 0.946884, size = 11, normalized size = 0.92

$$-\frac{1}{2\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2/(sqrt(a)*x^2)
```

Fricas [A] time = 1.28153, size = 27, normalized size = 2.25

$$-\frac{1}{2\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2/(sqrt(a)*x^2)
```

Sympy [A] time = 0.063928, size = 12, normalized size = 1.

$$-\frac{1}{2\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/a**(1/2),x)
```

```
[Out] -1/(2*sqrt(a)*x**2)
```

Giac [A] time = 1.20006, size = 11, normalized size = 0.92

$$-\frac{1}{2\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="giac")
```

```
[Out] -1/2/(sqrt(a)*x^2)
```

3.1038 $\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{3\sqrt{a}x^3}$$

[Out] -1/(3*Sqrt[a]*x^3)

Rubi [A] time = 0.0014429, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 30}

$$-\frac{1}{3\sqrt{a}x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -1/(3*Sqrt[a]*x^3)

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a}x^4}\,dx$$
$$= \frac{\int \frac{1}{x^4}\,dx}{\sqrt{a}}$$
$$= -\frac{1}{3\sqrt{a}x^3}$$

Mathematica [A] time = 0.0004325, size = 12, normalized size = 1.

$$-\frac{1}{3\sqrt{a}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/(3*Sqrt[a]*x^3)
```

Maple [A] time = 0.05, size = 9, normalized size = 0.8

$$-\frac{1}{3\,x^3}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/a^(1/2),x)
```

```
[Out] -1/3/x^3/a^(1/2)
```

Maxima [A] time = 0.940013, size = 11, normalized size = 0.92

$$-\frac{1}{3\,\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3/(sqrt(a)*x^3)
```

Fricas [A] time = 1.20896, size = 27, normalized size = 2.25

$$-\frac{1}{3\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3/(sqrt(a)*x^3)
```

Sympy [A] time = 0.062202, size = 12, normalized size = 1.

$$-\frac{1}{3\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/a**(1/2),x)
```

```
[Out] -1/(3*sqrt(a)*x**3)
```

Giac [A] time = 1.22938, size = 11, normalized size = 0.92

$$-\frac{1}{3\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="giac")
```

```
[Out] -1/3/(sqrt(a)*x^3)
```

3.1039 $\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{\text{EllipticF}\left(\sin^{-1}(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rubi [A] time = 0.0103334, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 419}

$$\frac{F\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx = 2\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.0174583, size = 18, normalized size = 1.5

$$-i\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right),-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

Maple [B] time = 0.044, size = 43, normalized size = 3.6

$$\frac{\text{EllipticF}\left(x,\frac{i}{3}\sqrt{3}\right)}{3}\sqrt{-x^2+1}\sqrt{3\,x^2+9}\frac{1}{\sqrt{-x^4-2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4-2*x^2+3)^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4-2\,x^2+3}}{x^4+2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 - 2*x^2 + 3)/(x^4 + 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4-2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 - 2*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

3.1040 $\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx$

Optimal. Leaf size=39

$$-\frac{\text{EllipticF}\left(\cos^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right),\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

```
[Out] -(EllipticF[ArcCos[Sqrt[2/(5 + Sqrt[21])]*x], (21 + 5*Sqrt[21])/42]/21^(1/4
))
```

Rubi [A] time = 0.0725563, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1095, 420}

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + 5*x^2 - x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[2/(5 + Sqrt[21])]*x], (21 + 5*Sqrt[21])/42]/21^(1/4
))
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = 2\int \frac{1}{\sqrt{5+\sqrt{21}-2x^2}\sqrt{-5+\sqrt{21}+2x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

Mathematica [B] time = 0.107016, size = 87, normalized size = 2.23

$$\frac{\sqrt{-2x^2-\sqrt{21}+5}\sqrt{\left(\sqrt{21}-5\right)x^2+2}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right),\frac{23}{2}-\frac{5\sqrt{21}}{2}\right)}{2\sqrt{-x^4+5x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + 5*x^2 - x^4],x]
```

```
[Out] (Sqrt[5 - Sqrt[21] - 2*x^2]*Sqrt[2 + (-5 + Sqrt[21])*x^2]*EllipticF[ArcSin[
Sqrt[(5 + Sqrt[21])/2]*x], 23/2 - (5*Sqrt[21])/2])/(2*Sqrt[-1 + 5*x^2 - x^4
])
```

Maple [A] time = 0.209, size = 82, normalized size = 2.1

$$\frac{\text{EllipticF}\left(x\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right),\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}}\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\sqrt{1-\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)x^2}\frac{1}{\sqrt{-x^4+5\,x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+5*x^2-1)^(1/2),x)
```

```
[Out] 1/(1/2*7^(1/2)-1/2*3^(1/2))*(1-(5/2-1/2*21^(1/2))*x^2)^(1/2)*(1-(5/2+1/2*21
^(1/2))*x^2)^(1/2)/(-x^4+5*x^2-1)^(1/2)*EllipticF(x*(1/2*7^(1/2)-1/2*3^(1/2
)),5/2+1/2*21^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+5\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4+5\,x^2-1}}{x^4-5\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 5*x^2 - 1)/(x^4 - 5*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+5x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+5*x**2-1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + 5*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+5\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

3.1041 $\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=31

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rubi [A] time = 0.0061682, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx &= \int \left(ax^{5/2}+bx^{9/2}+cx^{13/2}\right)\,dx\\ &= \frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.006172, size = 25, normalized size = 0.81

$$\frac{2x^{7/2}\left(165a+105bx^2+77cx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(7/2)*(165*a + 105*b*x^2 + 77*c*x^4))/1155
```

Maple [A] time = 0.045, size = 22, normalized size = 0.7

$$\frac{154\,c x^{4}+210\,b x^{2}+330\,a}{1155}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 2/1155*x^(7/2)*(77*c*x^4+105*b*x^2+165*a)
```

Maxima [A] time = 0.990398, size = 26, normalized size = 0.84

$$\frac{2}{15}\,c x^{\frac{15}{2}}+\frac{2}{11}\,b x^{\frac{11}{2}}+\frac{2}{7}\,a x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Fricas [A] time = 1.22206, size = 69, normalized size = 2.23

$$\frac{2}{1155}\left(77\,c x^{7}+105\,b x^{5}+165\,a x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*c*x^7 + 105*b*x^5 + 165*a*x^3)*sqrt(x)
```

Sympy [A] time = 5.76865, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{11}{2}}}{11} + \frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

Giac [A] time = 1.1642, size = 26, normalized size = 0.84

$$\frac{2}{15}\,cx^{\frac{15}{2}} + \frac{2}{11}\,bx^{\frac{11}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

3.1042 $\int x^{3/2} \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=31

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rubi [A] time = 0.0065908, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)dx &= \int \left(ax^{3/2}+bx^{7/2}+cx^{11/2}\right)dx\\ &= \frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0064249, size = 25, normalized size = 0.81

$$\frac{2}{585}x^{5/2}\left(117a+65bx^2+45cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(5/2)*(117*a + 65*b*x^2 + 45*c*x^4))/585
```

Maple [A] time = 0.043, size = 22, normalized size = 0.7

$$\frac{90\,cx^4+130\,bx^2+234\,a}{585}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 2/585*x^(5/2)*(45*c*x^4+65*b*x^2+117*a)
```

Maxima [A] time = 0.964942, size = 26, normalized size = 0.84

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Fricas [A] time = 1.30514, size = 66, normalized size = 2.13

$$\frac{2}{585}\left(45\,cx^6+65\,bx^4+117\,ax^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/585*(45*c*x^6 + 65*b*x^4 + 117*a*x^2)*sqrt(x)
```

Sympy [A] time = 2.73244, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{9}{2}}}{9} + \frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

Giac [A] time = 1.15261, size = 26, normalized size = 0.84

$$\frac{2}{13}\,cx^{\frac{13}{2}} + \frac{2}{9}\,bx^{\frac{9}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

3.1043 $\int \sqrt{x}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=31

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rubi [A] time = 0.0066384, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right)\,dx &= \int \left(a\sqrt{x}+bx^{5/2}+cx^{9/2}\right)\,dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0061114, size = 25, normalized size = 0.81

$$\frac{2}{231}x^{3/2}\left(77a+33bx^2+21cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2)*(77*a + 33*b*x^2 + 21*c*x^4))/231
```

Maple [A] time = 0.042, size = 22, normalized size = 0.7

$$\frac{42\,cx^{4}+66\,bx^{2}+154\,a}{231}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 2/231*x^(3/2)*(21*c*x^4+33*b*x^2+77*a)
```

Maxima [A] time = 0.945647, size = 26, normalized size = 0.84

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Fricas [A] time = 1.22361, size = 62, normalized size = 2.

$$\frac{2}{231}\left(21\,cx^{5}+33\,bx^{3}+77\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/231*(21*c*x^5 + 33*b*x^3 + 77*a*x)*sqrt(x)
```

Sympy [A] time = 1.69798, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{3}{2}}}{3} + \frac{2bx^{\frac{7}{2}}}{7} + \frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

Giac [A] time = 1.17217, size = 26, normalized size = 0.84

$$\frac{2}{11}\,cx^{\frac{11}{2}} + \frac{2}{7}\,bx^{\frac{7}{2}} + \frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

3.1044 $\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rubi [A] time = 0.0062927, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx &= \int \left(\frac{a}{\sqrt{x}}+bx^{3/2}+cx^{7/2}\right)dx\\ &= 2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0063804, size = 25, normalized size = 0.86

$$\frac{2}{45}\sqrt{x}\left(45a+9bx^2+5cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(45*a + 9*b*x^2 + 5*c*x^4))/45
```

Maple [A] time = 0.043, size = 22, normalized size = 0.8

$$\frac{10\,c x^{4}+18\,b x^{2}+90\,a}{45}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(1/2),x)
```

```
[Out] 2/45*x^(1/2)*(5*c*x^4+9*b*x^2+45*a)
```

Maxima [A] time = 0.946927, size = 26, normalized size = 0.9

$$\frac{2}{9}\,c x^{\frac{9}{2}}+\frac{2}{5}\,b x^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Fricas [A] time = 1.20527, size = 55, normalized size = 1.9

$$\frac{2}{45}\left(5\,c x^{4}+9\,b x^{2}+45\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*c*x^4 + 9*b*x^2 + 45*a)*sqrt(x)
```

Sympy [A] time = 0.716301, size = 27, normalized size = 0.93

$$2a\sqrt{x} + \frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

Giac [A] time = 1.16637, size = 26, normalized size = 0.9

$$\frac{2}{9}\,cx^{\frac{9}{2}} + \frac{2}{5}\,bx^{\frac{5}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

3.1045 $\int \frac{a+bx^2+cx^4}{x^{3/2}} dx$

Optimal. Leaf size=29

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rubi [A] time = 0.0062198, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx&=\int \left(\frac{a}{x^{3/2}}+b\sqrt{x}+cx^{5/2}\right)dx\\&=-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0082624, size = 25, normalized size = 0.86

$$\frac{2\left(-21a+7bx^2+3cx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*(-21*a + 7*b*x^2 + 3*c*x^4))/(21*Sqrt[x])
```

Maple [A] time = 0.045, size = 22, normalized size = 0.8

$$-\frac{-6\,c x^{4}-14\,b x^{2}+42\,a}{21}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(3/2),x)
```

```
[Out] -2/21*(-3*c*x^4-7*b*x^2+21*a)/x^(1/2)
```

Maxima [A] time = 0.948077, size = 26, normalized size = 0.9

$$\frac{2}{7}\,c x^{\frac{7}{2}}+\frac{2}{3}\,b x^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Fricas [A] time = 1.21412, size = 55, normalized size = 1.9

$$\frac{2\left(3\,c x^{4}+7\,b x^{2}-21\,a\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*c*x^4 + 7*b*x^2 - 21*a)/sqrt(x)
```

Sympy [A] time = 0.969157, size = 27, normalized size = 0.93

$$-\frac{2a}{\sqrt{x}}+\frac{2bx^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

Giac [A] time = 1.14864, size = 26, normalized size = 0.9

$$\frac{2}{7}cx^{\frac{7}{2}}+\frac{2}{3}bx^{\frac{3}{2}}-\frac{2a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

3.1046 $\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rubi [A] time = 0.0059993, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{\sqrt{x}}+cx^{3/2}\right)dx\\ &= -\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0082028, size = 25, normalized size = 0.86

$$\frac{2\left(-5a+15bx^2+3cx^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] (2*(-5*a + 15*b*x^2 + 3*c*x^4))/(15*x^(3/2))
```

Maple [A] time = 0.044, size = 22, normalized size = 0.8

$$-\frac{-6\,c x^4-30\,b x^2+10\,a}{15}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(5/2),x)
```

```
[Out] -2/15*(-3*c*x^4-15*b*x^2+5*a)/x^(3/2)
```

Maxima [A] time = 0.948782, size = 26, normalized size = 0.9

$$\frac{2}{5}\,c x^{\frac{5}{2}}+2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Fricas [A] time = 1.24963, size = 55, normalized size = 1.9

$$\frac{2\left(3\,c x^4+15\,b x^2-5\,a\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*x^4 + 15*b*x^2 - 5*a)/x^(3/2)
```

Sympy [A] time = 1.21285, size = 27, normalized size = 0.93

$$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) + 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

Giac [A] time = 1.17868, size = 26, normalized size = 0.9

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

3.1047 $\int \frac{a+bx^2+cx^4}{x^{7/2}} dx$

Optimal. Leaf size=29

$$-\frac{2a}{5x^{5/2}} - \frac{2b}{\sqrt{x}} + \frac{2}{3}cx^{3/2}$$

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rubi [A] time = 0.0062878, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{2a}{5x^{5/2}} - \frac{2b}{\sqrt{x}} + \frac{2}{3}cx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{x^{7/2}} dx &= \int \left(\frac{a}{x^{7/2}} + \frac{b}{x^{3/2}} + c\sqrt{x}\right) dx \\ &= -\frac{2a}{5x^{5/2}} - \frac{2b}{\sqrt{x}} + \frac{2}{3}cx^{3/2} \end{aligned}$$

Mathematica [A] time = 0.007854, size = 25, normalized size = 0.86

$$\frac{2\left(-3a-15bx^2+5cx^4\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (2*(-3*a - 15*b*x^2 + 5*c*x^4))/(15*x^(5/2))
```

Maple [A] time = 0.045, size = 22, normalized size = 0.8

$$-\frac{-10\,cx^4+30\,bx^2+6\,a}{15}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(7/2),x)
```

```
[Out] -2/15*(-5*c*x^4+15*b*x^2+3*a)/x^(5/2)
```

Maxima [A] time = 0.952902, size = 27, normalized size = 0.93

$$\frac{2}{3}cx^{\frac{3}{2}}-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

Fricas [A] time = 1.25229, size = 55, normalized size = 1.9

$$\frac{2\left(5\,cx^4-15\,bx^2-3\,a\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*c*x^4 - 15*b*x^2 - 3*a)/x^(5/2)
```

Sympy [A] time = 1.88949, size = 27, normalized size = 0.93

$$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}+\frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(7/2),x)
```

```
[Out] -2*a/(5*x**(5/2)) - 2*b/sqrt(x) + 2*c*x**(3/2)/3
```

Giac [A] time = 1.13365, size = 27, normalized size = 0.93

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\left(5\,bx^{2}+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

3.1048 $\int x^{5/2} \left(a + bx^2 + cx^4\right)^2 \, dx$

Optimal. Leaf size=64

$$\frac{2}{7}a^2x^{7/2} + \frac{2}{15}x^{15/2}\left(2ac + b^2\right) + \frac{4}{11}abx^{11/2} + \frac{4}{19}bcx^{19/2} + \frac{2}{23}c^2x^{23/2}$$

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/2))/15 + (
4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rubi [A] time = 0.0229386, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$\frac{2}{7}a^2x^{7/2} + \frac{2}{15}x^{15/2}\left(2ac + b^2\right) + \frac{4}{11}abx^{11/2} + \frac{4}{19}bcx^{19/2} + \frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/2))/15 + (
4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2+cx^4\right)^2 dx &= \int \left(a^2x^{5/2} + 2abx^{9/2} + \left(b^2+2ac\right)x^{13/2} + 2bcx^{17/2} + c^2x^{21/2}\right) dx\\ &= \frac{2}{7}a^2x^{7/2} + \frac{4}{11}abx^{11/2} + \frac{2}{15}\left(b^2+2ac\right)x^{15/2} + \frac{4}{19}bcx^{19/2} + \frac{2}{23}c^2x^{23/2}\end{aligned}$$

Mathematica [A] time = 3.54703, size = 64, normalized size = 1.

$$\frac{2}{7}a^2x^{7/2} + \frac{2}{15}x^{15/2}\left(2ac + b^2\right) + \frac{4}{11}abx^{11/2} + \frac{4}{19}bcx^{19/2} + \frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/2))/15 + (
4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Maple [A] time = 0.047, size = 49, normalized size = 0.8

$$\frac{43890\,c^2x^8+106260\,bcx^6+134596\,x^4ac+67298\,b^2x^4+183540\,abx^2+144210\,a^2}{504735}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/504735*x^(7/2)*(21945*c^2*x^8+53130*b*c*x^6+67298*a*c*x^4+33649*b^2*x^4+9
1770*a*b*x^2+72105*a^2)
```

Maxima [A] time = 0.974124, size = 59, normalized size = 0.92

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\left(b^2+2\,ac\right)x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*(b^2 + 2*a*c)*x^(15/2) + 4/11*
a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Fricas [A] time = 1.20814, size = 147, normalized size = 2.3

$$\frac{2}{504735}\left(21945\,c^2x^{11}+53130\,bcx^9+33649\left(b^2+2\,ac\right)x^7+91770\,abx^5+72105\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*c^2*x^11 + 53130*b*c*x^9 + 33649*(b^2 + 2*a*c)*x^7 + 91770*
a*b*x^5 + 72105*a^2*x^3)*sqrt(x)
```

Sympy [A] time = 21.3145, size = 70, normalized size = 1.09

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{4acx^{\frac{15}{2}}}{15}+\frac{2b^2x^{\frac{15}{2}}}{15}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 4*a*c*x**(15/2)/15 + 2*b**2*x**(15
/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

Giac [A] time = 1.18456, size = 62, normalized size = 0.97

$$\frac{2}{23}c^2x^{\frac{23}{2}}+\frac{4}{19}bcx^{\frac{19}{2}}+\frac{2}{15}b^2x^{\frac{15}{2}}+\frac{4}{15}acx^{\frac{15}{2}}+\frac{4}{11}abx^{\frac{11}{2}}+\frac{2}{7}a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2) + 4/15*a*c*x^(15/
2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

3.1049 $\int x^{3/2} \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=64

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*(b^2 + 2*a*c)*x^(13/2))/13 + (4*
b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rubi [A] time = 0.0222693, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*(b^2 + 2*a*c)*x^(13/2))/13 + (4*
b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2x^{3/2}+2abx^{7/2}+\left(b^2+2ac\right)x^{11/2}+2bcx^{15/2}+c^2x^{19/2}\right) dx\\ &= \frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}\left(b^2+2ac\right)x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.0563089, size = 66, normalized size = 1.03

$$2\left(\frac{1}{5}a^2x^{5/2}+\frac{1}{13}x^{13/2}\left(2ac+b^2\right)+\frac{2}{9}abx^{9/2}+\frac{2}{17}bcx^{17/2}+\frac{1}{21}c^2x^{21/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] 2*((a^2*x^(5/2))/5 + (2*a*b*x^(9/2))/9 + ((b^2 + 2*a*c)*x^(13/2))/13 + (2*b
*c*x^(17/2))/17 + (c^2*x^(21/2))/21)
```

Maple [A] time = 0.046, size = 49, normalized size = 0.8

$$\frac{6630\,c^2x^8+16380\,bcx^6+21420\,x^4ac+10710\,b^2x^4+30940\,abx^2+27846\,a^2}{69615}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/69615*x^(5/2)*(3315*c^2*x^8+8190*b*c*x^6+10710*a*c*x^4+5355*b^2*x^4+15470
*a*b*x^2+13923*a^2)
```

Maxima [A] time = 0.954168, size = 59, normalized size = 0.92

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\left(b^2+2\,ac\right)x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*(b^2 + 2*a*c)*x^(13/2) + 4/9*a
*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Fricas [A] time = 1.32872, size = 142, normalized size = 2.22

$$\frac{2}{69615}\left(3315\,c^2x^{10}+8190\,bcx^8+5355\left(b^2+2\,ac\right)x^6+15470\,abx^4+13923\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*c^2*x^10 + 8190*b*c*x^8 + 5355*(b^2 + 2*a*c)*x^6 + 15470*a*b*
x^4 + 13923*a^2*x^2)*sqrt(x)
```

Sympy [A] time = 11.8301, size = 70, normalized size = 1.09

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{4acx^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 4*a*c*x**(13/2)/13 + 2*b**2*x**(13/2
)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

Giac [A] time = 1.14748, size = 62, normalized size = 0.97

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{13}\,acx^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2) + 4/13*a*c*x^(13/
2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

3.1050 $\int \sqrt{x} \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=64

$$\frac{2}{3}a^2x^{3/2}+\frac{2}{11}x^{11/2}\left(2ac+b^2\right)+\frac{4}{7}abx^{7/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*(b^2 + 2*a*c)*x^(11/2))/11 + (4*
b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rubi [A] time = 0.0230685, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$\frac{2}{3}a^2x^{3/2}+\frac{2}{11}x^{11/2}\left(2ac+b^2\right)+\frac{4}{7}abx^{7/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*(b^2 + 2*a*c)*x^(11/2))/11 + (4*
b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2\sqrt{x}+2abx^{5/2}+\left(b^2+2ac\right)x^{9/2}+2bcx^{13/2}+c^2x^{17/2}\right) dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}\left(b^2+2ac\right)x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}\end{aligned}$$

Mathematica [A] time = 3.32368, size = 50, normalized size = 0.78

$$\frac{2x^{3/2}\left(7315a^2+1995x^4\left(2ac+b^2\right)+6270abx^2+2926bcx^6+1155c^2x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*(7315*a^2 + 6270*a*b*x^2 + 1995*(b^2 + 2*a*c)*x^4 + 2926*b*c*x^6
 + 1155*c^2*x^8))/21945
```

Maple [A] time = 0.045, size = 49, normalized size = 0.8

$$\frac{2310\,c^2x^8+5852\,bcx^6+7980\,x^4ac+3990\,b^2x^4+12540\,abx^2+14630\,a^2}{21945}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/21945*x^(3/2)*(1155*c^2*x^8+2926*b*c*x^6+3990*a*c*x^4+1995*b^2*x^4+6270*a
*b*x^2+7315*a^2)
```

Maxima [A] time = 0.956327, size = 59, normalized size = 0.92

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,\left(b^2+2\,ac\right)x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*(b^2 + 2*a*c)*x^(11/2) + 4/7*a
*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Fricas [A] time = 1.24976, size = 135, normalized size = 2.11

$$\frac{2}{21945}\,\left(1155\,c^2x^9+2926\,bcx^7+1995\,\left(b^2+2\,ac\right)x^5+6270\,abx^3+7315\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*c^2*x^9 + 2926*b*c*x^7 + 1995*(b^2 + 2*a*c)*x^5 + 6270*a*b*x^
3 + 7315*a^2*x)*sqrt(x)
```

Sympy [A] time = 3.66458, size = 63, normalized size = 0.98

$$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{11}{2}}\left(2ac+b^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2
)/19 + 2*x**(11/2)*(2*a*c + b**2)/11
```

Giac [A] time = 1.14697, size = 62, normalized size = 0.97

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{11}\,acx^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2) + 4/11*a*c*x^(11/
2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

3.1051 $\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*(b^2 + 2*a*c)*x^(9/2))/9 + (4*b*c*x^
(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rubi [A] time = 0.022, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```


```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*(b^2 + 2*a*c)*x^(9/2))/9 + (4*b*c*x^
(13/2))/13 + (2*c^2*x^(17/2))/17
```


Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}}+2abx^{3/2}+\left(b^2+2ac\right)x^{7/2}+2bcx^{11/2}+c^2x^{15/2}\right)dx\\ &= 2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}\left(b^2+2ac\right)x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.042085, size = 63, normalized size = 1.02

$$2\left(a^2\sqrt{x}+\frac{1}{9}x^{9/2}\left(2ac+b^2\right)+\frac{2}{5}abx^{5/2}+\frac{2}{13}bcx^{13/2}+\frac{1}{17}c^2x^{17/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] 2*(a^2*Sqrt[x] + (2*a*b*x^(5/2))/5 + ((b^2 + 2*a*c)*x^(9/2))/9 + (2*b*c*x^(
13/2))/13 + (c^2*x^(17/2))/17)
```

Maple [A] time = 0.046, size = 49, normalized size = 0.8

$$\frac{1170\,c^2x^8+3060\,bcx^6+4420\,x^4ac+2210\,b^2x^4+7956\,abx^2+19890\,a^2}{9945}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 2/9945*x^(1/2)*(585*c^2*x^8+1530*b*c*x^6+2210*a*c*x^4+1105*b^2*x^4+3978*a*b
*x^2+9945*a^2)
```

Maxima [A] time = 0.964708, size = 65, normalized size = 1.05

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}+2\,a^2\sqrt{x}+\frac{4}{45}\left(5\,cx^{\frac{9}{2}}+9\,bx^{\frac{5}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 2*a^2*sqrt(x) + 4
/45*(5*c*x^(9/2) + 9*b*x^(5/2))*a
```

Fricas [A] time = 1.31232, size = 130, normalized size = 2.1

$$\frac{2}{9945}\left(585\,c^2x^8+1530\,bcx^6+1105\left(b^2+2\,ac\right)x^4+3978\,abx^2+9945\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*c^2*x^8 + 1530*b*c*x^6 + 1105*(b^2 + 2*a*c)*x^4 + 3978*a*b*x^2
+ 9945*a^2)*sqrt(x)
```

Sympy [A] time = 4.93246, size = 68, normalized size = 1.1

$$2a^2\sqrt{x}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{4acx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{9}{2}}}{9}+\frac{4bcx^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 4*a*c*x**(9/2)/9 + 2*b**2*x**(9/2)/9 +
4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

Giac [A] time = 1.16505, size = 62, normalized size = 1.

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{9}\,acx^{\frac{9}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 4/9*a*c*x^(9/2) +
 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

3.1052 $\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{\sqrt{x}}+\frac{2}{7}x^{7/2}\left(2ac+b^2\right)+\frac{4}{3}abx^{3/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*(b^2 + 2*a*c)*x^(7/2))/7 + (4*b*c
*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rubi [A] time = 0.0222581, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$-\frac{2a^2}{\sqrt{x}}+\frac{2}{7}x^{7/2}\left(2ac+b^2\right)+\frac{4}{3}abx^{3/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*(b^2 + 2*a*c)*x^(7/2))/7 + (4*b*c
*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2}{x^{3/2}}+2ab\sqrt{x}+\left(b^2+2ac\right)x^{5/2}+2bcx^{9/2}+c^2x^{13/2}\right)dx\\ &= -\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}\left(b^2+2ac\right)x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.04392, size = 54, normalized size = 0.87

$$\frac{2\left(-1155a^2+110a\left(7bx^2+3cx^4\right)+165b^2x^4+210bcx^6+77c^2x^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*(-1155*a^2 + 165*b^2*x^4 + 210*b*c*x^6 + 77*c^2*x^8 + 110*a*(7*b*x^2 + 3
*c*x^4)))/(1155*Sqrt[x])
```

Maple [A] time = 0.048, size = 49, normalized size = 0.8

$$-\frac{-154\,c^2x^8-420\,bcx^6-660\,x^4ac-330\,b^2x^4-1540\,abx^2+2310\,a^2}{1155}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(3/2),x)
```

```
[Out] -2/1155*(-77*c^2*x^8-210*b*c*x^6-330*a*c*x^4-165*b^2*x^4-770*a*b*x^2+1155*a
^2)/x^(1/2)
```

Maxima [A] time = 0.964376, size = 59, normalized size = 0.95

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\left(b^2+2\,ac\right)x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*(b^2 + 2*a*c)*x^(7/2) + 4/3*a*b
*x^(3/2) - 2*a^2/sqrt(x)
```

Fricas [A] time = 1.28154, size = 124, normalized size = 2.

$$\frac{2\left(77\,c^2x^8+210\,bcx^6+165\left(b^2+2\,ac\right)x^4+770\,abx^2-1155\,a^2\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*c^2*x^8 + 210*b*c*x^6 + 165*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2 - 11
55*a^2)/sqrt(x)
```

Sympy [A] time = 5.53825, size = 68, normalized size = 1.1

$$-\frac{2a^2}{\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3}+\frac{4acx^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 4*a*c*x**(7/2)/7 + 2*b**2*x**(7/2)/7 +
 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

Giac [A] time = 1.16571, size = 62, normalized size = 1.

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{7}\,acx^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2) + 4/7*a*c*x^(7/2) +
 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

3.1053 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{5/2}} \, dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{3x^{3/2}}+\frac{2}{5}x^{5/2}\left(2ac+b^2\right)+4ab\sqrt{x}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*(b^2 + 2*a*c)*x^(5/2))/5 + (4*b*c
*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rubi [A] time = 0.0220935, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$-\frac{2a^2}{3x^{3/2}}+\frac{2}{5}x^{5/2}\left(2ac+b^2\right)+4ab\sqrt{x}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*(b^2 + 2*a*c)*x^(5/2))/5 + (4*b*c
*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{5/2}} \, dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{\sqrt{x}}+\left(b^2+2ac\right)x^{3/2}+2bcx^{7/2}+c^2x^{11/2}\right) dx\\ &= -\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}\left(b^2+2ac\right)x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0420915, size = 53, normalized size = 0.85

$$\frac{-390a^2 + 468a\left(5bx^2 + cx^4\right) + 234b^2x^4 + 260bcx^6 + 90c^2x^8}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (-390*a^2 + 234*b^2*x^4 + 260*b*c*x^6 + 90*c^2*x^8 + 468*a*(5*b*x^2 + c*x^4
))/(585*x^(3/2))
```

Maple [A] time = 0.046, size = 49, normalized size = 0.8

$$-\frac{-90\,c^2x^8 - 260\,bcx^6 - 468\,x^4ac - 234\,b^2x^4 - 2340\,abx^2 + 390\,a^2}{585}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(5/2),x)
```

```
[Out] -2/585*(-45*c^2*x^8-130*b*c*x^6-234*a*c*x^4-117*b^2*x^4-1170*a*b*x^2+195*a^
2)/x^(3/2)
```

Maxima [A] time = 0.972254, size = 59, normalized size = 0.95

$$\frac{2}{13}\,c^2x^{\frac{13}{2}} + \frac{4}{9}\,bcx^{\frac{9}{2}} + \frac{2}{5}\left(b^2 + 2\,ac\right)x^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*(b^2 + 2*a*c)*x^(5/2) + 4*a*b*sqr
t(x) - 2/3*a^2/x^(3/2)
```

Fricas [A] time = 1.32683, size = 123, normalized size = 1.98

$$\frac{2\left(45\,c^2x^8+130\,bcx^6+117\left(b^2+2\,ac\right)x^4+1170\,abx^2-195\,a^2\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*c^2*x^8 + 130*b*c*x^6 + 117*(b^2 + 2*a*c)*x^4 + 1170*a*b*x^2 - 19
5*a^2)/x^(3/2)
```

Sympy [A] time = 6.44588, size = 68, normalized size = 1.1

$$-\frac{2a^2}{3x^{\frac{3}{2}}}+4ab\sqrt{x}+\frac{4acx^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4bcx^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 4*a*c*x**(5/2)/5 + 2*b**2*x**(5/2)/5
 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

Giac [A] time = 1.14893, size = 62, normalized size = 1.

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,b^2x^{\frac{5}{2}}+\frac{4}{5}\,acx^{\frac{5}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2) + 4/5*a*c*x^(5/2) + 4
*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

3.1054 $\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{5x^{5/2}}+\frac{2}{3}x^{3/2}\left(2ac+b^2\right)-\frac{4ab}{\sqrt{x}}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*(b^2 + 2*a*c)*x^(3/2))/3 + (4*b
*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rubi [A] time = 0.0229676, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$-\frac{2a^2}{5x^{5/2}}+\frac{2}{3}x^{3/2}\left(2ac+b^2\right)-\frac{4ab}{\sqrt{x}}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*(b^2 + 2*a*c)*x^(3/2))/3 + (4*b
*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{7/2}}\,dx &= \int \left(\frac{a^2}{x^{7/2}}+\frac{2ab}{x^{3/2}}+\left(b^2+2ac\right)\sqrt{x}+2bcx^{5/2}+c^2x^{9/2}\right)dx\\ &= -\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}\left(b^2+2ac\right)x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0445557, size = 50, normalized size = 0.81

$$\frac{2\left(-231a^2+385x^4\left(2ac+b^2\right)-2310abx^2+330bcx^6+105c^2x^8\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*(-231*a^2 - 2310*a*b*x^2 + 385*(b^2 + 2*a*c)*x^4 + 330*b*c*x^6 + 105*c^2
*x^8))/(1155*x^(5/2))
```

Maple [A] time = 0.044, size = 49, normalized size = 0.8

$$-\frac{-210\,c^2x^8-660\,bcx^6-1540\,x^4ac-770\,b^2x^4+4620\,abx^2+462\,a^2}{1155}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(7/2),x)
```

```
[Out] -2/1155*(-105*c^2*x^8-330*b*c*x^6-770*a*c*x^4-385*b^2*x^4+2310*a*b*x^2+231*
a^2)/x^(5/2)
```

Maxima [A] time = 0.969837, size = 61, normalized size = 0.98

$$\frac{2}{11}\,c^2x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\left(b^2+2\,ac\right)x^{\frac{3}{2}}-\frac{2\left(10\,abx^2+a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*(b^2 + 2*a*c)*x^(3/2) - 2/5*(10*a
*b*x^2 + a^2)/x^(5/2)
```

Fricas [A] time = 1.30901, size = 126, normalized size = 2.03

$$\frac{2\left(105\,c^2x^8+330\,bcx^6+385\left(b^2+2\,ac\right)x^4-2310\,abx^2-231\,a^2\right)}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^2*x^8 + 330*b*c*x^6 + 385*(b^2 + 2*a*c)*x^4 - 2310*a*b*x^2 -
231*a^2)/x^(5/2)
```

Sympy [A] time = 8.94939, size = 68, normalized size = 1.1

$$-\frac{2a^2}{5x^{\frac{5}{2}}}-\frac{4ab}{\sqrt{x}}+\frac{4acx^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{3}{2}}}{3}+\frac{4bcx^{\frac{7}{2}}}{7}+\frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(7/2),x)
```

```
[Out] -2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 4*a*c*x**(3/2)/3 + 2*b**2*x**(3/2)/3
 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

Giac [A] time = 1.1804, size = 63, normalized size = 1.02

$$\frac{2}{11}\,c^2x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\,b^2x^{\frac{3}{2}}+\frac{4}{3}\,acx^{\frac{3}{2}}-\frac{2\left(10\,abx^2+a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2) + 4/3*a*c*x^(3/2) - 2
/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

3.1055 $\int x^{5/2} \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=103

$$\frac{6}{11}a^2bx^{11/2}+\frac{2}{7}a^3x^{7/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(15/2))/5 +
(2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(23/2))/23 + (2*b*c^2*
x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

Rubi [A] time = 0.0484514, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$\frac{6}{11}a^2bx^{11/2}+\frac{2}{7}a^3x^{7/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(15/2))/5 +
(2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(23/2))/23 + (2*b*c^2*
x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3x^{5/2}+3a^2bx^{9/2}+3a\left(b^2+ac\right)x^{13/2}+b\left(b^2+6ac\right)x^{17/2}+3c\left(b^2+ac\right)x^{21/2}+3bc^2x^{2}\right.\\ &= \frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}a\left(b^2+ac\right)x^{15/2}+\frac{2}{19}b\left(b^2+6ac\right)x^{19/2}+\frac{6}{23}c\left(b^2+ac\right)x^{23/2}+\frac{2}{9}\end{aligned}$$

Mathematica [A] time = 3.5143, size = 103, normalized size = 1.

$$\frac{6}{11}a^2bx^{11/2}+\frac{2}{7}a^3x^{7/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(15/2))/5 +
(2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(23/2))/23 + (2*b*c^2*
x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

Maple [A] time = 0.046, size = 90, normalized size = 0.9

$$\frac{3028410\,c^3x^{12}+10431190\,bc^2x^{10}+12245310\,x^8ac^2+12245310\,x^8b^2c+29646540\,x^6abc+4941090\,x^6b^3+18776142\,a^2c \ldots}{46940355}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/46940355*x^(7/2)*(1514205*c^3*x^12+5215595*b*c^2*x^10+6122655*a*c^2*x^8+6
122655*b^2*c*x^8+14823270*a*b*c*x^6+2470545*b^3*x^6+9388071*a^2*c*x^4+93880
71*a*b^2*x^4+12801915*a^2*b*x^2+6705765*a^3)
```

Maxima [A] time = 0.96827, size = 109, normalized size = 1.06

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\left(b^2c+ac^2\right)x^{\frac{23}{2}}+\frac{2}{19}\left(b^3+6\,abc\right)x^{\frac{19}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{5}\left(ab^2+a^2c\right)x^{\frac{15}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*(b^2*c + a*c^2)*x^(23/2) + 2/
19*(b^3 + 6*a*b*c)*x^(19/2) + 6/11*a^2*b*x^(11/2) + 2/5*(a*b^2 + a^2*c)*x^(
15/2) + 2/7*a^3*x^(7/2)
```

Fricas [A] time = 1.36986, size = 257, normalized size = 2.5

$$\frac{2}{46940355}\left(1514205\,c^3x^{15}+5215595\,bc^2x^{13}+6122655\left(b^2c+ac^2\right)x^{11}+2470545\left(b^3+6\,abc\right)x^9+12801915\,a^2bx^5+93$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/46940355*(1514205*c^3*x^15 + 5215595*b*c^2*x^13 + 6122655*(b^2*c + a*c^2)
*x^11 + 2470545*(b^3 + 6*a*b*c)*x^9 + 12801915*a^2*b*x^5 + 9388071*(a*b^2 +
 a^2*c)*x^7 + 6705765*a^3*x^3)*sqrt(x)
```

Sympy [A] time = 63.1449, size = 129, normalized size = 1.25

$$\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{6a^2bx^{\frac{11}{2}}}{11}+\frac{2a^2cx^{\frac{15}{2}}}{5}+\frac{2ab^2x^{\frac{15}{2}}}{5}+\frac{12abcx^{\frac{19}{2}}}{19}+\frac{6ac^2x^{\frac{23}{2}}}{23}+\frac{2b^3x^{\frac{19}{2}}}{19}+\frac{6b^2cx^{\frac{23}{2}}}{23}+\frac{2bc^2x^{\frac{27}{2}}}{9}+\frac{2c^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a**2*c*x**(15/2)/5 + 2*a*b**2
*x**(15/2)/5 + 12*a*b*c*x**(19/2)/19 + 6*a*c**2*x**(23/2)/23 + 2*b**3*x**(1
9/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/9 + 2*c**3*x**(31/2)/3
1
```

Giac [A] time = 1.15364, size = 117, normalized size = 1.14

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{6}{23}\,ac^2x^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}+\frac{12}{19}\,abcx^{\frac{19}{2}}+\frac{2}{5}\,ab^2x^{\frac{15}{2}}+\frac{2}{5}\,a^2cx^{\frac{15}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 6/23*a*c^2*x
^(23/2) + 2/19*b^3*x^(19/2) + 12/19*a*b*c*x^(19/2) + 2/5*a*b^2*x^(15/2) + 2
/5*a^2*c*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7/2)
```

3.1056 $\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{2}{3}a^2bx^{9/2}+\frac{2}{5}a^3x^{5/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*(b^2 + a*c)*x^(13/2))/13 + (
2*b*(b^2 + 6*a*c)*x^(17/2))/17 + (2*c*(b^2 + a*c)*x^(21/2))/7 + (6*b*c^2*x^
(25/2))/25 + (2*c^3*x^(29/2))/29
```

Rubi [A] time = 0.0434847, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$\frac{2}{3}a^2bx^{9/2}+\frac{2}{5}a^3x^{5/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*(b^2 + a*c)*x^(13/2))/13 + (
2*b*(b^2 + 6*a*c)*x^(17/2))/17 + (2*c*(b^2 + a*c)*x^(21/2))/7 + (6*b*c^2*x^
(25/2))/25 + (2*c^3*x^(29/2))/29
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx&=\int\left(a^3x^{3/2}+3a^2bx^{7/2}+3a\left(b^2+ac\right)x^{11/2}+b\left(b^2+6ac\right)x^{15/2}+3c\left(b^2+ac\right)x^{19/2}+3bc^2x^{23/}\right.\\&=\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}a\left(b^2+ac\right)x^{13/2}+\frac{2}{17}b\left(b^2+6ac\right)x^{17/2}+\frac{2}{7}c\left(b^2+ac\right)x^{21/2}+\frac{6}{25}bc\end{aligned}$$

Mathematica [A] time = 0.09244, size = 105, normalized size = 1.02

$$2\left(\frac{1}{3}a^2bx^{9/2}+\frac{1}{5}a^3x^{5/2}+\frac{1}{7}cx^{21/2}\left(ac+b^2\right)+\frac{1}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{3}{13}ax^{13/2}\left(ac+b^2\right)+\frac{3}{25}bc^2x^{25/2}+\frac{1}{29}c^3x^{29/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] 2*((a^3*x^(5/2))/5 + (a^2*b*x^(9/2))/3 + (3*a*(b^2 + a*c)*x^(13/2))/13 + (b
*(b^2 + 6*a*c)*x^(17/2))/17 + (c*(b^2 + a*c)*x^(21/2))/7 + (3*b*c^2*x^(25/2
))/25 + (c^3*x^(29/2))/29)
```

Maple [A] time = 0.046, size = 90, normalized size = 0.9

$$\frac{232050\,c^3x^{12}+807534\,bc^2x^{10}+961350\,x^8ac^2+961350\,x^8b^2c+2375100\,x^6abc+395850\,x^6b^3+1552950\,a^2cx^4+1552}{3364725}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/3364725*x^(5/2)*(116025*c^3*x^12+403767*b*c^2*x^10+480675*a*c^2*x^8+48067
5*b^2*c*x^8+1187550*a*b*c*x^6+197925*b^3*x^6+776475*a^2*c*x^4+776475*a*b^2*
x^4+1121575*a^2*b*x^2+672945*a^3)
```

Maxima [A] time = 0.976441, size = 109, normalized size = 1.06

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\left(b^2c+ac^2\right)x^{\frac{21}{2}}+\frac{2}{17}\left(b^3+6\,abc\right)x^{\frac{17}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{6}{13}\left(ab^2+a^2c\right)x^{\frac{13}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*(b^2*c + a*c^2)*x^(21/2) + 2/
17*(b^3 + 6*a*b*c)*x^(17/2) + 2/3*a^2*b*x^(9/2) + 6/13*(a*b^2 + a^2*c)*x^(1
3/2) + 2/5*a^3*x^(5/2)
```

Fricas [A] time = 1.19322, size = 246, normalized size = 2.39

$$\frac{2}{3364725}\left(116025\,c^3x^{14}+403767\,bc^2x^{12}+480675\left(b^2c+ac^2\right)x^{10}+197925\left(b^3+6\,abc\right)x^8+1121575\,a^2bx^4+776475\left(ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/3364725*(116025*c^3*x^14 + 403767*b*c^2*x^12 + 480675*(b^2*c + a*c^2)*x^1
0 + 197925*(b^3 + 6*a*b*c)*x^8 + 1121575*a^2*b*x^4 + 776475*(a*b^2 + a^2*c)
*x^6 + 672945*a^3*x^2)*sqrt(x)
```

Sympy [A] time = 36.9578, size = 129, normalized size = 1.25

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{6a^2cx^{\frac{13}{2}}}{13}+\frac{6ab^2x^{\frac{13}{2}}}{13}+\frac{12abcx^{\frac{17}{2}}}{17}+\frac{2ac^2x^{\frac{21}{2}}}{7}+\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{21}{2}}}{7}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a**2*c*x**(13/2)/13 + 6*a*b**2*
x**(13/2)/13 + 12*a*b*c*x**(17/2)/17 + 2*a*c**2*x**(21/2)/7 + 2*b**3*x**(17
/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/25 + 2*c**3*x**(29/2)/29
```

Giac [A] time = 1.14891, size = 117, normalized size = 1.14

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{7}\,ac^2x^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{12}{17}\,abcx^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{6}{13}\,a^2cx^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/7*a*c^2*x^
(21/2) + 2/17*b^3*x^(17/2) + 12/17*a*b*c*x^(17/2) + 6/13*a*b^2*x^(13/2) + 6
/13*a^2*c*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/2)
```

3.1057 $\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/2))/11 + (
2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19/2))/19 + (6*b*c^2*x
^(23/2))/23 + (2*c^3*x^(27/2))/27
```

Rubi [A] time = 0.0416628, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/2))/11 + (
2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19/2))/19 + (6*b*c^2*x
^(23/2))/23 + (2*c^3*x^(27/2))/27
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx&=\int \left(a^3\sqrt{x}+3a^2bx^{5/2}+3a\left(b^2+ac\right)x^{9/2}+b\left(b^2+6ac\right)x^{13/2}+3c\left(b^2+ac\right)x^{17/2}+3bc^2x^{21/2}\right.\\&=\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}a\left(b^2+ac\right)x^{11/2}+\frac{2}{15}b\left(b^2+6ac\right)x^{15/2}+\frac{6}{19}c\left(b^2+ac\right)x^{19/2}+\frac{6}{23}\end{aligned}$$

Mathematica [A] time = 3.31634, size = 103, normalized size = 1.

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/2))/11 + (
2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19/2))/19 + (6*b*c^2*x
^(23/2))/23 + (2*c^3*x^(27/2))/27
```

Maple [A] time = 0.046, size = 90, normalized size = 0.9

$$\frac{336490\,c^3x^{12}+1185030\,bc^2x^{10}+1434510\,x^8ac^2+1434510\,x^8b^2c+3634092\,x^6abc+605682\,x^6b^3+2477790\,a^2cx^4+24}{4542615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/4542615*x^(3/2)*(168245*c^3*x^12+592515*b*c^2*x^10+717255*a*c^2*x^8+71725
5*b^2*c*x^8+1817046*a*b*c*x^6+302841*b^3*x^6+1238895*a^2*c*x^4+1238895*a*b^
2*x^4+1946835*a^2*b*x^2+1514205*a^3)
```

Maxima [A] time = 0.97738, size = 109, normalized size = 1.06

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\left(b^2c+ac^2\right)x^{\frac{19}{2}}+\frac{2}{15}\left(b^3+6\,abc\right)x^{\frac{15}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{6}{11}\left(ab^2+a^2c\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*(b^2*c + a*c^2)*x^(19/2) + 2
/15*(b^3 + 6*a*b*c)*x^(15/2) + 6/7*a^2*b*x^(7/2) + 6/11*(a*b^2 + a^2*c)*x^(
11/2) + 2/3*a^3*x^(3/2)
```

Fricas [A] time = 1.29559, size = 244, normalized size = 2.37

$$\frac{2}{4542615}\left(168245\,c^3x^{13}+592515\,bc^2x^{11}+717255\left(b^2c+ac^2\right)x^9+302841\left(b^3+6\,abc\right)x^7+1946835\,a^2bx^3+1238895\,\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/4542615*(168245*c^3*x^13 + 592515*b*c^2*x^11 + 717255*(b^2*c + a*c^2)*x^9
 + 302841*(b^3 + 6*a*b*c)*x^7 + 1946835*a^2*b*x^3 + 1238895*(a*b^2 + a^2*c)
*x^5 + 1514205*a^3*x)*sqrt(x)
```

Sympy [A] time = 7.61993, size = 112, normalized size = 1.09

$$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{6bc^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}+\frac{2x^{\frac{19}{2}}\left(3ac^2+3b^2c\right)}{19}+\frac{2x^{\frac{15}{2}}\left(6abc+b^3\right)}{15}+\frac{2x^{\frac{11}{2}}\left(3a^2c+3ab^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*b*c**2*x**(23/2)/23 + 2*c**3*x*
*(27/2)/27 + 2*x**(19/2)*(3*a*c**2 + 3*b**2*c)/19 + 2*x**(15/2)*(6*a*b*c +
b**3)/15 + 2*x**(11/2)*(3*a**2*c + 3*a*b**2)/11
```

Giac [A] time = 1.15765, size = 117, normalized size = 1.14

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\,b^2cx^{\frac{19}{2}}+\frac{6}{19}\,ac^2x^{\frac{19}{2}}+\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{4}{5}\,abcx^{\frac{15}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{6}{11}\,a^2cx^{\frac{11}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 6/19*a*c^2*
x^(19/2) + 2/15*b^3*x^(15/2) + 4/5*a*b*c*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6
/11*a^2*c*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/2)
```

3.1058 $\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=101

$$\frac{6}{5}a^2bx^{5/2}+2a^3\sqrt{x}+\frac{6}{17}cx^{17/2}\left(ac+b^2\right)+\frac{2}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{2}{3}ax^{9/2}\left(ac+b^2\right)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*(b^2 + a*c)*x^(9/2))/3 + (2*b*(b
^2 + 6*a*c)*x^(13/2))/13 + (6*c*(b^2 + a*c)*x^(17/2))/17 + (2*b*c^2*x^(21/2
))/7 + (2*c^3*x^(25/2))/25
```

Rubi [A] time = 0.0437411, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$\frac{6}{5}a^2bx^{5/2}+2a^3\sqrt{x}+\frac{6}{17}cx^{17/2}\left(ac+b^2\right)+\frac{2}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{2}{3}ax^{9/2}\left(ac+b^2\right)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*(b^2 + a*c)*x^(9/2))/3 + (2*b*(b
^2 + 6*a*c)*x^(13/2))/13 + (6*c*(b^2 + a*c)*x^(17/2))/17 + (2*b*c^2*x^(21/2
))/7 + (2*c^3*x^(25/2))/25
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{\sqrt{x}}\,dx=\int \left(\frac{a^3}{\sqrt{x}}+3a^2bx^{3/2}+3a\left(b^2+ac\right)x^{7/2}+b\left(b^2+6ac\right)x^{11/2}+3c\left(b^2+ac\right)x^{15/2}+3bc^2x^{19/2}+c^3x^{23/2}\right)dx$$
$$=2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}a\left(b^2+ac\right)x^{9/2}+\frac{2}{13}b\left(b^2+6ac\right)x^{13/2}+\frac{6}{17}c\left(b^2+ac\right)x^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Mathematica [A] time = 0.0706392, size = 102, normalized size = 1.01

$$2\left(\frac{3}{5}a^2bx^{5/2}+a^3\sqrt{x}+\frac{3}{17}cx^{17/2}\left(ac+b^2\right)+\frac{1}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{1}{3}ax^{9/2}\left(ac+b^2\right)+\frac{1}{7}bc^2x^{21/2}+\frac{1}{25}c^3x^{25/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] 2*(a^3*Sqrt[x] + (3*a^2*b*x^(5/2))/5 + (a*(b^2 + a*c)*x^(9/2))/3 + (b*(b^2
+ 6*a*c)*x^(13/2))/13 + (3*c*(b^2 + a*c)*x^(17/2))/17 + (b*c^2*x^(21/2))/7
+ (c^3*x^(25/2))/25)
```

Maple [A] time = 0.045, size = 90, normalized size = 0.9

$$\frac{9282\,c^3x^{12}+33150\,bc^2x^{10}+40950\,x^8ac^2+40950\,x^8b^2c+107100\,x^6abc+17850\,x^6b^3+77350\,a^2cx^4+77350\,x^4b^2a+1}{116025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(1/2),x)
```

```
[Out] 2/116025*x^(1/2)*(4641*c^3*x^12+16575*b*c^2*x^10+20475*a*c^2*x^8+20475*b^2*
c*x^8+53550*a*b*c*x^6+8925*b^3*x^6+38675*a^2*c*x^4+38675*a*b^2*x^4+69615*a^
2*b*x^2+116025*a^3)
```

Maxima [A] time = 0.967681, size = 119, normalized size = 1.18

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}+2\,a^3\sqrt{x}+\frac{2}{15}\left(5\,cx^{\frac{9}{2}}+9\,bx^{\frac{5}{2}}\right)a^2+\frac{2}{663}\left(117\,c^2x^{\frac{17}{2}}+306\,bcx^{\frac{13}{2}}+221\,b^2x^{\frac{9}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2) + 2*a^3*sqrt(x) + 2/15*(5*c*x^(9/2) + 9*b*x^(5/2))*a^2 + 2/663*(117*c
^2*x^(17/2) + 306*b*c*x^(13/2) + 221*b^2*x^(9/2))*a
```

Fricas [A] time = 1.23518, size = 225, normalized size = 2.23

$$\frac{2}{116025}\left(4641\,c^3x^{12}+16575\,bc^2x^{10}+20475\left(b^2c+ac^2\right)x^8+8925\left(b^3+6\,abc\right)x^6+69615\,a^2bx^2+38675\left(ab^2+a^2c\right)x^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*c^3*x^12 + 16575*b*c^2*x^10 + 20475*(b^2*c + a*c^2)*x^8 + 89
25*(b^3 + 6*a*b*c)*x^6 + 69615*a^2*b*x^2 + 38675*(a*b^2 + a^2*c)*x^4 + 1160
25*a^3)*sqrt(x)
```

Sympy [A] time = 19.3961, size = 128, normalized size = 1.27

$$2a^3\sqrt{x}+\frac{6a^2bx^{\frac{5}{2}}}{5}+\frac{2a^2cx^{\frac{9}{2}}}{3}+\frac{2ab^2x^{\frac{9}{2}}}{3}+\frac{12abcx^{\frac{13}{2}}}{13}+\frac{6ac^2x^{\frac{17}{2}}}{17}+\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{17}{2}}}{17}+\frac{2bc^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a**2*c*x**(9/2)/3 + 2*a*b**2*x**(9
/2)/3 + 12*a*b*c*x**(13/2)/13 + 6*a*c**2*x**(17/2)/17 + 2*b**3*x**(13/2)/13
 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/7 + 2*c**3*x**(25/2)/25
```

Giac [A] time = 1.13289, size = 117, normalized size = 1.16

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{6}{17}\,ac^2x^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{12}{13}\,abcx^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{2}{3}\,a^2cx^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 6/17*a*c^2*x
^(17/2) + 2/13*b^3*x^(13/2) + 12/13*a*b*c*x^(13/2) + 2/3*a*b^2*x^(9/2) + 2/
3*a^2*c*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

3.1059 $\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=99

$$2a^2bx^{3/2}-\frac{2a^3}{\sqrt{x}}+\frac{2}{5}cx^{15/2}\left(ac+b^2\right)+\frac{2}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{6}{7}ax^{7/2}\left(ac+b^2\right)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*(b^2 + a*c)*x^(7/2))/7 + (2*b*(b^
2 + 6*a*c)*x^(11/2))/11 + (2*c*(b^2 + a*c)*x^(15/2))/5 + (6*b*c^2*x^(19/2))
/19 + (2*c^3*x^(23/2))/23
```

Rubi [A] time = 0.0425691, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$2a^2bx^{3/2}-\frac{2a^3}{\sqrt{x}}+\frac{2}{5}cx^{15/2}\left(ac+b^2\right)+\frac{2}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{6}{7}ax^{7/2}\left(ac+b^2\right)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*(b^2 + a*c)*x^(7/2))/7 + (2*b*(b^
2 + 6*a*c)*x^(11/2))/11 + (2*c*(b^2 + a*c)*x^(15/2))/5 + (6*b*c^2*x^(19/2))
/19 + (2*c^3*x^(23/2))/23
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^3}{x^{3/2}}+3a^2b\sqrt{x}+3a\left(b^2+ac\right)x^{5/2}+b\left(b^2+6ac\right)x^{9/2}+3c\left(b^2+ac\right)x^{13/2}+3bc^2x^{17/2}+c^3x\right.\\ &= -\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}a\left(b^2+ac\right)x^{7/2}+\frac{2}{11}b\left(b^2+6ac\right)x^{11/2}+\frac{2}{5}c\left(b^2+ac\right)x^{15/2}+\frac{6}{19}bc^2x^{19/2}+\end{aligned}$$

Mathematica [A] time = 0.0810406, size = 100, normalized size = 1.01

$$2\left(a^2bx^{3/2}-\frac{a^3}{\sqrt{x}}+\frac{1}{5}cx^{15/2}\left(ac+b^2\right)+\frac{1}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{3}{7}ax^{7/2}\left(ac+b^2\right)+\frac{3}{19}bc^2x^{19/2}+\frac{1}{23}c^3x^{23/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] 2*(-(a^3/Sqrt[x]) + a^2*b*x^(3/2) + (3*a*(b^2 + a*c)*x^(7/2))/7 + (b*(b^2 +
 6*a*c)*x^(11/2))/11 + (c*(b^2 + a*c)*x^(15/2))/5 + (3*b*c^2*x^(19/2))/19 +
 (c^3*x^(23/2))/23)
```

Maple [A] time = 0.046, size = 90, normalized size = 0.9

$$-\frac{-14630\,c^3x^{12}-53130\,bc^2x^{10}-67298\,x^8ac^2-67298\,x^8b^2c-183540\,x^6abc-30590\,x^6b^3-144210\,a^2cx^4-144210\,x^4b^2a}{168245}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(3/2),x)
```

```
[Out] -2/168245*(-7315*c^3*x^12-26565*b*c^2*x^10-33649*a*c^2*x^8-33649*b^2*c*x^8-
91770*a*b*c*x^6-15295*b^3*x^6-72105*a^2*c*x^4-72105*a*b^2*x^4-168245*a^2*b*
x^2+168245*a^3)/x^(1/2)
```

Maxima [A] time = 0.961496, size = 109, normalized size = 1.1

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\left(b^2c+ac^2\right)x^{\frac{15}{2}}+\frac{2}{11}\left(b^3+6\,abc\right)x^{\frac{11}{2}}+2\,a^2bx^{\frac{3}{2}}+\frac{6}{7}\left(ab^2+a^2c\right)x^{\frac{7}{2}}-\frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*(b^2*c + a*c^2)*x^(15/2) + 2/
11*(b^3 + 6*a*b*c)*x^(11/2) + 2*a^2*b*x^(3/2) + 6/7*(a*b^2 + a^2*c)*x^(7/2)
 - 2*a^3/sqrt(x)
```

Fricas [A] time = 1.19299, size = 228, normalized size = 2.3

$$\frac{2\left(7315\,c^3x^{12}+26565\,bc^2x^{10}+33649\left(b^2c+ac^2\right)x^8+15295\left(b^3+6\,abc\right)x^6+168245\,a^2bx^2+72105\left(ab^2+a^2c\right)x^4-16\right.}{168245\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*c^3*x^12 + 26565*b*c^2*x^10 + 33649*(b^2*c + a*c^2)*x^8 + 15
295*(b^3 + 6*a*b*c)*x^6 + 168245*a^2*b*x^2 + 72105*(a*b^2 + a^2*c)*x^4 - 16
8245*a^3)/sqrt(x)
```

Sympy [A] time = 20.8602, size = 126, normalized size = 1.27

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{\frac{3}{2}}+\frac{6a^2cx^{\frac{7}{2}}}{7}+\frac{6ab^2x^{\frac{7}{2}}}{7}+\frac{12abcx^{\frac{11}{2}}}{11}+\frac{2ac^2x^{\frac{15}{2}}}{5}+\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{2b^2cx^{\frac{15}{2}}}{5}+\frac{6bc^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a**2*c*x**(7/2)/7 + 6*a*b**2*x**(7/
2)/7 + 12*a*b*c*x**(11/2)/11 + 2*a*c**2*x**(15/2)/5 + 2*b**3*x**(11/2)/11 +
 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19 + 2*c**3*x**(23/2)/23
```

Giac [A] time = 1.13689, size = 117, normalized size = 1.18

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\,b^2cx^{\frac{15}{2}}+\frac{2}{5}\,ac^2x^{\frac{15}{2}}+\frac{2}{11}\,b^3x^{\frac{11}{2}}+\frac{12}{11}\,abcx^{\frac{11}{2}}+\frac{6}{7}\,ab^2x^{\frac{7}{2}}+\frac{6}{7}\,a^2cx^{\frac{7}{2}}+2\,a^2bx^{\frac{3}{2}}-\frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/5*a*c^2*x^
(15/2) + 2/11*b^3*x^(11/2) + 12/11*a*b*c*x^(11/2) + 6/7*a*b^2*x^(7/2) + 6/7
*a^2*c*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

3.1060 $\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}} dx$

Optimal. Leaf size=101

$$6a^2b\sqrt{x} - \frac{2a^3}{3x^{3/2}} + \frac{6}{13}cx^{13/2}\left(ac+b^2\right) + \frac{2}{9}bx^{9/2}\left(6ac+b^2\right) + \frac{6}{5}ax^{5/2}\left(ac+b^2\right) + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2}$$

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*(b^2 + a*c)*x^(5/2))/5 + (2*b
*(b^2 + 6*a*c)*x^(9/2))/9 + (6*c*(b^2 + a*c)*x^(13/2))/13 + (6*b*c^2*x^(17/
2))/17 + (2*c^3*x^(21/2))/21
```

Rubi [A] time = 0.0439858, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$6a^2b\sqrt{x} - \frac{2a^3}{3x^{3/2}} + \frac{6}{13}cx^{13/2}\left(ac+b^2\right) + \frac{2}{9}bx^{9/2}\left(6ac+b^2\right) + \frac{6}{5}ax^{5/2}\left(ac+b^2\right) + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*(b^2 + a*c)*x^(5/2))/5 + (2*b
*(b^2 + 6*a*c)*x^(9/2))/9 + (6*c*(b^2 + a*c)*x^(13/2))/13 + (6*b*c^2*x^(17/
2))/17 + (2*c^3*x^(21/2))/21
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}} + \frac{3a^2b}{\sqrt{x}} + 3a\left(b^2+ac\right)x^{3/2} + b\left(b^2+6ac\right)x^{7/2} + 3c\left(b^2+ac\right)x^{11/2} + 3bc^2x^{15/2} + c^3x^{19/2}\right) \\ &= -\frac{2a^3}{3x^{3/2}} + 6a^2b\sqrt{x} + \frac{6}{5}a\left(b^2+ac\right)x^{5/2} + \frac{2}{9}b\left(b^2+6ac\right)x^{9/2} + \frac{6}{13}c\left(b^2+ac\right)x^{13/2} + \frac{6}{17}bc^2x^{17/2} +\end{aligned}$$

Mathematica [A] time = 0.0718501, size = 103, normalized size = 1.02

$$2\left(3a^2b\sqrt{x}-\frac{a^3}{3x^{3/2}}+\frac{3}{13}cx^{13/2}\left(ac+b^2\right)+\frac{1}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{3}{5}ax^{5/2}\left(ac+b^2\right)+\frac{3}{17}bc^2x^{17/2}+\frac{1}{21}c^3x^{21/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] 2*(-a^3/(3*x^(3/2)) + 3*a^2*b*Sqrt[x] + (3*a*(b^2 + a*c)*x^(5/2))/5 + (b*(b
^2 + 6*a*c)*x^(9/2))/9 + (3*c*(b^2 + a*c)*x^(13/2))/13 + (3*b*c^2*x^(17/2))
/17 + (c^3*x^(21/2))/21)
```

Maple [A] time = 0.045, size = 90, normalized size = 0.9

$$-\frac{-6630\,c^3x^{12}-24570\,bc^2x^{10}-32130\,x^8ac^2-32130\,x^8b^2c-92820\,x^6abc-15470\,x^6b^3-83538\,a^2cx^4-83538\,x^4b^2a-}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(5/2),x)
```

```
[Out] -2/69615*(-3315*c^3*x^12-12285*b*c^2*x^10-16065*a*c^2*x^8-16065*b^2*c*x^8-4
6410*a*b*c*x^6-7735*b^3*x^6-41769*a^2*c*x^4-41769*a*b^2*x^4-208845*a^2*b*x^
2+23205*a^3)/x^(3/2)
```

Maxima [A] time = 0.967803, size = 109, normalized size = 1.08

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\left(b^2c+ac^2\right)x^{\frac{13}{2}}+\frac{2}{9}\left(b^3+6\,abc\right)x^{\frac{9}{2}}+6\,a^2b\sqrt{x}+\frac{6}{5}\left(ab^2+a^2c\right)x^{\frac{5}{2}}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*(b^2*c + a*c^2)*x^(13/2) + 2
/9*(b^3 + 6*a*b*c)*x^(9/2) + 6*a^2*b*sqrt(x) + 6/5*(a*b^2 + a^2*c)*x^(5/2)
- 2/3*a^3/x^(3/2)
```

Fricas [A] time = 1.25003, size = 224, normalized size = 2.22

$$\frac{2\left(3315\,c^3x^{12}+12285\,bc^2x^{10}+16065\left(b^2c+ac^2\right)x^8+7735\left(b^3+6\,abc\right)x^6+208845\,a^2bx^2+41769\left(ab^2+a^2c\right)x^4-2320\right.}{69615\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*c^3*x^12 + 12285*b*c^2*x^10 + 16065*(b^2*c + a*c^2)*x^8 + 773
5*(b^3 + 6*a*b*c)*x^6 + 208845*a^2*b*x^2 + 41769*(a*b^2 + a^2*c)*x^4 - 2320
5*a^3)/x^(3/2)
```

Sympy [A] time = 23.6497, size = 128, normalized size = 1.27

$$-\frac{2a^3}{3x^{\frac{3}{2}}}+6a^2b\sqrt{x}+\frac{6a^2cx^{\frac{5}{2}}}{5}+\frac{6ab^2x^{\frac{5}{2}}}{5}+\frac{4abcx^{\frac{9}{2}}}{3}+\frac{6ac^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2cx^{\frac{13}{2}}}{13}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a**2*c*x**(5/2)/5 + 6*a*b**2*x*
*(5/2)/5 + 4*a*b*c*x**(9/2)/3 + 6*a*c**2*x**(13/2)/13 + 2*b**3*x**(9/2)/9 +
 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17 + 2*c**3*x**(21/2)/21
```

Giac [A] time = 1.12786, size = 117, normalized size = 1.16

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{6}{13}\,ac^2x^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{4}{3}\,abcx^{\frac{9}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+\frac{6}{5}\,a^2cx^{\frac{5}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 6/13*a*c^2*
x^(13/2) + 2/9*b^3*x^(9/2) + 4/3*a*b*c*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6/5*a^
2*c*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

3.1061 $\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{5x^{5/2}}+\frac{6}{11}cx^{11/2}\left(ac+b^2\right)+\frac{2}{7}bx^{7/2}\left(6ac+b^2\right)+2ax^{3/2}\left(ac+b^2\right)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*(b^2 + a*c)*x^(3/2) + (2*b*(
b^2 + 6*a*c)*x^(7/2))/7 + (6*c*(b^2 + a*c)*x^(11/2))/11 + (2*b*c^2*x^(15/2)
)/5 + (2*c^3*x^(19/2))/19
```

Rubi [A] time = 0.0416287, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{5x^{5/2}}+\frac{6}{11}cx^{11/2}\left(ac+b^2\right)+\frac{2}{7}bx^{7/2}\left(6ac+b^2\right)+2ax^{3/2}\left(ac+b^2\right)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*(b^2 + a*c)*x^(3/2) + (2*b*(
b^2 + 6*a*c)*x^(7/2))/7 + (6*c*(b^2 + a*c)*x^(11/2))/11 + (2*b*c^2*x^(15/2)
)/5 + (2*c^3*x^(19/2))/19
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^{7/2}}\,dx=\int \left(\frac{a^3}{x^{7/2}}+\frac{3a^2b}{x^{3/2}}+3a\left(b^2+ac\right)\sqrt{x}+b\left(b^2+6ac\right)x^{5/2}+3c\left(b^2+ac\right)x^{9/2}+3bc^2x^{13/2}+c^3x^{17/2}\right.$$

$$=-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2a\left(b^2+ac\right)x^{3/2}+\frac{2}{7}b\left(b^2+6ac\right)x^{7/2}+\frac{6}{11}c\left(b^2+ac\right)x^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}$$

Mathematica [A] time = 0.0733973, size = 100, normalized size = 1.01

$$2\left(-\frac{3a^2b}{\sqrt{x}}-\frac{a^3}{5x^{5/2}}+\frac{3}{11}cx^{11/2}\left(ac+b^2\right)+\frac{1}{7}bx^{7/2}\left(6ac+b^2\right)+ax^{3/2}\left(ac+b^2\right)+\frac{1}{5}bc^2x^{15/2}+\frac{1}{19}c^3x^{19/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] 2*(-a^3/(5*x^(5/2)) - (3*a^2*b)/Sqrt[x] + a*(b^2 + a*c)*x^(3/2) + (b*(b^2 +
 6*a*c)*x^(7/2))/7 + (3*c*(b^2 + a*c)*x^(11/2))/11 + (b*c^2*x^(15/2))/5 + (
c^3*x^(19/2))/19)
```

Maple [A] time = 0.046, size = 90, normalized size = 0.9

$$-\frac{-770\,c^3x^{12}-2926\,bc^2x^{10}-3990\,x^8ac^2-3990\,x^8b^2c-12540\,x^6abc-2090\,x^6b^3-14630\,a^2cx^4-14630\,ab^2x^4+43890\,a}{7315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(7/2),x)
```

```
[Out] -2/7315*(-385*c^3*x^12-1463*b*c^2*x^10-1995*a*c^2*x^8-1995*b^2*c*x^8-6270*a
*b*c*x^6-1045*b^3*x^6-7315*a^2*c*x^4-7315*a*b^2*x^4+21945*a^2*b*x^2+1463*a^
3)/x^(5/2)
```

Maxima [A] time = 0.968151, size = 111, normalized size = 1.12

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\left(b^2c+ac^2\right)x^{\frac{11}{2}}+\frac{2}{7}\left(b^3+6\,abc\right)x^{\frac{7}{2}}+2\left(ab^2+a^2c\right)x^{\frac{3}{2}}-\frac{2\left(15\,a^2bx^2+a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*(b^2*c + a*c^2)*x^(11/2) + 2/
7*(b^3 + 6*a*b*c)*x^(7/2) + 2*(a*b^2 + a^2*c)*x^(3/2) - 2/5*(15*a^2*b*x^2 +
 a^3)/x^(5/2)
```

Fricas [A] time = 1.26604, size = 215, normalized size = 2.17

$$\frac{2\left(385\,c^3x^{12}+1463\,bc^2x^{10}+1995\left(b^2c+ac^2\right)x^8+1045\left(b^3+6\,abc\right)x^6-21945\,a^2bx^2+7315\left(ab^2+a^2c\right)x^4-1463\,a^3\right)}{7315\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*c^3*x^12 + 1463*b*c^2*x^10 + 1995*(b^2*c + a*c^2)*x^8 + 1045*(b
^3 + 6*a*b*c)*x^6 - 21945*a^2*b*x^2 + 7315*(a*b^2 + a^2*c)*x^4 - 1463*a^3)/
x^(5/2)
```

Sympy [A] time = 29.167, size = 124, normalized size = 1.25

$$-\frac{2a^3}{5x^{\frac{5}{2}}}-\frac{6a^2b}{\sqrt{x}}+2a^2cx^{\frac{3}{2}}+2ab^2x^{\frac{3}{2}}+\frac{12abcx^{\frac{7}{2}}}{7}+\frac{6ac^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(7/2),x)
```

```
[Out] -2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a**2*c*x**(3/2) + 2*a*b**2*x**(
3/2) + 12*a*b*c*x**(7/2)/7 + 6*a*c**2*x**(11/2)/11 + 2*b**3*x**(7/2)/7 + 6*
b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 + 2*c**3*x**(19/2)/19
```

Giac [A] time = 1.19079, size = 119, normalized size = 1.2

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,b^2cx^{\frac{11}{2}}+\frac{6}{11}\,ac^2x^{\frac{11}{2}}+\frac{2}{7}\,b^3x^{\frac{7}{2}}+\frac{12}{7}\,abcx^{\frac{7}{2}}+2\,ab^2x^{\frac{3}{2}}+2\,a^2cx^{\frac{3}{2}}-\frac{2\left(15\,a^2bx^2+a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 6/11*a*c^2*x
^(11/2) + 2/7*b^3*x^(7/2) + 12/7*a*b*c*x^(7/2) + 2*a*b^2*x^(3/2) + 2*a^2*c*
```

```
x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

3.1062 $\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=389

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{(b-}{}$$

```
[Out] (2*x^(3/2))/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-
b + Sqrt[b^2 - 4*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcT
anh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(
7/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c
])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3
/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.85992, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1115, 1367, 1510, 298, 205, 208}

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{(b-}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4),x]
```


```
[Out] (2*x^(3/2))/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-
b + Sqrt[b^2 - 4*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcT
anh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(
7/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c
])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3
```

```
/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^{10}}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2x^{3/2}}{3c} - \frac{2\,\text{Subst}\left(\int \frac{x^2\left(3a+3bx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{3c}$$

$$= \frac{2x^{3/2}}{3c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+c}\right.}{c}$$

$$= \frac{2x^{3/2}}{3c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}c^{3/2}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b+\sqrt{b^2}}}\right.}{\sqrt{2}c^{3/2}}$$

$$= \frac{2x^{3/2}}{3c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{2^{3/4}c^{7/4}}$$

Mathematica [C] time = 0.0496691, size = 80, normalized size = 0.21

$$\frac{4x^{3/2} - 3\text{RootSum}\left[\#1^4 b + \#1^8 c + a\&, \frac{\#1^4 b\log\left(\sqrt{x}-\#1\right)+a\log\left(\sqrt{x}-\#1\right)}{2\#1^5 c+\#1 b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*x^(3/2) - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] + b*Log
[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(6*c)
```

Maple [C] time = 0.312, size = 65, normalized size = 0.2

$$\frac{2}{3\,c}x^{\frac{3}{2}} - \frac{1}{2\,c}\sum_{\_R=\text{RootOf}\left(c\_Z^8+b\_Z^4+a\right)} \frac{\_R^6 b + \_R^2 a}{2\,\_R^7 c + \_R^3 b}\ln\left(\sqrt{x} - \_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/3*x^(3/2)/c-1/2/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R
=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2x^{\frac{3}{2}}}{3c} - \int \frac{bx^{\frac{5}{2}} + a\sqrt{x}}{c^2x^4 + bcx^2 + ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/c - integrate((b*x^(5/2) + a*sqrt(x))/(c^2*x^4 + b*c*x^2 + a*c)
, x)
```

Fricas [B] time = 20.8619, size = 14407, normalized size = 37.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*
c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^
6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^
2*c^8 + 16*a^2*c^9)))*arctan(1/2*((b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^
3*b^3*c^3 + 4*a^4*b*c^4 - (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3
*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*
b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*
c^16 - 64*a^3*c^17)))*sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 -
62*a^13*b^6*c^3 + 46*a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x - 1/2*sqr
t(1/2)*(a^7*b^17 - 17*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 9
24*a^11*b^9*c^4 - 1078*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 +
 12*a^15*b*c^8 - (a^7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a
^10*b^8*c^10 + 997*a^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 -
```

64*a^14*c^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8 *a*b^2*c^8 + 16*a^2*c^9))) + (a^5*b^15 - 14*a^6*b^13*c + 77*a^7*b^11*c^2 - 210*a^8*b^9*c^3 + 294*a^9*b^7*c^4 - 196*a^10*b^5*c^5 + 49*a^11*b^3*c^6 - 4* a^12*b*c^7 - (a^5*b^12*c^7 - 15*a^6*b^10*c^8 + 88*a^7*b^8*c^9 - 253*a^8*b^6 *c^10 + 362*a^9*b^4*c^11 - 224*a^10*b^2*c^12 + 32*a^11*c^13)*sqrt((b^12 - 1 0*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2* c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17))) *sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b* c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^ 2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^ 6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^ 2*c^8 + 16*a^2*c^9)))/(a^7*b^12 - 10*a^8*b^10*c + 37*a^9*b^8*c^2 - 62*a^10* b^6*c^3 + 46*a^11*b^4*c^4 - 12*a^12*b^2*c^5 + a^13*c^6)) - 12*c*sqrt(sqrt(1 /2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a* b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^ 6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^1 5 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))) *arctan(-1/2*((b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b* c^4 + (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2 *c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 - 62*a^13*b^6*c^3 + 46 *a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x - 1/2*sqrt(1/2)*(a^7*b^17 - 1 7*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 924*a^11*b^9*c^4 - 10 78*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 + 12*a^15*b*c^8 + (a^ 7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a^10*b^8*c^10 + 997*a ^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 - 64*a^14*c^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12 *a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^ 3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62* a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b ^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2* c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b ^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c ^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c ^8 + 16*a^2*c^9))) + (a^5*b^15 - 14*a^6*b^13*c + 77*a^7*b^11*c^2 - 210*a^8* b^9*c^3 + 294*a^9*b^7*c^4 - 196*a^10*b^5*c^5 + 49*a^11*b^3*c^6 - 4*a^12*b*c ^7 + (a^5*b^12*c^7 - 15*a^6*b^10*c^8 + 88*a^7*b^8*c^9 - 253*a^8*b^6*c^10 +

```
362*a^9*b^4*c^11 - 224*a^10*b^2*c^12 + 32*a^11*c^13)*sqrt((b^12 - 10*a*b^10
*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^
6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(x)
*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^
4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2
 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 -
12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 1
6*a^2*c^9))))/(a^7*b^12 - 10*a^8*b^10*c + 37*a^9*b^8*c^2 - 62*a^10*b^6*c^3
+ 46*a^11*b^4*c^4 - 12*a^12*b^2*c^5 + a^13*c^6)) - 3*c*sqrt(sqrt(1/2)*sqrt(
-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 +
 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 4
6*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^
2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(1/2*
sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^
4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17
*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^
5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a
^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b
^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a
*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b
^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^
2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^
10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*
c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 -
 a^8*c^3)*sqrt(x)) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a
*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b
^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c
 + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 +
 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9
- 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt
(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*
b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 -
8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^
3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4
*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^
```

```
9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x)) - 3*c*sqrt
(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*
b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2
*c^9)))*log(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*
b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7
+ (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b
^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^
3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4
*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^
5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*s
qrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4
- 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 6
4*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c
 + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt
((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 1
2*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a
^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c +
 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x)) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^
5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*s
qrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4
- 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 6
4*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^
14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 4
57*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 +
113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqr
t((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*
a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*
b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*
a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(
b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*
b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2
*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6
*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2
*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqr
t(x)) + 4*x^(3/2))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2 + a), x)
```

3.1063 $\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=385

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\cdots$$

```
[Out] (2*Sqrt[x])/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b - Sqrt[b
^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4
)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b +
Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*
c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.797352, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1115, 1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b - Sqrt[b
^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4
)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b +
Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*
```

```
c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\sqrt{x}}{c} - \frac{2\,\text{Subst}\left(\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{c}$$

$$= \frac{2\sqrt{x}}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\right.}{c}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b+\sqrt{b^2-}}}\right.}{c\sqrt{-b+\sqrt{b^2-}}}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\right.}$$

Mathematica [C] time = 0.0441358, size = 80, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^4b\log\left(\sqrt{x}-\#1\right)+a\log\left(\sqrt{x}-\#1\right)}{\#1^3b+2\#1^7c}\&\right]-4\sqrt{x}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(-4*Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] + b*Log
[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(2*c)
```

Maple [C] time = 0.249, size = 64, normalized size = 0.2

$$2\,\frac{\sqrt{x}}{c}+\frac{1}{2\,c}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{-\_R^4b-a}{2\,\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2*x^(1/2)/c+1/2/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootO
f(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 7.2757, size = 11297, normalized size = 29.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(-1/4*(sqrt(1/2)*(b^11 - 13
*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^
5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b
^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*
sqrt(4*(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*x
 + 2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125
*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 + (b^11*c^5 - 15*a*b^9*c^6 + 85*a
^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^
```

4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) -
 2*sqrt(1/2)*(a*b^15 - 16*a^2*b^13*c + 103*a^3*b^11*c^2 - 340*a^4*b^9*c^3 +
 605*a^5*b^7*c^4 - 554*a^6*b^5*c^5 + 224*a^7*b^3*c^6 - 32*a^8*b*c^7 + (a*b^
14*c^5 - 19*a^2*b^12*c^6 + 147*a^3*b^10*c^7 - 590*a^4*b^8*c^8 + 1290*a^5*b^
6*c^9 - 1464*a^6*b^4*c^10 + 736*a^7*b^2*c^11 - 128*a^8*c^12)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt(1/
2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c
^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*
c^6 + 16*a^2*c^7)))/(a^5*b^8 - 6*a^6*b^6*c + 11*a^7*b^4*c^2 - 6*a^8*b^2*c^3
 + a^9*c^4)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/4*(sqrt(1/2)*(b
^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*
a^5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 3
52*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))*sqrt(4*(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^
6*c^4)*x + 2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c
^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 - (b^11*c^5 - 15*a*b^9*c^
6 + 85*a^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt
(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^
2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b
^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*
a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))
/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) - 2*sqrt(1/2)*(a*b^15 - 16*a^2*b^13*
c + 103*a^3*b^11*c^2 - 340*a^4*b^9*c^3 + 605*a^5*b^7*c^4 - 554*a^6*b^5*c^5
+ 224*a^7*b^3*c^6 - 32*a^8*b*c^7 - (a*b^14*c^5 - 19*a^2*b^12*c^6 + 147*a^3*
b^10*c^7 - 590*a^4*b^8*c^8 + 1290*a^5*b^6*c^9 - 1464*a^6*b^4*c^10 + 736*a^7

```
*b^2*c^11 - 128*a^8*c^12)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 -
 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13
)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b
*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(a^5*b^8 - 6*a^
6*b^6*c + 11*a^7*b^4*c^2 - 6*a^8*b^2*c^3 + a^9*c^4)) - c*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt(x) + (b^6 - 7*a*b^
4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*s
qrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10
- 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^
5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*
b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2
*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5
 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c + a
^3*c^2)*sqrt(x) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5
- 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c
^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a
*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))
*log(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt(x) + (b^6 - 7*a*b^4*c + 13*a^2*
b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48
*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*
sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^
4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^
5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*sqrt(
```

```
x) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6
 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(s
qrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16
*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*
a*b^2*c^6 + 16*a^2*c^7)))) - 4*sqrt(x))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a), x)
```

3.1064 $\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\cdots\right.}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

```
[Out] -(((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c])) + ((-b + Sqrt
[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])
^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan
h[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/
4)*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.442259, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1115, 1374, 298, 205, 208}

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\cdots\right.}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c])) + ((-b + Sqrt
[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])
^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan
h[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/
4)*Sqrt[b^2 - 4*a*c])
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$
$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)+\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{}}\right.$$
$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}\sqrt{c}}+\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{}}\right.}{\sqrt{2}\sqrt{c}}$$
$$= -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b-\sqrt{}\right.}{}$$

Mathematica [C] time = 0.0281135, size = 48, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[\#1^4b+\#1^8c+a\&, \frac{\#1^3\log\left(\sqrt{x}-\#1\right)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1^3)/(b + 2*c*#1^4) & ]
/2
```

Maple [C] time = 0.246, size = 45, normalized size = 0.1

$$\frac{1}{2}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\_R^6}{2\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 3.20986, size = 8469, normalized size = 25.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*((b^4 -
 5*a*b^2*c + 4*a^2*c^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9
)))*sqrt((a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x - 1/2*sqrt(1/2)*(a^3*b^7 - 6*a
^4*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 + (a^3*b^8*c^3 - 13*a^4*b^6*c^4 + 60
*a^5*b^4*c^5 - 112*a^6*b^2*c^6 + 64*a^7*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*
a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*
b^2*c^4 + 16*a^2*c^5))) + (a^2*b^6 - 6*a^3*b^4*c + 9*a^4*b^2*c^2 - 4*a^5*c^
3 + (a^2*b^7*c^3 - 9*a^3*b^5*c^4 + 24*a^4*b^3*c^5 - 16*a^5*b*c^6)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c
^9)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4
 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 4
8*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a^3*b
^4 - 2*a^4*b^2*c + a^5*c^2)) + 2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4
*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)))*arctan(-1/2*((b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^5*c^3 - 8*a*b^3*
c^4 + 16*a^2*b*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^
7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x
```

```
- 1/2*sqrt(1/2)*(a^3*b^7 - 6*a^4*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 - (a^
3*b^8*c^3 - 13*a^4*b^6*c^4 + 60*a^5*b^4*c^5 - 112*a^6*b^2*c^6 + 64*a^7*c^7)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(sqrt(1/2)*sqrt(
-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))) + (a^2*b^6 - 6*a^3*b^4*c + 9*a^4*b^2*c^2
- 4*a^5*c^3 - (a^2*b^7*c^3 - 9*a^3*b^5*c^4 + 24*a^4*b^3*c^5 - 16*a^5*b*c^6)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^
4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
)))/(a^3*b^4 - 2*a^4*b^2*c + a^5*c^2)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*
a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*
b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2
 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^
6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 +
 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 -
 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x
)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*
b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*sqr
t(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b
^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3
- 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3
*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^
2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c
^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c
```

```
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sq
rt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*
a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2
*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14
*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))
*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c
 - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

3.1065 $\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt
[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^
2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1
/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(
2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*
Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.403399, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1115, 1374, 212, 208, 205}

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4),x]
```



```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt
[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^
2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1
/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(
2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*
Sqrt[b^2 - 4*a*c])
```


Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$
$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right) + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{}}\right.$$
$$= \frac{\sqrt{-b-\sqrt{b^2-4ac}}\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} + \frac{\sqrt{-b-\sqrt{b^2-4ac}}\,\text{Subst}\left(\int \frac{}{\sqrt{-b-\sqrt{b^2-}}}\right.}{\sqrt{b^2-4ac}}$$
$$= \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-b-\sqrt{b^2-}}}{\sqrt[4]{}}$$

Mathematica [C] time = 0.0254934, size = 46, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&, \frac{\#1\log\left(\sqrt{x}-\#1\right)}{2\#1^4 c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1)/(b + 2*c*#1^4) & ]/2
```

Maple [C] time = 0.259, size = 45, normalized size = 0.1

$$\frac{1}{2}\sum_{\_R=\text{RootOf}\left(\_Z^8 c+\_Z^4 b+a\right)} \frac{\_R^4}{2\_R^7 c+\_R^3 b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 1.76733, size = 5355, normalized size = 16.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)))*arctan(1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (b^7*c - 1
2*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b + (b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) + x)*sqrt(-(b + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*(b^4 - 8*a*b^2*
c + 16*a^2*c^2 - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqr
t(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x)*sqrt(-(b +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sq
rt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 2*sq
rt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)))*arctan(-1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5))*sqrt(sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b - (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) + x)*sqrt(-(b - (b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
```

```
3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x)*sqrt(-(b - (b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt(-
(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 1/2*sqrt
(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)))*log((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b
 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(-(b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log((b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) +
sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b
^2*c^2 + 16*a^2*c^3)))*log(-(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/
2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

3.1066 $\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$-\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] -((2^(1/4)*c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (2^(1/4)*c^
(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sq
rt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (2^(1/4)*c^(1/4)*ArcTanh[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*
c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (2^(1/4)*c^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.366469, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1115, 1375, 298, 205, 208}

$$-\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((2^(1/4)*c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (2^(1/4)*c^
(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sq
rt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (2^(1/4)*c^(1/4)*ArcTanh[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*
c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (2^(1/4)*c^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$
$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}}$$
$$= \frac{\left(\sqrt{2}\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{\left(\sqrt{2}\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}}$$
$$= -\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \dots$$

Mathematica [C] time = 0.0275613, size = 47, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&, \frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^5 c+\#1 b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1 + 2*c*#1^5) & ]/2
```

Maple [C] time = 0.257, size = 45, normalized size = 0.1

$$\frac{1}{2}\sum_{\_R=\text{RootOf}\left(\_Z^8 c+\_Z^4 b+a\right)} \frac{\_R^2}{2\_R^7 c+\_R^3 b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 2.08835, size = 5956, normalized size = 17.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)))*arctan(((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(c^2*x - 1/2*sqrt
(1/2)*(b^3*c - 4*a*b*c^2 - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*
a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-
(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - (a*b^4*c - 8*a^2*b^2*c^2 +
16*a^3*c^3)*sqrt(x)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))/c) + 2*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2)))*arctan(((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt
(c^2*x - 1/2*sqrt(1/2)*(b^3*c - 4*a*b*c^2 + (a*b^6*c - 12*a^2*b^4*c^2 + 48*
a^3*b^2*c^3 - 64*a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)))/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - (a
```

```
*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b - (a*b
^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/c) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b
^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a
^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*s
qrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a
*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*
a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqr
t(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sq
rt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*
c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c
^2)) + c*sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^
2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

3.1067 $\int \frac{1}{\sqrt{x}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=331

$$\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] (2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (2^(3/4)*c^(3/
4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[
b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (2^(3/4)*c^(3/4)*ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^
2 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.415109, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1115, 1347, 212, 208, 205}

$$\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (2^(3/4)*c^(3/
4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[
b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (2^(3/4)*c^(3/4)*ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^
2 - 4*a*c])^(3/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}}$$

$$= \frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\cdots}{\sqrt[4]{-b-\sqrt{b^2\cdots}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4\cdots}\right.}$$

Mathematica [C] time = 0.0307245, size = 49, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[\#1^4b+\#1^8c+a\&, \frac{\log\left(\sqrt{x}-\#1\right)}{\#1^3b+2\#1^7c}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1^3 + 2*c*#1^7) & ]/2
```

Maple [C] time = 0.254, size = 42, normalized size = 0.1

$$\frac{1}{2}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{1}{2\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(1/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{x}}{a} - \int \frac{cx^{\frac{7}{2}} + bx^{\frac{3}{2}}}{acx^4 + abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/a - integrate((c*x^(7/2) + b*x^(3/2))/(a*c*x^4 + a*b*x^2 + a^2),
x)
```

Fricas [B] time = 3.04883, size = 8471, normalized size = 25.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(-1/4*(sqrt(
1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (a^3*b^8 - 14*a^4*b
^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(
4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x + 2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 21*a
^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 - (a^3*b^9 - 13*a^4*b^7*c + 60*a^5*
b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5
*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + 2*sqrt(1/2)*(b
^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5 - (a^3
*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c
^5 - 128*a^8*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(x)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 -
8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^
```

7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^4*c^3 - 2*a*b
^2*c^4 + a^2*c^5)) + 2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))
)*arctan(1/4*(sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 +
(a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)))*sqrt(4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x + 2*sqrt(1/2)*(b^8
 - 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 + (a^3*b^9 - 13*
a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
 - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + 2*sqrt(1/2)*(b^9
*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5 + (a^3*b
^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5
 - 128*a^8*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
- (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^4*c^3 - 2*a*b^2
*c^4 + a^2*c^5)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))
)*log(-2*(b^2*c - a*c^2)*sqrt(x) + (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5
 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*
a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^
4*b^2*c + 16*a^5*c^2)))) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 -
12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a
^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) - (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(

```
b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)))) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
- (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) + (b^4 - 5*a*b^2*c + 4*a^2
*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 -
3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c
 + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(s
qrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x)), x)
```

3.1068 $\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=371

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

```
[Out] -2/(a*Sqrt[x]) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*
c])^(1/4)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^
(1/4)) + (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.567378, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1115, 1368, 1510, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*
c])^(1/4)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^
(1/4)) + (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{a\sqrt{x}} + \frac{2\,\mathrm{Subst}\left(\int \frac{x^2(-b-cx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{a\sqrt{x}} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int\right.}{a}$$

$$= -\frac{2}{a\sqrt{x}} + \frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}a} - \frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Su}}{}$$

$$= -\frac{2}{a\sqrt{x}} - \frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}} + -$$

Mathematica [C] time = 0.045767, size = 78, normalized size = 0.21

$$-\frac{\mathrm{RootSum}\left[\#1^4 b+\#1^8 c+a\&, \frac{\#1^4 c\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)}{2\#1^5 c+\#1 b}\&\right]+\frac{4}{\sqrt{x}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(4/Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] + c*Log[
Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(2*a)
```

Maple [C] time = 0.26, size = 65, normalized size = 0.2

$$-\frac{1}{2a}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\_R^6c+\_R^2b}{2\,\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)-2\frac{1}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*
c+_Z^4*b+a))-2/a/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{a\sqrt{x}} - \int \frac{cx^{\frac{5}{2}} + b\sqrt{x}}{acx^4 + abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2/(a*sqrt(x)) - integrate((c*x^(5/2) + b*sqrt(x))/(a*c*x^4 + a*b*x^2 + a^2
), x)
```

Fricas [B] time = 8.61433, size = 11429, normalized size = 30.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(1/2*((b^6 - 7*a*b^4*c +
13*a^2*b^2*c^2 - 4*a^3*c^3 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((b^8*c^8 - 6*a*b^6*c^9 +
11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x - 1/2*sqrt(1/2)*(b^13*c^5 -
13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 + 175*a^4*b^5*c^9 - 79*a^5
*b^3*c^10 + 12*a^6*b*c^11 + (a^5*b^12*c^5 - 16*a^6*b^10*c^6 + 100*a^7*b^8*c
^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b^2*c^10 + 64*a^11*c^11)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c
+ 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c
```

+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))) - (b^1
0*c^4 - 10*a*b^8*c^5 + 35*a^2*b^6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8 - 4
*a^5*c^9 + (a^5*b^9*c^4 - 11*a^6*b^7*c^5 + 41*a^7*b^5*c^6 - 56*a^8*b^3*c^7
+ 16*a^9*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^
4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(x)
)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 -
 6*a^3*b^2*c^8 + a^4*c^9)) - 4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-
1/2*((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (a^5*b^5 - 8*a^6*b^3*c
 + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((
b^8*c^8 - 6*a*b^6*c^9 + 11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x - 1/
2*sqrt(1/2)*(b^13*c^5 - 13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 +
175*a^4*b^5*c^9 - 79*a^5*b^3*c^10 + 12*a^6*b*c^11 - (a^5*b^12*c^5 - 16*a^6*
b^10*c^6 + 100*a^7*b^8*c^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b
^2*c^10 + 64*a^11*c^11)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*
c + 16*a^7*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^
5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a
^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))) - (b^10*c^4 - 10*a*b^8*c^
5 + 35*a^2*b^6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8 - 4*a^5*c^9 - (a^5*b^9
*c^4 - 11*a^6*b^7*c^5 + 41*a^7*b^5*c^6 - 56*a^8*b^3*c^7 + 16*a^9*b*c^8)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c +
16*a^7*c^2))))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 - 6*a^3*b^2*c^8 + a^
4*c^9)) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*
c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*
a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5
 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^
2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^

3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*s
qrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c
 + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a
^2*c^6)*sqrt(x)) + a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2
+ (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 -
 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(-1/2*sqrt(1/2)*(
b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32
*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 +
352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8
*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))
)/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^
4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b
^2*c^5 + a^2*c^6)*sqrt(x)) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12
*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sq
rt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3
*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b
^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^
5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a
^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^
4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x)) + a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b
^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*l
og(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 1
28*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 -
 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c +

```
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b
*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*
c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*
a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))
 + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x)) + 4*sqrt(x))/(a*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*x^(3/2)), x)
```

3.1069 $\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=371

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right.}{\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -2/(3*a*x^(3/2)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*
a*c])^(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c]
)^(3/4)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x]
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4
))
```

Rubi [A] time = 0.512135, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1115, 1368, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right.}{\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*
a*c])^(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c]
)^(3/4)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x]
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4
```

))

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{3ax^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{-3b-3cx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{3a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}}{}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{a\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Su}}{}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] time = 0.0527408, size = 82, normalized size = 0.22

$$-\frac{3\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^4c\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)}{\#1^3b+2\#1^7c}\&\right]+\frac{4}{x^{3/2}}}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(4/x^(3/2) + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] + c*Lo
g[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(6*a)
```

Maple [C] time = 0.261, size = 64, normalized size = 0.2

$$\frac{1}{2\,a}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{-\_R^4c-b}{2\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)-\frac{2}{3\,a}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^
4*b+a))-2/3/a/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(3\,b\sqrt{x}+\frac{a}{x^{\frac{3}{2}}}\right)}{3\,a^2}+\int\frac{bcx^{\frac{7}{2}}+\left(b^2-ac\right)x^{\frac{3}{2}}}{a^2cx^4+a^2bx^2+a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2/3*(3*b*sqrt(x) + a/x^(3/2))/a^2 + integrate((b*c*x^(7/2) + (b^2 - a*c)*x
^(3/2))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

Fricas [B] time = 15.2903, size = 14453, normalized size = 38.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 3
7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/4*(sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*
a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^
6*b^2*c^6 - 16*a^7*c^7 + (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a
^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(4*(b^1
2*c^4 - 10*a*b^10*c^5 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 -
12*a^5*b^2*c^9 + a^6*c^10)*x + 2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^
14*c^2 - 546*a^3*b^12*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6
```

*b^6*c^6 - 592*a^7*b^4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 + (a^7*b^15 - 19*a
^8*b^13*c + 148*a^9*b^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*
a^12*b^5*c^5 + 928*a^13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c
+ 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c
^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7
 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a
^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4
*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^
2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 -
7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^
4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c
^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) + 2*sqrt(1/2)*(b^
20*c^2 - 21*a*b^18*c^3 + 188*a^2*b^16*c^4 - 935*a^3*b^14*c^5 + 2821*a^4*b^1
2*c^6 - 5292*a^5*b^10*c^7 + 6083*a^6*b^8*c^8 - 4071*a^7*b^6*c^9 + 1449*a^8*
b^4*c^10 - 248*a^9*b^2*c^11 + 16*a^10*c^12 + (a^7*b^17*c^2 - 22*a^8*b^15*c^
3 + 204*a^9*b^13*c^4 - 1032*a^10*b^11*c^5 + 3075*a^11*b^9*c^6 - 5417*a^12*b
^7*c^7 + 5324*a^13*b^5*c^8 - 2480*a^14*b^3*c^9 + 320*a^15*b*c^10)*sqrt((b^1
2 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5
*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c
^3)))*sqrt(x)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*
b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*
a^9*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b
*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a
^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8
*b^2*c + 16*a^9*c^2)))/(b^12*c^7 - 10*a*b^10*c^8 + 37*a^2*b^8*c^9 - 62*a^3*
b^6*c^10 + 46*a^4*b^4*c^11 - 12*a^5*b^2*c^12 + a^6*c^13)) - 12*a*x^2*sqrt(s
qrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 -
 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c
^2)))*arctan(1/4*(sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^
3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^
7 - (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^1
1*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*
a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^1
5*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(4*(b^12*c^4 - 10*a*b^10*c^5
 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 - 12*a^5*b^2*c^9 + a^6*
c^10)*x + 2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^14*c^2 - 546*a^3*b^12
*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6*b^6*c^6 - 592*a^7*b^
4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 - (a^7*b^15 - 19*a^8*b^13*c + 148*a^9*b
^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*a^12*b^5*c^5 + 928*a^

13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) + 2*sqrt(1/2)*(b^20*c^2 - 21*a*b^18*c^3 + 188*a^2*b^16*c^4 - 935*a^3*b^14*c^5 + 2821*a^4*b^12*c^6 - 5292*a^5*b^10*c^7 + 6083*a^6*b^8*c^8 - 4071*a^7*b^6*c^9 + 1449*a^8*b^4*c^10 - 248*a^9*b^2*c^11 + 16*a^10*c^12 - (a^7*b^17*c^2 - 22*a^8*b^15*c^3 + 204*a^9*b^13*c^4 - 1032*a^10*b^11*c^5 + 3075*a^11*b^9*c^6 - 5417*a^12*b^7*c^7 + 5324*a^13*b^5*c^8 - 2480*a^14*b^3*c^9 + 320*a^15*b*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^12*c^7 - 10*a*b^10*c^8 + 37*a^2*b^8*c^9 - 62*a^3*b^6*c^10 + 46*a^4*b^4*c^11 - 12*a^5*b^2*c^12 + a^6*c^13)) - 3*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*sqrt(x) + (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*a*x^2*s

qrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*
b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*
a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*sqrt(x)
 - (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*
b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c
 + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*
c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt
(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*
b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^
4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)
))) - 3*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 3
7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a
^3*c^5)*sqrt(x) + (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^
4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^1
2 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5
*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c
^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
- (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)))) + 3*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2
*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 -
10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2
*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3))
)/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*
a^2*b^2*c^4 - a^3*c^5)*sqrt(x) - (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3
*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10
*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b
^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*
c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2
 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b
^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) - 4*sqrt(x))/(a*x^2)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*x^(5/2)), x)
```

3.1070 $\int \frac{1}{x^{7/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=412

$$\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) + (c^(1/4)*(b - (b^2 - 2*a*c)/Sqrt[b
^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2^(3/4)*a^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b -
(b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^
(1/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))
```

Rubi [A] time = 0.981175, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1368, 1504, 1510, 298, 205, 208}

$$\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) + (c^(1/4)*(b - (b^2 - 2*a*c)/Sqrt[b
^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2^(3/4)*a^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b -
(b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^
(1/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
```

```
])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1368

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{2}{5ax^{5/2}}+\frac{2\,\text{Subst}\left(\int \frac{-5b-5cx^4}{x^2\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)}{5a}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2\,\text{Subst}\left(\int \frac{x^2\left(-5\left(b^2-ac\right)-5bcx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{5a^2}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{\left(c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right.\right.}{}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{\left(\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}a^2}+\frac{\left(\sqrt{c}\left(b-\right.\right.}{}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a^2\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\sqrt[4]{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{-b+}}\right.}{2^{3/4}a^2\sqrt[4]{-b+\sqrt{b^2-4a}}}
\end{aligned}
$$

Mathematica [C] time = 0.0735676, size = 107, normalized size = 0.26

$$
-\frac{-5\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^4bc\log\left(\sqrt{x}-\#1\right)-ac\log\left(\sqrt{x}-\#1\right)+b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4a}{x^{5/2}}-\frac{20b}{\sqrt{x}}}{10a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((4*a)/x^(5/2) - (20*b)/Sqrt[x] - 5*RootSum[a + b*#1^4 + c*#1^8 & , (b^2*L
og[Sqrt[x] - #1] - a*c*Log[Sqrt[x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#
1 + 2*c*#1^5) & ])/(10*a^2)
```

Maple [C] time = 0.261, size = 82, normalized size = 0.2

$$\frac{1}{2a^2}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{bc\_R^6+\left(-ac+b^2\right)\_R^2}{2\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)-\frac{2}{5a}x^{-\frac{5}{2}}+2\frac{b}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a^2*sum((b*c*_R^6+(-a*c+b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=
RootOf(_Z^8*c+_Z^4*b+a))-2/5/a/x^(5/2)+2*b/a^2/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\frac{5b}{\sqrt{x}}-\frac{a}{x^{\frac{5}{2}}}\right)}{5a^2}+\int\frac{bcx^{\frac{5}{2}}+\left(b^2-ac\right)\sqrt{x}}{a^2cx^4+a^2bx^2+a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/5*(5*b/sqrt(x) - a/x^(5/2))/a^2 + integrate((b*c*x^(5/2) + (b^2 - a*c)*sq
rt(x))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

Fricas [B] time = 53.6437, size = 17871, normalized size = 43.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

[Out] 1/10*(20*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 3
0*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((
b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 -
314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 -
12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c +
16*a^11*c^2)))*arctan(1/2*((b^11 - 11*a*b^9*c + 43*a^2*b^7*c^2 - 70*a^3*b^5
*c^3 + 41*a^4*b^3*c^4 - 4*a^5*b*c^5 - (a^9*b^6 - 10*a^10*b^4*c + 32*a^11*b^
2*c^2 - 32*a^12*c^3)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b
^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*
c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))
*sqrt((b^16*c^14 - 14*a*b^14*c^15 + 79*a^2*b^12*c^16 - 230*a^3*b^10*c^17 +
367*a^4*b^8*c^18 - 314*a^5*b^6*c^19 + 130*a^6*b^4*c^20 - 20*a^7*b^2*c^21 +
a^8*c^22)*x - 1/2*sqrt(1/2)*(b^23*c^9 - 23*a*b^21*c^10 + 230*a^2*b^19*c^11
- 1311*a^3*b^17*c^12 + 4692*a^4*b^15*c^13 - 10947*a^5*b^13*c^14 + 16731*a^6
*b^11*c^15 - 16380*a^7*b^9*c^16 + 9711*a^8*b^7*c^17 - 3109*a^9*b^5*c^18 + 4
25*a^10*b^3*c^19 - 20*a^11*b*c^20 - (a^9*b^18*c^9 - 22*a^10*b^16*c^10 + 205
*a^11*b^14*c^11 - 1050*a^12*b^12*c^12 + 3206*a^13*b^10*c^13 - 5909*a^14*b^8
*c^14 + 6333*a^15*b^6*c^15 - 3580*a^16*b^4*c^16 + 880*a^17*b^2*c^17 - 64*a^
18*c^18)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 36
7*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^
8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(-(b^9
- 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*
a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*
a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7
*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c
^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2))) - (b^19*c^7 - 18*a*b^17*c^8
+ 135*a^2*b^15*c^9 - 546*a^3*b^13*c^10 + 1287*a^4*b^11*c^11 - 1782*a^5*b^9*
c^12 + 1386*a^6*b^7*c^13 - 540*a^7*b^5*c^14 + 81*a^8*b^3*c^15 - 4*a^9*b*c^1
6 - (a^9*b^14*c^7 - 17*a^10*b^12*c^8 + 117*a^11*b^10*c^9 - 416*a^12*b^8*c^1
0 + 805*a^13*b^6*c^11 - 810*a^14*b^4*c^12 + 352*a^15*b^2*c^13 - 32*a^16*c^1
4)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*
b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^
18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(x))*sqrt(sqr
t(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c
^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a
^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^
6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b
^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))/(b^16*c^9
- 14*a*b^14*c^10 + 79*a^2*b^12*c^11 - 230*a^3*b^10*c^12 + 367*a^4*b^8*c^13
- 314*a^5*b^6*c^14 + 130*a^6*b^4*c^15 - 20*a^7*b^2*c^16 + a^8*c^17)) - 20*a
^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*
c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*
a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^
4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^

2)))*arctan(-1/2*((b^11 - 11*a*b^9*c + 43*a^2*b^7*c^2 - 70*a^3*b^5*c^3 + 41*a^4*b^3*c^4 - 4*a^5*b*c^5 + (a^9*b^6 - 10*a^10*b^4*c + 32*a^11*b^2*c^2 - 32*a^12*c^3)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt((b^16*c^14 - 14*a*b^14*c^15 + 79*a^2*b^12*c^16 - 230*a^3*b^10*c^17 + 367*a^4*b^8*c^18 - 314*a^5*b^6*c^19 + 130*a^6*b^4*c^20 - 20*a^7*b^2*c^21 + a^8*c^22)*x - 1/2*sqrt(1/2)*(b^23*c^9 - 23*a*b^21*c^10 + 230*a^2*b^19*c^11 - 1311*a^3*b^17*c^12 + 4692*a^4*b^15*c^13 - 10947*a^5*b^13*c^14 + 16731*a^6*b^11*c^15 - 16380*a^7*b^9*c^16 + 9711*a^8*b^7*c^17 - 3109*a^9*b^5*c^18 + 425*a^10*b^3*c^19 - 20*a^11*b*c^20 + (a^9*b^18*c^9 - 22*a^10*b^16*c^10 + 205*a^11*b^14*c^11 - 1050*a^12*b^12*c^12 + 3206*a^13*b^10*c^13 - 5909*a^14*b^8*c^14 + 6333*a^15*b^6*c^15 - 3580*a^16*b^4*c^16 + 880*a^17*b^2*c^17 - 64*a^18*c^18)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2))) - (b^19*c^7 - 18*a*b^17*c^8 + 135*a^2*b^15*c^9 - 546*a^3*b^13*c^10 + 1287*a^4*b^11*c^11 - 1782*a^5*b^9*c^12 + 1386*a^6*b^7*c^13 - 540*a^7*b^5*c^14 + 81*a^8*b^3*c^15 - 4*a^9*b*c^16 + (a^9*b^14*c^7 - 17*a^10*b^12*c^8 + 117*a^11*b^10*c^9 - 416*a^12*b^8*c^10 + 805*a^13*b^6*c^11 - 810*a^14*b^4*c^12 + 352*a^15*b^2*c^13 - 32*a^16*c^14)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2))))/(b^16*c^9 - 14*a*b^14*c^10 + 79*a^2*b^12*c^11 - 230*a^3*b^10*c^12 + 367*a^4*b^8*c^13 - 314*a^5*b^6*c^14 + 130*a^6*b^4*c^15 - 20*a^7*b^2*c^16 + a^8*c^17)) - 5*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(1/2

sqrt(1/2)(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068
*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 26
4*a^8*b^2*c^8 - 16*a^9*c^9 - (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2
- 575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6
)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b
^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^1
8*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqr
t(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*
b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^
2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6
- 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 6
4*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7
*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*
c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*
c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7
+ a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^
9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*
c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x)) + 5*a^2*x^3*sqrt(sqrt(1/2)*sqrt(
-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^
4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2
- 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 -
20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*
a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(-1/2*sqrt(1/2)*(b^
18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4
- 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8
- 16*a^9*c^9 - (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7
*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 -
14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a
^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^1
9*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*
b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b
^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^
10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c
^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/
(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^
5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^
2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*
b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^
18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10
*b^2*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b
^2*c^10 + a^4*c^11)*sqrt(x)) - 5*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^
7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2
*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10
*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7
+ a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a

^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(1/2*sqrt(1/2)*(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8 - 16*a^9*c^9 + (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x)) + 5*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(-1/2*sqrt(1/2)*(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8 - 16*a^9*c^9 + (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x)) + 4*(5*b*x^2 - a)*sqrt(x))/(a^2*x^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*x^(7/2)), x)
```

3.1071 $\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=544

$$\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(-\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\cdots}{\sqrt[4]{\sqrt{b^2-4\cdots}}}\right.}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] -(b*x^(3/2))/(2*c*(b^2 - 4*a*c)) + (x^(7/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((3*b^3 - 20*a*b*c + (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*
a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b
^3 - 20*a*b*c - (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)
^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b^3 - 20*a*b*c + (3*b^2 - 14*a
*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4
*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*
c])^(1/4)) + ((3*b^3 - 20*a*b*c - (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTa
nh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^
(7/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 2.57734, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1365, 1502, 1510, 298, 205, 208}

$$\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(-\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\cdots}{\sqrt[4]{\sqrt{b^2-4\cdots}}}\right.}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*x^(3/2))/(2*c*(b^2 - 4*a*c)) + (x^(7/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((3*b^3 - 20*a*b*c + (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*
a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b
^3 - 20*a*b*c - (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
```

```
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)
^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b^3 - 20*a*b*c + (3*b^2 - 14*a
*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4
*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*
c])^(1/4)) + ((3*b^3 - 20*a*b*c - (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTa
nh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^
(7/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{14}}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^6\left(14a+3bx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(9ab+3\left(3b^2-14ac\right)x^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{6c\left(b^2-4ac\right)}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(3b^2-14ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{20abc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+ \cdots}\right.}{4c\left(b^2-4ac\right)}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^2-14ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{20abc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\sqrt{- \cdots}}\right.}{4\sqrt{2}c^{3/2}\left(b^2-4ac\right)}$$

$$= -\frac{bx^{3/2}}{2c\left(b^2-4ac\right)} + \frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(3b^3-20abc+\left(3b^2-14ac\right)\sqrt{b^2-4ac}\right)\tan^{-1} \cdots}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b-\sqrt{b^2- \cdots}}}$$

Mathematica [C] time = 0.261286, size = 144, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{-14\#1^4ac\log\left(\sqrt{x}-\#1\right)+3\#1^4b^2\log\left(\sqrt{x}-\#1\right)+3ab\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]-\frac{4x^{3/2}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{a+bx^2+cx^4}}{8c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*x^(3/2)*(b^2*x^2 + a*(b - 2*c*x^2)))/(a + b*x^2 + c*x^4) + RootSum[a +
 b*#1^4 + c*#1^8 & , (3*a*b*Log[Sqrt[x] - #1] + 3*b^2*Log[Sqrt[x] - #1]*#1^
4 - 14*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(8*c*(b^2 - 4*a*c
))
```

Maple [C] time = 0.266, size = 149, normalized size = 0.3

$$2\,\frac{1}{cx^4+bx^2+a}\left(-1/4\,\frac{\left(2\,ac-b^2\right)x^{7/2}}{c\left(4\,ac-b^2\right)}+1/4\,\frac{abx^{3/2}}{c\left(4\,ac-b^2\right)}\right)+\frac{1}{8\,c\left(4\,ac-b^2\right)}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(14\,ac-3\,b^2\right)\_R^6-3}{2\,\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*(2*a*c-b^2)/c/(4*a*c-b^2)*x^(7/2)+1/4*a*b/c/(4*a*c-b^2)*x^(3/2))/(c
*x^4+b*x^2+a)+1/8/c/(4*a*c-b^2)*sum(((14*a*c-3*b^2)*_R^6-3*_R^2*a*b)/(2*_R^
7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^2-2\,ac\right)x^{\frac{7}{2}}+abx^{\frac{3}{2}}}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}+\int\frac{\left(3\,b^2-14\,ac\right)x^{\frac{5}{2}}+3\,ab\sqrt{x}}{4\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^(7/2) + a*b*x^(3/2))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2
*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + integrate(1/4*((3*b^2 - 14*a*c)
*x^(5/2) + 3*a*b*sqrt(x))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
(b^3*c - 4*a*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1072 $\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=520

$$-\frac{\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right.}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)}$$

```
[Out] -(b*Sqrt[x])/(2*c*(b^2 - 4*a*c)) + (x^(5/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*
c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^
(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*
c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*
a*c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b +
Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 1.37058, antiderivative size = 520, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1365, 1502, 1422, 212, 208, 205}

$$-\frac{\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right.}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*Sqrt[x])/(2*c*(b^2 - 4*a*c)) + (x^(5/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*
c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^
(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*
c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/
```

```
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*
a*c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b +
Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{12}}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^4\left(10a+bx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{ab+\left(b^2-10ac\right)x^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2c\left(b^2-4ac\right)}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-10ac-\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}}\right.}{4c\left(b^2-4ac\right)}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2-10ac+\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4a}}}\right.}{4c\left(b^2-4ac\right)\sqrt{-b-\sqrt{b^2-4}}}$$

$$= -\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)} + \frac{x^{5/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2-10ac+\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] time = 0.248163, size = 144, normalized size = 0.28

$$\frac{\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{-10\#1^4 ac\log\left(\sqrt{x}-\#1\right)+\#1^4 b^2\log\left(\sqrt{x}-\#1\right)+ab\log\left(\sqrt{x}-\#1\right)}{\#1^3 b+2\#1^7 c}\&\right]-\frac{4\sqrt{x}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{a+bx^2+cx^4}}{8c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*Sqrt[x]*(b^2*x^2 + a*(b - 2*c*x^2)))/(a + b*x^2 + c*x^4) + RootSum[a +
 b*#1^4 + c*#1^8 & , (a*b*Log[Sqrt[x] - #1] + b^2*Log[Sqrt[x] - #1]*#1^4 -
10*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(8*c*(b^2 - 4*a*c))
```

Maple [C] time = 0.267, size = 146, normalized size = 0.3

$$2\frac{1}{cx^4+bx^2+a}\left(-1/4\frac{\left(2ac-b^2\right)x^{5/2}}{c\left(4ac-b^2\right)}+1/4\frac{ab\sqrt{x}}{c\left(4ac-b^2\right)}\right)+\frac{1}{8c\left(4ac-b^2\right)}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(10ac-b^2\right)\_R^4-ab}{2\_R^7c+\_R^3b}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*(2*a*c-b^2)/c/(4*a*c-b^2)*x^(5/2)+1/4*a*b/c/(4*a*c-b^2)*x^(1/2))/(c
*x^4+b*x^2+a)+1/8/c/(4*a*c-b^2)*sum(((10*a*c-b^2)*_R^4-a*b)/(2*_R^7*c+_R^3*
b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx^{\frac{9}{2}}+2\,ax^{\frac{5}{2}}}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}+\int-\frac{bx^{\frac{7}{2}}+10\,ax^{\frac{3}{2}}}{4\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^(9/2) + 2*a*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*x^2) + integrate(-1/4*(b*x^(7/2) + 10*a*x^(3/2))/((b^2*c - 4*
```

```
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1073 $\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=471

$$\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tanh}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-}}$$

```
[Out] (x^(3/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2 + 12*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b - (b^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*
a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(
3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((b - (b
^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4))
```

Rubi [A] time = 0.917352, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1115, 1365, 1510, 298, 205, 208}

$$\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tanh}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2 + 12*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b - (b^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*
a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]
```

```
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(
3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((b - (b
^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{10}}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(6a-bx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+12ac-b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)^{3/2}} +$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2+12ac-b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right.}{4\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2+12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b^2+12ac-b\right.}{4\,2^{3/4}c^{3/4}\left(\right.}$$

Mathematica [C] time = 0.195584, size = 124, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{\#1^4 b\log\left(\sqrt{x}-\#1\right)-6a\log\left(\sqrt{x}-\#1\right)}{2\#1^5 c+\#1 b}\&\right]}{8\left(b^2-4ac\right)} - \frac{-2ax^{3/2}-bx^{7/2}}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(-2*a*x^(3/2) - b*x^(7/2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootSum
[a + b*#1^4 + c*#1^8 & , (-6*a*Log[Sqrt[x] - #1] + b*Log[Sqrt[x] - #1]*#1^4
)/(b*#1 + 2*c*#1^5) & ]/(8*(b^2 - 4*a*c))
```

Maple [C] time = 0.267, size = 120, normalized size = 0.3

$$2\frac{1}{cx^4+bx^2+a}\left(-1/4\frac{bx^{7/2}}{4ac-b^2}-1/2\frac{ax^{3/2}}{4ac-b^2}\right)-\frac{1}{32ac-8b^2}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\_R^6b-6\_R^2a}{2\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/(4*a*c-b^2)*x^(7/2)-1/2*a/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)-1/
8/(4*a*c-b^2)*sum((_R^6*b-6*_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Roo
tOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx^{\frac{7}{2}}+2ax^{\frac{3}{2}}}{2\left(\left(b^2c-4ac^2\right)x^4+ab^2-4a^2c+\left(b^3-4abc\right)x^2\right)}-\int-\frac{bx^{\frac{5}{2}}-6a\sqrt{x}}{4\left(\left(b^2c-4ac^2\right)x^4+ab^2-4a^2c+\left(b^3-4abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^(7/2) + 2*a*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*x^2) - integrate(-1/4*(b*x^(5/2) - 6*a*sqrt(x))/((b^2*c - 4*a
*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Fricas [B] time = 169.428, size = 28295, normalized size = 60.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b
^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4
*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b
```

^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*arctan(-1/2*((b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 - (b^14*c^3 - 12*a*b^12*c^4 - 48*a^2*b^10*c^5 + 1600*a^3*b^8*c^6 - 11520*a^4*b^6*c^7 + 39936*a^5*b^4*c^8 - 69632*a^6*b^2*c^9 + 49152*a^7*c^10)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt((117649*a^4*b^20 + 9983358*a^5*b^18*c + 404714961*a^6*b^16*c^2 + 9897860448*a^7*b^14*c^3 + 158656107456*a^8*b^12*c^4 + 1707655509504*a^9*b^10*c^5 + 12338818573824*a^10*b^8*c^6 + 58812305154048*a^11*b^6*c^7 + 177024646692864*a^12*b^4*c^8 + 304679870005248*a^13*b^2*c^9 + 228509902503936*a^14*c^10)*x - 1/2*sqrt(1/2)*(2401*a^3*b^25 + 294294*a^4*b^23*c + 13335105*a^5*b^21*c^2 + 323354360*a^6*b^19*c^3 + 4269253584*a^7*b^17*c^4 + 24537890304*a^8*b^15*c^5 - 79436754432*a^9*b^13*c^6 - 1621756588032*a^10*b^11*c^7 - 3506876964864*a^11*b^9*c^8 + 27305557622784*a^12*b^7*c^9 + 100201644490752*a^13*b^5*c^10 - 142936235311104*a^14*b^3*c^11 - 677066377789440*a^15*b*c^12 - (2401*a^3*b^30*c^3 - 49049*a^4*b^28*c^4 - 1432760*a^5*b^26*c^5 - 6473264*a^6*b^24*c^6 + 373184512*a^7*b^22*c^7 - 319185152*a^8*b^20*c^8 - 27408852992*a^9*b^18*c^9 + 93871525888*a^10*b^16*c^10 + 774145638400*a^11*b^14*c^11 - 4486009651200*a^12*b^12*c^12 - 5590781263872*a^13*b^10*c^13 + 81717925773312*a^14*b^8*c^14 - 108093958520832*a^15*b^6*c^15 - 454721122861056*a^16*b^4*c^16 + 1497904875307008*a^17*b^2*c^17 - 1283918464548864*a^18*c^18)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))) - (343*a^2*b^19 + 21070*a^3*b^17*c + 600271*a^4*b^15*c^2 + 8903196*a^5*b^13*c^3 + 62719920*a^6*b^11*c^4 - 15909696*a^7*b^9*c^5 - 2396812032*a^8*b^7*c^6 - 6953610240*a^9*b^5*c^7 + 19591041024*a^10*b^3*c^8 + 78364164096*a^11*b*c^9 - (343*a^2*b^24*c^3 + 10437*a^3*b^22*c^4 + 90132*a^4*b^20*c^5 - 1028432*a^5*b^18*c^6 - 14041152*a^6*b^16*c^7 + 70390272*a^7*b^14*c^8 + 646137856*a^8*b^12*c^9 - 3121520640*a^9*b^10*c^10 - 11091935232*a^10*b^8*c^11 + 68335239168*a^11*b^6*c^12 + 24652283904*a^12*b^4*c^13 - 557256278016*a^13*b^2*c^14 + 743008370688*a^14*c^15

)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^
4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 3
2256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7
*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(x))*sqrt(sqrt(1/2
)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 1
7496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14
*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 3440
64*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^
15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840
*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))/(2401*a^3*b^16 + 179046*a
^4*b^14*c + 6354369*a^5*b^12*c^2 + 131902344*a^6*b^10*c^3 + 1713103344*a^7*
b^8*c^4 + 13740938496*a^8*b^6*c^5 + 65167421184*a^9*b^4*c^6 + 166523848704*
a^10*b^2*c^7 + 176319369216*a^11*c^8)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 -
 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 16
8*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^
5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*
sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*
c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 322
56*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b
^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^
4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^
8 + 4096*a^6*c^9)))*arctan(1/2*((b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*
a^3*b^3*c^3 + 5184*a^4*b*c^4 + (b^14*c^3 - 12*a*b^12*c^4 - 48*a^2*b^10*c^5
 + 1600*a^3*b^8*c^6 - 11520*a^4*b^6*c^7 + 39936*a^5*b^4*c^8 - 69632*a^6*b^2*
c^9 + 49152*a^7*c^10)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3
*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5
376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b
^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sq
rt((117649*a^4*b^20 + 9983358*a^5*b^18*c + 404714961*a^6*b^16*c^2 + 9897860
448*a^7*b^14*c^3 + 158656107456*a^8*b^12*c^4 + 1707655509504*a^9*b^10*c^5 +
 12338818573824*a^10*b^8*c^6 + 58812305154048*a^11*b^6*c^7 + 17702464669286
4*a^12*b^4*c^8 + 304679870005248*a^13*b^2*c^9 + 228509902503936*a^14*c^10)*
x - 1/2*sqrt(1/2)*(2401*a^3*b^25 + 294294*a^4*b^23*c + 13335105*a^5*b^21*c^
2 + 323354360*a^6*b^19*c^3 + 4269253584*a^7*b^17*c^4 + 24537890304*a^8*b^15
*c^5 - 79436754432*a^9*b^13*c^6 - 1621756588032*a^10*b^11*c^7 - 35068769648
64*a^11*b^9*c^8 + 27305557622784*a^12*b^7*c^9 + 100201644490752*a^13*b^5*c^
10 - 142936235311104*a^14*b^3*c^11 - 677066377789440*a^15*b*c^12 + (2401*a^
3*b^30*c^3 - 49049*a^4*b^28*c^4 - 1432760*a^5*b^26*c^5 - 6473264*a^6*b^24*c
^6 + 373184512*a^7*b^22*c^7 - 319185152*a^8*b^20*c^8 - 27408852992*a^9*b^18
*c^9 + 93871525888*a^10*b^16*c^10 + 774145638400*a^11*b^14*c^11 - 448600965
1200*a^12*b^12*c^12 - 5590781263872*a^13*b^10*c^13 + 81717925773312*a^14*b^
8*c^14 - 108093958520832*a^15*b^6*c^15 - 454721122861056*a^16*b^4*c^16 + 14
97904875307008*a^17*b^2*c^17 - 1283918464548864*a^18*c^18)*sqrt((b^8 + 54*a

*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))) - (343*a^2*b^19 + 21070*a^3*b^17*c + 600271*a^4*b^15*c^2 + 8903196*a^5*b^13*c^3 + 62719920*a^6*b^11*c^4 - 15909696*a^7*b^9*c^5 - 2396812032*a^8*b^7*c^6 - 6953610240*a^9*b^5*c^7 + 19591041024*a^10*b^3*c^8 + 78364164096*a^11*b*c^9 + (343*a^2*b^24*c^3 + 10437*a^3*b^22*c^4 + 90132*a^4*b^20*c^5 - 1028432*a^5*b^18*c^6 - 14041152*a^6*b^16*c^7 + 70390272*a^7*b^14*c^8 + 646137856*a^8*b^12*c^9 - 3121520640*a^9*b^10*c^10 - 11091935232*a^10*b^8*c^11 + 68335239168*a^11*b^6*c^12 + 24652283904*a^12*b^4*c^13 - 557256278016*a^13*b^2*c^14 + 743008370688*a^14*c^15)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))))/(2401*a^3*b^16 + 179046*a^4*b^14*c + 6354369*a^5*b^12*c^2 + 131902344*a^6*b^10*c^3 + 1713103344*a^7*b^8*c^4 + 13740938496*a^8*b^6*c^5 + 65167421184*a^9*b^4*c^6 + 166523848704*a^10*b^2*c^7 + 176319369216*a^11*c^8)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b

^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 537
6*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6
*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^1
2*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c
^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*log(1/2*sqrt(1/2)*(b^18 + 25*a*b^16
*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5
*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c
^8 + 71663616*a^9*c^9 - (b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 1171
2*a^3*b^17*c^6 + 195072*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*
b^11*c^9 - 50823168*a^7*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b
^5*c^12 + 250609664*a^10*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a
*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 -
36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10
- 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824
*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c +
168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8
*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^
9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a
^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 +
32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^
7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10
*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2
*c^8 + 4096*a^6*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3
*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3
840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c +
1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^
16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 12902
4*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2
*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 12
80*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (34
3*a^2*b^10 + 14553*a^3*b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 1
0077696*a^6*b^2*c^4 + 15116544*a^7*c^5)*sqrt(x)) - ((b^2*c - 4*a*c^2)*x^4 +
a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^
5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^
2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a
^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104
976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c
^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 5898
24*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a
*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^
5*b^2*c^8 + 4096*a^6*c^9)))*log(-1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^
2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 -
10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 716636
16*a^9*c^9 - (b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*
c^6 + 195072*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 -

50823168*a^7*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 2
50609664*a^10*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 13
77*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*
c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a
^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^
14 - 262144*a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^
3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280
*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^
8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^
18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b
^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13
+ 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*
a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096
*a^6*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b
^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4
*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b
^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 57
6*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c
^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262
144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10
+ 14553*a^3*b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6
*b^2*c^4 + 15116544*a^7*c^5)*sqrt(x)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a
^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 -
1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqr
t((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4
)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*
a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*
c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 +
240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 +
4096*a^6*c^9)))*log(1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 -
5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6
*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 +
(b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*c^6 + 195072
*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 - 50823168*a^7
*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 250609664*a^1
0*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c
^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^
2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11
+ 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*
a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024
*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6
+ 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6
*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*

a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 1
29024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8
*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5
- 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*
sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24
*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*
a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 174
96*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c
^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064
*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15
)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a
^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10 + 14553*a^3*
b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6*b^2*c^4 + 1
5116544*a^7*c^5)*sqrt(x)) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 +
3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^
6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*
a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6
- 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^1
0 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 58982
4*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8
*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^
9)))*log(-1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b
^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 +
44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 + (b^23*c^3
- 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*c^6 + 195072*a^4*b^15*c
^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 - 50823168*a^7*b^9*c^10 +
139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 250609664*a^10*b^3*c^13
- 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*
a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8
- 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^
6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))
*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3
- (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a
^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7
+ 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b
^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 -
262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*
b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*sqrt(-(b^7
+ 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4
+ 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8
+ 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*
c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a
^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^

```
12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c
^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7
- 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10 + 14553*a^3*b^8*c + 281
232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6*b^2*c^4 + 15116544*a^7
*c^5)*sqrt(x)) - 4*(b*x^3 + 2*a*x)*sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] Timed out
```

3.1074 $\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=483

$$\frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\right.}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}\right.}$$

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2 + 4
*a*c + 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2
- 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((3*b^2 + 4*a*c + 3*b*Sqrt
[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(
1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4
)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2
)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 1.03179, antiderivative size = 483, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1115, 1365, 1422, 212, 208, 205}

$$\frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\right.}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2 + 4
*a*c + 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2
- 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((3*b^2 + 4*a*c + 3*b*Sqrt
```

```
[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(
1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4
)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2
)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{2a-3bx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^2+4ac-3b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(3b^2+4ac-3b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x,\right.}{4\left(b^2-4ac\right)^{3/2}\sqrt{-b+\sqrt{b^2-4ac}}}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^2+4ac+3b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(3b^2+4ac\right.}{4\sqrt[4]{2}\sqrt[4]{c}}$$

Mathematica [C] time = 0.20085, size = 127, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&, \frac{3\#1^4 b\log\left(\sqrt{x}-\#1\right)-2a\log\left(\sqrt{x}-\#1\right)}{\#1^3 b+2\#1^7 c}\&\right]}{8\left(b^2-4ac\right)} - \frac{-2a\sqrt{x}-bx^{5/2}}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(-2*a*Sqrt[x] - b*x^(5/2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootSum
[a + b*#1^4 + c*#1^8 & , (-2*a*Log[Sqrt[x] - #1] + 3*b*Log[Sqrt[x] - #1]*#1
^4)/(b*#1^3 + 2*c*#1^7) & ]/(8*(b^2 - 4*a*c))
```

Maple [C] time = 0.263, size = 118, normalized size = 0.2

$$2\,\frac{1}{cx^4+bx^2+a}\left(-1/4\,\frac{bx^{5/2}}{4\,ac-b^2}-1/2\,\frac{a\sqrt{x}}{4\,ac-b^2}\right)+\frac{1}{32\,ac-8\,b^2}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{-3\,\_R^4b+2\,a}{2\,\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/(4*a*c-b^2)*x^(5/2)-1/2*a/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*x^2+a)+1/
8/(4*a*c-b^2)*sum((-3*_R^4*b+2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootO
f(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,cx^{\frac{9}{2}}+bx^{\frac{5}{2}}}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}-\int-\frac{2\,cx^{\frac{7}{2}}+5\,bx^{\frac{3}{2}}}{4\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^(9/2) + b*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) - integrate(-1/4*(2*c*x^(7/2) + 5*b*x^(3/2))/((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Fricas [B] time = 25.7689, size = 22881, normalized size = 47.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^
```

10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b
^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2
 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^
6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a
^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*
arctan(-1/2*(sqrt(1/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 -
1990720*a^3*b^9*c^3 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a
^6*b^3*c^6 - 180224*a^7*b*c^7 - (27*b^22*c - 820*a*b^20*c^2 + 10064*a^2*b^1
8*c^3 - 57024*a^3*b^16*c^4 + 44544*a^4*b^14*c^5 + 1505280*a^5*b^12*c^6 - 10
838016*a^6*b^10*c^7 + 38436864*a^7*b^8*c^8 - 79233024*a^8*b^6*c^9 + 9201254
4*a^9*b^4*c^10 - 49283072*a^10*b^2*c^11 + 4194304*a^11*c^12)*sqrt((6561*b^4
 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 -
 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b
^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt
((1476225*b^8 + 641520*a*b^6*c + 30816*a^2*b^4*c^2 - 8448*a^3*b^2*c^3 + 256
*a^4*c^4)*x + sqrt(1/2)*(111537*b^12 - 1375704*a*b^10*c + 5803760*a^2*b^8*c
^2 - 8961280*a^3*b^6*c^3 + 2522880*a^4*b^4*c^4 - 186368*a^5*b^2*c^5 + 4096*
a^6*c^6 + 8*(81*b^19*c - 2596*a*b^17*c^2 + 36416*a^2*b^15*c^3 - 292096*a^3*
b^13*c^4 + 1465856*a^4*b^11*c^5 - 4716544*a^5*b^9*c^6 + 9519104*a^6*b^7*c^7
 - 11075584*a^7*b^5*c^8 + 5832704*a^8*b^3*c^9 - 262144*a^9*b*c^10)*sqrt((65
61*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14
*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064
*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11))
)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 2
40*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4
096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^
16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024
*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^1
0 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3
*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(-(81*
b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c
^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)
*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576
*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7
 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a
^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3
840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)) + sqrt(1/2)*(2657205*b^
19 - 57028212*a*b^17*c + 502044480*a^2*b^15*c^2 - 2306152704*a^3*b^13*c^3 +
 5758457344*a^4*b^11*c^4 - 7169792000*a^5*b^9*c^5 + 2897625088*a^6*b^7*c^6
+ 946012160*a^7*b^5*c^7 - 111345664*a^8*b^3*c^8 + 2883584*a^9*b*c^9 - (3280
5*b^26*c - 989172*a*b^24*c^2 + 12010848*a^2*b^22*c^3 - 66614144*a^3*b^20*c^
4 + 38905600*a^4*b^18*c^5 + 1841587200*a^5*b^16*c^6 - 12771508224*a^6*b^14*
c^7 + 43815469056*a^7*b^12*c^8 - 85947383808*a^8*b^10*c^9 + 90262732800*a^9
*b^8*c^10 - 34319892480*a^10*b^6*c^11 - 9386852352*a^11*b^4*c^12 + 18958254

08*a^12*b^2*c^13 - 67108864*a^13*c^14)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(x)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))/(332150625*a*b^12 + 321489000*a^2*b^10*c + 107535600*a^3*b^8*c^2 + 12061440*a^4*b^6*c^3 - 463104*a^5*b^4*c^4 - 104448*a^6*b^2*c^5 + 4096*a^7*c^6)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*arctan(1/2*(sqrt(1/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^7*b*c^7 + (27*b^22*c - 820*a*b^20*c^2 + 10064*a^2*b^18*c^3 - 57024*a^3*b^16*c^4 + 44544*a^4*b^14*c^5 + 1505280*a^5*b^12*c^6 - 10838016*a^6*b^10*c^7 + 38436864*a^7*b^8*c^8 - 79233024*a^8*b^6*c^9 + 92012544*a^9*b^4*c^10 - 49283072*a^10*b^2*c^11 + 4194304*a^11*c^12)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt((1476225*b^8 + 641520*a*b^6*c + 30816*a^2*b^4*c^2 - 8448*a^3*b^2*c^3 + 256*a^4*c^4)*x + sqrt(1/2)*(111537*b^12 - 1375704*a*b^10*c + 5803760*a^2*b^8*c^2 - 8961280*a^3*b^6*c^3 + 2522880*a^4*b^4*c^4 - 186368*a^5*b^2*c^5 + 4096*a^6*c^6 - 8*(81*b^19*c - 2596*a*b^17*c^2 + 36416*a^2*b^15*c^3 - 292096*a^3*b^13*c^4 + 1465856*a^4*b^11*c^5 - 4716544*a^5*b^9*c^6 + 9519104*a^6*b^7*c^7 - 11075584*a^7*b^5*c^8 + 5832704*a^8*b^3*c^9 - 262144*a^9*b*c^10)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 +

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589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240
*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 +
 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a
*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^
3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 -
 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24
*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*
a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c -
240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 64
8*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376
*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^
8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c -
 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 61
44*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^
2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4
*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c +
16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^
5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a
^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c
^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c
^6 + 4096*a^6*c^7)) + sqrt(1/2)*(2657205*b^19 - 57028212*a*b^17*c + 5020444
80*a^2*b^15*c^2 - 2306152704*a^3*b^13*c^3 + 5758457344*a^4*b^11*c^4 - 71697
92000*a^5*b^9*c^5 + 2897625088*a^6*b^7*c^6 + 946012160*a^7*b^5*c^7 - 111345
664*a^8*b^3*c^8 + 2883584*a^9*b*c^9 + (32805*b^26*c - 989172*a*b^24*c^2 + 1
2010848*a^2*b^22*c^3 - 66614144*a^3*b^20*c^4 + 38905600*a^4*b^18*c^5 + 1841
587200*a^5*b^16*c^6 - 12771508224*a^6*b^14*c^7 + 43815469056*a^7*b^12*c^8 -
 85947383808*a^8*b^10*c^9 + 90262732800*a^9*b^8*c^10 - 34319892480*a^10*b^6
*c^11 - 9386852352*a^11*b^4*c^12 + 1895825408*a^12*b^2*c^13 - 67108864*a^13
*c^14)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3
 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b
^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 26
2144*a^9*c^11)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a
^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3
840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b
^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*
b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 5
89824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a
*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^
5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (
b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*
c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^
2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 3
2256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^
4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 +
```

240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))/(332150625*a*b^12 + 321489000*a^2*b^10*c + 107535600*a^3*b^8*c^2 + 12061440*a^4*b^6*c^3 - 463104*a^5*b^4*c^4 - 104448*a^6*b^2*c^5 + 4096*a^7*c^6)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqrt(x) + (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqrt(x) - (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6

```
144*a^5*b^2*c^6 + 4096*a^6*c^7)))) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*
a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 +
3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*
b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3
*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 -
589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*
a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a
^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqr
t(x) + (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4*(b^13*c -
24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 614
4*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/
(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^
4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 +
 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 76
0*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 128
0*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6
561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^1
4*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 34406
4*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)
))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) - ((b^2*c - 4*a*c^2)*x^4 + a*
b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b
^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3
*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b
^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4
 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6
*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b
^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c
^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a
^2*c^2)*sqrt(x) - (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4
*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^
5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 1
6*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5
 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^
7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(
81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^
8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c
^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 +
576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*
c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 26214
4*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) - 4*(b*x^2 + 2*a)*
sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1075 $\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=450

$$-\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (c^(1/4)*(
4*b + Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(4*b - Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4)) + (c^(1/4)*(4*b + Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(
3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(4*b - Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4
)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.709712, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1115, 1364, 1510, 298, 205, 208}

$$-\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (c^(1/4)*(
4*b + Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(4*b - Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4)) + (c^(1/4)*(4*b + Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(
```

```
3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(4*b - Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4
)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3b-2cx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(4b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)^{3/2}} - \ldots$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(\sqrt{c}\left(4b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{\ldots}\right.}{2\sqrt{2}\left(b^2-4ac\right)^{3/2}}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt[4]{c}\left(4b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-\ldots}\right.}{2\,2^{3/4}\left(b^2-4\ldots\right.}$$

Mathematica [C] time = 0.207011, size = 109, normalized size = 0.24

$$-\frac{\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{2\#1^4 c\log\left(\sqrt{x}-\#1\right)-3b\log\left(\sqrt{x}-\#1\right)}{2\#1^5 c+\#1 b}\&\right]+\frac{4x^{3/2}\left(b+2cx^2\right)}{a+bx^2+cx^4}}{8\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -((4*x^(3/2)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1
^8 & , (-3*b*Log[Sqrt[x] - #1] + 2*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1
^5) & ])/(8*(b^2 - 4*a*c))
```

Maple [C] time = 0.27, size = 121, normalized size = 0.3

$$2\frac{1}{cx^4+bx^2+a}\left(1/2\frac{cx^{7/2}}{4ac-b^2}+1/4\frac{bx^{3/2}}{4ac-b^2}\right)+\frac{1}{32ac-8b^2}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{2\_R^6c-3\_R^2b}{2\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(1/2*c/(4*a*c-b^2)*x^(7/2)+1/4*b/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)+1/8
/(4*a*c-b^2)*sum((2*_R^6*c-3*_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Ro
otOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,cx^{\frac{7}{2}}+bx^{\frac{3}{2}}}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}+\int-\frac{2\,cx^{\frac{5}{2}}-3\,b\sqrt{x}}{4\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^(7/2) + b*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) + integrate(-1/4*(2*c*x^(5/2) - 3*b*sqrt(x))/((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Fricas [B] time = 63.1198, size = 24712, normalized size = 54.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*
b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b
^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18
```

- 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^
4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a
^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2
- 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*
arctan(-((81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4*(a*b^13
- 24*a^2*b^11*c + 240*a^3*b^9*c^2 - 1280*a^4*b^7*c^3 + 3840*a^5*b^5*c^4 - 6
144*a^6*b^3*c^5 + 4096*a^7*b*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2
)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*
a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7
 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt((74733890625*b^16*c^2 + 11
2193100000*a*b^14*c^3 + 68088600000*a^2*b^12*c^4 + 20761920000*a^3*b^10*c^5
 + 3063744000*a^4*b^8*c^6 + 113909760*a^5*b^6*c^7 - 19021824*a^6*b^4*c^8 -
1179648*a^7*b^2*c^9 + 65536*a^8*c^10)*x - 1/2*sqrt(1/2)*(2989355625*b^21*c
- 23678649000*a*b^19*c^2 + 7135160400*a^2*b^17*c^3 + 277460328960*a^3*b^15*
c^4 - 338956033536*a^4*b^13*c^5 - 492326940672*a^5*b^11*c^6 - 183476674560*
a^6*b^9*c^7 - 21980119040*a^7*b^7*c^8 + 750059520*a^8*b^5*c^9 + 190316544*a
^9*b^3*c^10 - 7340032*a^10*b*c^11 + (36905625*a*b^28*c - 1159839000*a^2*b^2
6*c^2 + 15854324400*a^3*b^24*c^3 - 122710429440*a^4*b^22*c^4 + 584418357504
*a^5*b^20*c^5 - 1728949905408*a^6*b^18*c^6 + 2983008514048*a^7*b^16*c^7 - 2
317983285248*a^8*b^14*c^8 - 462348419072*a^9*b^12*c^9 + 1339972648960*a^10*
b^10*c^10 + 254402363392*a^11*b^8*c^11 - 161849802752*a^12*b^6*c^12 - 51220
840448*a^13*b^4*c^13 - 2550136832*a^14*b^2*c^14 + 268435456*a^15*c^15)*sqrt
((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*
b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 34
4064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c
^9)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c
 + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5
 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*
a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 12
9024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^
2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280
*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))) + (221
43375*b^14*c - 161619300*a*b^12*c^2 + 233100720*a^2*b^10*c^3 + 224213184*a^
3*b^8*c^4 + 48450816*a^4*b^6*c^5 + 185344*a^5*b^4*c^6 - 487424*a^6*b^2*c^7
+ 16384*a^7*c^8 - 4*(273375*a*b^21*c - 6355800*a^2*b^19*c^2 + 60732720*a^3*
b^17*c^3 - 301810176*a^4*b^15*c^4 + 798453248*a^5*b^13*c^5 - 951914496*a^6*
b^11*c^6 + 38461440*a^7*b^9*c^7 + 557711360*a^8*b^7*c^8 + 179503104*a^9*b^5
*c^9 + 11010048*a^10*b^3*c^10 - 1048576*a^11*b*c^11)*sqrt((6561*b^4 - 648*a
*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^
5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 -
 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(x))*sqr
t(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*
b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b
^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18
 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^

4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a
^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2
- 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))/
(332150625*b^12*c + 321489000*a*b^10*c^2 + 107535600*a^2*b^8*c^3 + 12061440
*a^3*b^6*c^4 - 463104*a^4*b^4*c^5 - 104448*a^5*b^2*c^6 + 4096*a^6*c^7)) - 4
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(
1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c
+ 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5
+ 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a
^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129
024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2
*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*
a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*arctan(
((81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(a*b^13 - 24*a^2
*b^11*c + 240*a^3*b^9*c^2 - 1280*a^4*b^7*c^3 + 3840*a^5*b^5*c^4 - 6144*a^6*
b^3*c^5 + 4096*a^7*b*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b
^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10
*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 58982
4*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt((74733890625*b^16*c^2 + 1121931000
00*a*b^14*c^3 + 68088600000*a^2*b^12*c^4 + 20761920000*a^3*b^10*c^5 + 30637
44000*a^4*b^8*c^6 + 113909760*a^5*b^6*c^7 - 19021824*a^6*b^4*c^8 - 1179648*
a^7*b^2*c^9 + 65536*a^8*c^10)*x - 1/2*sqrt(1/2)*(2989355625*b^21*c - 236786
49000*a*b^19*c^2 + 7135160400*a^2*b^17*c^3 + 277460328960*a^3*b^15*c^4 - 33
8956033536*a^4*b^13*c^5 - 492326940672*a^5*b^11*c^6 - 183476674560*a^6*b^9*
c^7 - 21980119040*a^7*b^7*c^8 + 750059520*a^8*b^5*c^9 + 190316544*a^9*b^3*c
^10 - 7340032*a^10*b*c^11 - (36905625*a*b^28*c - 1159839000*a^2*b^26*c^2 +
15854324400*a^3*b^24*c^3 - 122710429440*a^4*b^22*c^4 + 584418357504*a^5*b^2
0*c^5 - 1728949905408*a^6*b^18*c^6 + 2983008514048*a^7*b^16*c^7 - 231798328
5248*a^8*b^14*c^8 - 462348419072*a^9*b^12*c^9 + 1339972648960*a^10*b^10*c^1
0 + 254402363392*a^11*b^8*c^11 - 161849802752*a^12*b^6*c^12 - 51220840448*a
^13*b^4*c^13 - 2550136832*a^14*b^2*c^14 + 268435456*a^15*c^15)*sqrt((6561*b
^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2
- 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8
*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sq
rt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a
^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*
a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16
*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7
*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 -
262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6
*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(sqrt(1/2)
*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 24
0*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 40
96*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b
^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*

a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8
 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))) + (22143375
*b^14*c - 161619300*a*b^12*c^2 + 233100720*a^2*b^10*c^3 + 224213184*a^3*b^8
*c^4 + 48450816*a^4*b^6*c^5 + 185344*a^5*b^4*c^6 - 487424*a^6*b^2*c^7 + 163
84*a^7*c^8 + 4*(273375*a*b^21*c - 6355800*a^2*b^19*c^2 + 60732720*a^3*b^17*
c^3 - 301810176*a^4*b^15*c^4 + 798453248*a^5*b^13*c^5 - 951914496*a^6*b^11*
c^6 + 38461440*a^7*b^9*c^7 + 557711360*a^8*b^7*c^8 + 179503104*a^9*b^5*c^9
+ 11010048*a^10*b^3*c^10 - 1048576*a^11*b*c^11)*sqrt((6561*b^4 - 648*a*b^2*
c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^1
2*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 5898
24*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(x)*sqrt(sqrt
(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c
 + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5
 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*
a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 12
9024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^
2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280
*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))))/(3321
50625*b^12*c + 321489000*a*b^10*c^2 + 107535600*a^2*b^8*c^3 + 12061440*a^3*
b^6*c^4 - 463104*a^4*b^4*c^5 - 104448*a^5*b^2*c^6 + 4096*a^6*c^7)) - ((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sq
rt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a
^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*
a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16
*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7
*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 -
262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6
*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(1/2*sqrt(1
/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3
 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224
*a^7*b*c^7 - (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b
^16*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6
 + 38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 49
283072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16
*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3
+ 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9
*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(8
1*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8
*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^
6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 5
76*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c
^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144
*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*

a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*
a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((656
1*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*
c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*
a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))
/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^
4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^
2 + 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) + ((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt
(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3
*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^
7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c
 + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b
^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 26
2144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c
^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(-1/2*sqrt(1/
2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3
+ 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*
a^7*b*c^7 - (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^
16*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6
+ 38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 492
83072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*
a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 +
 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*
b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81
*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*
c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6
)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 57
6*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^
5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*
a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a
*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a
^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561
*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c
^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a
^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/
(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4
*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^2
 + 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) - ((b^2*c -
 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(
-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*
b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7
*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c
+ 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^

8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262
144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^
3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(1/2*sqrt(1/2)
*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 +
5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^
7*b*c^7 + (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^16
*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6 +
38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 49283
072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^
2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 3
2256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^
4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b
^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^
2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*
sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*
a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5
+ 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^
11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 38
40*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a*b
^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4
*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b
^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2
 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8
*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a
*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c
^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^2 +
 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) + ((b^2*c - 4
*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(
81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^
8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c
^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c +
576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*
c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 26214
4*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3
+ 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(-1/2*sqrt(1/2)*
(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 5
177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^7
*b*c^7 + (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^16*
c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6 + 3
8436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 492830
72*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2
*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32
256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4
*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b^
5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2

```
 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*s
qrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a
^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 +
 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^1
1*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 384
0*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a*b^
3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^
4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2
- 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*
b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*
b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^
4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 205200*a*b^6*c^2 +
47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) + 4*(2*c*x^3 +
 b*x)*sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x
^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1076 $\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=442

$$\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (c^(3/4)*(
3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 -
 4*a*c])^(3/4)) + (c^(3/4)*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(
-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 - 4*a*c])*Arc
Tanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*
(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.704618, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1115, 1364, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (c^(3/4)*(
3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 -
 4*a*c])^(3/4)) + (c^(3/4)*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(
```

```
-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 - 4*a*c])*Arc
Tanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*
(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{b-6cx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)} - \frac{\left(c\left(3\right.\right.}{}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{2\left(b^2-4ac\right)\sqrt{-b-\sqrt{b^2-4ac}}} + \cdots$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{c^{3/4}\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\text{ta}}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\right.}$$

Mathematica [C] time = 0.175239, size = 111, normalized size = 0.25

$$-\frac{\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&, \frac{6\#1^4 c\log\left(\sqrt{x}-\#1\right)-b\log\left(\sqrt{x}-\#1\right)}{\#1^3 b+2\#1^7 c}\&\right]+\frac{4\sqrt{x}\left(b+2cx^2\right)}{a+bx^2+cx^4}}{8\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -((4*Sqrt[x]*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1
^8 & , (-(b*Log[Sqrt[x] - #1]) + 6*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*
#1^7) & ])/(8*(b^2 - 4*a*c))
```

Maple [C] time = 0.264, size = 118, normalized size = 0.3

$$2\,{\frac {1}{c{x}^{4}+b{x}^{2}+a} \left(1/2\,{\frac {c{x}^{5/2}}{4\,ac-{b}^{2}}}+1/4\,{\frac {b\sqrt {x}}{4\,ac-{b}^{2}}} \right) }+{\frac {1}{32\,ac-8\,{b}^{2}}\sum _{\_R={\it RootOf} \left({\_Z}^{8}c+{\_Z}^{4}b+a \right) }{\frac {6\,{\_R}^{4}c-b}{2\,{\_R}^{7}c+{\_R}^{3}b}\ln \left(\sqrt {x}-\_R \right) }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(1/2*c/(4*a*c-b^2)*x^(5/2)+1/4*b/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*x^2+a)+1/8
/(4*a*c-b^2)*sum((6*_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z
^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx^{\frac{9}{2}} + \left(b^2 - 2\,ac\right)x^{\frac{5}{2}}}{2\left(\left(ab^2c - 4\,a^2c^2\right)x^4 + a^2b^2 - 4\,a^3c + \left(ab^3 - 4\,a^2bc\right)x^2\right)} + \int -\frac{bcx^{\frac{7}{2}} + \left(b^2 + 6\,ac\right)x^{\frac{3}{2}}}{4\left(\left(ab^2c - 4\,a^2c^2\right)x^4 + a^2b^2 - 4\,a^3c + \left(ab^3 - 4\,a^2bc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^(9/2) + (b^2 - 2*a*c)*x^(5/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-1/4*(b*c*x^(7/2) + (b
^2 + 6*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2), x)
```

Fricas [B] time = 58.7175, size = 26324, normalized size = 59.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a
^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4
```

*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b
^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 57
6*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*
c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262
144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*
c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*arctan(1/2*(sqr
t(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^
4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*
c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 - (a^3*b^23 - 20*a^4*b^21*c +
 432*a^5*b^19*c^2 - 11712*a^6*b^17*c^3 + 195072*a^7*b^15*c^4 - 1935360*a^8*
b^13*c^5 + 12214272*a^9*b^11*c^6 - 50823168*a^10*b^9*c^7 + 139788288*a^11*b
^7*c^8 - 245628928*a^12*b^5*c^9 + 250609664*a^13*b^3*c^10 - 113246208*a^14*
b*c^11)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104
976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c
^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 5898
24*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt((49*b^12*c^
2 + 3150*a*b^10*c^3 + 95985*a^2*b^8*c^4 + 1621296*a^3*b^6*c^5 + 15746400*a^
4*b^4*c^6 + 75582720*a^5*b^2*c^7 + 136048896*a^6*c^8)*x + 1/2*sqrt(1/2)*(b^
18 + 52*a*b^16*c + 1269*a^2*b^14*c^2 + 14294*a^3*b^12*c^3 + 48608*a^4*b^10*
c^4 - 679392*a^5*b^8*c^5 - 4209408*a^6*b^6*c^6 - 4105728*a^7*b^4*c^7 + 2149
90848*a^8*b^2*c^8 - 483729408*a^9*c^9 - (a^3*b^23 + 7*a^4*b^21*c - 152*a^5*
b^19*c^2 - 2960*a^6*b^17*c^3 + 44032*a^7*b^15*c^4 + 60928*a^8*b^13*c^5 - 44
44160*a^9*b^11*c^6 + 36855808*a^10*b^9*c^7 - 153681920*a^11*b^7*c^8 + 36352
8192*a^12*b^5*c^9 - 467140608*a^13*b^3*c^10 + 254803968*a^14*b*c^11)*sqrt((
b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(
a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^1
0*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^
7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*
a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2
 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sq
rt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^
4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256
*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^
4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c
 + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5
 + 4096*a^9*c^6)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^
3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a
^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377
*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*
c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^1
1*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8
 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)) - sqrt(1/2
)*(7*b^24*c + 400*a*b^22*c^2 + 7843*a^2*b^20*c^3 + 22574*a^3*b^18*c^4 - 139
5688*a^4*b^16*c^5 - 11961472*a^5*b^14*c^6 + 98703360*a^6*b^12*c^7 + 1408361

472*a^7*b^10*c^8 - 12100202496*a^8*b^8*c^9 + 1218281472*a^9*b^6*c^10 + 2412
19731456*a^10*b^4*c^11 - 812665405440*a^11*b^2*c^12 + 835884417024*a^12*c^1
3 - (7*a^3*b^29*c + 85*a^4*b^27*c^2 + 1764*a^5*b^25*c^3 - 37920*a^6*b^23*c^
4 - 103296*a^7*b^21*c^5 - 2564352*a^8*b^19*c^6 + 145468416*a^9*b^17*c^7 - 1
602797568*a^10*b^15*c^8 + 6543507456*a^11*b^13*c^9 + 7533166592*a^12*b^11*c
^10 - 193399619584*a^13*b^9*c^11 + 890247315456*a^14*b^7*c^12 - 20785209016
32*a^15*b^5*c^13 + 2556193406976*a^16*b^3*c^14 - 1320903770112*a^17*b*c^15)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32
256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(x)*sqrt(-(b^7 + 21
*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4
096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3
+ 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b
^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 -
589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5 + 4096*a^9*c^6)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 1
68*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c
^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32
256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10
*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c
^5 + 4096*a^9*c^6)))/(2401*b^16*c^3 + 179046*a*b^14*c^4 + 6354369*a^2*b^12*
c^5 + 131902344*a^3*b^10*c^6 + 1713103344*a^4*b^8*c^7 + 13740938496*a^5*b^6
*c^8 + 65167421184*a^6*b^4*c^9 + 166523848704*a^7*b^2*c^10 + 176319369216*a
^8*c^11)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^
2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^
3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a
^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377
*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*
c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^1
1*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8
- 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*arctan(-1
/2*(sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 -
2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a
^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 + (a^3*b^23 - 20*a^4*b
^21*c + 432*a^5*b^19*c^2 - 11712*a^6*b^17*c^3 + 195072*a^7*b^15*c^4 - 19353
60*a^8*b^13*c^5 + 12214272*a^9*b^11*c^6 - 50823168*a^10*b^9*c^7 + 139788288
*a^11*b^7*c^8 - 245628928*a^12*b^5*c^9 + 250609664*a^13*b^3*c^10 - 11324620
8*a^14*b*c^11)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^

3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9
*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6
 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt((49*
b^12*c^2 + 3150*a*b^10*c^3 + 95985*a^2*b^8*c^4 + 1621296*a^3*b^6*c^5 + 1574
6400*a^4*b^4*c^6 + 75582720*a^5*b^2*c^7 + 136048896*a^6*c^8)*x + 1/2*sqrt(1
/2)*(b^18 + 52*a*b^16*c + 1269*a^2*b^14*c^2 + 14294*a^3*b^12*c^3 + 48608*a^
4*b^10*c^4 - 679392*a^5*b^8*c^5 - 4209408*a^6*b^6*c^6 - 4105728*a^7*b^4*c^7
 + 214990848*a^8*b^2*c^8 - 483729408*a^9*c^9 + (a^3*b^23 + 7*a^4*b^21*c - 1
52*a^5*b^19*c^2 - 2960*a^6*b^17*c^3 + 44032*a^7*b^15*c^4 + 60928*a^8*b^13*c
^5 - 4444160*a^9*b^11*c^6 + 36855808*a^10*b^9*c^7 - 153681920*a^11*b^7*c^8
+ 363528192*a^12*b^5*c^9 - 467140608*a^13*b^3*c^10 + 254803968*a^14*b*c^11)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32
256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(-(b^7 + 21*a*b^5*c
 + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b
^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*
c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976
*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3
+ 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*
a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*
b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b
^2*c^5 + 4096*a^9*c^6)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b
^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b
^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a
^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10
*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7
 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240
*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 409
6*a^9*c^6)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (
a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^
4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*
b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 5
76*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8
*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 26
2144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6
*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)) - sqrt(1/2)*(7*
b^24*c + 400*a*b^22*c^2 + 7843*a^2*b^20*c^3 + 22574*a^3*b^18*c^4 - 1395688*
a^4*b^16*c^5 - 11961472*a^5*b^14*c^6 + 98703360*a^6*b^12*c^7 + 1408361472*a
^7*b^10*c^8 - 12100202496*a^8*b^8*c^9 + 1218281472*a^9*b^6*c^10 + 241219731
456*a^10*b^4*c^11 - 812665405440*a^11*b^2*c^12 + 835884417024*a^12*c^13 + (
7*a^3*b^29*c + 85*a^4*b^27*c^2 + 1764*a^5*b^25*c^3 - 37920*a^6*b^23*c^4 - 1
03296*a^7*b^21*c^5 - 2564352*a^8*b^19*c^6 + 145468416*a^9*b^17*c^7 - 160279
7568*a^10*b^15*c^8 + 6543507456*a^11*b^13*c^9 + 7533166592*a^12*b^11*c^10 -

193399619584*a^13*b^9*c^11 + 890247315456*a^14*b^7*c^12 - 2078520901632*a^15*b^5*c^13 + 2556193406976*a^16*b^3*c^14 - 1320903770112*a^17*b*c^15)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))/(2401*b^16*c^3 + 179046*a*b^14*c^4 + 6354369*a^2*b^12*c^5 + 131902344*a^3*b^10*c^6 + 1713103344*a^4*b^8*c^7 + 13740938496*a^5*b^6*c^8 + 65167421184*a^6*b^4*c^9 + 166523848704*a^7*b^2*c^10 + 176319369216*a^8*c^11)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) + 1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 - (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*

a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4
*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c +
240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 +
4096*a^9*c^6)))) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^
3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 +
3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c
+ 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7
*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 1290
24*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b
^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1
280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log
((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) - 1/2
*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 -
(a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*
b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8
+ 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6
*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b
^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 +
589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*
b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*
a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096
*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 1
04976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12
*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 58
9824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24
*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*
a^8*b^2*c^5 + 4096*a^9*c^6)))) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c +
(b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*
c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a
^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8
+ 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*
b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^
10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 +
589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^
5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a
^9*c^6)))*log((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*
sqrt(x) + 1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184
*a^4*b*c^4 + (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3
- 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c
^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*
a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 +
32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a
^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(
-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*

```
b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b
^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^
3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 -
5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12
*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(
a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^
4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) - ((b^2*c - 4*a*c^2)*x^4 + a*b^
2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c +
 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8
*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^
6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a
^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 +
32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^
13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
*c^5 + 4096*a^9*c^6)))*log((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11
664*a^3*c^4)*sqrt(x) - 1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b
^3*c^3 + 5184*a^4*b*c^4 + (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 160
0*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 +
 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*
c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a
^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c
^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sq
rt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b
^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4
 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c
^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^
8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5
+ 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*
a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3
+ 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) + 4*(2*c*x^2 + b)*
sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1077 $\int \frac{\sqrt{x}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=489

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\cdots\right.}{4\,2^{3/4}a\cdots}$$

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (c^(1/4)*(b - (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)
*a*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b - (b^2 - 20*
a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
- (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b +
Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.997934, antiderivative size = 489, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1115, 1366, 1510, 298, 205, 208}

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\cdots\right.}{4\,2^{3/4}a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (c^(1/4)*(b - (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)
*a*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b - (b^2 - 20*
```

```
a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
- (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b +
Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1366

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-b^2+10ac-bcx^4\right)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{4a\left(b^2-4ac\right)} + \frac{\left(c\left(b+\right.\right.}{}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(\sqrt{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{4\sqrt{2}a\left(b^2-4ac\right)} + -$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(b+\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-}}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-}}$$

Mathematica [C] time = 0.228606, size = 149, normalized size = 0.3

$$-\frac{\left(a+bx^2+cx^4\right)\text{RootSum}\left[\#1^4b+\#1^8c+a\&, \frac{\#1^4bc\log\left(\sqrt{x}-\#1\right)-10ac\log\left(\sqrt{x}-\#1\right)+b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+4x^{3/2}\left(-2ac+b^2+bcx^2\right)}{8a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(4*x^(3/2)*(b^2 - 2*a*c + b*c*x^2) + (a + b*x^2 + c*x^4)*RootSum[a + b*#1^
4 + c*#1^8 & , (b^2*Log[Sqrt[x] - #1] - 10*a*c*Log[Sqrt[x] - #1] + b*c*Log[
Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(8*a*(-b^2 + 4*a*c)*(a + b*x^2 +
 c*x^4))
```

Maple [C] time = 0.27, size = 146, normalized size = 0.3

$$2\frac{1}{cx^4+bx^2+a}\left(-1/4\,\frac{bcx^{7/2}}{a\left(4\,ac-b^2\right)}+1/4\,\frac{\left(2\,ac-b^2\right)x^{3/2}}{a\left(4\,ac-b^2\right)}\right)-\frac{1}{8\,a\left(4\,ac-b^2\right)}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{bc\_R^6+\left(-10\,ac+b^2\right.}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/a/(4*a*c-b^2)*c*x^(7/2)+1/4*(2*a*c-b^2)/a/(4*a*c-b^2)*x^(3/2))/(c
*x^4+b*x^2+a)-1/8/a/(4*a*c-b^2)*sum((b*c*_R^6+(-10*a*c+b^2)*_R^2)/(2*_R^7*c
+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx^{\frac{7}{2}}+\left(b^2-2\,ac\right)x^{\frac{3}{2}}}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}-\int-\frac{bcx^{\frac{5}{2}}+\left(b^2-10\,ac\right)\sqrt{x}}{4\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^(7/2) + (b^2 - 2*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - integrate(-1/4*(b*c*x^(5/2) + (b
^2 - 10*a*c)*sqrt(x))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b
^3 - 4*a^2*b*c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1078 $\int \frac{1}{\sqrt{x}(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=503

$$\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(-\right.}{}$$

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (c^(3/4)*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 -
 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b - Sq
rt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 - 4*a*c])
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1
/4)*a*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 1.2756, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1115, 1345, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (c^(3/4)*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 -
 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
```

```
*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b - Sq
rt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 - 4*a*c])
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1
/4)*a*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1345

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(
x*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^
2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + n*
(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^
(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*
c, 0] && ILtQ[p, -1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{b^2-2ac-4\left(b^2-4ac\right)-3bcx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,d\right.}{4a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}}\right.}{4a\left(b^2-4ac\right)^{3/2}\sqrt{-b-\sqrt{b^2-4ac}}}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{c^{3/4}\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^3}{}$$

Mathematica [C] time = 0.234815, size = 153, normalized size = 0.3

$$-\frac{\left(a+bx^2+cx^4\right)\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{3\#1^4bc\log\left(\sqrt{x}-\#1\right)-14ac\log\left(\sqrt{x}-\#1\right)+3b^2\log\left(\sqrt{x}-\#1\right)}{\#1^3b+2\#1^7c}\&\right]+4\sqrt{x}\left(-2ac+b^2+bcx\right.}{8a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(4*Sqrt[x]*(b^2 - 2*a*c + b*c*x^2) + (a + b*x^2 + c*x^4)*RootSum[a + b*#1^
4 + c*#1^8 & , (3*b^2*Log[Sqrt[x] - #1] - 14*a*c*Log[Sqrt[x] - #1] + 3*b*c*
Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(8*a*(-b^2 + 4*a*c)*(a + b
*x^2 + c*x^4))
```

Maple [C] time = 0.264, size = 144, normalized size = 0.3

$$2\frac{1}{cx^4+bx^2+a}\left(-1/4\,\frac{bcx^{5/2}}{a\left(4\,ac-b^2\right)}+1/4\,\frac{\left(2\,ac-b^2\right)\sqrt{x}}{a\left(4\,ac-b^2\right)}\right)+\frac{1}{8\,a\left(4\,ac-b^2\right)}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{-3\,\_R^4bc+14\,ac-3\,b^2}{2\,\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/a/(4*a*c-b^2)*c*x^(5/2)+1/4*(2*a*c-b^2)/a/(4*a*c-b^2)*x^(1/2))/(c
*x^4+b*x^2+a)+1/8/a/(4*a*c-b^2)*sum((-3*_R^4*b*c+14*a*c-3*b^2)/(2*_R^7*c+_R
^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,b^2c-14\,ac^2\right)x^{\frac{9}{2}}+\left(3\,b^3-13\,abc\right)x^{\frac{5}{2}}+4\left(ab^2-4\,a^2c\right)\sqrt{x}}{2\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a^3bc\right)x^2\right)}-\int\frac{\left(3\,b^2c-14\,ac^2\right)x^{\frac{7}{2}}+\left(3\,b^3-17\,abc\right)x^{\frac{3}{2}}}{4\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,a^3c^2\right)x^4+\left(a^2b^3-4\,a^3bc\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((3*b^2*c - 14*a*c^2)*x^(9/2) + (3*b^3 - 13*a*b*c)*x^(5/2) + 4*(a*b^2 -
 4*a^2*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*
b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((3*b^2*c - 14*a*c^2)*x^(7/2) + (3*b^
3 - 17*a*b*c)*x^(3/2))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (
a^2*b^3 - 4*a^3*b*c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] Timed out
```

3.1079 $\int \frac{1}{x^{3/2}(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=573

$$-\frac{5b^2-18ac}{2a^2\sqrt{x}\,(b^2-4ac)}+\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2-4ac}-\right.}{4\,2^{3/4}a^2\left(b^2-4a\right.}$$

```
[Out] -(5*b^2 - 18*a*c)/(2*a^2*(b^2 - 4*a*c)*Sqrt[x]) + (b^2 - 2*a*c + b*c*x^2)/(
2*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4)) + (c^(1/4)*(5*b^3 - 28*a*b*c
 - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt
[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(5*b^3 - 28*a*b*c + (5*b^2 - 18*a*c)*Sqrt[
b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/
4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (
c^(1/4)*(5*b^3 - 28*a*b*c - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2
 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^3 - 28*a*b*
c + (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sq
rt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 2.44638, antiderivative size = 573, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1366, 1504, 1510, 298, 205, 208}

$$-\frac{5b^2-18ac}{2a^2\sqrt{x}\,(b^2-4ac)}+\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2-4ac}-\right.}{4\,2^{3/4}a^2\left(b^2-4a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(5*b^2 - 18*a*c)/(2*a^2*(b^2 - 4*a*c)*Sqrt[x]) + (b^2 - 2*a*c + b*c*x^2)/(
2*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4)) + (c^(1/4)*(5*b^3 - 28*a*b*c
 - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt
```

```
[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(5*b^3 - 28*a*b*c + (5*b^2 - 18*a*c)*Sqrt[
b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/
4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (
c^(1/4)*(5*b^3 - 28*a*b*c - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2
 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^3 - 28*a*b*
c + (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sq
rt[b^2 - 4*a*c])^(1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1366

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-5b^2+18ac-5bcx^4}{x^2\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(-b\left(5b^2-23ac\right)-c\left(5b^2-18a\right.\right.}{a+bx^4+cx^8}\right.}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(5b^2-18ac+\frac{5b^3}{\sqrt{b^2-4ac}}-\frac{28abc}{\sqrt{b^2-4a}}\right.\right.}{4a^2}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} + \frac{\left(\sqrt{c}\left(5b^2-18ac+\frac{5b^3}{\sqrt{b^2-4ac}}-\frac{28a}{\sqrt{b^2-}}\right.\right.}{4}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\sqrt[4]{c}\left(5b^2-18ac-\frac{5b^3}{\sqrt{b^2-4ac}}+\frac{28ab}{\sqrt{b^2-4}}\right.}{4\,2^{3/4}a^2\left(b^2-4ac\right)\sqrt[4]{-}}$$

Mathematica [C] time = 0.311675, size = 190, normalized size = 0.33

$$-\frac{\frac{\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{-18\#1^4ac^2\log\left(\sqrt{x}-\#1\right)+5\#1^4b^2c\log\left(\sqrt{x}-\#1\right)-23abc\log\left(\sqrt{x}-\#1\right)+5b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{b^2-4ac}+\frac{4x^{3/2}\left(-3abc-2ac^2x^2+b^2cx^2+b^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{16}{\sqrt{x}}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(16/Sqrt[x] + (4*x^(3/2)*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + RootSum[a + b*#1^4 + c*#1^8 & , (5*b^3*Log[
Sqrt[x] - #1] - 23*a*b*c*Log[Sqrt[x] - #1] + 5*b^2*c*Log[Sqrt[x] - #1]*#1^4
 - 18*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(b^2 - 4*a*c))/(8
*a^2)
```

Maple [C] time = 0.305, size = 245, normalized size = 0.4

$$-\frac{c^2}{a\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}x^{\frac{7}{2}}+\frac{b^2c}{2a^2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}x^{\frac{7}{2}}-\frac{3bc}{2a\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}x^{\frac{3}{2}}+\frac{}{2a^2\left(cx^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^(7/2)+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a
*c-b^2)*x^(7/2)*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^(3/2)*c+1/2/a^2/(
c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)*x^(3/2)-1/8/a^2/(4*a*c-b^2)*sum((c*(18*a*c-5
*b^2)*_R^6+b*(23*a*c-5*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootO
f(_Z^8*c+_Z^4*b+a))-2/a^2/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(5b^2c-18ac^2\right)x^{\frac{7}{2}}+\left(5b^3-19abc\right)x^{\frac{3}{2}}+\frac{4\left(ab^2-4a^2c\right)}{\sqrt{x}}}{2\left(a^3b^2-4a^4c+\left(a^2b^2c-4a^3c^2\right)x^4+\left(a^2b^3-4a^3bc\right)x^2\right)}-\int\frac{\left(5b^2c-18ac^2\right)x^{\frac{5}{2}}+\left(5b^3-23abc\right)\sqrt{x}}{4\left(a^3b^2-4a^4c+\left(a^2b^2c-4a^3c^2\right)x^4+\left(a^2b^3-4a^3bc\right)x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((5*b^2*c - 18*a*c^2)*x^(7/2) + (5*b^3 - 19*a*b*c)*x^(3/2) + 4*(a*b^2
- 4*a^2*c)/sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2
*b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((5*b^2*c - 18*a*c^2)*x^(5/2) + (5*b
^3 - 23*a*b*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 +
(a^2*b^3 - 4*a^3*b*c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1080 $\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=621

$$-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(\right.}{}$$

```
[Out] (-3*(b^2 + 12*a*c)*Sqrt[x])/(16*c*(b^2 - 4*a*c)^2) + (x^(9/2)*(2*a + b*x^2)
)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(5/2)*(8*a*b + (b^2 + 12*a
*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (3*(b^3 - 28*a*b*c + (
b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^
2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c
- 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2
- 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c + (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqr
t[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1
/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 1.77181, antiderivative size = 621, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1115, 1365, 1498, 1502, 1422, 212, 208, 205}

$$-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] (-3*(b^2 + 12*a*c)*Sqrt[x])/(16*c*(b^2 - 4*a*c)^2) + (x^(9/2)*(2*a + b*x^2)
)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(5/2)*(8*a*b + (b^2 + 12*a
*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (3*(b^3 - 28*a*b*c + (
```

```
b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^
2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c
- 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2
- 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c + (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqr
t[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1
/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1498

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(f^(n - 1)*(f*x)^(m - n + 1)*(a +
 b*x^n + c*x^(2*n))^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^n))/(n*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
```

```
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{16}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^8\left(18a-3bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^4\left(-120ab-3\left(b^2+12ac\right)x^4\right.}{a+bx^4+cx^8}\right.}{16\left(b^2-4ac\right)^2}$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3\left(b^3-\right.\right.}{}$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3\left(b^3-\right.\right.}{}$$

$$= -\frac{3\left(b^2+12ac\right)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{3\left(b^3-2\right.}{32}$$

<u>**Mathematica**</u> **[C]** time = 0.448081, size = 254, normalized size = 0.41

$$\frac{3c\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^4b+\#1^8c+a\&, \frac{-28\#1^4abc\log\left(\sqrt{x}-\#1\right)+\#1^4b^3\log\left(\sqrt{x}-\#1\right)+12a^2c\log\left(\sqrt{x}-\#1\right)+ab^2\log\left(\sqrt{x}-\#1\right)}{\#1^3b+2\#1^7c}\&\right]+4\sqrt{}}{64c^2\left(b^2-4ac\right)^2\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*Sqrt[x]*(-4*b^4 + 21*a*b^2*c - 68*a^2*c^2 + b^3*c*x^2 - 28*a*b*c^2*x^2)*
(a + b*x^2 + c*x^4) + 16*(b^2 - 4*a*c)*Sqrt[x]*(-2*a^2*c + b^3*x^2 + a*b*(b
```

```
 - 3*c*x^2)) + 3*c*(a + b*x^2 + c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (a
*b^2*Log[Sqrt[x] - #1] + 12*a^2*c*Log[Sqrt[x] - #1] + b^3*Log[Sqrt[x] - #1]
*#1^4 - 28*a*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(64*c^2*(
b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

Maple [C] time = 0.297, size = 275, normalized size = 0.4

$$2\,\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(-\frac{3\,a^2\left(12\,ac+b^2\right)\sqrt{x}}{32\,c\left(16\,a^2c^2-8\,acb^2+b^4\right)}-3/16\,\frac{ab\left(8\,ac+b^2\right)x^{5/2}}{c\left(16\,a^2c^2-8\,acb^2+b^4\right)}-1/32\,\frac{\left(68\,a^2c^2+7\,acb^2+3\,b^4\right)x^{9/2}}{c\left(16\,a^2c^2-8\,acb^2+b^4\right)}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-3/32*a^2*(12*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-3/16*a/c*b*(
8*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)-1/32*(68*a^2*c^2+7*a*b^2*c+3*
b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)-1/32*b*(28*a*c-b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/c/(16*a^2*c^2-8*a*b^2*c+b^4)
*sum((b*(-28*a*c+b^2)*_R^4+12*a^2*c+b^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R)
,_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(b^2c+12\,ac^2\right)x^{\frac{17}{2}}+\left(7\,b^3+44\,abc\right)x^{\frac{13}{2}}+24\,a^2bx^{\frac{5}{2}}+\left(35\,ab^2+4\,a^2c\right)x^{\frac{9}{2}}}{16\left(\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^8+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^6+a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^6-6\,ab^4c+32\,a^3c^3\right)x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^2*c + 12*a*c^2)*x^(17/2) + (7*b^3 + 44*a*b*c)*x^(13/2) + 24*a^2*
b*x^(5/2) + (35*a*b^2 + 4*a^2*c)*x^(9/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2
*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2)*x^2) - integrate(3/32*((b^2 + 12*a*c)*x^(7/2) + 40*a*b*x
^(3/2))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1081 $\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=569

$$\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(\sqrt{b\right.}$$

```
[Out] (x^(7/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^(3/2)*
(24*a*b + (5*b^2 + 28*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b^2 + 28*a*c))*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2
- 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((5*b^2 + 28*a*c - (5*b^3
+ 172*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt
[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 -
 4*a*c])^(1/4)) - ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b^2 + 28*a*c))
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(
3/4)*c^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((5*b^2
+ 28*a*c - (5*b^3 + 172*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)
^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 1.90797, antiderivative size = 569, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1365, 1498, 1510, 298, 205, 208}

$$\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(\sqrt{b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(7/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^(3/2)*
(24*a*b + (5*b^2 + 28*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b^2 + 28*a*c))*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2
```

```
- 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((5*b^2 + 28*a*c - (5*b^3
+ 172*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt
[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 -
 4*a*c])^(1/4)) - ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b^2 + 28*a*c))
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(
3/4)*c^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((5*b^2
+ 28*a*c - (5*b^3 + 172*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)
^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

<u>Rule 1115</u>

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

<u>Rule 1365</u>

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

<u>Rule 1498</u>

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(f^(n - 1)*(f*x)^(m - n + 1)*(a +
 b*x^n + c*x^(2*n))^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^n))/(n*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

<u>Rule 1510</u>

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
```

```
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{14}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^6\left(14a-5bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(-72ab+\left(5b^2+28ac\right)x^4\right)}{a+bx^4+cx^8}\right.}{16\left(b^2-4ac\right)^2}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(5b^3+172abc+\sqrt{b^2-4ac}\,(5\right.}{}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(5b^3+172abc+\sqrt{b^2-4ac}\,(5\right.}{}$$

$$= \frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(24ab+\left(5b^2+28ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(5b^3+172abc+\sqrt{b^2-4ac}\,(5\right.}{32\,2^{3/4}c^{3/4}\left(b^2-4a\right.}$$

<u>Mathematica</u> [C] time = 0.420517, size = 216, normalized size = 0.38

$$\frac{c\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^4b+\#1^8c+a\&, \frac{28\#1^4ac\log\left(\sqrt{x}-\#1\right)+5\#1^4b^2\log\left(\sqrt{x}-\#1\right)-72ab\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+4x^{3/2}\left(8abc+28ac^2\right.}{64c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*x^(3/2)*(4*b^3 + 8*a*b*c + 5*b^2*c*x^2 + 28*a*c^2*x^2)*(a + b*x^2 + c*x^
4) - 16*(b^2 - 4*a*c)*x^(3/2)*(b^2*x^2 + a*(b - 2*c*x^2)) + c*(a + b*x^2 +
c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (-72*a*b*Log[Sqrt[x] - #1] + 5*b^2
*Log[Sqrt[x] - #1]*#1^4 + 28*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5)
& ])/(64*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

Maple [C] time = 0.285, size = 242, normalized size = 0.4

$$2\,\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(3/4\,\frac{a^2bx^{3/2}}{16\,a^2c^2-8\,acb^2+b^4}-1/32\,\frac{a\left(4\,ac-37\,b^2\right)x^{7/2}}{16\,a^2c^2-8\,acb^2+b^4}+\frac{9\,b\left(4\,ac+b^2\right)x^{11/2}}{512\,a^2c^2-256\,acb^2+32\,b^4}+1/32\,\frac{c\left(28\,ac\right.}{16\,a^2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/4*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)-1/32*a*(4*a*c-37*b^2)/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+9/32*b*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^(11/2)+1/32*c*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^4
+b*x^2+a)^2+1/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum(((28*a*c+5*b^2)*_R^6-72*_R^
2*a*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(5\,b^2c+28\,ac^2\right)x^{\frac{15}{2}}+9\left(b^3+4\,abc\right)x^{\frac{11}{2}}+24\,a^2bx^{\frac{3}{2}}+\left(37\,ab^2-4\,a^2c\right)x^{\frac{7}{2}}}{16\left(\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^8+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^6+a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^6-6\,ab^4c+32\,a^3c^3\right)x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^2*c + 28*a*c^2)*x^(15/2) + 9*(b^3 + 4*a*b*c)*x^(11/2) + 24*a^2*b
*x^(3/2) + (37*a*b^2 - 4*a^2*c)*x^(7/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*
c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*
c + 16*a^3*b*c^2)*x^2) + integrate(1/32*((5*b^2 + 28*a*c)*x^(5/2) - 72*a*b*
sqrt(x))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1082 $\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=569

$$\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(x^2\left(20ac+7b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{-\sqrt{\cdots}}}\right.}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] (x^(5/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[x]*
(24*a*b + (7*b^2 + 20*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) -
 (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*b^2 + 20*a*c))*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2
 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7*b^2 + 20*a*c - (7*b
^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2
 - 4*a*c])^(3/4)) - (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*b^2 + 20*a*
c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*
2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7
*b^2 + 20*a*c - (7*b^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*
a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 1.96457, antiderivative size = 569, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1365, 1498, 1422, 212, 208, 205}

$$\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(x^2\left(20ac+7b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{-\sqrt{\cdots}}}\right.}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(5/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[x]*
(24*a*b + (7*b^2 + 20*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) -
 (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*b^2 + 20*a*c))*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2
```

```
 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7*b^2 + 20*a*c - (7*b
^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2
 - 4*a*c])^(3/4)) - (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*b^2 + 20*a*
c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*
2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7
*b^2 + 20*a*c - (7*b^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*
a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1498

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(f^(n - 1)*(f*x)^(m - n + 1)*(a +
 b*x^n + c*x^(2*n))^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^n))/(n*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
```

```
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{12}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^4\left(10a-7bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{-24ab+3\left(7b^2+20ac\right)x^4}{a+bx^4+cx^8}\,d\right.}{16\left(b^2-4ac\right)^2}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3\left(7b^3+36abc+\sqrt{b^2-4ac}\right.\right.}{}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3\left(7b^3+36abc+\sqrt{b^2-4ac}\right.\right.}{32\left(b\right.}$$

$$= \frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{3\left(7b^3+36abc+\sqrt{b^2-4ac}\left(7\right.\right.}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)}$$

Mathematica [C] time = 0.403937, size = 219, normalized size = 0.38

$$\frac{3c\left(a+bx^2+cx^4\right)^2 \text{RootSum}\left[\#1^4 b+\#1^8 c+a\&, \frac{20\#1^4 ac\log\left(\sqrt{x}-\#1\right)+7\#1^4 b^2\log\left(\sqrt{x}-\#1\right)-8ab\log\left(\sqrt{x}-\#1\right)}{\#1^3 b+2\#1^7 c}\&\right]+4\sqrt{x}\left(8abc+20ac^2\right.}{64c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*Sqrt[x]*(4*b^3 + 8*a*b*c + 7*b^2*c*x^2 + 20*a*c^2*x^2)*(a + b*x^2 + c*x^
4) - 16*(b^2 - 4*a*c)*Sqrt[x]*(b^2*x^2 + a*(b - 2*c*x^2)) + 3*c*(a + b*x^2
+ c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (-8*a*b*Log[Sqrt[x] - #1] + 7*b^
2*Log[Sqrt[x] - #1]*#1^4 + 20*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^
7) & ])/(64*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

Maple [C] time = 0.277, size = 241, normalized size = 0.4

$$2\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(3/4\,\frac{a^2b\sqrt{x}}{16\,a^2c^2-8\,acb^2+b^4}-\frac{\left(12\,ac-39\,b^2\right)ax^{5/2}}{512\,a^2c^2-256\,acb^2+32\,b^4}+1/32\,\frac{b\left(28\,ac+11\,b^2\right)x^{9/2}}{16\,a^2c^2-8\,acb^2+b^4}+1/32\,\frac{c\left(20\,ac\right.}{16\,a^2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/4*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-3/32*(4*a*c-13*b^2)*a/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/32*b*(28*a*c+11*b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^(9/2)+1/32*c*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*
x^4+b*x^2+a)^2+3/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum(((20*a*c+7*b^2)*_R^4-8*a
*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{24\,bc^2x^{\frac{17}{2}}+\left(41\,b^2c-20\,ac^2\right)x^{\frac{13}{2}}+\left(13\,b^3+20\,abc\right)x^{\frac{9}{2}}+3\left(3\,ab^2+4\,a^2c\right)x^{\frac{5}{2}}}{16\left(\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^8+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^6+a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^6-6\,ab^4c+32\,a^3c^3\right)x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*b*c^2*x^(17/2) + (41*b^2*c - 20*a*c^2)*x^(13/2) + (13*b^3 + 20*a*
b*c)*x^(9/2) + 3*(3*a*b^2 + 4*a^2*c)*x^(5/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3
*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x^2) + integrate(3/32*(8*b*c*x^(7/2) + 5*(3*b^2 + 4*
a*c)*x^(3/2))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1083 $\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=533

$$-\frac{3x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-}}\right.}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

```
[Out] (x^(3/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*x^(3/2
)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (
3*c^(1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*
(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^
2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)
])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(
1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b
- Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^2 -
4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 1.45192, antiderivative size = 533, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1365, 1500, 1510, 298, 205, 208}

$$-\frac{3x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-}}\right.}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*x^(3/2
)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (
3*c^(1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*
(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^
```

```
2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)
])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(
1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b
- Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^2 -
4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1500

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2
 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^{10}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(6a-9bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a\left(7b^2+20ac\right)-24ab\right.}{a+bx^4+cx^8}\right.}{16a\left(b^2-4ac\right)^2}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(11b^2+20ac-4b\sqrt{b^2-\right.\right.}{1}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3\sqrt{c}\left(11b^2+20ac-4b\sqrt{b^2\right.\right.}{}$$

$$= \frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3x^{3/2}\left(5b^2-4ac+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{3\sqrt[4]{c}\left(11b^2+20ac+4b\sqrt{b^2-\right.}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}}$$

Mathematica [C] time = 0.391479, size = 176, normalized size = 0.33

$$\frac{-3\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{8\#1^4bc\log\left(\sqrt{x}-\#1\right)-20ac\log\left(\sqrt{x}-\#1\right)-7b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]-\frac{12x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{a+bx^2+cx^4}+\frac{16x^{3/2}\left(b^2-4ac\right)\left(2a+b\right.}{\left(a+bx^2+cx^4\right)^2}}{64\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((16*(b^2 - 4*a*c)*x^(3/2)*(2*a + b*x^2))/(a + b*x^2 + c*x^4)^2 - (12*x^(3/
2)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(a + b*x^2 + c*x^4) - 3*RootSum[a + b*#1^4
+ c*#1^8 & , (-7*b^2*Log[Sqrt[x] - #1] - 20*a*c*Log[Sqrt[x] - #1] + 8*b*c*L
og[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(64*(b^2 - 4*a*c)^2)
```

Maple [C] time = 0.302, size = 244, normalized size = 0.5

$$2\,\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(-1/32\,\frac{a\left(20\,ac+7\,b^2\right)x^{3/2}}{16\,a^2c^2-8\,acb^2+b^4}-1/32\,\frac{b\left(28\,ac+11\,b^2\right)x^{7/2}}{16\,a^2c^2-8\,acb^2+b^4}+\frac{\left(12\,ac-39\,b^2\right)cx^{11/2}}{512\,a^2c^2-256\,acb^2+32\,b^4}-3/4\,\frac{bc}{16\,a^2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-1/32*a*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)-1/32*b*(28*a*c
+11*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/32*(4*a*c-13*b^2)*c/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^(11/2)-3/4*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(
c*x^4+b*x^2+a)^2-3/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((8*b*c*_R^6+(-20*a*c-7
*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{24\,bc^2x^{\frac{15}{2}}+3\left(13\,b^2c-4\,ac^2\right)x^{\frac{11}{2}}+\left(11\,b^3+28\,abc\right)x^{\frac{7}{2}}+\left(7\,ab^2+20\,a^2c\right)x^{\frac{3}{2}}}{16\left(\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^8+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^6+a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^6-6\,ab^4c+32\,a^3c^3\right)x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*b*c^2*x^(15/2) + 3*(13*b^2*c - 4*a*c^2)*x^(11/2) + (11*b^3 + 28*a
*b*c)*x^(7/2) + (7*a*b^2 + 20*a^2*c)*x^(3/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3
*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x^2) - integrate(3/32*(8*b*c*x^(5/2) - (7*b^2 + 20*a
*c)*sqrt(x))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1084 $\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=533

$$\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\ldots$$

[Out] (Sqrt[x]*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (Sqrt[x]*(13*b^2 - 4*a*c + 24*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (c^(3/4)*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))

Rubi [A] time = 1.36301, antiderivative size = 533, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1365, 1430, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\ldots$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]

[Out] (Sqrt[x]*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (Sqrt[x]*(13*b^2 - 4*a*c + 24*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (c^(3/4)*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2

```
- 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*
2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1365

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*(a + b*x^n
 + c*x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^(2*n)/(n*(p +
 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*p
+ 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] &
& GtQ[m, 2*n - 1]
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{2a-11bx^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{a\left(5b^2+28ac\right)-72abcx^4}{a+bx^4+cx^8}\,d\right.}{16a\left(b^2-4ac\right)^2}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(41b^2+28ac-36b\sqrt{b^2-}\right.\right.}{1}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(41b^2+28ac-36b\sqrt{b^2-}\right.\right.}{16\left(b^2-\right.}$$

$$= \frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\sqrt{x}\left(13b^2-4ac+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{c^{3/4}\left(41b^2+28ac+36b\sqrt{b^2}\right.}{16\sqrt[4]{2}\left(b^2-4ac\right)^{5/2}\left(\right.}$$

Mathematica [C] time = 0.443112, size = 177, normalized size = 0.33

$$-\frac{\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{72\#1^4bc\log\left(\sqrt{x}-\#1\right)-28ac\log\left(\sqrt{x}-\#1\right)-5b^2\log\left(\sqrt{x}-\#1\right)}{\#1^3b+2\#1^7c}\&\right]+\frac{4\sqrt{x}\left(28a^2c+a\left(5b^2+36bcx^2-4c^2x^4\right)+bx^2\left(9b^2+37bcx\right.\right.}{\left(a+bx^2+cx^4\right)^2}}{64\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((4*Sqrt[x]*(28*a^2*c + a*(5*b^2 + 36*b*c*x^2 - 4*c^2*x^4) + b*x^2*(9*b^2
+ 37*b*c*x^2 + 24*c^2*x^4)))/(a + b*x^2 + c*x^4)^2 + RootSum[a + b*#1^4 + c
*#1^8 & , (-5*b^2*Log[Sqrt[x] - #1] - 28*a*c*Log[Sqrt[x] - #1] + 72*b*c*Log
[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(64*(b^2 - 4*a*c)^2)
```

Maple [C] time = 0.27, size = 237, normalized size = 0.4

$$2\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(-1/32\frac{a\left(28ac+5b^2\right)\sqrt{x}}{16a^2c^2-8acb^2+b^4}-\frac{9b\left(4ac+b^2\right)x^{5/2}}{512a^2c^2-256acb^2+32b^4}+1/32\frac{c\left(4ac-37b^2\right)x^{9/2}}{16a^2c^2-8acb^2+b^4}-3/4\frac{bc}{16a^2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-1/32*a*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-9/32*b*(4*a*c+
b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/32*c*(4*a*c-37*b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(9/2)-3/4*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*x^4
+b*x^2+a)^2+1/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((-72*_R^4*b*c+28*a*c+5*b^2)
/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(5b^2c^2+28ac^3\right)x^{\frac{17}{2}}+2\left(5b^3c+16abc^2\right)x^{\frac{13}{2}}+\left(5b^4+ab^2c+60a^2c^2\right)x^{\frac{9}{2}}+\left(ab^3+20a^2\right.}{16\left(\left(ab^4c^2-8a^2b^2c^3+16a^3c^4\right)x^8+a^3b^4-8a^4b^2c+16a^5c^2+2\left(ab^5c-8a^2b^3c^2+16a^3bc^3\right)x^6+\left(ab^6-6a^2b^4c+32a^4c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^2*c^2 + 28*a*c^3)*x^(17/2) + 2*(5*b^3*c + 16*a*b*c^2)*x^(13/2) +
 (5*b^4 + a*b^2*c + 60*a^2*c^2)*x^(9/2) + (a*b^3 + 20*a^2*b*c)*x^(5/2))/((a
*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5
*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*
c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) - integ
rate(1/32*((5*b^2*c + 28*a*c^2)*x^(7/2) + 5*(b^3 + 20*a*b*c)*x^(3/2))/(a^2*
b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x^4
 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1085 $\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=594

$$\frac{3x^{3/2}\left(cx^2\left(12ac+b^2\right)+b\left(4ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\sqrt[4]{c}\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tan^{-1}\left(\ldots\right.}{32\ 2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4ac}-\ldots}}$$

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(3/
2)*(b*(b^2 + 4*a*c) + c*(b^2 + 12*a*c)*x^2))/(16*a*(b^2 - 4*a*c)^2*(a + b*x
^2 + c*x^4)) + (3*c^(1/4)*(b^2 + 12*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c
)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) +
 (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 12*a*c))*ArcTan[(2^(
1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 -
 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(b^2 + 12*a*c -
b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2
*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 -
4*a*c]*(b^2 + 12*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4
*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))
```

Rubi [A] time = 2.3102, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1364, 1500, 1510, 298, 205, 208}

$$\frac{3x^{3/2}\left(cx^2\left(12ac+b^2\right)+b\left(4ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\sqrt[4]{c}\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tan^{-1}\left(\ldots\right.}{32\ 2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4ac}-\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(3/
2)*(b*(b^2 + 4*a*c) + c*(b^2 + 12*a*c)*x^2))/(16*a*(b^2 - 4*a*c)^2*(a + b*x
^2 + c*x^4)) + (3*c^(1/4)*(b^2 + 12*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c
```

```
)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) +
 (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 12*a*c))*ArcTan[(2^(
1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 -
 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(b^2 + 12*a*c -
b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2
*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 -
4*a*c]*(b^2 + 12*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4
*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1500

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2
 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
```

```
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3b-18cx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-3b\left(b^2-28\right.\right.}{a\cdots}\right.}{16a\left(\cdots\right.}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(b^2+12ac+\frac{b\cdots}{\sqrt{b^2\cdots}}\right.\right.}{\cdots}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3\sqrt{c}\left(b^2+12ac+\frac{\cdots}{\sqrt{\cdots}}\right.\right.}{\cdots}$$

$$= -\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b\left(b^2+4ac\right)+c\left(b^2+12ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt[4]{c}\left(b^2+12ac-\frac{\cdots}{\sqrt{b\cdots}}\right.}{32\ 2^{3/4}a\left(b\cdots\right.}$$

Mathematica [C] time = 0.411774, size = 222, normalized size = 0.37

$$\frac{3\left(a+bx^2+cx^4\right)^2 \text{RootSum}\left[\#1^4 b+\#1^8 c+a\&, \frac{12\#1^4 ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4 b^2 c\log\left(\sqrt{x}-\#1\right)-28abc\log\left(\sqrt{x}-\#1\right)+b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^5 c+\#1 b}\&\right]+12x^{3/\cdots}}{64a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-16*a*(b^2 - 4*a*c)*x^(3/2)*(b + 2*c*x^2) + 12*x^(3/2)*(b^3 + 4*a*b*c + b^
2*c*x^2 + 12*a*c^2*x^2)*(a + b*x^2 + c*x^4) + 3*(a + b*x^2 + c*x^4)^2*RootS
um[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 28*a*b*c*Log[Sqrt[x] -
#1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 12*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
 + 2*c*#1^5) & ])/(64*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

Maple [C] time = 0.272, size = 277, normalized size = 0.5

$$2\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(1/32\,\frac{b\left(28\,ac-b^2\right)x^{3/2}}{16\,a^2c^2-8\,acb^2+b^4}+1/32\,\frac{\left(68\,a^2c^2+7\,acb^2+3\,b^4\right)x^{7/2}}{a\left(16\,a^2c^2-8\,acb^2+b^4\right)}+3/16\,\frac{bc\left(8\,ac+b^2\right)x^{11/2}}{a\left(16\,a^2c^2-8\,acb^2+b^4\right)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*b*(28*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)+1/32*(68*a^2*c^2+
7*a*b^2*c+3*b^4)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/16/a*c*b*(8*a*c+b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/2)+3/32*c^2*(12*a*c+b^2)/a/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(15/2))/(c*x^4+b*x^2+a)^2+3/64/a/(16*a^2*c^2-8*a*b^2*c+b^4)
*sum((c*(12*a*c+b^2)*_R^6+b*(-28*a*c+b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2
)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(b^2c^2+12\,ac^3\right)x^{\frac{15}{2}}+6\left(b^3c+8\,abc^2\right)x^{\frac{11}{2}}+\left(3\,b^4+7\,ab^2c+68\,a^2c^2\right)x^{\frac{7}{2}}-\left(ab^3-28\,a\right.}{16\left(\left(ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4\right)x^8+a^3b^4-8\,a^4b^2c+16\,a^5c^2+2\left(ab^5c-8\,a^2b^3c^2+16\,a^3bc^3\right)x^6+\left(ab^6-6\,a^2b^4c+32\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^2*c^2 + 12*a*c^3)*x^(15/2) + 6*(b^3*c + 8*a*b*c^2)*x^(11/2) + (3
*b^4 + 7*a*b^2*c + 68*a^2*c^2)*x^(7/2) - (a*b^3 - 28*a^2*b*c)*x^(3/2))/((a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c
 + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + integr
ate(3/32*((b^2*c + 12*a*c^2)*x^(5/2) + (b^3 - 28*a*b*c)*sqrt(x))/(a^2*b^4 -
 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x^4 + (a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1086 $\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=594

$$-\frac{3c^{3/4}\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\left(44ac+b^2\right)-68abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\ldots}{\sqrt[4]{\sqrt{b\ldots}}}\right.}{32\sqrt[4]{2}a\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[x]
*(b*(b^2 + 20*a*c) + c*(b^2 + 44*a*c)*x^2))/(16*a*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) - (3*c^(3/4)*(b^2 + 44*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)
/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c
])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) -
(3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 44*a*c))*ArcTan[(2^(1
/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 -
4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^2 + 44*a*c - b
^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4
*a*c]*(b^2 + 44*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3
/4))
```

Rubi [A] time = 2.36606, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1364, 1430, 1422, 212, 208, 205}

$$-\frac{3c^{3/4}\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\left(44ac+b^2\right)-68abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\ldots}{\sqrt[4]{\sqrt{b\ldots}}}\right.}{32\sqrt[4]{2}a\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[x]
*(b*(b^2 + 20*a*c) + c*(b^2 + 44*a*c)*x^2))/(16*a*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) - (3*c^(3/4)*(b^2 + 44*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)
```

```
/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c
])^(1/4)]])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) -
(3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 44*a*c))*ArcTan[(2^(1
/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]])/(32*2^(1/4)*a*(b^2 -
4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^2 + 44*a*c - b
^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4
*a*c]*(b^2 + 44*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)]])/(32*2^(1/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3
/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1364

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*(a + b*x^n + c*
x^(2*n))^(p + 1))/(n*(p + 1)*(b^2 - 4*a*c)), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
```

```
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{b-22cx^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-3b\left(b^2-12ac\right)}{a+b}\right.}{16a\left(b^2\right.}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(b^2+44ac+\frac{b}{\sqrt{b^2}}\right.\right.}{}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3c\left(b^2+44ac-\frac{b}{\sqrt{b^2-}}\right.\right.}{}$$

$$= -\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(b\left(b^2+20ac\right)+c\left(b^2+44ac\right)x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{3c^{3/4}\left(b^2+44ac-\frac{}{\sqrt{b}}\right.}{32\sqrt[4]{2}a\left(b^2\right.}$$

Mathematica [C] time = 0.409663, size = 224, normalized size = 0.38

$$\frac{3\left(a+bx^2+cx^4\right)^2 \text{RootSum}\left[\#1^4 b+\#1^8 c+a\&, \frac{44\#1^4 ac^2\log\left(\sqrt{x}-\#1\right)+\#1^4 b^2 c\log\left(\sqrt{x}-\#1\right)-12abc\log\left(\sqrt{x}-\#1\right)+b^3\log\left(\sqrt{x}-\#1\right)}{\#1^3 b+2\#1^7 c}\&\right]+4\sqrt{x}}{64a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-16*a*(b^2 - 4*a*c)*Sqrt[x]*(b + 2*c*x^2) + 4*Sqrt[x]*(b^3 + 20*a*b*c + b^
2*c*x^2 + 44*a*c^2*x^2)*(a + b*x^2 + c*x^4) + 3*(a + b*x^2 + c*x^4)^2*RootS
um[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 12*a*b*c*Log[Sqrt[x] -
#1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 44*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
^3 + 2*c*#1^7) & ])/(64*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

Maple [C] time = 0.268, size = 270, normalized size = 0.5

$$2\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(\frac{3b\left(12ac-b^2\right)\sqrt{x}}{512a^2c^2-256acb^2+32b^4}+1/32\frac{\left(76a^2c^2+13acb^2+b^4\right)x^{5/2}}{a\left(16a^2c^2-8acb^2+b^4\right)}+1/16\frac{bc\left(32ac+b^2\right)x^{9/2}}{a\left(16a^2c^2-8acb^2+b^4\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/32*b*(12*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)+1/32*(76*a^2*c^2+
13*a*b^2*c+b^4)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/16/a*c*b*(32*a*c+b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)+1/32*c^2*(44*a*c+b^2)/a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/a/(16*a^2*c^2-8*a*b^2*c+b^4)*
sum((c*(44*a*c+b^2)*_R^4-12*a*b*c+b^3)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=
RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(b^3c^2-12abc^3\right)x^{\frac{17}{2}}+\left(6b^4c-71ab^2c^2+44a^2c^3\right)x^{\frac{13}{2}}+\left(3b^5-28ab^3c-8a^2bc^2\right)x^{\frac{9}{2}}+\left(7ab^4-\right.}{16\left(\left(a^2b^4c^2-8a^3b^2c^3+16a^4c^4\right)x^8+a^4b^4-8a^5b^2c+16a^6c^2+2\left(a^2b^5c-8a^3b^3c^2+16a^4bc^3\right)x^6+\left(a^2b^6-6a^3b^4c+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^3*c^2 - 12*a*b*c^3)*x^(17/2) + (6*b^4*c - 71*a*b^2*c^2 + 44*a^2*
c^3)*x^(13/2) + (3*b^5 - 28*a*b^3*c - 8*a^2*b*c^2)*x^(9/2) + (7*a*b^4 - 59*
a^2*b^2*c + 76*a^3*c^2)*x^(5/2))/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 +
 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + integrate(-3/32*((b^3*c - 12*a*b*c^2)*
x^(7/2) + (b^4 - 13*a*b^2*c - 44*a^2*c^2)*x^(3/2))/(a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a
^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1087 $\int \frac{\sqrt{x}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=658

$$\frac{x^{3/2}\left(52a^2c^2+bcx^2\left(5b^2-44ac\right)-45ab^2c+5b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\right.}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + b*c*(5*b^2 - 44*a*c)*x^2))/(
16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c
+ 520*a^2*c^2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^
(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*
a^2*c^2 + b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*
(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^
2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 + b
*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 5.48456, antiderivative size = 658, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1366, 1500, 1510, 298, 205, 208}

$$\frac{x^{3/2}\left(52a^2c^2+bcx^2\left(5b^2-44ac\right)-45ab^2c+5b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\right.}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + b*c*(5*b^2 - 44*a*c)*x^2))/(
```

```
16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c
+ 520*a^2*c^2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^
(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*
a^2*c^2 + b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*
(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^
2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 + b
*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))
```

<u>Rule 1115</u>

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

<u>Rule 1366</u>

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(
2*n))^(p + 1))/(a*d*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2
 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(p
+ 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0
] && IGtQ[n, 0] && ILtQ[p, -1]
```

<u>Rule 1500</u>

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2
 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

<u>Rule 1510</u>

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
```

```
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(-5b^2+26ac-9bcx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(5b^4-\right.\right.}{}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(\sqrt{c}\left(5b^4\right.\right.}{}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}\left(5b^4-45ab^2c+52a^2c^2+bc\left(5b^2-44ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\sqrt[4]{c}\left(5b^4-\right.}{}$$

Mathematica [C] time = 0.508161, size = 254, normalized size = 0.39

$$\frac{\text{RootSum}\left[\#1^4b+\#1^8c+a\&, \frac{-44\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+5\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+260a^2c^2\log\left(\sqrt{x}-\#1\right)-49ab^2c\log\left(\sqrt{x}-\#1\right)+5b^4\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4x}{}}{64a^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-16*a*(-b^2 + 4*a*c)*x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^2 + c*x^4)
^2 + (4*x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + 5*b^3*c*x^2 - 44*a*b*c^2
*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1^8 & , (5*b^4*Log[Sqr
t[x] - #1] - 49*a*b^2*c*Log[Sqrt[x] - #1] + 260*a^2*c^2*Log[Sqrt[x] - #1] +
 5*b^3*c*Log[Sqrt[x] - #1]*#1^4 - 44*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
+ 2*c*#1^5) & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

Maple [C] time = 0.273, size = 321, normalized size = 0.5

$$2\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(\frac{\left(84a^2c^2-69acb^2+9b^4\right)x^{3/2}}{\left(512a^2c^2-256acb^2+32b^4\right)a}-1/32\frac{b\left(8a^2c^2+36acb^2-5b^4\right)x^{7/2}}{a^2\left(16a^2c^2-8acb^2+b^4\right)}+1/32\frac{c\left(52a^2c^2-89acb^2+\right.}{a^2\left(16a^2c^2-8acb\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/32*(28*a^2*c^2-23*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(3/2)-
1/32*b*(8*a^2*c^2+36*a*b^2*c-5*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+
1/32/a^2*c*(52*a^2*c^2-89*a*b^2*c+10*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/
2)-1/32*b*c^2*(44*a*c-5*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^
4+b*x^2+a)^2-1/64/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((b*c*(44*a*c-5*b^2)*_R
^6+(-260*a^2*c^2+49*a*b^2*c-5*b^4)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_
R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(5b^3c^2-44abc^3\right)x^{\frac{15}{2}}+\left(10b^4c-89ab^2c^2+52a^2c^3\right)x^{\frac{11}{2}}+\left(5b^5-36ab^3c-8a^2bc^2\right)x^{\frac{7}{2}}+3\left(3ab^4-\right.}{16\left(\left(a^2b^4c^2-8a^3b^2c^3+16a^4c^4\right)x^8+a^4b^4-8a^5b^2c+16a^6c^2+2\left(a^2b^5c-8a^3b^3c^2+16a^4bc^3\right)x^6+\left(a^2b^6-6a^3b^4c+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^3*c^2 - 44*a*b*c^3)*x^(15/2) + (10*b^4*c - 89*a*b^2*c^2 + 52*a^2
*c^3)*x^(11/2) + (5*b^5 - 36*a*b^3*c - 8*a^2*b*c^2)*x^(7/2) + 3*(3*a*b^4 -
23*a^2*b^2*c + 28*a^3*c^2)*x^(3/2))/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*
c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^
2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b
^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) - integrate(-1/32*((5*b^3*c - 44*a*b*
c^2)*x^(5/2) + (5*b^4 - 49*a*b^2*c + 260*a^2*c^2)*sqrt(x))/(a^3*b^4 - 8*a^4
*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1088 $\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=658

$$\frac{\sqrt{x}\left(60a^2c^2+bcx^2\left(7b^2-52ac\right)-55ab^2c+7b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tan^{-1}\left(\right.}{32\sqrt[4]{2}a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + b*c*(7*b^2 - 52*a*c)*x^2))/(
16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*c^(3/4)*(7*b^4 - 66*a*b^2*
c + 280*a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c
)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c +
280*a^2*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5
/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*
a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a^2
*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[
x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-
b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 5.79179, antiderivative size = 658, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1115, 1345, 1430, 1422, 212, 208, 205}

$$\frac{\sqrt{x}\left(60a^2c^2+bcx^2\left(7b^2-52ac\right)-55ab^2c+7b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tan^{-1}\left(\right.}{32\sqrt[4]{2}a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + b*c*(7*b^2 - 52*a*c)*x^2))/(
```

```
16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*c^(3/4)*(7*b^4 - 66*a*b^2*
c + 280*a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c
)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c +
280*a^2*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5
/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*
a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a^2
*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[
x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-
b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1115

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + (b
*x^(2*k))/d^2 + (c*x^(4*k))/d^4)^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1345

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(
x*(b^2 - 2*a*c + b*c*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^
2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + n*
(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^
(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*
c, 0] && ILtQ[p, -1]
```

Rule 1430

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> -Simp[(x*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*(
a + b*x^n + c*x^(2*n))^(p + 1))/(a*n*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a
*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d*(
2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n + c
*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
```

```
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{b^2-2ac-8\left(b^2-4ac\right)-11bcx^4}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}}{}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(3c\left(7\right.\right.}{}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(3c\left(7\right.\right.}{}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(7b^4-55ab^2c+60a^2c^2+bc\left(7b^2-52ac\right)x^2\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3c^{3/4}\left(\right.}{}$$

Mathematica [C] time = 0.47207, size = 258, normalized size = 0.39

$$\frac{3\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&, \frac{-52\#1^4 abc^2\log\left(\sqrt{x}-\#1\right)+7\#1^4 b^3 c\log\left(\sqrt{x}-\#1\right)+140a^2c^2\log\left(\sqrt{x}-\#1\right)-59ab^2c\log\left(\sqrt{x}-\#1\right)+7b^4\log\left(\sqrt{x}-\#1\right)}{\#1^3 b+2\#1^7 c}\&\right]+}{64a^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-16*a*(-b^2 + 4*a*c)*Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^2 + c*x^4)
^2 + (4*Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + 7*b^3*c*x^2 - 52*a*b*c^2
*x^2))/(a + b*x^2 + c*x^4) + 3*RootSum[a + b*#1^4 + c*#1^8 & , (7*b^4*Log[S
qrt[x] - #1] - 59*a*b^2*c*Log[Sqrt[x] - #1] + 140*a^2*c^2*Log[Sqrt[x] - #1]
 + 7*b^3*c*Log[Sqrt[x] - #1]*#1^4 - 52*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#
```

```
1^3 + 2*c*#1^7) & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

Maple [C] time = 0.266, size = 316, normalized size = 0.5

$$2\,\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(1/32\,\frac{\left(92\,a^2c^2-79\,acb^2+11\,b^4\right)\sqrt{x}}{\left(16\,a^2c^2-8\,acb^2+b^4\right)a}-1/32\,\frac{b\left(8\,a^2c^2+44\,acb^2-7\,b^4\right)x^{5/2}}{a^2\left(16\,a^2c^2-8\,acb^2+b^4\right)}+1/32\,\frac{c\left(60\,a^2c^2-107\,ac\right.}{a^2\left(16\,a^2c^2-8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(92*a^2*c^2-79*a*b^2*c+11*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(1/2)
-1/32*b*(8*a^2*c^2+44*a*b^2*c-7*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)
+1/32/a^2*c*(60*a^2*c^2-107*a*b^2*c+14*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9
/2)-1/32*b*c^2*(52*a*c-7*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*x
^4+b*x^2+a)^2+3/64/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((b*c*(-52*a*c+7*b^2)*
_R^4+140*a^2*c^2-59*a*c*b^2+7*b^4)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Root
Of(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(7\,b^4c^2-59\,ab^2c^3+140\,a^2c^4\right)x^{\frac{17}{2}}+\left(42\,b^5c-347\,ab^3c^2+788\,a^2bc^3\right)x^{\frac{13}{2}}+\left(21\,b^6-121\,ab^4c-41\,a^2b^2c^2+900\,a^3c^3\right)x^{\frac{9}{2}}}{16\left(a^5b^4-8\,a^6b^2c+16\,a^7c^2+\left(a^3b^4c^2-8\,a^4b^2c^3+16\,a^5c^4\right)x^8+2\left(a^3b^5c-8\,a^4b^3c^2+16\,a^5bc^3\right)x^6+\left(a^3b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(7*b^4*c^2 - 59*a*b^2*c^3 + 140*a^2*c^4)*x^(17/2) + (42*b^5*c - 347
*a*b^3*c^2 + 788*a^2*b*c^3)*x^(13/2) + (21*b^6 - 121*a*b^4*c - 41*a^2*b^2*c
^2 + 900*a^3*c^3)*x^(9/2) + (49*a*b^5 - 398*a^2*b^3*c + 832*a^3*b*c^2)*x^(5
/2) + 32*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*sqrt(x))/(a^5*b^4 - 8*a^6*b^2
*c + 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^8 + 2*(a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*
c^3)*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2) - integrate(3/32*(
(7*b^4*c - 59*a*b^2*c^2 + 140*a^2*c^3)*x^(7/2) + (7*b^5 - 66*a*b^3*c + 192*
a^2*b*c^2)*x^(3/2))/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + (a^3*b^4*c - 8*a^
4*b^2*c^2 + 16*a^5*c^3)*x^4 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2),
```

```
x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.1089 $\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rubi [A] time = 0.207493, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int (dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.584016, size = 365, normalized size = 2.48

$$\frac{2d\sqrt{dx}\left(2x^2\left(10ac-3b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-10ab\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\cdots\right.}{225c\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*Sqrt[d*x]*(5*(2*b + 5*c*x^2)*(a + b*x^2 + c*x^4) - 10*a*b*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(-3*
b^2 + 10*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])]))/(225*c*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\,(dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}\sqrt{dx}dx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)*d*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\,(dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2), x)
```

3.1090 $\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rubi [A] time = 0.12568, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int \sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.431395, size = 342, normalized size = 2.33

$$\frac{2x\sqrt{dx}\left(6bx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+28a\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2}{\sqrt{b^2-4ac}+}}\right.}{147\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[d*x]*(21*(a + b*x^2 + c*x^4) + 28*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 6*b*x^2*Sqrt[(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(147*Sqr
t[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.271, size = 0, normalized size = 0.

$$\int \sqrt{dx}\sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\sqrt{dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.1091 $\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[1
 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])
```

Rubi [A] time = 0.124622, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[1
 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{dx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.379736, size = 342, normalized size = 2.36

$$\frac{2x\left(2bx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+20a\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1}{25\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*x*(5*(a + b*x^2 + c*x^4) + 20*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(25*Sqrt[d*x]*Sqrt[a
 + b*x^2 + c*x^4])
```

Maple [F] time = 0.274, size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}\frac{1}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
```

```
[Out] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\sqrt{dx}}{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

3.1092 $\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (-2*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])
```

Rubi [A] time = 0.126113, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$-\frac{2\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{(dx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{2\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.365195, size = 345, normalized size = 2.38

$$\frac{x\left(24cx^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+28bx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\right.}{21(dx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(d*x)^(3/2),x]
```

```
[Out] (x*(-42*(a + b*x^2 + c*x^4) + 28*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 24*c*x^4*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*(d*x)^(3/2
)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.27, size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}\,(dx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
```

```
[Out] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\sqrt{dx}}{d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(d*x)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1093 $\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=148

$$\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])])
```

Rubi [A] time = 0.127711, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])])
```


Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}dx=\frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int (dx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.856215, size = 459, normalized size = 3.1

$$2d\sqrt{dx}\Bigg(4x^2\left(260a^2c^2-157ab^2c+21b^4\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+5\left(a^2c\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*x]*(5*(-28*b^4*x^2 - 8*b^3*c*x^4 + 305*b^2*c^2*x^6 + 480*b*c^3*
x^8 + 195*c^4*x^10 + a^2*c*(176*b + 455*c*x^2) + a*(-28*b^3 + 196*b^2*c*x^2
 + 916*b*c^2*x^4 + 650*c^3*x^6)) + 20*a*b*(7*b^2 - 44*a*c)*Sqrt[(b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 4*(21*b^4
- 157*a*b^2*c + 260*a^2*c^2)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(16575*c^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.315, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \left(c x^4 + b x^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^4 + b x^2 + a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c d x^5 + b d x^3 + a d x\right) \sqrt{c x^4 + b x^2 + a} \sqrt{d x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*d*x^5 + b*d*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \left(a + b x^2 + c x^4\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2), x)
```

3.1094 $\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])])
```

Rubi [A] time = 0.130043, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int \sqrt{dx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$
$$=\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.728047, size = 417, normalized size = 2.82

$$\frac{2x\sqrt{dx}\left(7\left(209a^2c+12ab^2+328abcx^2+286ac^2x^4+131b^2cx^4+12b^3x^2+196bc^2x^6+77c^3x^8\right)+12bx^2\left(36ac-5b^2\right)\sqrt{\frac{-\cdots}{\cdots}}\cdots\right)}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*(7*(12*a*b^2 + 209*a^2*c + 12*b^3*x^2 + 328*a*b*c*x^2 + 131*
b^2*c*x^4 + 286*a*c^2*x^4 + 196*b*c^2*x^6 + 77*c^3*x^8) - 28*a*(3*b^2 - 44*
a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2
, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])] + 12*b*(-5*b^2 + 36*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(8085*c*Sqrt[a + b*x^2 + c*x^4
])
```

Maple [F] time = 0.28, size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.1095 $\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=146

$$\frac{2a\sqrt{dx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])
```

Rubi [A] time = 0.127411, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2a\sqrt{dx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/Sqrt[d*x],x]
```


```
[Out] (2*a*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])
```


Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{dx}}\,dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{\sqrt{dx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2a\sqrt{dx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.732367, size = 415, normalized size = 2.84

$$\frac{2x\left(5\left(51a^2c+4ab^2+76abcx^2+66ac^2x^4+29b^2cx^4+4b^3x^2+40bc^2x^6+15c^3x^8\right)-4bx^2\left(3b^2-28ac\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\right.}{97}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*x*(5*(4*a*b^2 + 51*a^2*c + 4*b^3*x^2 + 76*a*b*c*x^2 + 29*b^2*c*x^4 + 66*
a*c^2*x^4 + 40*b*c^2*x^6 + 15*c^3*x^8) - 20*a*(b^2 - 36*a*c)*Sqrt[(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 4*b*(3*b
^2 - 28*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])]))/(975*c*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int \left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}} \frac{1}{\sqrt{d x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}}{\sqrt{d x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}} \sqrt{d x}}{d x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)/(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

3.1096 $\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2a\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (-2*a*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rubi [A] time = 0.129358, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$-\frac{2a\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x]
```


```
[Out] (-2*a*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```


Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{(dx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{2a\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.74838, size = 384, normalized size = 2.63

$$\frac{x\left(14\left(-77a^2-64abx^2-70acx^4+13b^2x^4+20bcx^6+7c^2x^8\right)+24x^4\left(28ac+b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\right.\right.}{539(dx)^{3/2}\sqrt{a+b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (x*(14*(-77*a^2 - 64*a*b*x^2 + 13*b^2*x^4 - 70*a*c*x^4 + 20*b*c*x^6 + 7*c^2
*x^8) + 896*a*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])] + 24*(b^2 + 28*a*c)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(539*(d*x)^(3/2)*Sqr
t[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.287, size = 0, normalized size = 0.

$$\int \left(c x^4+b x^2+a\right)^{\frac{3}{2}} (d x)^{-\frac{3}{2}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)
```

```
[Out] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c x^4+b x^2+a\right)^{\frac{3}{2}}}{(d x)^{\frac{3}{2}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c x^4+b x^2+a\right)^{\frac{3}{2}} \sqrt{d x}}{d^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)/(d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(d*x)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.1097 $\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*Sqrt[a + b*x^2
+ c*x^4])
```

Rubi [A] time = 0.126073, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*Sqrt[a + b*x^2
+ c*x^4])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^{3/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2(dx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.0934703, size = 173, normalized size = 1.18

$$\frac{2x(dx)^{3/2}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{5\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])])/(5*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}dx}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d*x)**(3/2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.1098 $\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*Sqrt[a + b*x^2
+ c*x^4])
```

Rubi [A] time = 0.121396, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*Sqrt[a + b*x^2
+ c*x^4])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{dx}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$
$$= \frac{2(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.102948, size = 173, normalized size = 1.18

$$\frac{2x\sqrt{dx}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{3\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[d*x]*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF
1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \sqrt{dx}\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.1099 $\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[a + b*x^2 + c*
x^4])
```

Rubi [A] time = 0.122257, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[a + b*x^2 + c*
x^4])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$
$$= \frac{2\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.0812958, size = 171, normalized size = 1.18

$$\frac{2x\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2
, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])])/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}}\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\sqrt{dx}}{cdx^5 + bdx^3 + adx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c*d*x^5 + b*d*x^3 + a*d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

3.1100 $\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sqrt[a + b*x^2 +
c*x^4])
```

Rubi [A] time = 0.124865, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sqrt[a + b*x^2 +
c*x^4])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] time = 0.501232, size = 348, normalized size = 2.4

$$\frac{x\left(18cx^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+14bx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}}{21a(dx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (x*(-42*(a + b*x^2 + c*x^4) + 14*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 18*c*x^4*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*a*(d*x)^(3
/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.285, size = 0, normalized size = 0.

$$\int (dx)^{-\frac{3}{2}}\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\,(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\sqrt{dx}}{cd^2x^6 + bd^2x^4 + ad^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c*d^2*x^6 + b*d^2*x^4 + a*d^2*x
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\,\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\,(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)), x)
```

3.1101 $\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*d*Sqrt[a + b*x^
2 + c*x^4])
```

Rubi [A] time = 0.128344, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*d*Sqrt[a + b*x^
2 + c*x^4])
```


Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^{3/2}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2(dx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] time = 0.370293, size = 348, normalized size = 2.32

$$\frac{d\sqrt{dx}\left(2cx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+5b\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\right.}{5\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (d*Sqrt[d*x]*(-5*(b + 2*c*x^2) + 5*b*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*c*x^2*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*(b^2 - 4*a*c)*S
qrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\sqrt{dx}dx}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)*d*x/(c^2*x^8 + 2*b*c*x^6 + (b^2
+ 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.1102 $\int \frac{\sqrt{dx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*d*Sqrt[a + b*x^
2 + c*x^4])
```

Rubi [A] time = 0.126062, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*d*Sqrt[a + b*x^
2 + c*x^4])
```


Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{dx}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] time = 0.522798, size = 367, normalized size = 2.45

$$\frac{x\sqrt{dx}\left(9bcx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+7\left(2ac+b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\cdots}}{21a\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[d*x]*(-21*(b^2 - 2*a*c + b*c*x^2) + 7*(b^2 + 2*a*c)*Sqrt[(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*b*c*x^2
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/
2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])]))/(21*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\sqrt{dx}}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*
a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.1103 $\int \frac{1}{\sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*Sqrt[a + b*x^2 +
c*x^4])
```

Rubi [A] time = 0.125796, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*Sqrt[a + b*x^2 +
c*x^4])
```


Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{dx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] time = 0.496438, size = 366, normalized size = 2.47

$$\frac{x\left(bcx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-5\left(b^2-6ac\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b\cdots}{\sqrt{b^2-4ac}\cdots}}\cdots}{5a\sqrt{dx}\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x*(-5*(b^2 - 2*a*c + b*c*x^2) - 5*(b^2 - 6*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + b*c*x^2*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*a*(
-b^2 + 4*a*c)*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\sqrt{dx}}{c^2dx^9+2\,bcdx^7+\left(b^2+2\,ac\right)dx^5+2\,abdx^3+a^2dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*d*x^9 + 2*b*c*d*x^7 + (b^2
+ 2*a*c)*d*x^5 + 2*a*b*d*x^3 + a^2*d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

3.1104 $\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*Sqrt[d*x]*Sqrt[a + b*x^2
+ c*x^4])
```

Rubi [A] time = 0.127419, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1141, 510}

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```



```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*Sqrt[d*x]*Sqrt[a + b*x^2
+ c*x^4])
```


Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{(dx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] time = 0.728795, size = 409, normalized size = 2.76

$$-\frac{x\left(-7\left(8a^2c+a\left(-2b^2+11bcx^2+10c^2x^4\right)-3b^2x^2\left(b+cx^2\right)\right)+3cx^4\left(10ac-3b^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\right.\right.}{7a^2(dx)^{3/2}\left(b^2-4\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -(x*(-7*(8*a^2*c - 3*b^2*x^2*(b + c*x^2) + a*(-2*b^2 + 11*b*c*x^2 + 10*c^2*
x^4)) - 7*b*(b^2 - 3*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 3*c*(-3*b^2 + 10*a*c)*x^4*Sqrt[(b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(7*a^2
*(b^2 - 4*a*c)*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.343, size = 0, normalized size = 0.

$$\int (dx)^{-\frac{3}{2}} \left(c x^{4}+b x^{2}+a \right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{4} + bx^{2} + a\right)^{\frac{3}{2}} \left(dx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{\sqrt{cx^{4} + bx^{2} + a} \sqrt{dx}}{c^{2} d^{2} x^{10} + 2 \, bcd^{2} x^{8} + {\left(b^{2} + 2 \, ac\right)} d^{2} x^{6} + 2 \, abd^{2} x^{4} + a^{2} d^{2} x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*d^2*x^10 + 2*b*c*d^2*x^8 +
(b^2 + 2*a*c)*d^2*x^6 + 2*a*b*d^2*x^4 + a^2*d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}} \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)), x)
```

3.1105 $\int (dx)^m \left(a + bx^2 + cx^4\right)^3 \, dx$

Optimal. Leaf size=156

$$\frac{3a^2b(dx)^{m+3}}{d^3(m+3)} + \frac{a^3(dx)^{m+1}}{d(m+1)} + \frac{3a\left(ac+b^2\right)(dx)^{m+5}}{d^5(m+5)} + \frac{b\left(6ac+b^2\right)(dx)^{m+7}}{d^7(m+7)} + \frac{3c\left(ac+b^2\right)(dx)^{m+9}}{d^9(m+9)} + \frac{3bc^2(dx)^{m+11}}{d^{11}(m+11)} + \frac{c^3(}{d^{13}}$$

```
[Out] (a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
3*a*(b^2 + a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (b*(b^2 + 6*a*c)*(d*x)^(7 +
m))/(d^7*(7 + m)) + (3*c*(b^2 + a*c)*(d*x)^(9 + m))/(d^9*(9 + m)) + (3*b*c^
2*(d*x)^(11 + m))/(d^11*(11 + m)) + (c^3*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rubi [A] time = 0.0999336, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$\frac{3a^2b(dx)^{m+3}}{d^3(m+3)} + \frac{a^3(dx)^{m+1}}{d(m+1)} + \frac{3a\left(ac+b^2\right)(dx)^{m+5}}{d^5(m+5)} + \frac{b\left(6ac+b^2\right)(dx)^{m+7}}{d^7(m+7)} + \frac{3c\left(ac+b^2\right)(dx)^{m+9}}{d^9(m+9)} + \frac{3bc^2(dx)^{m+11}}{d^{11}(m+11)} + \frac{c^3(}{d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] (a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
3*a*(b^2 + a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (b*(b^2 + 6*a*c)*(d*x)^(7 +
m))/(d^7*(7 + m)) + (3*c*(b^2 + a*c)*(d*x)^(9 + m))/(d^9*(9 + m)) + (3*b*c^
2*(d*x)^(11 + m))/(d^11*(11 + m)) + (c^3*(d*x)^(13 + m))/(d^13*(13 + m))
```


Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 dx = \int \left(a^3(dx)^m + \frac{3a^2b(dx)^{2+m}}{d^2} + \frac{3a\left(b^2+ac\right)(dx)^{4+m}}{d^4} + \frac{b\left(b^2+6ac\right)(dx)^{6+m}}{d^6} + \frac{3c\left(b^2+ac\right)}{d^8}\right.$$
$$= \frac{a^3(dx)^{1+m}}{d(1+m)} + \frac{3a^2b(dx)^{3+m}}{d^3(3+m)} + \frac{3a\left(b^2+ac\right)(dx)^{5+m}}{d^5(5+m)} + \frac{b\left(b^2+6ac\right)(dx)^{7+m}}{d^7(7+m)} + \frac{3c\left(b^2+ac\right)(}{d^9(9+m}$$

Mathematica [A] time = 0.130296, size = 111, normalized size = 0.71

$$x(dx)^m\left(\frac{3a^2bx^2}{m+3}+\frac{a^3}{m+1}+\frac{3cx^8\left(ac+b^2\right)}{m+9}+\frac{bx^6\left(6ac+b^2\right)}{m+7}+\frac{3ax^4\left(ac+b^2\right)}{m+5}+\frac{3bc^2x^{10}}{m+11}+\frac{c^3x^{12}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^2)/(3 + m) + (3*a*(b^2 + a*c)*x^4)/(5 +
 m) + (b*(b^2 + 6*a*c)*x^6)/(7 + m) + (3*c*(b^2 + a*c)*x^8)/(9 + m) + (3*b*
c^2*x^10)/(11 + m) + (c^3*x^12)/(13 + m))
```

Maple [B] time = 0.048, size = 782, normalized size = 5.

$$\frac{\left(c^3m^6x^{12}+36\,c^3m^5x^{12}+3\,bc^2m^6x^{10}+505\,c^3m^4x^{12}+114\,bc^2m^5x^{10}+3480\,c^3m^3x^{12}+3\,ac^2m^6x^8+3\,b^2cm^6x^8+1665\,bc^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] x*(c^3*m^6*x^12+36*c^3*m^5*x^12+3*b*c^2*m^6*x^10+505*c^3*m^4*x^12+114*b*c^2
*m^5*x^10+3480*c^3*m^3*x^12+3*a*c^2*m^6*x^8+3*b^2*c*m^6*x^8+1665*b*c^2*m^4*
x^10+12139*c^3*m^2*x^12+120*a*c^2*m^5*x^8+120*b^2*c*m^5*x^8+11820*b*c^2*m^3
*x^10+19524*c^3*m*x^12+6*a*b*c*m^6*x^6+1839*a*c^2*m^4*x^8+b^3*m^6*x^6+1839*
b^2*c*m^4*x^8+42117*b*c^2*m^2*x^10+10395*c^3*x^12+252*a*b*c*m^5*x^6+13584*a
*c^2*m^3*x^8+42*b^3*m^5*x^6+13584*b^2*c*m^3*x^8+68706*b*c^2*m*x^10+3*a^2*c*
m^6*x^4+3*a*b^2*m^6*x^4+4074*a*b*c*m^4*x^6+49881*a*c^2*m^2*x^8+679*b^3*m^4*
x^6+49881*b^2*c*m^2*x^8+36855*b*c^2*x^10+132*a^2*c*m^5*x^4+132*a*b^2*m^5*x^
4+31752*a*b*c*m^3*x^6+83064*a*c^2*m*x^8+5292*b^3*m^3*x^6+83064*b^2*c*m*x^8+
3*a^2*b*m^6*x^2+2259*a^2*c*m^4*x^4+2259*a*b^2*m^4*x^4+122010*a*b*c*m^2*x^6+
```

```
45045*a*c^2*x^8+20335*b^3*m^2*x^6+45045*b^2*c*x^8+138*a^2*b*m^5*x^2+18840*a
^2*c*m^3*x^4+18840*a*b^2*m^3*x^4+209916*a*b*c*m*x^6+34986*b^3*m*x^6+a^3*m^6
+2505*a^2*b*m^4*x^2+77937*a^2*c*m^2*x^4+77937*a*b^2*m^2*x^4+115830*a*b*c*x^
6+19305*b^3*x^6+48*a^3*m^5+22620*a^2*b*m^3*x^2+142308*a^2*c*m*x^4+142308*a*
b^2*m*x^4+925*a^3*m^4+104277*a^2*b*m^2*x^2+81081*a^2*c*x^4+81081*a*b^2*x^4+
9120*a^3*m^3+219162*a^2*b*m*x^2+48259*a^3*m^2+135135*a^2*b*x^2+129072*a^3*m
+135135*a^3)*(d*x)^m/(13+m)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.46044, size = 1474, normalized size = 9.45

$$\frac{\left(\left(c^3m^6 + 36\,c^3m^5 + 505\,c^3m^4 + 3480\,c^3m^3 + 12139\,c^3m^2 + 19524\,c^3m + 10395\,c^3\right)x^{13} + 3\left(bc^2m^6 + 38\,bc^2m^5 + 555\,bc^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^6 + 36*c^3*m^5 + 505*c^3*m^4 + 3480*c^3*m^3 + 12139*c^3*m^2 + 19524
*c^3*m + 10395*c^3)*x^13 + 3*(b*c^2*m^6 + 38*b*c^2*m^5 + 555*b*c^2*m^4 + 39
40*b*c^2*m^3 + 14039*b*c^2*m^2 + 22902*b*c^2*m + 12285*b*c^2)*x^11 + 3*((b^
2*c + a*c^2)*m^6 + 40*(b^2*c + a*c^2)*m^5 + 613*(b^2*c + a*c^2)*m^4 + 4528*
(b^2*c + a*c^2)*m^3 + 15015*b^2*c + 15015*a*c^2 + 16627*(b^2*c + a*c^2)*m^2
 + 27688*(b^2*c + a*c^2)*m)*x^9 + ((b^3 + 6*a*b*c)*m^6 + 42*(b^3 + 6*a*b*c)
*m^5 + 679*(b^3 + 6*a*b*c)*m^4 + 5292*(b^3 + 6*a*b*c)*m^3 + 19305*b^3 + 115
830*a*b*c + 20335*(b^3 + 6*a*b*c)*m^2 + 34986*(b^3 + 6*a*b*c)*m)*x^7 + 3*((
a*b^2 + a^2*c)*m^6 + 44*(a*b^2 + a^2*c)*m^5 + 753*(a*b^2 + a^2*c)*m^4 + 628
0*(a*b^2 + a^2*c)*m^3 + 27027*a*b^2 + 27027*a^2*c + 25979*(a*b^2 + a^2*c)*m
^2 + 47436*(a*b^2 + a^2*c)*m)*x^5 + 3*(a^2*b*m^6 + 46*a^2*b*m^5 + 835*a^2*b
*m^4 + 7540*a^2*b*m^3 + 34759*a^2*b*m^2 + 73054*a^2*b*m + 45045*a^2*b)*x^3
+ (a^3*m^6 + 48*a^3*m^5 + 925*a^3*m^4 + 9120*a^3*m^3 + 48259*a^3*m^2 + 1290
```

```
72*a^3*m + 135135*a^3)*x)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 573
79*m^3 + 177331*m^2 + 264207*m + 135135)
```

Sympy [A] time = 7.34987, size = 4451, normalized size = 28.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-a**3/(12*x**12) - 3*a**2*b/(10*x**10) - 3*a**2*c/(8*x**8) - 3*
a*b**2/(8*x**8) - a*b*c/x**6 - 3*a*c**2/(4*x**4) - b**3/(6*x**6) - 3*b**2*c
/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*log(x))/d**13, Eq(m, -13)), ((-a**3/(1
0*x**10) - 3*a**2*b/(8*x**8) - a**2*c/(2*x**6) - a*b**2/(2*x**6) - 3*a*b*c/
(2*x**4) - 3*a*c**2/(2*x**2) - b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**2
*log(x) + c**3*x**2/2)/d**11, Eq(m, -11)), ((-a**3/(8*x**8) - a**2*b/(2*x**
6) - 3*a**2*c/(4*x**4) - 3*a*b**2/(4*x**4) - 3*a*b*c/x**2 + 3*a*c**2*log(x)
 - b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4)/d**9, E
q(m, -9)), ((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a**2*c/(2*x**2) - 3*a*b
**2/(2*x**2) + 6*a*b*c*log(x) + 3*a*c**2*x**2/2 + b**3*log(x) + 3*b**2*c*x*
*2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6)/d**7, Eq(m, -7)), ((-a**3/(4*x**4) -
3*a**2*b/(2*x**2) + 3*a**2*c*log(x) + 3*a*b**2*log(x) + 3*a*b*c*x**2 + 3*a*
c**2*x**4/4 + b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8)/
d**5, Eq(m, -5)), ((-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a**2*c*x**2/2 + 3*
a*b**2*x**2/2 + 3*a*b*c*x**4/2 + a*c**2*x**6/2 + b**3*x**4/4 + b**2*c*x**6/
2 + 3*b*c**2*x**8/8 + c**3*x**10/10)/d**3, Eq(m, -3)), ((a**3*log(x) + 3*a*
*2*b*x**2/2 + 3*a**2*c*x**4/4 + 3*a*b**2*x**4/4 + a*b*c*x**6 + 3*a*c**2*x**
8/8 + b**3*x**6/6 + 3*b**2*c*x**8/8 + 3*b*c**2*x**10/10 + c**3*x**12/12)/d,
 Eq(m, -1)), (a**3*d**m*m**6*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48*a**3*d**m*m**5*x*x**m
/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 26420
7*m + 135135) + 925*a**3*d**m*m**4*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 1004
5*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9120*a**3*d**m*m**
3*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
 + 264207*m + 135135) + 48259*a**3*d**m*m**2*x*x**m/(m**7 + 49*m**6 + 973*m
**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 129072*a
**3*d**m*m*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17
7331*m**2 + 264207*m + 135135) + 135135*a**3*d**m*x*x**m/(m**7 + 49*m**6 +
973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a
**2*b*d**m*m**6*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m
**3 + 177331*m**2 + 264207*m + 135135) + 138*a**2*b*d**m*m**5*x**3*x**m/(m*
```

*7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 2505*a**2*b*d**m*m**4*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 22620*a**2*b*d**m*m**3*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 104277*a**2*b*d**m*m**2*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 219162*a**2*b*d**m*m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**2*b*d**m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a**2*c*d**m*m**6*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 132*a**2*c*d**m*m**5*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 2259*a**2*c*d**m*m**4*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 18840*a**2*c*d**m*m**3*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 77937*a**2*c*d**m*m**2*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 142308*a**2*c*d**m*m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 81081*a**2*c*d**m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a*b**2*d**m*m**6*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 132*a*b**2*d**m*m**5*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 2259*a*b**2*d**m*m**4*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 18840*a*b**2*d**m*m**3*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 77937*a*b**2*d**m*m**2*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 142308*a*b**2*d**m*m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 81081*a*b**2*d**m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 6*a*b*c*d**m*m**6*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 252*a*b*c*d**m*m**5*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 4074*a*b*c*d**m*m**4*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 31752*a*b*c*d**m*m**3*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 122010*a*b*c*d**m*m**2*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 209916*a*b*c*d**m*m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 115830*a*b*c*d**m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a*c**2*d**m*m**6*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 120*a*c**2*d**m*m**5*x**

9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
 264207*m + 135135) + 1839*a*c**2*d**m*m**4*x**9*x**m/(m**7 + 49*m**6 + 973
*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 13584*
a*c**2*d**m*m**3*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*
m**3 + 177331*m**2 + 264207*m + 135135) + 49881*a*c**2*d**m*m**2*x**9*x**m/
(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207
*m + 135135) + 83064*a*c**2*d**m*m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 1
0045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 45045*a*c**2*d*
*m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*
m**2 + 264207*m + 135135) + b**3*d**m*m**6*x**7*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 42*b**3
*d**m*m**5*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
 177331*m**2 + 264207*m + 135135) + 679*b**3*d**m*m**4*x**7*x**m/(m**7 + 49
*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 5292*b**3*d**m*m**3*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 20335*b**3*d**m*m**2*x**7
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 34986*b**3*d**m*m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5
 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 19305*b**3*
d**m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 3*b**2*c*d**m*m**6*x**9*x**m/(m**7 + 49*m**6
+ 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 1
20*b**2*c*d**m*m**5*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 573
79*m**3 + 177331*m**2 + 264207*m + 135135) + 1839*b**2*c*d**m*m**4*x**9*x**
m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 2642
07*m + 135135) + 13584*b**2*c*d**m*m**3*x**9*x**m/(m**7 + 49*m**6 + 973*m**
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 49881*b**2
*c*d**m*m**2*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3
 + 177331*m**2 + 264207*m + 135135) + 83064*b**2*c*d**m*m*x**9*x**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 45045*b**2*c*d**m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*b*c**2*d**m*m**6*x**11
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 114*b*c**2*d**m*m**5*x**11*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 1665*b*
c**2*d**m*m**4*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m
3 + 177331*m2 + 264207*m + 135135) + 11820*b*c**2*d**m*m**3*x**11*x**m/
(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207
*m + 135135) + 42117*b*c**2*d**m*m**2*x**11*x**m/(m**7 + 49*m**6 + 973*m**5
 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 68706*b*c**
2*d**m*m*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 36855*b*c**2*d**m*x**11*x**m/(m**7 + 49*
m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + c**3*d**m*m**6*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 573
79*m**3 + 177331*m**2 + 264207*m + 135135) + 36*c**3*d**m*m**5*x**13*x**m/(

```
m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*
m + 135135) + 505*c**3*d**m*m**4*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10
045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3480*c**3*d**m*m
**3*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 12139*c**3*d**m*m**2*x**13*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 19524*c**3*d**m*m*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57
379*m**3 + 177331*m**2 + 264207*m + 135135) + 10395*c**3*d**m*x**13*x**m/(m
**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
 + 135135), True))
```

Giac [B] time = 1.18059, size = 1528, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^3*m^6*x^13 + 36*(d*x)^m*c^3*m^5*x^13 + 3*(d*x)^m*b*c^2*m^6*x^11
+ 505*(d*x)^m*c^3*m^4*x^13 + 114*(d*x)^m*b*c^2*m^5*x^11 + 3480*(d*x)^m*c^3*
m^3*x^13 + 3*(d*x)^m*b^2*c*m^6*x^9 + 3*(d*x)^m*a*c^2*m^6*x^9 + 1665*(d*x)^m
*b*c^2*m^4*x^11 + 12139*(d*x)^m*c^3*m^2*x^13 + 120*(d*x)^m*b^2*c*m^5*x^9 +
120*(d*x)^m*a*c^2*m^5*x^9 + 11820*(d*x)^m*b*c^2*m^3*x^11 + 19524*(d*x)^m*c^
3*m*x^13 + (d*x)^m*b^3*m^6*x^7 + 6*(d*x)^m*a*b*c*m^6*x^7 + 1839*(d*x)^m*b^2
*c*m^4*x^9 + 1839*(d*x)^m*a*c^2*m^4*x^9 + 42117*(d*x)^m*b*c^2*m^2*x^11 + 10
395*(d*x)^m*c^3*x^13 + 42*(d*x)^m*b^3*m^5*x^7 + 252*(d*x)^m*a*b*c*m^5*x^7 +
 13584*(d*x)^m*b^2*c*m^3*x^9 + 13584*(d*x)^m*a*c^2*m^3*x^9 + 68706*(d*x)^m*
b*c^2*m*x^11 + 3*(d*x)^m*a*b^2*m^6*x^5 + 3*(d*x)^m*a^2*c*m^6*x^5 + 679*(d*x
)^m*b^3*m^4*x^7 + 4074*(d*x)^m*a*b*c*m^4*x^7 + 49881*(d*x)^m*b^2*c*m^2*x^9
+ 49881*(d*x)^m*a*c^2*m^2*x^9 + 36855*(d*x)^m*b*c^2*x^11 + 132*(d*x)^m*a*b^
2*m^5*x^5 + 132*(d*x)^m*a^2*c*m^5*x^5 + 5292*(d*x)^m*b^3*m^3*x^7 + 31752*(d
*x)^m*a*b*c*m^3*x^7 + 83064*(d*x)^m*b^2*c*m*x^9 + 83064*(d*x)^m*a*c^2*m*x^9
 + 3*(d*x)^m*a^2*b*m^6*x^3 + 2259*(d*x)^m*a*b^2*m^4*x^5 + 2259*(d*x)^m*a^2*
c*m^4*x^5 + 20335*(d*x)^m*b^3*m^2*x^7 + 122010*(d*x)^m*a*b*c*m^2*x^7 + 4504
5*(d*x)^m*b^2*c*x^9 + 45045*(d*x)^m*a*c^2*x^9 + 138*(d*x)^m*a^2*b*m^5*x^3 +
 18840*(d*x)^m*a*b^2*m^3*x^5 + 18840*(d*x)^m*a^2*c*m^3*x^5 + 34986*(d*x)^m*
b^3*m*x^7 + 209916*(d*x)^m*a*b*c*m*x^7 + (d*x)^m*a^3*m^6*x + 2505*(d*x)^m*a
^2*b*m^4*x^3 + 77937*(d*x)^m*a*b^2*m^2*x^5 + 77937*(d*x)^m*a^2*c*m^2*x^5 +
19305*(d*x)^m*b^3*x^7 + 115830*(d*x)^m*a*b*c*x^7 + 48*(d*x)^m*a^3*m^5*x + 2
2620*(d*x)^m*a^2*b*m^3*x^3 + 142308*(d*x)^m*a*b^2*m*x^5 + 142308*(d*x)^m*a^
2*c*m*x^5 + 925*(d*x)^m*a^3*m^4*x + 104277*(d*x)^m*a^2*b*m^2*x^3 + 81081*(d
```

*x)^m*a*b^2*x^5 + 81081*(d*x)^m*a^2*c*x^5 + 9120*(d*x)^m*a^3*m^3*x + 219162
*(d*x)^m*a^2*b*m*x^3 + 48259*(d*x)^m*a^3*m^2*x + 135135*(d*x)^m*a^2*b*x^3 +
 129072*(d*x)^m*a^3*m*x + 135135*(d*x)^m*a^3*x)/(m^7 + 49*m^6 + 973*m^5 + 1
0045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)

3.1106 $\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}$$

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + ((b
^2 + 2*a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (2*b*c*(d*x)^(7 + m))/(d^7*(7 +
m)) + (c^2*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rubi [A] time = 0.0528217, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1108}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + ((b
^2 + 2*a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (2*b*c*(d*x)^(7 + m))/(d^7*(7 +
m)) + (c^2*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{2+m}}{d^2}+\frac{\left(b^2+2ac\right)(dx)^{4+m}}{d^4}+\frac{2bc(dx)^{6+m}}{d^6}+\frac{c^2(dx)^{8+m}}{d^8}\right) dx\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{\left(b^2+2ac\right)(dx)^{5+m}}{d^5(5+m)}+\frac{2bc(dx)^{7+m}}{d^7(7+m)}+\frac{c^2(dx)^{9+m}}{d^9(9+m)}\end{aligned}$$

Mathematica [A] time = 0.0526927, size = 70, normalized size = 0.69

$$x(dx)^m\left(\frac{a^2}{m+1}+\frac{x^4(2ac+b^2)}{m+5}+\frac{2abx^2}{m+3}+\frac{2bcx^6}{m+7}+\frac{c^2x^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + ((b^2 + 2*a*c)*x^4)/(5 + m)
+ (2*b*c*x^6)/(7 + m) + (c^2*x^8)/(9 + m))
```

Maple [B] time = 0.048, size = 301, normalized size = 3.

$$\frac{(c^2m^4x^8+16\,c^2m^3x^8+2\,bcm^4x^6+86\,c^2m^2x^8+36\,bcm^3x^6+176\,c^2mx^8+2\,acm^4x^4+b^2m^4x^4+208\,bcm^2x^6+105\,c^2x^8+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(c^2*m^4*x^8+16*c^2*m^3*x^8+2*b*c*m^4*x^6+86*c^2*m^2*x^8+36*b*c*m^3*x^6+1
76*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+208*b*c*m^2*x^6+105*c^2*x^8+40*a*c*m
^3*x^4+20*b^2*m^3*x^4+444*b*c*m*x^6+2*a*b*m^4*x^2+260*a*c*m^2*x^4+130*b^2*m
^2*x^4+270*b*c*x^6+44*a*b*m^3*x^2+600*a*c*m*x^4+300*b^2*m*x^4+a^2*m^4+328*a
*b*m^2*x^2+378*a*c*x^4+189*b^2*x^4+24*a^2*m^3+916*a*b*m*x^2+206*a^2*m^2+630
*a*b*x^2+744*a^2*m+945*a^2)*(d*x)^m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.47672, size = 587, normalized size = 5.81

$$\frac{\left(\left(c^2m^4 + 16\,c^2m^3 + 86\,c^2m^2 + 176\,c^2m + 105\,c^2\right)x^9 + 2\left(bcm^4 + 18\,bcm^3 + 104\,bcm^2 + 222\,bcm + 135\,bc\right)x^7 + \left(\left(b^2 + 2\,\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^9 + 2*(b*c*m^4
 + 18*b*c*m^3 + 104*b*c*m^2 + 222*b*c*m + 135*b*c)*x^7 + ((b^2 + 2*a*c)*m^4
 + 20*(b^2 + 2*a*c)*m^3 + 130*(b^2 + 2*a*c)*m^2 + 189*b^2 + 378*a*c + 300*(
b^2 + 2*a*c)*m)*x^5 + 2*(a*b*m^4 + 22*a*b*m^3 + 164*a*b*m^2 + 458*a*b*m + 3
15*a*b)*x^3 + (a^2*m^4 + 24*a^2*m^3 + 206*a^2*m^2 + 744*a^2*m + 945*a^2)*x)
*(d*x)^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [A] time = 2.76449, size = 1486, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) -
b*c/x**2 + c**2*log(x))/d**9, Eq(m, -9)), ((-a**2/(6*x**6) - a*b/(2*x**4) -
 a*c/x**2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/d**7, Eq(m, -7)), (
(-a**2/(4*x**4) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c**2*x
**4/4)/d**5, Eq(m, -5)), ((-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*
x**2/2 + b*c*x**4/2 + c**2*x**6/6)/d**3, Eq(m, -3)), ((a**2*log(x) + a*b*x*
*2 + a*c*x**4/2 + b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8)/d, Eq(m, -1)), (a
**2*d**m*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 24*a**2*d**m*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 206*a**2*d**m*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 744*a**2*d**m*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 945*a**2*d**m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 2*a*b*d**m*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 44*a*b*d**m*m**3*x**3*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 328*a*b*d**m*m**2*x**3*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a*b*d**m*m*x**3*x**m/(m**5
```

```
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a*b*d**m*x**3*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*a*c*d**m*m**4*x**5
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 40*a*c*d**m*m
**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 260*a
*c*d**m*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945
) + 600*a*c*d**m*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 378*a*c*d**m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 16
89*m + 945) + b**2*d**m*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**
2 + 1689*m + 945) + 20*b**2*d**m*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 130*b**2*d**m*m**2*x**5*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 300*b**2*d**m*m*x**5*x**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 189*b**2*d**m*x**5*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*b*c*d**m*m**4*x**7*x**
m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 36*b*c*d**m*m**3*
x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 208*b*c*d
**m*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
444*b*c*d**m*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 270*b*c*d**m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + c**2*d**m*m**4*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945) + 16*c**2*d**m*m**3*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 95
0*m**2 + 1689*m + 945) + 86*c**2*d**m*m**2*x**9*x**m/(m**5 + 25*m**4 + 230*
m**3 + 950*m**2 + 1689*m + 945) + 176*c**2*d**m*m*x**9*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*c**2*d**m*x**9*x**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

Giac [B] time = 1.1323, size = 606, normalized size = 6.

$$\frac{(dx)^m c^2m^4x^9 + 16\,(dx)^m c^2m^3x^9 + 2\,(dx)^m bcm^4x^7 + 86\,(dx)^m c^2m^2x^9 + 36\,(dx)^m bcm^3x^7 + 176\,(dx)^m c^2mx^9 + (dx)^m b^2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^2*m^4*x^9 + 16*(d*x)^m*c^2*m^3*x^9 + 2*(d*x)^m*b*c*m^4*x^7 + 86*
(d*x)^m*c^2*m^2*x^9 + 36*(d*x)^m*b*c*m^3*x^7 + 176*(d*x)^m*c^2*m*x^9 + (d*x
)^m*b^2*m^4*x^5 + 2*(d*x)^m*a*c*m^4*x^5 + 208*(d*x)^m*b*c*m^2*x^7 + 105*(d*
x)^m*c^2*x^9 + 20*(d*x)^m*b^2*m^3*x^5 + 40*(d*x)^m*a*c*m^3*x^5 + 444*(d*x)^
m*b*c*m*x^7 + 2*(d*x)^m*a*b*m^4*x^3 + 130*(d*x)^m*b^2*m^2*x^5 + 260*(d*x)^m
*a*c*m^2*x^5 + 270*(d*x)^m*b*c*x^7 + 44*(d*x)^m*a*b*m^3*x^3 + 300*(d*x)^m*b
^2*m*x^5 + 600*(d*x)^m*a*c*m*x^5 + (d*x)^m*a^2*m^4*x + 328*(d*x)^m*a*b*m^2*
x^3 + 189*(d*x)^m*b^2*x^5 + 378*(d*x)^m*a*c*x^5 + 24*(d*x)^m*a^2*m^3*x + 91
6*(d*x)^m*a*b*m*x^3 + 206*(d*x)^m*a^2*m^2*x + 630*(d*x)^m*a*b*x^3 + 744*(d*
```

```
x)^m*a^2*m*x + 945*(d*x)^m*a^2*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*
m + 945)
```

3.1107 $\int (dx)^m \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}+\frac{c(dx)^{m+5}}{d^5(m+5)}$$

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(3 + m))/(d^3*(3 + m)) + (c*(d*x)^
(5 + m))/(d^5*(5 + m))
```

Rubi [A] time = 0.0195031, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}+\frac{c(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(3 + m))/(d^3*(3 + m)) + (c*(d*x)^
(5 + m))/(d^5*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^2+cx^4\right) dx &= \int \left(a(dx)^m+\frac{b(dx)^{2+m}}{d^2}+\frac{c(dx)^{4+m}}{d^4}\right) dx\\ &= \frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{3+m}}{d^3(3+m)}+\frac{c(dx)^{5+m}}{d^5(5+m)}\end{aligned}$$

Mathematica [A] time = 0.0296029, size = 35, normalized size = 0.67

$$x(dx)^m \left(\frac{a}{m+1}+\frac{bx^2}{m+3}+\frac{cx^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + (b*x^2)/(3 + m) + (c*x^4)/(5 + m))
```

Maple [A] time = 0.043, size = 78, normalized size = 1.5

$$\frac{\left(cm^2x^4+4\,cmx^4+bm^2x^2+3\,cx^4+6\,bmx^2+am^2+5\,bx^2+8\,am+15\,a\right)x\,(dx)^m}{(5+m)\,(3+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(c*m^2*x^4+4*c*m*x^4+b*m^2*x^2+3*c*x^4+6*b*m*x^2+a*m^2+5*b*x^2+8*a*m+15*a
)*(d*x)^m/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29335, size = 159, normalized size = 3.06

$$\frac{\left(\left(cm^2+4\,cm+3\,c\right)x^5+\left(bm^2+6\,bm+5\,b\right)x^3+\left(am^2+8\,am+15\,a\right)x\right)(dx)^m}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 4*c*m + 3*c)*x^5 + (b*m^2 + 6*b*m + 5*b)*x^3 + (a*m^2 + 8*a*m + 1
5*a)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [A] time = 0.874611, size = 314, normalized size = 6.04

$$\begin{cases} \frac{-\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x)}{d^5} \\ \frac{-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}}{d^3} \\ \frac{a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}}{d} \\ \frac{ad^m m^2 x x^m}{m^3+9m^2+23m+15}+\frac{8ad^m m x x^m}{m^3+9m^2+23m+15}+\frac{15ad^m x x^m}{m^3+9m^2+23m+15}+\frac{bd^m m^2 x^3 x^m}{m^3+9m^2+23m+15}+\frac{6bd^m m x^3 x^m}{m^3+9m^2+23m+15}+\frac{5bd^m x^3 x^m}{m^3+9m^2+23m+15}+\frac{cd^m m^2 x^5 x^m}{m^3+9m^2+23m+15} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise((((-a/(4*x**4) - b/(2*x**2) + c*log(x))/d**5, Eq(m, -5)), ((-a/(2*
x**2) + b*log(x) + c*x**2/2)/d**3, Eq(m, -3)), ((a*log(x) + b*x**2/2 + c*x*
*4/4)/d, Eq(m, -1)), (a*d**m*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8*a*
d**m*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a*d**m*x*x**m/(m**3 + 9*m**2
 + 23*m + 15) + b*d**m*m**2*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 6*b*d**
m*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*b*d**m*x**3*x**m/(m**3 + 9*m*
*2 + 23*m + 15) + c*d**m*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*c*d
**m*m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 3*c*d**m*x**5*x**m/(m**3 + 9*
m**2 + 23*m + 15), True))
```

Giac [B] time = 1.10666, size = 161, normalized size = 3.1

$$\frac{(dx)^m cm^2x^5 + 4\,(dx)^m cmx^5 + (dx)^m bm^2x^3 + 3\,(dx)^m cx^5 + 6\,(dx)^m bmx^3 + (dx)^m am^2x + 5\,(dx)^m bx^3 + 8\,(dx)^m amx +}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*c*m^2*x^5 + 4*(d*x)^m*c*m*x^5 + (d*x)^m*b*m^2*x^3 + 3*(d*x)^m*c*x^
5 + 6*(d*x)^m*b*m*x^3 + (d*x)^m*a*m^2*x + 5*(d*x)^m*b*x^3 + 8*(d*x)^m*a*m*x
 + 15*(d*x)^m*a*x)/(m^3 + 9*m^2 + 23*m + 15)
```

3.1108 $\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=173

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m
)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(
1 + m))
```

Rubi [A] time = 0.250873, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1131, 364}

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m
)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(
1 + m))
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^2), x]
, x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx = \frac{c\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

Mathematica [C] time = 0.0418205, size = 59, normalized size = 0.34

$$\frac{x(dx)^m \text{RootSum}\left[\#1^2 b+\#1^4 c+a\&,\frac{{}_2F_1\left(1,m+1;m+2;\frac{x}{\#1}\right)}{\#1^2 b+2a}\&\right]}{2(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*(d*x)^m*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2F1[1, 1 + m, 2 +
m, x/#1]/(2*a + b*#1^2) & ])/(2*(1 + m))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a),x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.1109 $\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=315

$$\frac{c(dx)^{m+1}\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(1-m)\sqrt{b^2-4ac}\right.}{2ad(m}$$

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c
*x^4)) + (c*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
 - (c*(b^2*(1 - m) - b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi [A] time = 0.695568, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1121, 1285, 364}

$$\frac{c(dx)^{m+1}\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(1-m)\sqrt{b^2-4ac}\right.}{2ad(m}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c
*x^4)) + (c*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
 - (c*(b^2*(1 - m) - b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
```

```
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1285

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{(dx)^{1+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} - \frac{\int \frac{(dx)^m\left(-b^2(1-m)+2ac(3-m)-bc(1-m)x^2\right)}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2(1-m)-b\sqrt{b^2-4ac}(1-m)-4ac(3-m)\right)\right)\int \frac{(dx)^{m}}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}\cdots}}{4a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{(dx)^{1+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} + \frac{c\left(b^2(1-m)+b\sqrt{b^2-4ac}(1-m)-4ac(3-m)\right)(dx)^{1+m}\,{}_2F_1\left(\cdots\right)}{2a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+\cdots)}$$

Mathematica [C] time = 0.0799011, size = 78, normalized size = 0.25

$$\frac{x(dx)^m F_1\left(\frac{m+1}{2};2,2;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{a^2(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(a^2*(1 + m))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^4 + bx^2 + a\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^4 + bx^2 + a\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{c^2x^8 + 2\,bcx^6 + \left(b^2 + 2\,ac\right)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^4 + bx^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

3.1110 $\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=158

$$\frac{a(dx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.145507, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1141, 510}

$$\frac{a(dx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```


Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int (dx)^m \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{a(dx)^{1+m}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1+m}{2};-\frac{3}{2},-\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] time = 0.277901, size = 357, normalized size = 2.26

$$\frac{x(dx)^m\sqrt{a+bx^2+cx^4}\left(a\left(m^2+8m+15\right)F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+(m+1)x^2\left(c(m+3)x^2F_1\left(\frac{m+5}{2};-\right.\right.\right.}{(m+1)(m+3)(m+5)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^2 + c*x^4]*(a*(15 + 8*m + m^2)*AppellF1[(1 + m)/2,
-1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])] + (1 + m)*x^2*(b*(5 + m)*AppellF1[(3 + m)/2, -1/2, -1/2
, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])] + c*(3 + m)*x^2*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 +
m)*(3 + m)*(5 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^{\frac{3}{2}} (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

3.1111 $\int (dx)^m \sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3 +
 m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.136412, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1141, 510}

$$\frac{(dx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3 +
 m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int (dx)^m\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{(dx)^{1+m}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A] time = 0.0997378, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3 + m)/
2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int (dx)^m\sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a} (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \sqrt{a + bx^2 + cx^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\,(dx)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

3.1112 $\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^2+cx^4}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 +
m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.135529, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1141, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 +
m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.109496, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

3.1113 $\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^2+cx^4}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1
+ m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.142262, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1141, 510}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1
+ m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A] time = 0.171554, size = 221, normalized size = 1.38

$$\frac{x(dx)^m\left(\sqrt{b^2-4ac}-b-2cx^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{3/2}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{(m+1)\left(\sqrt{b^2-4ac}-b\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqr
t[b^2 - 4*a*c])*(1 + m)*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int (dx)^m\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\,(dx)^m}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*
c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.1114 $\int (dx)^m \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=155

$$\frac{(dx)^{m+1}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{m+1}{2};-p,-p;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p, (3 + m)/2,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.101973, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1141, 510}

$$\frac{(dx)^{m+1}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{m+1}{2};-p,-p;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p, (3 + m)/2,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int (dx)^m \left(1+\frac{2cx^2}{b-\sqrt{b^2-}}\right.$$
$$= \frac{(dx)^{1+m}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1+m}{2};-p,-p;\frac{3+m}{2};-\frac{2cx}{b-\sqrt{b^2}}\right.}{d(1+m)}$$

Mathematica [A] time = 0.243679, size = 179, normalized size = 1.15

$$\frac{x(dx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{m+1}{2};-p,-p;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p, (3 + m)/2, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((1 +
 m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^4+bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

3.1115 $\int x^7 \left(a + bx^2 + cx^4\right)^p \, dx$

Optimal. Leaf size=257

$$-\frac{b2^{p-2}\left(6ac-b^2(p+3)\right)\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^3(p+1)(2p+3)\sqrt{b^2-4ac}}+\frac{\left(-2ac(2p+\right.}{}$$

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(1 + p))/(4*c*(2 + p)) + ((b^2*(2 + p)*(3 + p) - 2
*a*c*(3 + 2*p) - 2*b*c*(1 + p)*(3 + p)*x^2)*(a + b*x^2 + c*x^4)^(1 + p))/(8
*c^3*(1 + p)*(2 + p)*(3 + 2*p)) - (2^(-2 + p)*b*(6*a*c - b^2*(3 + p))*(-((b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c
*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c^3*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rubi [A] time = 0.373316, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 742, 779, 624}

$$-\frac{b2^{p-2}\left(6ac-b^2(p+3)\right)\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^3(p+1)(2p+3)\sqrt{b^2-4ac}}+\frac{\left(-2ac(2p+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2 + c*x^4)^p,x]
```


```
[Out] (x^4*(a + b*x^2 + c*x^4)^(1 + p))/(4*c*(2 + p)) + ((b^2*(2 + p)*(3 + p) - 2
*a*c*(3 + 2*p) - 2*b*c*(1 + p)*(3 + p)*x^2)*(a + b*x^2 + c*x^4)^(1 + p))/(8
*c^3*(1 + p)*(2 + p)*(3 + 2*p)) - (2^(-2 + p)*b*(6*a*c - b^2*(3 + p))*(-((b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c
*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c^3*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int x^7\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\,\text{Subst}\left(\int x^3\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$

$$=\frac{x^4\left(a+bx^2+cx^4\right)^{1+p}}{4c(2+p)}+\frac{\text{Subst}\left(\int x(-2a-b(3+p)x)\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)}{4c(2+p)}$$

$$=\frac{x^4\left(a+bx^2+cx^4\right)^{1+p}}{4c(2+p)}+\frac{\left(b^2(2+p)(3+p)-2ac(3+2p)-2bc(1+p)(3+p)x^2\right)\left(a+bx^2+cx\right.}{8c^3(1+p)(2+p)(3+2p)}$$

$$=\frac{x^4\left(a+bx^2+cx^4\right)^{1+p}}{4c(2+p)}+\frac{\left(b^2(2+p)(3+p)-2ac(3+2p)-2bc(1+p)(3+p)x^2\right)\left(a+bx^2+cx\right.}{8c^3(1+p)(2+p)(3+2p)}$$

Mathematica [C] time = 0.242301, size = 162, normalized size = 0.63

$$\frac{1}{8}x^8\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(4;-p,-p;5;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^8*(a + b*x^2 + c*x^4)^p*AppellF1[4, -p, -p, 5, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x^7\left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^7*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^p x^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p x^7, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^7, x)
```

3.1116 $\int x^5 \left(a + bx^2 + cx^4\right)^p \, dx$

Optimal. Leaf size=223

$$\frac{2^{p-1}\left(2ac - b^2(p+2)\right)\left(a + bx^2 + cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p, p+1; p+2; \frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}} - \frac{b(p+2)\left(a + b\right.}{4c^2(p+1)(}$$

```
[Out] -(b*(2 + p)*(a + b*x^2 + c*x^4)^(1 + p))/(4*c^2*(1 + p)*(3 + 2*p)) + (x^2*(
a + b*x^2 + c*x^4)^(1 + p))/(2*c*(3 + 2*p)) + (2^(-1 + p)*(2*a*c - b^2*(2 +
 p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a +
 b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(
3 + 2*p))
```

Rubi [A] time = 0.211161, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1114, 742, 640, 624}

$$\frac{2^{p-1}\left(2ac - b^2(p+2)\right)\left(a + bx^2 + cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p, p+1; p+2; \frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}} - \frac{b(p+2)\left(a + b\right.}{4c^2(p+1)(}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2 + c*x^4)^p,x]
```


```
[Out] -(b*(2 + p)*(a + b*x^2 + c*x^4)^(1 + p))/(4*c^2*(1 + p)*(3 + 2*p)) + (x^2*(
a + b*x^2 + c*x^4)^(1 + p))/(2*c*(3 + 2*p)) + (2^(-1 + p)*(2*a*c - b^2*(2 +
 p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a +
 b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(
3 + 2*p))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int x^5\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\operatorname{Subst}\left(\int x^2\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$

$$=\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}+\frac{\operatorname{Subst}\left(\int(-a-b(2+p)x)\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)}{2c(3+2p)}$$

$$=-\frac{b(2+p)\left(a+bx^2+cx^4\right)^{1+p}}{4c^2(1+p)(3+2p)}+\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}-\frac{\left(2ac-b^2(2+p)\right)\operatorname{Subst}\left(\int\left(a+bx\right.\right.}{4c^2(3+2p)}$$

$$=-\frac{b(2+p)\left(a+bx^2+cx^4\right)^{1+p}}{4c^2(1+p)(3+2p)}+\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}+\frac{2^{-1+p}\left(2ac-b^2(2+p)\right)\left(-\frac{b-\sqrt{b^2-4ac}+}{\sqrt{b^2-4a}}\right.}{}$$

Mathematica [C] time = 0.23252, size = 162, normalized size = 0.73

$$\frac{1}{6}x^6\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(3;-p,-p;4;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^6*(a + b*x^2 + c*x^4)^p*AppellF1[3, -p, -p, 4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(6*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int x^5 \left(c x^4 + b x^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^5*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^4 + b x^2 + a\right)^p x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c x^4 + b x^2 + a\right)^p x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^5, x)
```

3.1117 $\int x^3 \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=160

$$\frac{b2^{p-1}\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx^2+cx^4\right)^{p+1}}{4c(p+1)}$$

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(4*c*(1 + p)) + (2^(-1 + p)*b*(-((b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c*x^4)^(1 + p
)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*
Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rubi [A] time = 0.125484, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 640, 624}

$$\frac{b2^{p-1}\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx^2+cx^4\right)^{p+1}}{4c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(4*c*(1 + p)) + (2^(-1 + p)*b*(-((b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c*x^4)^(1 + p
)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*
Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int x^3 \left(a+bx^2+cx^4\right)^p dx = \frac{1}{2}\operatorname{Subst}\left(\int x\left(a+bx+cx^2\right)^p dx, x, x^2\right)$$
$$= \frac{\left(a+bx^2+cx^4\right)^{1+p}}{4c(1+p)} - \frac{b\operatorname{Subst}\left(\int \left(a+bx+cx^2\right)^p dx, x, x^2\right)}{4c}$$
$$= \frac{\left(a+bx^2+cx^4\right)^{1+p}}{4c(1+p)} + \frac{2^{-1+p}b\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^2+cx^4\right)^{1+p}{}_2F_1\left(-p,1+p;2+p;\frac{b}{}\right.}{c\sqrt{b^2-4ac}(1+p)}$$

Mathematica [C] time = 0.19605, size = 162, normalized size = 1.01

$$\frac{1}{4}x^4\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(2;-p,-p;3;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^p*AppellF1[2, -p, -p, 3, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int x^3\left(cx^4+bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^3*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^3, x)
```

3.1118 $\int x\left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=126

$$-\frac{2^p\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^2+cx^4\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

```
[Out] -((2^p*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a
 + b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rubi [A] time = 0.0757463, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 624}

$$-\frac{2^p\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^2+cx^4\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^p,x]
```


```
[Out] -((2^p*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a
 + b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)))
```


Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int x\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\,\text{Subst}\left(\int\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$
$$=-\frac{2^p\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^2+cx^4\right)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx^2}{2\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}(1+p)}$$

Mathematica [A] time = 0.099281, size = 135, normalized size = 1.07

$$\frac{2^{p-2}\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{-2cx^2-b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (2^(-2 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(a + b*x^2 + c*x^4)^p*Hyperge
ometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)/(2*Sqrt[b^2
 - 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c
])^p)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int x\left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^4+bx^2+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x, x)
```

3.1119 $\int \frac{(a+bx^2+cx^4)^p}{x}\,dx$

Optimal. Leaf size=152

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{p}$$

```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -(b - Sqr
t[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/(p*((b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c
*x^2))^p)
```

Rubi [A] time = 0.148887, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 758, 133}

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x,x]
```


```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -(b - Sqr
t[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/(p*((b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c
*x^2))^p)
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
```

```
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^p}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^p}{x}\,dx, x, x^2\right)$$

$$= -\left(\left(2^{-1+2p}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\text{Sub}$$

$$= \frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4a}}{2cx^2}\right.}{p}$$

Mathematica [A] time = 0.213018, size = 152, normalized size = 1.

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{\sqrt{b^2-4ac}-b}{2cx^2}\right)}{p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x,x]
```

```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -(b + Sqr
t[b^2 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/(p*((b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c
*x^2))^p)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{\left(c x^{4}+b x^{2}+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c x^{4}+b x^{2}+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c x^{4}+b x^{2}+a\right)^{p}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**p/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x, x)
```

3.1120 $\int \frac{(a+bx^2+cx^4)^p}{x^3} dx$

Optimal. Leaf size=166

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-2p)x^2}$$

```
[Out] -((2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p),
-(b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((
1 - 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```

Rubi [A] time = 0.130781, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 758, 133}

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-2p)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^3,x]
```


```
[Out] -((2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p),
-(b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((
1 - 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```


Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
```

```
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^p}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^p}{x^2}\,dx, x, x^2\right)$$

$$= -\left(\left(2^{-1+2p}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\text{Sub}$$

$$= -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-}{}\right.}{(1-2p)x^2}$$

Mathematica [A] time = 0.210106, size = 163, normalized size = 0.98

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(1-2p;-p,-p;2-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{\sqrt{b^2-4ac}-b}{2cx^2}\right)}{(2p-1)x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^3,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -(b
+ Sqrt[b^2 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((-1 +
 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(c*x^2))^p)
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\left(c x^4+b x^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^3,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c x^4+b x^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c x^4+b x^2+a\right)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^3, x)
```

3.1121 $\int \frac{(a+bx^2+cx^4)^p}{x^5} dx$

Optimal. Leaf size=164

$$-\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-p)x^4}$$

```
[Out] -((4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*p, -(
b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((1
- p)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(c*x^2))^p))
```

Rubi [A] time = 0.133008, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 758, 133}

$$-\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-p)x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^5,x]
```

```
[Out] -((4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*p, -(
b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((1
- p)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(c*x^2))^p))
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
```

```
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^p}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^p}{x^3}\,dx, x, x^2\right)$$

$$= -\left(\left(2^{-1+2p}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\text{Sub}\right.$$

$$= -\frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-}{}\right.}{(1-p)x^4}$$

Mathematica [A] time = 0.225437, size = 159, normalized size = 0.97

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(2-2p;-p,-p;3-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{\sqrt{b^2-4ac}-b}{2cx^2}\right)}{(p-1)x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^5,x]
```

```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -(b +
Sqrt[b^2 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((-1 + p
)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(c*x^2))^p)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{\left(c x^{4}+b x^{2}+a\right)^{p}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^5,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c x^{4}+b x^{2}+a\right)^{p}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c x^{4}+b x^{2}+a\right)^{p}}{x^{5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^5, x)
```

3.1122 $\int x^4 \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{5}x^5\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.103532, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1141, 510}

$$\frac{1}{5}x^5\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^4 \left(a+bx^2+cx^4\right)^p \,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int x^4\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right.$$

$$= \frac{1}{5}x^5\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{}{b-}\right.$$

Mathematica [A] time = 0.184573, size = 166, normalized size = 1.2

$$\frac{1}{5}x^5\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4a}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int x^4\left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^4*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^4, x)
```

3.1123 $\int x^2 \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{3}x^3\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*(1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.0913095, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1141, 510}

$$\frac{1}{3}x^3\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*(1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int x^2 \left(a+bx^2+cx^4\right)^p dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int x^2\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^p$$

$$= \frac{1}{3}x^3\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2}{b-\sqrt{}}\right.$$

Mathematica [A] time = 0.160621, size = 166, normalized size = 1.2

$$\frac{1}{3}x^3\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int x^2\left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^2*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^4+bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^2, x)
```

3.1124 $\int \left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=133

$$x\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.0600732, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1105, 429}

$$x\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
+ (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^p dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^p \left(1-\right.$$
$$= x\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-}}\right.$$

Mathematica [A] time = 0.178141, size = 161, normalized size = 1.21

$$x\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^2 + cx^4\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p, x)
```

3.1125 $\int \frac{(a+bx^2+cx^4)^p}{x^2} dx$

Optimal. Leaf size=136

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{x}$$

```
[Out] -(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rubi [A] time = 0.0856047, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1141, 510}

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^2,x]
```

```
[Out] -(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^p}{x^2}\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^p}{x^2}\,dx$$
$$= -\frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{x}$$

Mathematica [A] time = 0.16109, size = 164, normalized size = 1.21

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^2,x]
```

```
[Out] -(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^2,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^2, x)
```

3.1126 $\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3x^3}$$

```
[Out] -((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*x^3*(1 + (2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.0870568, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1141, 510}

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^4,x]
```

```
[Out] -((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*x^3*(1 + (2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^p}{x^4}\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx}{b+\sqrt{b^2}}\right.}{x^4}$$

$$= -\frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4a}}\right.}{3x^3}$$

Mathematica [A] time = 0.170957, size = 166, normalized size = 1.2

$$-\frac{\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^4,x]
```

```
[Out] -((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*x^3*((b - Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^4,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^4 + bx^2 + a)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
                         #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```